

ANALYSIS OF BOUNDARY LAYER FLOW OF NANOFLUID WITH
THE CHARACTERISTICS OF HEAT AND MASS TRANSFER

OLANREWaju ANJOLUWAPO MARY

CAPE PENINSULA
UNIVERSITY OF TECHNOLOGY
Library and Information Services
Dewey No. ARC 621.4022 OCA

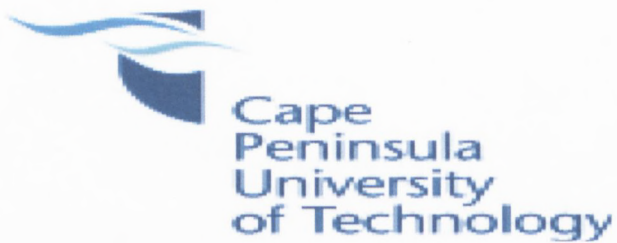
①

CAPE PENINSULA
UNIVERSITY OF TECHNOLOGY



20117198

CPT ARC 621.4022 OLA
(Green)
Not for loan



**ANALYSIS OF BOUNDARY LAYER FLOW OF NANOFUID WITH THE
CHARACTERISTICS OF HEAT AND MASS TRANSFER**

By

Olanrewaju Anuoluwapo Mary

211269573

**Thesis submitted in fulfilment of the requirements for the degree of Master
of Technology**

In the Department of Mechanical Engineering

Cape Peninsula University of Technology

Bellville Campus

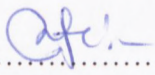
South Africa

Supervisor: Senior Professor O. D. Makinde

November 2011

DECLARATION

I, Olanrewaju Anuoluwapo Mary, declare that the content of this thesis represent my own work and has not been submitted previously for academic examination towards any qualification. Furthermore, it represents my own opinions and not necessarily those of the Cape Peninsula University of Technology.

Signed: 

Date: 13/02/2012

ABSTRACT

Nanofluid, which was first discovered by the Argonne laboratory, is a nanotechnology- based heat transfer fluid. This fluid consists of particles which are suspended inside conventional heat transfer liquid or base fluid. The purpose of this suspension is for enhancing thermal conductivity and convective heat transfer performance of this base fluid. The name nanofluid came about as a result of the nanometer- sized particles of typical length scales 1-100nm which are stably suspended inside of the base fluids. These nanoparticles are of both physical and chemical classes and are also produced by either the physical process or the chemical process.

Nanofluid has been discovered to be the best option towards accomplishing the enhancement of heat transfer through fluids in different unlimited conditions as well as reduction in the thermal resistance by heat transfer liquids. Various manufacturing industries and engineering processes such as transportation, electronics, food, medical, textile, oil and gas, chemical, drinks e.t.c, now aim at the use of this heat transfer enhancement fluid. Advantages such organisations can obtain from this fluid includes, reduced capital cost, reduction in size of heat transfer system and improvement of energy efficiencies.

This research has been able to solve numerically, using Maple 12 which uses a fourth- fifth order Runge –kutta- Fehlberg algorithm alongside shooting method, a set of nonlinear coupled differential equations together with their boundary conditions, thereby modelling the heat and mass transfer characteristics of the boundary layer flow of the nanofluids. Important properties of these nanofluids which were considered are viscosity, thermal conductivity, density, specific heat and heat transfer coefficients and microstructures (particle shape,

volume concentration, particle size, distribution of particle, component properties and matrix-particle interface).

Basic fluid dynamics equations such as the continuity equation, linear momentum equation, energy equation and chemical species concentration equations have also been employed.

Furthermore, the effects of various parameters controlling the flow system such as Prandtl number (Pr), Lewis number (Le), Biot number (Bi), Thermophoresis (Nt), Brownian motion (Nb) Suction/Injection β , the local skin friction coefficient, the reduced Nusselt number (Nur) and reduced Sherwood number (Shr) are worked out as well. In order to significantly provide a better understanding on the dynamics of nanofluids and to augment its usefulness in all fields applicable. This research has established mathematically the usefulness of nanofluids over conventional fluids.

DEDICATION

I dedicate this thesis to God Almighty and to all who indeed are less privileged all over the world.

ACKNOWLEDGEMENTS

My utmost appreciation goes to the Almighty God, my maker and the master planner of my life. Thank you Jesus.

How can I ever appreciate Professor O.D Makinde, Senior Professor and Director of the centre for post graduate studies, Cape Peninsula University of Technology, You have been my driving force, my motivation and inspiration. The drillings, fatherly roles and your advice have gone farther than you may imagine. I will forever acknowledge you. Thank you Sir.

My husband, Dr Olanrewaju Oladapo Philips, I never can say enough to appreciate you for your support, endurance and encouragements throughout this period. Thank you.

Oyindamola and Samuel, my lovely kids, thank you for your endurances and sacrifices. I love you both.

My loved ones, home and abroad, I appreciate every bit of the encouragements you all gave me.

I thank National Research Foundation (NRF) for their financial support during the course of this research work

NOMENCLATURE

Symbol	Description
C	nanoparticle volume fraction
C_f	skin- friction coefficient
C_w	nanoparticle volume fraction at the wall
C_∞	nanoparticle volume fraction at free stream
D_B	Brownian diffusion
D_T	thermophoretic diffusion coefficient
k	thermal conductivity
Le	Lewis number
Pr	Prandtl number
Nb	Brownian motion parameter
Nt	thermophoresis parameter
P	fluid pressure
q_w	surface heat flux
q_m	surface heat flux
t	time
T	fluid temperature
T_∞	ambient temperature
T_f	temperature of the fluid
Nur	reduced Nusselt number

Shr	reduced Sherwood number
T_w	surface temperature
h	convective heat transfer coefficient
u, v	velocity component along x and y directions respectively
∞	free stream values
α	thermal diffusivity of the base fluid
ϕ	concentration at the sheet surface
θ	temperature at the sheet surface
ρ	fluid density
ρ_f	density of the base fluid
ρ_p	density of the particle
$(\rho c)\rho$	heat capacity of the base fluid
$(\rho c)_p$	heat capacity of the nanoparticle material
τ	rate of heat capacity
ν	kinematic viscosity of the base fluid
ψ	stream function
Bi	Biot number
β	suction/injection parameter
\vec{q}	velocity vector
Φ	dissipation function

TABLE OF CONTENTS

DECLARATION.....	ii
ABSTRACT	iii
DEDICATION	v
ACKNOWLEDGEMENTS.....	vi
NOMENCLATURE	vii
TABLE OF CONTENTS	ix
LIST OF FIGURES	xi
LIST OF TABLES	xii
CHAPTER 1	1
1. Introduction	1
1.1 Applications of nanofluids.....	5
1.1.1 Transportation.....	5
1.1.2 Electronics Cooling	7
1.1.3 Defence.....	7
1.1.4 Space.....	8
1.1.5 Nuclear Systems Cooling	8
1.1.6 Biomedicine.....	9
1.2 Properties of nanofluids.....	9
1.3 Literature Review	10
1.4 Statement of the problem.....	13
1.5 Research aims and Objective.....	14
1.6 Methodology.....	14
1.6.1 Shooting Methods.....	15
1.7 Significance	17
CHAPTER 2.....	18
2 Fluid Dynamics Equations.....	18
2.1 Continuity Equation.....	18
2.2 Momentum Equation	21
2.3 Energy Equation	25

2.3.1	The Zeroth Law of Thermodynamics	25
2.3.2	The First Law of Thermodynamics	25
2.3.3	Fourier Heat Conduction	28
2.4	Concentration Equation	31
CHAPTER 3.....		34
3	Modelling boundary layer flow of a nanofluid past a permeable flat surface with Newtonian heating	34
3.1	Summary.....	34
3.2	Introduction	34
3.3	Convective transport equations.....	37
3.4	Important Physical quantities.	47
CHAPTER 4.....		50
4	Computational Analysis of the Model Problem	50
4.1	Summary.....	50
4.2	Numerical Procedure	50
4.3	Graphical Results.....	54
CHAPTER 5.....		64
5.1	Analysis and discussion of the results	64
5.2	Discussion of the tables.....	64
5.3	Discussion of the graphs.....	66
5.4	Conclusions	71
BIBLIOGRAPHY		72

LIST OF FIGURES

Figure Number	Description	Page
1.1	Images of (a) amorphous carbonic nanofluid and (b) alumina nanofluids	2
1.2	Semi micrograph of ZnO nanoparticles	3
1.3	Sketch of Boundary layer	4
1.4	Diagram of boundary layer flow in laminar and turbulent regime	5
2.1	Flow continuity diagram	18
3.1	Geometry of the problem	38
4.1	Effects of suction/injection parameter (β) on the velocity profile	55
4.2	Effects of suction/injection parameter on the temperature profiles	55
4.3	Effects of suction/injection parameter on the concentration profile	56
4.4	Effects of Lewis number (Le) on the temperature profile	56
4.5	Effects of Lewis number on the concentration profiles	57
4.6	Effects of Brownian motion (Nb) and thermophoresis (Nt) parameters on the temperature profile	57
4.7	Effects of Brownian motion (Nb) and thermophoresis (Nt) parameters on the concentration profiles	58
4.8	Effects of Biot number (Bi) on the temperature profiles	58
4.9	Effects of Biot number on concentration profiles	59
4.10	Effects of Prandtl number (Pr) on the temperature profiles	59
4.11	Effects of suction/injection parameter on skin- friction	60
4.12	Effects of thermophoresis and Brownian motion parameter on the reduced Nusselt number	60
4.13	Effects of thermophoresis and Brownian motion parameter on the reduced Sherwood number	61
4.14	Effects of Biot number (Bi) and suction/injection on the Nusselt number	61
4.15	Effects of Biot number (Bi) and suction/injection on the Sherwood number	62
4.16	Effects of Lewis number on the reduced Sherwood number	62
4.17	Effects of Prandtl number on the reduced Nusselt number	63

LIST OF TABLES

Table	Description	Page
1.1	Models showing the effective heat transfer coefficient of nanofluids	11
3.1	The physical properties of water, Cu, Al ₂ O ₃ and TiO ₂	38
4.1	Computation showing the comparison with Makinde and Aziz[112] for Nur, Shr and surface temperature($\theta(0)$)	53
4.2	Computations showing comparison with Makinde and Aziz[112] for Nur	54
4.3	Computations showing comparison with Makinde and Aziz [112] for Shr	54

CHAPTER

1

1. Introduction

Nanofluids are nanotechnology-based heat transfer fluids, which consists of particles suspended inside of liquids (typically conventional heat transfer liquids). They have been shown to enhance thermal conductivity and convective heat transfer performance of the base liquids. The name was discovered by Argonne National Laboratory to describe this fluid because of nanometer- sized particles (of typical length scales 1 to 100nm) that are stably suspended inside of the base liquid. Moreover, many industrial processes require the transfer of heat through a flowing fluid either in laminar or turbulent regime, flowing or stagnant boiling fluids in a large range of temperature and pressure. Reduction in thermal resistance of heat transfer liquids will serve as a benefit to many of these applications in terms of reduced capital cost, reduction of size of heat transfer system and improvement of energy efficiencies. These can be achieved in nanofluids and various industries and manufacturing organizations such as transportation, electronics, medical and food stand to benefit a lot in these improved heat transfer fluids.

Nanoparticles are regarded as a class of materials with unique properties both chemical and physical and they are fabricated materials at nanometer scale. The production of these particles can then be classified into two, namely the physical process and chemical processes.

Examples of nanoparticles which have been commonly used are oxide ceramics (Al_2O_3 ,

Analysis of Boundary Layer Flow of Nanofluid with the Characteristics of Heat and Mass Transfer

Olanrewaju A.M

CuO), Nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Ag, Au, Cu, Fe), semiconductors (TiO₂), single-, double- or multi-walled carbon nanotubes (SWCNT, DWCNT, MWCNT), and the composite materials such as nanoparticle core-polymer shell composites. However, new materials and structures are attractive for use in nanofluids where the particle-liquid interface is doped with various molecules. Nanoparticles are mostly produced in the form of powder and are dispersed in aqueous or organic host liquids to form nanofluids for specific applications. Physical processes include mechanical grinding method and the inert-gas-condensation technique (developed by Granqvist and Buhrman [55]). The chemical processes include chemical precipitation, chemical vapour deposition, micro-emulsions, spray pyrolysis, and thermal spraying. A sonochemical method has been developed to make suspensions of iron nanoparticles stabilized by oleic acid (Kenneth, et al [76]).

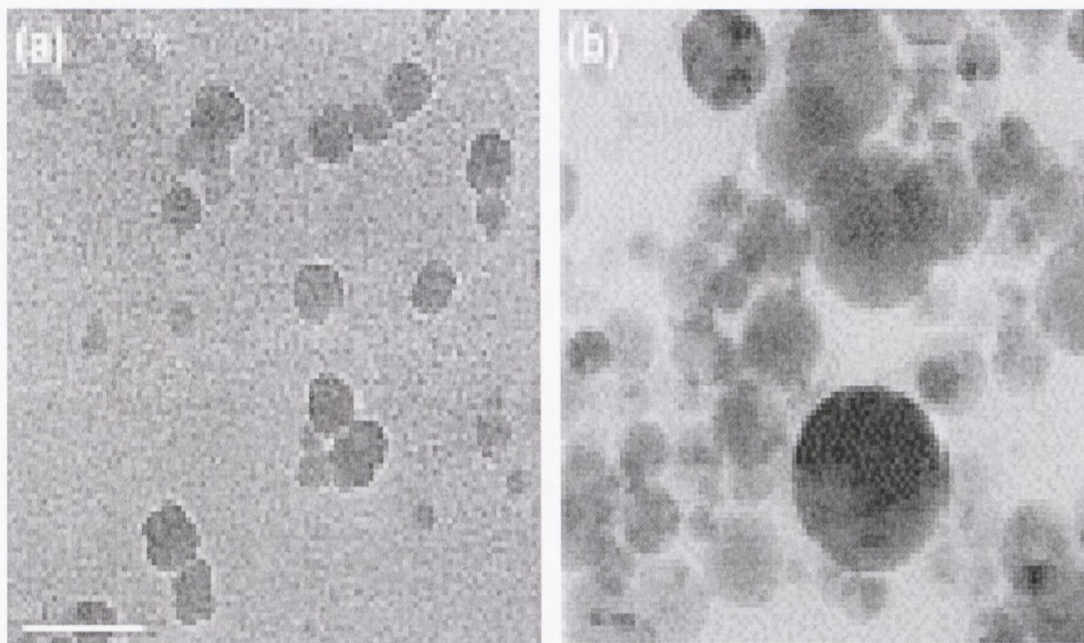


Figure 1.1: (a) is the image of amorphous carbonic nanofluids and (b) is the image of alumina nanofluids

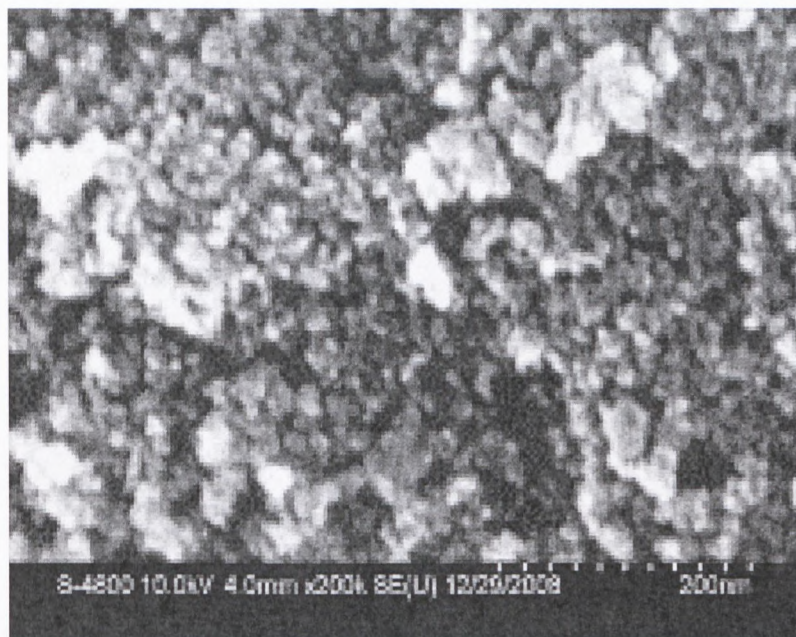


Figure 1.2: Semi micrograph of ZnO nanoparticles

Meanwhile, boundary layer refers to that region of a fluid which forms in the vicinity of a surface bounding a fluid. It occurs when fluid flows in pipes and channels, wind moves around structures or buildings, over the wings of aeroplanes, the surface of moving vehicles, e.t.c. The movement of fluid molecules over surfaces creates regions near the surfaces which greatly affects the dynamics of both the fluid and the object. The flow in the region surrounding a body can be divided into the boundary layer and anywhere outside of the boundary. Outside of the boundary layer, the viscosity on the body is negligible, and the flow may be treated as inviscid. Therefore, only flow inside the boundary layer is important for determining frictional resistance, Fox et al [47].

The most important development in the study of fluid mechanics occurred during the 20th century when Prandtl [136] introduced his concepts of boundary layer. Prandtl's theory states that due to the no-slip condition, the velocity of fluid molecules at the surface of a stationary

body is zero, but the velocity given by the inviscid flow theory is reached within a thin layer closed to the surface called the ‘boundary layer’. David et al [34] and Tulapurkara [164] indicated that Prandtl’s 1904 paper on the subject formed the basis for future works on skin friction, heat and mass transfer and separation in the boundary layer. The details of flow within the boundary layer are very important for many problems in aerodynamics. Prandtl’s concepts led to improve designs of objects and bodies moving in fluids such as aeroplanes, ships, rockets, cars, pipelines, etc. The present study investigates mathematically the effects of various controlling parameters on the boundary layer flow of nanofluids with heat and mass transfer characteristics.

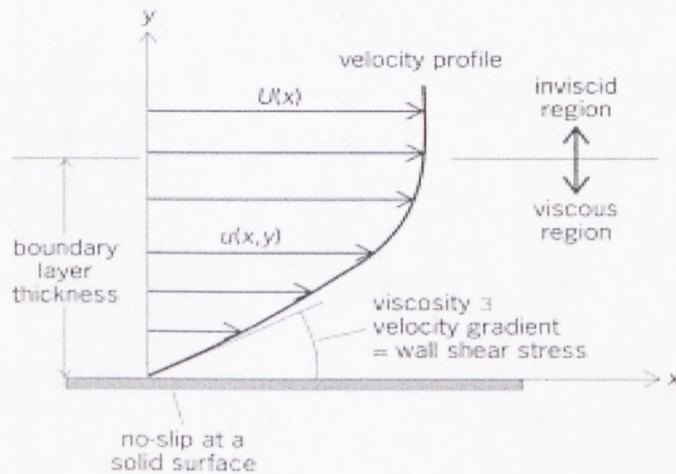


Figure 1.3: Sketch of boundary layer

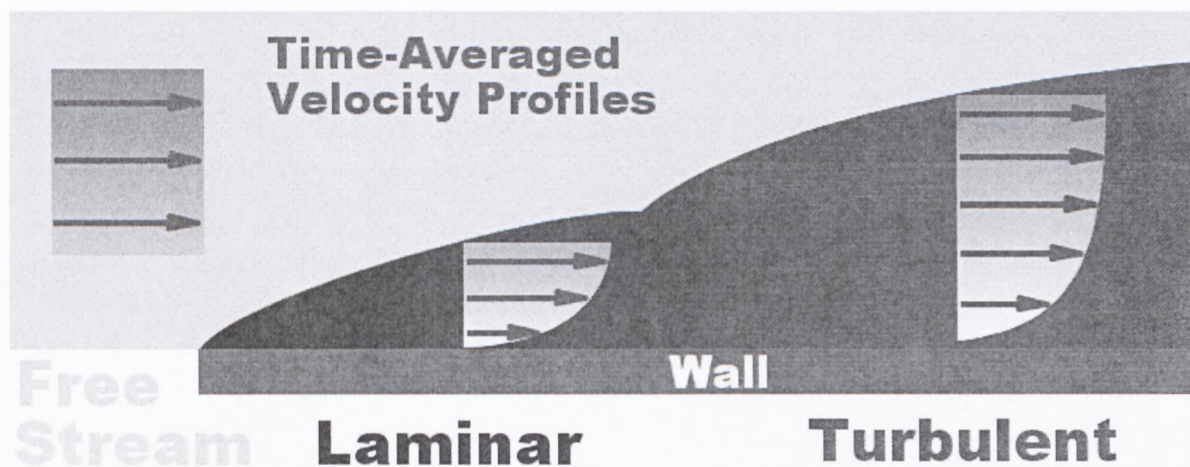


Figure 1.4: Diagram of boundary layer flow in laminar and turbulent regime.

1.1 Applications of nanofluids.

Various applications of nanofluids can be found in transportation, microelectronics, defence, nuclear, space, and biomedical as highlighted below:

1.1.1 Transportation

Conventional or base fluids such as, water, ethylene glycol, water mixture and engine oil in the order of their performance in heat transfer from the highest to the lowest are yet poor heat transfer fluids. When nanoparticles are stably dispersed into standard engine coolant it tends to improve automotive and heavy-duty engine cooling rates. This improvement serves a lot of benefits, examples, reduction in coolant system size and weight and this benefit all parts of the vehicle and its performance also fuel conservation. Furthermore, improved cooling rates

for truck engines and any other automotives can remove more heat from higher horsepower engines with the same size of coolant system. As earlier stated, pure ethylene glycol is a poor heat transfer fluid compared to a mixture of ethylene glycol and water in the same proportion. Nevertheless, the addition of nanoparticles improves the enhancement. The higher boiling point characteristic of nanofluid coolant can be used to increase the normal coolant operating temperature, and then reject more heat through the existing coolant system. The more heat is being rejected, the more variety of design enhancements is being allowed in engines and with ones of higher horsepower.

Tzeng et al. [165] performed an experiment on the platform of transmission of a four-wheel drive; this transmission had an advanced rotary blade coupling, where high local temperatures occurred at high rotating speeds using engine transmission oil in which he suspended CuO and Al₂O₃ nanoparticles. The temperature distribution on the exterior of the rotary-blade-coupling transmission was measured at four engine operating speeds (400, 800, 1200, and 1600rpm), and the optimum composition of nanofluids with regard to heat transfer performance was investigated. From the results it was deduced that CuO nanofluids produced the lowest transmission hence, the application of nanofluids in transmission is of great advantage in terms of thermal conductivity.

Zhang and Que [203] point out that surface-modified nanoparticles are detected to be effective in enhancing tribological properties such as reducing wear and reducing the friction between moving mechanical components when these particles are stably dispersed in mineral oils. This accounts for the application of nanofluid in automotive lubrication.

1.1.2 Electronics Cooling

The increase in microprocessors and integrated circuits have been in the recent years a dramatic one. It was predicted by International Technology Roadmap for Semiconductors that by 2018, high performance integrated circuits will contain more than 9.8 billion transistors on a chips of 90-nm node size. Also processors for high-performance computers and servers have been predicted to dissipate higher power in the range of 100-300W/cm²ⁱ in future. Single-phase fluids are candidate replacements for air-cooling techniques for removing heat as liquid cooling technology investigations for heat transfer increases.

Tsai et al [162], experimented a circular heat pipe designed as heat spreader to be used in a CPU in a notebook or a desktop PC using water- based nanofluid and the experiment shows a significant decrease in thermal resistance of the heat pipe compared with deionised water. The results showed the effects of nanoparticles size on the thermal resistance of a vertical meshed heat pipe. Ma et al.[106] also investigated the effect of nanofluids on the heat transport capability of an oscillating heat pipe experimentally, it has been proven that, a nanofluid containing 1 vol.% nanoparticles with input power of 80W, reduced the temperature difference between the evaporator and the condenser is reduced from 40.9^oc to 24.3^oc. Hence this result and others encourages more research on the application of nanofluids.

1.1.3 Defence

A high-heat-flux cooling of about tens of MW/m² is required in quite a number of military devices, as a result of this, the use of convention fluids as cooling fluids is challenging. For examples, the cooling of power electronics and energy directed- weapons which involve high

heat fluxes more than 500-1000 W/cm². Nanofluids have a potential of providing adequate cooling for them and the associated power electronics as well as providing the required cooling in other military equipments and machines such as military vehicles, submarines, and high-power laser diodes. Other areas of nanofluid research for defence applications includes multifunctional nanofluids with added thermal energy storage or energy harvesting feat, transformer cooling in the navy as well as in power generation industry having the objective of reducing the size and weight of transformers. The projective demand for greater production of electricity can also lead to the replacement or improvement of transformer on a large scale and at a high cost. Nanofluid as it has been demonstrated, could significantly improve the heat transfer properties of transformer oils.

1.1.4 Space

You [198] and Vassalo et al. [170] reported order of magnitude increasing in the critical heat flux in a pool boiling, with comparing nanofluids to base fluids. The tendency of raising chip power in electronic components or simplifying cooling requirements for space applications is being presented. High critical heat fluxes which allows for boiling to higher qualities with increased heat removal and wider safety margin from film boiling makes nanofluids attractive in general electronic cooling as well as space applications with very high power density.

1.1.5 Nuclear Systems Cooling

An interdisciplinary centre for nanofluid technology for the nuclear energy industry has been established by the Massachusetts Institute of Technology. In this institute, evaluations of the

impact and use of nanofluids on safety, neutronic and economic functions of nuclear systems are being examined. The use of nanofluids in the place of water in a Pressurized Water Reactor (PWR), makes the fuel rod becomes coated with nanoparticles such as alumina and prevents the formation of a layer vapour around the rod and subsequently increasing the critical heat flux significantly. Nanofluids are also useful in the geothermal power and other energy sources.

1.1.6 Biomedicine

In order to circumvent some side effects of local methods of cancer treatment, iron- based nanoparticle could be used as delivery vehicles for drugs or radiation without affecting the surrounding tissues, hence reducing organ damage. Nanoparticles could be guided in the bloodstream to a tumour using magnets external to the body. More so, nanofluids could be used to provide effective cooling around the surgical region hence improving safe surgery which thereby enhances patient's chance of survival.

Jordan, et al. [71] however reported that nanofluid could also be used to produce a higher temperature around the tumours to kill cancerous cells without affecting nearby healthy cells.

1.2 Properties of nanofluids

The properties of nanofluids such as viscosity, thermal conductivity, density and specific heat and heat transfer coefficient are effectively measured. And this is done when details of their microstructures (such as particle shape, component volume concentration, particle size, distribution of particle, component properties, and matrix-particle interfacial effects) are fully

known. The term “Matrix” is used to define nanofluids for theoretical modelling purpose. Prandtl number (named after the German physics Ludwig Prandtl) is a dimensionless quantity of momentum diffusivity (Kinematic viscosity) to thermal diffusivity and is hence defined as,

$$\text{Pr} = \frac{\nu}{\alpha} = \frac{C_p \mu}{k} \frac{\text{Viscosity diffusion rate}}{\text{thermal diffusion rate}},$$

Where ν = Kinematic viscosity, $\nu = \frac{\mu}{\rho}$ (S.I.units : m^2/s),

α = Thermal diffusivity, $\frac{k}{\rho C_p}$ (S.I.units : m^2/s),

μ = Dynamic viscosity, (S.I.units : $w/(mk)$),

C_p = Specific heat, (S.I.units : $J/(kg.k)$),

ρ = Density, (S.I.units : kg/m^3).

1.3 Literature Review

Heat transfer enhancement brought about by the excellent potentials of nanofluids has propelled both industry and universities to launch research and development efforts in nanotechnology after the pioneering demonstrations and experiments by Argonne Laboratory. Nanofluids exhibit increases in thermal performances as compared to the conventional heat transfer fluids. Choi et al. [25] in their study reveal that the addition of less than 1% volume of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluids approximately twice. Another important property of nanofluids is a stronger

Analysis of Boundary Layer Flow of Nanofluid with the Characteristics of Heat and Mass Transfer

Olanrewaju A.M

temperature-dependent thermal conductivity than in the ordinary base fluid (Das et al. [29]). Many researchers have determined the thermal conductivity enhancement of different nanoparticles in a variety of liquids with volume concentrations to be in the range of 0.5-4%. At these low particle volume concentrations, typical enhancement has been in the 25% range over the base fluid parameter in forced-convection cooling applications, including engines and thermal systems. More importantly, experiments have shown that nanoparticles significantly enhance the heat transfer coefficient of flowing liquids as reported by (Faulkner, et al. [46]).

Table 1.1: Models showing the effective heat transfer coefficient of nanofluids:

Pak and Cho [130] (for Al_2O_3 -in-water and TiO_2 -in-water nanofluids, turbulent flow) $Nu = 0.021Re^{0.8} Pr^{0.5}$
Das, et al. [30] (Al_2O_3 -in-water nanofluids, pool boiling $Nu = c Re_b^m Pr^{0.4}$, where c and m are particle volume concentration dependent parameters.
Xuan and Li [188](for CuO-in-water nanofluids, turbulent flow) $Nu = 0.0059(1.0 + 7.6286v_p^{0.6886} Pe_p^{0.001}) Re^{0.9238} Pr^{0.4}$
Yang, et al. [193], (for graphite-in-transmission fluid and graphite-in-synthetic oil mixture nanofluids, laminar flow) $Nu = c Re^m Pr(D/L)^{1/3} (\mu_b / \mu_w)^{0.14}$ where c and m are nanofluid and temperature dependent empirical parameters.
Buongiorno [20] (turbulent), $Nu = \frac{(f/8)(Re-1000)Pr}{1 + \delta_v^+ (f/8)^{1/2} (Pr_v^{2/3} - 1)}$ where the dimensionless thickness of the laminar sublayer δ_v^+ is an empirical.

Furthermore, the increase of viscosity of nanofluids as compared to that of conventional fluids has been investigated by many researchers. Although experimental data for effective viscosity of nanofluids are limited to certain nanofluids, such as;

- Al_2O_3 in water (Park and Cho [129]; Das, et al. [30], Putra, et al. [141]; Li, et al. [95]; Heris, et al. [63]),
- CuO in water (Heris, et al [63]; Kulkarni, et al.[87]),
- TiO_2 in water (Park and Cho [129]),
- MWCNT in water (Ding, et al. [38]),
- Al_2O_3 in octane (Liu, et al. [96]).

This experimental data show the trend that the effective viscosities of nanofluids are higher than the theoretical predictions and the ranges of the parameters (the particle volume concentration, temperature, etc.) are limited. Researchers then proposed equations applied to specific applications e.g., Al_2O_3 in water (Ma?ga, et al. [108]), Al_2O_3 in ethylene glycol (Ma?ga, et al. [108]), and TiO_2 in water (Tseng and Lin [163]). Kulkarni et al [87] proposed an equation for CuO in water taking temperature into consideration. Wang and Wei [174] developed an exact equivalence between the dual phase-lagging heat conduction in nanofluids. Abu-Nada [1] investigated numerically the heat transfer over a backward step (BFS) using nanofluids. Oztop and Abu-Nada [129] studied theoretically the problem of steady boundary layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. Duangthongsuk and Wongwises [33] investigated the effects of the thermophysical properties on the predicted values of convective heat transfer coefficient in TiO_2 -water nanofluid flowing through a horizontal double-tube counter flow heat exchangers under turbulent flow conditions. Oztop and Abu-Nada [129] performed a numerical study to investigate the effect of using different nanofluids on natural convection flow field and temperature distributions in partially heated square enclosure from the left vertical wall.

Nield and Kuznetsov [128] examined the influence of nanoparticles on natural convection

Analysis of Boundary Layer Flow of Nanofluid with the Characteristics of Heat and Mass Transfer

Olanrewaju A.M

boundary layer flow in a porous medium past a vertical plate. Dongsheng Wen et al. [40] conducted a critical review of the state-of-the-art nanofluids research for heat transfer applications and conclude that current understanding on nanofluids is still quite limited. Syakila and Loan [157] studied the steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids using different nanoparticles such as Al_2O_3 , Cu, TiO_2 . Hamad et al. [59] studied the hydromagnetic free convection boundary layer flow past a semi-infinite flat plate embedded in water filled with a nanofluid. Kuznetsov and Nield [90] examined the natural convective boundary layer flow of nanofluids over a vertical plate. The model used for the nanofluids incorporates the effect of Brownian motion and thermophoresis. Cyril Okhio et al. [28] investigated the flow characteristics and heat transfer coefficient of nanofluids under laminar forced convection over a flat plate. Khan and Pop [79] studied the problem of boundary layer flow of nanofluids over a stretching flat surface numerically.

1.4 Statement of the problem

In many industrial and engineering processes, where improved heat transfer in terms of capital and cost of production, the quality and quantity, nanofluid is a better option for the accomplishment of the enhancement in heat transfer performance in various different and unlimited conditions. An illustration is the reservation of energy in a heating, ventilating and air conditioning systems which is obtained without necessarily increasing the pumping power in buildings as a result of nanofluid application while providing environmental benefits. Furthermore, nanofluid coolant also have application potentials in major industrial systems such as material, chemical, food, textile, paper printing, drink, oil and gas. In order to

improve the effectiveness of nanofluid, it is extremely important to understand the inherent complex dynamics of the flow systems. In this research project a mathematical model for boundary layer flow of nanofluids with heat and mass characteristics will be investigated.

1.5 Research aims and Objective

The main aim of this research is to provide a mathematical model for boundary layer flow of nanofluids with heat and mass transfer characteristics. In order to achieve this goal, our objectives include;

- to derive a mathematical model for boundary layer flow of nanofluids
- to analyse the model using appropriate numerical technique
- to investigate the effects of various embedded parameters controlling the flow system on skin friction together with heat and mass transfer rate
- to establish mathematically the usefulness of nanofluids over conventional fluids in industrial and engineering processes.

1.6 Methodology

Modelling real systems like boundary layer flow of nanofluids do involve solving a system of nonlinear differential equations. The model will be derived from the basic equations governing the boundary layer flow of nanofluids which include: the continuity, momentum, energy and concentration balance equations. The problem will be solved numerically using shooting method coupled with fourth order Runge-Kutta integration scheme. Relevant

software like Maple will be utilised to develop a numerical code that will be implemented on a computer.

1.6.1 Shooting Methods

Shooting method is a numerical technique of analysing some common two-point boundary value problems. This iterative algorithm technique is usually used to guess the initial conditions for a related boundary value problem (BVP), hence transforming a BVP to an initial value problem (IVP). Fourth order Runge-Kutta Iteration scheme is then employed to solve the resulting IVP. Shooting methods reforms the two-point value problems as a nonlinear parameter estimation problem which is majorly the need for this technique. Solution for the reformulated problem is expected to satisfy the approximated boundary conditions at the other end point with the initial conditions chosen. This process is repeated until the limit of the iteration is attained or the expected accuracy is achieved. A mathematical illustration is expressed below;

Consider the two- point BVP for a system of n first- order ODEs.

$$\begin{aligned} \frac{dy}{dt} &= f(t, y(t)), \\ y_i(a) &= \alpha_i, \quad i = 1, 2, \dots, m_1. \\ y_{m_1+j}(b) &= \beta_j, \quad j = 1, 2, \dots, m_2. \end{aligned} \tag{1.1}$$

The vector y contains the n unknown functions of the independent variable t . The initial m_1 ($0 < m_1 < n$) components of y have first order boundary conditions as the unknown functions are ordered. The remaining $m_2 := n - m_1$ components of the solution have first

order boundary conditions specified at a second point, $t = b^1$ (note; (1.1) is an initial value problem if $m_2 = 0$).

The shooting method recognises the vector of parameters $s \in R^{m_2}$ so that the solution, denoted by $y(t; s)$, to the initial value problem

$$\begin{aligned} \frac{dy}{dt} &= f(t, y(t; s)), \\ y_i(a; s) &= \alpha_i, \quad i = 1, 2, \dots, m_1, \\ y_{m_1+j}(a; s) &= s_j, \quad j = 1, 2, \dots, m_2. \end{aligned} \tag{1.2}$$

agrees with the solution to (1.1). Note that (1.2) is the same as (1.1) with the boundary conditions at present ($t = b$) replaced with unknown initial conditions at $t = a$. to determine the correct initial values, consider the ‘‘objective function’’ F with components

$$F_j(s) := y_{m_1+j}(b; s) - \beta_j, \quad j = 1, 2, \dots, m_2. \tag{1.3}$$

This implies that (1.1) can be solved if and only if there exists $s \in R^{m_2}$ such that $F(s) = 0$. Obtaining the desired result of this process depends majorly on the iterative procedure used to construct a sequence of parameter vectors that converges to a zero of F . Any numerical root-finding algorithm could be used for this step but the mostly used method is one step of the Newton-Raphson method. This means that given an initial guess $s^0 \in R^{m_2}$, define the sequence of initial conditions $\{s^k\}$ by

$$s^{k+1} := s^k - (\nabla F(s^k))^{-1} F(s^k) \tag{1.4}$$

for all $k \geq 0$. In order to implement this, it is important to note that the vector $F(s^k)$ is directly obtained from the solution of (1.2), but the Jacobian matrix $\nabla F(s^k)$ require the

values of $\frac{\partial y_{m+1}}{\partial s_j}(b; s^k)$ for all $i, j = 1, 2, \dots, m_2$. these values can be obtained by solving the n

IVPs in (1.2) alongside the nm_2 sensitivity equations:

$$\frac{d}{dt} \left(\frac{\partial y_i}{\partial s_j} \right) = \frac{\partial f}{\partial y_i} \frac{\partial y_i}{\partial s_j}, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m_2, \quad 1.5$$

With corresponding initial conditions

$$\begin{aligned} \frac{\partial y_i}{\partial s_j} &= 0, & \text{for all } i = 1, \dots, m_1, \quad j = 1, 2, \dots, m_2 \\ \frac{\partial y_{m+1}}{\partial s_j} &= \delta_{ij}, & \text{for all } i, j = 1, \dots, m_2. \end{aligned} \quad 1.6$$

1.7 Significance

The output of this study will provide a better understanding on the dynamics of nanofluids and thereby augment its usefulness in various industrial and engineering processes like chemical and manufacturing industries, defence, biomedical, transportation etc.

CHAPTER

2

2 Fluid Dynamics Equations

2.1 Continuity Equation

In continuous fluid motion, if we consider a surface S fixed in a space containing a volume V , it is clear that the increase in the mass of the fluid that flows in the system is the same as the mass of the fluid that flows out. The mass of the fluid within the system is

$$\int \rho dv . \quad (2.1)$$

The rate of increase of the mass within the surface is given as

$$\frac{\partial}{\partial t} \int_v \rho dv = \int_v \frac{\partial \rho}{\partial t} dv . \quad (2.2)$$

Consider the following diagram and the fluid within the surface,

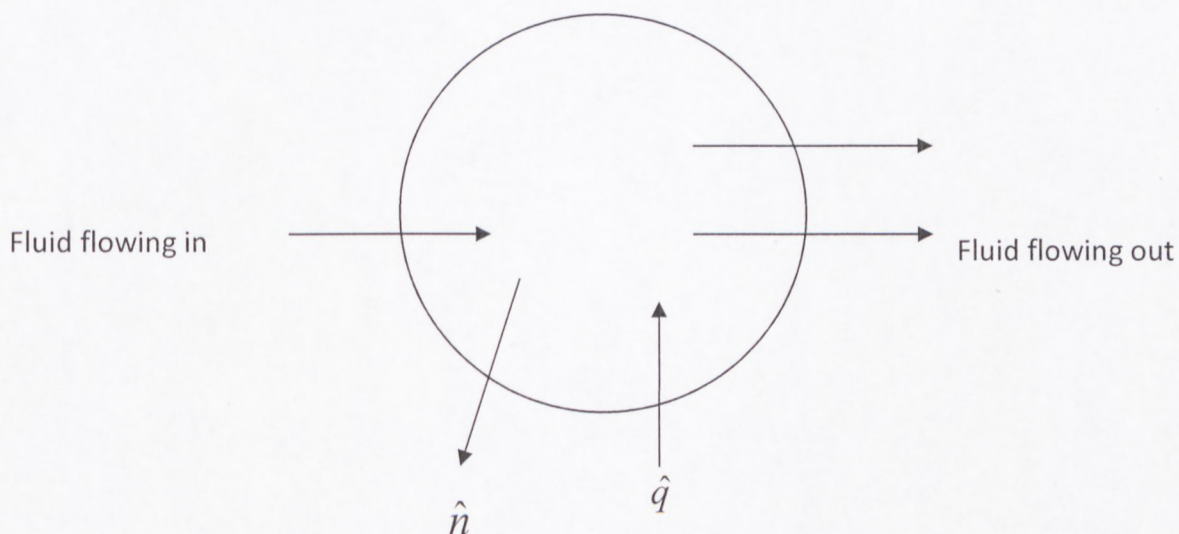


Figure 2.1: Flow continuity diagram.

Since volume V does not vary with time, the rate of flow through S is,

$$\int_S (\vec{q} \cdot \vec{n}) \rho ds = \int_V \nabla \cdot (\rho \vec{q}) dv, \quad (2.3)$$

$$\int_V \frac{\partial \rho}{\partial t} dv = - \int_V \nabla \cdot (\rho \vec{q}) dv, \quad (2.4)$$

$$\int_V \frac{\partial \rho}{\partial t} dv + \int_V \nabla \cdot (\rho \vec{q}) dv = 0, \quad (2.5)$$

$$\int_V \left(\frac{\partial \rho}{\partial t} + \text{div} (\rho \vec{q}) \right) dv = 0. \quad (2.6)$$

This is valid for any arbitrary volume V , the integral must identically vanish. Thus, we obtain,

$$\frac{\partial \rho}{\partial t} + \text{div} (\rho \vec{q}) = 0. \quad (2.7)$$

Equation (2.7) is called continuity equation for any fluid and it can be written as,

$$\frac{\partial \rho}{\partial t} + \rho \text{div} \vec{q} + (\vec{q} \cdot \nabla) \rho = 0. \quad (2.8)$$

But

$$\frac{D}{Dt} + \rho \text{div} \vec{q} = 0. \quad (2.9)$$

Where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{q} \cdot \nabla,$$

is the material derivative, which represents differentiation following fluid motion. For incompressible motion we have

$$\text{div } \vec{q} = 0. \quad (2.10)$$

If

$$\vec{q} = (u, v, w),$$

then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (2.11)$$

If the fluid is incompressible, but two dimensional we have,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2.12)$$

then one can (or may) introduce the function ψ given by,

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \quad (2.13)$$

This equation satisfies the continuity equation:

$$u_x + u_y = (\psi_y)_x + (-\psi_x)_y = \psi_{yx} - \psi_{xy} = 0,$$

$$-\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial x \partial y} = 0. \quad (2.14)$$

Now if the flow is irrotational then,

$$\nabla \wedge \vec{q} = 0. \quad (2.15)$$

This implies that there exists a scalar function ϕ such that,

$$\vec{q} = -\nabla \phi. \quad (2.16)$$

In two dimensions, we have,

$$u = \frac{\partial \phi}{\partial x}, v = -\frac{\partial \phi}{\partial y}. \quad (2.17)$$

From equation (2.12), we obtain,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0. \quad (2.18)$$

2.2 Momentum Equation

Consider a small parallel pipe of volume $dV = dx_1 \times dx_2 \times dx_3$, isolated instantaneous from the fluid with centre (x_1, x_2, x_3) we shall denote by σ_{ij} the stress acting in the X_i direction on the force whose normal lies in the X_j direction. The stress component $\sigma_{11}, \sigma_{12}, \sigma_{13}, \dots$ from a second order stress tensor σ_{ij}

i.e

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}. \quad (2.19)$$

This is called the stress matrix. The components σ_{ij} are the normal stress while the components $\sigma_{ij(i \neq j)}$ are shearing stresses. The shear tensor and the corresponding matrix are symmetric i.e., $\sigma_{ij} = \sigma_{ji}$. In relation to a surface at the right angles to the axis x_i , the stress per unit area at $(x_1 + x_2 + x_3)$ and $\sigma_{11}, \sigma_{12}, \sigma_{13}$. The corresponding stresses as the centre of the forces are given as:

$$\begin{aligned}\sigma_{11} + \frac{1}{2} \frac{\partial \sigma_{11}}{\partial x_1} dx_1 \\ \sigma_{12} + \frac{1}{2} \frac{\partial \sigma_{12}}{\partial x_1} dx_1, \\ \sigma_{13} + \frac{1}{2} \frac{\partial \sigma_{13}}{\partial x_1} dx_1\end{aligned}\tag{2.20}$$

using Taylor expansion in the form

$$\sigma_{11}(dx_1, x_2, x_3) = \sigma_{11}(x_1, x_2, x_3) + \frac{1}{2} \frac{\partial \sigma_{11}}{\partial x_1} dx_1.\tag{2.21}$$

At the centre of the opposite force, the corresponding stresses are

$$\begin{aligned}\sigma_{11} - \frac{1}{2} \frac{\partial \sigma_{11}}{\partial x_1} dx_1 \\ \sigma_{12} - \frac{1}{2} \frac{\partial \sigma_{12}}{\partial x_1} dx_1, \\ \sigma_{13} - \frac{1}{2} \frac{\partial \sigma_{13}}{\partial x_1} dx_1\end{aligned}\tag{2.22}$$

acting on the fluid in parallel compound. The stresses on a pair of opposite force may be compounded into

$$\begin{aligned}
& \frac{\partial \sigma_{11}}{\partial x} dx_1, dx_2, dx_3, \\
& \frac{\partial \sigma_{12}}{\partial x} dx_1, dx_2, dx_3, \\
& \frac{\partial \sigma_{13}}{\partial x} dx_1, dx_2, dx_3,
\end{aligned} \tag{2.23}$$

acting at $(x_1 + x_2 + x_3)$ parallel to $(0x_1 + 0x_2 + 0x_3)$. The stress on the other two pairs opposite forces may be compounded into similar forces at $(x_1 + x_2 + x_3)$. The resultant force in x_1 the direction becomes

$$\left(\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} \right) dx_1 dx_2 dx_3. \tag{2.24}$$

If X, Y, Z are the forces per unit area due to variation of stress along $0x_1 + 0x_2 + 0x_3$, then

$$\begin{aligned}
X &= \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} = \frac{\partial \sigma_{\alpha 1}}{\partial x_\alpha} \\
Y &= \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} = \frac{\partial \sigma_{\alpha 2}}{\partial x_\alpha}, \alpha = 1, 2, 3. \\
Z &= \frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} = \frac{\partial \sigma_{\alpha 3}}{\partial x_\alpha}
\end{aligned} \tag{2.25}$$

Generally, we denote the body of forces by $F = (F_1, F_2, F_3) = F_i, i = 1, 2, 3$ then the equation of motion is given by:

$$\rho \frac{\partial u_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + F_i. \tag{2.26}$$

In tensor notation we have $\vec{q} = (U_1, U_2, U_3) = U_i, i = 1, 2, 3$ i.e., the velocity vector. It is convenient to regard the stress σ_{ij} as the sum of inviscid part $-p\sigma_{ij}\delta_{ij}$ and a viscous part σ_{ij} where p is the average of the three normal stresses for any orthogonal set of axes

$$p = \frac{\sigma_{ij}}{3} \delta_{ij} = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}), \quad (2.27)$$

where δ_{ij} is the substitution tensor.

Now separate the second tensor $\frac{\partial U_i}{\partial x_j}$ as follows:

$$\begin{aligned} \frac{\partial U_i}{\partial x_j} &= \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_i}{\partial x_j} \right) + \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_i}{\partial x_i} \right), \\ &= \frac{1}{2} e_{ij} + \frac{1}{2} \gamma_{ij}. \end{aligned} \quad (2.28)$$

e_{ij} is the symmetric tensor while γ_{ij} is the anti-symmetric rotational tensor defining the velocity of the motion. In uniform media viscous stress are induced only by deformation and not by rotation e_{ij} is called rate of strain tensor. It represents the rate of change in the size and shape of a fluid element. The momentum equation now becomes

$$\rho \frac{d\vec{q}}{dt} + (\vec{q} \cdot \nabla)\vec{q} = -\nabla P + \nabla \cdot \sigma + \rho F. \quad (2.29)$$

2.3 Energy Equation

In order to derive energy equation, we need to consider the zeroth and the first law of thermodynamics.

2.3.1 The Zeroth Law of Thermodynamics

There exists the temperature T , such that when two systems that are in contact are in thermal equilibrium, then T is the same in both systems. T is called the absolute temperature.

2.3.2 The First Law of Thermodynamics

This law expresses the principle of conservation of energy such that energy can not be created or destroyed. The law states that there exists a variable of state E such that if a system is transformed from one state of equilibrium to another by the process in which an amount of work W is done on the system by its surroundings and an amount of heat Q is added to the system from the surrounding. The difference between the initial and final values of energy of the systems E_i and E_f is given by

$$E_f - E_i = Q + W, \quad (2.30)$$

in differential form we have

$$dE = dQ + dW, \quad (2.31)$$

or

$$\frac{dE}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}. \quad (2.32)$$

In order to determine the work, we consider the contribution from the component σ_{ij} of stress. The work in unit time is given by

$$\frac{\partial (W \sigma_{xx})}{\partial t} = dydz \left\{ -U \sigma_{xx} + \left(U + \frac{\partial U}{\partial x} dx \right) \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx \right) \right\} = \Delta V \frac{\partial (U \sigma_{xx})}{\partial x}, \quad (2.33)$$

where,

$$\nabla V = dx dy dz .$$

The total work done by the stress per unit mass on deforming elements of fluid ΔV is given by

$$\frac{1}{\rho} \frac{\partial (\sigma_{ij} U_j)}{\partial x_i} = \frac{1}{2} \left(\frac{\partial \sigma_{ij}}{\partial x_i} + \sigma_{ij} \frac{\partial U_j}{\partial x_i} \right), \quad (2.34)$$

where σ_{ij} is the stress acting in the x_i on the force normal lies in the x_j direction

For the equation of fluid motion, we have

$$\rho \frac{\partial U_i}{\partial t} = \frac{\partial (\sigma_{ij})}{\partial x_i} \quad (2.35)$$

and

$$U_j \frac{\partial \sigma_{ij}}{\partial x_i} = \rho \frac{\partial U_i}{\partial t} = \frac{\rho}{2} \frac{d(U_i U_j)}{dt}, \quad (2.36)$$

which is clearly the change in kinematic energy of fluid element following motion. The remaining term in equation (2.35) represents the rate of dissipation of energy per unit mass.

Substituting $\sigma_{ij} = p\delta_{ij} + \mu e_{ij}$,

$$\frac{\partial W}{\partial t} = \frac{1}{2} \frac{\partial(U_i U_j)}{\partial t} - \frac{p}{\rho} \sigma_{ij} \frac{\partial U_j}{\partial x_i} + \nu \frac{\partial U_i}{\partial x_i} \left(\frac{\partial U_i}{\partial x_i} + \frac{U_i}{\partial x_i} \right), \quad (2.37)$$

but
$$\sigma_{ij} \frac{\partial U_i}{\partial x_i} = \frac{\partial U_i}{\partial x_i} = 0.$$

It is clear that only viscous force and not pressure forces contribute to energy dissipation, now define the energy dissipation function Φ by

$$\Phi = \frac{\partial U_i}{\partial x_i} \left(\frac{\partial U_i}{\partial x_i} + \frac{\partial U_j}{\partial x_i} \right) = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_i} + \frac{\partial U_j}{\partial x_i} \right)^2. \quad (2.38)$$

Which shows that $\Phi \geq 0$ and we have

$$\frac{dW}{dt} = \frac{1}{2} \frac{d}{dt} (U_i U_j) + \nu \Phi. \quad (2.39)$$

Similarly the heat transferred to the system from the surrounding is Q . We shall neglect the transfer of heat by radiation and consider only that by conduction. If we consider the element

of the volume, $dv = dx dy dz$ of the mass ρdv , then the change in the total dE is equal to the change in the internal energy $\rho dv e$ and a change in kinetic energy of an amount

$$d \left[\frac{1}{2} \rho dv (U_1^2 + U_2^2 + U_3^2) \right], \quad (2.40)$$

neglecting change in potential energy, we then have

$$\frac{dE}{dt} = \rho dv \left\{ \frac{de}{dt} + \frac{1}{2} \frac{d(U_1^2 + U_2^2 + U_3^2)}{dt} \right\}. \quad (2.41)$$

where e is the thermal energy per unit mass.

2.3.3 Fourier Heat Conduction

This law states that the heat flux q per unit area is proportional to the temperature gradient, i.e.,

$$q = \frac{1}{A} \frac{\partial Q}{\partial t} = -k \frac{\partial T}{\partial n} \quad (2.42)$$

or,

$$q = -k \nabla T, \text{ where } k \text{ is the thermal conductivity}$$

The negative sign signifies that the heat flux is reckoned as positive in the direction of temperature gradient (i.e. heat flux in the direction of decreasing temperature). Hence the

amount of heat transferred into the volume dv through the surface elements which are normal to the x direction is equal to

$$\left(-k \frac{\partial T}{\partial n} \right) dydz, \quad (2.43)$$

The amount of heat leaving the volume is given by

$$\left[k \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx \right] dydz, \quad (2.44)$$

Thus the amount of heat added by conduction in the x direction during time dt to volume dv is

$$dt \cdot dv \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right), \quad (2.45)$$

Hence, the total amount of heat added in all directions is given by

$$\frac{\partial Q}{\partial t} = dv \left[\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) \right], \quad (2.46)$$

Using equations (2.44) and (2.1.3.6), we obtain

$$\rho \frac{\partial e}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + v\Phi, \quad (2.47)$$

where,

$$\Phi = 2 \left\{ \left(\frac{\partial U_1}{\partial x} \right)^2 + \left(\frac{\partial U_2}{\partial y} \right)^2 + \left(\frac{\partial U_3}{\partial z} \right)^2 \right\} + \left(\frac{\partial U_1}{\partial y} + \frac{\partial U_2}{\partial x} \right)^2 + \left(\frac{\partial U_3}{\partial y} + \frac{\partial U_2}{\partial z} \right)^2 + \left(\frac{\partial U_1}{\partial z} + \frac{\partial U_3}{\partial x} \right)^2,$$

Equation (2.47) holds for an incompressible fluid. For perfect fluid,

$$\frac{de}{dt} = C_v \frac{dT}{dt}, (de = C_v dT), \quad (2.48)$$

Equation (2.47) now takes the form

$$\rho C_v \frac{dT}{dt} = \nabla \cdot (k \nabla T) + \mu \Phi + H(T), \quad (2.49)$$

where C_v is the specific heat at constant volume and $H(t)$ is the heat source/sink term added to the equation depending whether it is positive or negative. From differential calculus, the left hand side of equation (2.49) can be written as:

$$\rho C_v \frac{dT}{dt} = \rho C_v \left(\frac{dT}{dt} + (\vec{q} \cdot \nabla) T \right), \quad (2.50)$$

where \vec{q} is the velocity vector.

Incorporating equation (2.50) into (2.49), we obtain the energy balance equation as

$$\rho C_v \left(\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right) = \nabla \cdot (k \nabla T) + \mu \Phi + H(T). \quad (2.51)$$

2.4 Concentration Equation

Mass transfer is another important concept in the field of chemical engineering and science in general. Mass transfer is the process responsible for the conversion of one component of species to another. In many systems, mass transfer process is made up of advection, diffusion and chemical kinetics. Advective transport is a function of the average linear velocity due to fluid flow. Diffusion is the macroscopic result of random thermal motion on a macroscopic scale. For example, in the diagram below, oxygen and nitrogen molecules move in random directions. If there are more oxygen molecules on the left side of the plane A-A than on the right, more molecules will cross to the right than the left: there will be a net flux even though the motion of each individual molecule is completely random.

Following the Fick's Law, the flux is proportional to the gradient in concentration (C) and is given by

$$J = -D\nabla C, \quad (2.52)$$

where

- J = The mass flux. Flux is the movement of objects from one point to another in a given time. The flux is what we are measuring when studying diffusion.
- D = Mass Diffusivity. The diffusivity is the constant that describes how fast or slow an object diffuses.

The chemical kinetics can be the sum of sorption, desorption, decay, abiotic reaction and metabolism by bacteria. It may also be a function of multiple solutes. Moreover, the rate of chemical reaction typically kinetically limit mass transfer processes. Consider a given volume

in which mass transfer process takes place. The change in the number of molecules with time is the difference between the flux going in and the flux going out.

Using equation (2.52), the time rate of accumulation of chemical species in the volume is given as:

$$\frac{dC}{dt} = \nabla \cdot (D \nabla C) + R(C), \quad (2.53)$$

where $R(C)$ term in equation (2.53) represents the conversion of one component species to another by chemical kinetics. When $R(C)$ is negative, the reactant species is been consumed through the process. From differential calculus, the left hand side of equation (2.53) can be written as:

$$\frac{dC}{dt} = \frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla) C, \quad (2.54)$$

where \vec{q} is the velocity vector. Incorporating equation (2.53) into (2.54), we obtain the fundamental equation for advective-dispersive-reactive transport of chemicals and microbes as:

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = \nabla \cdot (D \nabla C) + R(C). \quad (2.55)$$

Finally, the basic fluid equations to be investigated under some assumptions in the following chapters are as follows:

CONTINUITY EQUATION

$$\frac{\partial \rho}{\partial t} + \vec{q} \cdot \nabla \rho = 0.$$

LINEAR MOMENTUM EQUATION

$$\rho \left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right) = \nabla p + \nabla \cdot \sigma + \rho F.$$

ENERGY EQUATION

$$\rho C_p \left(\frac{\partial T}{\partial t} + \vec{q} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \mu \Phi + H(T).$$

CHEMICAL SPECIES CONCENTRATION EQUATION

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = \nabla \cdot (D \nabla C) + R(C).$$

CHAPTER

3

3 Modelling boundary layer flow of a nanofluid past a permeable flat surface with Newtonian heating

3.1 Summary

The boundary layer stagnation point flow of a nanofluid past a permeable flat surface with Newtonian heating is studied. The model used for the nanofluid is the one which incorporates with the effects of Brownian motion and thermophoresis. The basic partial differential equations are reduced to an ordinary differential equation using similarity transformation approach. The model equations for the nanofluid velocity, temperature and nanoparticle concentration distributions depend on six parameters; Prandtl number Pr , Lewis number Le Biot number Bi , the Brownian motion parameter Nb , the thermophoresis parameter Nt , and the suction/ injection parameter β . The expressions for the local skin friction, local Nusselt number and the local Sherwood number are obtained and discussed.

3.2 Introduction

Conventional fluids, such as water, mineral oils, engine oil and ethylene glycol are normally used as heat transfer fluids and play an important role in many industry processes including

power generation, chemical processes, heating and cooling processes, transportation, microelectronics and other micro-sized applications. The poor heat transfer properties of these fluids constitute a limitation in improving the heat transfer augmentation and compactness of the heat exchangers.

Moreover, the use of solid particles as additive suspended into the base fluid is a technique for the heat transfer enhancement. The term nanofluid refers to a solid-liquid mixture with a continuous phase which is a nanometer sized particle dispersed in conventional base fluids. The particles are different from conventional particles (millimetre or micro-scale) in that they keep suspended in the fluid and no sedimentation occurs. Some numerical and experimental studies on nanofluids include thermal conductivity (Kang et al. [74]), separated flow (Abu-Nada [1]) and convective heat transfer (Khanafar et al. [80], Maiga et al. [109], Jou and Tzeng [72], Hwang et al. [66], Tiwari and Das [160], Oztop and Abu-Nada [129], Abu-Nada and Oztop [2] and Muthamilselvan et al. [124]). Daungthongsuk and Wongwises [33] studied the influence of thermophysical properties of nanofluids on the convective heat transfer and summarized various models used in literature for predicting the thermophysical properties of nanofluids. They analyzed the convective heat transfer coefficient in double-tube counter flow heat exchanger in the presence of TiO_2 - water nanofluid. Studies on natural convection using nanofluids are very limited and they are related with differentially heated enclosures. Eastman et al. [41] used pure copper nanoparticle of less than 10nm sized and achieved 40% increase in thermal conductivity for only 0.3% volume fraction of the dispersed in ethylene glycol. They showed with some preliminary experiments with suspended nanoparticle, the conductivity of approximately 60% can be obtained with 5 vol. % CuO nanoparticles in the based fluid of water. Heat transfer coefficient is the determining

factor in the forced convection cooling-heating applications of heat exchange equipments including engines and engine systems. Such enhancements mainly depend upon factors such as particle volume concentration, particle material, particle size, particle shape, base fluid material, temperature and additives. The comprehensive references on nanofluid can be found in the recent book by Das et al. [30] and the review papers by Daungthongsuk and Wongwises [32], Trisaksri and Wongwise [161], Wang and Mujumdar [173], Kaka? and Pramuanjaroenkij [73]. The principal aim of this dissertation is to study the mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with a nanofluid, the base fluid being water. Based on the literature survey only the papers by Nield and Kuznetsov [128] have extended the paper by Cheng and Minkowycz [24] on free convection boundary layer flow past a vertical flat plate embedded in a porous medium to the case when the porous medium is filled with a nanofluid. Nield and Kuznetsov [127] have also studied the classical problem of free convection boundary layer flow of a viscous and incompressible fluid (Newtonian fluid) past a vertical flat plate to the case of nanofluids. In both of these papers, the authors have used the nanofluid model proposed by Buongiorno [20]. Although this author discovered that seven slip mechanisms take place in convective transport in nanofluids, it is only the Brownian diffusion and the thermophoresis that are the most important when the turbulent flow effects are absent, Khan and Pop [79]. Meanwhile, Aziz [8] has previously used a convective boundary condition to study the Blassius flow over a flat plate. Following his paper, several authors have used the convective boundary condition to revisit the problems that were previously studied with the isothermal or the isoflux boundary conditions. A few examples are the papers by Bateller [11], Yao et al [197], Makinde and Aziz [112], and Makinde [110]. Moreover, it is interesting to note that studies related the influence of a convective boundary condition on the boundary layer flow of

nanofluids are still very scarce in the literature. In this chapter, a model of boundary layer stagnation point flow of nanofluids with heat and mass transfer characteristics over a permeable flat surface in the presence of Newtonian heating will be presented, analysed and discussed.

Table 3.1: The physical properties of water, Cu, Al₂O₃ and TiO₂

Physical Property	base fluid (water)	Cu	Al ₂ O ₃	TiO ₂
C _p (J/KgK)	4179	385	765	686.2
ρ (Kg/m ³)	997.1	8933	3970	4250
k((W/mK)	0.613	400	40	8.9538

3.3 Convective transport equations.

Consider steady state two-dimensional boundary layer stagnation point flow of a nanofluid towards a permeable flat surface with a uniform suction velocity $v_m = -V$ as shown in Figure 3.1 below:

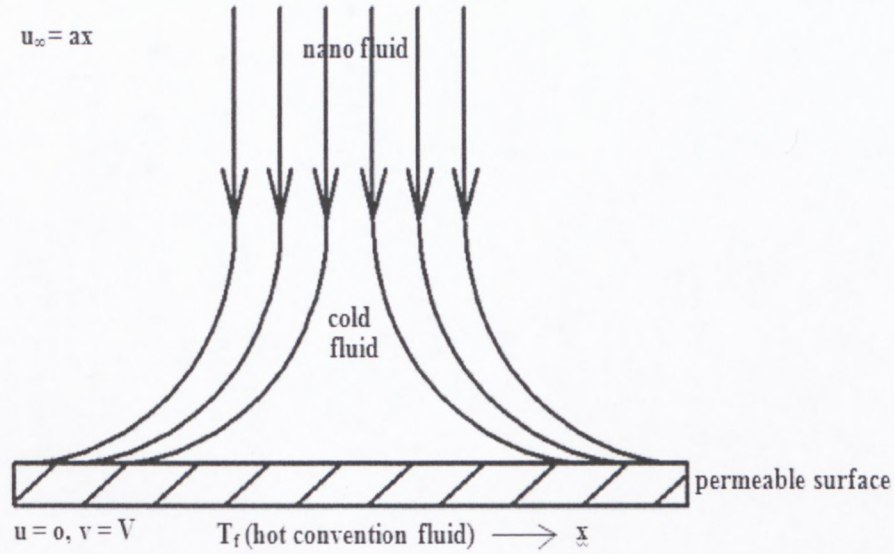


Figure 3.1: Geometry of the problem

The surface temperature T_w , to be determined later, is the result of a convective heating process which is characterised by a temperature T_f and a heat transfer coefficient h . The nanoparticle volume fraction C at the wall is C_w , while at large values of y , the value is C_∞ . The Boungiorno [20] model may be modified for this problem to give the following continuity, momentum, energy and volume fraction equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + v \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (3.2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_B \left(\frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] \right\}, \quad (3.3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (3.4)$$

with boundary conditions at the surface:

$$u = 0, \quad v = V, \quad C = C_w, \quad -k \frac{\partial T}{\partial y} = h(T_f - T), \quad \text{at} \quad y = 0. \quad (3.5)$$

Far away from the wall i.e. as y tends to infinity:

$$u = U_\infty(x), \quad c = C_\infty, \quad T = T_\infty \quad \text{as} \quad y \rightarrow \infty, \quad (3.6)$$

where u and v are the velocity components along the x and y directions respectively, p is the fluid pressure, ρ_f is the density of base fluid, ν is the kinematic viscosity of the base fluid, α is the thermal diffusivity of the base fluid, $\tau = (\rho c)_p / (\rho c)_f$ is the ratio of nanoparticle heat capacity and the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and T is the local temperature, $V > 0$ represents uniform suction, while $V < 0$ represents uniform injection. The case of $V = 0$ corresponds to impermeable plate surface. The subscript ∞ denotes the free stream values at large values of y where the fluid is assumed to be accelerating with the velocity $U_\infty(x) = ax$. we introduce the following dimensionless quantities

$$\eta = y \sqrt{\frac{a}{\nu}}, \quad \psi = (a\nu)^{1/2} x f(\eta), \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}. \quad (3.7)$$

From equation (3.1),

$$\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial y} \left(-\frac{\partial \psi}{\partial x} \right) = 0.$$

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0. \quad (3.8)$$

Hence the continuity equation is satisfied by

$$v = -\frac{\partial \psi}{\partial x}, \quad u = \frac{\partial \psi}{\partial y}, \quad (3.9)$$

now,

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \eta} \frac{\partial \eta}{\partial x}, \quad \text{“and”} \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial \eta} \frac{\partial \eta}{\partial y}, \quad (3.10)$$

then,

$$\frac{\partial \eta}{\partial y} = \sqrt{\frac{a}{\nu}}, \quad \frac{\partial \eta}{\partial x} = 0, \quad (3.11)$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial y} = \sqrt{\frac{a}{\nu}} (a\nu)^{\frac{1}{2}} x f'(\eta),$$

$$= ax f'(\eta),$$

$$v = -\frac{\partial \psi}{\partial x} = - (a\nu)^{1/2} f(\eta), \quad (3.12)$$

where prime symbol represent derivatives with respect to η .

Now
$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y},$$

Recall equation (3.11)

$$\frac{\partial u}{\partial y} = \sqrt{\frac{a}{\nu}} (a\nu)^{1/2} x f'''(\eta). \quad (3.13)$$

$$\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial \eta} \frac{\partial \eta}{\partial y} \left[\sqrt{\frac{a}{\nu}} \cdot a x f'''(\eta) \right], \\ &= \frac{\partial}{\partial \eta} \sqrt{\frac{a}{\nu}} \left[\sqrt{\frac{a}{\nu}} \cdot a x f'''(\eta) \right], \end{aligned}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{a^2 x}{\nu} \cdot f''''(\eta). \quad (3.14)$$

Recall,

$U_\infty(x) = ax$, then differentiating U at ∞ with respect to x

$$U_\infty \frac{dU_\infty}{dx} = ax \cdot a = a^2 x. \quad (3.15)$$

Now substitute equations (3.13), (3.14) and (3.15) into equation (3.2)

$$\text{where } \frac{\partial u}{\partial x} = a f'(\eta) \quad \text{“and”} \quad \frac{\partial^2 u}{\partial x^2} = 0 \quad (3.16)$$

$$a x f''(\eta) \cdot \frac{\partial}{\partial x} (a x f'(\eta)) + (-a\nu)^{1/2} f(\eta) \sqrt{\frac{a}{\nu}} a x f'''(\eta) = a^2 x + \nu^2 \cdot \frac{a^2 x}{\nu^2} f''''(\eta),$$

$$a x f''(\eta) \cdot a f'(\eta) - (a\nu)^{1/2} f(\eta) \cdot \sqrt{\frac{a}{\nu}} \cdot a x f'''(\eta) = a^2 x + \nu^2 \cdot \frac{a^2 x}{\nu} f''''(\eta), \quad (3.17)$$

cancelling ν^2 and x , then the equation below is derived,

$$f''' + ff'' - f'^2 + 1 = 0. \quad (3.18)$$

Similarly,

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial x} = 0, \quad \text{and} \quad \frac{\partial^2 T}{\partial x^2} = 0, \quad (3.19)$$

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial y}, \quad (3.20)$$

but,

$$T = (T_f - T_\infty)\theta + T_\infty. \quad (3.21)$$

Recall equation (3.11),

therefore,

$$\frac{\partial T}{\partial \eta} = \frac{\partial}{\partial \eta} (T_f - T_\infty)\theta + T_\infty,$$

$$\frac{\partial T}{\partial \eta} = (T_f - T_\infty)\theta'(\eta). \quad (3.22)$$

$$\frac{\partial T}{\partial y} = \sqrt{\frac{a}{\nu}} (T_f - T_\infty)\theta'(\eta). \quad (3.23)$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \cdot \sqrt{\frac{a}{\nu}} (T_f - T_\infty)\theta'(\eta),$$

Recall equation (3.10),

therefore,

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial \eta} \sqrt{\frac{a}{\nu}} \cdot \sqrt{\frac{a}{\nu}} (T_f - T_\infty) \theta'(\eta) = \frac{\partial}{\partial \eta} \cdot \frac{a}{\nu} (T_f - T_\infty) \theta'(\eta),$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{a}{\nu} (T_f - T_\infty) \theta''(\eta). \quad (3.24)$$

Similarly;

$$\frac{\partial C}{\partial y} = \frac{\partial C}{\partial \eta} \frac{\partial \eta}{\partial y}, \quad (3.25)$$

$$\text{but } C = (C_w - C_\infty) \phi + C_\infty. \quad (3.26)$$

$$\frac{\partial C}{\partial \eta} = \frac{\partial}{\partial \eta} (C_w - C_\infty) \phi + C_\infty. \quad (3.27)$$

Therefore;

$$\frac{\partial C}{\partial y} = \sqrt{\frac{a}{\nu}} \cdot \frac{\partial}{\partial \eta} ((C_w - C_\infty) \phi + C_\infty),$$

$$\frac{\partial C}{\partial y} = \sqrt{\frac{a}{\nu}} (C_w - C_\infty) \phi'(\eta). \quad (3.28)$$

Substitute equations (3.23), (3.24) and (3.28) into equation (3.3).

Recall equation (3.19)

$$\begin{aligned} -(a\nu)^{1/2} f(\eta) \cdot \sqrt{\frac{a}{\nu}} (T_f - T_\infty) \theta''(\eta) &= \alpha \cdot \frac{a}{\nu} (T_f - T_\infty) \theta''(\eta) + D_B \tau \cdot \sqrt{\frac{a}{\nu}} (C_w - C_\infty) \phi'(\eta) \cdot \sqrt{\frac{a}{\nu}} (T_f - T_\infty) \theta'(\eta) \\ &+ \frac{D_T}{T_\infty} \left[\sqrt{\frac{a}{\nu}} (T_f - T_\infty) \theta'(\eta) \right], \end{aligned} \quad (3.29)$$

$$-af(\eta)\theta'(\eta) = \frac{a}{P_r}\theta''(\eta) + D_B \frac{\tau\alpha}{\nu}(C_w - C_\infty)\phi'(\eta)\theta'(\eta) + \tau \frac{D_T}{T_\infty} \frac{a}{\nu}(T_f - T_\infty)\theta'^2(\eta),$$

$$-af(\eta)\theta'(\eta) = \frac{a}{P_r}\theta''(\eta) + D_B \frac{\tau\alpha}{\nu}(C_w - C_\infty)\phi'(\eta)\theta'(\eta) + \tau \frac{D_T}{T_\infty} \frac{a}{\nu}(T_f - T_\infty)\theta'^2(\eta),$$

after cancelling a

$$-f(\eta)\theta'(\eta) = \frac{1}{P_r}\theta''(\eta) + D_B \frac{\tau}{\nu}(C_w - C_\infty)\phi'(\eta)\theta'(\eta) + \tau \frac{D_T}{T_\infty \nu}(T_f - T_\infty)\theta'^2(\eta). \quad (3.30)$$

But;

$$P_r = \frac{\nu}{\alpha}, \quad Nb = \tau \frac{D_B}{\nu}(C_w - C_\infty), \quad Nt = \tau \frac{D_T}{T_\infty \nu}(T_f - T_\infty). \quad (3.31)$$

Then multiply through by Pr and substitute equation (3.31) into equation (3.30),

$$-Pr(\eta)\theta'(\eta) = \theta''(\eta) + Pr Nb \phi'(\eta)\theta'(\eta) + Pr Nt \theta'^2(\eta), \quad (3.32)$$

$$\theta'' + Pr Nb \phi' \theta' + Pr Nt \theta'^2 + Pr \theta = 0. \quad (3.33)$$

Finally,

$$\frac{\partial C}{\partial x} = \frac{\partial^2 C}{\partial x^2} = 0. \quad (3.34)$$

$$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y} \frac{\partial C}{\partial y}. \quad (3.35)$$

Recall from equation (3.28)

$$\frac{\partial C}{\partial y} = \sqrt{\frac{a}{\nu}} \cdot (C_w - C_\infty) \phi', \quad (3.36)$$

also recall equation (3.10),

$$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y} \cdot \left[\sqrt{\frac{a}{\nu}} \cdot (C_w - C_\infty) \phi'(\eta) \right],$$

$$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial \eta} \frac{\partial \eta}{\partial y} \left[\sqrt{\frac{a}{\nu}} \cdot (C_w - C_\infty) \phi'(\eta) \right] = \frac{\partial}{\partial \eta} \left[\sqrt{\frac{a}{\nu}} \sqrt{\frac{a}{\nu}} (C_w - C_\infty) \phi'(\eta) \right],$$

$$\frac{\partial^2 C}{\partial y^2} = \frac{a}{\nu} (C_w - C_\infty) \phi''(\eta). \quad (3.37)$$

Now substitute equations (3.24) and (3.26) into equation (3.4),

$$\begin{aligned} - (a\nu)^{1/2} f(\eta) \cdot \sqrt{\frac{a}{\nu}} (C_w - C_\infty) \phi'(\eta) &= D_B \frac{a}{\nu} (C_w - C_\infty) \phi''(\eta) + \frac{D_T}{T_\infty} \left[\frac{a}{\nu} (T_f - T_\infty) \theta''(\eta) \right], \\ - f(\eta) \cdot \phi'(\eta) &= D_B \frac{1}{\nu} \phi''(\eta) + \frac{D_T}{T_\infty} \left[\frac{1}{(C_w - C_\infty)} (T_f - T_\infty) \theta''(\eta) \right]. \end{aligned} \quad (3.38)$$

But,

$$Le = \frac{\nu}{D_B}. \quad (3.39)$$

Therefore,

$$\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' = 0. \quad (3.40)$$

The basic model equations to solve are the following set of nonlinear ordinary differential equations:

$$f''' + ff'' + f'^2 + 1 = 0. \quad (3.18)$$

$$\theta'' + \text{Pr} f\theta' + \text{Pr} Nb \phi' \theta' + \text{Pr} Nt \theta'^2 = 0, \quad (3.33)$$

$$\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' = 0, \quad (3.40)$$

Subject to the following boundary conditions;

$$f(0) = -\beta, f'(0) = 0, \theta'(0) = -Bi[1 - \theta(0)], \phi(0) = 1, \quad (3.41)$$

$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0, \quad (3.42)$$

where primes denote differentiation with respect to η and the six parameters appearing in equations (3.31) and (3.39) are defined as follows:

$$\text{Pr} = \frac{\nu}{\alpha}, \quad Le = \frac{\nu}{D_B}, \quad Nb = \frac{(\rho c)_p D_B (\phi_w - \phi_\infty)}{(\rho c)_f \nu T_\infty},$$

$$Nt = \frac{(\rho c)_p D_T (T_f - T_\infty)}{(\rho c)_f \nu T_\infty}, \quad Bi = \frac{h(\nu/a)^{1/2}}{k}, \quad \beta = \frac{V}{\sqrt{a\nu}}. \quad (3.43)$$

In equation (3.40), Pr , Le , Nb , Nt , Bi , and β denote the Prandtl number, the Lewis number, the Brownian motion parameter, the thermophoresis parameter, Biot number and suction ($\beta > 0$)/ injection ($\beta < 0$) parameter, respectively.

3.4 Important Physical quantities.

The reduced Nusselt number Nur and the Sherwood Shr may be found in terms of the dimensionless temperature at the sheet surface, $\theta'(0)$ and the dimensionless concentration at the sheet surface, $\phi'(0)$, respectively i.e.

$$\begin{aligned} Nur &= (\text{Re}_x)^{-\frac{1}{2}} Nu = -\theta'(0), \\ Shr &= (\text{Re}_x)^{-\frac{1}{2}} Sh = -\phi'(0). \end{aligned} \quad (3.44)$$

Where,

$$Nu = \frac{q_w x}{k(T_f - T_\infty)}, Sh = \frac{q_m x}{D_B(C_w - C_\infty)}, \text{Re}_x = \frac{U_\infty x}{\nu}, q_w = -k \frac{\partial T}{\partial y} \Big|_{y=0}, q_m = -D_B \frac{\partial C}{\partial y} \Big|_{y=0}, \quad (3.45)$$

and q_m is the surface heat flux and q_w is the surface mass flux.

Skin friction is given as;

$$\tau_w = \frac{\partial u}{\partial y} \Big|_{y=0}. \quad (3.46)$$

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y} (axf') = \frac{\partial}{\partial \eta} (axf') \frac{\partial \eta}{\partial y}, \\ &= axf'' \sqrt{\frac{a}{\nu}} = \frac{a^{3/2} x}{\nu^{1/2}} f''(\eta). \end{aligned} \quad (3.47)$$

$$C_f = \frac{\tau_w \nu^{1/2}}{a^{3/2} x} = f''|_{\eta=0} = f''(0). \quad (3.48)$$

The above reduced Nusselt number and reduced Sherwood number are derived as follows;

Nusselt number (Heat transfer rate at the plate surface).

Recall equation (3.45),

where;

$$q_w = -\frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial y} = -(T_f - T_\infty) \theta' \sqrt{\frac{a}{\nu}}. \quad (3.48)$$

$$Nu = -x \sqrt{\frac{a}{\nu}} \theta'(0) = -\frac{(xa)^{1/2} x^{1/2}}{\nu^{1/2}} \theta'(0) = -\left(\frac{U_\infty x}{\nu}\right)^{1/2} \theta'(0),$$

$$Nu = -(\text{Re}_x)^{1/2} \theta'(0). \quad (3.49)$$

Therefore

$$Nur = \text{Re}_x^{-1/2} Nu = -\theta'(0). \quad (3.50)$$

Similarly, recall from equation (3.45),

$$Sh = \frac{q_m x}{C_w - C_\infty}.$$

Where,

$$q_m = -\frac{\partial C}{\partial y} \Big|_{y=0},$$

$$q_m = -\frac{\partial C}{\partial \eta} \frac{\partial \eta}{\partial y} = -(C_w - C_\infty) \phi' \sqrt{\frac{a}{\nu}}. \quad (3.51)$$

$$Sh = -x \sqrt{\frac{a}{\nu}} \phi'(0) = -\frac{(xa)^{1/2} x^{1/2}}{\nu^{1/2}} = -\left(\frac{U_\infty x}{\nu}\right)^{1/2} \phi'(0) = -\text{Re}_x^{1/2} \phi'(0),$$

$$Shr = \text{Re}_x^{-1/2} Sh = -\phi'(0). \quad (3.52)$$

CHAPTER

4

4 Computational Analysis of the Model Problem

4.1 Summary

In this chapter, the set of nonlinear coupled ordinary differential equations together with their corresponding boundary conditions modelling the heat and mass transfer characteristics of a nanofluid stagnation point flow over a flat surface obtained in chapter 3 are solved numerically using Maple 12 which uses a fourth-fifth order Runge–kutta- Fehlberg algorithm together with shooting method for solving nonlinear boundary value problems. Numerical results are presented both in tabular and graphical forms illustrating the effects of these parameters on velocity, thermal and concentration boundary layers.

4.2 Numerical Procedure

In Chapter 3, we obtained the following set of nonlinear boundary value problems;

$$f''' + f' - f'^2 + 1 = 0, \quad (3.18)$$

$$\theta'' + \text{Pr} f\theta' + \text{Pr} Nb\phi'\theta' + \text{Pr} Nt\theta'^2 = 0, \quad (3.33)$$

$$\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' = 0, \quad (3.40)$$

Subject to the following boundary conditions;

$$f(0) = -\beta, \quad f'(0) = 0, \quad \theta'(0) = -Bi[1 - \theta(0)], \quad \phi(0) = 1, \quad (3.41)$$

$$f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \quad (3.42)$$

Equations (3.18), (3.33) and (3.40) together with the boundary conditions in equations (3.41) and (3.42) have been solved numerically using fourth-fifth order Runge –Kutta-Fehlberg algorithm together with shooting method implemented on Maple 12. The numerical procedure is as follows;

$$\text{Let; } f = x_1, \quad f' = x_2, \quad f'' = x_3, \quad \theta = x_4, \quad \theta' = x_5, \quad \phi = x_6, \quad \phi' = x_7. \quad (4.1)$$

Equations (3.18), (3.33) and (3.40) are transformed into systems of first order differential equations as follows;

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= x_3, \\ x_3' &= x_2^2 - x_1 x_3, \\ x_4' &= x_5, \\ x_5' &= -Pr x_1 x_5 - Pr Nb x_7 x_5 - Pr Nt x_5^2, \\ x_6' &= x_7, \\ x_7' &= -Lex_1 x_7 + \frac{Nb}{Nt} (Pr x_1 x_5 + Pr Nb x_7 x_5 + Nt x_5^2), \end{aligned} \quad (4.2)$$

Subject to the following initial conditions;

$$x_1(0) = -\beta, \quad x_2(0) = 0, \quad x_3(0) = 0, \quad x_4(0) = s_2,$$

$$x_5(0) = -Bi(1 - s_2), \quad x_6(0) = 1, \quad x_7(0) = s_3. \quad (4.3)$$

In shooting method, the unspecified initial conditions; s_1 , s_2 and s_3 in equation (4.3) are assumed. Equation (4.2) is then integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, improved values of the missing initial conditions must be obtained and the process is repeated. The computations were done by a written program which was implemented on Maple. A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The maximum value of η_∞ to each group of parameters Pr , Le , Nb , Nt , Bi and β is determined when the values of unknown boundary conditions at $\eta = 0$ did not change to successful loop with error with less than 10^{-7} . From the process of numerical computation, the local skin friction coefficient, the reduced Nusselt number Nur and the reduced Sherwood number Shr , which are respectively proportional to $f''(0)$, $-\theta(0)$ and $-\phi'(0)$ are worked out and their numerical values presented in a tabular form. The accuracy of this numerical method was validated by direct comparison with the numerical results reported by Makinde and Aziz [112] for the boundary layer flow of a nanofluid past stretching sheet with a convective modelled as

$$f''' + ff'' - f'^2 + 1 = 0, \quad (3.18)$$

$$\theta'' + Pr f\theta' + Pr Nb\phi'\theta' + Pr Nt\theta'^2 = 0, \quad (3.33)$$

$$\phi'' + Le f\phi' + \frac{Nt}{Nb}\theta'' = 0, \quad (3.40)$$

with

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -Bi[1 - \theta(0)], \quad \phi(0) = 1, \quad (3.41)$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad (3.42)$$

and a perfect agreement is observed as demonstrated in tables (4.1)-(4.3).

Table 4.1: Computations showing comparison with Makinde and Aziz [112] for Nur, Shr and the surface temperature ($\theta(0)$) with $Nt = Nb = 0.5$

Pr	Bi	Le	Nur Makinde & Aziz [112]	Shr Makinde & Aziz [112]	$\theta(0)$ Makinde & Aziz [112]	Nur Present	Shr Present	$\theta(0)$ Present
1	0.1	5	0.0789	1.5477	0.2107	0.0789	1.5477	0.2107
2	0.1	5	0.0806	1.5554	0.1938	0.0806	1.5554	0.1938
5	0.1	5	0.0735	1.5983	0.2655	0.0735	1.5983	0.2655
10	0.1	5	0.0387	1.7293	0.6132	0.0387	1.7293	0.6132
5	1.0	5	0.1476	1.6914	0.8524	0.1476	1.6914	0.8524
5	10.0	5	0.1550	1.7122	0.9845	0.1550	1.7122	0.9845
5	100	5	0.1557	1.7144	0.9984	0.1557	1.7144	0.9984
5	∞	5	0.1557	1.7146	1.0000	0.1557	1.7146	1.0000
5	0.1	10	0.0647	2.3920	0.3531	0.0647	2.3920	0.3531
5	0.1	15	0.0600	2.9899	0.4001	0.0600	2.9899	0.4001
5	0.1	20	0.0570	3.4881	0.4296	0.0570	3.4881	0.4296

Table 4.2: Computations showing comparison with Makinde and Aziz [112] for Nur when $Le=10$, $Pr=10$, $Bi=0.1$

Nt	Nur ($Nb=0.1$) Makinde & Aziz [112]	Nur ($Nb=0.2$) Makinde & Aziz [112]	Nur ($Nb=0.3$) Makinde & Aziz [112]	Nur ($Nb=0.1$) Present	Nur ($Nb=0.2$) Present	Nur ($Nb=0.3$) Present
0.1	0.0929	0.0873	0.0769	0.0929	0.0873	0.0769
0.2	0.0927	0.0868	0.0751	0.0927	0.0868	0.0751
0.3	0.0925	0.0861	0.0729	0.0925	0.0861	0.0729
0.4	0.0923	0.0854	0.0703	0.0923	0.0854	0.0703
0.5	0.0921	0.0845	0.0700	0.0921	0.0845	0.0700

Table 4.3: Computations showing comparison with Makinde and Aziz [112] for Shr when $Le=10$, $Pr=10$, $Bi=0.1$

Nt	Shr ($Nb=0.1$) Makinde & Aziz [112]	Shr ($Nb=0.2$) Makinde & Aziz [112]	Shr ($Nb=0.3$) Makinde & Aziz [112]	Shr ($Nb=0.1$) Present	Shr ($Nb=0.2$) Present	Nur ($Nb=0.3$) Present
0.1	2.2774	2.3109	2.3299	2.2774	2.3109	2.3299
0.2	2.2490	2.3168	2.3569	2.2490	2.3168	2.3569
0.3	2.2228	2.3261	2.3900	2.2228	2.3261	2.3900
0.4	2.1992	2.3392	2.4303	2.1992	2.3392	2.4303
0.5	2.1783	2.3570	2.4792	2.1783	2.3570	2.4792

4.3 Graphical Results

The numerical computations of the results are demonstrated in the Figures 4.1- 4.17 below. In order to have greater insight in the qualitative analysis of the results, we have taken the values of various parameters controlling the flow systems as $1 = Pr = 10$, $-0.3 = \beta = 1$, $5 = Le = 20$, $0.1 = Bi = 10$, $0.1 = Nt = 0.5$ and $0.1 = Nb = 0.5$

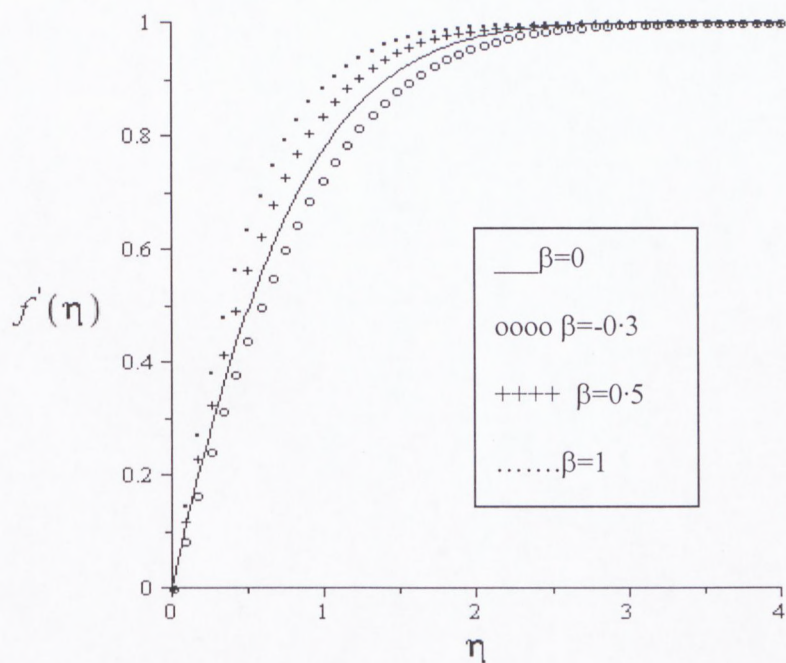


Figure 4.1: Effect of suction/injection parameter on the velocity profiles when $Le=5$, $Pr=5$, $Bi=Nt=Nb=0.1$

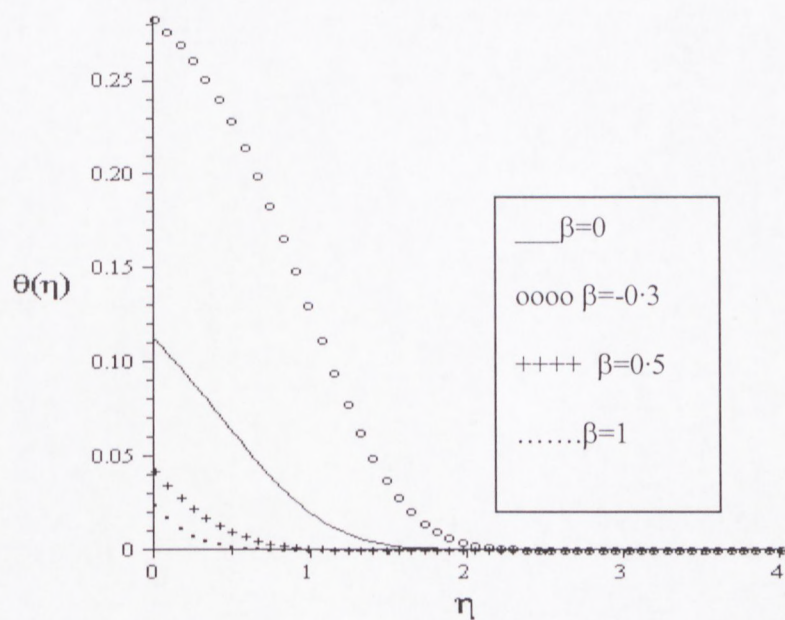


Figure 4.2: Effect of suction/injection parameter on the temperature profiles when $Le=5$, $Pr=5$, $Bi=Nt=Nb=0.1$

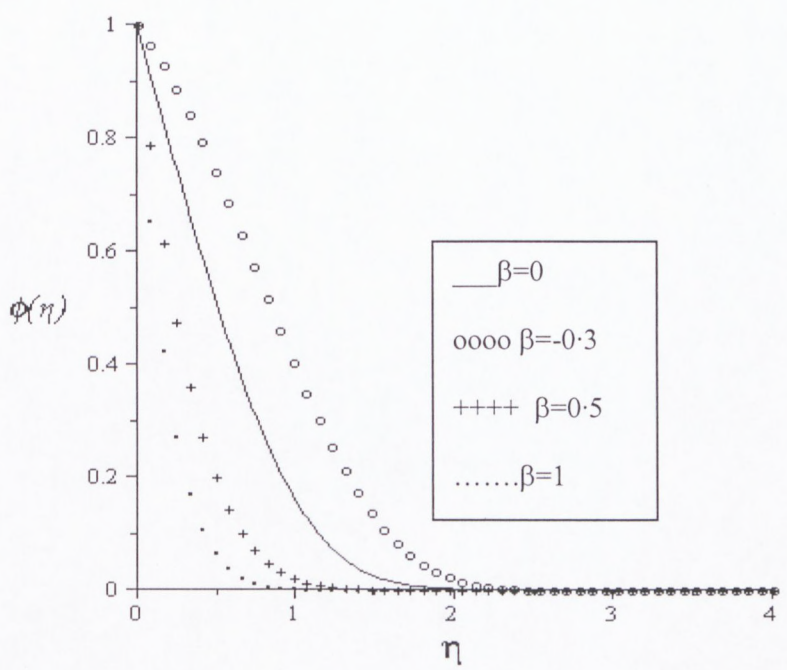


Figure 4.3: Effect of suction/injection parameter on the concentration profiles when $Le=5, Pr=5, Bi=Nt=Nb=0.1$

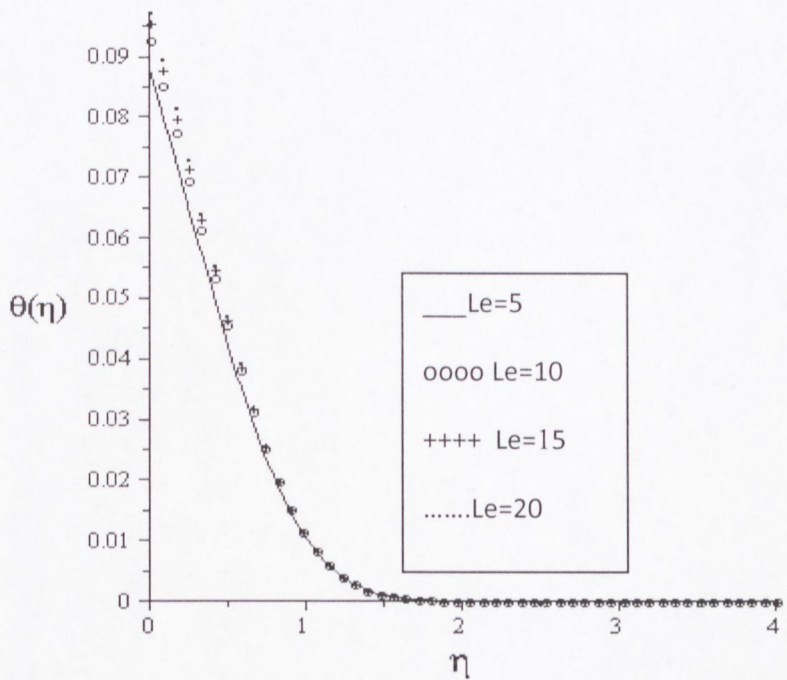


Figure 4.4: Effect of Le on the temperature profiles when $Pr=5, \beta=Bi=Nt=Nb=0.1$

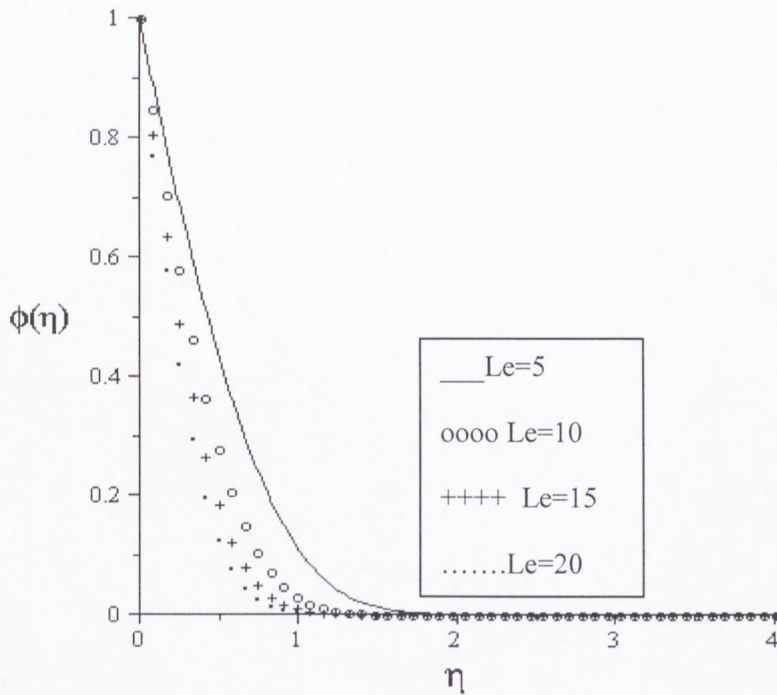


Figure 4.5: Effect of Le on the concentration profiles when $Pr=5$, $\beta=Bi=Nt=Nb=0.1$

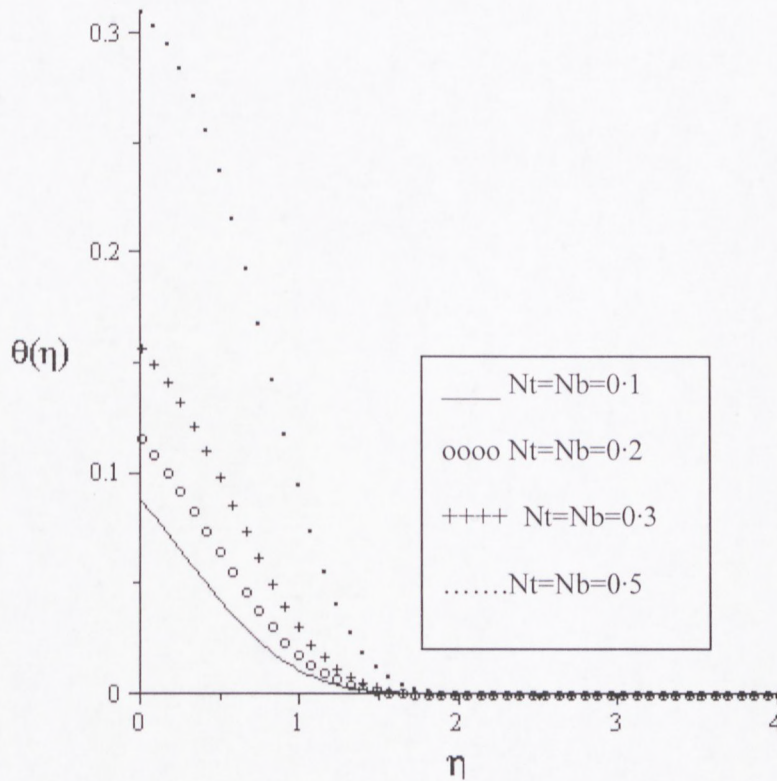


Figure 4.6: Effect of Nt and Nb on the temperature profiles when $Le=Pr=5$, $\beta=Bi=0.1$

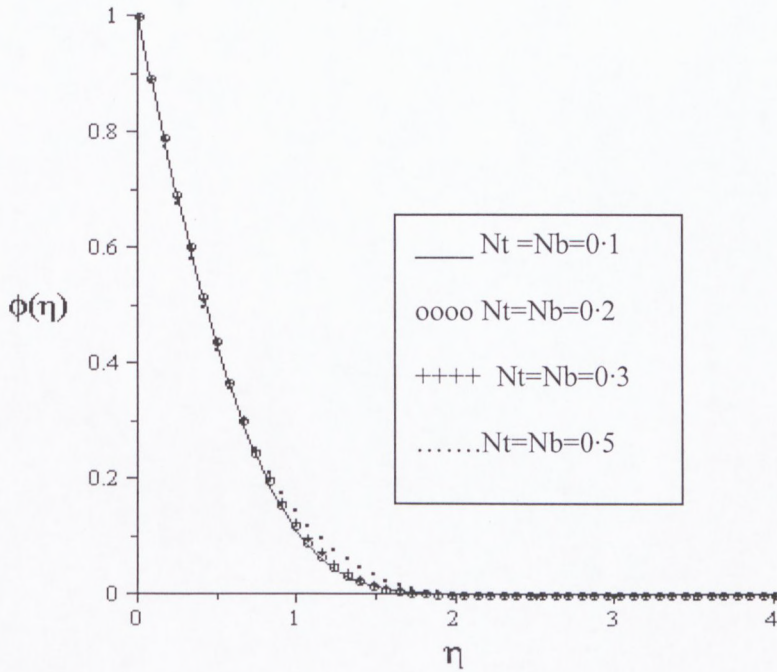


Figure 4.7: Effect of Nt and Nb on the concentration profiles when $Le=Pr=5$, $\beta=Bi=0.1$

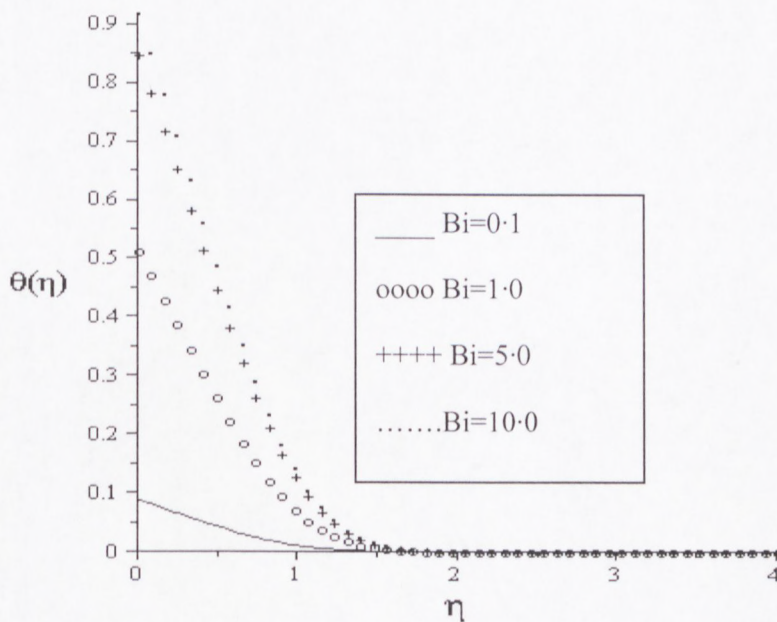


Figure 4.8: Effect of Bi on the temperature profiles when $Le=Pr=5$, $Nt=Nb=\beta=0.1$

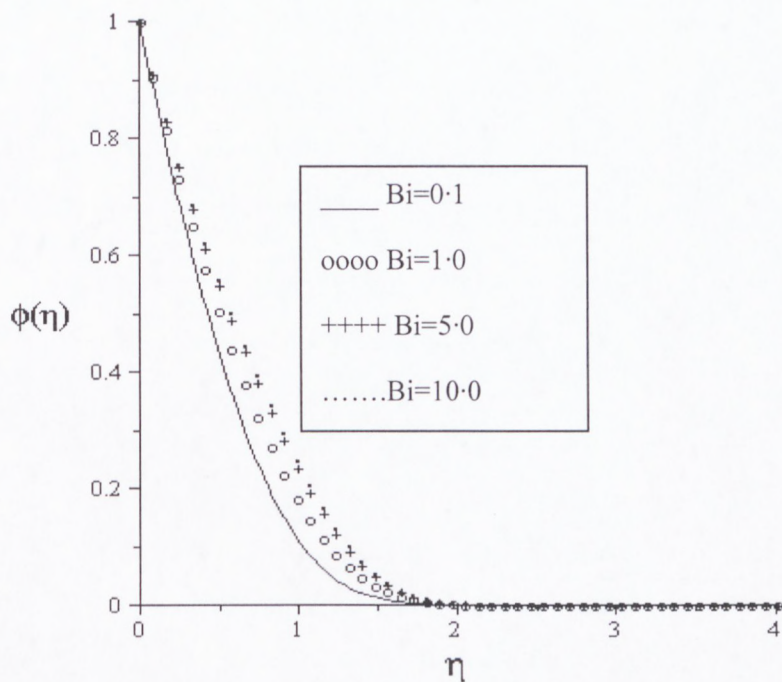


Figure 4.9: Effect of Bi on the concentration profiles when $Le=Pr=5$, $Nt=Nb=\beta=0.1$

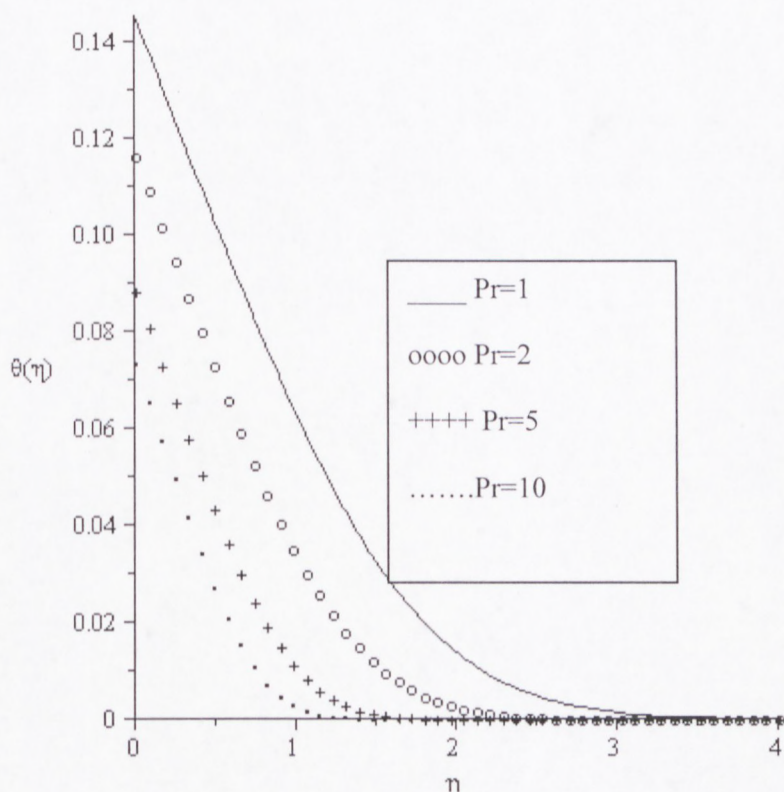


Figure 4.10: Effect of Pr on the temperature profiles when $Le=5$, $Bi=Nt=Nb=\beta=0.1$

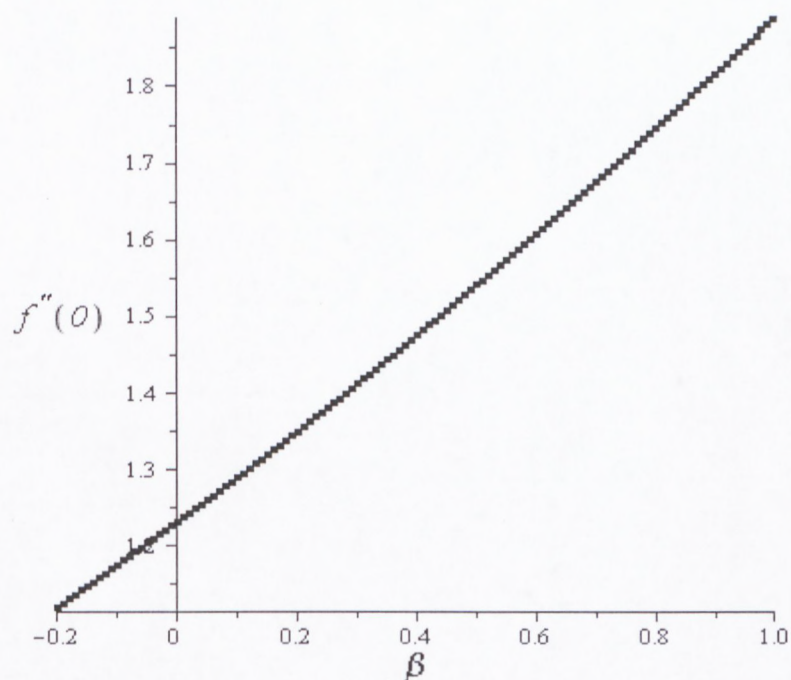


Figure 4.11: Effect of suction / injection parameter β on the skin friction.

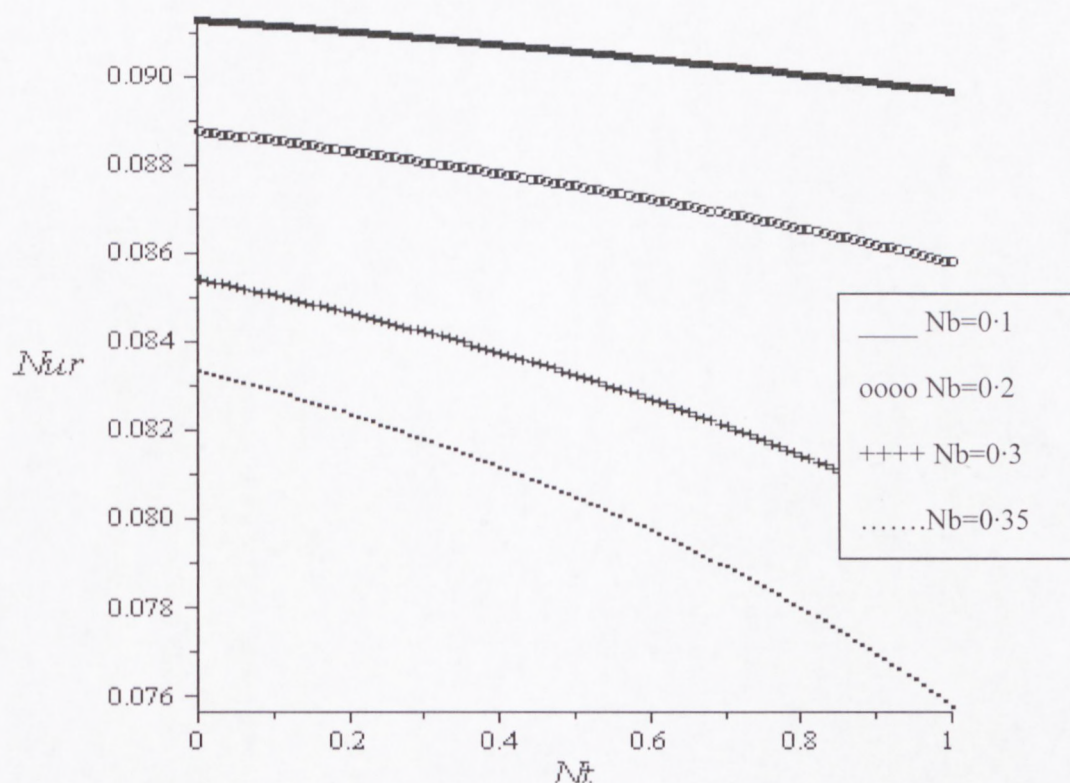


Figure 4.12: Effect of Nt and Nb on the reduced Nusselt number when $Pr=Le=5$, $Bi=\beta=0.1$

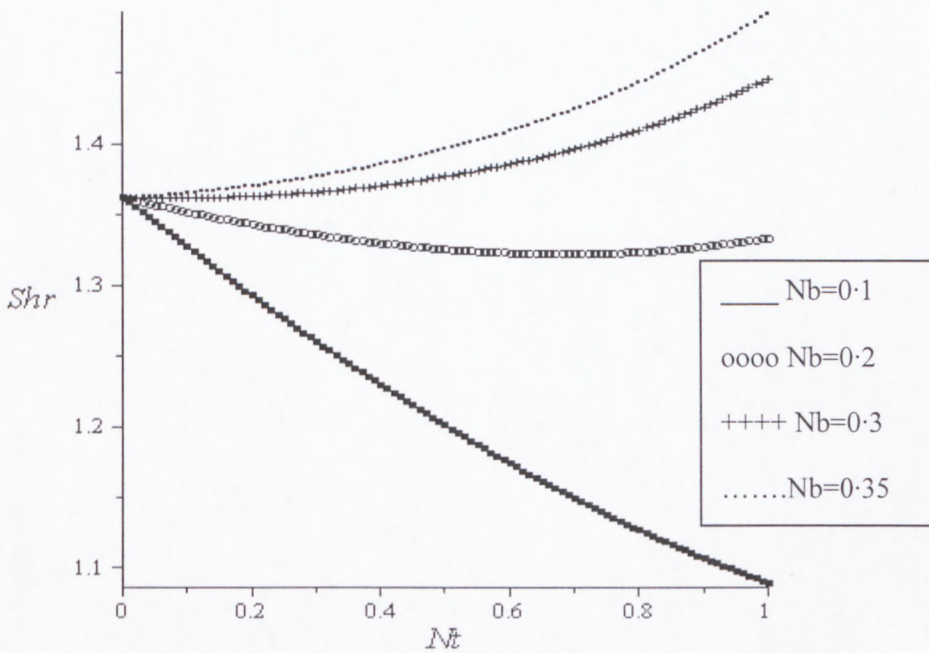


Figure 4.13: Effect of Nt and Nb on the reduced Sherwood number when $Pr=Le=5$, $Bi=\beta=0.1$

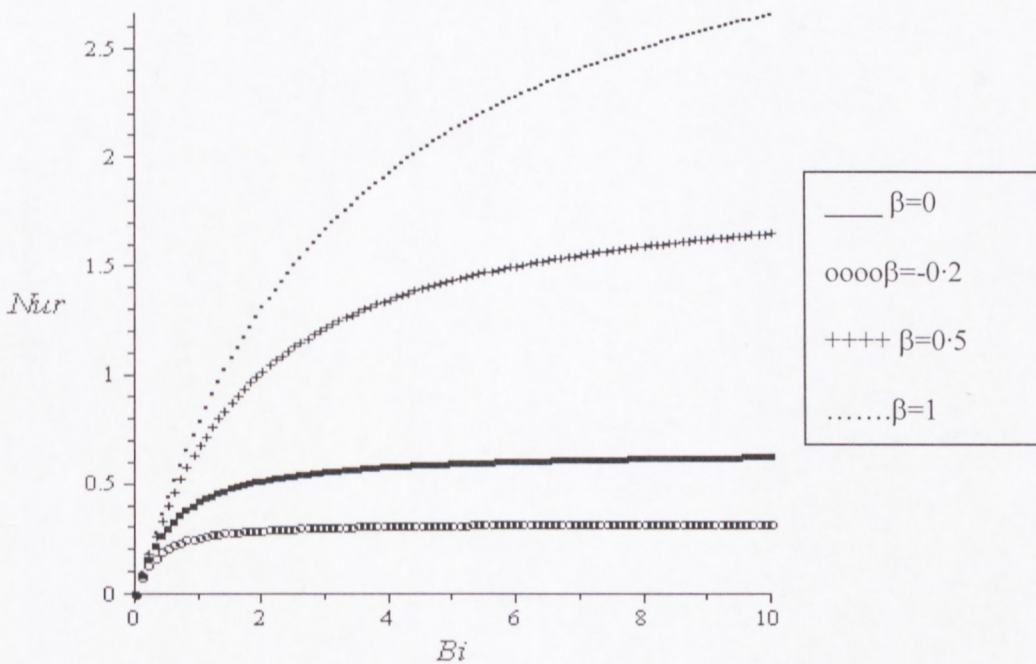


Figure 4.14: Effect of Bi and β on the reduced Nusselt number when $Pr=Le=5$, $Nt=Nb=0.1$

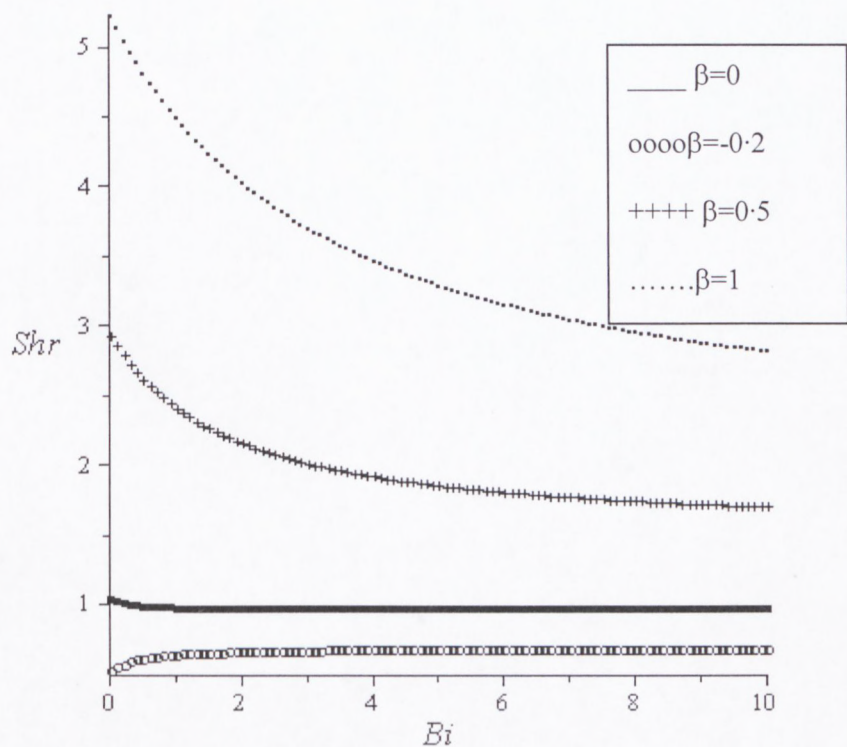


Figure 4.15: Effect of Bi and β on the reduced Sherwood number when $Pr=Le=5$, $Nt=Nb=0.1$

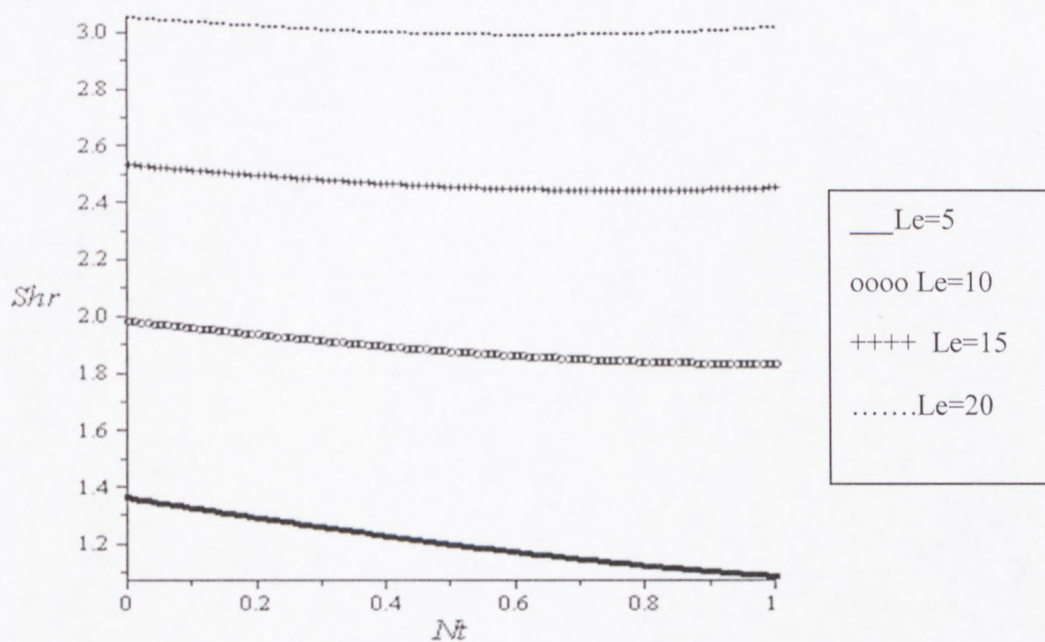


Figure 4.16: Effect of Le on the reduced Sherwood number when $Pr=5$, $\beta=Bi=Nb=0.1$

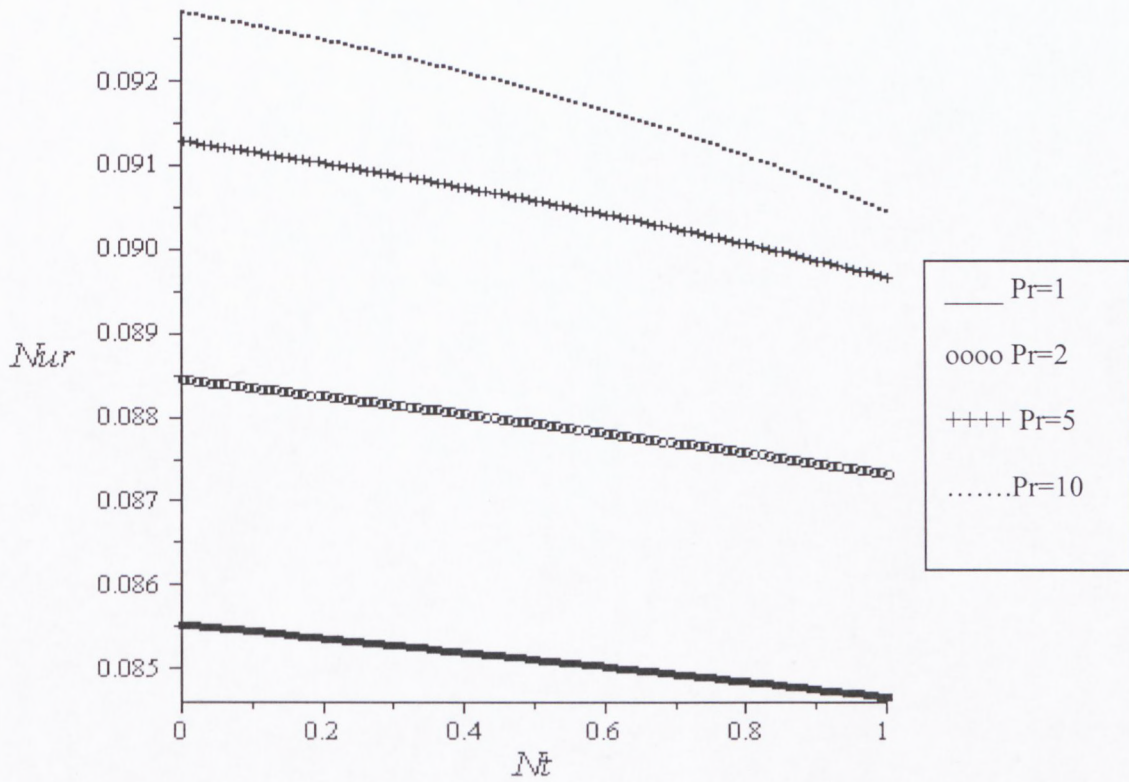


Figure 4.17: Effect of Pr on the reduced Nusselt number when $Le=5$, $\beta=Bi=Nb=0.1$

Detail discussion on the graphical results outlined in section 4.3 will be presented in the following Chapter.

CHAPTER

5

5.1 Analysis and discussion of the results

The numerical computations of the results are demonstrated in the Figures 4.1- 4.17 and Tables 1-3 in chapter 4. In order to have greater insight in the qualitative analysis of the results, the values of various parameters controlling the flow systems as $1 = Pr = 10$, $-0.3 = \beta = 1$, $5 = Le = 20$, $0.1 = Bi = 10$, $0.1 = Nt = 0.5$ and $0.1 = Nb = 0.5$ have been taken. As a further check on the accuracy of the numerical computations of this work, the results of this work are being compared with that of Makinde and Aziz [112] in Tables 4.1- 4.3 and an excellent agreement is established. In this chapter, the graphical results that provide additional insights into the problem under consideration is discussed.

5.2 Discussion of the tables

Table 4.1 contains the comparison with Makinde and Aziz [112] for reduced Nusselt number (Nur), reduced Sherwood number (Shr) and the surface temperature ($\theta(0)$) with thermophoresis parameter (i.e, the rate of heat transfer by the nanoparticle) and Brownian motion parameter (i.e, the rate of Brownian motion) $0.5 (Nt \leq Nb \leq 0.5)$ given as and varying Prandtl number, Lewis number and Biot number. It is clearly seen from the table that

as Prandtl number Pr increases from 1 to 2, it enhances reduced Nusselt number (heat transfer rate) and the reduced Sherwood number (mass transfer rate) while the surface wall temperature reduces. It is interesting to note that as Prandtl number increases from 5 to 10, the heat transfer rate decreases while the mass transfer rate and the surface wall temperature were enhanced. As expected, when the rate of convective heat transfer (Bi) was increased, enhances the rate of heat transfer at the wall and the surface wall temperature while reduces the rate of mass transfer at the wall surface. Furthermore, as we increase the Lewis number, the rate of heat and mass transfer reduces while it enhances the wall surface temperature as expected.

Table 4.2 shows the comparison with Makinde and Aziz[112] for reduced Nusselt number when Lewis number, Prandtl number and the Biot number is given as 10, 10 and 0.1. It was noted that as we increase the thermophoresis and kept the Brownian motion, it was established that the heat transfer rate reduces but the highest values was observed when the Brownian motion Nb is 10.

Table 4.3 contains the comparison with Makinde and Aziz [112] for reduced Sherwood number when Lewis and Prandtl numbers are given as 10 and Biot number is 0.1. The same values of the thermophoresis Nt and Brownian motion Nb was used. As the thermophoresis parameter increases, the rate of mass transfer reduces for $Nb = 0.1$, while for $Nb = 0.2$ and 0.3, it enhances the mass transfer rate. It was interesting to note that the maximum values were obtained for $Nb = 0.3$ different from Table 4.2.2.

5.3 Discussion of the graphs

The samples of velocity, temperature and the nanoparticle volume fraction profiles are given in Figures 4.1- 4.17, respectively. These profiles satisfy the far field boundary conditions (4.12- 4.13) asymptotically, which support the numerical results obtained.

Figure 4.1 shows the effect of suction/ injection parameter β on the velocity profile. For suction, as β increases, the boundary layer thickness increases and as for injection, the velocity boundary layer thickness reduces and thins out. This implies that as suction parameter is been increased, the rate of heat transfer is been enhanced and reduces the surface wall temperature.

Figure 4.2 describes the effect of suction/injection parameter on the temperature profile such that as suction parameter is increased, the thermal boundary layer decreases. as injection increases, the thermal boundary layer increases. This implies that convective transfer of heat between the cold fluid (i.e nanofluid) on top of the surface and the hot fluid (ordinary fluid) below the permeable surface brings about a gradual decrease in temperature of the hot fluid as the cold fluid is being sucked down through the surface, while the temperature increases at the top of the surface as the hot fluid is being injected up to the cold fluid through the surface. The rate of heat transfer also is enhanced but reduces the wall surface temperature.

Figure 4.3 shows the effect of suction/injection parameter on the concentration profile, as suction parameter increases, concentration boundary layer thickness decreases while as injection increases; the concentration boundary layer thickness thickens across the wall plate. This explains that concentration is lower when the cold fluid flows down through the surface

to the hot fluid but higher concentration is observed as the hot fluid flows up to the surface top and becomes cold.

Figure 4.4 shows the effect of Lewis number on temperature profile such that as the Lewis number increases, the thickness of the thermal boundary layer increases only at the wall of the plate. This indicates that as the rate of thermal diffusivity of the fluid increases due to the nanoparticles, the temperature at the surface also increases. An increase in Lewis number causes the reduced Nusselt number to decrease but enhances the reduced Sherwood number and the surface temperature

Figure 4.5 discusses the significant effect of the Lewis number on concentration distribution. As the Lewis number increases, the volume fraction boundary layer thickness thins out across the wall plate. It implies that as the thermal diffusivity of the fluid increases, which also increases the temperature of the surface, the concentration of the fluid becomes very low. Also, the rate of mass transfer at the wall surface is enhanced and the concentration across the wall surface plate is reduced.

Figure 4.6 shows temperature distribution in the thermal boundary layer for different values of the Brownian motion and the thermophoresis parameter. As both N_b and N_t increase, the boundary layer thickness increases, as noted earlier in discussing the tabular data, the surface temperature increases, and the curves become less steep indicating a diminution of the reduced Nusselt number.

Figure 4.7 shows the curve of concentration profile against the coordinate η for various values of the rate of Brownian motion Nb and thermophoresis Nt parameters. From the figure, it was observed that as the thermophoresis and Brownian motion parameters increases, significant increase in concentration boundary layer thickness away from the wall plate is observed.

Figure 4.8 shows the effect of Biot number on the thermal boundary layer which indicates that as it increases; the thermal boundary layer thickness increases across the plate. This implies that convective heat transfer is enhanced as convective heat transfer coefficient increases, which also increases the thermal penetration as expected.

In figure 4.9, because of the concentration distribution which is driven by the temperature field, it is suggested that an increase in Biot number, i.e the convective heat transfer coefficient, increases the rate of penetration of the concentration. Which implies that as the Biot number increases, the concentration increases as well.

Figure 4.10 represents the effect of Prandtl number on thermal boundary layer thickness as it increases. The thermal boundary layer thickness decreases as the curves become increasingly steeper. As a result of this, the reduced Nusselt number, being proportional to the initial slope, increases. This pattern is reminiscent of the free convective boundary layer flow in a regular fluid.

Figure 4.11 represents the effect of suction/injection on the skin-friction. From the graph, it is noted that suction/injection has no influence on the skin-friction coefficient. It is clearly

seen that as suction parameter increases across the plate the skin-friction coefficient increases. Similarly as injection increases, the skin-friction decreases across the plate.

Figure 4.12 shows the effect of the Brownian motion parameter and the thermophoresis parameter on the reduced Nusselt number. As the Brownian motion parameter increases, the thermal boundary layer increases leading to a decrease in the rate of heat transfer. Also as the thermophoresis parameter increases, the fluid temperature decreases leading to an increase in the heat transfer rate across the plate surface.

Figure 4.13 shows the effect of Brownian motion parameter and the thermophoresis parameter for reduced Sherwood number. As the Brownian motion parameter increases from 0.1 to 0.2, the rate of mass transfer decreases across the wall plate as thermophoresis parameter increases. It is interesting to note that as the Brownian motion parameter increases from 0.3 to 0.35, the mass transfer rate across the wall plate increases. As the thermophoresis parameter increases, the fluid concentration decreases to an increase in the mass transfer rate across the plate surface.

Figure 4.14 shows the effect of suction/injection and Biot number on reduced Nusselt number. As the suction parameter increases, the rate of heat transfer increases across the wall plate and the convective heat transfer (Biot number) also increases. Similarly for injection parameter, reverse was the case. Also as the Biot number increases, the fluid temperature increases leading to a decrease in the heat transfer rate across the plate surface.

Figure 4.15 shows the effect of the suction/injection parameter and the Biot number on the reduced Sherwood number. As suction parameter is increasing, the rate of mass transfer increases across the wall plate increases. As injection parameter increases, the mass transfer rate across the wall plate decreases and the convective transfer rate is increased. It is interesting to note that for each values of suction/injection parameter used, the mass transfer rate decreases across the wall plate. And the convective heat transfer rate increases across the wall plate.

Figure 4.16 shows the effect of Lewis number and the thermophoresis parameter on the reduced Sherwood number. As the Lewis number increases, the rate of mass transfer increases across the wall plate as the thermophoresis parameter increases. It is interesting to note that as the Lewis number increases, the mass transfer rate across the wall plate increases. And as the thermophoresis parameter increases, the fluid concentration decreases to an increase in the mass transfer rate across the plate surface.

Figure 4.17 shows the effect of Prandtl number and thermophoresis parameter on the reduced Nusselt. As the Prandtl number increases, the thermal boundary layers decreases leading to an increase in the rate of heat transfer. Also as the thermophoresis parameter increases, the fluid temperature increases leading to a decrease in the heat transfer rate across the plate surface.

5.4 Conclusions

We examined the theoretical analysis of boundary layer flow of nanofluid with the characteristics of heat and mass transfer with suction/injection. The governing momentum, energy and concentration equations are solved using the fourth-fifth order of Runge- Kutta method alongside with shooting iteration scheme and our results show that the local skin friction, heat transfer rate and the mass transfer rate were affected by the embedded flow parameters. It is interesting to note that as Brownian motion parameter increases, the mass transfer rate decreases. Similarly, increasing the suction parameter increases the heat transfer rate and the convective heat transfer. It is noteworthy to mention that as the Lewis number increases, the mass transfer rate (Sherwood number) and the thermophoresis parameter increases across the wall plate. Enhanced Prandtl number decreases the thermal boundary layer thickness across the wall plate.

BIBLIOGRAPHY

- [1]. Abu-Nada, E. (2007), 'Application of Nanofluid for Heat Transfer Enhancement of Separated flows encountered in a backward facing step' *International Journal of Heat Fluid Flow* 29:242-249.
- [2]. Abu-Nada, E. and Oztop H. F. (2009), 'Effects of Inclination angle on Natural Convection in enclosures filled with Cu-Water nanofluid', *International Journal of Heat and Fluid Flow* 30:669-678.
- [3]. Akbarinia, A. and Behzadmehr, A. (2007), 'Numerical Study of Laminar mixed Convection of a nanofluid in Horizontal Curved tubes', *Applied Thermal Engineering* 27:1327-1337.
- [4]. Ali, A., Vafai, K. and Khaled, A.-R. A. (2003), 'Comparative Study between Parallel and Counter Flow Configurations between Air and Falling Film Desiccant in the Presence of Nanoparticles Suspensions', *International Journal of Energy Research* 27:725-745.
- [5]. Ali, A., Vafai, K. and Khaled, A.-R. A. (2004), 'Analysis of heat and Mass Transfer between Air and Falling film in a cross Flow Configuration', *International Journal of Heat and Mass Transfer* 47:743-755.
- [6]. Assael, M. J., Chen, C. F., Metaxa, I. and Wakeham, W. A. (2004), 'Thermal Conductivity of Suspensions of Carbon nanotubes in water', *International Journal of Thermophysics* 25:971-985.
- [7]. Assael, M. J., Metaxa, I., Arvanitidis, J., Christofilos, D. and Lioutas, C. (2005), 'Thermal Conductivity Enhancement in Aqueous Suspensions of Carbon Multi-Walled and Double Walled Nanotubes in the presence of Two Different Dispersants' *International Journal of Thermophysics* 26:647-664.
- [8]. Aziz, A. (2009), 'A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition', *Communication in Nonlinear science and Numerical Simulation*.14:1064-1068.
- [9]. Bang, I. C. and Chang S, H. (2005), 'Boiling Heat transfer Performance and Phenomena of Al₂O₃-water nanofluids from a plain Surface in a pool', *International Journal of Heat and Mass Transfer* 48:2407-2419.
- [10]. Batchelor, G. K. (1977), 'The Effect of Brownian Motion on the Bulk stress in a Suspension of Spherical Particles', *Journal of Fluid Mechanics* 83:97-117.
- [11]. Bateller, R. C. (2008), 'Radiation effects for the Blasius and Sakiadis flow with a convective surface boundary condition', *Applied Mathematics and Computation* 206:832-840.

- [12]. Bég, O. A. and Makinde, O.D. (2011), 'Viscoelastic flow and species transfer in a Darcian high- permeability channel,' *Journal of Petroleum science and Engineering*. 76(3-4):93-99.
- [13]. Benveniste, Y. (1987), 'Effective Thermal Conductivity of Composites with a Thermal Contact Resistance between the Constituents: Nondilute case', *Journal of Applied Physics* 61:2840-2843.
- [14]. Benveniste, Y. and Miloh, T. (1991), 'Effective Thermal Conductivity of Coated Short-Fibre Composites', *Journal of Applied Physics* 69:1337-1334.
- [15]. Bhattacharya, P., Saha, S. K., Yadav, A., Phelan, P.E. and Prasher, R. S. (2004), 'Brownian Dynamics Simulation to determine the Effective Thermal Conductivity of Nanofluids', *Journal of Applied Physics* 95:6492-6494.
- [16]. Bilboul, R. R. (1969), 'A note on the permittivity of a double-layer Ellipsoid', *British Journal of Applied Physics* 2:921-923.
- [17]. Böttcher, C.J. F. (1945), 'The Dielectric Constant of Crystalline powders', *Recueil des Travaux Chimiques des Pays- Bas* 64:47-51.
- [18]. Brinkman, H. C. (1952), 'The viscosity of Concentrated Suspensions and Solutions', *The Journal of Chemical Physics* 20:571.
- [19]. Bruggeman, D. A. G. (1935), 'Berechnung verschiedener physikalischer Konstanten von heterogenen Substanzen: I.', *Dielektrizitätskonstanten und Leitfähigkeiten der Mischkörper aus isotropen Substanzen, annalen der Physik* 24:636-664.
- [20]. Buonigiorno, J. (2006), 'Convective Transport in Nanofluids transactions of the ASME', *Journal of Heat Transfer* 128:240-250.
- [21]. Cao, H. L., Qian, X. F., Gong, Q., Du, W. M., Ma, X. D. and Zhu, Z. K. (2006), 'Shape- and size- Controlled Synthesis of nanometer ZnO from a Simple Solution Route at Room Temperature,' *Nanotechnology* 17:3632-3636.
- [22]. Cao, Q. and Tavares J. (2006), 'Dual- plasma Synthesis of coated nanoparticles and nanofluids', <http://aiche.confex.com/aiche/2006/teachprogram/P65561.HTM>.
- [23]. Chang, H., Tsung, T. T., Yang, Y. C., Chen, L. C., Lin, H. M., Lin, C. K. and Jwo, C.S. (2005), 'Nanoparticle Suspension Preparation Using the Arc Spray Nanoparticles Synthesis System combined with Ultrasonic Vibration and Rotating Electrode', *The International Journal of Advanced Manufacturing Technology* 26:552-558.
- [24]. Cheng, P. and Minkowycz, W. J. (1977), 'Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike', *Journal of Geophysical Research* 84:2040-2044.

- [25]. Choi, S. U. S., Zhang, Z. G., Yu, W., Lockwood, F. E. and Grulke, E. A. (2001), 'Anomalous Thermal Conductivity Enhancement in nanotube Suspensions', *Applied Physics Letters* 79:2252-2254.
- [26]. Chon, C. H., Kihm, K.D., Lee, S. P. and Choi, S. U. S. (2005), 'Empirical Correlation finding the Role of Temperature and Particle Size for Nanofluid (Al_2O_3) Thermal Conductivity Enhancement', *Applied Physics Letters* 87:153107.
- [27]. Chopkar, M., Das, P. K. and Manna, I. (2006), 'Synthesis and Characterization of a nanofluid for advanced Heat Transfer applications', *Scripta Materialia* 55:549-552.
- [28]. Cyril, O., Diedra, H. and Black, J. (2010), 'Review of literature on nanofluid flow and heat transfer properties', *Journal of selected Areas in Nanotechnology. (JSAN)*.
- [29]. Das, S. K., Putra, N., Thiesen, P. and Roetzel, W. (2003), 'Temperature Dependence of Thermal Conductivity Enhancement for Nanofluids', *Journal of Heat Transfer* 125:567-574.
- [30]. Das, S. K., Putra, N. and Roetzel, W. (2003a), 'Pool Boiling Characteristics of nanofluids', *International Journal of Heat and Mass Transfer* 46:851-862.
- [31]. Das, S. K., Putra, N. and Roetzel, W. (2003b), 'Pool Boiling of nanofluids on Horizontal Narrow Tubes', *International Journal of Multiphase Flow* 29:1237-1247.
- [32]. Daungthongsuk, W. and Wongwise, S. (2007), 'A Critical Review of Convective Heat Transfer of Nanofluid. Renewable Sustainable Energy', *Review* 11:797-817.
- [33]. Daungthongsuk, W. and Wongwise, S. (2008), 'Effect of Thermophysical Properties Models on the predicting of the Convective Heat Transfer Coefficient for Low Concentration Nanofluid', *International Communication in Heat and Mass Transfer* 35:1320-1326.
- [34]. David, E., Lauriat, G. and Cheng, P. (1991), 'A Numerical Solution of Variable Porosity Effects on Natural Convection in Packed -Spheric Cavity', *Journal of heat transfer* 113:391-399.
- [35]. Davis, R. H. (1986), 'The Effective Thermal Conductivity of a composite Material with Spherical Inclusions', *International Journal of Thermophysics* 7:609-620.
- [36]. De Bruijin, H. (1942), 'The Viscosity of Suspension of Spherical Particles', *Recueil des travaux chimiques des Pays- Bas* 61:863-874.
- [37]. Ding, Y. and Wen D. (2005), 'Particle Migration in a flow of Nanoparticle suspensions', *Power Technology (Institute of Particle Science and Engineering, University of Leeds)* 149:84-94.
- [38]. Ding, Y., Alias, H., Wen, D. and Williams, A. R. (2006), 'Heat Transfer of Aqueous Suspensions of Carbon Nanotubes (CNT nanofluids)', *International Journal of Heat and Mass Transfer* 49:240-250.

- [39]. Dittus, F. W. and. Boelter I. M. K. (1930), 'Heat Transfer in Automobile Radiators of the Tubular Type', *University of California Publications in Engineering* 2:443-461.
- [40]. Dongsheng, W., Lin, G., Vafaei, S. and Zhang, K. (2009), 'Review of nanofluids for heat transfer applications', *International Journal of Heat and Fluid flow* 7:141-150.
- [41]. Eastman, J. A., Choi, S. U. S., Li, S., Yu, W. and Thompson, L. J. (2001), 'Anomalous Increase Effective Thermal Conductivity of Ethylene Glycol-Based Nanofluids Containing Copper Nanoparticles', *Applied Physics Letter* 78:718-720.
- [42]. Eilers, V. H. (1941), 'Die Viskosität von Emulsionen hochviskoser Stoffe als Funktion der Konzentration', *Kolloid- Zeitschrift* 97:313-321.
- [43]. Eistein, A. (1906), 'Eine neue Bestimmung der Moleküldimensionen', *Annalen der Physik* 19:289-306.
- [44]. Eshelby, J. D. (1957), 'The Determination of the Elastic Field of an Ellipsoidal Inclusion, and Related Problems', *Proceedings of Royal Society of London* 241:376396.
- [45]. Evans, W., Fish, J. and Koblinski, P. (2006), 'Role of Brownian Motion Hydrodynamics on nanofluid Thermal Conductivity', *Applied Physics Letters* 88:093116.
- [46]. Faulkner, D. J., Rector, D. R., Davidson, J. and Shekarriz, R. (2004), 'Enhanced Heat Transfer through the use of Nanofluids in Forced Convection', *Proceedings of IMECE Anaheim California, USA*.
- [47]. Fox, V. G. (1969), 'The laminar boundary layer on a moving continuous flat sheet immersed in a non-Newton fluid', *American Institute of Chemical Engineers* 15:327-333.
- [48]. Frankel, N. A., and Acrivos, A. (1967), 'On the viscosity of Concentrated Suspension of Solid Spheres', *Chemical Engineering Science* 22:847-853.
- [49]. Fricke, H. (1924), 'A mathematical Treatment of the Electric Conductivity and Capacity of Disperse Systems', *International Journal of the Electric Conductivity of a Suspension of Homogeneous Spheroids. Physical review* 24:575-587.
- [50]. Fricke, H. (1953), 'The Maxwell-Wagner Dispersion in a Suspension of Ellipsoids', *Journal of Physical Chemistry* 57:934-937.
- [51]. Gnielinski, V. (1976), 'New Equations for Heat and Mass Transfer in Turbulent Pipe and Channel Flow', *International Chemical Engineering* 16:359-368.
- [52]. Gosselin, L. and Da Silva, A. K. (2004), 'Combined "Heat Transfer and Power Dissipation" Optimization of nanofluid Flows', *Applied Physics Letters* 85:4160-4162.

- [53]. Graham, A. L. (1981), 'On the viscosity of Suspensions of solid Spheres', *Applied Scientific Research* 37:275-286.
- [54]. Granqvist, C. G. and Buhrman R. A. (1976), 'Ultrafine Metal Particles', *Journal of Applied Physics* 47:2200-2219.
- [55]. Granqvist, C. G. and Hunderi, O. (1977), 'Optical properties of Ultrafine Gold Particles', *Physical Review* 16:3513-3534.
- [56]. Granqvist, C. G. and Hunderi, O. (1978), 'Conductivity of In Homogeneous Materials: Effective-Medium Theory with Dipole-Dipole Interaction,' *Physical Review* 18:1554-1561.
- [57]. Gupte, S. K., Advani, G.S. and Huq, P. (1995), 'Role of Micro-Convection due to Non-Affine Motion of Particles in a Mono-Disperse Suspension', *International Journal of Heat and Mass Transfer* 38:2945-2958.
- [58]. Hakan, F. O. and Abu-Nada, E. (2008), 'The numerical study of Natural Convection in partially heated Rectangular enclosure filled with Nanofluid', *International Journal Heat and Fluid Flow* 29:1326-1336.
- [59]. Hamad, M. A. A., Pop, I. and Md Ishmail, A. I. (2010), 'Magnetic Field Effects on free Convection flow of a Nanofluid past a vertical semi- infinite flat plate', *Nonlinear Analysis Real world application* 12:1338-1346.
- [60]. Hamilton, R. L. and Crosser, O. K. (1962), 'Thermal Conductivity of Heterogeneous Two-plate component Systems', *Industrial and Engineering Chemistry Fundamentals* 1:187-191.
- [61]. Hasselman, D. P. H. and Johnson L. F. (1987), 'Effective Thermal Conductivity of Composites with Thermal Barrier Resistance', *Journal of Composites Materials* 21:508-515.
- [62]. Henderson, J. R. and Van Swol, F. (1984), 'On the Interface between a Fluid and a Planner Wall', Theory and Simulations of a Hard sphere Fluid at a Hard Wall', *Molecular Physics* 51:991-1010.
- [63]. Heris, S. Z., Etemad, S. Gh. and Esfahany, N. M. (2006), 'Experimental Investigation of Oxide nanofluids Laminar Flow Convective Heat Transfer', *International Communications in Heat and Mass Transfer* 33:529-535.
- [64]. Hong, T.-K., Yang, H. -S. and Choi, C. J. (2005), 'Study of the Enhanced Thermal Conductivity of Fe Nanofluids', *Journal of applied Physics* 97:064311.
- [65]. Hong, K. S., Hong, T. -K. and Yang, H. -S. (2006), 'Thermal Conductivity of Fe Nanofluids Depending on the Cluster size of Nanoparticles', *Applied Physics Letters* 88:031901.
- [66]. Hwang, Y., Park, H. S., Lee, J. K. and Jung, W. H. (2006), 'Thermal Conductivity and Lubrication Characteristics of Nanofluids', *Current Applied Physics* 6:67-71.

- [67]. Hwang, K.S., Lee, J, -H. and Jang, P. S. (2007), 'Buoyancy driven Heat Transfer of Water- based Al₂O₃ nanofluids in a Rectangular Cavity', *International Journal of Heat and Mass Transfer* 50:4003-4010.
- [68]. Ibrahim, S. Y. and Makinde, O. D. (2010), 'Chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction', *Scientific Research and Essays* 5: 2875-2882.
- [69]. Jang, S. P. and Choi, S. U. S. (2004), 'Role of Brownian motion in the Enhanced Thermal Conductivity of Nanofluids', *Applied Physics Letters* 84:4316-4318.
- [70]. Jeffrey, D. J. (1973), 'Conduction through a Random Suspension of Spheres', *The Proceedings of Royal Society of London* 335:355-367.
- [71]. Jordan, A., Scholz, R., Wust, P., Föhling, H. and Felix, R. (1999), 'Magnetic Fluid Hyperthermia (MFH): Cancer Treatment with AC Magnetic Field Induced Excitation of Biocompatible Super paramagnetic Nanoparticles', *Journal of Magnetism and Magnetic Materials* 201:413-419.
- [72]. Jou, R. -Y. and Tzeng, S. C. (2006), 'Numerical Research of Nature Convective Heat Transfer Enhancement Filled with Nanofluids in Rectangular Enclosures', *International Communications in Heat and Mass Transfer* 33:727-736.
- [73]. Kaka? , S. and Pramuanjaroenkij, A. (2009), 'Review of convective heat transfer enhancement with nanofluids', *International Journal of Thermal Science* 52:3187-3196.
- [74]. Kang, H. U., Kim, S. H. and Oh, J. M. (2006), 'Estimation of Thermal Conductivity of Nanofluid Using Experimental Effective Particle Volume', *Experimental Heat Transfer* 19:181-191.
- [75]. Keblinski, P. and Thomin, J. (2006), 'Hydrodynamic Field around a Brownian Particle', *Physical Review* 73:010502.
- [76]. Kenneth, S. S., Mingming, F. and Taeghwan, H. (1996), 'Sonochemical Synthesis of Iron colloids', *Journal of the American Chemical society* 118:11960-11961.
- [77]. Kerner, E. H. (1956), 'The Electrical Conductivity of Composite Media', *The Proceedings of the Physical Society* 69:802-807.
- [78]. Khaled, A.-R. A. and Vafai, K. (2005), 'Heat Transfer Enhancement through Control of Thermal Dispersion Effects', *International Journal of Heat and Mass Transfer* 48: 2172-2185.
- [79]. Khan, W. A. and Pop, I. (2010), 'Boundary layer flow of a nanofluid past a stretching sheet', *International Journal of heat and mass transfer* 53:2477-2483.

- [80]. Khanafer, K., Vafai, K. and Lightstone, M. (2003), 'Buoyancy- Driven Heat Transfer Enhancement in a Two-Dimensional Enclosure Utilizing nanofluid', *International Journal of Heat and Mass Transfer* 46:3639-3653.
- [81]. Kim, J., Kang, Y. J. and Choi, C. K. (2004), 'Analysis of Convective Instability and Heat Transfer Characteristics of Nanofluids', *Physics of Fluids* 16:2395-2401.
- [82]. Kim, J., Kang, Y. T. and Choi, C. K. (2006), 'Effects of Thermodiffusion Nanoparticles on convective Instabilities in Binary Nanofluids', *International Journal of heat and fluid flow* 30:771-776.
- [83]. Koo, J. and Kleinstreuer, C. (2004), 'A New Thermal Conductivity Model for Nanofluids', *Journal of Nanoparticle Research* 6:577-588.
- [84]. Koo, J. and Kleinstreuer C. (2005), 'Laminar Nanofluid Flow in Microheat-Sinks', *International Journal of Heat and Mass Transfer* 48:2652-2661.
- [85]. Krieger, I. M. (1972), 'Rheology of monodisperse Lattices', *Advances in Colloid & Interface Sciences* 3:111-136.
- [86]. Kuhn, W. and Kuhn H. (1945), 'Die Ahangigkeit der Viskositat vom Str? mungsgefalle bei hochverdunnten Suspensionen und L? sungen', *Helvetica Chimica Acta* 28:97-127.
- [87]. Kulkarni, D. P., Das, D. K. and Patil, S. L. (2006), 'Effects of Temperature on Rheological Property of Copper Oxide Nanoparticles dispersed in propylene glycol and water mixture', *Journal of Nanoscience and Nanotechnology* 6:1150-1154.
- [88]. Kumar, D. H., Patel, E. H., Kumar, V. R. R., Sundararajan, T., Pradeep, T. and Das, S. K. (2004), 'Model for Heat Conduction in Nanofluids', *Physical Review Letters* 93:144301.
- [89]. Kumar, S. and Murthy, J. Y. (2005), 'A Numerical Technique for Computing Effective Thermal Conductivity of Fluid-Particle Mixtures', *Numerical Heat Transfer (Part B)* 47:555-572.
- [90]. Kuznetsov, A. V. and Nield, D. A. (2010), 'Natural convective boundary layer flow of a nanofluid past a vertical plate', *International Journal of Thermal Science* 49:243-247.
- [91]. Lamb, W., Wood, D. M. and Ashcroft, N. W. (1978), 'Optical Properties of small Particle Composites', *American Institute of Physics* 40:240-255.
- [92]. Ladauer, R. (1952), 'The Electrical Resistance of Binary Metallic Mixtures', *Journal of Applied Physics* 23:779-784.
- [93]. Ladauer, R. (1978), 'Electrical Conductivity in Inhomogeneous Media', *American Institute of Physics* 40:2-45.

- [94]. Lee, D., Kim, J. W. and Kim, B. J. (2006), 'A New Parameter to Control Heat Transport in Nanofluids: Surface Charge State of the particle in Suspension', *Journal of Physical Chemistry* 110:4323-4328.
- [95]. Lee, S., Choi, S. U. S., Li, S. and Eastman, J. A. (1999), 'Measuring Thermal Conductivity of Fluids Containing Oxide Nanoparticles', *Journal of Heat Transfer* 121:280-289.
- [96]. Li, C., Akinc, M., Wiench, J., Pruski, M. and Schilling, C. H. (2005), 'Relationship between water Mobility and Viscosity of Nanometric Alumina Suspensions', *Journal of the American Ceramic Society* 88:2762-2768.
- [97]. Li, C. H. and Peterson, G. P. (2006), 'Experimental Investigation of Temperature and Volume Fraction Variations on the Effective Thermal Conductivity of Nanoparticle Suspensions (Nanofluids)', *Journal of Applied Physics* 99:084314.
- [98]. Liu, M.-S., Lin, M. C.-C., Huang, I. -T. and Wang, C. -C. (2005), 'Enhancement of Thermal Conductivity with Carbon Nanotube for Nanofluids', *International Communications in Heat and Mass Transfer* 32:1202-1210.
- [99]. Liu, M., Lin, M. C.-C., Tsai, C. Y. and Wang, C. -C. (2006), 'Enhancement of Thermal Conductivity with Cu for Nanofluids Using Chemical Reduction Method', *International Journal of Heat and Mass Transfer* 49:3028-3033.
- [100]. Liu, S. and Masliyah, J. H., (1996), 'Rheology of suspensions', *American Chemical Society Advances in Chemistry* 251:107-176.
- [101]. Liu, Z.-H. and Qiu, Y. H. (2006), 'Boiling Heat Transfer Characteristics of Nanofluids Jet Impingement on a Plate Surface', *Heat and Mass Transfer* 43:699-706.
- [102]. Lo, C.-H., (2005a), 'Fabrication of Copper Oxide Nanofluid Using Submerged Arc Nanoparticles Synthesis System (SANSS)', *Journal of Nanoparticle Research* 7:313-320.
- [103]. Lo, C.-H., Tsung, T. -T. and Chen, L. -C. (2005b), 'Shaped-Controlled synthesis of Cu-Based Nanofluid Using Submerged Arc Nanoparticle System (SANSS)', *Journal of Crystal Growth* 277:636-642.
- [104]. Lu, S. -Y. and Song J. -L. (1996), 'Effective Conductivity of Composites with Spherical Inclusions, Effective of Coating and Detachment', *Journal of Applied Physics* 79:609-618.
- [105]. Lundgren, T. S. (1972), 'Slow Flow through Stationary Random Beds and suspensions of Spheres', *Journal of Fluid Mechanics* 51:273-299.
- [106]. Ma, H. B., Wilson, C., Borgmeyer, B., Park, K., Yu, Q., Choi, S. U. S. and Tirumala, M. (2006), 'Effect of Nanofluid on the heat Transport Capability in an Oscillating Heat Pipe', *Applied Physics Letters* 88:143116.

- [107]. Mahdy, A. and Hady, F. M. (2009), 'Effect of Thermophoretic particle deposition in non-Newtonian free convection flow over a vertical plate with a magnetic field effect,' *Journal of Non-Newtonian Fluid Mechanics* 161:37-41.
- [108]. Ma?ga, S. E. B., Nguyen, C. -T., Roy, G. and Galonis, N. (2004), 'Heat Transfer Behaviours of Nanofluids in a uniformly Heated Tube', *Superlattices and Microstructures* 35:543-557.
- [109]. Ma?ga, S. E. B., Palm, S. J., Nguyen, C. -T., Roy, G. and Galonis, N. (2005), 'Heat Transfer Enhancement by Using Nanofluids in Forced Convection Flows', *International Journal of Heat and Fluid Flow* 26:530-546.
- [110]. Makinde, O. D. (2009), 'On MagnetoHydrodynamic boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux', *International Journal of Numerical Methods for heat and fluid flow* 19:546-554.
- [111]. Makinde, O. D. (2011), 'Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition', *Thermal Science* 15:137-143.
- [112]. Makinde, O. D. and Aziz, A. (2011), 'Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition', *International Journal of thermal science* 50:1326-1332.
- [113]. Makinde, O.D. and Aziz, A. (2010), 'MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition', *International Journal of Thermal Science* 49:1813-1820
- [114]. Makinde, O. D. and Onyejekwe, O.O. (2011), 'A numerical study of MHD generalized couette flow and heat transfer with variable viscosity and electrical conductivity', *Journal of Magnetism and Magnetic Materials* 32 :2757-2763.
- [115]. Makinde, O. D. and Sibanda, P. (2011), 'Effects of chemical reaction on boundary layer flow past a vertical stretching surface in the presence of internal heat generation', *International Journal of Numerical Methods for Heat and Fluid flow* 21:779- 792.
- [116]. Mansour, R. B. (2006), 'Effect of Uncertainties in Physical Properties on Forced Convection Heat Transfer with Nanofluids', *Applied Thermal Engineering* 27:240- 249.
- [117].Marquis, F. D. S. and Chibante, L. P. F. (2005). Improving the Heat Transfer of nanofluids and nanolubricants with Carbon nanotubes', *Journal of the minerals, Metals & Materials society* 57:32-43.
- [118]. Masuda, H., Ebata, A., Teramae, K. and Hishinuma, N. (1993), 'Alternation of Thermal conductivity and Viscosity of Liquid by Dispersing Ultra-Fine Particles (Dispersion of γ - Al_2O_3 , SiO_2 and TiO_2 Ultra-Fine Particles)', *Netsu Bussei* 7:227-233.

- [119]. Maxwell, J. C. (1873), 'Treatise on Electricity and Magnetism', *Oxford: Clarendon Press*.
- [120]. Mianova, D. and Kumar, R. (2005), 'Role of Ions in Pool Boiling Heat Transfer of Pure and Silica Nanofluids', *Applied Physics Letters* 87:233107.
- [121]. Miles, J. B. and Robertson, H. P. (1932), 'The Dielectric Behaviour of Colloidal Particles with an Electric Double-Layer', *Physical Review* 40:583-591.
- [122]. Mooney, M. (1951), 'The Viscosity of a Concentrated Suspension of Spherical Particles', *Journal of Colloid Science* 6:162-170.
- [123]. Murshed, S. M. S., Leong, K. C. and Yang, C. (2005), 'Enhanced Thermal Conductivity of TiO₂-water Based Nanofluids', *International Journal of Thermal Sciences* 44:367-373.
- [124]. Muthamilselvan, M., Kandaswamy, P. and Lee, J. (2010), 'Heat Transfer enhancement of Copper-Water Nanofluids in a Lid-driven enclosure', *Communication in Non-linear Science and Numerical Simulation* 15:1501-1510.
- [125]. Nan, C.-W., Shi, Z. and Lin, Y. (2003), 'A Simple Model for Thermal Conductivity of Carbon Nanotubes- Based Composites. *Chemical Physics Letters* 375:666-669.
- [126]. Nan, C.-W., Liu, G., Lin, Y. and Li, M. (2004), 'Interface Effect on Thermal Conductivity of Carbon Nanotube Composites', *Applied Physics Letters* 85:3549-3551.
- [127]. Nield, D. A. and Kuznetsov, A. V. (2009a), 'Thermal Instability in a porous medium layer Saturated by a Nanofluid', *International Journal of Heat and Mass Transfer* 52:5796-5801.
- [128]. Nield, D. A. and Kuznetsov, A. V. (2009b), 'The Cheng- Minkowycz Problem for Natural Convective boundary layer flow in a porous Medium Saturated by Nanofluid', *International Journal of Heat and Mass Transfer* 52:5792- 5795.
- [129]. Oztop, H. F. and Abu-Nada, E. (2008), 'Numerical study of natural convection in a partially heated rectangular enclosure filled with nanofluids', *International Journal of Heat and Fluid flow* 29:1326-1336.
- [130]. Pak, B. C. and Cho, Y. I (1998), 'Hydrodynamic and Heat Transfer Study of Dispersed Fluids with submicron Metallic Oxide Particles', *Experimental Heat Transfer* 11:151-170.
- [131]. Patel, H. E., Das, S. K., Sundarajan, T., Nair, A. S., George, B. and Pradeep, T. (2003), 'Thermal Conductivity of Naked and Monolayer Protected Metal Nanoparticle Based Nanofluids, Manifestation of Anomalous Enhancement and Chemical Effects', *Applied Physics Letters* 83:2931-2933.
- [132]. Pauly, H. (1959), 'über die Impedanz einer Suspension von Kugelförmigen Teilchen mit einer Schale', *Zeitschrift für Naturforschung* 146:125-131.

- [133]. Phan-Thien, N. and Graham, A. L. (1991), 'A New Constitutive Model for Fibre Suspensions, Flow past a Sphere', *Rheologica Acta* 30:44-57.
- [134]. Polder, D. and Van Santen, J. H. (1946), 'The Effective Permeability of Mixtures of solids', *Physica* 12:257-271.
- [135]. Pozhar, L. A. (2000), 'Structur and Dynamics of Nanofluids, Theory and simulations to Calculate Viscosity', *Physical Review E* 61:1432-1446.
- [136]. Prantl, L. (1904), 'Verhandlugen des dritten Internationale Mathematiker- Kongresses, Heidelberg.
- [137]. Prasher, R., Bhattacharya, P. and Phelan, P. E. (2005), 'Thermal conductivity of Nanoscale Colloidal Solutions (Nanofluids)', *Physical Review Letters* 94:025901.
- [138]. Prasher, R., Bhattacharya, P. and Phelan, P. E. (2006a). Brownian- Motion-Based Convective- Conductive Model for the Thermal Conductivity of Nanofluids. Transaction of the ASME', *Journal of Heat Transfer* 128:588- 595.
- [139]. Prasher, R., Phelan, P. and Bhattacharya, P.(2006b), 'Effect of Aggregation Kinetics on the Thermal Conductivity of Nanoscale Colloidal Solutions (Nanofluid)', *Nano Letters* 6:1529-1534.
- [140]. Prasher, R., Evans, W., Meakin, P., Fish, J., Phelan, P. and Keblinski, P. (2006c), 'Effect of Aggregation on Thermal on Thermal Condition in Colloidal Nanofluids', *Applied Physics Letters* 89:143119.
- [141]. Putnam, S. A. (2006), 'Thermal Conductivity of Nanoparticle Suspensions', *Journal of applied Physics* 99:084308.
- [142]. Putra, N. (2003), 'Natural Convection of Nano-fluids', *Journal of Heat and Mass Transfer* 39:775-784.
- [143]. Rayleigh, L. (1892), 'On the Influence of Obstacles Arranged in Rectangular Order upon the Properties of a Medium', *Philosophical Magazine* 34: 481-502.
- [144]. Ren, Y., Xie, H. and Cai, A. (2005), 'Effective Thermal Conductivity of Nanofluids Containing Spherical Nanoparticles', *Journal of Physics D, Applied Physics* 38:3958- 3961.
- [145]. Robinson, J. V. (1949), 'The Viscosity of Suspensions of Spheres', *Journal of Physical & Colloid Chemistry* 53:1042-1056.
- [146]. Romano, J. M., Parker, J. C. and Ford, Q. B. (1997), 'Application Opportunities for Nanoparticles Made from Condensation of Physical Vapour', *Proceedings in the International Conference on Power Metallurgy and particulate Materials, Chicago, Ill, U.S.A* 2:12-13.

- [147]. Roscoe, R. (1952), 'The Viscosity of Suspensions of Rigid Spheres', *British Journal of Applied Physics* 3:267-269.
- [148]. Routbort, J. L. and Singh, D. (2006), 'Erosion of Materials in Nanofluids', *In the U.S. Department of Energy Annual Progress Report, Heavy Vehicle Systems Optimization Program*.
- [149]. Roy, G., Nguyen, C. T. and Lajoie, P. -R. (2004), 'Numerical Investigation of Laminar Flow and Heat Transfer in a Radial Flow Cooling System with the Use of Nanofluids', *Superlattices and Microstructures* 35:497- 511.
- [150]. Saitô, N. (1950), 'Concentration Dependence of the Viscosity of High Polymer solutions', *International Journal of the Physical Society of Japan* 5:4-8.
- [151]. Sato, Y., Deutsch, E. and Simonin, O. (1998), 'Direct Numerical simulations of Heat Transfer by Solid Particles suspended in Homogeneous Isotropic Turbulence', *International Journal of Heat and Fluid Flow* 19:187-192.
- [152]. Schwan, H. P., Schwarz, G., Maczuk, J. and Pauly, H. (1962), 'On the Low- Frequency Dielectric Dispersion of Colloidal Particles in Electrolyte Solution', *Journal of Physical Chemistry* 66:2626-2635.
- [153]. Shenogin, S., Xue, L., Ozisik, R., Keblinsk, P. and Cahil, D.G. (2004), 'Role of Thermal Boundary Resistance on the heat Flow in Carbon- Nanotubes Composites', *Journal of Applied Physics* 95:8136-8144.
- [154]. Simha, R. (1940), 'The Influence of Brownian Movement on the Viscosity of solutions' *Journal of Physical Chemistry* 44:25-34.
- [155]. Simha, R. (1952), 'A Treatment of the Viscosity of Concentrated Suspensions', *Journal of Applied Physics* 23:1020- 1024.
- [156]. Srdic, V. V., Winterer, M., Müller, A., Miede, G. and Hahn, H. (2001), 'Nanocrystalline Zirconia Surface- Doped with Alumina: Chemical Vapour Synthesis, Characterization, and Properties', *Journal of the American Ceramic society* 84:2771-2776.
- [157]. Syakila, A. and Loan, P. (2010), 'Mixed Convection Boundary Layer Flow from a Vertical flat plate embedded in a porous medium filled with Nanofluid', *International Communication in Heat and mass Transfer* 37:987-991.
- [158]. Taylor, L. S. (1965), 'Dielectrics Properties of Mixtures', *IEEE Transactions on Antennas and Propagation* 13:943-947.
- [159]. Taylor, L.S. (1966), 'Dielectrics Loaded with Anisotropic Materials', *IEEE Transactions on Antennas and Propagation AP-14:669-670*.

- [160]. Tiwari, R. J. and Das, M. K. (2007), 'Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids', *International Journal of Heat and Mass Transfer* 50:2002-2018.
- [161]. Trisaksri, V. and Wongwises, S. (2007), 'Critical Review of Heat Transfer Characteristics of Nanofluids', *Renewable Sustainable Energy Review*. 11:512-523.
- [162]. Tsai, C. Y. (2004), 'Effect of structural Character of Gold Nanoparticles in Nanofluid on Heat Pipe Thermal Performance', *Materials Letters* 58:1461-1465.
- [163]. Tseng, W. J. and Lin, K. -C. (2003), 'Rheology and Colloidal Structure of Aqueous TiO₂ Nanoparticles Suspensions', *Materials Science and Engineering* 355:186-192.
- [164]. Tulapurkara, E. G. (2005), 'Hundred years of boundary layer', *Sādahnā* 30:499-5.
- [165]. Tzeng, S. -C., Lin, C. -W. and Huang, K. D. (2005), 'Heat Transfer Enhancement of Nanofluids in Rotary Blade Coupling of Four-Wheel-Drive Vehicles', *Acta Mechanica* 179:11-23.
- [166]. Tzou, D. A. (2008), 'Thermal Instability of Nanofluids in Natural Convection', *International Journal of Heat Transfer* 51:2967-2979.
- [167]. Tzou, D. A. (2008), 'Instability of Nanofluids in Natural Convection', *International Journal of Heat and Mass Transfer* 130:401.1-401.9.
- [168]. Hendrik, C. H. and Van de Hulst, H. C. (1957), 'Light Scattering by Small Particles', *New York: John Wiley & sons*.
- [169]. Vand, V. (1948), 'Viscosity of Solutions and Solutions', *International Journal of Theory Physical and Colloid Chemistry* 52:277-299.
- [170]. Vassallo, P., Kumar, R. and Damisco, S. (2004), 'Pool Boiling Heat Transfer Experiments in Silica-Water Nano-Fluids', *International Journal of Heat and Mass Transfer* 47: 407-411.
- [171]. Wang, B. -X., Zhou, L. -P. and Peng, X. F. (2003), 'A Fractal Model for Predicting the Effective Thermal Conductivity of Liquid with suspensions of Nanoparticles', *International Journal of Heat and Mass Transfer* 46:2665-2672.
- [172]. Wang, X. W., Xu, X. F. and Choi, S. U. S. (1999), 'Thermal Conductivity of Nanoparticles-Fluid Mixture', *Journal of Thermophysics and Heat Transfer* 13: 474-480.
- [173]. Wang, X. Q. and Majumdar, A. S. (2007), 'Heat Transfer Characteristics of Nanofluids', *A Review of International Journal of Thermal Science* 46:1-119.
- [174]. Wang, L. and Wei, X. (2007), 'Equivalence between dual phase-lagging and two-phase system heat conduction', *International Journal of Heat and Mass transfer* 51:1751-1756 .

- [175]. Wen, D. and Ding, Y. (2004a), 'Experimental Investigation into Convective Heat Transfer of Nanofluids at the Entrance Region under Laminar Flow Conditions', *International Journal of Heat mass Transfer* 47:5181-5188.
- [176]. Wen, D. and Ding, Y. (2004b), 'Effective Thermal Conductivity of Aqueous Suspensions of Carbon Nanotubes (Carbon Nanotube Nanofluids)', *Journal of Thermophysics and Heat Transfer* 18:481-485.
- [177]. Wen, D. and Y. Ding (2005a), 'Effect of Particle Migration on Heat Transfer in Suspensions of Nanoparticles Flowing through Minichannels', *Microfluid 1*:183-189.
- [178]. Wen, D. and Ding, Y. (2005b), 'Experimental Investigation into pool Boiling Heat Transfer of aqueous Based γ - Alumina Nanofluids', *Journal of Nanoparticle Research* 7:265-274.
- [179]. Wen, D. and Ding, Y. (2005c), 'Formulation of Nanofluids for Natural Convective Heat Transfer Applications', *International Journal of Heat and Fluid Flow* 26:855-864.
- [180]. Wen, D. and Y. Ding (2006), 'Natural Convective Heat Transfer of Suspensions of Titanium Dioxide Nanoparticles (Nanofluids)', *IEEE Transactions on Nanotechnology* 5: 220-227.
- [181]. Wiener, O. (1912), 'Die Theorie des Mischk? rpers f? r das feld der station? ren str? mung, Die Mittelwerts? tze f? r Kraft, Polarisation und Energie', *Der Abhandlungen der Mathematisch-Physischen Klasse der K? niglich S? chsischen Gessellschaft der Wissenschaften* 32:507-604.
- [182]. Xie, H., Wang, J., Xi, T. G. and Liu, Y. (2002), 'Thermal Conductivity of Suspensions Containing Nanosized SiC Particles', *International Journal of Thermophysics* 23:571- 580.
- [183]. Xie, H., Wang, J., Xi, T. G., Liu, Y. and Ai, F. (2002a), 'Thermal Conductivity Enhancement of suspensions Containing Nanosized Alumina Particles', *Journal of Applied Physics* 91: 4568-4572.
- [184]. Xie, H., Wang, J., Xi, T. G., Liu, Y. and Ai, F. (2002b), 'Dependence of the Thermal Conductivity of Nanoparticle-Fluid Mixture on the Base Fluid', *Journal of Materials Science Letters* 21:1469- 1471.
- [185]. Xie, H., Lee, H., Youn, W. and Choi, M. (2003), 'Nanofluids Containing Multiwalled Carbon Nanotubes and their Enhanced Thermal Conductivities', *Journal of Applied Physics* 94: 4967-4971.
- [186]. Xie, H., Fujii, M. and Zhang, X. (2005), 'Effect of Interfacial Nanolayer on the Effective Thermal Conductivity of Nanoparticle-Fluid Mixture', *International Journal of Heat and Mass Transfer* 48:-2926-2932.

- [187]. Xuan, Y. and Li, Q. (2000), 'Heat Transfer Enhancement of Nanofluids', *International Journal of Heat and Fluid Flow* 21:58-64.
- [188]. Xuan, Y. and Li, Q. (2003), 'Investment on Convective Heat Transfer and Flow Features of Nanofluids', *Transactions of the ASME, Journal of Heat Transfer* 125:151-155.
- [189]. Xuan, Y., Li, Q. and Hu, W. (2003), 'Aggregation Structure and Thermal Conductivity of Nanofluids', *American Institute of Chemical Engineering Journal* 49: 1038-1043.
- [190]. Xuan, Y. and Yao, Z. (2005), 'Lattice Boltzmann Model for Nanofluids', *Heat and Mass Transfer* 41: 199-205.
- [191]. Xue, L., Keblinski, P., Phillpot, S. R., Choi, S.U. S. and Eastman, J. A. (2004), 'Effect of Liquid layering at the Liquid-solid Interface on thermal Transport', *International Journal of Heat and Mass Transfer* 47:4277-4284.
- [192]. Xue, Q. (2000), 'Effective -Medium Theory for Two-Phase Random Composite with an Interfacial Shell', *Journal of Material science and Technology* 16:367-369.
- [193]. Xue, Q. Z. (2005), 'Model for Thermal Conductivity of Carbon Nanotube- Based Composites', *Physica B*368:302-307.
- [194]. Yang, Y., Zhang, Z. G., Grulke, E. A., Anderson, W. B, and Wu, G. (2005), 'Heat Transfer Properties of Nanoparticles-in-Fluid Dispersions (Nanofluids) in Laminar Flow', *International Journal of Heat and Mass Transfer* 48:1107-1116.
- [195]. Yang, B. and Han, Z. H. (2006), 'Temperature-Dependent Thermal Conductivity of Nanorod-Based Nanofluids', *Applied Physics Letters* 89:083111.
- [196]. Yang, Y., Grulke, E. A., Zhang, G. Z. and Wu, G. (2006), 'Thermal and Rheological Properties of Carbon Nanotube-in-Oil Dispersions', *Journal of Applied Physics* 99:114307.
- [197]. Yao, S., Fang, T. and Zhong, Y. (2011), 'Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions', *Communication in Nonlinear Science and Numerical Simulation* 16:752-760.
- [198]. You, S. M. (2003), 'Effect of Nanoparticles on Critical Heat Flux of water in Pool Boiling Heat Transfer', *Applied Physics Letters* 83:3374-3376.
- [199]. Yu, C. -J., Richter, A. G., Datta, A., Durbin, M. K. and Dutta, P. (2000), 'Molecular Layering in a Liquid on a Solid Substrate, an X-Ray Reflectivity Study', *Physica* 283:27-31.
- [200]. Yu, W. and Choi, S. U. S. (2003), 'The Role of Interfacial Layers in the Enhanced Thermal Conductivity of Nanofluids: A Renovated Maxwell Model', *Journal of Nanoparticle Research* 5: 167-171.

- [201]. Yu, W. and Choi, S. U. S. (2004), 'The Role of Interfacial Layers in the Enhanced Thermal Conductivity of Nanofluids, A Renovated Hamilton-Crosser Model', *Journal of Nanoparticle Research* 6:355-361.
- [202]. Yu, W. and Choi, S. U. S. (2005), 'An Effective Thermal Conductivity Model of Nanofluids with a Cubic Arrangement of Spherical Particles', *Journal of Nanoscience and Technology* 5: 580-586.
- [203]. Zhang, J., Que, Q. and Zhang, Z. (1997), 'Synthesis, Structure and Lubricating Properties of Dialkylidithiophosphate-Modified Mo-S Compound Nanoclusters' *Wear* 209:8-12.
- [204]. Zhu, H. -T., Lin, Y. S. and Yin, Y. S. (2004), 'A Novel One-step Chemical Method for Preparation of Copper Nanofluids', *Journal of Colloid and Interface Science* 277:100-103.

CAPE PENINSULA
UNIVERSITY OF TECHNOLOGY

