## SHADED-POLE FLAT LINEAR INDUCTION MOTOR

A Thesis submitted to the cape Technikon in fulfilment of the requirements for the Master's

Diploma in Technology (Heavy Current).

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## ABSTRACT

Linear induction motors are built for numerous applications. Their robustness, gearfree-link to the load and the fact that speed is not a function of the number of poles (as in round machines) are only a few advantages. As every single phase motor needs some starting aid, so does a linear induction drive. For round machines, capacitors or shaded poles are usually used. A capacitor can be damaged electrically, which is a rare occurrence for shading rings. A useful combination of these advantages is a linear induction motor which is gearless and uses the shaded pole principle as a starting aid.

In order to simplify the designing process, a computer program (Fortran) was written. The main objective on the electrical side was to obtain low input power, high power factor and high efficiency.

In order to find the performance curves, torque measurements were made. Although the principles of torque measurements are well known the device to measure torque for this machine was not available and had to be constructed.

Material obtainable in South Africa was used, so that the motor could be reproduced at any time. This work should provide a valuable foundation for further research in this direction.

## TERMS OF REFERENCE

The objective is to design and build a shaded-pole flat linear induction motor. A computer program is to be written in order to simulate the motor and to simplify the design process, as well as finding the necessary performance curves.

The above mentioned points are to be carried out as a thesis, to be submitted in July 1991, for the completion of a Master's Diploma at the Cape Technikon.

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## TABLE OF CONTENTS

Page No.NOMENCLATURE ..... 1

1. INTRODUCTION ..... 6
2. CONSTRUCTION
2.1 Construction ..... 8
2.2 Moving Magnetic Field ..... 12
3. DESIGN OF SHADED-POLE FLAT LINEAR INDUCTION MOTOR
3.1 Formulae used in Design ..... 13
3.2 Calculations ..... 21
3.3 Flowchart ..... 31
4. EXAMPLE OF CALCULATIONS
4.1 Input Data ..... 34
4.2 Results ..... 36
4.3 Nameplate ..... 38
4.4 Characteristic Curves ..... 39
4.5 Figures ..... 42
5. TESTING
5.1 No-Load Test ..... 45
5.1.1 Measured Results ..... 45
5.1.2 Characteristic curves ..... 46
5.2 Short-Circuit Test ..... 48
5.2.1 Input Data ..... 49
5.2.2 Calculations ..... 49
5.2.3 Results ..... 50
5.2.4 Characteristic Curves ..... 51
5.3 Load Test ..... 53
5.3.1 Input Data ..... 54
5.3.2 Calculations ..... 54
5.3.3 Results ..... 55
5.3.4 Characteristic Curves ..... 57
5.4 Inertia ..... 63
5.4.1 Input Data ..... 64
5.4.2 Calculations ..... 64
5.4.3 Run-out Time ..... 66
5.4.4 Measured Run-out Time ..... 68
5.4.4.1 Results ..... 68
5.4.4.2 Graphs ..... 69
6. CONCLUSION ..... 70
REFERENCES ..... 72
APPENDIX 1 ..... 73
APPENDIX 2 ..... 85
APPENDIX 3 ..... 90

## TABLE OF CONTENTS (Contd.)

## LIST OF FIGURES

Figure Page No.
Construction of shaded-pole flat
Iinear induction motor ..... 8
Assembly of whole shaded-pole flat LIM ..... 11
Equivalent circuit for a linearinduction motor18
Efficiency versus slip (graph) ..... 39
Magnetizing current versus slip ..... 39
Power factor versus slip ..... 40
Current density versus slip ..... 40
Thrust versus slip ..... 41
Power versus slip ..... 41
Total assembly with dimensions of motor ..... 42
Construction of stator core ..... 43
Secondary rotor ..... 43
Assembly of shaft ..... 44
Power factor versus input voltage ..... 46
Input current versus input voltage ..... 47
Input power versus input current ..... 47
Short-circuit test ..... 48
Power factor versus input voltage ..... 51
Input current versus input voltage ..... 51
Input power versus input current ..... 52
Torque versus input power ..... 52
Load test ..... 53
Efficiency versus speed ..... 57
Power factor versus slip ..... 57
Input power versus speed ..... 58
Output power versus speed ..... 58
Input current versus speed ..... 59
Torque versus speed ..... 59
Efficiency versus torque ..... 60
Power factor versus torque ..... 60
Input power versus torque ..... 61
Output power versus torque ..... 61
Input current versus torque ..... 62
Inertia test ..... 63
Run-out time ..... 69

## LIST OF TABLES

## TABLE

1 Results from simulating program . . . . . 36
2 Results from simulating program . . . . . . 37
3 Typical nameplate values used in design . . 38
4 Results for the no-load test . . . . . . . 45
5 Results for the short-circuit test . . . . 50
6 Results for the load test . . . . . . . . . 55
7 Results for the load test . . . . . . . . . 56
8 Measured run-out time . . . . . . . . . . . 68

## NOMENCLATURE



| J | $=$ current density ( $\mathrm{A} / \mathrm{m}^{2}$ ) |
| :---: | :---: |
| $\mathrm{k}_{\mathrm{c}}$ | = Carter's factor |
| $\mathrm{k}_{\mathrm{d}}$ | $=$ distribution factor |
| $\mathrm{k}_{\mathrm{n}}$ | $=$ saturation factor |
| $\mathrm{k}_{\mathrm{p}}$ | $=$ pitch factor |
| $\mathrm{k}_{\mathrm{rn}}$ | = Russell and Norsworthy factor |
| $k_{t r}$ | $=$ transfer coeficient |
| $\mathrm{K}_{\mathrm{w}}$ | = winding factor |
| 1 | $=$ width of core (m) |
| $l_{\text {av }}$ | $=$ average length of one turn (m) |
| $1_{1}$ | = effective winding space (m) |
| $\mathrm{I}_{2}$ | = available winding space (m) |
| $L_{i}$ | $=$ length of core ( m ) |
| $\mathrm{m}_{\mathrm{Fe}}$ | $=$ mass of core (kg) |
| $\mathrm{m}_{\mathrm{I}}$ | $=$ mass used in inertia test (kg) |
| $\mathrm{m}_{s}$ | ```= mass needed to start rotating disk with uniform velocity (kg)``` |
| $\mathrm{m}_{\mathrm{t}}$ | $=$ total mass of disk and shaft (kg) |
| $\mathrm{m}_{1}$ | $=$ number of phases |
| N | $=$ speed (rpm) |
| $\mathrm{N}_{\mathrm{p}}$ | $=$ number of turns per pole |
| $\mathrm{N}_{1}$ | $=$ total number of turns |
| $p$ | $=$ number of pole pairs |
| $\mathrm{P}_{\text {elect }}$ | $=$ secondary $I^{2} \mathrm{R}$ losses (W) |
| $\mathrm{P}_{\mathrm{Fe}}$ | = iron loss in core (W) |
| $P_{\text {I }}$ | $=$ input power ( $W$ ) |
| $\mathrm{P}_{\mathrm{L}}$ | $=$ total power loss (W) |
| $\mathrm{P}_{1 \mathrm{cu}}$ | $=$ copper loss in primary (W) |


| $\mathrm{P}_{2 \mathrm{cu}}$ | $=$ total transfered power in air gap (W) |
| :---: | :---: |
| r | $=$ radius of shaft used for inertia test (m) |
| $\mathrm{r}_{\mathrm{g}}$ | = radius of gyration (m) |
| $\mathrm{R}_{\mathrm{Fe}}$ | $=$ iron loss resistance ( $\Omega$ ) |
| $\mathrm{R}_{1}$ | = primary resistance ( $\Omega$ ) |
| s | = siip |
| $\mathrm{S}_{\mathrm{d}}$ | ```= distance that the weight for inertia test has dropped (m)``` |
| t | ```= time that the mass took to drop in the inertia test (s)``` |
| $t_{p}$ | ```= thickness of insulation paper between windigs (m)``` |
| u | $=$ initial velocity (m/s) |
| $\mathrm{V}_{\mathrm{Fe}}$ | $=$ core volume ( $\mathrm{m}^{3}$ ) |
| $\mathrm{V}_{1}$ | $=$ input voltage (V) |
| $\mathrm{w}_{\text {s }}$ | = width of shading pole slot (m) |
| $\mathrm{w}_{1}$ | ```= initial angular velocity, inertia test (rad/s}\mp@subsup{}{}{2}``` |
| $\mathrm{w}_{2}$ | ```= final angular velocity, inertia test \(\left(\mathrm{rad} / \mathrm{s}^{2}\right)\)``` |
| $\mathrm{X}_{\mathrm{m}}$ | = mutual inductance ( $\Omega$ ) |
| $\mathrm{X}_{1}$ | = primary leakage reactance ( $\Omega$ ) |
| $\mathrm{Z}_{\mathrm{AI}}$ | ```= impedance of aluminium reaction rail refered to primary (\Omega)``` |
| $\mathrm{Z}^{\prime} \mathrm{Fe}$ | ```= impedance of iron reaction rail refered to primary (\Omega)``` |
| $z_{t}$ | = total impedance of motor ( $\Omega$ ) |
| $\mathrm{Z}^{\prime} 2$ | $=$ total impedance of reaction rail refered to |


|  | primary ( $\Omega$ ) |
| :---: | :---: |
| $\alpha$ | $=$ angular acceleration (rad/s ${ }^{2}$ ) |
| $\alpha_{\text {Al }}$ | $=$ propagation constant for aluminium ( $\mathrm{m}^{-1}$ ) |
| $\alpha_{\text {Fe }}$ | $=$ propagation constant for iron ( $\mathrm{m}^{-1}$ ) |
| $\gamma_{c u}$ | $=$ specific weight of aluminium ( $\mathrm{kp} / \mathrm{m}^{3}$ ) |
| $\gamma_{\mathrm{Fe}}$ | $=$ specific weight of sheet steel ( $\mathrm{kp} / \mathrm{m}^{3}$ ) |
| $\Delta \mathrm{P}_{\mathrm{Fe}}$ | $=$ unit loss based on 0.4 T in the air gap |
|  | ( $\mathrm{W} / \mathrm{kg}$ ) |
| n | = efficiency |
| $\mu_{\mathrm{Fe}}$ | = complex permeability in reaction rail ( Hm ) |
| $\mu_{0}$ | = permeability of free space ( $\mathrm{H} / \mathrm{m}$ ) |
| $\mu_{r s}$ | = relative surface permeability iron disk |
| $\mu^{\prime}$ | $=$ real component of $\mu_{r s}$ |
| $\mu^{\prime \prime}$ | $=$ imaginary component of $\mu_{r s}$ |
| $\rho_{\mathrm{Fe}}$ | $=$ density of steel ( $\mathrm{kg} / \mathrm{m}^{3}$ ) |
| $\sigma_{\text {Al }}$ | $=$ conductivity of aluminium ( $\mathrm{S} / \mathrm{m}$ ) |
| $\sigma_{\text {cu }}$ | = conductivity of copper ( $\mathrm{S} / \mathrm{m}$ ) |
| $\sigma_{\mathrm{Fe}}$ | $=$ conductivity of steel ( $\mathrm{S} / \mathrm{m}$ ) |
| $\sigma_{1}$ | = leakage factor |
| $\sigma^{\prime}{ }_{\text {AI }}$ | ```= equivalent conductivity of aluminium disk (S/m)``` |
| $\tau$ | $=$ pole pitch (m) |
| $\tau_{\text {Al }}$ | $=$ temperature constant of aluminium at $t=20^{\circ} \mathrm{C}$ |
| $\tau_{\text {cu }}$ | $=$ temperature constant of copper at $t=20^{\circ} \mathrm{C}$ |
| ${ }^{T}$ Fesh | $=$ temperature constant of sheet steel at |
|  | $t=20^{\circ} \mathrm{C}$ |
| $\tau_{\text {Feso }}$ | $=$ temperature constant of solid steel at |
|  | $\mathrm{t}=20^{\circ} \mathrm{C}$ |


| $\Phi_{G}$ | $=$ air gap flux $(\mathrm{Wb})$ |
| :--- | :--- |
| $\Phi_{t}$ | $=$ total flux in core $(\mathrm{Wb})$ |
| $\omega$ | $=$ angular velocity $(\mathrm{rad} / \mathrm{s})$ |

Just as electromagnetic forces produce a rotary motion in an electric machine, electromagnetic forces may be used to produce linear motion. In principle there is a linear electric machine for every rotary machine.

The main difference between linear electric machines and its rotary counterpart is best explained by an imaginary process whereby an induction motor is transformed into a linear induction motor (LIM) by cutting the stator and the rotor of the rotary motor in a radial plane and unrolling it. This produces an open air gap with an entry point and an exit point in the linear inductionmotor.

I believe that a need exists for the development and manufacture of a single-phase linear machine, because three-phase power is not normally available for domestic use. The three-phase induction motor, the two phase induction motor and some single-phase induction motors are similarly constructed. The essential elements for the production of a rotating or moving field in these induction motors are the time displacement of the exciting current and the space displacement of the winding. The single-phase induction motor that falls into this category usually uses a capacitor for the phase displacement.

The shaded-pole induction motor is a very popular choice for use where only fractional horse-power is required. It is very simple in construction, low in cost, extremely rugged and very reliable because it does not need commutator brushes, a centrifugal switch, a starting capacitor, collector rings, governor or contacts of any sort. All these advantages and the fact that the shaded-pole principle has not been investigated in conjunction with linear induction motors, justifies this theses.

The stator core with slots and teeth of linear induction motors can be replaced by a simpler flat salient pole magnetic circuit. The secondary magnetic circuit can be a simple conductive disk which is much cheaper and simpler than the rotor of an inductive machine. An additional advantage, especially in turntables, is the elimination of a gear box. These advantages in a linear induction motor are reason enough to adapt its performance characteristics to drive various large and small machines.

The disadvantages of the linear induction motors are a small power factor and a small starting torque. This is due to a large air gap and an open air gap. For the shaded-pole linear induction motor the starting torque is even lower. This is due to the phase split being less than 90 degrees as a result of various practical imperfections.

### 2.1 Construction

Figure 1 shows the basic construction of the shaded-pole flat linear induction motor. It consists basically of two parts, namely a flat magnetic core (6) and a round conductive disk (1;2).


Figure 1 Basic construction of the shaded-pole flat linear induction motor

A laminated core was constructed out of standard H18, 0.5 mm , Non Orientated Silicon Steel (N.O.S.S.) transformer laminations. The laminations, which are aligned in the direction of the field lines, are insulated from each other by a thin layer of varnish.

Laminating an iron core means an increase in permeability and decrease in conductivity, this consequently reduces the magnitude of the eddy current paths.

A shaded four pole flat linear induction motor core (Figure 1,6), was constructed by laminating two blocks of laminations under high pressure with strong epoxy glue (HY4076 and AV4076). A support frame was then built to keep the laminations together and to prevent them from fringing while slots where milled into the poles. The slots have to be open, as this reduces the leakage flux between the auxiliary and main windings and therefore allows more effective flux to flow through the air gap.

Shaded-pole copper rings (Figure 1,4) were then constructed to fit tightly into the slots. This was done to keep the main windings in place and to ensure that the shaded-pole copper rings do not become detached when vibrations occur. The shaded-pole copper rings were soldered in a very clean environment with silver solder. It is important to solder the seams with a material that has a high melting point to prevent disintegration during extreme operating conditions. The main windings were then wound (Figure 1,5 ), taking the space needed for the shaded-pole copper rings into consideration. The direction in which the coils are
wound, is important in creating north and south poles.

The assembly of the core was as follows. First the coils were slipped over the limbs, taking care that the correct south and north poles were created. The coil was then insulated with special insulating paper (DMDMitron) from the limb and from the yoke. After this the shaded-pole copper rings where forced into the slots.

The following procedure was followed to secure all the various parts and thereby preventing magnetic vibrations. First the core was placed into the oven and heated to 160 degrees, so as to obtain a homogeneous temperature throughout the core. Then the core was dipped in a special transformer resin (Isonol 31) until all air bubbles disappeared. The high temperature of the core ensures that the resin in the vicinity of the core becomes thinner and is able to fill up every air gap that exists between laminations of the core and the windings of the armature coils. Finally the core was baked to dry and harden the resin. This is necessary to keep all the parts in place during extreme running conditions.

The round conductive disk is much cheaper and easer to construct than its counterpart, the squirrel cage induction motor rotor. The secondary disk was constructed out of a 10 mm mild steel disk to which a 3 mm
aluminium cap was laminated.

Figure 2 shows the whole assembly of the motor. The magnetic core and the conductive disk are mounted on the supportive frame in such a way that the air gap and distance of the core from the centre of the disk to the edge of the disk can be changed for different tests.


Figure 2 Assembly of the shaded-pole flat linear induction motor

### 2.2 Moving Magnetic field

The induced current in the shading-ring-coil causes the flux in the shaded portion of the pole to lag the flux in the other portion. The result is a travelling magnetic field from the direction of the unshaded portion to the shaded portion of the pole. This travelling magnetic field induces currents to flow in the round aluminium disk. The secondary currents in turn create their own magnetic field, and the interaction between the two magnetic fields causes the thrust. The 10 mm mild steel disk has a flux depth penetration of 7.11 mm (Appendix 3) and ensures that the flux lines return by the shortest path into the core and therefor reduce the total reluctance of the field circuit which reduces the excitation for the flux.

To enable eddy currents to be fully established, the secondary disk should at least be half a pole pitch larger than the primary core in all direction. Another consideration when deciding on the size of the disk, is the distance of the primary core from the centre of the disk. The core should be situated as far away as physically possible, to ensure that the braking forces do not become to large.

### 3.1 Formulae used in the design

To start the design of the shaded-pole flat linear induction motor the main dimensions of the core have to be chosen tentatively from the output graphs. After a standard transformer lamination has been chosen most dimensions like $L_{i}, \tau, b_{p}, h_{y}, h_{s}, h_{p}$ and $l$ are fixed. (i) volume of the core

$$
\begin{equation*}
V_{F e}=L_{i}\left(1 h_{y}+4 \tau\left(\tau-b_{p}\right)-3 h_{s} w_{s}\right) \tag{1}
\end{equation*}
$$

mass of core

$$
\begin{equation*}
m_{F e}=V_{F e} \rho_{F e} \tag{2}
\end{equation*}
$$

iron core loss (multiplied by 1.5 to take additional core losses like those for bolts and sharp edges into account)

$$
\begin{equation*}
P_{F e}=\Delta P_{F e} m_{F e} 1.5 \tag{3}
\end{equation*}
$$

(ii) total flux (the flux of the air gap was chosen tentatively from the final current density, i.e. when the air gap flux is to high the current density will become to high)

$$
\begin{equation*}
\Phi_{t}=b_{p} L_{i} B_{g} \sigma_{I} \tag{4}
\end{equation*}
$$

total number of turns ( $E_{1}$ was determined via the iterative method - refer to Flowchart in section 3.3, page 31)

$$
\begin{equation*}
N_{1}=\frac{\left|E_{1}\right|}{4.44 K_{w} f \Phi_{t}} \tag{5}
\end{equation*}
$$

number of turns for one pole

$$
\begin{equation*}
N_{p}=\frac{N_{1}}{2 p} \tag{6}
\end{equation*}
$$

transfer coefficient (reference 4)

$$
\begin{equation*}
k_{t r}=\frac{2 m_{1}\left(N_{1} K_{w}\right)^{2}}{p} \tag{7}
\end{equation*}
$$

(iv) available winding space

$$
\begin{equation*}
I_{2}=\frac{\tau-b_{p}}{2} \tag{8}
\end{equation*}
$$

effective winding space

$$
\begin{equation*}
I_{1}=\frac{N_{p} d_{w}\left(d_{w}+t_{p}\right)}{h_{p}} \tag{9}
\end{equation*}
$$

average length of one turn

$$
\begin{equation*}
I_{a v}=2\left(L_{i}+b_{F}+2 I_{1}\right) \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
R_{1}=\frac{I_{\mathrm{av}} N_{1}}{\sigma_{c u} A_{c u}} \tag{11}
\end{equation*}
$$

primary resistance at a temperature of $75^{\circ}$

$$
\begin{equation*}
R_{1}=R_{1} \frac{\tau_{c u}+75}{\tau_{c u}+20} \tag{12}
\end{equation*}
$$

mutual inductance (reference 1)

$$
\begin{equation*}
X_{m}=12.8 \mathrm{fp}\left(N_{p} K_{w}\right)^{2} \frac{\tau L_{i}}{k_{n} k_{c} g} 10^{-6} \tag{13}
\end{equation*}
$$

primary leakage reactance (reference 1)

$$
X_{1}=0.8\left(\sigma_{1}-1\right) X_{n}
$$

iron loss resistance

$$
\begin{equation*}
R_{F \theta}=\frac{E_{1}^{2}}{P_{F \theta}} \tag{15}
\end{equation*}
$$

(reference 4)

$$
\begin{equation*}
\beta=\frac{\pi}{\tau} \tag{16}
\end{equation*}
$$

winding overhang

$$
\begin{equation*}
h_{o v}=\frac{\tau}{2} \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
w=\tau+L_{i} \tag{18}
\end{equation*}
$$

Russel - Norsworthy factor (this factor takes care of edge effects in high conductivity nonmagnetic layers, reference 4)

$$
\begin{equation*}
k_{r n}=1-\frac{\tanh \left(\beta \frac{w}{2}\right)}{\beta \frac{w}{2}\left(1+\tanh \left(\beta \frac{w}{2}\right) \tanh \left(\beta h_{o v}\right)\right)} \tag{19}
\end{equation*}
$$

conductivity of aluminium

$$
\begin{equation*}
\sigma_{A I}=\sigma_{A I}\left(\frac{\tau_{A I}+20}{\tau_{A 1}+75}\right) \tag{20}
\end{equation*}
$$

equivalent conductivity of aluminium disk (reference 2)

$$
\begin{equation*}
\sigma_{A 1}=k_{r n} \sigma_{\dot{A} 1} \tag{21}
\end{equation*}
$$

propagation constant for aluminium (reference 2)

$$
\begin{equation*}
\chi_{A 1}=\sqrt{\left(j s \omega \mu_{c} \sigma_{A 1}\right)+\left(\frac{\pi}{\tau}\right)^{2}} \tag{22}
\end{equation*}
$$

impedance of the aluminium reaction rail referred to the primary (reference 2)

$$
\begin{equation*}
Z_{A 1}^{\prime}=k t I \frac{j s \omega \mu_{0}}{\chi_{A 1}} \frac{1}{\tanh \left(\chi_{A 1} d\right)} \frac{L_{i}}{\tau} \tag{23}
\end{equation*}
$$

complex permeability in reaction rail (takes into account saturation and hysteresis and is a function of the magnetic field strength, (reference $2 \& 3$ )

$$
\begin{equation*}
\mu_{F e}=\mu_{0} \mu_{I s}\left(\mu^{\prime}-\mu^{\prime}\right) \tag{24}
\end{equation*}
$$

propagation constant for iron (reference 2)

$$
\begin{equation*}
\chi_{F e}=\sqrt{\left(j s \omega \mu_{F e} \sigma_{F e}\right)+\frac{\pi}{\tau}} \tag{25}
\end{equation*}
$$

the influence of the transverse edge effect on the secondary impedance can be taken into account by the coefficient $\mathrm{k}_{\mathrm{z}}$ (reference 2)

$$
\begin{equation*}
k_{z}=1-\left[\frac{g}{L_{i}}\right] \frac{2 \tau}{\pi w}\left[1-\exp \left[\frac{-\langle\pi w\rangle}{2 L_{i}}\right]\right] \tag{26}
\end{equation*}
$$

impedance of laminated ferromagnetic reaction rail reverted to the primary (reference 2 )

$$
\begin{equation*}
Z_{F e}^{\prime}=k_{t r} \frac{j s \omega \mu_{F e}}{\chi_{F \varepsilon}} \frac{1}{\tanh \left(\chi_{F e} h\right)} \frac{L_{i} k_{z}}{\tau} \tag{27}
\end{equation*}
$$

total impedance of reaction rail referred to primary

$$
\begin{equation*}
Z_{2}^{\prime}=\frac{Z_{\dot{A} 1}^{\prime} Z_{\dot{F e}}^{\prime}}{Z_{\dot{A} 1}^{\prime}+Z_{\dot{F e}}^{\prime}}\left[\frac{1}{S}\right] \tag{28}
\end{equation*}
$$



Figure 3 Equivalent circuit for a linear inductive motor with negligible small end effect (low speed).
(vi) total impedance of motor (derived from figure 3)

$$
\begin{equation*}
Z_{t}=\left(R_{2}+j X_{i}\right)+\left[\frac{R_{F e}\left[\frac{j X_{m} z_{2}^{\prime}}{j X_{m}+Z_{2}^{\prime}}\right]}{R_{F e}+\left[\frac{j X_{m} z_{2}^{\prime}}{j X_{m}+Z_{2}^{\prime}}\right]}\right] \tag{29}
\end{equation*}
$$

(vii) primary current

$$
\begin{equation*}
I_{1}=\frac{V_{1}}{Z_{t}} \tag{30}
\end{equation*}
$$

current density

$$
\begin{equation*}
J=\frac{\left|I_{1}\right|}{A_{c u}} \tag{31}
\end{equation*}
$$

(viii) EMF induced in the primary

$$
\begin{equation*}
E_{1}=V_{1}-\left|I_{1}\right|\left|R_{1}+j X_{1}\right| \tag{32}
\end{equation*}
$$

(ix) magnetizing current

$$
\begin{equation*}
I_{\mu}=\frac{\left|E_{1}\right|}{X_{m}} \tag{33}
\end{equation*}
$$

iron loss current

$$
\begin{equation*}
I_{F \theta}=\frac{\left|E_{1}\right|}{R_{F E}} \tag{34}
\end{equation*}
$$

no load current

$$
I_{0}=I_{F \epsilon}-j I_{\mu}
$$

secondary current

$$
\begin{equation*}
I_{2}^{\prime}=I_{1}-I_{0} \tag{36}
\end{equation*}
$$

(x) iron loss in the core

$$
\begin{equation*}
P_{F e}=I_{F e}^{2} R_{F e} \tag{37}
\end{equation*}
$$

copper loss in the primary

$$
\begin{equation*}
P_{\text {Icu }}=\left|I_{1}\right| R_{1} \tag{38}
\end{equation*}
$$

total power transferred across the air gap from the secondary

$$
\begin{equation*}
P_{2 c u}=\left|I_{2}^{\prime}\right| R_{2}^{\prime} S \tag{39}
\end{equation*}
$$

secondary $I^{2} R$ losses

$$
\begin{equation*}
P_{\text {elect }}=\left|I_{2}^{\prime 2}\right| R_{2}^{\prime} \tag{40}
\end{equation*}
$$

Thrust

$$
\begin{equation*}
\text { Thrust }=\frac{P_{\text {elect }}}{2 E \tau} \tag{41}
\end{equation*}
$$

(xi) total power loss

$$
\begin{equation*}
P_{Z}=P_{F e}+P_{1 c u}+P_{2 c u} \tag{42}
\end{equation*}
$$

(xii) power factor

$$
\begin{equation*}
\cos \phi=\cos \left[\left[L(I 1) \frac{180}{\pi}\right] \frac{\pi}{180}\right] \tag{43}
\end{equation*}
$$

(xiii) Input power

$$
\begin{equation*}
P_{I}=V_{1}\left|I_{1}\right| \cos \phi \tag{44}
\end{equation*}
$$

(xvi) efficiency

$$
\begin{equation*}
\eta=1-\frac{P_{L}}{P_{I}} \tag{45}
\end{equation*}
$$

### 3.2 CALCULATIONS

The following calculations are based on the formulas in section 3.1 , where the calculation numbers correspond to the formula numbers. The single-phase shaded-pole LIM was constructed according to the dimensions used in the calculations. Refer to the input values in section 4.1 .
(i) Volume according to formula (1).

$$
\begin{aligned}
V_{F e}= & 0.09(0.192 * 0.016+4 * 0.048 * \\
& (0.048-0.032)-3 * 0.005 * 0.005) \\
= & 5.462 * 10^{-4}\left(\mathrm{~m}^{3}\right)
\end{aligned}
$$

mass of core according to formula (2)

$$
\begin{align*}
m_{F e} & =5.462 * 10^{-6} * 7.8 * 10^{3}  \tag{2}\\
& =4.26(\mathrm{~kg})
\end{align*}
$$

iron core loss

$$
\begin{align*}
P_{F e} & =0.6 * 4.26 * 1.5  \tag{3}\\
& =3.834(\mathrm{~W})
\end{align*}
$$

(ii) total flux

$$
\begin{align*}
\Phi_{t} & =.032 * 0.09 * 0.4 * 1.15  \tag{4}\\
& =0.001(\mathrm{~Wb})
\end{align*}
$$

(iii) The induced voltage was found via the iterative method. Refer to the flowchart in section 3.3 to follow the iterative loop.

$$
E_{1}=153.597(\mathrm{~V})
$$

total number of turns

$$
\begin{align*}
N_{1} & =\frac{153.597}{4.44 * 1 * 50 * 0.001}  \tag{5}\\
& =522.251
\end{align*}
$$

number of turns for one pole

$$
\begin{align*}
N_{F} & =\frac{522.251}{2 * 2}  \tag{6}\\
& =130.563
\end{align*}
$$

transfer coefficient

$$
\begin{align*}
k_{t r} & =\frac{2 * 2 *(522.251 * 1)^{2}}{2}  \tag{7}\\
& =5.455 * 10^{5}
\end{align*}
$$

(iv) available winding space

$$
\begin{align*}
I_{2} & =\frac{0.048-0.032}{2}  \tag{8}\\
& =0.008(\mathrm{~m})
\end{align*}
$$

effective winding space

$$
\begin{align*}
I_{1} & =\frac{130.563 * 0.001 *\left(0.001+6 * 10^{-4}\right)}{0.043}  \tag{9}\\
& =0.007(\mathrm{~m})
\end{align*}
$$

average length of one turn

$$
\begin{align*}
I_{a v} & =2 *(0.09+0.032+2 * 0.007)  \tag{10}\\
& =0.272(\mathrm{~m})
\end{align*}
$$

(v) primary resistance

$$
\begin{align*}
R_{1} & =\frac{0.272 * 522.251}{5.6 * 10^{7} * 1.227 * 10^{-6}}  \tag{11}\\
& =2.068(\Omega)
\end{align*}
$$

primary resistance at a temperature of $75^{\circ}$

$$
\begin{align*}
R_{1} & =2.068 * \frac{235+75}{235+20}  \tag{12}\\
& =2.514(\Omega)
\end{align*}
$$

primary inductance

$$
\begin{align*}
X_{m}= & 12.8 * 50 * 2 *(130.563 * 1)^{2} \\
& * \frac{0.048 * 0.009}{1.1 * 1 * 0.002} * 10^{-6}  \tag{13}\\
= & 42.846 \text { ( } \Omega)
\end{align*}
$$

primary leakage reactance

$$
\begin{align*}
X_{1} & =0.8 *(1.15-1) * 42.846  \tag{14}\\
& =5.142(\Omega)
\end{align*}
$$

iron loss resistance

$$
\begin{align*}
R_{F e} & =\frac{153.597^{2}}{3.834}  \tag{15}\\
& =6.153 * 10^{3}(\Omega)
\end{align*}
$$

$$
\begin{equation*}
\beta=\frac{3.142}{0.048} \tag{16}
\end{equation*}
$$

$$
=65.45(\mathrm{rad} / \mathrm{m})
$$

winding overhang

$$
\begin{align*}
h_{o v} & =\frac{0.048}{2}  \tag{17}\\
& =0.024(\mathrm{~m})
\end{align*}
$$

$$
\begin{align*}
W & =0.048+0.09  \tag{18}\\
& =0.138(\mathrm{~m})
\end{align*}
$$

Russel - Norsworthy factor

$$
\begin{aligned}
k_{r n} & =1-\frac{\tanh \left(65.45 * \frac{0.138}{2}\right)}{65.45 * \frac{0.138}{2}} * \\
& \frac{1}{\left[1+\tanh \left(65.45 * \frac{0.138}{2}\right) * \tanh (65.45 * 0.024)\right]} \\
& =0.885
\end{aligned}
$$

conductivity of aluminium

$$
\begin{align*}
\sigma_{A 1} & =3.6 * 10^{7} *\left[\frac{245+20}{245+75}\right]  \tag{20}\\
& =2.981 * 10^{7}(\mathrm{~S} / \mathrm{m})
\end{align*}
$$

equivalent conductivity of the aluminium disk

$$
\begin{align*}
\sigma_{A 1} & =0.885 * 2.981 * 10^{7}  \tag{21}\\
& =2.637 * 10^{7}(\mathrm{~s} / \mathrm{m})
\end{align*}
$$

propagation constant for aluminium

$$
\begin{aligned}
\chi_{A I} & =\sqrt{\left(j * 50 * 314.159 * 1.257 * 10^{-6} * 2.637 * 10^{7}\right)+\left[\frac{\pi}{0.048}\right]^{2}} \\
& =88.15+59.049 j(1 / \mathrm{m})
\end{aligned}
$$

impedance of aluminium reaction rail

$$
\begin{align*}
Z_{A 1}= & 5.455 * 10^{5} * \frac{\left(j * 1 * 314.159 * 1.257 * 10^{-6}\right)}{(88.15+59.049 j)} * \\
& \frac{1}{\tanh [(88.15+59.049 j) * 0.002]} * \frac{0.09}{0.048}  \tag{23}\\
= & 16.586+7.094 j(\Omega)
\end{align*}
$$

complex permeability in reaction rail

$$
\begin{align*}
\mu_{F e} & =1.257 * 10^{-6} * 200 *(1.23-0.69 j)  \tag{24}\\
& =3.091 * 10^{-4}-1.734 * 10^{-4} j(\mathrm{H} / \mathrm{m})
\end{align*}
$$

propagation constant for iron

$$
\begin{align*}
\chi_{F e} & =\sqrt{\left(j * 50 * 314.159 *\left(3.091 * 10^{-4}-1.734 * 10^{-4} j\right)\right)+\left[\frac{\pi}{0.048}\right]^{2}}  \tag{25}\\
& =646.361+375.63 j(1 / \mathrm{m})
\end{align*}
$$

transverse edge effect coefficient

$$
\begin{align*}
k_{z}= & 1-\left[\frac{0.002}{0.09}\right] * \frac{2 * 0.048}{\pi * 0.138} * \\
& {\left[1-\exp \left[\frac{-\pi * 0.138}{2 * 0.09}\right]\right] }  \tag{26}\\
= & 1.179
\end{align*}
$$

$$
\begin{align*}
Z_{F e}^{\prime}= & 5.455 * 10^{5} * \frac{j * 1 * 314.159 *\left(3.091 * 10^{-4}-1.734 * 10^{-4}\right)}{(6.46 .361+375.63 j)} * \\
& \frac{1}{\tanh ((646.361+375.63 j) * 0.005)} * \frac{0.09 * 1.179}{0.048}  \tag{27}\\
= & 154.173+91.355 j(\Omega)
\end{align*}
$$

total impedance of reaction rail

$$
\begin{aligned}
Z_{2}^{\prime} & =\frac{(16.586+7.094 j) *(154.173+91.355 j)}{(16.586+7.094 j)+(154.173+91.355 j)} *\left[\frac{1}{1}\right] \\
& =15.002+6.629 j(\Omega)
\end{aligned}
$$

(iv) total impedance of motor

$$
\begin{aligned}
Z_{t} & =(2.068+5.142 j)+ \\
& {\left[\frac{6.153 * 10^{3} *\left[\frac{42.846 j *(15.002+6.629 j)}{42.846 j+(15.002+6.629 j)}\right]}{6.153 * 10^{3}+\left[\frac{42.846 j+(15.002+6.629 j)}{42.846 j+(15.002+6.629 j)}\right]}\right] } \\
& =12.813+13.977 j(\Omega)
\end{aligned}
$$

(vii) primary current

$$
\begin{aligned}
I_{1} & =\frac{220}{(12.813+13.977 j)} \\
& =7.84-8.553 j \\
& =11.603 \angle-47.49 \text { (A) }
\end{aligned}
$$

current density

$$
\begin{align*}
J & =\frac{11.603}{1.227 * 10^{6}}  \tag{31}\\
& =9.455 * 10^{-6}\left(\mathrm{~A} / \mathrm{m}^{2}\right)
\end{align*}
$$

(viii) EMF induced in primary

$$
\begin{align*}
E_{1} & =220-11.603 *(|2.514+5.142 j|) \\
& =220-11.603 * 5.723  \tag{32}\\
& =153.597(\mathrm{~V})
\end{align*}
$$

(ix) magnetizing current

$$
\begin{align*}
I_{\mu} & =\frac{153.597}{42.846}  \tag{33}\\
& =3.585(\mathrm{~A})
\end{align*}
$$

iron loss current

$$
\begin{align*}
I_{F E} & =\frac{153.597}{6.153 * 10^{2}}  \tag{34}\\
& =0.025(A)
\end{align*}
$$

no load current

$$
\begin{equation*}
I_{0}=0.025-3.585 j(A) \tag{35}
\end{equation*}
$$

secondary current

$$
\begin{align*}
I_{2}^{\prime} & =(7.84-8.553 j)-(0.025-3.585 j)  \tag{36}\\
& =7.8 i 6-4.968 j(A)
\end{align*}
$$

(x) iron loss in the core

$$
\begin{align*}
P_{F e} & =0.025^{2} * 6.153 * 10^{3}  \tag{37}\\
& =3.834(\mathrm{~W})
\end{align*}
$$

copper loss in the primary

$$
\begin{align*}
P_{1 c u} & =11.603^{2} * 2.514  \tag{38}\\
& =338.394(\mathrm{~W})
\end{align*}
$$

total power transferred across the air gap from the secondary

$$
\begin{align*}
P_{2 c u} & =9.261^{2} * 15.002 * 1  \tag{39}\\
& =1.287 * 10^{3}(\text { W) }
\end{align*}
$$

secondary $I^{2} R$ losses

$$
\begin{align*}
P_{\text {elect }} & =9.26 I^{2} * 15.002  \tag{40}\\
& =1.287 * 10^{3}(\mathrm{~W})
\end{align*}
$$

Thrust

$$
\begin{align*}
\text { Thrust } & =\frac{1.287 \times 10^{3}}{2 * 50 * 0.048}  \tag{41}\\
& =268.039(\mathrm{~N})
\end{align*}
$$

(xi) total power loss

$$
\begin{align*}
P_{t} & =3.834+338.394+1.287 * 10^{3}  \tag{42}\\
& =1.629 * 10^{3}(\mathrm{~W})
\end{align*}
$$

(xii) power factor

$$
\begin{align*}
\cos \phi & =\cos \left[\left[-0.829 * \frac{180}{\pi}\right] * \frac{\pi}{180}\right]  \tag{43}\\
& =0.676
\end{align*}
$$

(xiii) input power

$$
\begin{align*}
P_{I} & =220 * 11.603 * 0.676  \tag{44}\\
& =1.725 * 10^{3}(\mathrm{~W})
\end{align*}
$$

(xvi) efficiency

$$
\begin{align*}
\eta & =1-\frac{1.629 * 10^{3}}{1.725 * 10^{3}}  \tag{45}\\
& =0.056
\end{align*}
$$

### 3.3 Elowchart





The computer program was written in Fortran and can be found in Appendix 1 . The computed results can be found in Appendix 2.
4.1 Input data

$$
\begin{aligned}
& A_{\mathrm{cu}}=1.2272 * 10^{-6}\left(\mathrm{~m}^{2}\right) \\
& \mathrm{B}_{\mathrm{g}}=0.4(\mathrm{~T}) \\
& b_{p}=0.032(\mathrm{~m}) \\
& \mathrm{d}=0.002(\mathrm{~m}) \\
& \mathrm{d}_{\mathrm{w}}=0.00125(\mathrm{~m}) \\
& \mathrm{E}_{1}=153.597 \text { (V) } \\
& \mathrm{f}=50(\mathrm{~Hz}) \\
& \mathrm{g}=0.002(\mathrm{~m}) \\
& \mathrm{h}=0.005(\mathrm{~m}) \\
& h_{p}=0.043(\mathrm{~m}) \\
& \mathrm{h}_{\mathrm{s}}=0.005(\mathrm{~m}) \\
& \mathrm{h}_{\mathrm{y}}=0.016(\mathrm{~m}) \\
& \mathrm{H}_{\mathrm{l}}=112(\mathrm{~A} / \mathrm{m}) \\
& \mathrm{H}_{\mathrm{s}}=6000(\mathrm{~A} / \mathrm{m}) \\
& \mathrm{k}_{\mathrm{c}}=1 \\
& k_{n}=1.1 \\
& K_{W}=1 \\
& 1=0.192(\mathrm{~m}) \\
& L_{i}=0.09(\mathrm{~m}) \\
& m_{1}=2 \\
& p=2 \\
& s \quad=1 \\
& t_{p}=0.0006(\mathrm{~m}) \\
& \mathrm{V}_{1}=220(\mathrm{~V})
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{w}_{\mathrm{s}} & =0.005(\mathrm{~m}) \\
\gamma_{\mathrm{cu}} & =8920\left(\mathrm{kp} / \mathrm{m}^{3}\right) \\
\gamma_{\mathrm{Fe}} & =7800\left(\mathrm{kp} / \mathrm{m}^{3}\right) \\
\Delta \mathrm{P}_{\mathrm{Fe}} & =0.6(\mathrm{~W} / \mathrm{kg}) \\
\mu_{\mathrm{O}} & =4 * \pi * 10^{-7}(\mathrm{H} / \mathrm{m}) \\
\mu_{\mathrm{rs}} & =200 \\
\mu^{\prime} & =1.23 \\
\mu^{\prime} \prime & =0.69 \\
\rho_{\mathrm{Fe}} & =7800\left(\mathrm{~kg} / \mathrm{m}^{3}\right) \\
\sigma_{\mathrm{Al}} & =36 * 10^{6}(\mathrm{~S} / \mathrm{m}) \\
\sigma_{\mathrm{Cu}} & =56 * 10^{6}(\mathrm{~S} / \mathrm{m}) \\
\sigma_{\mathrm{Fe}} & =5 * 10^{6}(\mathrm{~S} / \mathrm{m}) \\
\sigma_{1} & =1.15 \\
\tau & =0.048(\mathrm{~m}) \\
\tau_{\mathrm{Al}} & =245\left({ }^{\circ} \mathrm{C}\right) \\
\tau_{\mathrm{Cu}} & =235\left({ }^{\circ} \mathrm{C}\right) \\
\tau_{\mathrm{Fesh}} & =202\left({ }^{\circ} \mathrm{C}\right) \\
\tau_{\mathrm{Feso}} & =230\left({ }^{\circ} \mathrm{C}\right) \\
\omega & =2 * \pi * \mathrm{f}
\end{aligned}
$$

Results for the slip between 1 and 0 are given in
tabular form below. They were calculated with the simulating program found in Appendix 1.

Table 1 Results from simulating program in Appendix 1

| Slip | $\cos \phi$ | n | $I_{\mu}$ <br> (A) | $\begin{gathered} J \\ \left(A / m^{2}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1.00 | 0.676 | 0.056 | 3.584 | $9.45 e+06$ |
| 0.95 | 0.665 | 0.091 | 3.619 | $9.24 e+06$ |
| 0.90 | 0.654 | 0.127 | 3.653 | $9.03 e+06$ |
| 0.85 | 0.641 | 0.162 | 3.687 | $8.82 e+06$ |
| 0.80 | 0.627 | 0.198 | 3.721 | $8.62 e+06$ |
| 0.75 | 0.612 | 0.233 | 3.754 | $8.42 e+06$ |
| 0.70 | 0.595 | 0.268 | 3.786 | $8.22 e+06$ |
| 0.65 | 0.576 | 0.303 | 3.817 | $8.03 \mathrm{e}+06$ |
| 0.60 | 0.555 | 0.337 | 3.848 | $7.84 \mathrm{e}+06$ |
| 0.55 | 0.532 | 0.371 | 3.877 | $7.66 e+06$ |
| 0.50 | 0.507 | 0.403 | 3.905 | $7.49 \mathrm{e}+06$ |
| 0.45 | 0.480 | 0.435 | 3.932 | $7.33 e+06$ |
| 0.40 | 0.449 | 0.464 | 3.957 | $7.18 e+06$ |
| 0.35 | 0.416 | 0.491 | 3.980 | $7.04 e+06$ |
| 0.30 | 0.381 | 0.514 | 4.001 | $6.91 e+06$ |
| 0.25 | 0.342 | 0.531 | 4.020 | $6.79 \mathrm{e} \div 06$ |
| 0.20 | 0.300 | 0.538 | 4.036 | $6.70 e+06$ |
| 0.15 | 0.255 | 0.528 | 4.050 | $6.61 e+06$ |
| 0.10 | 0.206 | 0.486 | 4.060 | $6.55 \mathrm{e} \div 06$ |

Table 2 Results from simulating program in Appendix 1

| Slip | $\mathrm{P}_{1}$ <br> (W) | $\mathrm{P}_{\text {In }}$ <br> (W) | $\begin{aligned} & \text { Pelect } \\ & (W) \end{aligned}$ | Thrust <br> (N) |
| :---: | :---: | :---: | :---: | :---: |
| 1.00 | 1628.185 | 1724.30 | 1286.21 | 267.96 |
| 0.95 | 1508.547 | 1660.17 | 1243.65 | 259.09 |
| 0.90 | 1392.372 | 1594.79 | 1199.72 | 249.94 |
| 0.85 | 1279.859 | 1528.16 | 1154.39 | 240.50 |
| 0.80 | 1171.21 | 1460.27 | 1107.59 | 230.75 |
| 0.75 | 1066.628 | 1391.11 | 1059.27 | 220.68 |
| 0.70 | 966.3248 | 1320.69 | 1009.37 | 210.29 |
| 0.65 | 870.5121 | 1248.98 | 957.79 | 199.54 |
| 0.60 | 779.4088 | 1175.99 | 904.43 | 188.42 |
| 0.55 | 693.2388 | 1101.70 | 849.14 | 176.90 |
| 0.50 | 612.2343 | 1026.09 | 791.75 | 164.95 |
| 0.45 | 536.6393 | 949.13 | 732.03 | 152.51 |
| 0.40 | 466.7172 | 870.79 | 669.67 | 139.51 |
| 0.35 | 402.7597 | 791.00 | 604.28 | 125.89 |
| 0.30 | 345.1045 | 709.67 | 535.37 | 111.53 |
| 0.25 | 294.1567 | 626.65 | 462.31 | 96.31 |
| 0.20 | 250.4183 | 541.70 | 384.37 | 80.08 |
| 0.15 | 214.5179 | 454.41 | 300.70 | 62.65 |
| 0.10 | 187.2295 | 364.02 | 210.34 | 43.82 |

### 4.3 Nameplate

Table 3 Typical nameplate values used in design

| Dimensions |  |  |
| :---: | :---: | :---: |
| length of core |  | 0.09 (m) |
| hight of core |  | 0.064 (m) |
| width of core |  | 0.192 (m) |
| weight of core |  | 4.26 (kg) |
| weight of aluminium and mild steel disk |  | 9.5 (kg) |
| diameter of aluminium and mild steel disk |  | 0.52 (m) |
| thickness of aluminium cap |  | 3 (mm) |
| thickness of mild steel disk |  | 10 (mm) |
| diameter of secondary rotor |  | 0.52 (m) |
| cross sectional area of shaded pole ring |  | $\begin{aligned} & 0.0024 \\ & \left(\mathrm{~m}^{2}\right) \\ & \hline \end{aligned}$ |
| Electrical Information |  |  |
| number of phases |  | 1 |
| frequency |  | $50(\mathrm{~Hz})$ |
| voltage |  | 220 (V) |
| current |  | 11.603 (A) |
| resistance |  | 12.813 ( $\Omega$ ) |
| speed |  | 84 (rpm) |
| Motor Information |  |  |
| stator motor | single-phase shaded-pole |  |
| stator core | laminated (H18, 0.5 mm , Non orientated Silicon steel) |  |
| rotor disk | aluminium cap laminated to mild steel disk |  |
| slot insulation | DMD-Mitron (6510) |  |
| shaded pole | copper soldered wi solder | silver |

### 4.4 Characteristic curves

The followig graphs are based on the tabulated values in section 4.2.


Figure 4 Efficiency versus slip


Figure 5 Magnetizing current versus slip


Figure 6 Power factor versus slip


Figure 7 Current density versus slip


Figure 8 Thrust versus slip


Figure 9 Power versus slip

### 4.5 Fiqures



Figure 10 Sectional view of the single-phase shadedpole linear induction motor.


Figure 11 Constuction of stator core, aluminium cap and mild steel rotor disk


Figure 12 Aluminium and mild steel disk forming secondary rotor


Figure 13 Sectional view of the shaft. The shaft was constructed in this specific way, so as to withstand the attraction force exerted onto the aluminium and mild steel disk by the stator core.

To determine the characteristic curves of the single-phase single-pole LIM, a no-load, a short-circuit and a torque test was done. The inertia of the disk was found by applying a test normally used for flywheels.

### 5.1 No-load test

The power factor was calculated from readings of the input power and the input current as the supply of the motor was increased in steps.
5.1.1 Measured results

Table 4 No-load test

| $V_{1}$ <br> $(\mathrm{~V})$ | $I_{1}$ <br> $(\mathrm{~A})$ | $P_{I}$ <br> $(\mathrm{~W})$ | POWER <br> FACTOR |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 |  |
| 10 | 0.56 | 5.8 | 1.00 |
| 20 | 1.1 | 16 | 0.73 |
| 30 | 1.62 | 32 | 0.66 |
| 40 | 2.18 | 54 | 0.62 |
| 50 | 2.79 | 78 | 0.56 |
| 60 | 3.33 | 105 | 0.53 |
| 70 | 3.8 | 140 | 0.53 |
| 80 | 4.45 | 180 | 0.51 |
| 90 | 4.9 | 225 | 0.51 |
| 100 | 5.46 | 270 | 0.49 |
| 110 | 5.9 | 320 | 0.49 |
| 120 | 6.52 | 380 | 0.49 |


| 130 | 6.9 | 450 | 0.50 |
| :---: | :---: | :---: | :---: |
| 140 | 7.4 | 520 | 0.50 |
| 150 | 8 | 600 | 0.50 |
| 160 | 8.5 | 660 | 0.49 |
| 170 | 9 | 760 | 0.50 |
| 180 | 9.5 | 840 | 0.49 |
| 190 | 10 | 920 | 0.48 |
| 200 | 10.25 | 1020 | 0.50 |
| 210 | 10.68 | 1100 | 0.49 |
| 220 | 11.06 | 1200 | 0.49 |

### 5.1.2 Characteristic curves

The following graphs show clearly that a supply of 50 V is needed to overcome windage and friction losses.


Figure 14 Power factor versus input voltage


Figure 15 Input current versus input voltage


Figure 16 Input power versus input voltage

An adjustable scale was connected to the rotor disk (figure 17). Each reading was taken with the scale perpendicular to the center of the disk. As the supply voltage was raised in small increments, readings of the input power, input current and of the scale were taken.


Figure 17 Torque measurement for blocked rotor

### 5.2.1 Input data

$$
\begin{aligned}
& \mathrm{g}=9.81\left(\mathrm{~m} / \mathrm{s}^{2}\right) \\
& \mathrm{I}_{1}=11.9(\mathrm{~A}) \\
& \mathfrak{m}=1.24(\mathrm{~kg}) \\
& \mathrm{P}_{\mathrm{I}}=1200(\mathrm{~W}) \\
& \mathrm{I}=0.25(\mathrm{~m}) \\
& \mathrm{V}_{1}=220(\mathrm{~V})
\end{aligned}
$$

### 5.2.2 Calculations

## Force

$$
\begin{aligned}
F & =m g \\
& =1.24 * 9.81 \\
& =12.16(\mathrm{~N})
\end{aligned}
$$

Torque

$$
\begin{aligned}
T & =F r \\
& =12.16 * 0.25 \\
& =3.0411(\mathrm{Nm})
\end{aligned}
$$

Power Factor

$$
\begin{align*}
\cos \dot{\varphi} & =\frac{P_{I}}{V_{1} I_{1}} \\
& =\frac{1200}{220 * 11.9}  \tag{3}\\
& =0.485
\end{align*}
$$

Table 5 Short-circuit test

| $\begin{aligned} & \mathrm{V}_{1} \\ & (\mathrm{~V}) \end{aligned}$ | $\begin{gathered} I_{1} \\ (\mathrm{~A}) \end{gathered}$ | $\begin{aligned} & P_{I} \\ & (\mathrm{~W}) \end{aligned}$ | $\begin{gathered} \mathrm{m} \\ (\mathrm{~kg}) \end{gathered}$ | Power <br> Factor | $\begin{gathered} \mathrm{T} \\ (\mathrm{Nm}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 |  |  |  |
| 10 | 0.57 | 3 |  | 0.526 |  |
| 20 | 1.11 | 7.2 |  | 0.324 |  |
| 30 | 1.64 | 24.5 |  | 0.498 |  |
| 40 | 2.18 | 42 |  | 0.482 |  |
| 50 | 2.79 | 67 | 0.010 | 0.480 | 0.025 |
| 60 | 3.33 | 100 | 0.050 | 0.501 | 0.123 |
| 70 | 3.88 | 135 | 0.102 | 0.497 | 0.250 |
| 80 | 4.5 | 185 | 0.145 | 0.514 | 0.356 |
| 90 | 4.92 | 230 | 0.195 | 0.519 | 0.478 |
| 100 | 5.42 | 280 | 0.255 | 0.517 | 0.625 |
| 110 | 5.94 | 345 | 0.320 | 0.528 | 0.785 |
| 120 | 6.47 | 400 | 0.385 | 0.515 | 0.944 |
| 130 | 7 | 460 | 0.470 | 0.505 | 1.153 |
| 140 | 7.52 | 520 | 0.545 | 0.494 | 1.337 |
| 150 | 8.01 | 600 | 0.620 | 0.499 | 1.521 |
| 160 | 8.51 | 660 | 0.700 | 0.485 | 1.717 |
| 170 | 9.09 | 750 | 0.790 | 0.485 | 1.937 |
| 180 | 9.62 | 840 | 0.880 | 0.485 | 2.158 |
| 190 | 10.1 | 920 | 0.960 | 0.479 | 2.354 |
| 200 | 10.7 | 1020 | 1.050 | 0.477 | 2.575 |
| 210 | 11.4 | 1100 | 1.150 | 0.459 | 2.820 |
| 220 | 11.9 | 1200 | 1.240 | 0.458 | 3.041 |

### 5.2.4 Characteristic curves



Figure 18 Power factor versus input voltage


Figure 19 Input current versus input voltage


Figure 20 Input power versus input current


Figure 21 Torque versus input power

A rope was wound around a part of the shaft and connected perpendicularly to the axis of the shaft and to the scale. The other side of the rope was connected to a known weight. The weight was situated directly beneath the scale (Figure 22).


Figure 22 Load test

### 5.3.1 Input Data

$$
\begin{array}{ll}
\mathrm{I}_{1} & =8.009(\mathrm{~A}) \\
\mathrm{P}_{\mathrm{I}} & =598.75(\mathrm{~W}) \\
\mathrm{N} & =25(\mathrm{rpm}) \\
\mathrm{V}_{1} & =150(\mathrm{~V}) \\
\mathrm{W}_{\mathrm{s}} & =3.8(\mathrm{~kg}) \\
\mathrm{W}_{\mathrm{g}} & =1.5(\mathrm{~kg})
\end{array}
$$

### 5.3.2 Calculations

Force used for braking the disk

$$
\begin{aligned}
F_{b} & =\left(W_{s}-W_{g}\right) g \\
& =(3.8-1.5) * 9.81 \\
& =22.56(N)
\end{aligned}
$$

Torque due to braking force

$$
\begin{aligned}
T & =F_{b} r_{b} \\
& =22.56 * 0.0425 \\
& =0.959(\mathrm{Nm})
\end{aligned}
$$

Output power

$$
\begin{align*}
P_{I} & =\frac{2 \pi N T}{60} \\
& =\frac{2 * \pi * 25 * 0.959}{60}  \tag{3}\\
& =2.5 I(W)
\end{align*}
$$

efficiency

$$
\begin{aligned}
\eta & =\frac{P_{L}}{P_{I}} \\
& =\frac{2.51}{598.75} \\
& =0.0042
\end{aligned}
$$

power factor

$$
\begin{aligned}
\cos \phi & =\frac{P_{I}}{V_{1} I_{1}} \\
& =\frac{598.75}{150 * 8.009} \\
& =0.498
\end{aligned}
$$

### 5.3.3 Results

Table 6 Load test

| T <br> $(\mathrm{Nm})$ | Speed <br> $(\mathrm{rpm})$ | $\mathrm{P}_{\mathrm{L}}$ <br> $(\mathrm{W})$ | $\mathrm{P}_{\mathrm{I}}$ <br> $(\mathrm{W})$ | $\mathrm{I}_{1}$ <br> $(\mathrm{~A})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.000 | 90 | 0.00 | 580 | 8.000 |
| 0.117 | 84.9 | 1.04 | 581.25 | 8.001 |
| 0.183 | 80.5 | 1.55 | 582.5 | 8.001 |
| 0.233 | 76.5 | 1.87 | 583.75 | 8.002 |
| 0.296 | 74 | 2.29 | 585 | 8.003 |
| 0.342 | 70 | 2.51 | 586.25 | 8.003 |
| 0.467 | 64 | 3.13 | 587.5 | 8.004 |
| 0.542 | 59 | 3.35 | 588.75 | 8.004 |
| 0.592 | 54.5 | 3.38 | 590 | 8.005 |
| 0.650 | 51 | 3.47 | 591.25 | 8.006 |


| 0.688 | 48 | 3.46 | 592.5 | 8.006 |
| :---: | :---: | :---: | :---: | :---: |
| 0.750 | 43 | 3.38 | 593.75 | 8.007 |
| 0.834 | 39.5 | 3.45 | 595 | 8.008 |
| 0.876 | 35 | 3.21 | 596.25 | 8.008 |
| 0.934 | 30 | 2.93 | 597.5 | 8.009 |
| 0.959 | 25 | 2.51 | 598.75 | 8.009 |

Table 7
Load test

| $\begin{gathered} \mathrm{T} \\ (\mathrm{Nm}) \end{gathered}$ | Speed <br> (rpm) | Efficiency | Power factor |
| :---: | :---: | :---: | :---: |
| 0.000 | 90 | 0.0000 | 0.483 |
| 0.117 | 84.9 | 0.0018 | 0.484 |
| 0.183 | 80.5 | 0.0027 | 0.485 |
| 0.233 | 76.5 | 0.0032 | 0.486 |
| 0.296 | 74 | 0.0039 | 0.487 |
| 0.342 | 70 | 0.0043 | 0.488 |
| 0.467 | 64 | 0.0053 | 0.489 |
| 0.542 | 59 | 0.0057 | 0.490 |
| 0.592 | 54.5 | 0.0057 | 0.491 |
| 0.650 | 51 | 0.0059 | 0.492 |
| 0.688 | 48 | 0.0058 | 0.493 |
| 0.750 | 43 | 0.0057 | 0.494 |
| 0.834 | 39.5 | 0.0058 | 0.495 |
| 0.876 | 35 | 0.0054 | 0.496 |
| 0.934 | 30 | 0.0049 | 0.497 |
| 0.959 | 25 | 0.0042 | 0.498 |

### 5.3.4 Characteristic curves



Figure 23 Efficiency versus speed


Figure 24 Power factor versus speed


Figure 25 Input power versus speed


Figure 26 output power versus speed


Figure 27 Input current versus speed


Figure 28 Torque versus speed


Figure 29 Efficiency versus torque


Figure 30 Power factor versus torque


Figure 31 Input power versus torque


Figure 32 output power versus torque


Figure 33 Input current versus torque

The windage and frictional resistance was found by winding a rope around the shaft and attaching weights of various masses to the loose end. The force of these resistances is given by the (mass * g) that only sets the shaft in motion and does not accelerate it.

The inertia was found by adding a greater known mass to the rope and timing the fall through a known vertical distance with a stopwatch. This experiment was done with various weights to compare the results.


Figure 34 Diagram for the Inertia test.

### 5.4.1 Input data

$$
\begin{aligned}
& \mathrm{d}_{\mathrm{a}}=0.042(\mathrm{~m}) \\
& \mathrm{m}_{\mathrm{t}}=9.5(\mathrm{~kg}) \\
& \mathrm{m}_{\mathrm{s}}=0.2(\mathrm{~kg}) \\
& \mathrm{m}_{\mathrm{I}}=0.6(\mathrm{~kg}) \\
& \mathrm{r}
\end{aligned}
$$

### 5.4.2 Calculations

(i) acceleration of the 0.6 kg mass

$$
\begin{equation*}
S_{d}=u t+0.5 a t^{2} \tag{1}
\end{equation*}
$$

therefore

$$
\begin{aligned}
a & =\frac{2 *(0.542-0 * 10.82)}{10.82^{2}} \\
& =9.259 * 10^{-3}(\mathrm{~m} / \mathrm{s})
\end{aligned}
$$

(ii) acceleration of the secondary disk

$$
\begin{equation*}
\alpha=\frac{a}{r} \tag{2}
\end{equation*}
$$

$$
\begin{aligned}
\alpha & =\frac{9.259 * 10^{-3}}{0.042} \\
& =0.22\left(\mathrm{rad} / \mathrm{s}^{2}\right)
\end{aligned}
$$

(iii) friction and windage resistance force

$$
\begin{equation*}
F_{I}=m_{2} g \tag{3}
\end{equation*}
$$

$$
\begin{aligned}
F_{r} & =0.25 * 9.81 \\
& =2.453(\mathrm{~N})
\end{aligned}
$$

(iv) Resultant force due to the motion and the 0.6 kg mass

$$
\begin{equation*}
F=m_{3}(g-a) \tag{4}
\end{equation*}
$$

$$
\begin{aligned}
F & =0.6 *\left(9.8 I-9.259 * 10^{-3}\right) \\
& =5.88(\mathrm{~N})
\end{aligned}
$$

(v) force in the cord to accelerate the disk

$$
\begin{equation*}
F_{a}=F-F_{I} \tag{5}
\end{equation*}
$$

$$
\begin{aligned}
F_{a} & =5.88-2.453 \\
& =3.428(\mathrm{~N})
\end{aligned}
$$

(vi) accelerating torque

$$
\begin{equation*}
T_{a}=F_{a} r \tag{6}
\end{equation*}
$$

$$
\begin{aligned}
T_{a} & =3.428 * 0.042 \\
& =0.144(\mathrm{Nm})
\end{aligned}
$$

(vii) Moment of Inertia

$$
\begin{equation*}
I=\frac{T}{\boldsymbol{\alpha}} \tag{7}
\end{equation*}
$$

$$
\begin{aligned}
I & =\frac{3.428}{0.22} \\
& =0.653\left(\mathrm{~kg} / \mathrm{m}^{2}\right)
\end{aligned}
$$

(viii) Radius of gyration

$$
\begin{equation*}
r_{g}=\sqrt{\frac{I}{m_{1}}} \tag{8}
\end{equation*}
$$

$$
\begin{aligned}
I_{g} & =\sqrt{\frac{0.653}{9.5}} \\
& =0.262(\mathrm{~m})
\end{aligned}
$$

### 5.4.3 calculate run-out-time

(i) final angular velocity

$$
\begin{equation*}
\omega_{1}=\frac{2 \pi N}{60} \tag{9}
\end{equation*}
$$

$$
\begin{aligned}
\omega_{1} & =\frac{2 * \pi * 84}{60} \\
& =8.692(\mathrm{rad} / \mathrm{s})
\end{aligned}
$$

(ii) Torque due to friction

$$
\begin{equation*}
T_{T}=m_{2} g r \tag{10}
\end{equation*}
$$

$$
\begin{aligned}
T_{r} & =0.25 * 9.81 * 0.042 \\
& =0.103(\mathrm{Nm})
\end{aligned}
$$

(iii) angular velocity

$$
\begin{equation*}
\alpha=\frac{T_{r}}{I} \tag{11}
\end{equation*}
$$

$$
\begin{aligned}
\alpha & =\frac{0.103}{0.653} \\
& =0.158\left(\mathrm{rad} / \mathrm{s}^{2}\right)
\end{aligned}
$$

(iv) time for the disk to come to a stand still

$$
\begin{equation*}
t=\frac{\omega_{I}}{\alpha} \tag{12}
\end{equation*}
$$

$$
\begin{aligned}
t & =\frac{8.692}{0.158} \\
& =55.77 \quad(\mathrm{~s})
\end{aligned}
$$

### 5.4.4 Measured run-out time

The motor was run up to rated speed, after which the input power was cut and the speed measured every 5 seconds.

### 5.4.4.1 Measured results

Table 8 Run-out-time

| Time <br> (s) | Speed <br> (RPM) | Velocity <br> (m/s) |
| :---: | :---: | :--- |
| 0 | 84 | 2.3750 |
| 5 | 77 | 2.1771 |
| 10 | 69 | 1.9509 |
| 15 | 62 | 1.7530 |
| 20 | 54 | 1.5268 |
| 25 | 47 | 1.3288 |
| 30 | 40 | 1.1309 |
| 35 | 33 | 0.9330 |
| 40 | 25 | 0.7068 |
| 45 | 18 | 0.5089 |
| 50 | 12 | 0.3392 |
| 55 | 6 | 0.1696 |
| 60 | 0 | 0 |



Figure 35 Speed and velocity versus time

In this thesis the characteristics of the shaded-pole induction motor and the 3 phase single-sided linear induction motor were used to design the shaded-pole flat linear induction motor.

Since the design was limited to materials that were available locally, compromises could not be avoided. For example when a standard transformer lamination is chosen, most dimensions are fixed, including the pole pitch and the winding space.

Due to the fact that an iterative method was used to find the induced EMF and to alleviate the decision making process, the CAD approach was of great help.

The characteristic curves show clearly that the disadvantages of this motor are a low power factor, a small starting torque and a low efficiency. This is due to a large and open air gap and the fact that the shaded poles do not produce a 90 degree phase shift.

It was found that the distance from the core to the centre of the secondary disk plays an important role in the performance of the motor. The braking forces induced by the core onto the aluminium cap reduce as the distance between the two is increased.

Although efficiency, power factor and starting torque of this motor are worse then for normal induction motors, I think that this design holds a lot of potential where only a single phase supply is available and the advantages of a single-sided induction motor are needed.

## LIST OF REFERENCES

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2. Jacek F. Gieras, Design of Single-Sided Linear Induction Motors for propulsion of Wheel-On-Rail Vehicles, Dept. of Electrical \& Electronic Engineering, University of Cape Town
3. Davy Linear Motors Limited, Equipment Specifications
4. Jacek F. Gieras, Graham E. Dawson and Anthony R. Eastham, Performance Calculation for Single-Sided Linear Induction motors with a Double-Layer Reaction Rail Under Constant Current Excitation, IEEE Transactions on Magnetics, voll. mag 22, No 1, January 1986
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## APPENDIX 1

C LINEAR MOTOR
REAL I, TAU, CFE,Li,ws,hy,hs,bp,Bg,ol,ocu,Acu, lav, RI REAL VFE, V1, E1,N1,Kw,f,p,Np,ktr,mi,dw,hp,ll,tp,tcu REAL $12, \mathrm{mFE}, \mathrm{PFE}, \mathrm{Xm}, \mathrm{kn}, \mathrm{kc}, \mathrm{g}, \mathrm{X} 1, \mathrm{BETA}, \mathrm{PI}, \mathrm{hov}, \mathrm{w}, \mathrm{krn}$ REAL OAL,tAL, s,wav, FLUXd, PERrs, PER1,PER2,oFE,kz,e REAL c,h,RFE, uPFE,E1new,J1T,R,nl,nTAU,nhy,nhs,nws REAL nbp, nhp,ALi,AACu,Adw,Iu,IFe,P2cu,Pelect REAL Thrust, PL, cosA, eff, Pin, PERo,P1cu

COMPLEX xAL,j, ZAL, a,b,PERFE, XFE, ZFE,Zt,Z2,I1,Z1
COMPLEX Io,I2
INTEGER COUNT,LOOP, CHANGE,ANSWER,VALUE,SWITCH,SLIP INTEGER ANDER,NICHT,NOCH,CHOICE, ANTWORD, SCHALTER

INTEGER WERT,SKAKEL
OPEN (1, STATUS ='SCRATCH')
OPEN (3,STATUS='NEW')
OPEN (4,STATUS='NEW')
OPEN (7,STATUS='NEW')
OPEN ( $8, \operatorname{STATUS}=$ 'NEW')
ANDER=0
$\mathrm{NICHT}=0$
$\mathrm{NOCH}=0$
SKAKEL=0
SWITCH $=0$
SCHALTER=0
WERT=0
$\operatorname{CoUNT}=0$

LOOP $=0$
$C H A N G E=0$
$E 1=170.00$
20 IF (ANDER.EQ.0) $L i=0.09$
30 IF (CHANGE.EQ.0) THEN

```
                                    ACU=0.0000012272
                                    dw=0.00125
```

ENDIF
IF (SWITCH.EQ.1) GOTO 70
$40 \quad 1=0.192$
$T A U=0.048$
$W S=0.005$
$h y=0.016$
$h s=0.005$
$h p=0.043$
$\mathrm{bp}=0.032$
IF (SKAKEL.EQ.I) GOTO 70
IF (NOCH.EQ.1) GOTO 70
PRINT*, 'DO YOU WANT TO KEEP CORE DIMENSIONS
$[Y E S=1 / N O=2]^{\prime}$
READ*, DYW

IF (DYW.EQ.1.0) THEN $\mathrm{NOCH}=1$ GOTO 70

ENDIF
IF (DYW.EQ.2) THEN
PRINT*,'THE CURRENT DIMENSIONS ARE GIVEN IN THE FOLLOWING ORDER: "

```
    PRINT*,' 1, TAU, hy, hs, hp, bp, ws'
    PRINT*,l,TAU,hy,hs,hp,bp,ws
    PRINT*,'PLEASE ENTER NEW DIMENSIONS, KEEPING THE
        SAME ORDER AND SEPERATED BY A COMMA'
    READ*,l,TAU,hy,hs,hp,bp,ws
    WRITE (7,*) l,TAU,hy,hs,hp,bp,ws
    SWITCH=1
    ENDIF
CONTINUE
IF (SWITCH.EQ.1) THEN
    REWIND 7
    READ (7,*) l,TAU,hy,hs,hp,bp,ws
ENDIF
IF (ANDER.EQ.1) THEN
    REWIND 3
    READ (3,*) Li
ENDIF
IF (CHANGE.EQ.1) THEN
    REWIND 4
    READ (4,*) Acu,dw
ENDIF
IF (SCHALTER.EQ.1) THEN
    REWIND 8
    READ (8,*) Bg,upFE
    ENDIF
IF (SCHALTER.EQ.I) GOTO 78
Bg=0.4
```

UPFE=0.6
IF (WERT.EQ.1) GOTO 78
PRINT*,'AT THE MOMENT Bg = ',Bg,'
AND $\mathrm{W} / \mathrm{Kg}=$ ', uPFE
PRINT*,'DO YOU WANT TO CHANGE Bg AND W/Kg
$[\mathrm{YES}=1 / \mathrm{NO}=2]^{\prime}$
READ*, ANTWORD
IF (ANTWORD.EQ.1) THEN
PRINT*,'ENTER NEW VALUE FOR Bg AND $\mathrm{W} / \mathrm{Kg}{ }^{\prime}$
READ*, Bg, uPFE
WRITE (8,*) Bg,uPFE
SCHALTER=1
GOTO 20
ELSE IF (ANTWORD.EQ.2) THEN
WERT=1
GOTO 78
ENDIF
CONTINUE
ol=1.15
$\mathrm{V} 1=220.0$
$s=1.0$
$\mathrm{CFE}=7800.0$
$K w=1.0$
$\mathrm{f}=50.0$
$\mathrm{p}=2.0$
$\mathrm{m} 1=2.0$
$t p=0.0006$
ocu $=56000000.0$

```
    tcu=235.0
    kc=1.0
    kn=1.1
    g=0.002
    PIE=3.141593
    OAL=36000000.0
    tAL=245.0
    wav=2*PIE*f
    PERO=4.0E-7*PIE
    j=CSQRT(CMPLX(-1))
    d=0.002
    PERrs=200.0
    PER1=1.23
    PER2=0.69
    OFE=5.0E6
    h=0.005
C FORMULA ORIENTATED
VFE=Li*(1*hy+4*TAU*(TAU-bp) -3*hs*Ws)
mFE=VFE*CFE
PFE=UPFE*mFE*1.5
FLUX=bp*Li*Bg*ol
N1=EI/(4.44*Kw*f*FLUX)
Np=N1/(2*p)
ktr=(2*ml*(NI*Kw)**2)/p
II=(Np*dw/hp)* (dw+tp)
lav=2*(Li+bp+(2*l1))
12=(TAU-bp)/2
80 IF (II.GE.12) THEN
```

PRINT*,'THERE IS NOT ENOUGH STAKINGSPACE' PRINT*,'AND $11=1,11, '$ AND $12=1,12$ PRINT*,'AT THIS MOMENT Li = ',Li,' AND $A C u=1, A C u$

PRINT*,'YOU ARE ALLOWED TO CHANGE ONLY Li OR Acu'

PRINT*,'WHICH ONE DO YOU WANT TO CHANGE?' PRINT*,'ENTER 1 FOR Li OR 2 FOR ACu' READ*, CHOICE

CLOSE (3)
PRINT*,'ENTER NEW VALUE FOR Li'
READ*, Li
OPEN (3)
WRITE $(3, *) L i$
ANDER=1
SKAKEL=1
GOTO 20
ELSE IF (CHOICE.EQ.2) THEN
CLOSE (4)
PRINT*,' ENTER NEW VALUE FOR Acu
AND FOR $d w^{\prime}$
READ*,ACu, dw
OPEN (4)
WRITE (4,*) ACu, dw
CHANGE=1
SKAKEL=1
GOTO 20

ENDIF
110 ELSE IF (11.LT.12) THEN
IF (NICHT.EQ.I) GOTO 150
PRINT*,'AT THIS MOMENT Li = ', Li,' AND $A C L=1, A c u$

PRINT*,'AND $11=1,11, ' \operatorname{AND~} 12=1,12$

PRINT*,'DO YOU WANT TO CHANGE THE VALUES? $[\mathrm{YES}=1 / \mathrm{NO}=2]^{\prime}$

READ*, VALUE

120

IF (VALUE.EQ.1) THEN PRINT*,'YOU ARE ALLOWED TO CHANGE ONLY Li OR ACu' PRINT*,'WHICH ONE DO YOU WANT TO CHANGE?' PRINT*,'ENTER 1 FOR Li OR 2 FOR Acu' READ*,ANSWER

IF (ANSWER.EQ.1) THEN CLOSE (3) PRINT*,'ENTER NEW VALUE FOR Li' READ*, Li OPEN (3) WRITE $(3, *) \mathrm{Li}$ ANDER=1

SKAKEL=1
GOTO 20
ELSE IF (ANSWER.EQ.2) THEN
CLOSE (4)
PRINT*,'ENTER NEW VALUE FOR Acu

AND FOR $d w^{\prime}$
READ*,Acu,dw
OPEN (4)
WRITE $(4, *)$ Acu,dw
$\mathrm{CHANGE}=1$
SKAKEL=1
GOTO 20
ENDIF
ELSE IF (VALUE.EQ.2) THEN
$\mathrm{NICHT}=1$
GOTO 150
ENDIF
ENDIF
CONTINUE
$R I=(1 a v * N 1) /(o c u * A C u)$
$R 1=R 1 *(t c u+75.0) /(t c u+20.0)$
Xm=12.8*f*p*(Np*Kw)**2*TAU*Li/(kn*kc*g*1000000.0)
X1=0.8*(ol-1.0)*Xm
$\mathrm{BETA}=\mathrm{PIE} / \mathrm{TAU}$
hov $=\mathrm{TAU} / 2$
$\mathrm{w}=\mathrm{TAU}+\mathrm{Li}$
$\mathrm{krn}=1-(($ TANH $(\operatorname{BETA} * \mathrm{w} / 2)) /$
1((BETA*W/2)*(1+TANH(BETA*W/2)*TANH(BETA*hov))))

- AL=OAL*krn* ((tAL+20)/(tAL+75))
$x A L=C S Q R T((s * j * w a v * P E R o * O A L)+((P I E / T A U) *(P I E / T A U)))$
$\mathrm{b}=\mathrm{xAL} * \mathrm{~d}$
$a=(\operatorname{CEXP}(b)-\operatorname{CEXP}((-1) * b)) /$
$1(\operatorname{CEXP}(b)+\operatorname{CEXP}((-1) * b))$

```
    ZAL=(ktr*j*S*Wav*PERo*Li)/(XAL*TAU*a)
    PERFE=PERo*PERrS*(PER1-PER2*j)
    xFE=CSQRT ((s*j*Wav*PERFE*oFE)+
    1/((PIE/TAU)*(PIE/TAU)))
    kz=1-(g/Li)+((2*TAU)/(PIE*W))*(1-
    1/(EXP(((-1)*PIE*W)/(2*Li))))
        e=xFE*h
        c=(\operatorname{CEXP}(e)-\operatorname{CEXP}((-1)*e))/
        1(\operatorname{CEXP}(e)+\operatorname{CEXP}((-1)*e))
    ZFE=(ktr*j*s*Wav*PERFE*Li*kz)/(xFE*TAU*C)
    Z2=((ZAL*ZFE)/(ZAL+ZFE))*(1/s)
    RFE=(E1*EI)/PFE
    Zt=(RI+(j*X1))+((RFE*((j*Xm*Z2)/((j*Xm)+Z2)))
    1/(RFE+((j*Xm*Z2)/((j*Xm)+Z2))))
    II=V1/Zt
    J1=CABS (I1)/ACu
    R=CABS (I1)
    ZI=RI+(j*XI)
    T=CABS (Z1)
    E1new=V1-(R*T)
    PRINT*,'WHEN S=1, E1new = ',Elnew
160 IF (EInEW.NE.E1) THEN
        El=Elnew
        COUNT=COUNT+1
        GOTO 20
    ENDIF
CONTINUE
PRINT*,'
```

```
    PRINT*,'ENTER A VALUE FOR S [0<S<1]'
    READ*,S
190 IF (S.NE.1.0) THEN
195 XAL=CSQRT((s*j*Wav*PERo*OAL) +
```

1

1

1

1

1

1

```
    ((PIE/TAU)*(PIE/TAU)))
    b=xAL*d
    a=(\operatorname{CEXP}(b)-\operatorname{CEXP}((-1)*b))/
    (CEXP(b)+\operatorname{CEXP}((-1)*b))
    ZAL=(ktr*j*s*wav*PERO*Li)/(xAL*TAU*a)
    PERFE=PERo*PERrs*(PER1-PER2*j)
    XFE=CSQRT ((S*j*Wav*PERFE*OFE) +
        ((PIE/TAU)*(PIE/TAU)))
    kz=1-(g/Li)+((2*TAU)/(PIE*W))*
    (1-(EXP(((-1)*PIE*W)/(2*Li))))
    e=xFE*h
    c=(\operatorname{CEXP}(e)-\operatorname{CEXP}((-1)*e))/
        (CEXP (e) + CEXP ((-1)*e))
    ZFE=(ktr*j*s*Wav*PERFE*Li*kz)/(xFE*TAU*C)
    Z2=((ZAL*ZFE)}/(ZAL+ZFE))*(1/s
    RFE=(E1*E1)/PFE
    Zt=(R1+(j*X1)) +((RFE* ((j*Xm*Z2) /((j*Xm)+Z2)))
    /(RFE+((j*Xm*Z2)/((j*Xm)+Z2))))
    II =V1/2t
    JI=CABS(I1)/ACu
    R=CABS (II)
    Z1=R1+(j*X1)
    T=CABS (21)
    Elnew=V1-(R*T)
```

PRINT*,'THE NEW ITERATION FOR EInew $=$ ', E1new
IF (EInew.NE.E1) THEN
E1=E1new
GOTO 195
ELSE
GOTO 210
ENDIF
ELSE
GOTO 210
ENDIF

WRITE(*, 220) Elnew, COUNT
FORMAT (' EInew $=$ ',F10.6,' AFTER',I3,' INTERATIONS')

WRITE (*, 240)N1,Np
FORMAT(' NI = ',F4.0,' AND $N p=$ ',F4.0)
Iu $=$ EInew/Xm
IFe=EInew/RFe
Io $=I F e-(I u * j)$
$I 2=I 1-I o$
$\mathrm{PFE}=(\mathrm{IFe} * * 2) * \mathrm{RFE}$
Plcu=CABS (II*II)*RI
P2cu=CABS (I2*I2)*REAL (Z2)*S
Pelect $=$ CABS (I2*I2)*REAL(Z2)
Thrust=Pelect/(2*f*TAU)
$\mathrm{PL}=\mathrm{PFe}+\mathrm{P} 1 \mathrm{Cu}+\mathrm{P} 2 \mathrm{Cu}$
$\cos A=\operatorname{COS}((\operatorname{ATAN}(A I M A G(I 1) / \operatorname{REAL}(I 1)) *$
$1(180 /$ PIE $)) * P I E / 180)$
$\mathrm{Pin}=\mathrm{V} 1 * \mathrm{R} * \cos \mathrm{~A}$
eff=i-(PL/Pin)
PRINT*,' '
PRINT*,' '
PRINT*,' '
PRINT*,' II =', R,' Iu =',Iu
PRINT*,'Pelect =',Pelect,' PL =',PL,' Pin $=1$, Pin

PRINT*,' Thrust=', Thrust
PRINT*,' eff =',eff
PRINT*,' $\cos A=', \cos A$
PRINT*,' J =', JI
PRINT*,' '
PRINT*,' '
PRINT*,'DO YOU WANT TO CHANGE THE SLIP $[\mathrm{YES}=1 / \mathrm{NO}=2] \quad$ '

READ*,SLIP
IF (SLIP.EQ.1) THEN
GOTO 180
ELSE IF (SLIP.EQ.2) THEN
CLOSE(1)
CLOSE (3)
CLOSE(4)
CLOSE(7)
CLOSE (8)
ENDIF

## APPENDIX 2

## Sample run of program

```
A:\>LM
DO YOU WANT TO KEEP CORE DIMENSIONS [YES=1/NO=2]
I
AT THE MOMENT Bg = 4.0000E-01 AND W/Kg = 6.0000E-01
DO YOU WANT TO CHANGE Bg AND W/Kg [YES=1/NO=2]
2
AT THIS MOMENT Li = 9.0000E-02 AND ACu = 1.2272E-06
AND 11 = 7.7713E-03 AND 12 = 7.9999E-03
DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2]
2
ENTER A VALUE FOR S [0<S<1]
1
EInew = 153.6131 AFTER 5 INTERATIONS
N1 = 522. AND Np = 131.
I1 = 11.5976 Iu = 3.5844
Pelect = 1286.2110 PL = 1628.1850 Pin = 1724.2960
Thrust= 267.9607
eff = 5.5739E-02
cosA=6.7580E-01
J = 9450505.00
```

DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1
ENTER A VALUE FOR S [ $0<S<1]$
0.8

```
E1new = 159.448700 AFTER 5 INTERATIONS
N1 = 522. AND Np = 131.
I1 = 10.5781
    Iu = 3.7206
Pelect = 1107.5890
PL = 1171.2100 Pin = 1460.2690
Thrust= 230.7478
eff = 1.9794E-01
cosA = 6.274791E-01
J = 8619766.00
DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1
ENTER A VALUE FOR S [0<S<1]
0.55
E1new = 166.163000 AFTER 5 INTERATIONS
N1 = 522. AND Np = 131.
II = 9.4052 Iu = 3.8773
Pelect = 849.1395 PL = 693.2388 Pin = 1101.7000
Thrust= 176.9041
eff = 3.7075E-01
cos}\textrm{A}=5.3244\textrm{E}-0
J = 7663960.00
DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1
ENTER A VALUE FOR S [0<S<1]
0.1
Elnew = 173.9980 AFTER 5 INTERATIONS
N1 = 522. AND NP = 131.
II=8.0364 Iu = 4.0601
Pelect = 210.3370 PL = 187.2295 Pin = 364.0162
```

```
Thrust = 43.8202
eff = 4.8565E-01
cosA = 2.0588E-01
J = 6548609.00
DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1
```


## Sample run for different Design

DO YOU WANT TO KEEP CORE DIMENSIONS [YES=1/NO=2]

2

THE CURRENT DIMENSIONS ARE GIVEN IN THE FOLLOWING ORDER:
1, TAU, hy, hs, hp, bp, ws

| $1.9200 \mathrm{E}-01$ | $4.8000 \mathrm{E}-02$ | $1.6000 \mathrm{E}-02$ | $5.0000 \mathrm{E}-03$ |
| :--- | :--- | :--- | :--- |
| $4.3000 \mathrm{E}-02$ | $3.2000 \mathrm{E}-02$ | $5.0000 \mathrm{E}-03$ |  |

PLEASE ENTER NEW DIMENSIONS, KEEPING THE SAME ORDER AND SEPERATED BY A COMMA
$0.192,0.048,0.018,0.005,0.043,0.032,0.005$
AT THE MOMENT $\mathrm{Bg}=4.0000 \mathrm{E}-01$ AND $\mathrm{W} / \mathrm{Kg}=6.0000 \mathrm{E}-01$
DO YOU WANT TO CHANGE Bg AND W/Kg [YES=1/NO=2]
1
ENTER NEW VALUE FOR Bg AND W/Kg
$0.6,0.92$
AT THIS MOMENT Li $=9.0000 \mathrm{E}-02$ AND Acu $=1.2272 \mathrm{E}-06$
AND $11=5.1809 \mathrm{E}-03$ AND $12=7.9999 \mathrm{E}-03$
DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2]
1
YOU ARE ALLOWED TO CHANGE ONLY Li OR Acu

```
WHICH ONE DO YOU WANT TO CHANGE?
ENTER 1 FOR Li OR 2 FOR ACu
ENTER NEW VALUE FOR Bg AND W/Kg
0.6,0.92
AT THIS MOMENT Li = 9.0000E-02 AND ACU = 1.2272E-06
AND 11 = 5.1809E-03 AND 12 = 7.9999E-03
DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2]
1
YOU ARE ALLOWED TO CHANGE ONLY Li OR Acu
WHICH ONE DO YOU WANT TO CHANGE?
ENTER 1 FOR Li OR 2 FOR ACu
1
ENTER NEW VALUE FOR Li
0.04
THERE IS NOT ENOUGH STAKINGSPACE
AND 11 = 1.1657E-02 AND 12 = 7.9999E-03
AT THIS MOMENT Li = 4.0000E-02 AND Acu = 1.2272E-06
YOU ARE ALLOWED TO CHANGE ONLY Li OR Acu
WHICH ONE DO YOU WANT TO CHANGE?
ENTER 1 FOR Li OR 2 FOR ACu
2
ENTER NEW VALUE FOR Acu AND FOR dw
0.0000005176,0.00089
AT THIS MOMENT Li = 4.0000E-02 AND ACu = 5.1760E-07
AND 11 = 6.6847E-03 AND 12 = 7.9999E-03
DO YOU WANT TO CHANGE THE VALUES? [YES=1/NO=2]
2
ENTER A VALUE FOR S [0<S<1]
```

E1new $=141.815300$ AFTER 9 INTERATIONS
$\mathrm{N} 1=723 . \quad \mathrm{AND} \quad \mathrm{Np}=181$.
$I 1=11.6990$ Iu $=3.8826$

Pelect $=1227.2710$ $\mathrm{PL}=1920.5430 \quad \mathrm{Pin}=1911.2130$

Thrust $=255.6814$
eff $=-4.8817 E-03$
$\cos A=7.4256 \mathrm{E}-01$
$J=2.2602 \mathrm{E}+07$
DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]
1
ENTER A VALUE FOR $S[0<S<1]$
0.1

EInew $=159.0340$ AFTER 9 INTERATIONS
$\mathrm{NI}=723 . \quad \mathrm{AND} \quad \mathrm{Np}=181$.
$I 1=9.1225$
$\mathrm{Iu}=4.54112$
Pelect $=261.6733 \quad \mathrm{PL}=448.7939 \quad \mathrm{Pin}=625.0469$
Thrust $=54.5152$
eff $=2.8198 \mathrm{E}-01$
$\cos A=3.1143 E-01$
$J=1.7624 \mathrm{E}+07$
DO YOU WANT TO CHANGE THE SLIP [YES=1/NO=2]

From the above sample run it can be seen that the current desity is much to high for the selected wire diameter.

## APPENDIX 3

## Depth of Penetration

The depth of penetration of the magnetic field into the mild steel disk was calculated in the following way.

$$
\begin{aligned}
\delta & =\frac{1}{\sqrt{f \pi \mu_{0} \sigma_{F e}}} \\
& =\frac{1}{\sqrt{50 * \pi * 200 * 4 * \pi * 10^{-7} * 5 * 10^{5}}} \\
& =0.00711(\mathrm{~m}) \\
& =7.11(\mathrm{~mm})
\end{aligned}
$$

As the mild steel disk is 10 mm thick, the disk is not saturated by the magnetic field.

