SHADED-POLE FLAT LINEAR INDUCTION MOTOR

A Thesis submitted to the Cape Technikon in fulfilment of the requirements for the Master's Diploma in Technology (Heavy Current).

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ABSTRACT

Linear induction motors are built for numerous applications. Their robustness, gearfree-link to the load and the fact that speed is not a function of the number of poles (as in round machines) are only a few advantages. As every single phase motor needs some starting aid, so does a linear induction drive. For round machines, capacitors or shaded poles are usually used. A capacitor can be damaged electrically, which is a rare occurrence for shading rings. A useful combination of these advantages is a linear induction motor which is gearless and uses the shaded pole principle as a starting aid.

In order to simplify the designing process, a computer program (Fortran) was written. The main objective on the electrical side was to obtain low input power, high power factor and high efficiency.

In order to find the performance curves, torque measurements were made. Although the principles of torque measurements are well known the device to measure torque for this machine was not available and had to be constructed.
Material obtainable in South Africa was used, so that the motor could be reproduced at any time. This work should provide a valuable foundation for further research in this direction.
TERMS OF REFERENCE

The objective is to design and build a shaded-pole flat linear induction motor. A computer program is to be written in order to simulate the motor and to simplify the design process, as well as finding the necessary performance curves.

The above mentioned points are to be carried out as a thesis, to be submitted in July 1991, for the completion of a Master's Diploma at the Cape Technikon.
ACKNOWLEDGEMENT

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Finally I want to thank my wonderful wife in encouraging and assisting me throughout.
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**NOMENCLATURE**

\(a\) = acceleration of mass, inertia test \((m/s^2)\)

\(A_{cu}\) = area of copper wire \((m^2)\)

\(b_p\) = width of pole \((m)\)

\(B_g\) = flux density in air gap \((T)\)

\(\cos \phi\) = power factor

\(d\) = thickness of aluminium disk \((m)\)

\(d_w\) = diameter of wire with insulation \((m)\)

\(E_1\) = assumed initial primary EMF \((V)\)

\(f\) = frequency \((Hz)\)

\(F_r\) = force to overcome friction and windage resistance \((N)\)

\(g\) = air gap \((m)\)

\(h\) = thickness of iron disk \((m)\)

\(h_{ov}\) = winding overhang \((m)\)

\(h_p\) = height of winding space \((m)\)

\(h_s\) = height of shading ring slot \((m)\)

\(h_y\) = height of yoke \((m)\)

\(H_l\) = magnetic field intensity in the limb \((A/m)\)

\(H_s\) = magnetic field intensity in the iron disk \((A/m)\)

\(H_y\) = magnetic field intensity in the yoke \((A/m)\)

\(I_{Fe}\) = iron loss current \((A)\)

\(I_c\) = no load current \((A)\)

\(I_i\) = primary current \((A)\)

\(I_{\mu}\) = magnetization current \((A)\)

\(j = \sqrt{-1}\) = imaginary unit
\( J \)  
\[ \text{current density (A/m}^2) \]

\( k_c \)  
\[ \text{Carter's factor} \]

\( k_d \)  
\[ \text{distribution factor} \]

\( k_n \)  
\[ \text{saturation factor} \]

\( k_p \)  
\[ \text{pitch factor} \]

\( k_{rn} \)  
\[ \text{Russell and Norsworthy factor} \]

\( k_{tr} \)  
\[ \text{transfer coefficient} \]

\( K_w \)  
\[ \text{winding factor} \]

\( l \)  
\[ \text{width of core (m)} \]

\( l_{av} \)  
\[ \text{average length of one turn (m)} \]

\( l_1 \)  
\[ \text{effective winding space (m)} \]

\( l_2 \)  
\[ \text{available winding space (m)} \]

\( L_i \)  
\[ \text{length of core (m)} \]

\( m_{Fe} \)  
\[ \text{mass of core (kg)} \]

\( m_i \)  
\[ \text{mass used in inertia test (kg)} \]

\( m_s \)  
\[ \text{mass needed to start rotating disk with uniform velocity (kg)} \]

\( m_t \)  
\[ \text{total mass of disk and shaft (kg)} \]

\( m_{l} \)  
\[ \text{number of phases} \]

\( N \)  
\[ \text{speed (rpm)} \]

\( N_p \)  
\[ \text{number of turns per pole} \]

\( N_t \)  
\[ \text{total number of turns} \]

\( p \)  
\[ \text{number of pole pairs} \]

\( P_{\text{elect}} \)  
\[ \text{secondary } I^2R \text{ losses (W)} \]

\( P_{Fe} \)  
\[ \text{iron loss in core (W)} \]

\( P_I \)  
\[ \text{input power (W)} \]

\( P_L \)  
\[ \text{total power loss (W)} \]

\( P_{\text{1cu}} \)  
\[ \text{copper loss in primary (W)} \]
\( P_{2cu} \) = total transferred power in air gap (W)
\( r \) = radius of shaft used for inertia test (m)
\( r_g \) = radius of gyration (m)
\( R_{Fe} \) = iron loss resistance (\( \Omega \))
\( R_1 \) = primary resistance (\( \Omega \))
\( s \) = slip
\( S_d \) = distance that the weight for inertia test has dropped (m)
\( t \) = time that the mass took to drop in the inertia test (s)
\( t_p \) = thickness of insulation paper between windings (m)
\( u \) = initial velocity (m/s)
\( V_{Fe} \) = core volume (m\(^3\))
\( V_i \) = input voltage (V)
\( w_s \) = width of shading pole slot (m)
\( w_i \) = initial angular velocity, inertia test (rad/s\(^2\))
\( w_2 \) = final angular velocity, inertia test (rad/s\(^2\))
\( X_m \) = mutual inductance (\( \Omega \))
\( X_1 \) = primary leakage reactance (\( \Omega \))
\( Z_{Al} \) = impedance of aluminium reaction rail referred to primary (\( \Omega \))
\( Z'_{Fe} \) = impedance of iron reaction rail referred to primary (\( \Omega \))
\( Z_t \) = total impedance of motor (\( \Omega \))
\( Z'_{2} \) = total impedance of reaction rail referred to
\( \alpha \) = angular acceleration (rad/s\(^2\))

\( \alpha_{Al} \) = propagation constant for aluminium (m\(^{-1}\))

\( \alpha_{Fe} \) = propagation constant for iron (m\(^{-1}\))

\( \gamma_{cu} \) = specific weight of aluminium (kp/m\(^3\))

\( \gamma_{Fe} \) = specific weight of sheet steel (kp/m\(^3\))

\( \Delta P_{Fe} \) = unit loss based on 0.4T in the air gap (W/kg)

\( \eta \) = efficiency

\( \mu_{Fe} \) = complex permeability in reaction rail (H/m)

\( \mu_o \) = permeability of free space (H/m)

\( \mu_{rs} \) = relative surface permeability iron disk

\( \mu' \) = real component of \( \mu_{rs} \)

\( \mu'' \) = imaginary component of \( \mu_{rs} \)

\( \rho_{Fe} \) = density of steel (kg/m\(^3\))

\( \sigma_{Al} \) = conductivity of aluminium (S/m)

\( \sigma_{cu} \) = conductivity of copper (S/m)

\( \sigma_{Fe} \) = conductivity of steel (S/m)

\( \sigma_l \) = leakage factor

\( \sigma'_{Al} \) = equivalent conductivity of aluminium disk (S/m)

\( \tau \) = pole pitch (m)

\( \tau_{Al} \) = temperature constant of aluminium at \( t=20^\circ C \)

\( \tau_{cu} \) = temperature constant of copper at \( t=20^\circ C \)

\( \tau_{FeSh} \) = temperature constant of sheet steel at \( t=20^\circ C \)

\( \tau_{FeSo} \) = temperature constant of solid steel at \( t=20^\circ C \)
\( \Phi_g \) = air gap flux (Wb)
\( \Phi_t \) = total flux in core (Wb)
\( \omega \) = angular velocity (rad/s)
1. **INTRODUCTION**

Just as electromagnetic forces produce a rotary motion in an electric machine, electromagnetic forces may be used to produce linear motion. In principle there is a linear electric machine for every rotary machine.

The main difference between linear electric machines and its rotary counterpart is best explained by an imaginary process whereby an induction motor is transformed into a linear induction motor (LIM) by cutting the stator and the rotor of the rotary motor in a radial plane and unrolling it. This produces an open air gap with an entry point and an exit point in the linear induction-motor.

I believe that a need exists for the development and manufacture of a single-phase linear machine, because three-phase power is not normally available for domestic use. The three-phase induction motor, the two phase induction motor and some single-phase induction motors are similarly constructed. The essential elements for the production of a rotating or moving field in these induction motors are the time displacement of the exciting current and the space displacement of the winding. The single-phase induction motor that falls into this category usually uses a capacitor for the phase displacement.
The shaded-pole induction motor is a very popular choice for use where only fractional horse-power is required. It is very simple in construction, low in cost, extremely rugged and very reliable because it does not need commutator brushes, a centrifugal switch, a starting capacitor, collector rings, governor or contacts of any sort. All these advantages and the fact that the shaded-pole principle has not been investigated in conjunction with linear induction motors, justifies this theses.

The stator core with slots and teeth of linear induction motors can be replaced by a simpler flat salient pole magnetic circuit. The secondary magnetic circuit can be a simple conductive disk which is much cheaper and simpler than the rotor of an inductive machine. An additional advantage, especially in turntables, is the elimination of a gear box. These advantages in a linear induction motor are reason enough to adapt its performance characteristics to drive various large and small machines.

The disadvantages of the linear induction motors are a small power factor and a small starting torque. This is due to a large air gap and an open air gap. For the shaded-pole linear induction motor the starting torque is even lower. This is due to the phase split being less than 90 degrees as a result of various practical imperfections.
2. CONSTRUCTION

2.1 Construction

Figure 1 shows the basic construction of the shaded-pole flat linear induction motor. It consists basically of two parts, namely a flat magnetic core (6) and a round conductive disk (1;2).

A laminated core was constructed out of standard H18, 0.5mm, Non Orientated Silicon Steel (N.O.S.S.) transformer laminations. The laminations, which are aligned in the direction of the field lines, are insulated from each other by a thin layer of varnish.
Laminating an iron core means an increase in permeability and decrease in conductivity, this consequently reduces the magnitude of the eddy current paths.

A shaded four pole flat linear induction motor core (Figure 1,6), was constructed by laminating two blocks of laminations under high pressure with strong epoxy glue (HY4076 and AV4076). A support frame was then built to keep the laminations together and to prevent them from fringing while slots where milled into the poles. The slots have to be open, as this reduces the leakage flux between the auxiliary and main windings and therefore allows more effective flux to flow through the air gap.

Shaded-pole copper rings (Figure 1,4) were then constructed to fit tightly into the slots. This was done to keep the main windings in place and to ensure that the shaded-pole copper rings do not become detached when vibrations occur. The shaded-pole copper rings were soldered in a very clean environment with silver solder. It is important to solder the seams with a material that has a high melting point to prevent disintegration during extreme operating conditions. The main windings were then wound (Figure 1,5), taking the space needed for the shaded-pole copper rings into consideration. The direction in which the coils are
wound, is important in creating north and south poles.

The assembly of the core was as follows. First the coils were slipped over the limbs, taking care that the correct south and north poles were created. The coil was then insulated with special insulating paper (DMD-Mitron) from the limb and from the yoke. After this the shaded-pole copper rings were forced into the slots.

The following procedure was followed to secure all the various parts and thereby preventing magnetic vibrations. First the core was placed into the oven and heated to 160 degrees, so as to obtain a homogeneous temperature throughout the core. Then the core was dipped in a special transformer resin (Isonol 31) until all air bubbles disappeared. The high temperature of the core ensures that the resin in the vicinity of the core becomes thinner and is able to fill up every air gap that exists between laminations of the core and the windings of the armature coils. Finally the core was baked to dry and harden the resin. This is necessary to keep all the parts in place during extreme running conditions.

The round conductive disk is much cheaper and easier to construct than its counterpart, the squirrel cage induction motor rotor. The secondary disk was constructed out of a 10mm mild steel disk to which a 3mm
aluminium cap was laminated.

Figure 2 shows the whole assembly of the motor. The magnetic core and the conductive disk are mounted on the supportive frame in such a way that the air gap and distance of the core from the centre of the disk to the edge of the disk can be changed for different tests.

Figure 2 Assembly of the shaded-pole flat linear induction motor
2.2 Moving Magnetic field

The induced current in the shading-ring-coil causes the flux in the shaded portion of the pole to lag the flux in the other portion. The result is a travelling magnetic field from the direction of the unshaded portion to the shaded portion of the pole. This travelling magnetic field induces currents to flow in the round aluminium disk. The secondary currents in turn create their own magnetic field, and the interaction between the two magnetic fields causes the thrust. The 10mm mild steel disk has a flux depth penetration of 7.11mm (Appendix 3) and ensures that the flux lines return by the shortest path into the core and therefore reduce the total reluctance of the field circuit which reduces the excitation for the flux.

To enable eddy currents to be fully established, the secondary disk should at least be half a pole pitch larger than the primary core in all direction. Another consideration when deciding on the size of the disk, is the distance of the primary core from the centre of the disk. The core should be situated as far away as physically possible, to ensure that the braking forces do not become too large.
3. DESIGN OF SHADED-POLE FLAT LIM

3.1 Formulae used in the design

To start the design of the shaded-pole flat linear induction motor the main dimensions of the core have to be chosen tentatively from the output graphs. After a standard transformer lamination has been chosen most dimensions like \( L_i, \tau, b_p, h_y, h_s, h_p \) and \( l \) are fixed.

(i) volume of the core

\[
V_{Fe} = L_i \left( l h_y + 4 \tau (\tau - b_p) - 3 h_s w_s \right) \quad (1)
\]

mass of core

\[
m_{Fe} = V_{Fe} \rho_{Fe} \quad (2)
\]

iron core loss (multiplied by 1.5 to take additional core losses like those for bolts and sharp edges into account)

\[
P_{Fe} = \Delta P_{Fe} \ m_{Fe} \times 1.5 \quad (3)
\]

(ii) total flux (the flux of the air gap was chosen tentatively from the final current density, i.e. when the air gap flux is too high the current density will become too high)

\[
\Phi_t = b_p L_i B_g \sigma I \quad (4)
\]
(iii) total number of turns ($E_1$ was determined via the iterative method - refer to Flowchart in section 3.3, page 31)

$$N_1 = \frac{|E_1|}{4.44 K_w f \Phi_t}$$  (5)

number of turns for one pole

$$N_p = \frac{N_1}{2 \rho}$$  (6)

transfer coefficient (reference 4)

$$k_{tr} = \frac{2 m_1 (N_1 K_w)^2}{p}$$  (7)

(iv) available winding space

$$l_2 = \frac{\tau - b_p}{2}$$  (8)

effective winding space

$$l_1 = \frac{N_p d_w (d_w + t_p)}{h_p}$$  (9)

average length of one turn

$$l_{av} = 2 (L_i + h_p + 2 l_1)$$  (10)
(v) primary resistance

\[ R_1 = \frac{I_{av} N_1}{\sigma_{cu} A_{cu}} \]  \hspace{1cm} (11)

primary resistance at a temperature of 75°

\[ R_1 = R_1 \frac{\tau_{cu} + 75}{\tau_{cu} + 20} \]  \hspace{1cm} (12)

mutual inductance (reference 1)

\[ X_m = 12.8 f p (N_p K_p)^2 \frac{\tau L_i}{k_n k_c} 10^{-6} \]  \hspace{1cm} (13)

primary leakage reactance (reference 1)

\[ X_1 = 0.8 (\sigma_1 - 1) X_m \]  \hspace{1cm} (14)

iron loss resistance

\[ R_{fe} = \frac{E_i^2}{p_{fe}} \]  \hspace{1cm} (15)

(reference 4)

\[ \beta = \frac{\pi}{\tau} \]  \hspace{1cm} (16)
winding overhang

\[ h_{ov} = \frac{\tau}{2} \]  \hspace{1cm} (17)

\[ w = \tau + L_d \]  \hspace{1cm} (18)

Russel - Norsworthy factor (this factor takes care of edge effects in high conductivity nonmagnetic layers, reference 4)

\[ k_{rn} = 1 - \frac{\tanh(\beta \frac{w}{2})}{\beta \frac{w}{2} \left(1 + \tanh(\beta \frac{w}{2}) \tanh(\beta h_{ov})\right)} \]  \hspace{1cm} (19)

conductivity of aluminium

\[ \sigma_{Al} = \sigma_{Al} \left(\frac{\tau_{Al} + 20}{\tau_{Al} + 75}\right) \]  \hspace{1cm} (20)

equivalent conductivity of aluminium disk (reference 2)

\[ \sigma_{Al} = k_{rn} \sigma_{Al} \]  \hspace{1cm} (21)

propagation constant for aluminium (reference 2)

\[ \chi_{Al} = \sqrt{(j \omega \mu_c \sigma_{Al}) + \left(\frac{\pi}{\tau}\right)^2} \]  \hspace{1cm} (22)
impedance of the aluminium reaction rail referred to the primary (reference 2)

\[ Z_{Al} = ktr \frac{j \omega \mu_0}{\chi_{Al}^{\prime}} \frac{1}{\tanh(\chi_{Al} d)} \frac{L_d}{\tau} \]  

(23)

complex permeability in reaction rail (takes into account saturation and hysteresis and is a function of the magnetic field strength, (reference 2 & 3)

\[ \mu_{Fe} = \mu_0 \mu_{Fe}^{\prime} (\mu^{\prime} - \mu^{\prime\prime}) \]  

(24)

propagation constant for iron (reference 2)

\[ \chi_{Fe} = \sqrt{(j \omega \mu_{Fe} \sigma_{Fe}) + \frac{\pi}{\tau}} \]  

(25)

the influence of the transverse edge effect on the secondary impedance can be taken into account by the coefficient \( k_z \) (reference 2)

\[ k_z = 1 - \left[ \frac{g}{L_1} \right] \left[ \frac{2 \tau}{\pi \omega} \left[ 1 - \exp \left[ \frac{-\pi w}{2 L_1} \right] \right] \right] \]  

(26)

impedance of laminated ferromagnetic reaction rail reverted to the primary (reference 2)

\[ Z_{Fe}^{\prime} = k_{tt} \frac{j \omega \mu_{Fe}}{\chi_{Fe}^{\prime}} \frac{1}{\tanh(\chi_{Fe} h)} \frac{L_z}{\tau} \frac{k_z}{\tau} \]  

(27)
total impedance of reaction rail referred to primary

\[ Z_i = \frac{Z_{Al} Z_{Fe}}{Z_{Al} + Z_{Fe}} \begin{bmatrix} 1 \\ S \end{bmatrix} \]  \tag{28}

\textbf{Figure 3} Equivalent circuit for a linear inductive motor with negligible small end effect (low speed).

(vi) total impedance of motor (derived from figure 3)

\[ Z_e = (R_e + jX_e) - \begin{bmatrix} R_{Fe} & jX_m Z_i \\ jX_m Z_i & R_{Fe} \end{bmatrix} \begin{bmatrix} jX_m Z_i' \\ jX_m Z_i \end{bmatrix} \]  \tag{29}
(vii) primary current

\[ I_1 = \frac{V_1}{Z_c} \]  

(30)

current density

\[ J = \frac{|I_1|}{A_{cu}} \]  

(31)

(viii) EMF induced in the primary

\[ E_1 = V_1 - |I_1| |R_1 + jX_1| \]  

(32)

(ix) magnetizing current

\[ I_{\mu} = \frac{|E_1|}{X_m} \]  

(33)

iron loss current

\[ I_{Fe} = \frac{|E_1|}{R_{Fe}} \]  

(34)

no load current

\[ I_o = I_{Fe} - jI_{\mu} \]  

(35)

secondary current

\[ I_1' = I_1 - I_o \]  

(36)
iron loss in the core

\[ P_{Fe} = I_{Fe}^2 R_{Fe} \] (37)

copper loss in the primary

\[ P_{1cu} = |I_1| R_1 \] (38)

total power transferred across the air gap from the secondary

\[ P_{2cu} = |I_2| R_2 S \] (39)

secondary \( I^2 R \) losses

\[ P_{\text{elect}} = |I_2^2| R_2 \] (40)

Thrust

\[ \text{Thrust} = \frac{P_{\text{elect}}}{2 f \tau} \] (41)

total power loss

\[ P_L = P_{Fe} + P_{1cu} + P_{2cu} \] (42)

power factor

\[ \cos \phi = \cos \left[ L (I_1) \frac{180}{\pi} \frac{\pi}{180} \right] \] (43)
3.2 CALCULATIONS

The following calculations are based on the formulas in section 3.1, where the calculation numbers correspond to the formula numbers. The single-phase shaded-pole LIM was constructed according to the dimensions used in the calculations. Refer to the input values in section 4.1.

(i) Volume according to formula (1).

\[ V_{Fe} = 0.09 \times (0.192 \times 0.016 + 4 \times 0.048 \times (0.048 - 0.032) - 3 \times 0.005 \times 0.005) \]  \hspace{1cm} (1)

\[ = 5.462 \times 10^{-4} \text{ (m}^3\text{)} \]

mass of core according to formula (2)

\[ m_{Fe} = 5.462 \times 10^{-4} \times 7.8 \times 10^3 \]  \hspace{1cm} (2)

\[ = 4.26 \text{ (kg)} \]
iron core loss

\[ P_{Fe} = 0.6 \times 4.26 \times 1.5 \]
\[ = 3.834 \text{ (W)} \]  

(ii) total flux

\[ \Phi_c = 0.032 \times 0.09 \times 0.4 \times 1.15 \]
\[ = 0.001 \text{ (Wb)} \]  

(iii) The induced voltage was found via the iterative method. Refer to the flowchart in section 3.3 to follow the iterative loop.

\[ E_i = 153.597 \text{ (V)} \]  

total number of turns

\[ N_e = \frac{153.597}{4.44 \times 1 \times 50 \times 0.001} \]
\[ = 522.251 \]  

number of turns for one pole

\[ N_r = \frac{522.251}{2 \times 2} \]
\[ = 130.563 \]
transfer coefficient

\[ k_{tr} = \frac{2 \times 2 \times (522.251 \times 1)^2}{2} \]
\[ = 5.455 \times 10^5 \]  \hspace{1cm} (7)

(iv) available winding space

\[ I_2 = \frac{0.048 - 0.032}{2} \]
\[ = 0.008 \text{ (m)} \]  \hspace{1cm} (8)

effective winding space

\[ I_1 = \frac{130.563 \times 0.001 \times (0.001 + 6 \times 10^{-4})}{0.043} \]
\[ = 0.007 \text{ (m)} \]  \hspace{1cm} (9)

average length of one turn

\[ I_{av} = 2 \times (0.09 + 0.032 + 2 \times 0.007) \]
\[ = 0.272 \text{ (m)} \]  \hspace{1cm} (10)

(v) primary resistance

\[ R_1 = \frac{0.272 \times 522.251}{5.6 \times 10^7 \times 1.227 \times 10^{-6}} \]
\[ = 2.068 \text{ (\Omega)} \]  \hspace{1cm} (11)
primary resistance at a temperature of 75°

\[ R_1 = 2.068 \times \frac{235+75}{235+20} \]
\[ = 2.514 \, (\Omega) \]  \hspace{1cm} (12)

primary inductance

\[ X_m = 12.8 \times 50 \times 2 \times (130.563 \times 1)^2 \times \frac{0.048 \times 0.009 \times 10^{-6}}{1.1 \times 1 \times 0.002} \]
\[ = 42.846 \, (\Omega) \]  \hspace{1cm} (13)

primary leakage reactance

\[ X_1 = 0.8 \times (1.15 - 1) \times 42.846 \]
\[ = 5.142 \, (\Omega) \]  \hspace{1cm} (14)

iron loss resistance

\[ R_{fe} = \frac{153.597^2}{3.834} \]
\[ = 6.153 \times 10^3 \, (\Omega) \]  \hspace{1cm} (15)

\[ \beta = \frac{3.142}{0.048} \]
\[ = 65.45 \, (rad/m) \]  \hspace{1cm} (16)
winding overhang

\[ h_{ov} = \frac{0.048}{2} \]
\[ = 0.024 \text{ (m)} \] (17)

\[ w = 0.048 + 0.09 \]
\[ = 0.138 \text{ (m)} \] (18)

Russel - Norsworthy factor

\[ k_{rn} = 1 - \frac{\tanh\left(65.45 \times \frac{0.138}{2}\right)}{65.45 \times \frac{0.138}{2}} \]
\[ \times \frac{1}{\left[1 + \tanh\left(65.45 \times \frac{0.138}{2}\right) \tanh(65.45 \times 0.024)\right]} \] (19)
\[ = 0.885 \]

conductivity of aluminium

\[ \sigma_{Al} = 3.6 \times 10^7 \times \left[\frac{245 + 20}{245 + 75}\right] \]
\[ = 2.981 \times 10^7 \text{ (S/m)} \] (20)

equivalent conductivity of the aluminium disk

\[ \sigma_{Al} = 0.885 \times 2.981 \times 10^7 \]
\[ = 2.637 \times 10^7 \text{ (S/m)} \] (21)
The propagation constant for aluminium is given by:

$$\chi_{Al} = \sqrt{(j \cdot 50 \cdot 314.159 \cdot 1.257 \cdot 10^{-6} \cdot 2.637 \cdot 10^7) + \left(\frac{\pi}{0.048}\right)^2}$$

$$= 88.15 + 59.049j \text{ (1/m)}$$

(22)

The impedance of the aluminium reaction rail is:

$$Z_{Al} = \frac{5.455 \cdot 10^5 \cdot (j \cdot 1 \cdot 314.159 \cdot 1.257 \cdot 10^{-6}) \cdot 1}{(88.15 + 59.049j) \cdot \tanh [(88.15 + 59.049j) \cdot 0.002]} \cdot 0.09 \cdot 0.048$$

$$= 16.586 + 7.094j \text{ (} \Omega \text{)}$$

(23)

The complex permeability in the reaction rail is:

$$\mu_{Fe} = 1.257 \cdot 10^{-6} \cdot 200 \cdot (1.23 - 0.69j)$$

$$= 3.091 \cdot 10^{-4} - 1.734 \cdot 10^{-4}j \text{ (} \text{H/m} \text{)}$$

(24)

The propagation constant for iron is:

$$\chi_{Fe} = \sqrt{(j \cdot 50 \cdot 314.159 \cdot (3.091 \cdot 10^{-4} - 1.734 \cdot 10^{-4}j)) + \left(\frac{\pi}{0.048}\right)^2}$$

$$= 646.361 + 375.63j \text{ (1/m)}$$

(25)

The transverse edge effect coefficient is:

$$k_z = 1 - \left[\frac{0.002}{0.09}\right] \cdot \frac{2 \cdot 0.048}{\pi \cdot 0.138} \cdot \left[1 - \exp\left(-\frac{\pi \cdot 0.138}{2 \cdot 0.09}\right)\right]$$

$$= 1.179$$

(26)
impedance of laminated ferromagnetic material

\[
Z'_{Fe} = \frac{5.455 \times 10^5 \times j \times 314.159 \times (3.091 \times 10^{-4} - 1.734 \times 10^{-4}) \times 1}{(6.46.361 + 375.63j) \times \tanh((646.361 + 375.63j) \times 0.005) \times 0.09 \times 1.179 \times 0.048}
\]

\[
= 154.173 + 91.355j \quad (\Omega)
\]

total impedance of reaction rail

\[
Z'_2 = \frac{(16.586 + 7.094j) \times (154.173 + 91.355j) \times 1}{(16.586 + 7.094j) + (154.173 + 91.355j)} \times 1
\]

\[
= 15.002 + 6.629j \quad (\Omega)
\]
(iv) total impedance of motor

\[
Z_t = (2.068 + 5.142j) + \left[ \frac{6.153 \times 10^3 \times [42.846j \times (15.002 + 6.629j)]}{6.153 \times 10^3 + 42.846j + (15.002 + 6.629j)} \right]
\]

\[
= 12.813 + 13.977j \quad (\Omega)
\]
(vii) primary current

\[
I_t = \frac{220}{(12.813 + 13.977j)}
\]

\[
= 7.84 - 8.553j
\]

\[
= 11.603 \angle -47.49 \quad (A)
\]
current density

\[ J = \frac{11.603}{1.227 \times 10^6} \]
\[ = 9.455 \times 10^{-6} \text{ (A/m}^2\text{)} \]  

(viii) EMF induced in primary

\[ E_1 = 220 - 11.603 \times \left( |2.514 + 5.142j| \right) \]
\[ = 220 - 11.603 \times 5.723 \]
\[ = 153.597 \text{ (V)} \]  

(ix) magnetizing current

\[ I_{\mu} = \frac{153.597}{42.846} \]
\[ = 3.585 \text{ (A)} \]  

iron loss current

\[ I_{fe} = \frac{153.597}{6.153 \times 10^2} \]
\[ = 0.025 \text{ (A)} \]  

no load current

\[ I_o = 0.025 - 3.585j \text{ (A)} \]  

secondary current

\[ I_s' = (7.84 - 8.553j) - (0.025 - 3.585j) \]
\[ = 7.816 - 4.968j \text{ (A)} \]
iron loss in the core

\[ P_{fe} = 0.025^2 \times 6.153 \times 10^3 \]
= 3.834 (W) \hspace{1cm} (37)

copper loss in the primary

\[ P_{1Cu} = 11.603^2 \times 2.514 \]
= 338.394 (W) \hspace{1cm} (38)

total power transferred across the air gap from the secondary

\[ P_{2cu} = 9.261^2 \times 15.002 \times 1 \]
= 1.287 \times 10^3 (W) \hspace{1cm} (39)

secondary I^2R losses

\[ P_{elect} = 9.261^2 \times 15.002 \]
= 1.287 \times 10^3 (W) \hspace{1cm} (40)

Thrust

\[ \text{Thrust} = \frac{1.287 \times 10^3}{2 \times 50 \times 0.048} \]
= 268.039 (N) \hspace{1cm} (41)

(total power loss)

\[ P_L = 3.834 + 338.394 + 1.287 \times 10^3 \]
= 1.629 \times 10^3 (W) \hspace{1cm} (42)
(xii) power factor

\[
\cos \phi = \cos \left( \frac{-0.829 \times 180}{\pi} \times \frac{\pi}{180} \right) = 0.676
\]

(xiii) input power

\[
P_I = 220 \times 11.603 \times 0.676 = 1.725 \times 10^3 \ (\text{W})
\]

(xvi) efficiency

\[
\eta = 1 - \frac{1.629 \times 10^3}{1.725 \times 10^3} = 0.056
\]
3.3 Flowchart

START

DEFINE VALUES

OPEN STORAGE FILES

A

WANT TO KEEP CORE DIMENSIONS

B

NO

PRINT CURRENT CORE DIMENSIONS

GET NEW CORE DIMENSIONS

B

PRINT CURRENT FLUX DENSITY IN AIR GAP

32
31

WANT TO CHANGE THE UNIT POWER LOSS AND THE FLUX DENSITY IN THE AIR GAP

YES → A

NO

CALCULATE VALUES FOR CORE; NUMBER OF TURNS; WINDING SPACE

YES → 32

NO

11 ≥ 12

YES → B

NO

PRINT CURRENT VALUES OF 11, 12, Acu, dw AND Li

PRINT CURRENT VALUES OF 11, 12, Acu, dw AND Li

NOT ENOUGH STAKING SPACE; GET NEW VALUES FOR Li OR Acu AND dw

A

WANT TO CHANGE AREA, DIAMETER OF COPPER OR LENGTH OF CORE

YES → A

NO

CALCULATE VALUE FOR TOTAL IMPEDANCE OF MOTOR

E1 = E1new

33
The computer program was written in Fortran and can be found in Appendix 1. The computed results can be found in Appendix 2.
4. 

EXAMPLE OF CALCULATIONS

4.1 Input data

\[ A_{cu} = 1.2272 \times 10^{-6} \text{ (m}^2) \]
\[ B_g = 0.4 \text{ (T)} \]
\[ b_p = 0.032 \text{ (m)} \]
\[ d = 0.002 \text{ (m)} \]
\[ d_w = 0.00125 \text{ (m)} \]
\[ E_1 = 153.597 \text{ (V)} \]
\[ f = 50 \text{ (Hz)} \]
\[ g = 0.002 \text{ (m)} \]
\[ h = 0.005 \text{ (m)} \]
\[ h_p = 0.043 \text{ (m)} \]
\[ h_s = 0.005 \text{ (m)} \]
\[ h_y = 0.016 \text{ (m)} \]
\[ H_1 = 112 \text{ (A/m)} \]
\[ H_s = 6000 \text{ (A/m)} \]
\[ k_c = 1 \]
\[ k_n = 1.1 \]
\[ K_w = 1 \]
\[ l = 0.192 \text{ (m)} \]
\[ L_1 = 0.09 \text{ (m)} \]
\[ m_1 = 2 \]
\[ p = 2 \]
\[ s = 1 \]
\[ t_p = 0.0006 \text{ (m)} \]
\[ V_1 = 220 \text{ (V)} \]
\[ w_s = 0.005 \text{ (m)} \]
\[ \gamma_{cu} = 8920 \text{ (kp/m}^3\text{)} \]
\[ \gamma_{Fe} = 7800 \text{ (kp/m}^3\text{)} \]
\[ \Delta P_{Fe} = 0.6 \text{ (W/kg)} \]
\[ \mu_0 = 4 \times \pi \times 10^{-7} \text{ (H/m)} \]
\[ \mu_{rs} = 200 \]
\[ \mu' = 1.23 \]
\[ \mu'' = 0.69 \]
\[ \rho_{Fe} = 7800 \text{ (kg/m}^3\text{)} \]
\[ \sigma_{Al} = 36 \times 10^6 \text{ (S/m)} \]
\[ \sigma_{cu} = 56 \times 10^6 \text{ (S/m)} \]
\[ \sigma_{Fe} = 5 \times 10^6 \text{ (S/m)} \]
\[ \sigma_1 = 1.15 \]
\[ \tau = 0.048 \text{ (m)} \]
\[ \tau_{Al} = 245 \text{ (°C)} \]
\[ \tau_{cu} = 235 \text{ (°C)} \]
\[ \tau_{Fesh} = 202 \text{ (°C)} \]
\[ \tau_{FeSO} = 230 \text{ (°C)} \]
\[ \omega = 2 \times \pi \times f \]
4.2 Results

Results for the slip between 1 and 0 are given in tabular form below. They were calculated with the simulating program found in Appendix 1.

Table 1  Results from simulating program in Appendix 1

<table>
<thead>
<tr>
<th>Slip</th>
<th>cos φ</th>
<th>n</th>
<th>$I_{\mu}$ (A)</th>
<th>$J$ (A/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.676</td>
<td>0.056</td>
<td>3.584</td>
<td>9.45e+06</td>
</tr>
<tr>
<td>0.95</td>
<td>0.665</td>
<td>0.091</td>
<td>3.619</td>
<td>9.24e+06</td>
</tr>
<tr>
<td>0.90</td>
<td>0.654</td>
<td>0.127</td>
<td>3.653</td>
<td>9.03e+06</td>
</tr>
<tr>
<td>0.85</td>
<td>0.641</td>
<td>0.162</td>
<td>3.687</td>
<td>8.82e+06</td>
</tr>
<tr>
<td>0.80</td>
<td>0.627</td>
<td>0.198</td>
<td>3.721</td>
<td>8.62e+06</td>
</tr>
<tr>
<td>0.75</td>
<td>0.612</td>
<td>0.233</td>
<td>3.754</td>
<td>8.42e+06</td>
</tr>
<tr>
<td>0.70</td>
<td>0.595</td>
<td>0.268</td>
<td>3.786</td>
<td>8.22e+06</td>
</tr>
<tr>
<td>0.65</td>
<td>0.576</td>
<td>0.303</td>
<td>3.817</td>
<td>8.03e+06</td>
</tr>
<tr>
<td>0.60</td>
<td>0.555</td>
<td>0.337</td>
<td>3.848</td>
<td>7.84e+06</td>
</tr>
<tr>
<td>0.55</td>
<td>0.532</td>
<td>0.371</td>
<td>3.877</td>
<td>7.66e+06</td>
</tr>
<tr>
<td>0.50</td>
<td>0.507</td>
<td>0.403</td>
<td>3.905</td>
<td>7.49e+06</td>
</tr>
<tr>
<td>0.45</td>
<td>0.480</td>
<td>0.435</td>
<td>3.932</td>
<td>7.33e+06</td>
</tr>
<tr>
<td>0.40</td>
<td>0.449</td>
<td>0.464</td>
<td>3.957</td>
<td>7.18e+06</td>
</tr>
<tr>
<td>0.35</td>
<td>0.416</td>
<td>0.491</td>
<td>3.980</td>
<td>7.04e+06</td>
</tr>
<tr>
<td>0.30</td>
<td>0.381</td>
<td>0.514</td>
<td>4.001</td>
<td>6.91e+06</td>
</tr>
<tr>
<td>0.25</td>
<td>0.342</td>
<td>0.531</td>
<td>4.020</td>
<td>6.79e+06</td>
</tr>
<tr>
<td>0.20</td>
<td>0.300</td>
<td>0.538</td>
<td>4.036</td>
<td>6.70e+06</td>
</tr>
<tr>
<td>0.15</td>
<td>0.255</td>
<td>0.528</td>
<td>4.050</td>
<td>6.61e+06</td>
</tr>
<tr>
<td>0.10</td>
<td>0.206</td>
<td>0.486</td>
<td>4.060</td>
<td>6.55e+06</td>
</tr>
</tbody>
</table>
Table 2  Results from simulating program in Appendix 1

<table>
<thead>
<tr>
<th>Slip</th>
<th>$P_1$ (W)</th>
<th>$P_{In}$ (W)</th>
<th>$P_{elect}$ (W)</th>
<th>Thrust (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1628.185</td>
<td>1724.30</td>
<td>1286.21</td>
<td>267.96</td>
</tr>
<tr>
<td>0.95</td>
<td>1508.547</td>
<td>1660.17</td>
<td>1243.65</td>
<td>259.09</td>
</tr>
<tr>
<td>0.90</td>
<td>1392.372</td>
<td>1594.79</td>
<td>1199.72</td>
<td>249.94</td>
</tr>
<tr>
<td>0.85</td>
<td>1279.859</td>
<td>1528.16</td>
<td>1154.39</td>
<td>240.50</td>
</tr>
<tr>
<td>0.80</td>
<td>1171.21</td>
<td>1460.27</td>
<td>1107.59</td>
<td>230.75</td>
</tr>
<tr>
<td>0.75</td>
<td>1066.628</td>
<td>1391.11</td>
<td>1059.27</td>
<td>220.68</td>
</tr>
<tr>
<td>0.70</td>
<td>966.3248</td>
<td>1320.69</td>
<td>1009.37</td>
<td>210.29</td>
</tr>
<tr>
<td>0.65</td>
<td>870.5121</td>
<td>1248.98</td>
<td>957.79</td>
<td>199.54</td>
</tr>
<tr>
<td>0.60</td>
<td>779.4088</td>
<td>1175.99</td>
<td>904.43</td>
<td>188.42</td>
</tr>
<tr>
<td>0.55</td>
<td>693.2388</td>
<td>1101.70</td>
<td>849.14</td>
<td>176.90</td>
</tr>
<tr>
<td>0.50</td>
<td>612.2343</td>
<td>1026.09</td>
<td>791.75</td>
<td>164.95</td>
</tr>
<tr>
<td>0.45</td>
<td>536.6393</td>
<td>949.13</td>
<td>732.03</td>
<td>152.51</td>
</tr>
<tr>
<td>0.40</td>
<td>466.7172</td>
<td>870.79</td>
<td>669.67</td>
<td>139.51</td>
</tr>
<tr>
<td>0.35</td>
<td>402.7597</td>
<td>791.00</td>
<td>604.28</td>
<td>125.89</td>
</tr>
<tr>
<td>0.30</td>
<td>345.1045</td>
<td>709.67</td>
<td>535.37</td>
<td>111.53</td>
</tr>
<tr>
<td>0.25</td>
<td>294.1567</td>
<td>626.65</td>
<td>462.31</td>
<td>96.31</td>
</tr>
<tr>
<td>0.20</td>
<td>250.4183</td>
<td>541.70</td>
<td>384.37</td>
<td>80.08</td>
</tr>
<tr>
<td>0.15</td>
<td>214.5179</td>
<td>454.41</td>
<td>300.70</td>
<td>62.65</td>
</tr>
<tr>
<td>0.10</td>
<td>187.2295</td>
<td>364.02</td>
<td>210.34</td>
<td>43.82</td>
</tr>
</tbody>
</table>
### 4.3 Nameplate

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>length of core</td>
<td>0.09 (m)</td>
</tr>
<tr>
<td>height of core</td>
<td>0.064 (m)</td>
</tr>
<tr>
<td>width of core</td>
<td>0.192 (m)</td>
</tr>
<tr>
<td>weight of core</td>
<td>4.26 (kg)</td>
</tr>
<tr>
<td>weight of aluminium and mild steel disk</td>
<td>9.5 (kg)</td>
</tr>
<tr>
<td>diameter of aluminium and mild steel disk</td>
<td>0.52 (m)</td>
</tr>
<tr>
<td>thickness of aluminium cap</td>
<td>3 (mm)</td>
</tr>
<tr>
<td>thickness of mild steel disk</td>
<td>10 (mm)</td>
</tr>
<tr>
<td>diameter of secondary rotor</td>
<td>0.52 (m)</td>
</tr>
<tr>
<td>cross sectional area of shaded pole ring</td>
<td>0.0024 (m²)</td>
</tr>
</tbody>
</table>

### Electrical Information

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of phases</td>
<td>1</td>
</tr>
<tr>
<td>frequency</td>
<td>50 (Hz)</td>
</tr>
<tr>
<td>voltage</td>
<td>220 (V)</td>
</tr>
<tr>
<td>current</td>
<td>11.603 (A)</td>
</tr>
<tr>
<td>resistance</td>
<td>12.813 (Ω)</td>
</tr>
<tr>
<td>speed</td>
<td>84 (rpm)</td>
</tr>
</tbody>
</table>

### Motor Information

<table>
<thead>
<tr>
<th>Component</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>stator motor</td>
<td>single-phase shaded-pole</td>
</tr>
<tr>
<td>stator core</td>
<td>laminated (H18, 0.5mm, Non Orientated Silicon Steel)</td>
</tr>
<tr>
<td>rotor disk</td>
<td>aluminium cap laminated to mild steel disk</td>
</tr>
<tr>
<td>slot insulation</td>
<td>DMD-Mitron (6510)</td>
</tr>
<tr>
<td>shaded pole</td>
<td>copper soldered with silver solder</td>
</tr>
</tbody>
</table>
4.4 Characteristic curves

The following graphs are based on the tabulated values in section 4.2.

Figure 4 Efficiency versus slip

Figure 5 Magnetizing current versus slip
Figure 6  Power factor versus slip

Figure 7  Current density versus slip
Figure 8  Thrust versus slip

Figure 9  Power versus slip
Figure 10 Sectional view of the single-phase shaded-pole linear induction motor.
Figure 11 Construction of stator core, aluminium cap and mild steel rotor disk

Figure 12 Aluminium and mild steel disk forming secondary rotor
Figure 13 Sectional view of the shaft. The shaft was constructed in this specific way, so as to withstand the attraction force exerted onto the aluminium and mild steel disk by the stator core.
To determine the characteristic curves of the single-phase single-pole LIM, a no-load, a short-circuit and a torque test was done. The inertia of the disk was found by applying a test normally used for flywheels.

5.1 No-load test

The power factor was calculated from readings of the input power and the input current as the supply of the motor was increased in steps.

5.1.1 Measured results

**Table 4**  
No-load test

<table>
<thead>
<tr>
<th>$V_1$ (V)</th>
<th>$I_1$ (A)</th>
<th>$P_1$ (W)</th>
<th>POWER FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.56</td>
<td>5.8</td>
<td>1.00</td>
</tr>
<tr>
<td>20</td>
<td>1.1</td>
<td>16</td>
<td>0.73</td>
</tr>
<tr>
<td>30</td>
<td>1.62</td>
<td>32</td>
<td>0.66</td>
</tr>
<tr>
<td>40</td>
<td>2.18</td>
<td>54</td>
<td>0.62</td>
</tr>
<tr>
<td>50</td>
<td>2.79</td>
<td>78</td>
<td>0.56</td>
</tr>
<tr>
<td>60</td>
<td>3.33</td>
<td>105</td>
<td>0.53</td>
</tr>
<tr>
<td>70</td>
<td>3.8</td>
<td>140</td>
<td>0.53</td>
</tr>
<tr>
<td>80</td>
<td>4.45</td>
<td>180</td>
<td>0.51</td>
</tr>
<tr>
<td>90</td>
<td>4.9</td>
<td>225</td>
<td>0.51</td>
</tr>
<tr>
<td>100</td>
<td>5.46</td>
<td>270</td>
<td>0.49</td>
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<tr>
<td>110</td>
<td>5.9</td>
<td>320</td>
<td>0.49</td>
</tr>
<tr>
<td>120</td>
<td>6.52</td>
<td>380</td>
<td>0.49</td>
</tr>
</tbody>
</table>
5.1.2 Characteristic curves

The following graphs show clearly that a supply of 50V is needed to overcome windage and friction losses.

Figure 14 Power factor versus input voltage
Figure 15 Input current versus input voltage

Figure 16 Input power versus input voltage
5.2 Short-circuit test

An adjustable scale was connected to the rotor disk (figure 17). Each reading was taken with the scale perpendicular to the center of the disk. As the supply voltage was raised in small increments, readings of the input power, input current and of the scale were taken.

Figure 17 Torque measurement for blocked rotor
5.2.1 **Input data**

\[ g = 9.81 \text{ (m/s}^2\text{)} \]
\[ I_1 = 11.9 \text{ (A)} \]
\[ m = 1.24 \text{ (kg)} \]
\[ P_I = 1200 \text{ (W)} \]
\[ r = 0.25 \text{ (m)} \]
\[ V_1 = 220 \text{ (V)} \]

5.2.2 **Calculations**

**Force**

\[ F = m g \]
\[ = 1.24 \times 9.81 \]
\[ = 12.16 \text{ (N)} \] \hspace{1cm} (1)

**Torque**

\[ T = F r \]
\[ = 12.16 \times 0.25 \]
\[ = 3.0411 \text{ (Nm)} \] \hspace{1cm} (2)

**Power Factor**

\[ \cos \phi = \frac{P_I}{\frac{V_1}{I_1}} \]
\[ = \frac{1200}{220 \times 11.9} \]
\[ = 0.485 \] \hspace{1cm} (3)
5.2.3 Results

Table 5  Short-circuit test

<table>
<thead>
<tr>
<th>$V_1$ (V)</th>
<th>$I_1$ (A)</th>
<th>$P_I$ (W)</th>
<th>$m$ (kg)</th>
<th>Power Factor</th>
<th>$T$ (Nm)</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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<td>0</td>
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<td></td>
</tr>
<tr>
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<td>3</td>
<td></td>
<td>0.526</td>
<td></td>
</tr>
<tr>
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<td>1.11</td>
<td>7.2</td>
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<td>0.324</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.64</td>
<td>24.5</td>
<td></td>
<td>0.498</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>2.18</td>
<td>42</td>
<td></td>
<td>0.482</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2.79</td>
<td>67</td>
<td>0.010</td>
<td>0.480</td>
<td>0.025</td>
</tr>
<tr>
<td>60</td>
<td>3.33</td>
<td>100</td>
<td>0.050</td>
<td>0.501</td>
<td>0.123</td>
</tr>
<tr>
<td>70</td>
<td>3.88</td>
<td>135</td>
<td>0.102</td>
<td>0.497</td>
<td>0.250</td>
</tr>
<tr>
<td>80</td>
<td>4.5</td>
<td>185</td>
<td>0.145</td>
<td>0.514</td>
<td>0.356</td>
</tr>
<tr>
<td>90</td>
<td>4.92</td>
<td>230</td>
<td>0.195</td>
<td>0.519</td>
<td>0.478</td>
</tr>
<tr>
<td>100</td>
<td>5.42</td>
<td>280</td>
<td>0.255</td>
<td>0.517</td>
<td>0.625</td>
</tr>
<tr>
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<td>345</td>
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<td>0.528</td>
<td>0.785</td>
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<tr>
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<td>0.385</td>
<td>0.515</td>
<td>0.944</td>
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<tr>
<td>130</td>
<td>7</td>
<td>460</td>
<td>0.470</td>
<td>0.505</td>
<td>1.153</td>
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<tr>
<td>140</td>
<td>7.52</td>
<td>520</td>
<td>0.545</td>
<td>0.494</td>
<td>1.337</td>
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<tr>
<td>150</td>
<td>8.01</td>
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<td>0.620</td>
<td>0.499</td>
<td>1.521</td>
</tr>
<tr>
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<td>8.51</td>
<td>660</td>
<td>0.700</td>
<td>0.485</td>
<td>1.717</td>
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<tr>
<td>170</td>
<td>9.09</td>
<td>750</td>
<td>0.790</td>
<td>0.485</td>
<td>1.937</td>
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<tr>
<td>180</td>
<td>9.62</td>
<td>840</td>
<td>0.880</td>
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<tr>
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<td>1.050</td>
<td>0.477</td>
<td>2.575</td>
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<td>1100</td>
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<tr>
<td>220</td>
<td>11.9</td>
<td>1200</td>
<td>1.240</td>
<td>0.458</td>
<td>3.041</td>
</tr>
</tbody>
</table>
5.2.4 Characteristic curves

Figure 18 Power factor versus input voltage

Figure 19 Input current versus input voltage
Figure 20 Input power versus input current

Figure 21 Torque versus input power
5.3 **Load test**

A rope was wound around a part of the shaft and connected perpendicularly to the axis of the shaft and to the scale. The other side of the rope was connected to a known weight. The weight was situated directly beneath the scale (Figure 22).

**Figure 22** Load test
5.3.1 Input Data

\[ I_1 = 8.009 \text{ (A)} \]
\[ P_I = 598.75 \text{ (W)} \]
\[ N = 25 \text{ (rpm)} \]
\[ V_I = 150 \text{ (V)} \]
\[ W_s = 3.8 \text{ (kg)} \]
\[ W_g = 1.5 \text{ (kg)} \]

5.3.2 Calculations

Force used for braking the disk

\[ F_b = (W_s - W_g) g \]
\[ = (3.8 - 1.5) \times 9.81 \]
\[ = 22.56 \text{ (N)} \]

Torque due to braking force

\[ T = F_b x_b \]
\[ = 22.56 \times 0.0425 \]
\[ = 0.959 \text{ (Nm)} \]

Output power

\[ P_L = \frac{2 \pi N T \times 60}{60} \]
\[ = \frac{2 \times \pi \times 25 \times 0.959}{60} \]
\[ = 2.51 \text{ (W)} \]
efficiency

\[ \eta = \frac{P_L}{P_I} \]

\[ = \frac{2.51}{598.75} \]

\[ = 0.0042 \]

power factor

\[ \cos \phi = \frac{P_L}{V_1 I_1} \]

\[ = \frac{598.75}{150 \ast 8.009} \]

\[ = 0.498 \]

5.3.3 Results

**Table 6** Load test

<table>
<thead>
<tr>
<th>T (Nm)</th>
<th>Speed (rpm)</th>
<th>P_L (W)</th>
<th>P_I (W)</th>
<th>I_1 (A)</th>
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</thead>
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<tr>
<td>0.000</td>
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<td>0.00</td>
<td>580</td>
<td>8.000</td>
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<tr>
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<td>84.9</td>
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<td>581.25</td>
<td>8.001</td>
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<td>80.5</td>
<td>1.55</td>
<td>582.5</td>
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<td>1.87</td>
<td>583.75</td>
<td>8.002</td>
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<td>74</td>
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<tr>
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<td>70</td>
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<td>586.25</td>
<td>8.003</td>
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<td>587.5</td>
<td>8.004</td>
</tr>
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<td>588.75</td>
<td>8.004</td>
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<td>590</td>
<td>8.005</td>
</tr>
<tr>
<td>0.650</td>
<td>51</td>
<td>3.47</td>
<td>591.25</td>
<td>8.006</td>
</tr>
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<td>T (Nm)</td>
<td>Speed (rpm)</td>
<td>Efficiency</td>
<td>Power factor</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>------------</td>
<td>--------------</td>
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</tr>
<tr>
<td>0.000</td>
<td>90</td>
<td>0.0000</td>
<td>0.483</td>
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<td>80.5</td>
<td>0.0027</td>
<td>0.485</td>
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<td>0.486</td>
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<td>0.489</td>
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<td>0.542</td>
<td>59</td>
<td>0.0057</td>
<td>0.490</td>
<td></td>
</tr>
<tr>
<td>0.592</td>
<td>54.5</td>
<td>0.0057</td>
<td>0.491</td>
<td></td>
</tr>
<tr>
<td>0.650</td>
<td>51</td>
<td>0.0059</td>
<td>0.492</td>
<td></td>
</tr>
<tr>
<td>0.688</td>
<td>48</td>
<td>0.0058</td>
<td>0.493</td>
<td></td>
</tr>
<tr>
<td>0.750</td>
<td>43</td>
<td>0.0057</td>
<td>0.494</td>
<td></td>
</tr>
<tr>
<td>0.834</td>
<td>39.5</td>
<td>0.0058</td>
<td>0.495</td>
<td></td>
</tr>
<tr>
<td>0.876</td>
<td>35</td>
<td>0.0054</td>
<td>0.496</td>
<td></td>
</tr>
<tr>
<td>0.934</td>
<td>30</td>
<td>0.0049</td>
<td>0.497</td>
<td></td>
</tr>
<tr>
<td>0.959</td>
<td>25</td>
<td>0.0042</td>
<td>0.498</td>
<td></td>
</tr>
</tbody>
</table>
5.3.4 Characteristic curves

Figure 23 Efficiency versus speed

Figure 24 Power factor versus speed
Figure 25 Input power versus speed

Figure 26 Output power versus speed
Figure 27 Input current versus speed

Figure 28 Torque versus speed
Figure 29 Efficiency versus torque

Figure 30 Power factor versus torque
Figure 31 Input power versus torque

Figure 32 Output power versus torque
**Figure 33** Input current versus torque
The windage and frictional resistance was found by winding a rope around the shaft and attaching weights of various masses to the loose end. The force of these resistances is given by the \((\text{mass} \times g)\) that only sets the shaft in motion and does not accelerate it.

The inertia was found by adding a greater known mass to the rope and timing the fall through a known vertical distance with a stopwatch. This experiment was done with various weights to compare the results.

Figure 34 Diagram for the Inertia test.
5.4.1 Input data

d_a = 0.042 (m)

m_t = 9.5 (kg)

m_s = 0.2 (kg)

m_I = 0.6 (kg)

r = 0.042 (m)

S_d = 0.542 (m)

\( t = 10.82 \) (s)

\( \omega_1 = 0 \) (rad/s)

5.4.2 Calculations

(i) acceleration of the 0.6kg mass

\[ S_d = u t + 0.5 a t^2 \]

therefore

\[ a = \frac{2 \times (0.542 - 0 \times 10.82)}{10.82^2} \]

\[ = 9.259 \times 10^{-3} \text{ (m/s)} \]

(ii) acceleration of the secondary disk

\[ \alpha = \frac{a}{r} \]

\[ \alpha = \frac{9.259 \times 10^{-3}}{0.042} \]

\[ = 0.22 \text{ (rad/s)} \]
(iii) friction and windage resistance force

\[ F_r = m_2 g \]  

\[ F_r = 0.25 \times 9.81 \]
\[ = 2.453 \ (N) \]  

(iv) Resultant force due to the motion and the 0.6kg mass

\[ F = m_3 (g - a) \]  

\[ F = 0.6 \times (9.81 - 9.259 \times 10^{-3}) \]
\[ = 5.88 (N) \]  

(v) force in the cord to accelerate the disk

\[ F_a = F - F_r \]  

\[ F_a = 5.88 - 2.453 \]
\[ = 3.428 \ (N) \]  

(vi) accelerating torque

\[ T_a = F_a r \]  

\[ 65 \]
\[ T_a = 3.428 \times 0.042 \]
\[ = 0.144 \, (Nm) \]

(vii) Moment of Inertia

\[ I = \frac{T}{\alpha} \]  
\[ I = \frac{3.428}{0.22} \]
\[ = 0.653 \, (kg/m^2) \]

(viii) Radius of gyration

\[ r_g = \sqrt{\frac{I}{m_1}} \]  
\[ r_g = \sqrt{\frac{0.653}{9.5}} \]
\[ = 0.262 \, (m) \]

5.4.3 Calculate run-out-time

(i) final angular velocity

\[ \omega_1 = \frac{2 \pi N}{60} \]
\[ \omega_1 = \frac{2 \pi \times 84}{60} \]
\[ = 8.692 \text{ (rad/s)} \]

(ii) Torque due to friction

\[ T_r = m_g r \quad \text{(10)} \]

\[ T_r = 0.25 \times 9.81 \times 0.042 \]
\[ = 0.103 \text{ (Nm)} \]

(iii) Angular velocity

\[ \alpha = \frac{T_r}{I} \quad \text{(11)} \]

\[ \alpha = \frac{0.103}{0.653} \]
\[ = 0.158 \text{ (rad/s}^2) \]

(iv) Time for the disk to come to a stand still

\[ t = \frac{\omega}{\alpha} \quad \text{(12)} \]

\[ t = \frac{8.692}{0.158} \]
\[ = 55.77 \text{ (s)} \]
5.4.4 Measured run-out time

The motor was run up to rated speed, after which the input power was cut and the speed measured every 5 seconds.

5.4.4.1 Measured results

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Speed (RPM)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>84</td>
<td>2.3750</td>
</tr>
<tr>
<td>5</td>
<td>77</td>
<td>2.1771</td>
</tr>
<tr>
<td>10</td>
<td>69</td>
<td>1.9509</td>
</tr>
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<td>15</td>
<td>62</td>
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<tr>
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5.4.4.2 Graph

Figure 35 Speed and velocity versus time
CONCLUSION

In this thesis the characteristics of the shaded-pole induction motor and the 3 phase single-sided linear induction motor were used to design the shaded-pole flat linear induction motor.

Since the design was limited to materials that were available locally, compromises could not be avoided. For example when a standard transformer lamination is chosen, most dimensions are fixed, including the pole pitch and the winding space.

Due to the fact that an iterative method was used to find the induced EMF and to alleviate the decision making process, the CAD approach was of great help.

The characteristic curves show clearly that the disadvantages of this motor are a low power factor, a small starting torque and a low efficiency. This is due to a large and open air gap and the fact that the shaded poles do not produce a 90 degree phase shift.

It was found that the distance from the core to the centre of the secondary disk plays an important role in the performance of the motor. The braking forces induced by the core onto the aluminium cap reduce as the distance between the two is increased.
Although efficiency, power factor and starting torque of this motor are worse than for normal induction motors, I think that this design holds a lot of potential where only a single phase supply is available and the advantages of a single-sided induction motor are needed.
LIST OF REFERENCES


2. Jacek F. Gieras, Design of Single-Sided Linear Induction Motors for propulsion of Wheel-On-Rail Vehicles, Dept. of Electrical & Electronic Engineering, University of Cape Town

3. Davy Linear Motors Limited, Equipment Specifications


5. J. D. Walker, Applied Mechanics, SI edition
Appendix 1

C Linear Motor

REAL 1, TAU, CFE, Li, ws, hy, hs, bp, Bg, ol, ocu, Acu, lav, R1
REAL VFE, V1, E1, N1, Kw, f, p, Np, ktr, ml, dw, hp, ll, tp, tcu
REAL 12, mFE, PFE, Xm, kn, kc, g, X1, BETA, PI, hov, w, krn
REAL oAL, tAL, s, wav, FLUXd, PERrs, PER1, PER2, oFE, kz, e
REAL c, h, RFE, uPFE, E1new, J1T, R, n1, nTAU, nhy, nhs, nws
REAL nbp, nhp, ALi, AAcu, Adw, Iu, IFe, P2cu, Pelect
REAL Thrust, PL, cosA, eff, Pin, PERo, P1cu
COMPLEX xAL, j, ZAL, a, b, PERFE,xFE, ZFE, Zt, Z2, I1, Z1
COMPLEX Io, I2
INTEGER COUNT, LOOP, CHANGE, ANSWER, VALUE, SWITCH, SLIP
INTEGER ANDER, NICHT, NOCH, CHOICE, ANTWORD, SCHALTER
INTEGER WERT, SKAKEL
OPEN(1, STATUS='SCRATCH')
OPEN(3, STATUS='NEW')
OPEN(4, STATUS='NEW')
OPEN(7, STATUS='NEW')
OPEN(8, STATUS='NEW')
ANDER=0
NICHT=0
NOCH=0
SKAKEL=0
SWITCH=0
SCHALTER=0
WERT=0
COUNT=0

73
LOOP=0
CHANGE=0
E1=170.00
20 IF (ANDER.EQ.0) Li=0.09
30 IF (CHANGE.EQ.0) THEN
   Acu=0.0000012272
   dw=0.00125
ENDIF
IF (SWITCH.EQ.1) GOTO 70
40 l=0.192
   TAU=0.048
   ws=0.005
   hy=0.016
   hs=0.005
   hp=0.043
   bp=0.032
IF (SKAKEL.EQ.1) GOTO 70
IF (NOCH.EQ.1) GOTO 70
PRINT*, 'DO YOU WANT TO KEEP CORE DIMENSIONS [YES=1/NO=2]'
READ*,DYW
50 IF (DYW.EQ.1.0) THEN
   NOCH=1
   GOTO 70
ENDIF
60 IF (DYW.EQ.2) THEN
   PRINT*, 'THE CURRENT DIMENSIONS ARE GIVEN IN THE FOLLOWING ORDER:'
PRINT*, ' l, TAU, hy, hs, hp, bp, ws'
PRINT*, l, TAU, hy, hs, hp, bp, ws
PRINT*, 'PLEASE ENTER NEW DIMENSIONS, KEEPING THE SAME ORDER AND SEPERATED BY A COMMA'
READ*, l, TAU, hy, hs, hp, bp, ws
WRITE (*,7) l, TAU, hy, hs, hp, bp, ws
SWITCH=1
ENDIF
70 CONTINUE
IF (SWITCH.EQ.1) THEN
   REWIND 7
   READ (7,*) l, TAU, hy, hs, hp, bp, ws
ENDIF
IF (ANDER.EQ.1) THEN
   REWIND 3
   READ (3,*) Li
ENDIF
IF (CHANGE.EQ.1) THEN
   REWIND 4
   READ (4,*) Acu, dw
ENDIF
IF (SCHALTER.EQ.1) THEN
   REWIND 8
   READ (8,*) Bg, uPFE
ENDIF
75 CONTINUE
IF (SCHALTER.EQ.1) GOTO 78
Bg=0.4
uPFE=0.6
IF (WERT.EQ.1) GOTO 78
PRINT*, 'AT THE MOMENT Bg = ', Bg,
       AND W/Kg = ', uPFE
PRINT*, 'DO YOU WANT TO CHANGE Bg AND W/Kg
       [YES=1/NO=2]'
READ*,ANTWORD
IF (ANTWORD.EQ.1) THEN
   PRINT*, 'ENTER NEW VALUE FOR Bg AND W/Kg'
   READ*,Bg,uPFE
   WRITE (8,*) Bg,uPFE
   SCHALTER=1
   GOTO 20
ELSE IF (ANTWORD.EQ.2) THEN
   WERT=1
   GOTO 78
ENDIF
78 CONTINUE
o1=1.15
v1=220.0
s=1.0
CFE=7800.0
Kw=1.0
f=50.0
p=2.0
m1=2.0
tp=0.0006
ocu=56000000.0
tcu=235.0
c=1.0
kn=1.1
g=0.002
PIE=3.141593
caL=36000000.0
tAL=245.0
w=2*PIE*f
PER0=4.0E-7*PIE
j=CSQRT(CMPLX(-1))
d=0.002
PERrs=200.0
PER1=1.23
PER2=0.69
caFE=5.0E6
h=0.005

C FORMULA ORIENTATED
VFE=L*(1*hy+4*TAU*(TAU-bp)-3*hs*ws)
mFE=VFE*CFE
PF=UPFE*mFE*1.5
FLUX=bp*L*Bg*ol
N1=E1/(4.44*Kw*f*FLUX)
Np=N1/(2*p)
ktr=(2*m1*(N1*Kw)**2)/p
l1=(Np*dw/hp)*(dw+tp)
l=2*(L+bp+(2*l))
l2=(TAU-bp)/2

80 IF (l1.GE.l2) THEN
PRINT*,'THERE IS NOT ENOUGH STAKINGSPACE'
PRINT*,'AND 11 = ',11,' AND 12 = ',12
PRINT*,'AT THIS MOMENT Li = ',Li,'
      AND Acu = ',Acu
PRINT*,'YOU ARE ALLOWED TO CHANGE ONLY
      Li OR Acu'
PRINT*,'WHICH ONE DO YOU WANT TO CHANGE?'
PRINT*,'ENTER 1 FOR Li OR 2 FOR Acu'
READ*,CHOICE
IF (CHOICE.EQ.1) THEN
   CLOSE (3)
   PRINT*,'ENTER NEW VALUE FOR Li'
   READ*,Li
   OPEN (3)
   WRITE (3,*) Li
   ANDER=1
   SKAKEL=1
   GOTO 20
ELSE IF (CHOICE.EQ.2) THEN
   CLOSE (4)
   PRINT*,'ENTER NEW VALUE FOR Acu
      AND FOR dw'
   READ*,Acu,dw
   OPEN (4)
   WRITE (4,*) Acu,dw
   CHANGE=1
   SKAKEL=1
   GOTO 20
ENDIF

110 ELSE IF (11.LT.12) THEN

IF (NICHT.EQ.1) GOTO 150
PRINT*, 'AT THIS MOMENT Li = ', Li,
AND Acu = ', Acu
PRINT*, 'AND 11 = ', 11, ' AND 12 = ', 12

PRINT*, 'DO YOU WANT TO CHANGE THE VALUES?
[YES=1/NO=2]' READ*, VALUE

120 IF (VALUE.EQ.1) THEN

PRINT*, 'YOU ARE ALLOWED TO CHANGE ONLY
Li OR Acu'
PRINT*, 'WHICH ONE DO YOU WANT TO CHANGE?'
PRINT*, 'ENTER 1 FOR Li OR 2 FOR Acu'
READ*, ANSWER

130 IF (ANSWER.EQ.1) THEN

CLOSE(3)
PRINT*, 'ENTER NEW VALUE FOR Li'
READ*, Li
OPEN(3)
WRITE (3,*) Li
ANDER=1
SKAKEL=1
GOTO 20

140 ELSE IF (ANSWER.EQ.2) THEN

CLOSE(4)
PRINT*, 'ENTER NEW VALUE FOR Acu
AND FOR dw'

READ*,Acu,dw
OPEN(4)
WRITE (4,*) Acu,dw
CHANGE=1
SKAKEL=1
GOTO 20
ENDIF

ELSE IF (VALUE.EQ.2) THEN
NICT=1
GOTO 150
ENDIF

150 CONTINUE

Rl=(lav*Nl)/(ocu*Acu)
Rl=Rl*(tcu+75.0)/(tcu+20.0)
Xm=12.8*f*p*(Np*Kw)**2*TAU*Li/(kn*kc*g*1000000.0)
Xl=0.8*(ol-1.0)*Xm
BETA=PIE/TAU
hov=TAU/2
w=TAU+Li
krn=1-((TANH(BETA*w/2))/
1((BETA*w/2)*1+TANH(BETA*w/2)*TANH(BETA*hov)))
oAL=oAL*krn*(((tAL+20)/(tAL+75))
xAL=CSQRT((s*j*wav*PERo*oAL)+((PIE/TAU)*(PIE/TAU)))
b=xAL*d
a=(CEXP(b)-CEXP((-1)*b))/
1(CEXP(b)+CEXP((-1)*b))
ZAL = \( \frac{ktr \cdot j \cdot s \cdot \text{wav} \cdot \text{PERo} \cdot \text{Li}}{(xAL \cdot \text{TAU} \cdot a)} \)

\( \text{PERFE} = \text{PERo} \cdot \text{PERrs} \cdot (\text{PER1} - \text{PER2} \cdot j) \)

\( xFE = \text{CSQR}T((s \cdot j \cdot \text{wav} \cdot \text{PERFE} \cdot oFE) + \frac{1}{(\text{PIE}/\text{TAU}) \cdot (\text{PIE}/\text{TAU})}) \)

\( kZ = 1 - \left( \frac{g}{\text{Li}} \right) + \left( \frac{2 \cdot \text{TAU}}{\text{PIE} \cdot \text{w}} \right) \cdot (1 - \frac{1}{\exp\left(-\frac{1}{\text{PIE} \cdot \text{w}}\right) / (2 \cdot \text{Li})}) \)

\( e = xFE \cdot h \)

\( c = (\text{CEXP}(e) - \text{CEXP}((-1) \cdot e)) / (\text{CEXP}(e) + \text{CEXP}((-1) \cdot e)) \)

\( ZFE = \frac{(ktr \cdot j \cdot s \cdot \text{wav} \cdot \text{PERFE} \cdot \text{Li} \cdot kZ)}{(xfe \cdot \text{TAU} \cdot c)} \)

\( Z2 = \left( \frac{ZAL \cdot ZFE}{(ZAL + ZFE)} \right) \cdot (1/s) \)

\( \text{RFE} = \frac{E1 \cdot E1}{PFE} \)

\( Zt = (R1 + (j \cdot X1)) + ((RFE \cdot ((j \cdot Xm \cdot Z2) / ((j \cdot Xm) + Z2))) / (RFE + ((j \cdot Xm \cdot Z2) / ((j \cdot Xm) + Z2)))) \)

\( I1 = V1 / Zt \)

\( J1 = \text{CABS}(I1) / \text{Acu} \)

\( R = \text{CABS}(I1) \)

\( Z1 = R1 + (j \cdot X1) \)

\( T = \text{CABS}(Z1) \)

\( E1_{\text{new}} = V1 - (R \cdot T) \)

\( \text{PRINT*}, 'WHEN S=1, E1_{\text{new}} = ', E1_{\text{new}} \)

160 \text{IF} (E1_{\text{new}} \neq E1) \text{THEN} \)

\( E1 = E1_{\text{new}} \)

\( \text{COUNT} = \text{COUNT} + 1 \)

\( \text{GOTO 20} \)

\( \text{ENDIF} \)

180 \text{CONTINUE} \)

\( \text{PRINT*}, ' \)
PRINT*, 'ENTER A VALUE FOR S [0<S<1]' 
READ*, S

190 IF (S.NE.1.0) THEN
195   xAL=CSQRT((S*J*WAV*PER0*OAL)+
1        ((PI/TAU)*(PI/TAU)))
   b=xAL*d
   a=(CEXP(b)-CEXP((-1)*b))/
1        (CEXP(b)+CEXP((-1)*b))
   ZAL=(KTR*J*S*WAV*PER0*LI)/(XAL*TAU*a)
   PERF=PER0*PERR*(PER1-PER2*j)
   XFE=CSQRT((S*J*WAV*PERFE*OFE)+
1        ((PI/TAU)*(PI/TAU)))
   kZ=1-(G/LI)+((2*TAU)/(PI*W)*
1        (1-(EXP(((-1)*PI*W)/(2*LI))))
   e=XFE*h
   C=(CEXP(e)-CEXP((-1)*e))/
1        (CEXP(e)+CEXP((-1)*e))
   ZFE=(KTR*J*S*WAV*PERFE*LI*KZ)/(XFE*TAU*C)
   Z2=((ZAL*ZFE)/(ZAL+ZFE))*(1/S)
   RFE=(E1*E1)/PFE
   ZT=(R1+(J*X1))+((RFE*((J*Xm*Z2))/((J*Xm)+Z2))
1        /(RFE+((J*Xm*Z2))/((J*Xm)+Z2)))
   II=V1/ZT
   J1=CABS(II)/ACU
   R=CABS(II)
   Z1=R1+(J*X1)
   T=CABS(Z1)
   ELNEW=V1-(R*T)
PRINT*, 'THE NEW ITERATION FOR Elnew = ', Elnew

200 IF (Elnew.NE.E1) THEN
    E1=Elnew
    GOTO 195
ELSE
    GOTO 210
ENDIF
ELSE
    GOTO 210
ENDIF

210 WRITE(*,220)Elnew,COUNT
220 FORMAT(' Elnew= ',F10.6,' AFTER',I3,' INTERATIONS')

230 WRITE(*,240)N1,Np
240 FORMAT(' N1 = ',F4.0,' AND Np = ',F4.0)

Iu=Elnew/Xm
If=Elnew/RFe
Io=If-(Iu*j)
I2=I1-Io
PFe=(If**2)*RFe
Plcu=CABS(I1*I1)*Rl
P2cu=CABS(I2*I2)*REAL(Z2)*s
Pelect=CABS(I2*I2)*REAL(Z2)
Thrust=Pelect/(2*f*TAU)
PL=PFe+Plcu+P2cu
cosA=COS((ATAN(AIMAG(I1)/REAL(I1)))*
1 (180/PIE))*PIE/180)
Pin=V1*R*cosA
eff = 1 - (PL/Pin)
PRINT*, ' ' 
PRINT*, ' ' 
PRINT*, ' ' 
PRINT*, ' Iu = ', Iu
PRINT*, ' Pelect = ', Pelect, ' PL = ', PL, ' 
Pin = ', Pin
PRINT*, ' Thrust = ', Thrust
PRINT*, ' eff = ', eff
PRINT*, ' cosA = ', cosA
PRINT*, ' J = ', J1
PRINT*, ' ' 
PRINT*, ' DO YOU WANT TO CHANGE THE SLIP
[YES=1/NO=2] ' 
READ*, SLIP
IF (SLIP.EQ.1) THEN 
    GOTO 180
ENDIF 
ELSE IF (SLIP.EQ.2) THEN 
    CLOSE(1)
    CLOSE(3)
    CLOSE(4)
    CLOSE(7)
    CLOSE(8)
ENDIF 
END
APPENDIX 2

Sample run of program

A:\>LM

DO YOU WANT TO KEEP CORE DIMENSIONS [YES=1/NO=2]
1

AT THE MOMENT $b_g = 4.0000E-01$ AND $w/Kg = 6.0000E-01$
DO YOU WANT TO CHANGE $b_g$ AND $w/Kg$ [YES=1/NO=2]
2

AT THIS MOMENT $l_i = 9.0000E-02$ AND $a_{cu} = 1.2272E-06$
AND $l_1 = 7.7713E-03$ AND $l_2 = 7.9999E-03$
DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2]
2

ENTER A VALUE FOR $s$ [0<$s$<1]
1

$E_{\text{new}} = 153.6131$ AFTER 5 INTERATIONS

$N_I = 522.$ AND $N_p = 131.$

$I_1 = 11.5976$ $I_u = 3.5844$

$P_{\text{elected}} = 1286.2110$ $P_L = 1628.1850$ $P_{in} = 1724.2960$

$P_{\text{thrust}} = 267.9607$

$e_f = 5.5739E-02$

$\cos A = 6.7580E-01$

$J = 9450505.00$

DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1

ENTER A VALUE FOR $s$ [0<$s$<1]
0.8
Elnew = 159.448700  AFTER  5 INTERATIONS
N1 = 522.  AND  Np = 131.
I1  = 10.5781  Iu  = 3.7206
Pelect  = 1107.5890  PL  = 1171.2100  Pin  = 1460.2690
Thrust= 230.7478
eff  = 1.9794E-01
cosA  = 6.27479E-01
J  = 8619766.00
DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1
ENTER A VALUE FOR S [0<S<1]
0.55
Elnew = 166.163000  AFTER  5 INTERATIONS
N1 = 522.  AND  Np = 131.
I1  = 9.4052  Iu  = 3.8773
Pelect  = 849.1395  PL  = 693.2388  Pin  = 1101.7000
Thrust= 176.9041
eff  = 3.7075E-01
cosA  = 5.3244E-01
J  = 7663960.00
DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1
ENTER A VALUE FOR S [0<S<1]
0.1
Elnew = 173.9980  AFTER  5 INTERATIONS
N1 = 522.  AND  Np = 131.
I1  = 8.0364  Iu  = 4.0601
Pelect  = 210.3370  PL  = 187.2295  Pin  = 364.0162
Thrust = 43.8202
\text{eff} = 4.8565\times 10^{-1}
\cos A = 2.0588\times 10^{-1}
J = 6548609.00

DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2] 
1

Sample run for different Design

DO YOU WANT TO KEEP CORE DIMENSIONS [YES=1/NO=2] 
2

THE CURRENT DIMENSIONS ARE GIVEN IN THE FOLLOWING ORDER:

1, \tau, h_y, h_s, h_p, b_p, w_s

\begin{align*}
1.9200\times 10^{-1} & & 4.8000\times 10^{-2} & & 1.6000\times 10^{-2} & & 5.0000\times 10^{-3} \\
4.3000\times 10^{-2} & & 3.2000\times 10^{-2} & & 5.0000\times 10^{-3} \\
\end{align*}

PLEASE ENTER NEW DIMENSIONS, KEEPING THE SAME ORDER AND
SEPERATED BY A COMMA

0.192, 0.048, 0.018, 0.005, 0.043, 0.032, 0.005

AT THE MOMENT \text{B}_g = 4.0000\times 10^{-1} \text{ AND } W/K_g = 6.0000\times 10^{-1}

DO YOU WANT TO CHANGE \text{B}_g \text{ AND } W/K_g [YES=1/NO=2] 
1

ENTER NEW VALUE FOR \text{B}_g \text{ AND } W/K_g
0.6, 0.92

AT THIS MOMENT L_i = 9.0000\times 10^{-2} \text{ AND } A_{cu} = 1.2272\times 10^{-6}

AND \text{l}_1 = 5.1809\times 10^{-3} \text{ AND } \text{l}_2 = 7.9999\times 10^{-3}

DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2] 
1

YOU ARE ALLOWED TO CHANGE ONLY L_i OR A_{cu}
WHICH ONE DO YOU WANT TO CHANGE?
ENTER 1 FOR Li OR 2 FOR Acu
ENTER NEW VALUE FOR Bg AND W/Kg
0.6, 0.92
AT THIS MOMENT Li = 9.0000E-02 AND Acu = 1.2272E-06
AND 11 = 5.1809E-03 AND 12 = 7.9999E-03
DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2]
1
YOU ARE ALLOWED TO CHANGE ONLY Li OR Acu
WHICH ONE DO YOU WANT TO CHANGE?
ENTER 1 FOR Li OR 2 FOR Acu
1
ENTER NEW VALUE FOR Li
0.04
THERE IS NOT ENOUGH STAKINGSPACE
AND 11 = 1.1657E-02 AND 12 = 7.9999E-03
AT THIS MOMENT Li = 4.0000E-02 AND Acu = 1.2272E-06
YOU ARE ALLOWED TO CHANGE ONLY Li OR Acu
WHICH ONE DO YOU WANT TO CHANGE?
ENTER 1 FOR Li OR 2 FOR Acu
2
ENTER NEW VALUE FOR Acu AND FOR dw
0.0000005176, 0.00089
AT THIS MOMENT Li = 4.0000E-02 AND Acu = 5.1760E-07
AND 11 = 6.6847E-03 AND 12 = 7.9999E-03
DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2]
2
ENTER A VALUE FOR S [0<S<1]
1

E1new = 141.815300 AFTER 9 INTERATIONS

N1 = 723. AND Np = 181.

I1 = 11.6990 Iu = 3.8826

Pelect = 1227.2710 PL = 1920.5430 Pin = 1911.2130

Thrust = 255.6814
eff = -4.8817E-03
cosA = 7.4256E-01

J = 2.2602E+07

DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]

1

ENTER A VALUE FOR S [0<S<1]

0.1

E1new = 159.0340 AFTER 9 INTERATIONS

N1 = 723. AND Np = 181.

I1 = 9.1225 Iu = 4.54112

Pelect = 261.6733 PL = 448.7939 Pin = 625.0469

Thrust = 54.5152
eff = 2.8198E-01
cosA = 3.1143E-01

J = 1.7624E+07

DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]

From the above sample run it can be seen that the current density is much too high for the selected wire diameter.
APPENDIX 3

Depth of Penetration

The depth of penetration of the magnetic field into the mild steel disk was calculated in the following way.

\[ \delta = \frac{1}{\sqrt{\mu_0 \mu_r \sigma}} \]

\[ = \frac{1}{\sqrt{50 \times \pi \times 200 \times 4 \times \pi \times 10^{-7} \times 5 \times 10^5}} \]

\[ = 0.00711(m) \]

\[ = 7.11(mm) \]

As the mild steel disk is 10mm thick, the disk is not saturated by the magnetic field.