WRITING AND MATHEMATICAL PROBLEM-SOLVING IN GRADE 3

by

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ABSTRACT

The mathematics curriculum currently used in South African classrooms emphasises problem-solving to develop critical thinking. However, based on the local performance of South African Foundation Phase learners as well as performance in comparative international studies in mathematics, there is concern regarding their competence when solving mathematical problems and their use of meaningful strategies. This qualitative research study explores how writing can support Grade 3 learners' mathematical problem-solving abilities. Writing in mathematics is examined as a tool to support learners when they solve mathematical problems to develop their critical thinking and deepen their conceptual understanding. The study followed a case study design. Social constructivist theory formed the theoretical framework and scaffolding was provided by various types of writing tasks. These writing tasks, specifically those promoted by Burns (1995a) and Wilcox and Monroe (2011), were modelled to learners and implemented by them while solving mathematical problems. Writing tasks included writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes and shared writing. Data were gathered through learners' written work, field notes, audio-recordings of ability group discussions and interviews. Data were analysed to determine the usefulness of Burns' writing methodology to support learners' problem-solving strategies in the South African context. The analysis process involved developing initial insights, coding, interpretations and drawing implications to establish whether there was a relation between the use of writing in mathematics and development of learners' problem-solving strategies. This study revealed an improvement in the strategies and explanations learners used when solving mathematical problems. At the end of the eight week data collection period, a sample of eight learners showed marked improvement in verbal and written explanations of their mathematical problem-solving strategies than before the writing tasks were implemented.

DECLARATION

I, Belinda Petersen, declare that the content of this thesis represents my own unaided work and that the thesis has not previously been submitted for academic examination towards any qualification. Furthermore, it represents my own opinions and not necessarily those of the Cape Peninsula University of Technology.

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Signed Date

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DEDICATION

In memory of my sister, Natasha Fortune, and grandmother, Olive Pelling

Your tenacity and faith have taught me to remain steadfast and reach for my dreams.

I dedicate this work to you, together with all family and friends, especially Gerald, Nina and Emily.

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GLOSSARY

Above average	AA
Annual National Assessments	ANA
Average	A
Below Average	BA
Balanced Language Approach	BLA
Backward Number Word Sequences	BNWS
Cape Peninsula University of Technology	CPUT
Curriculum and Assessment Policy Statement	CAPS
Conceptual Place Value	CPV
Department of Basic Education	DBE
Early Multiplication and Division	EMD
Field Notes	FN
Forward Number Word Sequences	FNWS
Learning Framework In Number	LFIN
More knowledgeable other	MKO
Noting Collecting Thinking	NCT
Outcomes-Based Education	OBE
Post-test	Post
Pre-test	Pre
Question 1	Q1
Question 2	Q2
Question 3	Q3
Question 4	Q4
Question 5	Q5
Revised National Curriculum Statement	RNCS
Southern African Consortium for Monitoring Educational Quality	SACMEQ
Stages of Early Arithmetic Learning	SEAL
Structuring Number Strand	SNS
Trends in International Mathematics and Science Study	TIMSS
Western Cape Education Department	WCED
Zone of Proximal Development	ZPD

CHAPTER 1
INTRODUCTION AND OVERVIEW

Chapter one provides the background and rationale for this study. The research question and sub-questions are outlined and the methodological and theoretical orientations of the study are presented. In addition, the significance and limitations of the study are set out.

1.1 BACKGROUND TO THE STUDY

This study was prompted by the low standard of mathematics results in South Africa. The country has participated in international studies such as Trends in International Mathematics and Science Study (TIMSS) and Southern Africa Consortium for Monitoring Educational Quality (SACMEQ). According to Reddy (2013:16), these international studies provide an external benchmark against other countries, providing a reliable insight into the state of the education system. Participation in these studies shows that South Africa has consistently performed below international levels. In Ndlovu and Mji's (2012:189) comparison between the Revised National Curriculum Statement (RNCS) and South African learners' performance in TIMSS, it was found that "learners performed worst in (the category) Using Concepts, suggesting little conceptual understanding being achieved by the curriculum". This result implies learners had difficulty using mathematical concepts that they are expected to know according to the curriculum. Consequently, there seems to be a discrepancy between the intended curriculum, that which is expressed through its intended outcomes, and the implemented curriculum, that which is taught daily in South African classrooms. The intended curriculum encourages critical thinking in the application of mathematical knowledge to problem-solving. If critical thinking were practised daily, it is likely that learners could achieve better results in use of concepts tested in international studies such as TIMSS.

South Africa nationally uses the Annual National Assessments (ANA) to monitor learner performance while the Western Cape Education Department (WCED) continues to use the systemic evaluations in Grades 3, 6 and 9 to assess learner achievement. The WCED systemic evaluations are referred to because the site for the study is a school located in the Western Cape province of South Africa. The results of the ANA and systemic evaluations over the past few years have categorized the performances of schools and influenced curriculum delivery and coverage at local and provincial levels. Results have determined implementation of the curriculum by teachers and WCED officials. Learners are expected to know about, but do not necessarily fully understand, content areas within the curriculum.

The results of these assessments have raised concerns because learners often do not achieve minimum requirements at their grade levels. Results reflect a stronger ability to use procedural knowledge than conceptual knowledge. This imbalance is especially evident in the systemic evaluation which largely tests learners' problem-solving abilities. Learners consistently perform lowest in the area of Measurement compared to other content areas such as Numbers, Operations and Relationships and Space and Shape. Mathematical word problems are often used to test a content area such as measurement. Learners require conceptual knowledge rather than procedural knowledge in these instances. The site for this study displayed this trend since 2012 with learners performing at an average pass rate of approximately 60%. The pass rate refers to the number of learners reaching a minimum pass requirement of 50%.

Additionally, Siyepu (2013) suggests poor performance of South African learners is related to the quality of learning and teaching support materials (or lack thereof) as well as lack of qualifications, knowledge and skills of teachers. In large parts of the country there is an over reliance on textbooks and other support materials as resources for lessons. Siyepu (2013:8) claims that "South African textbooks encourage mainly lower order skills (such as recall) as opposed to the higher order skills (such as problem-solving)". Therefore, the standard and availability of textbooks could directly affect many learners' mathematical understanding and ability to solve problems. The researcher, drawing on experience as a Foundation Phase teacher, found this relation between textbooks and results to be true. In observing fellow teachers during mathematics lessons prior to this study, the researcher found a pattern of textbook teaching. In workshops and meetings regarding mathematics teaching, the same over-reliance on textbook learning or visible pedagogy was noted. These instances provide valuable insights into the way teachers use problem-solving in mathematics lessons. There is an overemphasis on procedural knowledge; where learners are taught how to solve mathematical problems. Teachers appeared to assist learners by teaching them tools such as looking for keywords in the context of the problem. Added to this, teachers were sometimes prescriptive by insisting on a specific operation that applies to a particular problem. Learners were often expected to use a number sentence to find their solution. Learners were generally not encouraged to try their own methods which would develop their critical thinking. All learners in a class were expected to solve a problem in the same way as prescribed by the teacher.

Teachers find it difficult to include problem-solving in daily mathematics lessons. Often the demands of the curriculum create an environment in which lessons focus on procedural knowledge rather than conceptual knowledge. Problem-solving is perceived as a time-

consuming activity that achieves little. However, it is through problem-solving that learners make sense of mathematical concepts: they learn new concepts and practise learned skills as they apply and develop their mathematical knowledge (Kilpatrick, Swafford & Findell, 2001:420; Schoenfeld, 2013).

1.2 RATIONALE OF THE STUDY

As a Foundation Phase teacher, the researcher has become increasingly concerned with learners' general ability to think, reason and solve problems in mathematics. Learners often lack competence in solving mathematical problems and explaining what they have done in their attempts to reach a solution. Learners tend to rely on the teacher's instruction to solve mathematical problems. Learners use too little writing: words, pictures and symbols in the mathematics classroom to track the processes followed when solving problems. It may be possible that learners are reluctant to do so due to a lack of exposure to writing in mathematics. Learners are taught too rigidly to solve problems using specific methods and procedures given by the teacher. In discussions with various Foundation Phase teachers, it has become clear that a disparity exists between their thinking and understanding of problem-solving and the daily use of problem-solving in mathematics lessons.

These concerns led to this research into different aspects of problem-solving: the role of problem-solving within the curriculum as well as different approaches to implementing problem-solving in the classroom. Learners were observed carefully in the researcher/teacher's class: the way they solved problems during mathematics lessons was examined. It became apparent that learners had difficulty writing their strategies and solutions when they solved problems. Some learners' writing did not reflect the problem being solved while other learners seemed to wait for instructions from the teacher to solve the problem. Most learners were unable to explain their solutions to the teacher or their peers. It was at this point that Burns's (1995a) work on the use of writing in mathematics became pertinent. Further investigation into research in this area led to questioning whether the use of writing could have an impact on learners' ability to solve mathematical problems. The research questions and purpose of this study emerged from this context.

1.3 THE PURPOSE OF THE STUDY

This research study seeks to investigate the use of writing in the mathematics classroom as a way of supporting learners in the process of problem-solving and learning mathematics. The research question is as follows:

Research question:

How do various types of writing tasks support Grade 3 learners in solving mathematical problems?

Sub-questions:

1. What support do writing tasks give to the development of conceptual understanding?
2. What support do writing tasks give to the development of problem-solving strategies?
3. How are writing tasks useful in the Foundation Phase mathematics classroom?
4. What challenges do learners encounter when implementing writing tasks in the Foundation Phase mathematics classroom?

Different types of writing tasks in mathematics are explored as methods that can enhance creative and critical thinking as well as encourage reflective thought, so deepening conceptual understanding in order to support mathematical problem-solving skills. Vygotsky's theory of social constructivism and, in particular, the Zone of Proximal Development (ZPD) (Vygotsky, 1978) and scaffolding (Bruner & Haste, 1987), underpin this research. Cognitive constructivist theory emphasises that children construct their own understanding and, therefore, construct their own strategies to solve mathematical problems. Social constructivist theory stipulates that the teacher and learners collaborate to build knowledge and construct the individual's understanding. Social constructivism and scaffolding clarify the use of writing in this study as a valuable tool to scaffold learners' understanding when solving mathematical problems. The work of other theorists is incorporated to support the overarching theory of social constructivism. Skemp's theory on the development of schemas (Skemp, 1987, 1989) is used to explain how learners construct and reconstruct their mathematical knowledge through problem-solving. In addition, Sfard's theory of the process and object of mathematical conceptions (Sfard, 1991) as it relates to problem-solving is discussed.

1.4 THE OBJECTIVES OF THE STUDY

An objective of the study was to determine the usefulness of writing in mathematics. It sought to gauge the support writing could give to the development of strategies learners used to solve mathematical problems. The question was whether learners displayed conceptual development in their ability to connect appropriate mathematical knowledge and skills to particular problems.

Another objective of the study was to conclude whether the systematic implementation of specific writing tasks would be beneficial to learners' problem-solving strategies. This objective would be evident if learners were to show increased development of more advanced strategies by the end of the data collection period. The aim was to determine whether there was a significant improvement in the written strategies and explanations learners used when solving mathematical problems to enable better verbal explanations of their solutions. This study was used to determine whether all the writing tasks could be relevant and beneficial to Foundation Phase learners in the South African context.

1.5 THE IMPORTANCE OF PROBLEM-SOLVING IN MATHEMATICS

The Revised National Curriculum Statement (RNCS) (South Africa DBE, 2011) is currently the curriculum in use in South Africa. The RNCS was restructured to be more prescriptive in the form of the Curriculum and Assessment Policy Statement (CAPS) (South Africa DBE, 2011) for individual subjects and was implemented in the Foundation Phase in 2012.

Education in South Africa has been governed by various curricula since the dawn of democracy in 1994. Beginning with the introduction of Outcomes-Based Education (OBE), the critical outcomes of the curricula have mentioned the importance of critical thinking. In mathematics, critical thinking is developed through problem-solving that encompasses all the content areas of this subject.

Problem-solving, which is discussed in further detail in the literature review in Chapter two, involves critical thinking and reasoning to find a solution and is generally considered a life skill that should be developed. Heddens and Speer (2006:82) define problem-solving as "the (interdisciplinary) process an individual uses to respond to and overcome obstacles or barriers when a solution or method of solution to a problem is not immediately obvious". Mathematical problems and, in particular, word problems should form part of problem-solving. Solving mathematical problems can be used either as a consolidation activity once a particular concept has been taught or as a starting point from which conceptual knowledge can be developed.

1.6 THE IMPORTANCE OF WRITING IN MATHEMATICS

As is discussed in the literature review, writing is essential in supporting the development of mathematical knowledge and its application to problem-solving strategies. Through the use of writing, learners express their thinking and extend their understanding of mathematical

ideas (Burns, 2007:38). This comprehension allows them to reflect critically on their conceptual understanding. Writing helps learners to make sense of mathematical problems: learners learn how to represent and communicate their thinking.

In this study, various writing tasks were modelled and implemented in a Grade 3 class to cultivate the use of writing in mathematics. These writing tasks included writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes (Burns, 1995a) and shared writing (Wilcox & Monroe, 2011). Through implementation of the tasks, learners could be encouraged to explain their thinking. It could then be determined whether the use of writing supports learners in mathematical problem-solving. Results of this study are presented in Chapter 4.

1.7 OVERVIEW OF THE RESEARCH METHODOLOGY

A case study approach was used in this qualitative study. A primary school in Cape Town, South Africa, was selected as the site for the study. The population of the study was one of the Grade 3 classes from the school. A sample of eight learners was purposively selected from the class. Data collection instruments included interviews, audio-recordings, field notes and learners' written pieces from the pre-test, post-test and writing intervention.

The four-step approach to analysing data described by Dana & Yendol-Hoppey (2009) was employed for this investigation. This approach included description, sense making, interpretation and implication drawing. Learners' problem-solving strategies were analysed using the Learning Framework In Number (Wright, Martland & Stafford, 2006; Wright, Martland, Stafford & Stanger, 2006; Wright, 2013). This framework, together with the theoretical framework of the study, guided the process of analysis.

1.8 SIGNIFICANCE OF THE STUDY

This study is significant both for the activities within the mathematics classroom and in terms of curriculum implementation. As far as the mathematics classroom is concerned, the use of writing should be included in lessons as stated in the curriculum. The use of writing tasks may be intentionally implemented to address this requirement. This study may enhance the teaching of mathematics as well as learners' problem-solving abilities by giving teachers tools to incorporate writing in mathematics.

This study is significant for implementation of the current curriculum in South African schools. The CAPS Mathematics for Foundation Phase stipulates that writing is essential in mathematics for learners to communicate their thinking (South Africa DBE, 2011:9). Kuzle (2013:43) agrees that writing is a valuable tool for learning and communicating mathematics. In order for writing to be used in mathematics classrooms across South Africa, teachers need to be trained how to develop their own writing skills and implement them successfully during their pre-service training. In-service teachers should be given the knowledge and tools to implement writing in their mathematics classroom when they engage in ongoing professional development. Teachers model good writing practices by explaining and justifying solutions for the mathematical problems they encounter.

1.9 LIMITATIONS OF THE STUDY

One of the limitations of this study was the researcher's position as teacher. Creswell and Miller (2000:127) state that researchers should "acknowledge and describe their entering beliefs and biases early in the research process". As the teacher of the selected Grade 3 class, the researcher for this project was close to events and interactions (Hamilton & Corbett-Whittier, 2013:129). Being researcher and teacher could have created bias since relations were created with subject learners participating in this study. Morrell and Carroll (2010:79) posit that "the researcher's initial opinion or impressions of a subject colour subsequent observations". The researcher/teacher had to be aware continually of discarding personal thoughts and views, especially when selecting the sample of eight learners as well as during the data analysis process. According to Morrell and Carroll (2010:80), being both teacher and researcher could jeopardise the validity of the study. This difficulty was addressed by making the researcher's dual role explicit in the context of the study. Multiple opportunities to collect and display data were used in conjunction with audio-recordings which helped to ensure validity of the data.

The sample for this study was relatively small: participants were from one Grade 3 class. Eight learners were selected from this class for the purpose of interviews and analysis of learners' written pieces. Findings of this study are limited to this particular class and group of learners in the sample and cannot be generalised to a broader population in a different setting regarding the impact of writing tasks in supporting problem-solving.

Another limitation of this study was the number ranges used in the mathematical problems learners solved during the data collection period. The mathematical problems were differentiated for the three mathematical ability groups present in the participating Grade 3

class. The problems shared the same context across the groups. However, the number ranges differed. A higher number range was employed for the above average (AA) ability group while the below average (BA) ability group solved problems with a lower number range. The number range for the average (A) ability group was considered to be typical for learners in this grade. The results concerning number ranges of mathematical problems will be discussed in Chapter five.

A further limitation concerned implementation of the number of writing tasks during the data collection. Before data collection commenced, it was planned to do three writing episodes per week over a period of ten weeks. These writing episodes included modelled writing lessons as well as opportunities for learners to implement the writing tasks. Added to this, a pre-test and post-test before and after the implementation of the writing tasks were envisaged. Data collection did not follow as planned because the school programme did not always afford the time to collect data on certain key days. The school's assessment programme needed to be taken into account. More data was collected in some weeks than others. Although the writing intervention was shortened to eight weeks, the same number of writing episodes took place as planned. Being well prepared for potential pitfalls is essential when conducting research.

1.10 ORGANISATION OF THE STUDY

1.10.1 Chapter One

The background and rationale for this study, as well as the purpose of the study, are presented. Chapter One provides a brief overview of problem-solving and the use of writing in mathematics. It includes an overview of the methodology as well as the significance of the study. The limitations of the study are also mentioned.

1.10.2 Chapter Two

The theoretical framework and literature review for this study are outlined. The chapter begins with defining Vygotsky's theory of social constructivism as the overarching theory with an emphasis on the Zone of Proximal Development (ZPD) and scaffolding. Particular theories of Skemp and Sfard are presented as they relate to the abovementioned theories. The literature review focuses on problem-solving in mathematics; placing it in the context of this study. Writing in mathematics is then explained, paying particular attention to the work of Burns (1995a).

1.10.3 Chapter Three

In this chapter the research design for this study is delineated as a qualitative case study. The research plan is presented describing the data collection plan. This includes the pilot study, pre-test, implementation of the writing tasks and the post-test. Subsequently the site and sample are discussed. The data collection instruments include learners' written pieces, audio-recordings of ability group discussions, field notes and interviews with eight learners selected from the Grade 3 class. The process of data analysis is explained.

1.10.4 Chapter Four

The findings of this study are presented. These provide evidence that writing supports learners when engaged in mathematical problem-solving. Results from the pre-test, implementation of the writing tasks and the post-test are given with examples from learners' written pieces as well as from interviews conducted with selected learners. Results from field notes and audio-recordings of the ability group discussions were considered.

1.10.5 Chapter Five

Themes are extracted from the data as they relate to the research questions of this study. These are discussed in answer to the research questions. Lastly, this study makes recommendations for mathematics education: possible areas of further research are highlighted.

The theoretical framework that underpins this study is discussed in the next chapter and the relevant literature that addresses the research.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

The purpose of this research study is to investigate how various types of writing tasks support Grade 3 learners when they attempt to solve mathematical problems. Learners often find difficulty solving word problems because they require a deeper conceptual understanding of mathematical ideas. The literature review discusses theories of learning and schools of thought in mathematics that relate to the research question.

The literature review begins with the theoretical framework that underpins this study. Vygotsky's social constructivist theory, in particular the Zone of Proximal Development (ZPD), scaffolding and inner speech is employed. Skemp's theory on the development and restructuring of schema and Sfard's theory on the process and object of mathematical conceptions relevant to this research are referred to throughout. Literature on mathematical problems pertaining to this investigation includes levels of problem-solving strategies, writing in mathematics with particular reference to Burns (1995a) and types of writing tasks that can be employed in the mathematics classroom to support problem-solving thinking strategies.

2.2 THEORETICAL FRAMEWORK

This research study involves support provided by the teacher and peers in order for learners to solve mathematical problems. Learners are required to use their existing knowledge of mathematical concepts when they engage in problem-solving. Through this application of knowledge, learners develop and broaden their skills. Constructivism is used in this investigation as an umbrella theory to which various theories can be linked. Vygotsky's theories of social constructivism, ZPD, scaffolding and inner speech are discussed. Theories of Skemp and Sfard are presented as they relate to the study. Key aspects of their theories are highlighted and linked to Vygotsky's theories.

Constructivist theory is based on the notion that knowledge is acquired by building on previous knowledge in order to construct new knowledge or concepts. According to Selley (1999:3), constructivism is "a theory of learning which holds that every learner constructs his or her ideas, as opposed to receiving them, complete and correct, from a teacher or authority source". Selley describes constructivism as internal and personal, enabling the learner to

build his or her knowledge by “reinterpreting bits and pieces of knowledge” gained from others. Sperry Smith (2013:10) concurs by stating that constructivism is “a theory that views the child as creating knowledge by acting on experience gained from the world and then finding meaning in it”. A learner assimilates and owns knowledge more thoroughly and completely when he or she is able to apply and re-configure knowledge as opposed to learning facts off by heart: what Freire terms ‘banking’. According to Ernest (1994:63), “social processes and individual sense making” are imperatives within this theory. Conceptual knowledge that is individually constructed is rooted in the individual conscience and experience (Skemp, 1989:203). Through learning constructively, the learner is an active participant in the process of testing, applying and appropriating knowledge (Selley, 1999:6). Learners make sense of the knowledge they have gained, and can own and apply it, when such knowledge has been shaped through their own experiences of life and interactions with others.

According to Piaget, children construct increasingly complex ‘maps’ of their world in an attempt to organize, understand and adapt to it (Donald, Lazarus & Lolwana, 2010:49). Piaget’s developmental stages provide a progression in terms of the learner’s ability to move from the concrete, pre-operational stage to the abstract. Carruthers and Worthington (2006:22) refer to Piaget’s idea of readiness where appropriate developmental stages need to be reached before certain concepts can be understood. Piaget’s theory is more concerned with the physical aspects of cognitive development in the construction of knowledge than interaction and culture. Vygotsky’s theories on social constructivism focus on the role of others in the construction of knowledge. For the purpose of this research, social constructivist theory supports interaction and collaborative learning that writing and mathematical problems stimulate. Vygotsky’s theory of social constructivism provides a theoretical underpinning: the works of other theorists are drawn upon to corroborate and contrast aspects of central theoretical concern.

2.2.1 Social constructivism

Vygotsky’s social constructivist theory explains that meanings are social constructions, built up and passed on between people in social contexts, each of which has a history and culture with its own set of ‘meanings’ (Donald et al., 2010:54). Similarly, Fosnot and Dolk (2001:6) suggest that “the process of constructing meaning is the process of learning”. When referring to socio-cultural theory, Sutherland (2007:5) states that “students bring informal perspectives on mathematics to any new learning situation and these influence what they pay attention to and thus the knowledge they construct”. The learning situation is interactive:

the teacher and learners collaborate to facilitate the individual's construction of knowledge (Schoenfeld, 2013:20). Learning is influenced by reflective thinking, social interaction and effective use of models or tools. Learning environments in which learners engage in explaining their thinking greatly affects the knowledge they construct (Schoenfeld, 2013:28). Learners need to be socially engaged when they solve mathematical problems (Schoenfeld, 2013:15). The role of the teacher is pivotal: Sutherland (2007:5) argues that teachers should be aware of the informal approaches learners bring to the mathematics classroom in order to exploit such prior and valid skills as a basis for acquiring and assimilating new mathematical ideas.

2.2.2 Zone of Proximal Development (ZPD)

A fundamental impact of Vygotsky's thought upon the development of educational theory is the concept of the zone of proximal development (ZPD). Vygotsky (1978:86) defines ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers". Wright, Martland, Stafford & Stanger (2006:28) explain ZPD as the "knowledge that the learner is capable of learning under the influence of appropriate teaching, and this zone is regarded as more extensive than that consisting of the knowledge that the learner is capable of learning without assistance". Learning within the ZPD makes use of the knowledge the learner already possesses as the foundation on which to construct prospective knowledge. What the learner is initially able to do collaboratively, he is later able to do independently. In the ZPD, teaching represents the means through which development is advanced (Vygotsky, 1978:131). Daniels (2001:56) describes Vygotsky's theory of the ZPD as an "attempt to understand the operation of contradiction between internal possibilities and external needs that constitutes the driving force of development". In this study, writing activities create the opportunity for a ZPD to be established. Different types of writing tasks are used as a method to determine the support writing could give to mathematical problem-solving strategies and explanations within the ZPD. Vygotsky's theory of ZPD leads to the theory of scaffolding.

2.2.3 Scaffolding

Vygotsky and Bruner's work discusses the theory of scaffolding which builds on the notion of the ZPD. The more knowledgeable other (MKO), be it the peer, parent or teacher, scaffolds understanding through individually tailored pacing of the problem-solving process (Bruner &

Haste, 1987:8). The gap between what the learner can do, given the constraints of her/his cognitive functioning, and what s/he can achieve with the intercession and scaffolding of adults or peers, describes the concept of the ZPD (Bruner & Haste, 1987:9). Scaffolding occurs when the MKO provides more manageable steps in the process that lead to the ability to solve the problem. These first realistic and attainable steps comprise beneficial teaching and learning situations that promote the construction of knowledge (Skemp, 1989:73). Through these manageable steps, the role or involvement of the learner is simplified rather than the task itself (Daniels, 2001:107). Orton and Frobisher (1996:18) add that the teacher's role within a constructivist learning environment is a vital contribution to the learner's construction of knowledge. In this study, in the ZPD, the teacher provides stimulation through writing activities that support, prompt and stimulate individual learning. Such writing tasks are "instructional strategies" (Daniels, 2001:108) that provide scaffolding for learners when they solve mathematical problems. Skemp (1989:76) concurs that, when learners talk with their peers in pairs or groups in a mathematical situation through co-operative learning, they have the opportunity to explain and discuss mathematical concepts. These situations help to develop and extend mathematical thinking; learners construct their knowledge socially and interactively. Through such situations, scaffolding occurs which leads, in turn, to the learner's construction of independent knowledge.

Sperry Smith (2013:10) explains scaffolding as support given by the teacher using prompts that eventually lead to the learner's ability to work independently. Similarly, Siyepu (2013:5) describes scaffolding within the ZPD as learning activities that the teacher employs to develop knowledge. Through creation of the ZPD, thinking can be tested and challenged without fear: knowledge and skills are enhanced through learning activities with the help of the teacher or significant other. This appropriation, ownership and assimilation of own knowledge requires assistance through social interaction between the MKO and the learner as well as between peer learners. Such interactions create opportunities for teachers and learners to "pause (and) comment on their problem-solving efforts in oral or written reflections" (Siyepu, 2013:8). In the ZPD, activities can be used to consolidate and organise the learner's informal knowledge into a more highly organised knowledge structure (Skemp, 1989:75). In this study, scaffolding occurred through implementing different types of writing tasks that learners may use to support their strategies when solving mathematical problems. Burns's (1995a) methodology of using writing in mathematics is introduced and implemented as a tool to scaffold learners' understanding and support them when solving mathematical problems.

Fosnot and Dolk (2001:18) describe scaffolding as support given where the teacher designs activities to develop understanding. Once the learner acquires the necessary knowledge or skill and performs a task or solves a problem within the ZPD, assistance is decreased and eventually removed to encourage independent thinking. Learners become more independent as they progress through the ZPD: they become less reliant on the support given through scaffolding. Daniels (2001: 109) explains that "the learner actually decreases the level of dependence upon the support structure as the learning sequence progresses". As soon as the learner understands the mathematical knowledge, the landmark is shifted and other questions are raised (Fosnot & Dolk, 2001:18). The ZPD is extended through further scaffolding to develop new mathematical knowledge: learners can proceed to engage with more challenging mathematical problems.

Vygotsky's overarching theory of social constructivism is detailed through the theories of the ZPD and scaffolding. In the next section, another tenet of Vygotsky's theories, inner speech, is discussed.

2.2.4 Inner speech

In this research study, when learners engage in personal writing, their engagement is similar to inner speech as theorized by Vygotsky. The role of inner speech is placed within the broader spectrum of language development. Vygotsky (1986:30) describes inner speech as fulfilling a similar role to egocentric speech, as theorized by Piaget. Both types of speech are used to comprehend a situation and, in essence, perform the same function of conversing with the self. Egocentric speech appears to be commonly experienced by younger children when they voiced their thinking: while silent inner speech was evident in older school children. Vygotsky (1986:33) explains that egocentric speech does not fall away as Piaget suggests but rather it turns into inner speech when a child reaches school-going age. He further argues (Vygotsky, 1986:36) that speech and, more importantly, thought development move from the social to the individual: "inner speech is speech for oneself (whereas) external speech is for others" (Vygotsky, 1986:225).

In relation to this study, learners often construct meaning socially, especially within a problem-solving context in a mathematics classroom. Learners make use of external speech when they engage in discourse around the mathematical problems presented. Learners attempt to construct their knowledge and make sense of problem-solving strategies when they engage with the MKO and their peers. Within this social constructivist setting, learners move through their ZPD's according to their individual conceptual understanding and

mathematical abilities. Learners progress through their ZPD's when they construct knowledge socially by engaging with others and individually by writing in mathematics. Learners employ writing tasks as a means of inner speech in order to make sense of mathematical ideas and express their thinking: their use of writing reveals their individual development of thought.

2.2.5 Social constructivism within CAPS

The CAPS Mathematics curriculum (DBE, 2011:10) states that, in the Foundation Phase, learners "should be exposed to mathematical experiences that give them many opportunities to do, talk and record their mathematical thinking". By doing this, mathematics lessons become interactive (DBE, 2011:12): learners work in groups or as a whole class. This constructive interaction provides ample opportunities for learners to construct mathematical knowledge socially: they engage with one another and the teacher. This interaction was elaborated on in the discussion of the zone of proximal development earlier in this chapter.

The current curriculum sets out a platform for collaborative mathematics lessons where knowledge is constructed and shared. This type of learning occurs when learners grapple with mathematical problems, and apply and develop their mathematical knowledge. This development links the theories of social constructivism and the ZPD mentioned earlier. The next section explains Skemp's theory of the development of schemas which relates to Vygotsky's pedagogical theories that underpin this study.

2.2.6 The development of schemas

Skemp (1987:24) discusses schemas as the development of conceptual structures which build on fundamental notions of constructivist learning. The function of a schema is to integrate existing knowledge in order to acquire new knowledge and so enhance understanding (Skemp, 1987:24). Skemp (1987:25) refers to the suitability of existing schema when building new knowledge. In order to construct new knowledge, there has to be a link to available schemas that exist. New knowledge cannot be constructed in isolation. Such linking of prior and new knowledge requires the learner to test, apply and imaginatively/cognitively assimilate new knowledge within an existing schema: any existing schema needs to be restructured to develop concepts further. Skemp (1987:28) refers to such further development as reconstruction. Fülöp (2015:40) concurs that engaging in problem-solving provides opportunities for learners to "refine, combine, and modify knowledge they have already learned". When this individual appropriation or ownership of

knowledge occurs, it is likely that a deeper conceptual understanding has developed through long-term schemas that are appropriate and adaptable (Skemp, 1987:34).

Sutherland (2007:53) adds that, in the construction of knowledge, not all learners are "focusing on the same processes or constructing the same knowledge, but that through dialogue, actions and interactions a sort of common knowledge emerges". Conceptual development may occur in a whole class or group setting where learners are developing and restructuring similar schemas through social constructivism. Such development relates to the previous discussion regarding Vygotsky's theory of the ZPD.

According to Skemp (1989:53), "concept formation has to happen in the learner's own mind…as teachers…help along the natural learning processes". Skemp (1989:62) describes formation of concepts by stating that "the process of abstraction involves becoming aware of something in common among a number of experiences, and if a learner does not have available in his own mind the concepts which provide the experiences, clearly he cannot form a new higher order concept from them". At this point, the role of the teacher becomes crucial in guiding learners and providing scaffolding within the ZPD. Skemp (1989:63) explains that knowledge is often constructed by combining and relating concepts which the learner has already mastered and owned through a process of explanation and use of examples. Skemp adds that learners are required to learn many higher order concepts in mathematics but that it is essential that learner already possesses the necessary lower order concepts. Learners may become confused by higher order concepts if their lower order concepts are incorrect or restricted, especially when such concepts are closely related. Such issues may be addressed within the ZPD when learners grapple with mathematical problem-solving.

According to Barnes and Venter (2008:11), "knowing what to do in a specific situation, but not necessarily understanding why it works, may limit the transfer of that procedure or skill". The individual learner learns to make connections and construct knowledge of mathematics in a flexible and coherent way which is fundamental to the development of schemas and, in turn, further mathematical knowledge. Countryman (1993) states that learners need to construct mathematics by "exploring, justifying, representing, discussing, using, describing, investigating and predicting". These elements can be incorporated and assimilated successfully when learners are engaged in solving mathematical problems that encourage development of mathematical knowledge while they progress through designated phases of ZPD.

In this study, learners use writing tasks to support and explain their mathematical problem-solving strategies. In order to solve problems, learners require certain mathematical knowledge and skills. If the necessary lower order concepts are incorrect or inadequate, learners experience difficulty later: they lack the essential schemas to engage in more advanced problem-solving. Mathematical problems develop knowledge: learners apply existing knowledge to the problem. This study investigates what kind of writing supports learners best when they solve and explain problems. Writing allows learners to clarify their thinking when they apply mathematical knowledge and reconstruct schemas.

Sfard's theory regarding mathematical ideas is relevant at this point: it relates to learners' understanding of mathematical ideas which is essential to solving mathematical problems.

2.2.7 The process and object of mathematical ideas

Sfard's theory on mathematical conceptions describes the interplay between the process and object of the same mathematical idea (Sfard, 1991:28). The process, or operational conception, is the dynamic action where an idea is conceived at a lower level. The object, or structural conception, is conceived at higher levels that underlie relational understanding (Sfard, 1991:16). Solving mathematical problems requires an existing knowledge of mathematical ideas: the objects. However, engaging in problem-solving may necessitate that a process be used to solve the problem which, in turn, may lead to the conception and development of other mathematical ideas.

Sfard (1991:19) explains the nature of moving from operational conception to structural conception where active, visual representations develop into a more abstract understanding through mental representations. Orton (2004:25) describes mathematics as a product (organised body of knowledge) and a process (learner participation in a creative activity). Problem-solving allows for movement between these concepts in order to use knowledge proficiently (Sfard, 1991:28). In this study, solving mathematical problems is supported by writing about the processes and solutions. In order to do so, learners engage in operational and structural conceptions as required by the problems they attempt to solve.

The literature review that follows focuses primarily on two areas: problem-solving and writing in mathematics. The nature and use of problem-solving develops mathematical knowledge and skills. Problem types and levels of strategies learners use when solving mathematical problems are discussed as they relate to number learning. Use of writing in mathematics is examined as well as types of writing tasks that can be used to encourage critical thinking and

support learners to solve mathematical problems. The role of language in mathematical problem-solving is argued.

2.3 SOLVING MATHEMATICAL PROBLEMS

The purpose of this study is to examine how writing tasks can support learners when they solve mathematical problems. In this section of the literature review, different perspectives of mathematical problems are examined. The use of problem-solving in the mathematics classroom is explained as well as the use of word problems as a type of problem-solving exercise. The role of previous knowledge and conceptual development is elaborated upon: both relate to mathematical problem-solving. Various types of word problems are dealt with as they are presented in a mathematics lesson.

2.3.1 Problem-solving

Problem-solving refers to real-life problems that encourage the use of skills such as prediction and analysis. Problem-solving makes use of novel problems that encourage critical thinking: learners engage with problems in an intelligent rather than routine manner (Orton & Frobisher, 1996:20). The problems are novel in that learners have not encountered the problem situation or context in previous mathematics lessons. Problem-solving encourages a higher cognitive demand: the context and the solution are not obvious (O'Donnell, 2006:349). According to Kuzle (2013:45), problem-solving is process-oriented: learners take an active role in generating ideas to solve problems. The ability to generate ideas further enhances the understanding that problem-solving requires higher order, critical thinking because solutions are, by definition, not immediately observable. The process of problem-solving may require learners to work through various possible solutions in order to solve problems (Marzano, 2014:85). Fülöp (2015:40) agrees that, in problem-solving, "students cannot directly apply methods and algorithms to solve it or… it is a task with multiple solutions where the students are asked to come up with different ways of solving the problem". Wright, Martland, Stafford and Stanger (2006:37) explain that learners could experience cognitive reorganisation when they generate more sophisticated strategies during problem-solving. Cognitive reorganisation links to Skemp's theory of constructing and reconstructing schemas when new mathematical knowledge is acquired. Problem-solving provides opportunities for such links to occur.

Word problems are a type of problem-solving. Burns (2007:16) explains how problem-solving and word problems are different but can be linked together to build the learner's use of mathematical knowledge. The following section addresses word problems.

2.3.2 Word problems

Burns (2007:16) states that traditional word problems require learners to "focus on the meaning of the arithmetic operations (where they need) to translate the situation into an arithmetic problem…and then perform the computation called for". She defines a mathematical word problem as a situation requiring that mathematical skills, concepts, or processes be used to arrive at the goal (Burns, 2007:17). This definition concurs with Frobisher's (1994:152) explanation that, "in a word problem, a task or situation is presented in words, and a question is asked which sets out the goal that the solver has to attain". Word problems are a particular way of presenting problems using words that provide a context or situation in which mathematical knowledge is required to find a solution. Burns (2007:16) links word problems and problem-solving when she proposes that problem-solving abilities can be raised through the use of word problems.

2.3.3 Problem-solving and previous knowledge

Problem-solving, as explained by Orton and Frobisher (1996:20), is "the use of novel problems which require children to draw on previously acquired knowledge expertise in an intelligent rather than random or routine way". There appears to be a common thread in this area of research: prior knowledge is a necessary starting point to problem-solving. In earlier research on problem-solving, Polya (1957:110) explains that, "in order to obtain the solution, we have to extract relevant elements from our memory, we have to mobilize the pertinent parts of our dormant knowledge…any feature of the present problem that played a role in the solution of some other problem may play again a role". Orton (2004:24) adds that "problem-solving is now normally intended to imply a process in which the learner combines previously learned elements of knowledge, rules, techniques, skills and concepts to provide a solution to a situation not encountered before". The same is true when learners encounter a word problem.

In order for learners to solve mathematical problems, they need to have some mathematical knowledge as a background on which to build. Polya (1957:9) explains that "the materials necessary for solving a mathematical problem are certain relevant items of our formerly acquired mathematical knowledge, as formerly solved problems". Learners use what they

know in order to solve that which is unknown: learners make connections with previous knowledge and mathematical problems in order to construct new meaning. Polya (1957:15) describes this connection as part of the problem-solving process where the learner looks back at a previous solution to make connections for solving a newer, harder problem.

Before presenting a learner with a problem, the teacher needs to establish what previous knowledge already exists. Orton (2004:25) posits that the knowledge and constructions the learner has mastered, linked to the knowledge required by the problem, can result in a successful solution to the problem.

2.3.4 Problem-solving and conceptual development

Problem-solving is described as a process of thinking and reasoning that helps conceptual development rather than procedural development (O'Donnell, 2006:351). It develops the learner's understanding of mathematical concepts rather than entrenching a set of procedures to reach an answer. Fosnot and Dolk (2001:9) concur learning in mathematics occurs through different contextual situations which generate various mathematical models, strategies and big ideas that involve schematizing, structuring and modelling. Big ideas are the structures of mathematics a learner grasps when making a shift in mathematical reasoning (Fosnot & Dolk, 2001:10), much like conceptual development. Models can represent mathematical ideas and be used as tools to express mathematical thought (Fosnot & Dolk, 2001:11). Learners may sometimes need to use mathematical tools or manipulatives to solve problems in order to make sense of the problem. However, they are still required to construct their mathematical knowledge by using certain tools as models of the mathematical relations that exist (Russell, 2000). It is through problem-solving that these big ideas and models are advanced.

Kennedy, Tipps and Johnson (2008:115) argue there is no problem if the answer and procedure are already known. If the procedure is known and applied to find the answer, an exercise is followed. Solving a problem requires a reflection and possibly an original step (Musser, Burger & Peterson, 2011:4). The problem needs to encourage a higher cognitive demand (O'Donnell, 2006:349) where the mathematical content embedded in the problem may not be immediately obvious to the learner. The learner needs to gain insight and perform analysis before finding a solution to a problem that could require decision-making (Heddens & Speer, 2006: 82). Kolovou, van den Heuvel-Panhuizen and Bakker (2009:35) posit that "the solution process often requires many steps back and forth until the student is able to unravel the complexity of the problem situation". In order to do this, the learner

ponders the problem thoroughly, tries out different approaches and connects a whole range of possible and appropriate techniques and methods (Orton, 2004:25). Problem-solving does not have a single direct path to a single fixed answer. A problem has to be deconstructed in order to understand and use the mathematical skills and tools required to find a solution. Luneta (2013:80) states that a problem is "a question (that) is posed to a person who initially does not know what direction to take to solve a problem (and) there may be many possible paths to a solution". This definition of a problem requires learners to become more flexible in their thinking, deepening their conceptual understanding in order to solve mathematical problems (Kennedy et al., 2008:5). When learners are engaged in problem-solving in this way, they become aware that one problem may be solved using different strategies: such thinking should be encouraged to explore alternatives (Kilpatrick et al., 2001:344). This view is supported by Sperry Smith (2013:65) who claims that as learners think about problems and create their own strategies, they become confident in using and enjoying mathematics in creative and original ways.

> *"Problem-solving ability is enhanced when students have opportunities to solve problems themselves and to see problems being solved. Further, problem-solving can provide the site for learning new concepts and for practicing learned skills" (Kilpatrick et al., 2001:420).*

Heddens and Speer (2006:84) argue the opportunity to apply conceptual knowledge through problem-solving is as important as understanding the concepts themselves because it provides more meaning and purpose to the knowledge and skills the learner has acquired: "mathematical thinking is nurtured through problem-solving experiences that do not restrict a child's avenues of success to a single route" (Heddens & Speer, 2006:85). This process allows learners to deepen their conceptual understanding. By solving mathematical problems, learners engage in the process of sense-making: they apply and develop their mathematical knowledge (Schoenfeld, 2013). This development occurs as a result of a learner's ability to "notice patterns, raise conjectures, and then defend them to one another" (Fosnot & Dolk, 2001:2). Learners learn to think critically about their own strategies as well as the strategies of others. Through sharing their strategies, learners are exposed to multiple strategies when they explain and compare their solutions and develop mathematical relations (Russell, 2000). Through this discussion and interaction with fellow learners, conceptual understanding is revealed (Campbell, Rowan & Suarez, 1998:50).

2.3.5 Invented strategies in problem-solving

Fülöp (2015:49) defines a strategy as the thinking aspect of problem-solving that is invented and flexible. She adds that it is "an overarching idea involving arranging or combining what is otherwise discrete and independent with a particular end in view". Strategy thinking involves making decisions while the doing aspect (methods and algorithms) entails implementing the decisions made. Added to this, Campbell et al. (1998:49) suggest that, when learners invent their own strategies, they enhance their learning. In reference to a project, Campbell et al. (1998:49) find that "students often solved problems by inventing algorithms on the basis of their interpretations of the problems, their understanding of arithmetic operations, and their representation of numerical relationships". Learners should be encouraged to explain their invented strategies (Campbell et al., 1998:50). This verbalisation of personal strategies displays an ability to arrive at the solution, demonstrating the conceptual and procedural knowledge needed in the process. Murphy (2006:219) adds that, when using their invented strategies, learners often rely on established mathematical ideas such as commutativity and associativity while they develop their mathematical reasoning abilities. In a study conducted by Fülöp (2015:51), it was found that instruction about different strategies was not a quick, easy process but that it was beneficial to learners' problem-solving abilities.

2.3.6 Types of problem-solving

There is a distinction between problem-solving and solving problems, whether they are word problems or mathematical problems. According to Orton (2004:84), "there are different kinds of problems in mathematics…routine practice problems, word problems, real-life applications and novel situations". Heddens and Speer (2006:82) concur by stating that there are four types of word problems: traditional textbook word problem; multistep textbook word problem; non-traditional word problem and a real-life problem situation. Routine practice problems may be incorporated at the end of a chapter or unit on a particular mathematical concept. Word problems, traditional word problems and multistep word problems may refer to problems traditionally presented where learners need to ascertain the operation required to solve the problem. Real-life problems and novel situations could be more realistic and relate to learners' own personal experiences, an example of which could involve planning a class outing and all the logistical aspects involved.

Use of word problems links to the process of problem-solving. Word problems play a crucial role in the mathematics classroom because they allow learners to develop the skills to engage in problem-solving. Hansen (2011:71) explains that word problems can have multiple purposes including the practice of mathematical skills, motivating children, assessing

attainment and developing problem-solving abilities and mathematical concepts and skills. Added to this, Kilpatrick et al. (2001:183) claim the use of word problems provide opportunities for learners to use more advanced levels of counting and procedures for computation. Different levels of counting and procedures that learners may use when solving problems as far as they relate to this study are discussed later. Such levels are referred to in analysis of learners' work described in the findings (chapter 4).

Although it is widely acknowledged that the use of word and/or mathematical problems and problem-solving in mathematics is essential to building conceptual understanding, problems are often not presented in a way that supports this. Heddens and Speer (2006:83) discuss these common shortcomings of the use of problem-solving in mathematics as:

- Not being constantly present throughout a unit;
- Not integrating topics from different units and/or subjects;
- Focused only on a specific interpretation of an operation;
- Looking for key words rather than contextual clues; and
- Oversimplified application of knowledge.

There is a difference between routine and non-routine (word) problems. Routine problems can be likened to solving procedures or exercises as discussed earlier. On the other hand, non-routine problems are more complex and puzzle-like. In a study conducted by Kolovou et al. (2009:45), on problem-solving in Dutch textbooks, it was found that the number of non-routine problems that encourage deeper conceptual understanding was irrelevant. The number of puzzle-like tasks presented in textbooks that the majority of Dutch teachers use are minimal which may be related to learners' underperformance in the area of problem-solving. It is possible that this lack of exposure to more complex problem-solving is a decisive factor in South African mathematics classrooms. Learners may not be presented with enough opportunities to deepen their conceptual understanding and develop better problem-solving abilities through challenging word problems.

2.3.7 Problem-solving in CAPS

According to the CAPS mathematics curriculum in the Foundation Phase (DBE, 2011:8), learners need to develop specific skills in mathematics, especially because they relate to problem-solving. These specific skills include: "learn to listen, communicate, think, reason logically and apply the mathematical knowledge gained; learn to investigate, analyse,

represent and interpret information; (and) learn to pose and solve problems". CAPS states, in the Foundation Phase, "solving problems in context enables learners to communicate their own thinking orally and in writing through drawings and symbols" (DBE, 2011:9). The curriculum does not specifically mention the use of writing in words when solving mathematical problems. Researchers such as Burns, however, advocate the use of writing in words. This study sought to determine whether the use of writing, including words, can support learners' mathematical problem-solving strategies. Luneta (2013:81) describes problem-solving, as indicated in CAPS, as "non-routine problems, higher order understanding and the ability to break a problem down into its component parts". It is imperative that learners use writing in the mathematics class to provide written explanations of their thinking when solving mathematical problems. It not only provides a means of clarifying their thinking and the strategies they choose to use but it can comprise an informative assessment of the learners' understanding. Later in this chapter, the purpose of writing in mathematics is examined: what it entails and the various types of writing tasks that can be used in mathematics.

2.4 PROBLEM TYPES

This study tests the use of different word problems to stimulate and develop learners' problem-solving skills and their ability to solve problems. The word problems relate to the basic operations (addition, subtraction, multiplication and division) using whole numbers. Naudé and Meier (2004:105) distinguish three problem types as they relate to the basic operations. These include problems that involve adding and subtracting, repeated addition as a means to conceptualise multiplication as well as grouping and sharing as a means to conceptualise division.

The purpose of combining addition, subtraction, multiplication and division for the purpose of this study is that mathematical problems are often presented in such a way that learners may use either operation as strategies: they are inverse operations. Some learners may use addition as a strategy while other learners may use subtraction to solve the same problem. This duality applies to problems where multiplication or division may be used as a strategy to solve a particular problem. Problem-solving usually has multiple paths to a solution; as previously mentioned in this chapter.

2.4.1 Addition and subtraction problem types

According to Kilpatrick et al. (2001:184), there are four types of problems that involve addition and subtraction: joining, separating, part-part-whole relations and comparison relations depending on which quantity is unknown. There are three quantities involved in addition and subtraction problems: the initial amount, the changed amount and the result (Naudé & Meier, 2004:108). Word problems provide contexts for adding and using different addition procedures to facilitate learners' reasoning and improve their understanding of addition processes (Kilpatrick et al., 2001:190). Kilpatrick et al. (2001:191) explain the relation between addition and subtraction as follows:

"Students examine a join or separate situation and identify which number represents the whole quantity and which numbers represent the parts. These experiences help students to see how addition and subtraction are related and help them to recognize when to add and when to subtract".

2.4.2 Multiplication and division problem types

Van den Heuvel-Panhuizen, Kühne and Lombard (2012:52) explain that "multiplication and division involve operations where objects or numbers are either grouped together or broken up into equal groupings". The concepts of repeated addition and grouping are usually associated with multiplication and the concepts of repeated subtraction, halving, sub-dividing and sharing are usually associated with division (Van den Heuvel-Panhuizen et al., 2012:52). Although multiplication and division are inverse operations that are closely linked, there are differences in their underlying strategies.

Division can use two main strategies, that is, distributing and chunking (Van den Heuvel-Panhuizen et al., 2012:53). Distributing involves sharing objects or numbers equally one by one, whereas chunking shares groups of objects equally. Fosnot and Dolk (2001:11) discuss unitizing when multiplying and dividing as the concept that requires children to "use number to count not only objects but also groups – and to count them both simultaneously". In other words, eight objects, for example, can concurrently be seen as one group.

Fosnot and Dolk (2001:53) refer to division problems as being quotitive or partitive. In quotitive problems, the whole is given in the problem and the learner needs to determine how many groups fit into the whole. One of the pre-test problems of this study (problem 2 in Appendix E) uses quotitive sharing where learners were given the amount in each group (platters of 7 doughnuts each) along with the total number of objects (e.g. 56 doughnuts). Learners needed to determine the number of platters needed: how many groups were

needed to fit into the whole. Partitive problems allow learners to distribute the whole amount between the number of groups given. For example, learners are given a problem where the whole amount of 35 needs to be distributed between 7 groups. A partitive strategy requires learners to share the whole amount of 35 items either by distributing or chunking as described earlier. Learners often find difficulty with partitive problems because they "need to comprehend the one-to-one correspondence involved…and consider the number of groups, the number in the groups, and the whole…simultaneously" (Fosnot & Dolk, 2001:53).

Splitting is another strategy that may be used when solving multiplication and division problems: learners break down the problem into smaller problems (Van den Heuvel-Panhuizen et al., 2012:162). Van den Heuvel-Panhuizen et al. (2012:154) refer to splitting as a form of decomposing into hundreds, tens and ones where learners require some understanding of the place-value structure of numbers.

2.4.3 Problem types in CAPS Mathematics

According to the CAPS Mathematics curriculum (DBE, 2011:79), there are certain problem types which should be posed at Grade 3 level. Learners should be solving problems such as grouping where the remainder is discarded or incorporated as well as sharing where the remainder is discarded. They should solve problems that involve repeated addition as well as addition and subtraction. These problem types specifically encompass the four basic operations which generally reflect problems used in this study. Other problem types such as sharing leading to fractions, grids, rate, proportional sharing and problem situations with different functional relations are mentioned in the CAPS Mathematics curriculum (DBE, 2011:79). However, these problem types are not included here: this research study focuses only on problems involving addition, subtraction, multiplication and division.

2.5 LEVELS OF PROBLEM-SOLVING STRATEGIES

`A variety of research has been conducted in the area of understanding the strategies and levels of conceptual knowledge that learners present when solving mathematical problems (Van Den Heuvel-Panhuizen et al., 2012; Wright, Martland & Stafford, 2006; Wright, Martland, Stafford & Stanger, 2006; Wright, 2013; Schoenfeld, 2013). Problem-solving strategies that learners have used, or are familiar with, are forms of knowledge they bring to a mathematical problem (Schoenfeld, 2013:18). The conceptual level that learners possess when solving a problem can be linked to the strategies they have previously used. This linkage implies that learners approach future problems with more knowledge than before

(Schoenfeld, 2013:20): they continually develop their mathematical knowledge and problem-solving strategies with each problem they solve.

For the purpose of this research study, the work of Wright, Martland and Stafford (2006) and Wright, Martland, Stafford and Stanger (2006) is fundamental in understanding learners' stages and levels of conceptual knowledge and the strategies used in tackling mathematical problems. The Learning Framework In Number (LFIN) is referred to as a description of early number learning.

2.5.1 Learning Framework in Number (LFIN)

The Learning Framework In Number (LFIN) "provides a blueprint for …assessment and indicates likely paths for children's learning" (Wright, Martland, Stafford & Stanger, 2006:7). LFIN encapsulates likely stages and levels of number learning that learners progress through as they develop their mathematical knowledge. The LFIN incorporates the following areas of number learning: the Stages of Early Arithmetical Learning (SEAL); number words and numerals; the Structuring Number Strand (SNS); conceptual place value knowledge and early multiplication and division (Wright, Martland & Stafford, 2006). The aspects of SEAL, conceptual place value knowledge and early multiplication and division, are most applicable to this study: all relate to strategies used by Grade 3 learners in solving mathematical problems. Descriptions of these aspects of the LFIN are given below. The majority of learners in the selected grade 3 class were at a stage of their number learning where they coped adequately with number words and numerals as well as SNS. These aspects of the LFIN are more applicable to the number learning required in lower grades: they were not areas of focus in defining and understanding the LFIN within this study.

2.5.1.1 Stages of Early Arithmetical Learning (SEAL)

The SEAL delineates the stages that learners pass through when they develop their knowledge of early arithmetic strategies and is, therefore, the primary aspect of LFIN. The term 'counting' is used to describe the SEAL. Wright (2013:28) clarifies the activity of counting as Forward Number Word Sequences (FNWS) and Backward Number Word Sequences (BNWS) in which the sequences of number words are recited. Wright, Martland, Stafford and Stanger (2006:20) posit that counting occurs when it is assumed that learners have a cognitive goal in determining the numerosity of a collection rather than reciting the FNWS or BNWS. Counting involves solving additive or subtractive problems (Wright,

Martland, Stafford & Stanger, 2006:10). The particular counting strategies that the learner uses in SEAL are demarcated in Table 2.1.

Table 2.1: Model for Stages of Early Arithmetic Learning (SEAL) (Wright, Martland, Stafford and Stanger, 2006:9)

Stage 0: Emergent Counting. Cannot count visible items. The child either does not know the number words or cannot coordinate the number words with items.
Stage 1: Perceptual Counting. Can count perceived items but not those in screened (that is concealed) collections. This may involve seeing, hearing or feeling items.
Stage 2: Figurative Counting. Can count the items in a screened collection but counting typically includes what adults might regard as redundant activity. For example, when presented with two screened collections, told how many in each collection, and asked how many counters in all, the child will count from 'one' instead of counting-on.
Stage 3: Initial Number Sequence. Child uses counting-on rather than counting from 'one', to solve addition or missing addend tasks (e.g. 6 + x = 9). The child may use a count-down-from strategy to solve removed items tasks (e.g. 17 – 3 as 16, 15, 14 – answer 14) but not count-down-to strategies to solve missing subtrahend tasks (e.g. 17 – 14 as 16, 15, 14 – answer 3).
Stage 4: Intermediate Number Sequence. The child counts-down-to to solve missing subtrahend tasks (e.g. 17 – 14 as 16, 15, 14 – answer 3). The child can choose the more efficient of count-down-from and count-down-to strategies.
Stage 5: Facile Number Sequence. The child uses a range of what are referred to as non-count-by-ones strategies. These strategies involve procedures other than counting-by-ones but may also involve some counting-by-ones. Thus in additive and subtractive situations, the child uses strategies such as compensation, using a known result, adding to ten, commutativity, subtraction as the inverse of addition, awareness of the 'ten' in a teen number.

As can be seen in Table 2.1 above, the strategies used in the SEAL become increasingly sophisticated. In the earlier stages, learners often need to see the items they are counting, whether they are physical objects or representations of objects, e.g. drawings or tallies. By the time learners reach stage 5, they are able to use advanced strategies such as those listed in Table 2.1. Facile number sequence includes a range of strategies that learners reach having gone through the development of number learning in the previous stages. Learners make use of procedures that display a deeper conceptual knowledge they apply their knowledge of compensation, commutativity, doubles and inverse operations, for example. The strategies which learners use when they solve problems display the stages of their conceptual development. When solving the same problem, some learners may use a strategy that reflects a lower or higher stage of number learning and development compared to other learners. Such distinctions may become evident in their writing when they solve and explain problems.

Learners progress to at least stage 3 of the SEAL, and generally begin to develop base-ten arithmetical strategies. Learners develop in their understanding of groups of ten within numbers as opposed to working with individual items: counting in ones. Learners develop

their conceptual understanding and more complex strategies could become more evident in their writing when they solve and explain problems. Wright, Martland and Stafford (2006:22) provide a detailed description of the levels of strategies depicted in Table 2.2 below.

Table 2.2: Model for development of base-ten arithmetical strategies (Wright, Martland, Stafford & Stanger, 2006:10)

Level 1: Initial Concept of Ten. The child does not see ten as a unit of any kind. The child focusses on the individual items that make up the ten. In addition or subtraction tasks involving tens, children count forward or backward by ones.
Level 2: Intermediate Concept of Ten. Ten is seen as a unit composed of ten ones. The child is dependent on re-presentations (like a mental replay or recollection) of units of ten such as hidden ten-strips or open hands of ten fingers. The child can perform addition and subtraction tasks involving tens where these are presented with materials such as covered strips of tens and ones. The child cannot solve addition and subtraction tasks involving tens and ones when presented as written number sentences.
Level 3: Facile Concept of Ten. The child can solve addition and subtraction tasks involving tens and ones without using materials or re-presentations of materials. The child can solve written number sentences involving tens and ones by adding or subtracting units of tens and ones.

Note: A necessary condition for attaining Level 1 is attainment of at least Stage 3 in the Stages of Early Arithmetical Learning.

2.5.1.2 Conceptual place value

According to Wright (2013:27), learners make use of materials such as bundling sticks, ten strips and hundred squares when they develop their understanding of place value. Learners' conceptual understanding of place value progresses by incrementing in tens on the decuple (adding using the multiples of ten): then off the decuple (adding ten to any number, e.g. 47). Following this process, learners decrement by tens off the decuple (subtracting ten from any number) and finally they are able to give ten more and ten less as well as a hundred more and a hundred less of given numbers.

At Grade 3 level, learners are expected to have some conceptual understanding of place value as stated in the CAPS Mathematics curriculum (South Africa DBE, 2011). Learners who have appropriately developed their understanding in this area make use of it as part of their strategies when solving mathematical problems.

2.5.1.3 Early multiplication and division

Learners progress through five levels when they develop their understanding of multiplication and division; part of the LFIN. The five levels of early multiplication and division are depicted in Table 2.3 below.

Table 2.3: Model for early multiplication and division levels (Wright, Martland, Stafford & Stanger, 2006:14)

Level 1: Initial Grouping. Uses perceptual counting (that is, by ones) to establish the numerosity of a collection of equal groups, to share items into groups of a given size (quotitive sharing) and to share items into a given number of groups (partitive sharing).
Level 2: Perceptual Counting in Multiples. Uses a multiplicative counting strategy to count visible items arranged in equal groups.
Level 3: Figurative Composite Grouping. Uses a multiplicative counting strategy to count items arranged in equal groups in cases where the individual items are not visible.
Level 4: Repeated Abstract Composite Grouping. Counts composite units in repeated addition or subtraction, that is, uses the composite unit a specified number of times.
Level 5: Multiplication and Division as Operations. Can regard both the number in each group and the number of groups as a composite unit. Can immediately recall or quickly derive many of the basic facts for multiplication and division.

As with the SEAL, learners progress through the levels of early multiplication and division. When they are at levels 1 and 2, they require individual items to be counted. At level 1, learners do not count in multiples whereas they use more advanced counting strategies at level 2. When learners have reached levels 4 and 5, their understanding of multiplication and division is more abstract because they do not require items, whether physical or drawn.

To conclude, levels of conceptual understanding according to the LFIN have been set out. This delineation of levels relates to this study in terms of analysis and reflection on learners' work, especially when solving mathematical problems. Such delineation allows a comparison to be made between the strategies learners used in the pre-test and the post-test. The next section of this chapter addresses writing in mathematics.

2.6 WRITING IN MATHEMATICS

The purpose of this study is to investigate how various types of writing tasks support Grade 3 learners in solving mathematical problems. In this section of the chapter, the purpose of writing in the mathematics classroom is examined. Reasons for including writing as an essential part of the mathematics lesson are given in supporting the development of mathematical knowledge that is crucial to effective problem-solving strategies.

2.6.1 The purpose of writing in mathematics

The importance of writing within mathematical problem-solving is to encourage children to develop a meaningful understanding of mathematical knowledge. Davison and Pearce

(1998:42) explain that performing a writing task requires learners to reflect on, analyse, and synthesize the material being studied in a thoughtful and precise way. Luneta (2013:87) adds that, when learners write a reflection on where they are stuck, it allows them to reflect on their mathematical understanding of concepts. Putting their strategies on paper allows learners to be mindful of their own strategies while verbal feedback can often be lost over time. Writing helps learners clarify and define their thinking as well as examine their ideas and reflect on what they have learned in order to deepen and extend their understanding (Burns, 1995a:13). Their final work is not meant to be a polished product but rather a provisional means of expressing and consolidating their understanding of mathematical ideas (Burns, 2007:38). Kuzle (2013:43) concurs by stating that writing is a tool for learning and communicating mathematics. Writing in mathematics is one of the means of representing and communicating understanding: it helps the learner to make sense of mathematical ideas in order to construct knowledge. According to Columba (2012:3), conceptual understanding develops when learners represent their understanding using words, symbols, graphs and discourse.

Researchers (Jurdak & Zein, 1998; Miller, 1991, 1992; Bagley & Gallenberger, 1992; Morgan, 1998) concur on the importance and purpose of writing in mathematics. Through the active process of writing, learners read the product of their thinking on paper: it is a way of knowing what they think and deepening their understanding. Learners reflect on, clarify and explain their thought processes.

When solving mathematical problems, writing forms a vital role in the learner's development of conceptual knowledge. According to Carruthers and Worthington (2006:13), this development occurs when learners make meaning personal: they make actions, marks, draw, model and play. Writing may take various forms in a mathematics lesson and, more so, in problem-solving because it encourages learners to engage actively with their previous knowledge to develop strategies and methods for solving a more difficult problem. Writing creates opportunities to make connections to the mathematical knowledge required by the problem. Writing in the form of words, pictures and numbers provides a platform for learners to explain their thinking to themselves and peers by placing emphasis on their process and not just the answer (Van de Walle & Lovin, 2006:16). Whitin and Whitin (2008:432) add that learners should be encouraged to write with increasing clarity and detail to demonstrate their understanding of a problem. In this study, learners are introduced to the use of writing in mathematics. Learners in this study had opportunities to engage with various writing tasks for different purposes. This study focuses on writing as a method to help learners solve

mathematical problems. The aim is to gauge whether the use of writing supports learners to make sense of mathematical problems when they solve and explain them.

2.6.2 Representing thinking through the use of writing

While learners are attempting to solve mathematical problems, they represent their thinking through what they write. They grapple with a problem and attempt to make sense of it. Their writing is a reflection of what is happening in their minds. Sperry Smith (2013:171) explains that "writing about math is one way to reflect on the process and to explain and defend ideas". Writing provides a key opportunity for learners to develop, clarify and communicate their thinking. Luneta (2013:109) claims that as learners represent their understanding by writing, they communicate mathematical ideas and understanding about concepts to themselves and to others. Writing in mathematics allows them to reflect, check, amend and understand what they have done (Orton, 2004:91).

Learners may use a variety of ways to represent their thinking when engaging with a particular problem. A mathematical problem can have multiple paths to a solution. A group of learners may have different representations of their thinking on paper: they may use various strategies to solve the same problem. The use of different strategies and representations may be due to the various mathematical abilities of the learners who understand mathematical concepts at varying levels. Some learners may write and solve problems at more sophisticated levels than others based on their levels of conceptual understanding. Through engaging with the writing of others, learners are able to compare and learn from the strategies of their peers. Luneta (2013:125) adds that learners gain better problem-solving skills when they are presented with both text and pictures. While learners write, they can represent their thinking through numbers, words and pictures and make sense of the mathematical problem.

As in a similar research study conducted by Amaral (2010), the thinking process is of importance when using writing in mathematics and not the presentation, spelling and/or grammar. The purpose of writing is to make sense of the mathematical problem and communicate thinking and understanding (Burns, 2007). The piece of writing is a product of their thinking and not a test of their writing abilities.

The use of representations in writing provides teachers with insights into learner thinking (Luneta, 2013:126). Through writing, the teacher is made more aware of individual learners' understanding, misconceptions and difficulties which may be responded to individually or

corporately (Borasi & Rose, 1989:358). Miller (1991:517) agrees: misconceptions can be dealt with through the use of writing. These insights may determine the direction of future lessons which address misconceptions timeously and appropriately. This provision helps the teacher to implement intervention strategies, either individually or as a class (Miller, 1991:519). This type of intervention may, in turn, improve the teacher's mathematics instruction as a whole.

In another study, conducted by Fluent (2006:43), it was found that learners' improvement in written explanations was nominal despite sharing strategies verbally. In the experience of the researcher/teacher of this particular study, learners had opportunities to solve problems individually and in pairs. They engaged in discourse in their mathematical ability groups concerning their strategies. The results of this study will be discussed in chapter 4 in comparison to Fluent's study.

2.6.3 Writing in mathematics lessons

Writing in mathematics may be implemented in different ways. Miller (1992:354) suggests engaging learners in a short writing activity at the start of a lesson to express their thinking and prepare for the lesson. These writing activities provide opportunities for a written dialogue between the teacher and learners. It is more likely that learners who find difficulty in mathematics may feel more at ease to express their confusion because this dialogue is private (Miller, 1991:518). However, the teacher needs to consider learners who have language difficulties: such learners may be less able to express themselves through writing. Alternatively, Elliot (1996:92) discusses the benefits of concluding a lesson with a writing activity to reflect on the day's lesson. This conclusion may guide the teacher in preparation for the next lesson if there are misconceptions which require correction.

In this study, specific writing tasks were introduced as a means to cultivate the use of writing in mathematics to support learners when they solve mathematical problems and explain their solution strategies. The type of writing task being implemented determined the point at which the task was used during the lesson. Certain writing tasks such as writing to record in a journal and writing to explain are likely to take place during the conclusion of a lesson. On the other hand, writing to solve mathematical problems is better suited to an earlier part of the lesson to give learners the opportunity to engage in group discussions on their strategies and explanations. The following section describes the various types of writing tasks that were implemented in this study.

2.7 TYPES OF WRITING TASKS IN MATHEMATICS

Writing in mathematics takes various forms which require learners to record their thinking in different ways. Writing may occur with or without revision (Wilcox & Monroe, 2011:521). It can be introduced through short, simple writing tasks at first to encourage and develop the use of writing tasks in the mathematics lesson (Meier & Rishel, 1998:7).

After reading the work of various researchers, writing in mathematics as explained by Burns (1995a) was used. Burns describes different types of writing tasks and their purpose in developing conceptual understanding. The writing tasks presented in her work were conducted with learners from different grades throughout the primary school years into early high school. Since this study focused on the Foundation Phase, Burns's writing tasks were most suitable: her research included learners from these grades. She provides a detailed methodology of each writing task that largely links to the aims of this study. There are five different writing tasks that will be described further below: writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes (Burns, 1995a) and shared writing (Wilcox & Monroe, 2011). Although not directly from Burns's work, shared writing was added to this study because it linked to the current curriculum guidelines in use in South Africa. Shared writing is an element of the Balanced Language Approach (BLA) in which learners and the teacher write together.

The overarching purpose of using writing in mathematics (discussed earlier in this chapter) is for learners to clarify, explain and communicate their thinking (Burns, 2007:38). Through the task of writing, early learners gain the opportunity to develop conceptual knowledge and build mathematical connections. Each writing task, according to Burns, has a different purpose and is used to develop specific areas where learners engage with a particular perspective of explaining and communicating their thinking. This study investigates how writing tasks can be used to support learners to solve word problems.

2.7.1 Writing to solve mathematical problems

The use of this writing task is specifically to solve mathematical problems: learners write to solve and explain their strategies. It is distinct from writing to explain, for example, which focuses on learners explaining their understanding of particular mathematical concepts. According to Kuzle (2013:44), writing is considered a method to support the acquisition and development of mathematical knowledge that enables an improvement in problem-solving

abilities. As learners write about their problem-solving strategies, they are able to make sense of their mathematical understanding. This concurrent writing reveals their understanding of the mathematical concepts concerned in the particular problem they are dealing with as well as their understanding of how mathematics relates to real life. Wadlington, Bitner, Partridge and Austin (1992:209) concur that writing about problem-solving connects mathematics to the world around the learners.

Burns (1995a:69) explains that, in order to solve problems, learners should use a variety of strategies, and verify and interpret results. This combination of strategies creates opportunities for them to develop and explain their thinking. In doing so, learners not only record their solutions but provide their reasoning as to why the answer made sense to them (Burns, 1995a:76). Jacobs and Ambrose (2009:265) emphasise that learners' representations of a strategy are linked with their interpretation of the problem and should reflect how they thought about and solved the problem. In a study conducted with pre-service teachers, Kuzle (2013:53) notes that writing about the problem-solving process enabled participants to better understand and justify their thinking when they reflected on their strategies and solutions. Although the participants of this research study are Grade 3 learners, the use of writing to solve mathematical problems encourages learners to consider and reflect on their strategies. This writing task engages learners in organising their thoughts: they write an explanation of the processes they followed to solve a mathematical problem.

Burns (1995b:41) encourages learners to discuss their ideas before engaging in writing: learners share possible strategies to solve the problem as a springboard for writing about their individual strategies and explanations. She adds that prompts displayed on the board help learners to start writing if they require such assistance. In the Foundation Phase, learners are often given opportunities to solve mathematical problems in pairs or groups. Despite working co-operatively, they should still write about the experience individually to develop and clarify their thinking (Burns, 2007:39). The think-write-share strategy develops learners' own understanding of the mathematical problem by thinking and recording their responses on paper on their own before participating in pair or group discussions (Wilcox & Monroe, 2011:522): the thinking and writing aspects of problem-solving happen individually and learners share their thinking and writing with others. On some occasions during this study, the think-write-share strategy was employed as a means to support learners in their writing tasks when they solved mathematical problems.

2.7.2 Writing to record (keeping a journal or log)

Burns (1995a:51) explains that journals or logs allow learners to keep ongoing records about what they are doing and learning in their mathematics class which can be used to record their thinking when they notice something, make an observation, or report a discovery. Keeping a journal or log serves to enrich the quality of discussions, review previous knowledge and construct meaning. Borasi and Rose (1989:348) refer to a journal as a "personal notebook where students can write down any thought related to their mathematics learning". Through writing in their journals, learners are actively engaged in the process of making connections and constructing meaning. In this way, the mathematical content makes more sense to the learner when they construct and internalise knowledge. Yang (2005:14) refers to journal writing as mathematical diary writing. He explains that, through mathematical diary writing, learners are able to communicate and reflect on their thinking by explaining what they learn in class in their own way. Bagley and Gallenberger (1992:661) describe the purpose of journal writing as allowing learners to summarise and associate ideas, define concepts, experiment with concepts, review and reflect on topics and strategies and express their feelings and frustrations with regards to their mathematics learning. Yang (2005:13) adds that the use of diary writing enables learners to enjoy problem-solving through writing as opposed to only representing their thinking. Jurdak and Zein (1998:416) find that there is a relation between journal writing and mathematics instruction. The results of their study show its positive effects on conceptual understanding, procedural knowledge and mathematical communication.

O'Donnell (2006:351) concurs that learners need daily opportunities to write about their mathematics lessons in a journal. The teacher may use prompts to focus their journal writing. Although Borasi and Rose's (1989:355) research involves university students, they acknowledge the need for prompts because some students find difficulty in writing spontaneously. A framework may be given that focuses on a specific lesson or mathematical concept that has been taught (Burns, 2007:39). Freed (1994:24) suggests a flexible use of the journal that allows opportunities for free writing as well as structured writing activities with the use of prompts. As learners write freely in this way, they do so without concern about spelling, punctuation and style (Bagley & Gallenberger, 1992:661).

Journals are ongoing records of learners' thinking which provide learners with regular opportunities to reflect on mathematics lessons and/or concepts and analyse their own learning. Bagley and Gallenberger (1992:660) explain that a journal allows the teacher to informally evaluate learners' levels of comprehension. Journals in mathematics provide a

record of learners' development for the teacher (Amaral, 2010:24). Amaral (2010: 67) shows that journals help to keep the teacher informed of learners' progress, drive instruction, improve learners' communication and increase learners' understanding of mathematical concepts. Keeping a journal creates a "private dialogue between the teacher and each student …(through) the exchange of questions, responses, comments and remarks" (Borasi & Rose, 1989:360). At a Foundation Phase level, a journal may prove more challenging because learners find it difficult to engage in a written dialogue. However, simplified comments and words of encouragement may stimulate individual learners to express their thinking. In this study, written dialogue was used at times to communicate with learners. Comments were written about their strategies when solving problems as well as when they engaged in other writing tasks. Learners were asked questions about their strategies in order to explain further or extend their thinking through their writing.

2.7.3 Writing to explain

The purpose of this writing task is for learners to explain what they understand about a specific mathematical topic or concept. This writing task could be referred to as note-taking or note-making where learners list the main points of a lesson as well as their reflections and perceptions (Wilcox & Monroe, 2011:522). Freed (1994:23) refers to note-taking as defining a concept where a term is explained in the learner's own words. Learners can "summarise what they learned and tell how to apply it" (Freed, 1994:23). When using this type of writing task, there is a focus on a particular mathematical concept that learners are required to clarify and explain. For example, after having learnt fractions, learners write an explanation of fractions in their own words. In their explanations, they are encouraged to write about what they have learned and understood.

2.7.4 Writing about thinking and learning processes

This form of writing task does not focus on a specific topic or mathematical concept. Burns (2007:40) suggests that learners write about their favourite or least favourite activities, qualities of a good problem-solving partner, directions for an activity or game or a letter to visitors describing mathematics activities in the classroom: "A letter to a friend, relative or teacher can combine reflective and communicative writing" (Freed, 1994:24). This type of writing task has a more general focus where learners engage in writing more freely. Writing in this way allows learners to think beyond the actual mathematics lesson and more on mathematics in general.

2.7.5 Shared Writing

In the CAPS curriculum for home language, shared writing is mentioned as a methodology to develop learners' writing skills in literacy. Shared writing involves the teacher and learners writing together or learners writing in pairs or groups. This kind of co-operation differs from the think-write-share strategy mentioned previously. Wilcox and Monroe (2011:526) suggest that teachers use this writing experience in the mathematics classroom to review and internalize mathematical concepts and ideas as well as develop mathematical communication. Together in the ZPD, the teacher and learners formulate a mathematical story reflecting their understanding of a particular concept. Learners may then take different sentences to write as a final draft and create representations for a class book. A similar approach could be used to make alphabet books about mathematics vocabulary. Freed (1994:23) suggests writing poetry about mathematical concepts, vocabulary or topics such as limericks, cinquains and concrete poems as well as a rap. Involving learners in activities such as these encourages learners to put their knowledge and understanding of mathematics across in a creative way and, at the same time, solidify that knowledge. Shared writing allows learners to collaborate as a class or group so encouraging the element of social interaction in a constructivist classroom.

Previous research (Burns, 1995a; Luneta, 2013; Jurdak & Zein, 1998; Miller, 1991, 1992; Bagley & Gallenberger, 1992; Morgan, 1998) shows that writing can be used to help develop understanding of mathematical concepts and processes. Although there are different writing tasks presented by Burns (1995a), there are similarities, overlappings and links between them. The use of writing tasks enables learners to clarify and represent their thinking. Writing may enhance their conceptual understanding generally when they are stimulated in this way to reflect on what they have done. Different types of writing tasks provide valuable tools to deepen the individual learner's knowledge while working collaboratively with the teacher and peers. In this study, such collaboration is manifested in examining learners' writing and the support it gives mathematical problem-solving.

2.8 THE ROLE OF LANGUAGE IN MATHEMATICAL PROBLEM-SOLVING

In this section of the literature review, the role of language in mathematical problem-solving is examined. When engaging with problems, learners are expected to read and understand as well as solve and explain their strategies. The language used in mathematics and how this is applied in problem-solving contexts is discussed.

According to Luneta (2013:105), learners should know and understand the language of mathematics and develop skills to apply it. Clemson and Clemson (1994:84) describe mathematics as a language of symbols which transcends words which: learners are expected to use in talking, reading and writing about what they have encountered in symbolic form. Often, words that are used in everyday language, take on a different and more specific meaning when used in a mathematical sense (Luneta, 2013:94). Such secondary meanings may cause confusion for learners, especially those who have limited language abilities. Often the errors in learners' thinking stem from their misunderstanding of the vocabulary of mathematics (Koshy, Ernest & Casey, 2000:177). It is imperative that time is spent teaching mathematical vocabulary linked to relevant concepts so that learners' understanding is enhanced. Learners acquire the language of mathematics through careful explanation, listening and practice (Sperry Smith, 2013:56). Burns (2007:372) explains that learners acquire mathematical language when words are used in contexts that bring meaning to them. However, Burns (2007:43) exhorts that "teaching knowledge of the mathematical ideas and relations must precede teaching vocabulary". It is only when doing so that learners connect their knowledge to mathematical language.

Clemson and Clemson (1994:98) add that reading competence needs to be achieved in order to solve mathematical problems. The wording of a problem could be read aloud and talked through before solving it in order to assist learners with language difficulties and to develop understanding. Learners may rely on keywords presented in the problem which may mislead them. Sperry Smith (2013:56) explains that, in order for learners to understand the mathematical concepts and processes required by the problem, their attention needs to be drawn to the way the problem is phrased. This strategy may support learners in developing a better understanding of the problem they are reading.

Language is important in this study: learners were engaged in reading, talking and writing in mathematics lessons. The impact of learners' level of language competency on their individual ability to solve problems will be discussed in Chapter 5.

2.9 CONCLUSION

The first section of this chapter focuses on the theoretical framework. Vygotsky is presented as the main theorist with a particular focus on social constructivism, the ZPD and scaffolding. Other theorists are drawn upon that link or elaborate upon to Vygotsky's theories. Bruner's ideas on scaffolding are included with Vygotsky's. Skemp's theory on the construction of schemas relates to the learners' development of conceptual knowledge. Learners'

understanding of mathematical concepts is discussed using Sfard's theory of the process and object of mathematical ideas.

The second section of chapter 2 reviews literature that relates to the research question. There is a discussion of research about mathematical problems and the use of problem-solving in the mathematics classroom. Types of problems that learners encounter in mathematics and the levels of understanding at which they solve these problems are set out. The work of Wright, Martland and Stafford (2006), Wright, Martland, Stafford and Stanger (2006) and Wright (2013) is pertinent in explaining the development of learners' early number learning through the Learning Framework In Number (LFIN). The literature review then focuses on writing in mathematics, drawing on various researchers' work. Detailed descriptions of the types of writing tasks as presented by Burns (1995a) and Wilcox and Monroe (2011) are given as they are used in this study. Finally, the role of language is examined as it pertains to learners' understanding of a mathematical problem. The methodology designed for this study is explained in chapter 3.

CHAPTER 3
RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

The purpose of this research study is to investigate how various types of writing tasks support Grade 3 learners in solving mathematical problems. In this chapter, the research methodology and design of the study are described. In order to answer the research questions, the study makes use of a qualitative research design in the form of a case study. The research site was a primary school in Cape Town, South Africa. The sample for this study was a Grade 3 class from which eight learners was purposively selected. The data collection instruments that were used included interviews, audio-recordings, field notes and learners' written work. In this chapter, the purpose of the instruments and the process of gathering and analysing the data are described. The trustworthiness, validity and reliability of the study are explained and the ethical considerations outlined.

The research design is the logical plan that guides the process of linking the data to be collected to the research questions and the conclusions of the study to ensure that the evidence addresses the research questions (Yin, 2009:26). The design could take the form of a quantitative, qualitative or mixed methods study. Quantitative research establishes generalisable trends and objective facts through the use of surveys, questionnaires and statistics; whereas qualitative research studies human beings and their behaviour in order to make sense of feelings, experiences, social situations and phenomena (Rule & John, 2011:60). Salkind (2009:12) refers to "the general purpose of qualitative research methods (as examining) human behavior in the social, cultural, and political contexts in which they occur". In addition, Denzin and Lincoln (2011:3) state that qualitative research attempts to make sense of phenomena and the meanings people bring to them. Mixed methods research combines these traditions to obtain a more holistic understanding of the data and, in turn, the research results. This research study makes use of a qualitative research design because the researcher investigates how writing tasks support learners in solving mathematical problems. Learners' experiences of writing tasks were observed during mathematics lessons. Lessons provided the social context for data to be collected. Data were collected from a pre- and post-test, as well as from interviews with selected learners. In this qualitative study, the researcher sought to interpret how writing could be used as a tool in the mathematics classroom to support and develop learners' mathematical problem-

solving skills and to determine whether these writing tasks could be implemented successfully in the Foundation Phase.

3.2 RESEARCH DESIGN

There are a number of design methodologies that are commonly used in qualitative research studies. These include action research, comparative research, case study, evaluation and experiment (Thomas, 2011:36). This research study makes use of a case study approach. Simons (2009:21) defines a case study as follows:

> *Case study is an in-depth exploration from multiple perspectives of the complexity and uniqueness of a particular project, policy, institution, programme or system in a 'real-life' context. It is research-based, inclusive of different methods and is evidence-led. The primary purpose is to generate in-depth understanding of a specific topic.*

Rule and John (2011:4) define a case study as a systematic, in-depth investigation of a particular instance in its context in order to generate knowledge. Writing tasks were introduced systematically during the data collection period beginning with the use of writing to solve mathematical problems. Learners already had prior experience of solving mathematical problems and were familiar with some elements of using writing to represent their thinking. They were progressively introduced to other writing tasks such as writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes and shared writing. While learners engaged with the different writing tasks, in-depth observations of their writing and the development of their writing strategies were conducted. The phenomenon, in the case of this study, was the learners' use of writing when solving mathematical problems which was monitored in its natural context; mathematics lessons, over a given period of time (Swanborn, 2010:13).

This case study has elements of a design experiment in which particular forms of learning are engineered and studied within a particular context (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003:9). A design experiment involves various elements such as tasks or problems, discourse, the established norms of participation, the tools provided, and the practical means by which classroom teachers can orchestrate relations among these elements (Cobb et al., 2003:9). In this study, Burns's (1995a) American-based use of writing in the mathematics class was applied in a South African Foundation Phase classroom. Based on Cobb et al.'s description above, the tools in this study were the writing tasks that were used to assist learners to express their mathematical thinking as they solved mathematical problems (the tasks) and to develop their mathematical understanding further

(Burns, 1995a:49). Relations among these elements were orchestrated by modelling the types of writing tasks to the learners and, noting how they can be used to solve and explain the thinking behind the solution of mathematical problems. In doing so, learners think, reason and make sense of mathematical ideas in order to support and enhance their problem-solving abilities (Burns, 1995a:13).

3.3 RESEARCH PLAN

This study incorporated various activities that sought to answer the research question. A pilot study was conducted with a different class of Grade 3 learners prior to the data collection. Details concerning the pilot study and its significance in the overall plan for the data collection are discussed later. A pre-test was given to the selected Grade 3 class followed by interviews of eight learners regarding their solutions in the pre-test. Writing tasks were introduced and implemented in the class as an intervention to support learners in solving mathematical problems. The data collection period concluded with a post-test and another set of interviews with the same eight learners. Together, these activities of the research plan sought to determine whether the writing tasks had supported the learners in their mathematical understanding and their ability to solve and explain problems. In the following sections the execution and purpose of the various activities of the research plan are elaborated.

3.3.1 Pilot Study

A pilot study was conducted with a different class of Grade 3 learners. Yin (2009:92) suggests that a pilot case study enables the researcher to refine the content of the data collection plan and the procedures to be followed. A pilot study provides conceptual clarification for the research design so that the researcher is able to develop relevant questions for the actual case study (Yin, 2009:92). The purpose of this pilot study was to give direction to the research plan by assisting in the design of the mathematical problems to be used for the actual study. This process took place in the year prior to the data collection period. The pilot study provided an opportunity to test various types of writing tasks while learners solved mathematical problems. Testing was a way of gauging the level of teacher support required. This technique helped to link particular mathematical problems to a suitable type of writing task that supports learners when they solve mathematical problems. A total of 35 learners participated in the pilot study. Before the pilot study, parents of the learners concerned were informed that the normal teaching and learning required by the

curriculum would not be adversely affected. They were made aware that none of the learners' written pieces would be used in the thesis report.

During the pilot study, learners used a journal only and were given eight writing tasks. The writing tasks used during the pilot study were: writing to solve mathematical problems, writing to record (keeping a journal or log) and writing to explain. Due to time constraints, learners did not have an opportunity to use shared writing or writing about thinking and learning processes. After reading their entries using the writing task, writing to explain, it became apparent that much more guidance and support were needed on the purpose of each writing episode. It was noted that instructions and expectations should be clarified better, especially when learners start using writing to explain their thinking. It was imperative to make learners aware that spelling, punctuation and grammar would not be taken into account when their writing was being read. After this explanation was conveyed to the class, a few of the struggling learners felt more at ease when writing in their journals and verbally expressed this greater security during the pilot study.

Learners had one experience of writing to record (keeping a journal or log) during which they reflected on the day's mathematics lesson. Before this writing episode, prompts were displayed and discussed during other lessons when learners gave verbal responses to the prompts. However, it was found that, when learners were expected to write their responses, they appeared more restricted in their written responses when compared to their prior verbal responses. This insight, gained from the pilot study, was valuable in the planning and implementation of the writing tasks in the actual study.

The pilot study was beneficial in preparing for the data collection period: it allowed the researcher to gain insight into the learner support required when introducing and implementing various types of writing tasks. The pilot study emphasized the importance of presenting the purpose of each writing task which was modelled to the learners. When learners were engaged in writing to solve mathematical problems, the researcher observed that verbal explanations of solutions did not reflect their written strategies when solving the problem. Verbal explanations were sometimes different to written solutions and explanations. The dichotomy between their verbal and written explanations demonstrated that they may not have understood the purpose of the writing task.

During the pilot study, a variety of mathematical problems were presented to learners. Some of the mathematical problems dealt with fractions. It was a concern that the variety of problems made the focus of the study too broad. In the activities of the research plan of the

actual study, mathematical problems using whole numbers that involved addition, subtraction, multiplication and division were used. Other mathematical problems were tackled during data collection as part of the normal teaching and learning programme. However, certain mathematical problems were purposively selected for this research study. These problems were selected because they used whole numbers and aimed to address the research questions of this study.

The insight gained from the pilot study made it possible to identify suitable mathematical problems to be used during the data collection period and to refine the data collection plan.

3.3.2 Data collection plan

Data were collected over a period of 10 weeks including a week at the beginning and the end for the pre-test and post-test respectively. This process spanned school terms: when I had collected data from the middle of one school term to the middle of the next. It was envisaged that there would be three writing episodes per week. However, in practice, this was not the case because the normal teaching and learning programme of the school needed to be considered which included time for assessments and related interventions. Some public holidays fell within the data collection period. Only one writing episode occurred in week 2 and week 6. In week 7 learners engaged in writing to explain geometric patterns. Although this exercise did not fall under whole numbers, it was decided to use this as a topic since it followed on from the lessons covered at the time. While this writing episode did not offer much evidence for this study, it provided insight into whether the use of writing tasks could be applied across content areas. The following table shows the data collection plan as it had been adjusted to accommodate the needs of the class and school.

Table 3.1: Data collection plan

	Researcher's actions	Learners' actions	Problems used
Week 1	Pre-test (five mathematical problems)		
Week 2	Model writing to solve mathematical problem		Problem 1
	Assessments		
	Assessments		Problem 1 continued
Week 3	Model writing to record (keeping a journal or log)	Write in journal	Problem 2 Problem 3
	End of first school term		
Week 4	Model writing to explain	Write to explain (empty number line)	Problem 4
Week 5	Model writing about thinking and learning processes (letter)	Write about thinking and learning processes (letter to principal)	Problem 5 Problem 6
Week 6			Problem 7
Week 7	Model shared writing	Shared writing (story) Writing to explain (geometric patterns)	Problem 8 Problem 9
Week 8		Write to explain (fractions) Write about thinking and learning processes (favourite activity)	Problem 10
Week 9		Write about thinking and learning processes (qualities of a good problem-solver)	Problem 11 Problem 12 Problem 13
Week 10	Post-test (five mathematical problems)		

The following sections describe the different aspects of the data collection period and how they unfolded during this study.

3.3.3 Pre-test

At the beginning of the data collection period, five mathematical problems (Appendix E) were given to the learners to solve as a pre-test over a period of five days. Three of these problems were addition/subtraction problems while two problems were multiplication/division problems. Structuring the pre-test in this way gave more opportunities to analyse the learners' strategies thoroughly. Their strategies would be used to select eight learners to be interviewed after the pre-test. The process and purpose of the pre-test were explained to the learners. The purpose of the pre-test was to gauge the learners' ability to solve and explain mathematical problems at the beginning of the data collection period. It was explained to the learners how the pre-test would be conducted. Learners were each given an A4 sheet of

paper to be used for the five problems presented to them during the pre-test. Although all the learners were presented with mathematical problems that shared the same context, the number ranges of the problems were differentiated according to the three mathematical ability groups in the class. The mathematical problems were read aloud before learners solved them to assist learners with reading difficulties. A brief class discussion took place before solving each problem but, for the most part, learners solved the problems individually so that the researcher could ascertain the mathematical knowledge, skills and strategies learners employed to solve the problems. While learners were solving the problems, the researcher moved around the classroom to observe their strategies and solutions without giving assistance.

3.3.4 First set of interviews

The solutions and strategies that learners employed during the pre-test were used to purposively select eight learners who were individually interviewed. The eight learners represented the different mathematical ability groups. Two learners were selected from the above average ability group and three learners each from the average and below average ability groups. The sample of learners was selected based on varying levels of success in solving the pre-test mathematical problems in order to obtain a variety of learners' written work. The purpose of the interviews was for learners to explain the written strategies they had used when solving the problems of the pre-test. The verbal explanations of their strategies and solutions were compared to their written strategies. The interviews helped to gauge learners' understanding of problems because learners verbally explained their strategies. (Appendix H lists the interview schedule.) The interviews were audio-recorded and transcribed.

3.3.5 Writing tasks

Following the pre-test and first set of interviews, the different types of writing tasks were introduced and implemented with the Grade 3 class. The researcher modelled the different types of writing tasks as presented by Burns (1995a) when they were introduced to the learners. Modelling allowed learners to see how each type of writing task could be used and provided them with opportunities to practise implementing them. During these writing episodes, the purpose of the writing task was communicated to learners as a means to support them while they were solving mathematical problems and help them to explain their thinking. From the insights gained through the pilot study, the researcher tried to ensure that

learners participating in the study understood the purpose and expectations of the various types of writing tasks.

Learners were encouraged to participate in the class discussion even though it was largely led by the researcher to help them understand how the writing tasks applied to mathematics in the context of problem-solving. After this phase, each type of writing task was modelled to the learners on a big sheet of paper which was displayed on the mathematics wall in the classroom for the rest of the data collection period. While each writing task was being modelled, the researcher continued to link the writing task to the mathematical content. Learners were invited to express their ideas as well as to enhance their understanding of writing in mathematics.

At Grade 3 level, the end of the Foundation Phase learning experience, learners are expected to have gained a certain level of competence when working with the basic operations. To this end, a variety of word problem types was used involving addition/subtraction and multiplication/division of whole numbers. The problems were used as tools to gauge learners' understanding of mathematical concepts within basic operations. The types of writing tasks taught to the learners during the data collection period aimed to support their understanding and ability to solve word problems. Learners used their journals to record and clarify their thinking when solving mathematical word problems. This technique enabled them to become accustomed to writing and the expectations of writing.

The mathematical problems (listed in Appendix G) were purposively selected prior to the data collection period. The results from the problems used in the pilot study assisted in selection of the problems for the actual study. In chapter 2, different problem types were discussed according to the basic operations the problems covered. Some criteria were used in the selection of the mathematical problems. The selected mathematical problems used whole numbers. The problem types used one or more of the four basic operations to arrive at the solution without incorporating other mathematical concepts such as fractions. The problems were presented in such a way that there would be more than one strategy to reach the solution. For some of the problems, learners could use inverse operations to reach the same solution. Four mathematical problems involved addition and/or subtraction. The remainder of the problems engaged learners in repeated addition/multiplication and division. One of these problems (problem 7) lent itself to using addition, subtraction and multiplication while another (problem 9) lent itself to using addition, multiplication and division. Problem 1 was a separation problem where the initial was unknown. Problem 2 was a joining problem where the result was unknown. Problem 6 was a comparison problem where the difference

was unknown. Problems 3, 4 and 5 were subtractive division problems. Problems 8, 10, 11 and 13 could be solved as repeated addition or repeated subtraction problem types. In problem 12, learners were presented with a strategy for solving a subtraction problem. Learners had to analyse the strategy to see where the mistake was made. One non-routine or context-free problem (problem 10) was included in the data collection. This problem was incorporated to see whether learners were able to explain their thinking when solving context-free problems as well.

3.3.5.1 Modelling writing to solve mathematical problems

The first writing task that was modelled for learners was writing to solve mathematical problems. In the Foundation Phase, learners should have already used writing in solving mathematical problems by the time they reach Grade 3. However, learners are generally not required to explain their thinking in writing. Often the expectation is simply to find the correct answer. It was decided that this assumption should be the first type of writing task introduced in this study because several learners were already familiar with the exercise. Jacobs and Ambrose (2009:265) suggest that learners write about their strategies and representations as a way of reflecting on the problem and how they solved it. The researcher explained that, when a problem is being solved, it should be possible to explain verbally and in writing what the solution is and how it was arrived at. The reader should realise that the answer is correct by explaining how the problem was solved so that the reader can understand why the answer and the solution made sense. What is written is as important as arriving at the answer. When writing, learners may use drawings, numbers and words to represent their thinking in a way that makes sense to all participants. Such a writing task was modelled specifically to solve and explain a problem (see Figures 3.1 and 3.2). Following this modelled lesson, learners were given an opportunity to solve and explain a mathematical problem before another type of writing task was introduced.

Figure 3.1: Modelling writing to solve mathematical problems

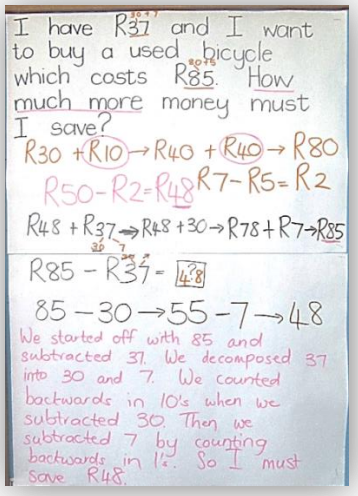


Figure 3.2: Display of writing to solve mathematical problems

3.3.5.2 Modelling writing to record (keeping a journal or log)

Writing to record (keeping a journal or log) was introduced next and the purpose of this writing task was explained to learners. According to Burns (1995a:51), this type of writing task should be an ongoing record of what learners are doing and learning in their mathematics class. It can be used to record their thinking as lessons occur. The journal prompts, called sentence starters, were displayed (see Appendix I). The prompts were discussed during which time learners suggested possible sentences orally. The ways in which writing prompts could be used as a starting point to explain learners' thinking were then modelled.

Learners were not expected to write in their journals daily during the data collection period. They were encouraged, however, to write as they noticed something or made a discovery which deepened their conceptual understanding of mathematics as they solved word problems (Burns, 1995a:51).

3.3.5.3 Modelling writing to explain

The next writing task, writing to explain, followed from a mathematics lesson on place value. At the end of the lesson, the researcher expounded on this type of writing task. A class discussion ensued around the purpose and process of this writing task. The researcher explained that this type of writing task is employed to explain a particular mathematical

concept: it helps to organise thinking and share what is understood with others. Learners in the class suggested sentences about place value upon which the class deliberated. As they did so, the researcher modelled writing to explain on a big sheet of paper (see Figure 3.3). Figure 3.4 shows the sentences that were written to explain the concept of place value. This record was the learners' first attempt at explaining place value. At a later stage, learners revisited this explanation to determine whether they could change or add to this explanation so that it made more sense. This alteration phase occurred after the data collection period once learners had more opportunities to engage with the concept of place value in mathematics lessons.

Figure 3.3 Modelling writing to explain

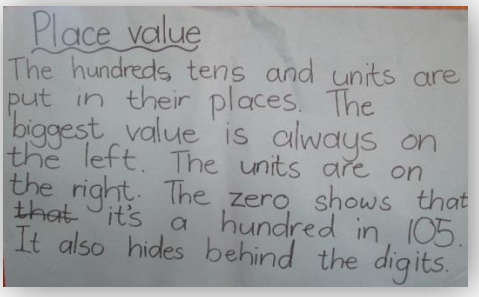


Figure 3.4 Display of writing to explain

3.3.5.4 Modelling writing about thinking and learning processes

The next writing task that was modelled was writing about thinking and learning processes. It was explained that this type of writing task communicates what happens in mathematics lessons. First a class discussion was held about what takes place in mathematics. Learners shared various thoughts and experiences of previous lessons such as mental mathematics, counting and group activities. After a few suggestions, learners decided that they would write about the problem-solving discussions that happen on the carpet once groups have solved a problem. Writing a letter to the principal was modelled explaining when and how the writing discussions happen on the carpet after learners write about their problem-solving strategies and explanations. A few learners provided sentences that were agreed upon by the class (see Figures 3.5 and 3.6).

Figure 3.5 Modelling writing about thinking and learning processes

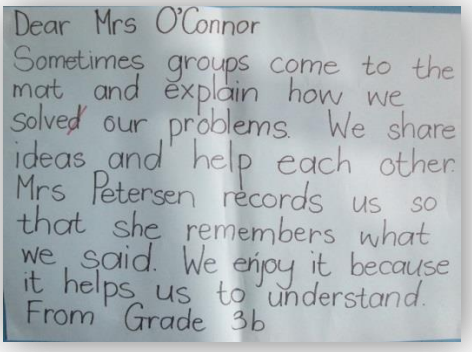


Figure 3.6 Display of writing about thinking nd learning processes

3.3.5.5 Modelling shared writing

Shared writing is not specifically drawn from Burns (1995a) but rather from researchers such as Wilcox and Monroe (2011). As mentioned in the literature review (paragraph 2.7.5), this type of writing task was included in the study because it is a methodology prescribed in the RNCS (CAPS) curriculum for languages currently used in South Africa. It was explained to learners that shared writing in mathematics can be used to review and internalize mathematical concepts and ideas and develop mathematical communication (Wilcox & Monroe 2011:526). The measurement concept of one centimetre was the context for the story modelled to the learners. Learners were encouraged to give ideas as the story developed which enabled the link to be made between the mathematical concept and the story.

3.3.5.6 Summary of implementation of writing tasks

Different types of writing tasks were introduced gradually and not all at once so that learners became accustomed to using each one. As Burns's (1995a) teaching methodologies suggest, class discussions took place to brainstorm ideas before learners engaged in writing. These discussions assisted learners to formulate their own thinking and extend their ideas.

After each writing task was introduced, learners engaged in writing episodes that allowed them to implement what was modelled to them. In all writing episodes, learners were urged to explain their thinking. At times, sample writing was discussed to deepen learners' understanding; allowing them to reflect on their thinking. Learners received feedback on their writing as notes were made in their journals or on their papers, or as the researcher

spoke to them during or after the writing episode. As Amaral (2010) suggests, any feedback given, whether verbal or written, should be positive to enhance the ability to appropriate and implement writing tasks. Learners were encouraged to write in their journals whenever they had an idea, made an observation or noticed something, and not only when instructed to do so. When learners wrote collaboratively as a pair or small group in a shared writing exercise, one piece of writing was on paper with individual learners' names recorded on it.

This study makes use of Burns's (1995a) different types of writing tasks in Grade 3 mathematics lessons. The types of writing tasks were introduced and implemented over a period of approximately eight weeks. They included writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes, and shared writing. Table 3.1 lists the writing episodes that learners engaged in throughout the data collection period. Learners predominantly made use of writing to solve mathematical problems since this writing task was applicable to answering the research questions of the study. However, other writing tasks were included as a means of supporting learners to develop the use of writing in mathematics generally.

In chapter 4, the findings of the writing episodes that learners engaged in after the different types of writing tasks were modelled are described.

3.3.6 Post-test

Learners solved five mathematical problems as a post-test at the end of the data collection period to gauge whether the use of writing supported them in solving mathematical problems. The problem types used in the post-test were similar to those used during the pre-test. Two of the problems in the post-test were addition/subtraction problem types while three problems were multiplication/division problem types. The differentiated mathematical problems are listed in Appendix F.

The post-test was conducted in a similar manner to the pre-test. The mathematical problems were read aloud to assist learners with language difficulties. However, learners solved the problems individually. The strategies and explanations learners wrote were compared to those in the pre-test in order to ascertain what improvements or changes occurred in their problem-solving strategies and abilities.

3.4 SITE AND SAMPLING

3.4.1 Site

The site for this research was a preparatory school in a suburban area in Cape Town, South Africa. It is a quintile 5, Foundation Phase school with classes from Grade R to Grade 3. The Language of Teaching and Learning (LOLT) at this school was English. The school is moderately well resourced in that learning and teaching support materials are readily available. The school's ANA mathematics results have been at an acceptable standard over the last few years. Results of the Grade 3 systemic tests monitored by the WCED have consistently been between 80% and 85% in mathematics. This percentage represents the number of learners who achieved the minimum requirement of 50%. The average percentage that learners scored per class was between 50% and 65% for the different content areas in mathematics. In the analysis of these results, learners at this particular school performed lowest in areas concerning problem-solving.

This school was selected as the site for this study because the researcher was a Grade 3 teacher at the school. Using another school as the site would have been disruptive to the normal teaching programme of the selected school.

3.4.2 Sample

In qualitative research, sampling methods may be random, convenient or purposive (Simons, 2009:35-36). For this study, convenient sampling was conducted because there were five Grade 3 classes at the school but the study was conducted in only one of the classes. This restriction kept the amount of data collection manageable because the researcher was the teacher of the selected class. The nature of the study involved collecting data at least three times per week over an eight week period which included the modelled writing lessons and excluded the pre-test and post-test. Obtaining data from the other classes was not realistic or manageable: it may have disrupted the normal teaching and learning programmes of the classes concerned. Conducting research in another class would have made it difficult to observe and make field notes when learners wrote in their journals at any time. For these reasons, the decision was made to select one of the Grade 3 classes for which I was the teacher for the purpose of this study. Doing so allowed for an in-depth exploration into the use of writing in mathematical problem-solving.

The population of the study constituted all the learners in the participating Grade 3 class where writing was introduced and implemented over a period of approximately eight weeks.

However, the researcher worked intensively with a sample of eight learners selected from the class. These learners were purposively selected. Purposive sampling involves a deliberate selection of settings, persons or activities to provide relevant information about the goals of this research (Maxwell, 2013:97). The eight learners were selected based on the solutions and strategies they used when solving the mathematical problems during the pre-test. They displayed varying abilities when solving and explaining mathematical problems and represented the three mathematical ability groups present in the Grade 3 class. The LOLT (English) was the home language of the eight sampled learners. These learners displayed varied literacy abilities in the classroom. Although the study focused on writing, its purpose was not to reflect on learners' literacy abilities. Writing in mathematics focuses on the conceptual understanding of mathematical concepts which is represented in numbers, words and pictures. The eight learners were selected based on their mathematical abilities and not their literacy abilities. These learners were interviewed following the pre-test and post-test to explore how writing was used as they solved mathematical problems. Due to the nature of the data collection in this study, it was decided that this study would not report on data from all the learners in the class because the data set would become too large. By purposively selecting eight learners, this thesis report could be more specific in answering the research questions.

The development of the eight learners' problem-solving abilities was determined through the mathematical problems presented to the learners at the beginning and the end of the data collection period. The development was gauged during this period while the researcher observed the learners' development of writing and how it was used to support their mathematical problem-solving abilities. It is important to note that learners were given generic mathematical problems with varying number ranges to accommodate the different mathematical ability groups in the class.

In the following section, data collection instruments used during this study are presented. These instruments were selected because they best suited the process of gathering data for the purpose of the study.

3.5 DATA COLLECTION INSTRUMENTS

The purpose of this research study is to investigate how various types of writing tasks support Grade 3 learners in solving mathematical problems. The data collection instruments used to answer the research questions included learners' written pieces, audio-recordings of the ability group discussions, field notes and interviews. In this section, each data collection

instrument and the purpose for including it in this study is described. The manner in which each instrument was used is outlined.

3.5.1 Learners' written work

In the case of this research study, the learners' written tasks were in the form of journals and work done on paper. The types of writing tasks included writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes, and shared writing.

According to Swanborn (2010:73), the advantage of documents, in this case the learners' written work, is that they provide a stable source of data: they are outside the researcher's influence. Rule and John (2011:67) suggest that the documents may prompt important questions which could be pursued further in interviews. In this study, learners' written work during the pre-test was used to select eight learners to be interviewed. The interview questions (Appendix H) referred to the writing these learners used when they solved mathematical problems of the pre-test. However, disadvantages of this data collection instrument include a biased selectivity as well as the possible bias of the researcher herself (Yin, 2003:86). As the teacher of the selected class, the researcher had to be aware of selecting learners based on their use of writing in the pre-test, considering their mathematical abilities and not their literacy abilities.

The eight learners' written work was collected over the duration of the data collection period when they engaged in various writing tasks. Written work was analysed to determine how writing tasks supported Grade 3 learners in mathematical problem-solving. Added to this, the mathematical problems learners solved during the pre-test and post-test were included as part of this data collection instrument.

3.5.2 Audio-recordings

In this study, audio-recordings were made throughout the data collection period since all the learners in the class engaged with writing to solve mathematical problems. Learners solved differentiated problems (listed in Appendix G) according to the mathematical ability group of which they were a part. They were given time to solve the problems and write their solutions and explanations in their journals. The different ability groups discussed their solutions and strategies on the carpet while the rest of the learners continued working on their solutions or completed other mathematics activities. Audio-recordings were made of the ability group

discussions. It was decided that audio-recordings would be beneficial as an instrument for collecting data because it may not have been possible to capture as much of the discussions as possible through taking field notes alone. Additionally, audio-recordings were inconspicuous (Creswell, 2014:192) since they allowed learners to explain their strategies and participate in discussions more freely. Learners were not as distracted by the audio-recordings as they would have been had field notes been made during the discussions. The use of audio-recordings allowed facilitation of the group discussions: it was not necessary to take field notes during discussions. The audio-recordings were transcribed, coded and analysed.

3.5.3 Field notes

Dana & Yendol-Hoppey (2009:74) explain that field notes capture what is occurring without commenting on or judging a particular act where no interpretations are made. Particular forms of field notes used in this study include scripting dialogue and conversation during the ability group discussions as well as when learners worked collaboratively in pairs. Field notes were used to record what learners were doing at particular times. How learners used different types of writing tasks in mathematics was noted. In this study, field notes captured the preferences learners had towards using different types of writing tasks, if any, and how writing can support mathematical problem-solving.

3.5.4 Interviews

Simons (2009:43) describes interviews as a means of exploring core issues quickly and in-depth. Interviews provide opportunities to ask follow-up questions and probe motivations. As a data collection method, “interviews can be time consuming to arrange, carry out and to analyse and yet interviews can also provide some of the richest data” (Hamilton & Corbett-Whittier, 2013:104).

The eight Grade 3 learners selected in this study were interviewed individually regarding their use of writing and how it influences and supports their thinking when solving mathematical problems. They were interviewed on two occasions. They were first interviewed after the pre-test and again after the post-test. Interviews were semi-structured with a flexible list of questions and key themes (Appendix H). Silverman (2011:162) highlights the skills needed to conduct semi-structured interviews as probing, rapport with the interviewee and understanding the aims of the study. As the teacher of the selected Grade 3 class, the researcher had an established rapport with the learners being interviewed. At the same time

care had to be taken not to influence their responses in a particular direction. The interview questions related to specific writing episodes the learners experienced during the pre-test and the post-test. Olsen (2012:33) explains the use of questions and prompts in semi-structured interviews: both need to be planned in advance. Learners were selected based on their use of writing during the pre-test, so their strategies were considered in the planning of possible prompts. Interviews were audio-recorded to ensure that analysis of interviews was not limited; rather that data were captured in their entirety. Interviews were transcribed, coded and analysed. This process is elaborated upon during discussion of the data analysis of this study.

3.6 DATA ANALYSIS

The process of analysing data makes sense of what has been collected. Rule and John (2011:75) state that the "key research questions…developed at the start of the study should serve as a guiding force in the analysis process". Data collected for this study were explored in relation to the research questions stated below.

Research question:

How do various types of writing tasks support Grade 3 learners in solving mathematical problems?

Sub-questions:

1. What support do writing tasks give to the development of conceptual understanding?
2. What support do writing tasks give to the development of problem-solving strategies?
3. How are writing tasks useful in the Foundation Phase mathematics classroom?
4. What challenges do learners experience when writing in the Foundation Phase mathematics classroom?

A four step approach to analysis of data was used: description, sense making, interpretation and implications which are commonly used in case studies (Dana & Yendol-Hoppey, 2009:120). This process enabled management of data analysis. The steps of description and sense making were used to organize and prepare the data for interpretation (Rule & John, 2011:76).

3.6.1 Description

Audio-recordings of the group discussions as well as interviews with the eight Grade 3 learners were transcribed in preparation for data analysis. Olsen (2012:39) describes

transcription as “writing down or typing out the text of an interview or other sound file”. Pseudonyms were used during transcription in order to maintain participants’ anonymity. Olsen (2012:35) adds that transcripts enable the researcher to have “insight into mechanisms, processes, reasons for actions, and social structures as well as many other phenomena”. Once data were organized and prepared, data were read and re-read to develop a descriptive sense of what was happening, describe initial insights and reflect on the overall meaning (Creswell, 2014:197). In doing so, empirical information was converted into a description of the data in order to draw meaning from them (Henning, 2004:6).

3.6.2 Sense making or coding

The next step, sense making, is referred to as coding. According to Cohen and Manion (1994:286), coding is the “translation of question responses and respondent information to specific categories for the purpose of analysis”. Rule and John (2011:77) refer to the use of codes as labels that are assigned to different themes or foci within the data. Moreover, Dana and Yendol-Hoppey (2009:118) refer to Schwandt’s definition of coding as “a procedure that disaggregates the data, breaks it down into manageable segments and identifies or names those segments”. Coding is “a database of connections between various terms and data items selected from among the whole basket of evidence” (Olsen, 2012:46). This understanding of the process of coding guided this part of the analysis process.

There are various steps to coding data including determining the size of text segments, developing a list of codes for basic retrieval and then detailed retrieval (Olsen, 2012:80). For the purpose of this study, data were coded using ATLAS.ti, a computer-assisted qualitative data analysis programme.

> *“Computer packages allow the user to store notes about the definition of their codes and to retrieve segments of data that have been assigned different codes, allowing you to gather together all instances of a particular code in order to compare these” (Barbour, 2014:262).*

ATLAS.ti proved a useful tool which provided a comprehensive overview of large amounts of text in the form of the learners’ written work as well as transcriptions of audio-recordings and interviews (Henning, 2004:126). Although computer-assisted analysis was used in this study, it remained the primary responsibility of the researcher in ensuring systematic, thorough analysis of the data (Barbour, 2014:260).

Friese (2014:12) refers to NCT analysis as an analytical procedure or approach. NCT analysis involves “noticing interesting things in the data, collecting these things and thinking

about them, and then coming up with insightful results" (Friese, 2014:12). Referring to NCT analysis, Friese (2014:13) describes noticing things as "the process of finding interesting things in the data…and nam(ing) them". In this study, as patterns were noticed in the data, they were assigned or attached to codes. The codes were largely determined before data were collected in order to provide a provisional coding frame for the data analysis. These codes developed from the relevant literature read in this field of research. The codes were established to attempt to answer the research questions of this study. The provisional coding frame included: representations of mathematical problems, demonstration of understanding through writing, individual writing and collaborative writing, the use of prompts when writing, the use of specific topics to develop writing and conceptual understanding, the usefulness of Burns's types of writing in a South African context and the development or change in learners' abilities to solve mathematical problems. However, these codes were adapted and additional codes considered while data were analysed. Barbour (2014:260) adds that one moves "back and forth between provisional and revised coding frames and transcripts or coded extracts in order to interrogate themes and build up explanations". Codes were developed deductively before data collection and inductively during data analysis. Some codes were merged into themes for the purpose of addressing the research questions in the findings and discussion in chapters 4 and 5 while others became themes on their own. Themes were used to search for a detailed description of the use of writing when solving mathematical problems (Creswell, 2014:199).

Barbour (2014: 278) argues theoretical frameworks that inform data analysis are often "referenced in terms of guiding the general approach taken in research, in formulating the questions to be asked and in determining what counts as 'data'". In this study, the theoretical framework largely concerned social constructivism since learners participated in collaborative work and group discussions where they engaged with the teacher and their peers. Through social constructivism, learners developed their mathematical problem-solving abilities in their ZPD's through the introduction and implementation of various writing tasks. Scaffolding was used to assist learners while they developed their use of writing in mathematics. Through encountering the writing tasks, learners' procedural knowledge and conceptual knowledge were drawn upon and developed. According to Thorn (2000:68), this theoretical lens determines how the researcher approaches and collects data which is relevant in answering the research questions so that raw data can be transformed to depict the focus of the study.

As discussed in the literature review in chapter 2, the Learning Framework in Number (LFIN) incorporates the Stages of Early Arithmetical Learning (SEAL), the Structuring Number Strand (SNS), conceptual place value and early multiplication and division (Wright, Martland

& Stafford, 2006). The LFIN, in conjunction with the theoretical framework, was used to guide the process of data analysis. Using the different stages and levels of the various components of LFIN, the researcher analysed the strategies that learners employed when solving mathematical problems, particularly when analysing their strategies in the pre-test and post-test. The LFIN supplied clear indicators of the stage or level of strategies that were used in the pre-test when compared to the post-test. This process made it possible to gauge what developments or changes there were in learners' strategies when solving mathematical problems.

3.6.3 Interpretation

During interpretation, statements were constructed that express and communicate the findings supported by data. Creswell (2013:187) describes interpretation as "abstracting out beyond the codes and themes to the larger meaning of the data". Olsen (2012:56) adds that interpretation is processing data by presenting it differently in order to deliver new meaning. In this study it was considered whether a writing intervention, through the implementation of writing tasks, made an appreciable difference to learners' use of strategies for solving problems (Creswell, 2014:178). The purpose was to determine whether writing tasks supported learners' development of mathematical problem-solving abilities.

3.6.4 Drawing implications

The final step, drawing implications, involves any change of action the study may bring about or any new questions generated for further research.

> *"Conclusions are likely to be strengthened by some further analysis that attempts to make sense of similarities and differences within the dataset and which also seeks to locate the study within the wider picture of what is already known about the topic in question" (Barbour, 2014:263).*

This study may lead to wider use of writing in mathematical problem-solving in the Foundation Phase in South African schools. It may support learners to develop their problem-solving strategies as well as their conceptual understanding and mathematical knowledge and skills. This study may highlight the usefulness of writing tasks in the Foundation Phase and the difficulties learners experience when implementing them.

The interpretations and implications drawn from the data are expounded upon in chapter 5 as far as they relate to discussion of findings in this study.

3.7 VALIDITY, TRUSTWORTHINESS AND RELIABILITY

According to Maxwell (2013:121), validity is assessed in terms of how it relates to the purposes and credibility of the research study. Validity can be achieved through explicitly reporting how research was conducted and locating any weak points within the study (Swanborn, 2010:37). Throughout this chapter, the researcher outlined in detail the research process by providing the research plan that answered the research questions of this study.

Triangulation was applied to the data collection and analysis of this study. Rule and John (2011:109) explain triangulation as "the process of using multiple sources and methods to support propositions or findings generated in a case study". This explanation confirms Yin's (2009:99) statement that the findings of the case study are corroborated through multiple measures of the same phenomenon. Craig (2009:108) adds that triangulation involves multiple sets of data to focus on views and perceptions of a particular phenomenon. This strategy eliminates bias and strengthens the validity of the study. In this study, data collection instruments include learners' written work, audio-recordings, field notes and interviews. According to Denzin and Lincoln (2011:5), using multiple methods such as those employed in this study adds rigour, complexity and depth. Such added objectivity was particularly evident because verbal explanations learners gave during interviews and group discussions added depth to the evidence provided from written work which displayed individual strategies and explanations.

Bias was reduced through the careful formulation of open-ended questions for the interviews (Cohen & Manion, 1994:282). The learners' responses were audio-recorded. Audio-recordings of the interviews and ability group discussions ensured the researcher's neutrality and the legitimacy of the learners' responses (Davies, 2007:157). These audio-recordings enabled the researcher to put aside prior assumptions in order to process data to determine the outcomes of the study. The use of multiple data collection instruments and audio-recordings ensured the results of the data analysis were accurate representations of the context in which data were collected (Davies, 2007:243).

The researcher for this study is also the teacher of the learners selected. As such, her position as teacher-researcher is acknowledged since it may have compromised the reliability and interpretation of data generated through various data collection instruments. Guba's concept of trustworthiness (Rule & John, 2011:107) refers to, amongst other things, the confirmability of the study where the researcher's influence and bias are disclosed. In this instance, the researcher's position as teacher is acknowledged as a possible bias and

limitation to the research study. The researcher's familiarity with the learners' mathematical ability may have presented bias in selection of the learners to be interviewed. Knowledge of learners' language abilities may have affected use of scaffolding; provided to those who were known to struggle to read, comprehend and interpret the mathematical problems. Some of the strategies learners used during the writing intervention may have been achieved through additional support. However, scaffolding was not provided during the pre-test and post-test where strategies and explanations were analysed and compared. The researcher limited the potential bias of being both teacher and researcher.

With regards to positionality, Creswell (2014:188) states that past experiences in the classroom "may cause researchers to lean toward certain themes, to actively look for evidence to support their positions and to create favourable or unfavourable conclusions about the sites or participants". Having previous knowledge of learners' abilities may have led to pre-empting the themes for discussion. According to Creswell (2014:188), such knowledge can compromise disclosure of information as well as create an imbalance of power between researcher/teacher and learners. The process of analysis could have been biased to meet the desired conclusions of this study. Being aware of this danger increased the researcher's care to collect data as accurately as possible: multiple instruments were used during this study. Trustworthiness could be ensured through triangulating the analysis and findings from the data collection instruments.

Reliability is reached through the precision of procedures and documentation. Henning (2004:151) describes reliability as follows:

> *"If all research steps are declared and documented, the research is potentially replicable and someone may then assess, by doing it all in the same way in a similar setting and with similar participants, whether the replicability is feasible".*

Creswell (2014: 203) explains that the reliability of a study is found in its consistency and stability concerning the steps and procedures followed in documenting the case study. In the research design and plan described earlier in this study, the steps and procedures were detailed to elaborate upon the reliability of the study. The research design and plan of this study could be replicated in a similar or different context to determine whether the results were consistent and reliable.

3.8 ETHICAL CONSIDERATIONS

According to Salkind (2009:80) anonymity should be observed during the research process by maintaining confidentiality: anything that is learned about the participant is held in the strictest of confidence (Salkind, 2009:82). In this research study, pseudonyms were used for the school and the names of all participants.

Salkind (2009:80) explains that a research project that relies upon human participants should have an informed consent form that is read and signed by each participant or the person granting participation (in the case of a minor child with the parent signing). For the purpose of this study, permission was sought from, and granted by, the Western Cape Education Department (WCED), Cape Peninsula University of Technology (CPUT), the principal of the school and the parents of all the learners in the participating Grade 3 class. The permission letters from the WCED and CPUT are provided in Appendix A and Appendix B. Samples of the consent letters to the principal and parents of the learners are provided in Appendix C and Appendix D.

Ethical concerns regarding implementation of the writing tasks were considered. Although eight learners were purposively selected for this study, all the learners in the participating Grade 3 class implemented the writing tasks as an intervention to support their mathematical problem-solving (Creswell, 2014:98).

3.9 CONCLUSION

In this chapter, the methodology used for this research study was explained. This qualitative study employed a case study approach because the use of writing tasks was investigated during mathematics lessons. Details of the research plan were elaborated. The pilot study was discussed as it gave meaningful assistance to the formulation of the research plan for the actual study. It afforded unique insights into the selection of mathematical problems for this study. The strategies and solutions of the pre-test conducted at the beginning of the data collection period were analysed and eight learners were selected to be interviewed. All the learners of the participating class were introduced to the writing tasks. Writing tasks were modelled to the learners who were given opportunities to use the writing tasks over a period of eight weeks. A post-test was conducted at the end of the data collection period to determine whether the use of writing supported learners in their problem-solving strategies. Throughout the data collection period, data were collected through learners' written work, audio-recordings, field notes and interviews. This compilation of data facilitated triangulation

and ensured the trustworthiness and validity of the study. Data collected from the data collection instruments were transcribed and coded using a provisional coding frame. Additional codes were added where necessary. Data analysis was conducted using the theoretical framework as well as the LFIN. Themes for the discussion emerged. This process occurred through merging codes into themes or taking a code as a particular theme in the discussion. The conclusion of the data analysis process was described with a focus on the interpretation of the findings and the implications drawn for the use of writing in the Foundation Phase of a typical South African mathematics classroom.

The following chapter presents the findings of this study as they relate to the research questions.

CHAPTER 4
FINDINGS

4.1 INTRODUCTION

In this chapter the findings of this research study into the use of writing tasks to develop learners' mathematical problem-solving skills are discussed (Burns 1995a). This study employed various types of writing tasks and investigated whether these writing tasks can be implemented in the Foundation Phase classroom in the South African context as a way of supporting learners when they engage in mathematical problem-solving.

Different frameworks were explored to describe learners' number learning and to find a suitable tool for analysis of the learners' problem-solving strategies. The framework of Wright, Martland and Stafford (2006), Wright, Martland, Stafford and Stanger (2006) and Wright (2013) on early number learning was used to describe different stages and levels of early number learning as outlined in the Learning Framework In Number (LFIN), explained in chapter 2. The stages and levels of the LFIN provide clear indicators of the conceptual development of number learning that could be linked to the problem-solving strategies learners used in this study. Chapter 2 dealt with different aspects of the LFIN that pertain to this study: Stages of Early Arithmetical Learning (SEAL), conceptual place value and early multiplication and division (see tables in Appendix J). Other aspects of LFIN include the Structuring Number Strand (SNS) and number words and numerals. Learners who participated in this study did not use strategies that reflected these two aspects because they already had a sound understanding of such concepts. The strategies they used showed that their conceptual development and number learning had moved beyond number words and numerals and the SNS. These aspects of the LFIN were not included in this study. The pre-test and post-test are linked to relevant stages of the SEAL, conceptual place value and early multiplication and division to describe the conceptual levels at which learners solved the problems. Strategies used before and after implementation of the writing tasks are compared to determine whether writing supports learners when solving mathematical problems. Although the writing tasks were implemented with all the learners of the selected Grade 3 class, the results are based on the written work and interviews of eight of these learners as explained in chapter 3. The eight learners represent the three mathematical ability groups in the class. Two learners were from the above average ability (AA) group while there were three learners each from the average (A) and below average (BA) ability groups.

This chapter begins with an overview of the results of the pre-test and the interviews at the beginning of the data collection period. Findings of the writing tasks that learners were engaged in during the period of implementation are presented. Findings of the post-test and the second round of interviews are also provided.

4.2 PRE-TEST

On the first day of the pre-test, the process and purpose of the pre-test were explained to learners of the selected Grade 3 class. Learners were expected to solve five problems as part of the pre-test over a period of five days. (The problems used for the pre-test are listed in Appendix E.) Every day of the week learners were presented with a different problem to solve. The problems were differentiated according to the three mathematical ability groups in the class. A brief class discussion took place before solving each problem to reiterate the purpose of solving these problems. But, for the most part, learners solved the problems on their own. It was important to determine what types of strategies learners would use when solving mathematical problems. While learners solved each problem, the researcher moved around the classroom to observe what various learners were doing. Although five different writing tasks were implemented during the study, the pre-test focused on writing to solve mathematical problems to address the research questions. The problems used in the pre-test are numbered below according to the three mathematical ability groups in the selected Grade 3 class. For each problem, the context or situation remained the same but the number range differed. There were problems for the above average ability group (1), the average ability group (2) and the below average ability group (3). The number range of the average ability group would be the expected level for the grade at the time data were collected. The above average ability group would have a higher number range while the below average ability group would have a lower number range than the average ability group.

Problem 1

1. *A cricket team needs 94 runs to win their match. They already have 47 runs. How many runs do they still need?*
2. *A cricket team needs 74 runs to win their match. They already have 49 runs. How many runs do they still need?*
3. *A cricket team needs 34 runs to win their match. They already have 19 runs. How many runs do they still need?*

All the learners used problem-solving strategies as reflected in SEAL which were appropriate in terms of the addition/subtraction problem type. As learners progress through the stages of SEAL, they develop increasingly sophisticated strategies from counting all, counting on and counting back to compensation and commutativity. The strategies used in the SEAL are applicable to addition and subtraction problem types. Learners' strategies were at different stages in their number learning as is reflected in their solutions. Five learners solved the problem at the level of perceptual counting: stage 1 of the SEAL. Their strategies involved counting visible or drawn items (as in Figure 4.2). Two learners were at stage 3 (initial number sequence) where counting on or counting down strategies was used; while one learner (Figure 4.1) solved the problem at stage 5 (facile number sequence) using strategies beyond counting-by-ones incorporating more advanced strategies and procedures.

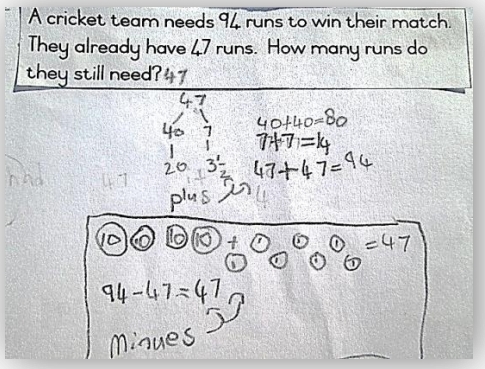


Figure 4.1: Learner 1 (AA) Pre Q1

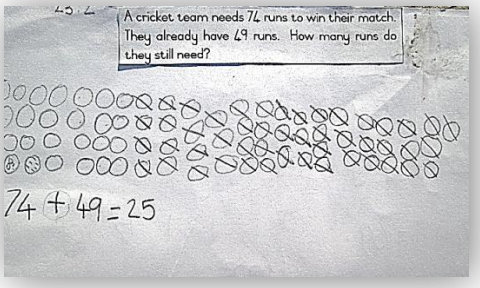


Figure 4.2: Learner 4 (A) Pre Q1

Learner 1 (AA) used three different representations or strategies to solve this problem at stage 5 of the SEAL (facile number sequence). It is unclear from his writing what the reason was for doing so. It is possible that he had used one strategy, found that it was not suitable to the problem and attempted a different strategy. This may be the case because there was evidence of halving 47 which did not apply to this addition/subtraction problem type. Another representation showed an understanding of doubling the decomposed number to find the solution. The third strategy, which used subtraction, was one of the possible strategies that suited this problem type. Learner 4 (A) tallied to represent the 74 runs mentioned in the problem. She subtracted 49 from 74 by drawing lines through the tallies to find the solution. This reflected a strategy at stage 1, which is perceptual counting. Learner 4 wrote down an addition sign in the sum as opposed to a subtraction sign. This error could have been due to a lack of conceptual understanding or simply a mistake in the learner's writing.

Problem 2

1. *Rodney is putting 56 doughnuts on platters for his party. He places 7 doughnuts on each platter. How many platters will he have?*
2. *Rodney is putting 42 doughnuts on platters for his party. He places 7 doughnuts on each platter. How many platters will he have?*
3. *Rodney is putting 28 doughnuts on platters for his party. He places 7 doughnuts on each platter. How many platters will he have?*

A few learners struggled with the context of the second problem. During observation, it was noted that some learners experienced difficulty reading and understanding the problem: they required some assistance in this regard. It is possible that this weakness was due to learners' limited vocabulary and/or reading comprehension skills. The problem was read and discussed briefly before learners solved it individually. The researcher/teacher did not provide much assistance to learners while solving the problem because this problem formed part of the pre-test. Assisting learners during the pre-test may have produced results that lacked validity when compared to the post-test.

When solving this problem, six learners appropriately used level 1 (initial grouping) of early multiplication and division which suited the multiplication/division problem type. The drawings learners made reflected quotitive sharing: learners arranged the items in the problem into groups. In quotitive sharing, items are shared into groups of a given size (Wright, Martland, Stafford & Stanger, 2006:14). This problem required learners to share the total number of doughnuts into platters of 7 doughnuts each to determine the number of platters needed. Three of the six learners identified at level 1 made errors when using this strategy because they had incorrect answers. In interviews after the pre-test, Learner 2 (AA) and Learner 4 (A) explained that they had used drawing but counted at a level of stage 1 of the SEAL (perceptual counting). Their drawings represented quotitive sharing which they combined with a counting strategy. Learners 6 and 7, both from the below average group, used incorrect strategies that did not fit the multiplication/division problem type.

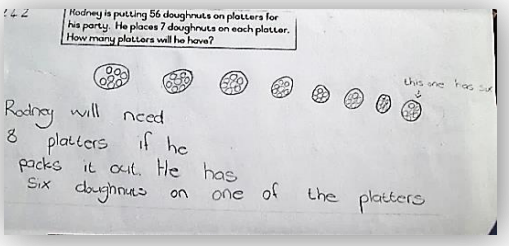


Figure 4.3: Learner 2 (AA) Pre Q2

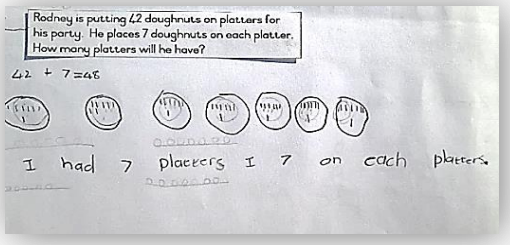


Figure 4.4: Learner 3 (A) Pre Q2

Learner 2 and Learner 3 used drawing as a strategy to solve this problem. These learners were from the above average and average ability groups respectively. Their use of drawings reflected quotitive sharing at level 1 of early multiplication and division as mentioned earlier. Learner 2 (AA) correctly drew eight platters with the doughnuts represented on each one (Figure 4.3). However, the number of doughnuts on two of the platters was incorrect. The learner explained that only six were drawn on the last platter. Learner 3's (A) drawing showed how he correctly solved the problem (Figure 4.4). However, the number sentence that was written did not match the drawing. The learner used the numbers given in the problem and added them together in the number sentence rather than divide the total number of items by the number in each group to reflect the multiplication/division problem type. This choice could indicate that the learner did not have an understanding of the problem as well as a deep conceptual understanding of division or sharing as it is known at Grade 3 level.

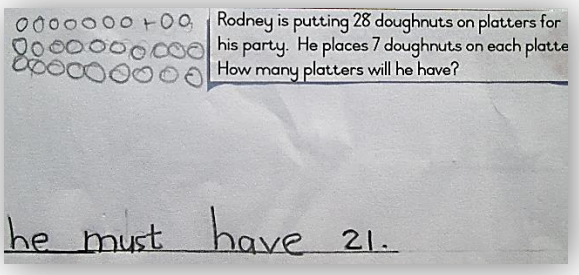


Figure 4.5: Learner 7 (BA) Pre Q2

Learner 7 (BA) did not use a strategy that fitted the multiplication/division problem type. As can be seen in Figure 4.5, this learner used addition/subtraction as a strategy. He used the numbers given in the problem and subtracted 7 from 28, although his tallies show that he added 7 and 21 to reach the total of 28 doughnuts. He used a combination of addition and subtraction that left him with the answer of 21. It is evident that Learner 7 (BA) did not understand the problem. Added to this, it is possible that he did not have the conceptual understanding of early multiplication and division as described in chapter 2.

Problem 3

1. *The school sports team has 68 runners, 16 long jumpers and 10 high jumpers. How many athletes are on the sports team?*
2. *The school sports team has 48 runners, 16 long jumpers and 10 high jumpers. How many athletes are on the sports team?*

3. *The school sports team has 28 runners, 16 long jumpers and 10 high jumpers. How many athletes are on the sports team?*

All eight learners used strategies that reflected the SEAL for solving the addition problem. However, various learners solved it at different stages. Two learners were at stage 1 using perceptual counting. Learner 3 (A) used initial number sequence (stage 3) and Learner 6 (BA) used intermediate number sequence (stage 4). At the stage of intermediate number sequence, learners use counting-down strategies to solve the missing subtrahend (Wright, Martland & Stafford, 2006:67). Learners 1 (AA), 4 (A) and 8 (BA) all used facile number sequence, stage 5 in the SEAL.

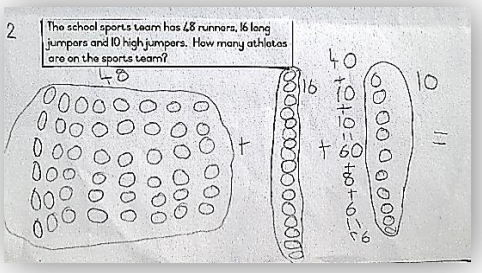


Figure 4.6: Learner 5 (A) Pre Q3

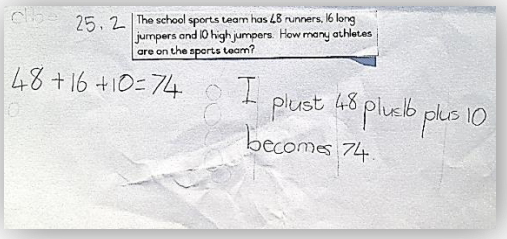


Figure 4.7: Learner 4 (A) Pre Q3

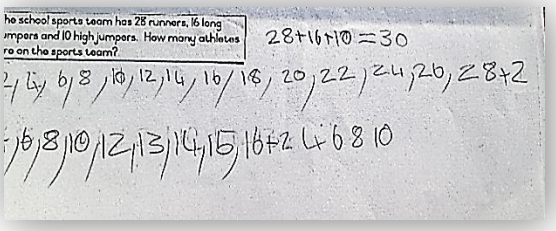


Figure 4.8: Learner 6 (BA) Pre Q3

The strategies in Figure 4.6 and 4.7 were used by learners from the average ability group. Learner 5 (A) combined perceptual counting (stage 2) and facile number sequence (stage 5) in his strategy. He drew circles to represent all the athletes in the problem: the visible items as described by Wright, Martland and Stafford (2006) which reflect stage 2. This learner wrote a number sentence vertically between the drawings. He used his knowledge of decomposing numbers using place value: the tens and units are evident. The learner did this to each number in the problem and then added them together to find the total number of athletes. This part of his strategy reflects stage 5.

Learner 4 (A) understood the context and question of the problem and applied a strategy relevant to the problem type represented in the problem. Although there was no evidence of a calculation, a sentence was written to explain the strategy that was used. At this stage, learners had not yet been exposed to the types of writing tasks that would be introduced as part of this research study once the pre-test and interviews were completed.

Learner 6 (BA) used counting in twos as a strategy for solving the same problem (Figure 4.8). This counting strategy was used within an addition sum which shows a combination of strategies at stage 4 of the SEAL (intermediate number sequence). However, another number sentence was written where the learner added incorrectly. The representations used in the strategy did not match the number sentence because the learner arrived at the incorrect answer. However, the strategy of adding by counting in two's suggests that Learner 6 (BA) understood that this was an addition problem type.

Problem 4

1. *There are 17 pins in a box. How many pins will there be in 6 boxes?*
2. *There are 17 pins in a box. How many pins will there be in 4 boxes?*
3. *There are 17 pins in a box. How many pins will there be in 2 boxes?*

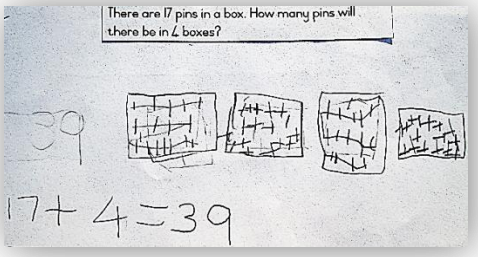


Figure 4.9: Learner 8 (BA) Pre Q4

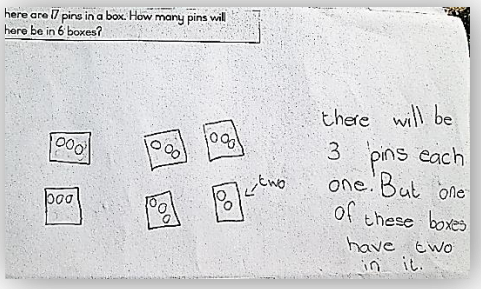


Figure 4.10: Learner 2 (AA) Pre Q4

Five learners used strategies that reflected early multiplication and division as explained by Wright, Martland and Stafford (2006a). Three learners were at level 1 (initial grouping) where Learner 2 (Figure 4.10) and Learner 6 (BA) had incorrectly used partitive sharing. These learners had taken the 17 pins and shared them between the number of groups in the problem. They had misunderstood the problem by using division instead of multiplication.

Learner 8 (BA) correctly used quotitive sharing: she assigned 17 pins to each group in her drawing of the items. Four blocks were drawn to represent the four boxes in the problem

with lines in each box. The quotitive sharing in her drawing showed that the learner conceptualised seventeen pins in each box. However, her number sentence did not reflect this strategy, as can be seen in Figure 4.9. Learner 8 (BA) added the numbers given in the problem (17 + 4) but came to the answer of 39. When counting the lines in all the boxes, it showed that she had drawn far more than 39 pins. It appeared that she had some conceptual understanding of the multiplication or repeated addition strategy required by the problem but was unable to follow this through the entire problem.

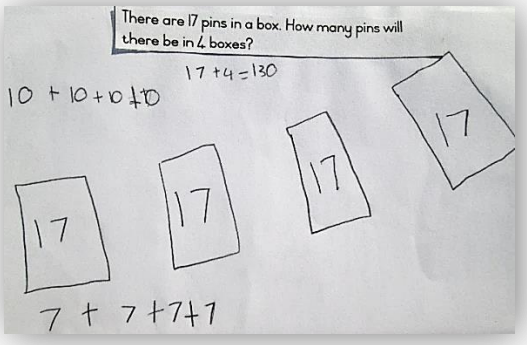


Figure 4.11: Learner 3 (A) Pre Q4

Learner 3 (A) used a stage 5 strategy (facile number sequence) of the SEAL shown in Figure 4.11. Despite this being a multiplication/division problem type, his strategy could have worked if he had used it correctly. He had decomposed 17 using place value (10 + 10 + 10 + 10; 7 + 7 + 7 + 7) four times which represented the 4 boxes in the problem. However, he did not continue using this strategy by adding these number sentences to find the total number of pins. The number sentence that he wrote does not match the rest of his strategy.

Problem 5

1. *Jack has some sweets. Sam gives him 28 more sweets. Now Jack has 73 sweets. How many sweets did he have in the beginning?*
2. *Jack has some sweets. Sam gives him 18 more sweets. Now Jack has 43 sweets. How many sweets did he have in the beginning?*
3. *Jack has some sweets. Sam gives him 18 more sweets. Now Jack has 33 sweets. How many sweets did he have in the beginning?*

Three learners used strategies reflecting initial number sequence (stage 3 of the SEAL). Two learners solved this problem at stage 1 of the SEAL (perceptual counting). One learner displayed a strategy using facile number sequence (stage 5 of the SEAL) while another learner had no visible strategy. On the last day of the pre-test, Learner 4 was absent.

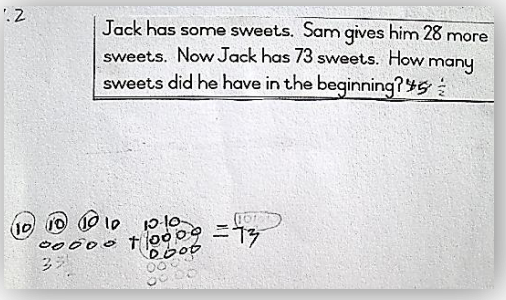


Figure 4.12: Learner 1 (AA) Pre Q5

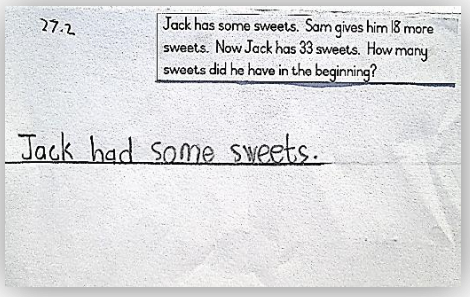


Figure 4.13: Learner 7 (BA) Pre Q5

In Figure 4.12 above, it is evident that Learner 1 (AA) used his previous knowledge of place value as a strategy. He decomposed 28 into tens and ones. He added tens and ones to find the missing addend in the problem until he reached 73 which was the total number of sweets. His strategy is indicative of facile number sequence which is stage 5 of the SEAL. It appeared that learner 7 (BA) did not comprehend the context of the problem: there was no visible strategy (Figure 4.13). He answered the question in the problem by stating the first sentence of the problem rather than finding out the number of sweets James would have had in the beginning.

When reflecting on learners' strategies and analysing their writing during the pre-test, most learners had difficulty solving mathematical word problems and communicating their thinking through the strategies they had written. Often, the strategy did not fit the problem or it showed their lack of deeper conceptual understanding of the problem. The strategies sometimes reflected the lower levels or stages of the aspects of the LFIN. There was little evidence of more advanced strategies typical of the higher levels of the LFIN, especially from learners in the average and below average mathematical ability groups.

Table 4.1: Summary of LFIN levels and number of learners for each problem in pre-test

Problem	LFIN level/stage	Number of learners
1	SEAL Stage 1	5
	SEAL Stage 3	2
	SEAL Stage 5	1
2	EMD Level 1	5
	Combined EMD Level 1 and SEAL Stage 1	1
	No clear strategy	2
3	SEAL Stage 1	2
	SEAL Stage 3	1
	SEAL Stage 4	1
	SEAL Stage 5	3
	Combined SEAL Stages 1 and 5	1
4	EMD Level 1	3
	EMD Level 2	2
	EMD Level 4	1
	SEAL Stage 5	1
	Absent	1
5	SEAL Stage 1	2
	SEAL Stage 3	3
	SEAL Stage 5	1
	No clear strategy	1
	Absent	1

4.3 FIRST SET OF INTERVIEWS

Interviews were conducted with the same eight learners discussed in the pre-test results above. The purpose of the interviews was to gauge learners' understanding of the problems when they verbally explained their strategies. Verbal explanations were considered against recordings of their solutions in the pre-test. Interview questions were structured in order to establish how learners were able to explain their solutions based on their writing when solving problems of the pre-test. Interviews helped to explore learners' thinking and understand what they were doing. There was evidence of scaffolding in some interviews: learners needed prompts to explain their strategies. Most of the selected learners found difficulty explaining their problem-solving methods used in the pre-test. Verbalisation of their

strategies did not always reflect what they had written on paper. Some learners, particularly from the average and below average groups, seemed to lack the mathematical vocabulary to explain what they had done.

Learner 1 (AA) was able to explain his strategies verbally even though this learner sometimes had the incorrect solution. At this stage of the data collection (pre-test), he had not written an explanation of his thinking when solving the problem because learners had not yet encountered the use of writing tasks. His strategies showed that he could use his conceptual understanding to represent his thinking. He was able to combine more than one method in certain strategies to reach his solution (Figure 4.1).

Learner 2 (AA) was able to explain her strategies verbally according to what she had done. This ability allowed an understanding of the problems of the pre-test compared to the writing she used in her strategies. In her writing she sometimes used tallies to represent her strategy. At the time of the pre-test, some mathematical problems had a low enough number range for tallies to be used as a strategy. Below is an excerpt from the interview where Learner 2 explained how she used tallies as a strategy.

Researcher: And the one with the cricket team?

Learner 2: I put, I had 90, I put 94 circles then I crossed out 47 of them and so I counted the rest of them and it gave me 47.

Learner 2 (AA) Pre Interview

When solving the first four problems of the pre-test, Learner 3 (A) used strategies that reflected the problem types represented. For example, the third problem required use of addition as a strategy which was reflected in his writing. However, he came to the incorrect solutions for these problems. As a result of this pre-test, it appeared that Learner 3 (A) was able to determine the underlying mathematical concepts needed to solve the problem but could not solve the problem. He needed many prompts during the interview to help explain or justify his strategies. He seemed to find difficulty applying the strategy to his writing in order to reach the solutions successfully. The following excerpt is from the pre-test interview conducted with Learner 3 (A).

Researcher: Let’s look at the problem with Rodney and the doughnuts. It says that Rodney’s putting 42 doughnuts on platters for his party. He places 7

doughnuts on each platter. How many platters will he have? Now what...how did you solve this problem?

Learner 3: I did put 7 platters then I put 7 on each platter.

Researcher: Ok, how did you know that you had to do that?

Learner 3: Because there's 7 of each.

Researcher: And when you put the 7 out, what did you come up with?

Learner 3: 48

Researcher: 48...but how many doughnuts does Rodney have?

Learner 3: 42.

Researcher: 42...ok, so how come you ended up with more doughnuts?

Learner 3: Because I add more.

Learner 3 (A) Pre Interview

His explanation of the strategy made sense according to the multiplication/division problem type (problem 2 of the pre-test). The recording of this strategy and his verbal explanation showed that he solved the problem at level 1 (initial grouping) of early multiplication and division. The above excerpt displays an understanding of the required operation or strategy to solve this problem. However, he did not follow through with this strategy and came to an incorrect answer.

During Learner 4's (A) interview, it appeared that there was an understanding of the mathematical concepts required by each problem. The learner generally represented her thinking by using drawings or tallies. This was often her strategy when recording her thinking while she solved mathematical problems. However, drawings were not used when solving the third problem about the school sports team. In the excerpt below, Learner 4 (A) explained that drawing would be time-consuming when solving this problem since the numbers were too high. In this case, she was able to adapt her strategy and change her usual method of representation to suit the needs of the problem. In the following excerpt she explains why drawings were not used as a strategy for this particular problem.

Researcher: Let's look at the school sports team problem. What did you do here?

Learner 4: Well, I usually... when it's a long, what I do is I just make a sum and then I use my brain to, um, add 16.

Researcher: Ok, so you added that without drawing?

Learner 4: Yes.

Researcher: Did you do it in your head?

Learner 4: Yes.

Researcher: Wow, that's good. Ok, because I see you didn't do a drawing there. I see you started and then you erased it.

Learner 4: Yes because I knew it would take too long.

Researcher: Is it? Why did you think it would take so long?

Learner 4: Because the numbers are high.

Learner 4 (A) Pre Interview

In the pre-test interviews with the selected learners from the above average and average ability groups, it seemed that they had the necessary conceptual understanding to solve mathematical problems according to the problem type as mentioned in chapter 2. Conversely, their solutions were not always correct: they had either misread on misinterpreted the problem. Some learners needed more prompting than others when verbally explaining their problems. Only one learner wrote a brief explanation of her strategy during the pre-test (see Figure 4.7) which was significant because learners had not been exposed to the various writing tasks at this stage of the data collection. They were not expected to write explanations of their problem-solving strategies but, in a few instances, learners wrote statements of their solutions without explanations of the strategies they used when solving the problems. During the pre-test, learners were asked to solve the problems showing their strategies and solutions. They were not asked to write explanations of their strategies.

The three learners from the below average ability group did not use strategies appropriate to the problem types presented in the pre-test. This failure showed their lack of conceptual understanding related to the mathematical problems. This lacking could be linked to their language ability: two of these learners (Learner 7 and Learner 8) had below average reading and comprehension abilities. When these learners were interviewed, they had difficulty explaining what they had done. Below is an excerpt from the interview conducted with Learner 7 (BA) which displays his difficulty in explaining his strategy.

Researcher: Let's look at the first problem that you did. Do you want to explain to me how you solved this problem?

Learner 7: I...I did dots there and I carried on.

Researcher: Ok, how did you carry on?

Learner 7: I did a plus there and I carried on.

Researcher: But how did you know in the problem that you had to carry on. When you read the problem, what made you think that you had to carry on adding?

Learner 7: I don't know.

Learner 7 (BA) Pre Interview

It was evident in their strategies that the three below average learners comprehended the third problem about the school sports team as an addition problem type even though their solutions or answers were incorrect: displayed in the interview with Learner 8 (excerpt below).

Researcher: Ok, and then let's look at the sports team. I see you just did a sum here. You started doing a drawing and then you erased it. So explain to me what were you thinking here.

Learner 8: I done a sum and then I made circles and then I erased it. Then I added the circles together and I kind of plussed it so I made 48 plus 10 plus 16 and it was equal to 73.

Researcher: Ok, so did you add this up in your head or did you use the drawing to help you?

Learner 8: I used my drawings to help me.

Learner 8 (BA) Pre Interview

Learner 8 did not elaborate upon her strategy for this problem. At first, she explained that she had erased her drawings, in which she used the tally method, and solved the problem using a number sentence. When asked how she added the numbers, she said that she used drawings. This explanation did not make sense since she had erased her drawing. It is possible that she may have erased the drawing after she arrived at the answer. Figure 4.14 shows some evidence of her erased drawings and the number sentence that she wrote to solve this problem.

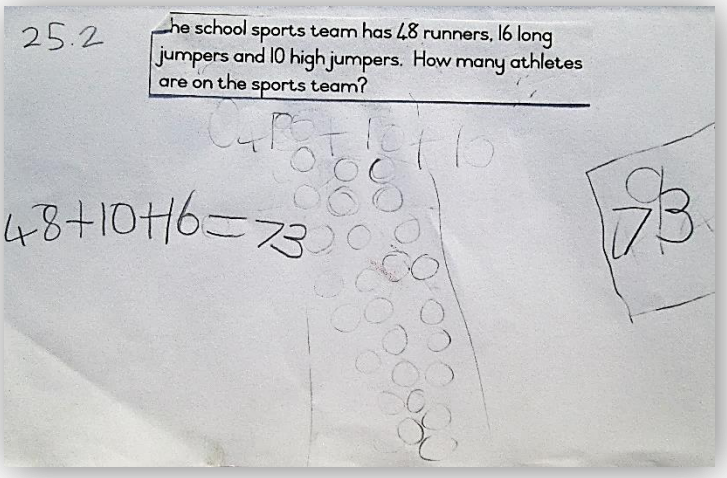


Figure 4.14: Learner 8 (BA) Pre Q3

During the pre-test interviews, learners often had difficulty giving verbal explanations of their problem-solving strategies. A possible factor in their inability to do so could have been a lack of appropriate mathematical vocabulary to clarify their thoughts. Another factor could have been that they had not previously explained their strategies verbally in the way that was expected during this study. Learners used limited details and explanation in their writing which may have led to the difficulty in their verbal explanations: they were not expected to use writing in this manner. Learners had not yet been exposed to using writing in mathematics through various writing tasks.

Once all the pre-test interviews were concluded, the various types of writing tasks, as modelled by Burns (1995a), were implemented in the selected Grade 3 class. Later, a post-test was conducted and the same learners were interviewed to compare their use of strategies and how the writing tasks supported them in reaching solutions. The findings of the post-test and interviews are elaborated later in this chapter.

4.4 WRITING TASKS

After the pre-test interviews were completed, various writing tasks (Burns, 1995a) were introduced to learners: writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes and shared writing. These tasks were modelled to learners to encourage them to clarify, justify and explain their thinking and to help in problem-solving. The writing tasks were implemented as

an intervention to support learners in mathematical problem-solving. In this section, findings are presented on the implementation of the writing tasks between the pre-test and post-test.

4.4.1 Writing to solve mathematical problems

Once writing to solve mathematical problems was modelled, learners solved thirteen mathematical problems covering various problem types involving different numerical operations. As in the pre-test, these mathematical problems were differentiated according to the three different mathematics ability groups present in the selected Grade 3 class (see Appendix G). Learners solved the problems and wrote about them in their journals. With each writing task, learners were encouraged to use writing to solve mathematical problems to clarify their thinking and explain their strategies. When doing so, they often needed questions or prompts to guide them in their writing. Some learners needed more assistance than others in this regard. This distinction may explain some of the reading and comprehension difficulties learners experience.

4.4.1.1 Verbal and written feedback

On some occasions, guidance was provided verbally while the researcher moved around the classroom observing the learners. At other times, written feedback was presented in the journals where learners solved the mathematical problems. This was often the case with learners who did not receive verbal feedback at the time they completed the writing task. Learners were requested to respond to the written feedback the following day by adding on to what they had already written. The aim of the verbal and written feedback was to guide their writing by drawing attention to the mathematical concept(s) within the problem. Figure 4.15 below is an example of writing to solve mathematical problems by Learner 2, an above average learner. The learner understood the problem type by using an appropriate strategy but made an error in her calculation. She counted by adding thirteen each time, not twelve. Feedback was written to guide the learner to check her counting again. After the learner followed the support given through the written feedback, she realised that she had counted incorrectly.

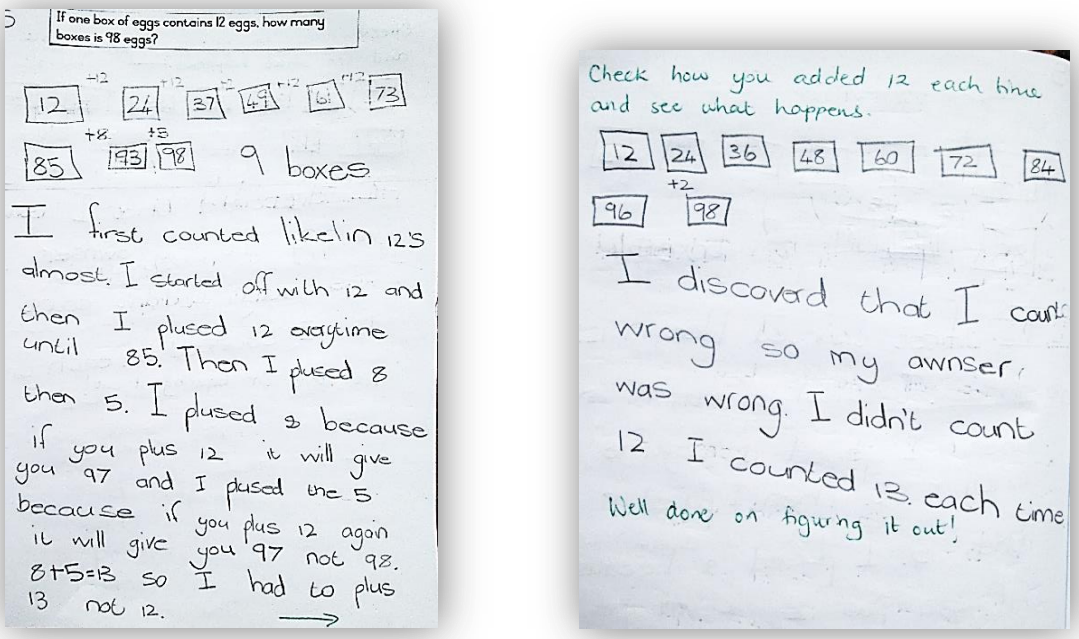


Figure 4.15: Learner 2 (AA) Writing - Problem 8

4.4.1.2 The use of effective strategies

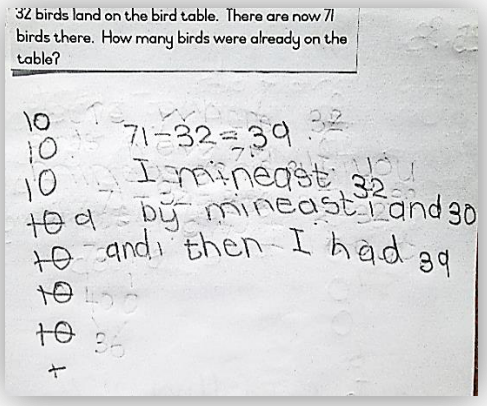


Figure 4.16: Learner 5 (A) Writing – Problem 1

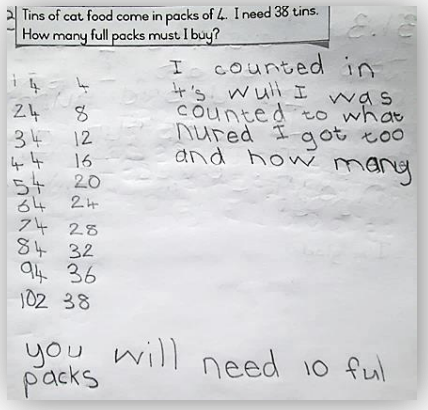


Figure 4.17: Learner 5 (A) Writing – Problem 3

Learner 5 (A) used inventive strategies to solve his mathematical problems: seen in Figures 4.16 and 4.17 above. The problem shown in Figure 4.16 was the first mathematical problem learners solved after the pre-test. Learners could work in pairs while they solved this problem. The strategy Learner 5 (A) used in Figure 4.16 reflected facile number sequence, stage 5 of the SEAL: he used a method that involved non counting-by-ones. Figure 4.17 displays level 3 of early multiplication and division, figurative composite grouping: he used skip counting and separate counting strategies at the same time in three columns. The left

column shows how he counted the number of packs. He kept track of the number of tins in each pack down the middle column, noting that there were only two tins needed from the last pack. The column on the right shows how he used skip counting for the total number of tins in the problem. This technique exhibited a deep conceptual understanding: he combined different counting strategies in his strategy in Figure 4.17. His conceptual understanding was evident in Figure 4.16 where he combined his mathematical knowledge of decomposing using place value and subtraction to find his solution.

4.4.1.3 The role of language when solving problems

The second problem learners were given to solve was comparing the height of the fence and the wall where they could use addition or subtraction as a strategy. It was noted that the below average ability group found the problem quite difficult: many of them did not understand the meaning of the word 'higher'. They needed guidance to be able to read and solve the problem. They thought that the wall was 18cm high as opposed to being 18cm higher than the fence. Most of the learners in this group had difficulty understanding the problem. Some scaffolding was needed to overcome this difficulty during the ability group discussion. Most learners in the group erased their initial strategies after the discussion. This erasure made it difficult to compare what they had done before and after deliberating over the problem. Figure 4.18 shows Learner 8's strategy after the group discussion. Even though she understood that she had to add to solve this problem, she added 3 in her number sentence and her drawing. Although she recorded 46 + 18 in her number sentence, she did not add 18. This omission did not make sense according to the problem.

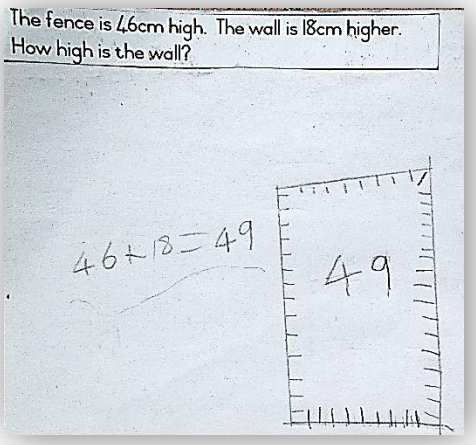


Figure 4.18: Learner 8 (BA) Writing – Problem 2

This problem was not the only mathematical one where learners had difficulty understanding the context of the problem. The following excerpt from the field notes below indicates the support learners needed to be able to solve a problem. Scaffolding through modelling was required to enable learners to make a connection to the mathematical knowledge in the problem.

The excerpt of the field notes was written while learners solved problem 4. The problem read as follows:

> *A medicine bottle holds 75 ml. A teaspoon holds 5 ml. How many teaspoons of medicine in the bottle?*
>
> *Using a bottle, teaspoon and water, I demonstrated the context of the problem. I poured one teaspoon of water into the bottle at a time. Each time the learners had to say how many teaspoons of water were already in the bottle as well as the amount of water that had been poured so far. As we did this, I guided them with questions to the fact that they needed to add five each time to work out the number of teaspoons needed. Learners worked on the problem again.*
> **FN 13/4/2015**

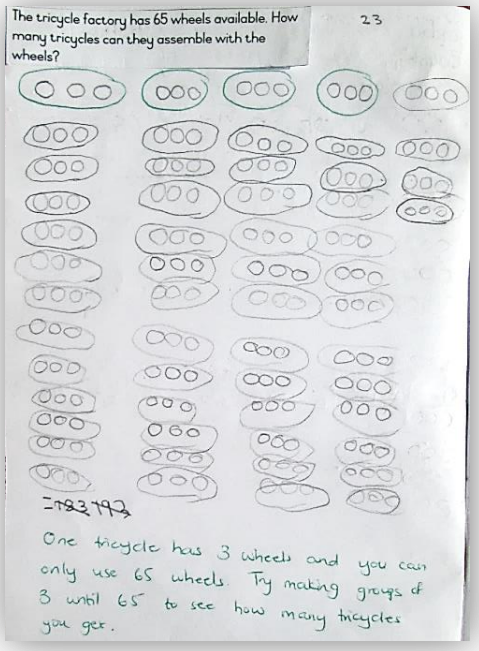


Figure 4.19: Learner 7 (BA) Writing – Problem 5

Figure 4.19 above is another example of a learner experiencing difficulty with the language in a mathematical problem. Initially, Learner 7 (BA) used perceptual counting (stage 1 of the SEAL) as a strategy to solve this problem. The researcher gave Learner 7 (BA) written feedback which he received the following day. When the researcher observed the learner responding to the feedback, it was clear that he was still uncertain of which strategy he

should use. We read through the written feedback together and the researcher probed him to explain what he thought this meant. As he explained his thinking, the researcher circled his tallies of the wheels to make a group of three wheels. He clarified that this represented one tricycle with three wheels. By doing this, he was being prompted to use quotitive sharing by arranging the items into groups. This technique was at level 1 (initial grouping) of early multiplication and division. The researcher proceeded to circle three more groups during which time he continued giving an appropriate verbal explanation that each group represented one tricycle with three wheels. He was reminded that there were only 65 wheels and then left him to continue the strategy on his own. Later, as the researcher analysed what he had done, it became clear that he had still misinterpreted the problem. He continued circling all his tallies into groups of three without counting his tally marks. This caused him to go beyond the 65 wheels mentioned in the problem. This reiterated the vital role language plays when reading, interpreting and solving mathematical problems. It appeared that Learner 7 (BA) may not have understood the problem due to misinterpreting the vocabulary/language and the context of the problem.

4.4.1.4 Solving multistep mathematical problems

Below are examples of learners solving multistep problems as mentioned in chapter 2 (Figure 4.20, Figure 4.21 and Figure 4.22). These types of problems have two or more questions where learners usually need to solve the first part and use that answer to solve the second part of the problem.

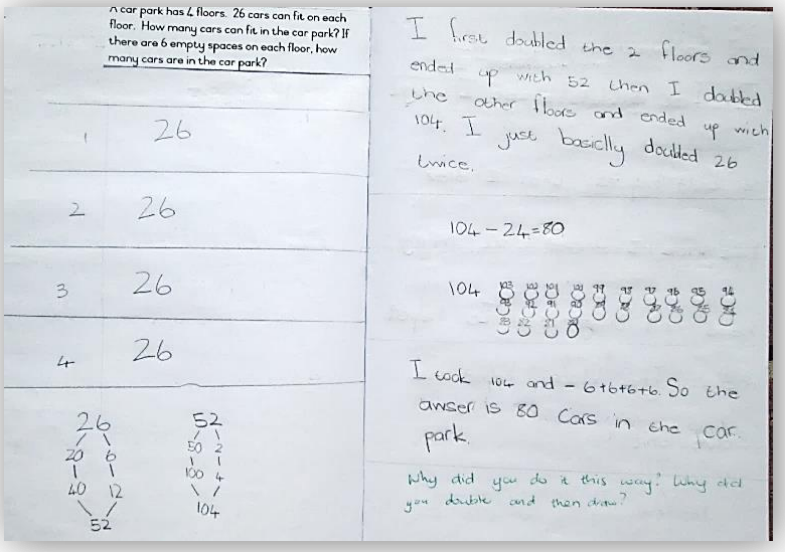
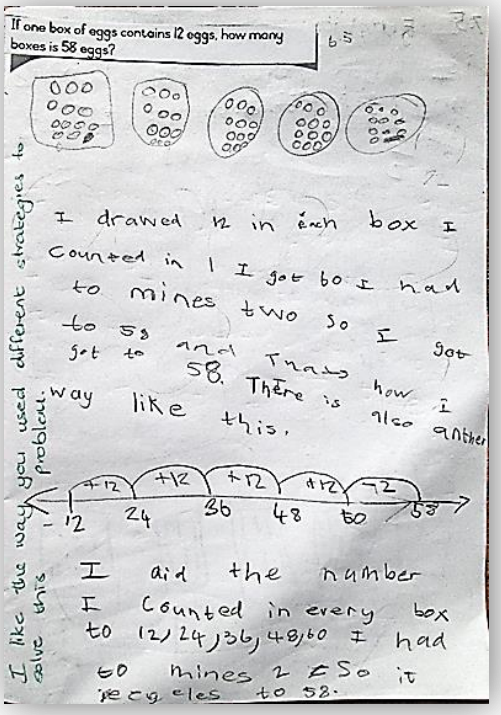


Figure 4.20: Learner 2 (AA) Writing – problem 7

The problem that Learner 2 (AA) solved in Figure 4.20 above is an example of a multistep problem. To solve the first part of the problem, she used her mathematical knowledge of doubling numbers. In her explanation, she states that she "doubled 26 twice". This strategy is at stage 5 of the SEAL (facile number sequence). She continued solving the problem and displayed repeated abstract composite grouping (level 4 of early multiplication and division) by using repeated subtraction.



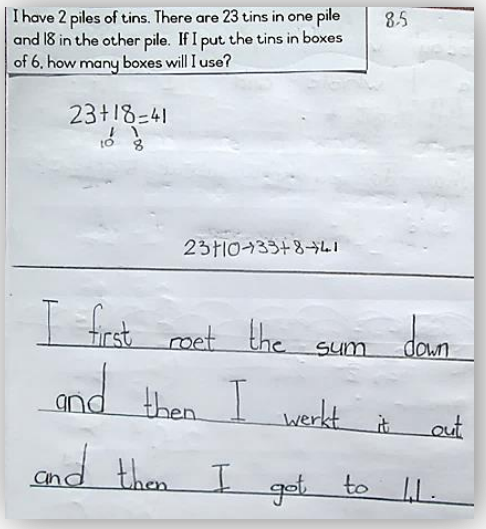


Figure 4.21: Learner 6 (BA) Writing– Problem 8 **Figure 4.22:** Learner 7 (BA) Writing– Problem 8

Figure 4.21 above shows Learner 6's (BA) strategy when solving a different multistep problem. In this multistep problem, there was one question only being posed to learners. However, learners generally use two steps in their strategies to reach a solution. They had to solve the problem using a particular strategy that reflected the problem type (multiplication/division). Subsequently, the knowledge they gained from this strategy enabled them to answer the question. To start, Learner 6 (BA) used quotitive sharing by dividing the eggs into groups. This sharing was at level 1 of early multiplication and division, namely initial grouping. She combined this strategy with repeated subtraction which reflected repeated abstract composite grouping (level 4 of early multiplication and division). She did not, however, continue by answering the question in the problem. The below average group discussed their strategies together to compare what they had done. Below is an excerpt from the field notes written during the group discussion.

Learner 6 - used the empty number line to solve this problem. Each jump on the number line represented one box of 12 eggs. Interesting to note – she applied a mathematical tool we had been learning about.
FN 6/5/2015

Figure 4.22 shows a similar error. When solving the first part of the strategy, Learner 7 (BA) reflected working at level 2 of conceptual place value when he was incrementing by tens off the decuple. He used this strategy to find the total number of tins in the problem but did not solve the second part of the problem concerning the number of boxes needed for all the tins.

4.4.1.5 Evidence of learners' errors

The researcher noticed that a number of the learners erased their work, especially when writing to solve mathematical problems. This erasure left minimal evidence of their thinking. In speaking to them about this, some learners appeared to feel that their strategies were not adequate or they did not want their mistakes to be seen. The researcher explained to all the learners that seeing their strategies as well as their errors helped to explain what they were thinking when they solved problems. This explanation was necessary to determine how writing supported their ability to solve problems. Evidence of their strategies, including their errors, was essential to assess learners' conceptual understanding and address misconceptions in their thinking.

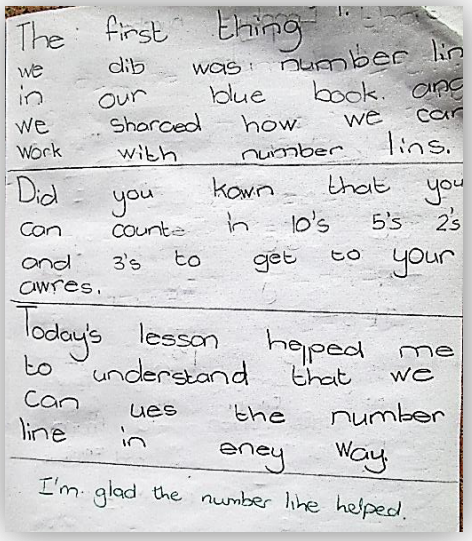
4.4.2 Writing to record (keeping a journal or log)

When this writing task was introduced to the class, it was explained to them that their journals were accessible at any time during mathematics lessons and not only when they were specifically asked to record in their journals. The purpose of doing this was to encourage learners to write about their experiences and thoughts while they occur and record them in a journal or log. This task was included to help learners make connections and think critically about the activities during a mathematics lesson. This task enhances learners' ability to make observations which is a necessary skill when solving mathematical problems.

Learners were instructed to record what they did and learned in mathematics lessons at any time to create an ongoing record (Burns, 1995a:51). As mentioned in chapter 2, learners should write in their journals whenever they notice or discover something but the researcher found that, unless this type of writing task was mentioned or time set aside to give them this

opportunity, most learners did not write in their journals spontaneously and regularly as had been envisaged. There was evidence of four writing assignments only of this nature in cases where the researcher had prompted learners to write.

Learners from the above average ability group used more detail when writing in their journals. They did not always use the prompts displayed in the classroom (see Appendix I) which were examples of ways to start sentences to guide learners' thinking when writing to record in their journals. This ability may be due to having an above average language competence as well as a greater ability to express their thinking in words. Their higher levels of reading and comprehension could have affected their conceptual understanding as discussed in chapter 2. There was more evidence of critical thinking when learners sometimes gave reasons for making certain statements. Most of the learners from the average ability group used the prompts to state what had happened in the day's lesson. They did not actually explain what they meant or extend their thinking through their writing. Similar findings emerged from learners in the below average ability group but most of them wrote less than learners from the other groups. Figures 4.23 and 4.24 are examples of two learners' use of writing through journaling. Both learners used prompts to start their sentences but Learner 4 (A) used more prompts than Learner 7 (BA) and included more detail in her writing.

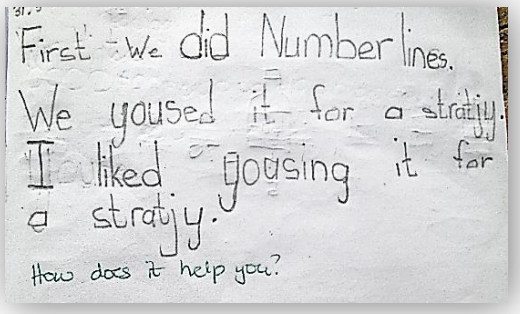

The first thing we did was number lin in our blue book. and we shared how we can work with number lins.

Did you kown that you can counte in 10's 5's 2's and 3's to get to your awres.

Today's lesson helped me to understand that we can ues the number line in eney way.

I'm glad the number like helped.

Figure 4.23: Learner 4 (A) Writing – Journal


First we did Number lines. We yoused it for a stratiy. I liked yousing it for a stratiy.

How does it help you?

Figure 4.24: Learner 7 (BA) Writing - Journal

4.4.3 Writing to explain

The purpose of this writing task is to explain a mathematical concept to show understanding. Learners clarify what they know through reflecting and summarising. By doing this, their writing is enhanced: they engage with mathematical concepts and develop their knowledge and understanding. This enhancement could be accomplished through writing a summary (Freed, 1994:23) or listing the main points of the lesson and their reflections (Wilcox & Monroe, 2011:522). In this study writing is used to support learners while they solve and explain mathematical problems and links to the purpose of this task: learners cultivate their use of writing to explain their thinking.

Learners were given three opportunities to use writing to explain their understanding of a mathematical concept. The mathematical concepts learners wrote about during these writing tasks included the empty number line, geometric patterns and fractions. All three topics had been covered in mathematics lessons in the weeks prior to the writing tasks as well as earlier in those particular days' lessons. Before delving into the writing tasks, learners engaged in discourse on the specific topic as a class. They provided verbal explanations in pairs or groups. Each learner was encouraged to say something during this time to support their writing afterwards. When writing, learners were instructed to explain their knowledge and understanding of the topic as well as give examples. In Figures 4.25 and 4.26, two learners were able to explain their understanding of the empty number line and geometric patterns respectively. Learner 5 (A), however, had to be prompted to provide an example to explain his thinking further.

Most of the learners needed written or verbal feedback when writing to explain a mathematical idea. Their explanations were often limited. Many required further prompting through feedback to provide more detailed explanations. The researcher encouraged learners to write more than one or two sentences when they could not explain everything they knew about a topic in their writing. Such writing tasks helped to gauge the level at which they understood a concept which reflected the knowledge they had constructed during previous mathematics lessons.

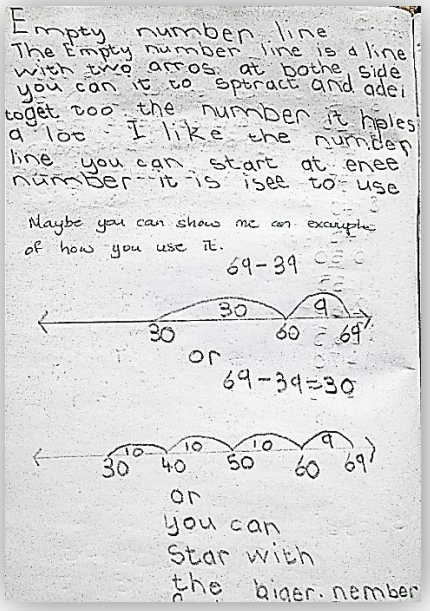


Figure 4.25: Learner 5 (A) Writing–Explanation

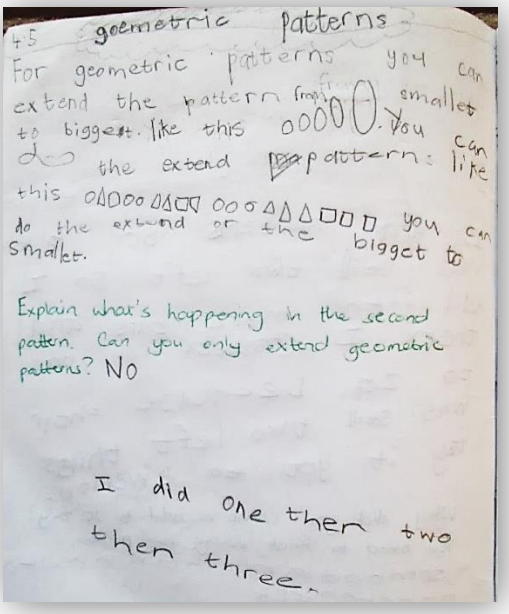


Figure 4.26: Learner 6 (BA) Writing-Explanation

4.4.4 Writing about thinking and learning processes

After several opportunities to use the types of writing tasks mentioned above, learners were introduced to writing about thinking and learning processes. When using this type of writing task, learners did not focus on a mathematical concept but rather explained mathematical ideas that could relate to their understanding in lessons. Freed (1994:24) suggests that this type of writing task encourages reflective and communicative writing. Learners used "writing about thinking and learning processes" on three occasions during the data collection period.

When learners first engaged with this writing task, they were given an opportunity to choose one of the ideas or topics mentioned during the class discussion prior to engaging in writing. They chose to write a letter to the principal. During this writing task, learners were given the option to write in pairs. Learners were reminded that the focus of their writing was not on their spelling and grammar but rather on communicating their thinking through their writing. In this way, writing about thinking and learning processes related to the study since it enhanced their ability to put thoughts into words. Seven pairs of learners focused more on mathematical concepts and how they are used in mathematics lessons. The rest of the learners wrote as if they were writing to record in a journal or log. They stated what happened and what they did during that particular day's lesson. The example in Figure 4.27

below shows how this pair of learners wrote about the day's lesson and included an explanation of the mathematical concept covered that day.

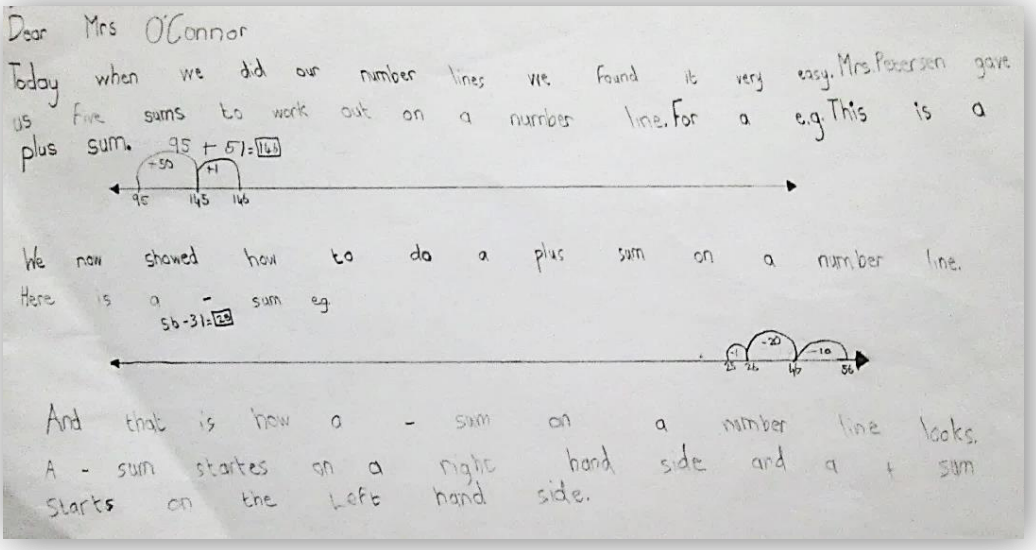


Figure 4.27: Learner 6 (BA) paired with another learner (A)

On another occasion, learners engaged in writing about their favourite mathematics activities. Learner 2's (AA) writing task in Figure 4.28 shows that she understood the purpose of this type of writing task because she focused on a general mathematics activity that occurred regularly during lessons. The researcher observed the learners while they were writing but did not engage with them or prompt them. Assistance was given to those learners who had language and/or vocabulary difficulties. It was noticed that some learners may not have understood the instructions regarding this type of writing. Their writing seemed to reflect writing to record (keeping a journal or log): they focused on the day's mathematics lesson rather than looking at general mathematics activities taking place during any lesson. In Figure 4.29, Learner 3 (A) wrote about mathematics in general. He did not write about a specific activity in the way this type of writing task requires.

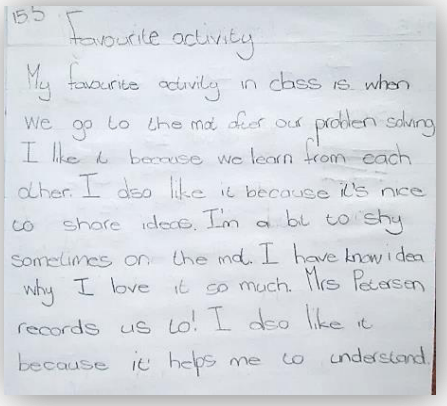


Figure 4.28: Learner 2 (AA) Writing–processes

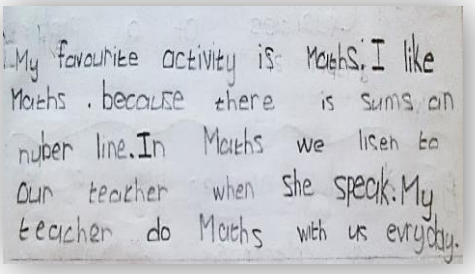


Figure 4.29: Learner 3 (A) Writing-processes

On the third occasion when learners used this type of writing task, they wrote about the qualities of a good problem-solver. Learners were given some time to think about problem-solving. The class was guided by questioning them about their experiences while solving problems. Examples of the questions asked during the class discussion were:

- What do you need to be a good problem-solver?
- What do you need to know to be a good problem-solver?
- What makes someone a good problem-solver?

Learners were given a few minutes to discuss their thoughts in groups before engaging in individual writing for approximately ten minutes. Learners were observed while they wrote but they were not assisted or prompted.

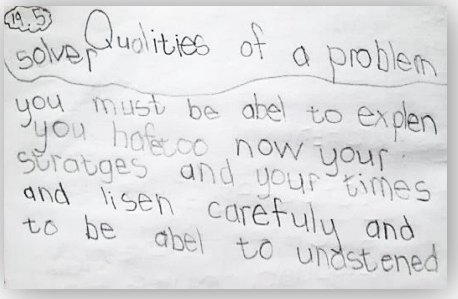


Figure 4.30: Learner 5 (A) Writing–processes

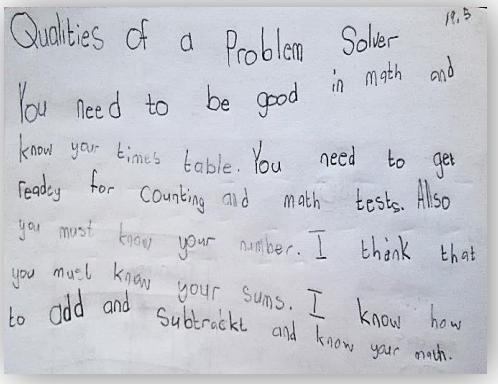


Figure 4.31: Learner 1 (AA) Writing-processes

Learner 5 (A) described what a problem-solver would need to be like without mentioning specific concepts or skills (Figure 4.30). His use of writing showed that he understood the purpose of the task. In Figure 4.31, Learner 1 (AA) listed the mathematical skills and concepts one may need to use when solving problems. This particular writing task supported learners because they focused on the process of problem-solving rather than solving mathematical problems themselves.

4.4.5 Shared writing

The last writing task presented to learners was shared writing. This writing task was included in the study because it develops learners' writing skills in mathematics. Shared writing provides opportunities to expand and clarify learners' thinking in a different way when compared to the other writing tasks. Learners had to use their knowledge of a mathematical concept in a creative manner. This technique was applied to the context of problem-solving because learners were required to explain and communicate their thinking.

There was classroom discourse around topics, however learners had difficulty finding a concept to write about that they could communicate. They worked in pairs and created a story of their experience: how they would feel if they shrank down to one centimetre tall. The same measurement concept was used in the shared writing piece which was modelled to the class. The topic was familiar to the learners but they were encouraged to write a different story to the modelled story displayed on the mathematics wall in the classroom. The researcher moved around; prompting learners through questioning since some of them were uncertain of the task. The concept of one centimetre was dealt with incidentally on a few occasions but the stories that the learners wrote reflected their understanding of measurement. There were some descriptions of what things looked like around them in comparison to their size. Some stories were more detailed in their descriptions which could be related to their language abilities. Figure 4.32 below is an example of a shared writing piece by two learners from the selected Grade 3 class. These learners were not part of the eight learners selected for the purposes of the pre-test, post-test and interviews. Consent was given by parents as noted in chapter 3 (paragraph 3.8).

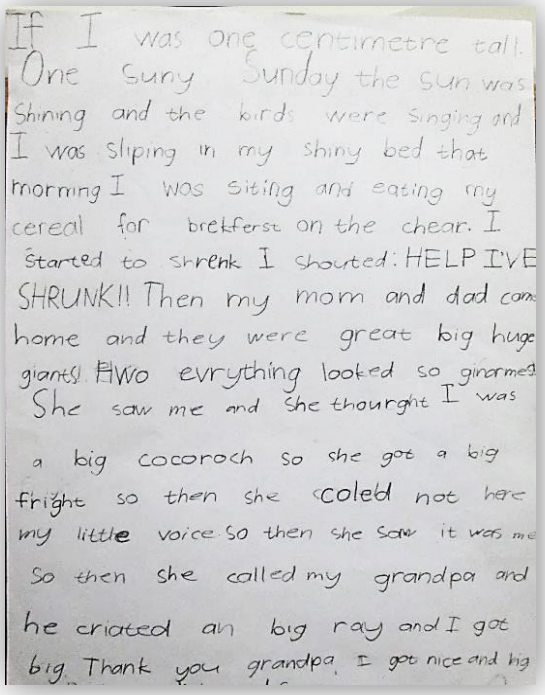

If I was one centimetre tall.
One Suny Sunday the Sun was
Shining and the birds were singing and
I was sliping in my shiny bed that
morming I was siting and eating my
cereal for bretferst on the chear. I
started to shrenk I shouted: HELP I'VE
SHRUNK!! Then my mom and dad com
home and they were great big huge
giants! Awo evrything looked so ginarmes
She saw me and she thourght I was
a big cocoroch so she got a big
fright so then she coleld not here
my little voice so then she saw it was me
So then she called my grandpa and
he criated an big ray and I got
big Thank you grandpa. I got nice and hig

Figure 4.32: A pair of non-selected learners shared written work

Various writing tasks were implemented in the Grade 3 class over a period of eight weeks. During implementation of writing tasks, the researcher often provided scaffolding to assist learners in their use of the writing tasks. Scaffolding occurred through verbal and written feedback, prompts and probing questions. By doing so, learners were encouraged to write in a more detailed manner. The findings of the post-test are now reviewed to gauge whether the implementation of such writing tasks supported learners in solving mathematical problems.

4.5 POST-TEST

The post-test was conducted in a similar manner to the pre-test. (Appendix F lists problems given to different mathematical ability groups in the post-test). The researcher reminded learners of the pre-test and the types of writing tasks that were implemented during the lessons throughout the data collection period. Learners were given an opportunity to talk about different types of writing tasks that they used. They were each given an A4 sheet of paper to be used for the five problems presented to them during the post-test.

As with the problems used in the pre-test, post-test problems are numbered below according to the three mathematical ability groups in the selected Grade 3 class. There were

differentiated problems for the above average ability group (1), the average ability group (2) and the below average ability group (3).

Problem 1

1. *Anwar has planted 19 seedlings in the vegetable garden. James has planted 16 seedlings. Thandi has planted twice as many as James. How many seedlings have they planted in the vegetable garden?*
2. *Anwar has planted 15 seedlings in the vegetable garden. James has planted 12 seedlings. Thandi has planted twice as many as James. How many seedlings have they planted in the vegetable garden?*
3. *Anwar has planted 13 seedlings in the vegetable garden. James has planted 9 seedlings. Thandi has planted twice as many as James. How many seedlings have they planted in the vegetable garden?*

The researcher did not instruct learners to use writing in mathematics: writing tasks were implemented during the data collection period. It was evident that many learners used "writing to solve mathematical problems" as can be seen in the examples below.

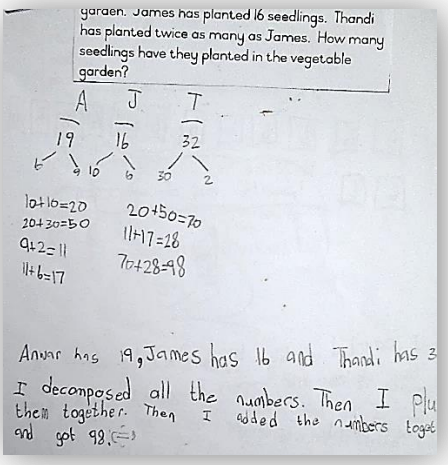


Figure 4.33: Learner 1 (AA) Post Q1

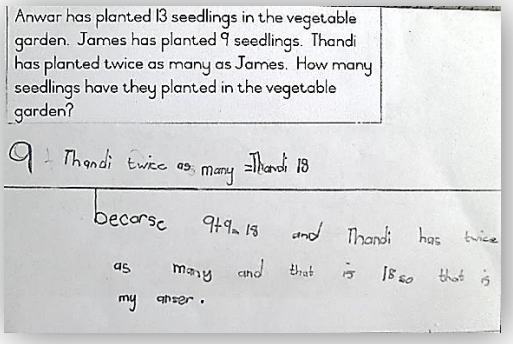


Figure 4.34: Learner 7 (BA) Post Q1

In Figure 4.33, Learner 1 (AA) used a strategy that was applicable to the addition problem type and gave a suitable explanation of what he had done. He had applied the type of writing task, writing to solve mathematical problems, successfully because he was able to make sense of how he solved the problem. His explanation described each step that he had followed and included doubling numbers, decomposing into tens and units and adding to

solve the problem. This ability was indicative of facile number sequence (stage 5 of the SEAL).

It seemed that Learner 7 (BA) understood only the first part of the problem: he doubled the number of seedlings to find Thandi's amount (shown in Figure 4.34). He was able to write some explanation of what he did using writing to solve mathematical problems. This fact reflects facile number sequence: he doubled the numbers which displays a non-counting-by-ones strategy: he did not continue solving the rest of this problem. It is possible that he misinterpreted the question in the problem. He may have thought he had to find out how much Thandi had rather than the total number of seedlings in the garden.

Problem 2

1. *There will be a parent meeting at school tomorrow evening. 81 parents will be coming. The big tables will be used with six chairs around each. How many tables will need to be set up?*
2. *There will be a parent meeting at school tomorrow evening. 65 parents will be coming. The big tables will be used with six chairs around each. How many tables will need to be set up?*
3. *There will be a parent meeting at school tomorrow evening. 39 parents will be coming. The big tables will be used with six chairs around each. How many tables will need to be set up?*

When solving this problem, Learner 7 (BA) was the only one to use the SEAL in his strategy (facile number sequence – stage 5). The rest of the selected learners were at various levels of early multiplication and division. Three learners were at level 1 (initial grouping), one learner was at level 3 (figurative composite grouping) and three learners were at level 4 (repeated abstract composite grouping), the highest level at which this problem was solved.

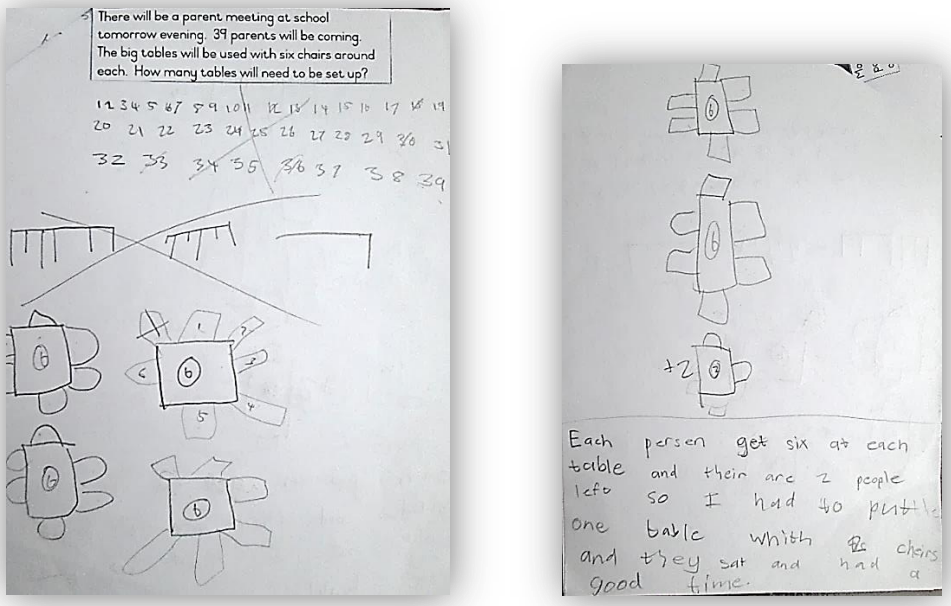


Figure 4.35: Learner 6 (BA) Post Q2

Learner 6 (BA) made three attempts at a strategy to solve the second problem in the post-test (Figure 4.35). In the first strategy, she had written out the numbers of the parents at the meeting. When she felt this strategy would not work, she began drawing tables with six legs. This drawing represented the six parents seated at each table. The third attempt at a strategy was a drawing of tables with six chairs around each. The last table had two chairs only. This strategy was at level 1 of early multiplication and division, initial grouping, in which she exhibited quotitive sharing. Learner 6 (BA) used a writing task, writing to solve mathematical problems, to explain the strategy she used. The first two attempts were not written about or explained, but the final strategy and the explanation showed an understanding of the problem as well as the problem type.

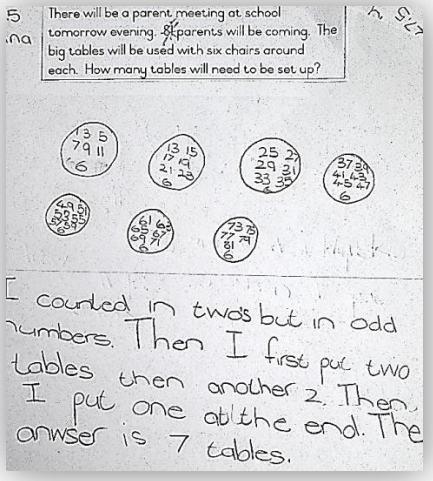


Figure 4.36: Learner 2 (AA) Post Q2

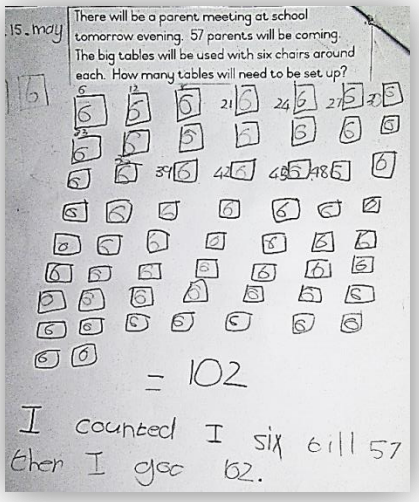


Figure 4.37: Learner 3 (A) Post Q2

Learner 2 (AA) generally displays good reading and comprehension skills. Yet, when solving this problem, it was evident that she had misread or misinterpreted the problem (Figure 4.36). She understood the mathematical concept of placing six parents at each table as stated in the problem but she represented the parents by counting the odd numbers up to 81. Even numbers are not recorded in Figure 4.36. Later, during the post-test interview, Learner 2 came to realise the error in her thinking.

In Figure 4.37, Learner 3 (A) used his knowledge of counting as his strategy. He reflected level 2 of early multiplication and division (perceptual counting in multiples) when he attempted to count in multiples of six. He misread or misinterpreted the problem; as can be seen above. He explained that he counted in sixes but reached a total of 102. This explanation did not make sense since the problem had a total of 57 parents.

The following two problems used the same context as the previous problem in the post-test. These problems extended the context by focusing on a different aspect of the problem.

<u>Problem 3</u>

1. *Mark and Martha packed out 81 chairs. Mark packed out 48 chairs. How many did Martha pack out?*
2. *Mark and Martha packed out 65 chairs. Mark packed out 38 chairs. How many did Martha pack out?*
3. *Mark and Martha packed out 39 chairs. Mark packed out 24 chairs. How many did Martha pack out?*

When solving this problem, some learners wrote simple explanations while others showed more detail. This comparison can be seen in the three examples below from learners representing different mathematical ability groups.

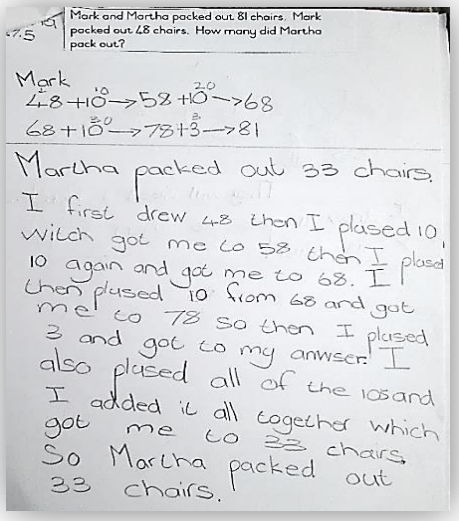


Figure 4.38: Learner 2 (AA) Post Q3

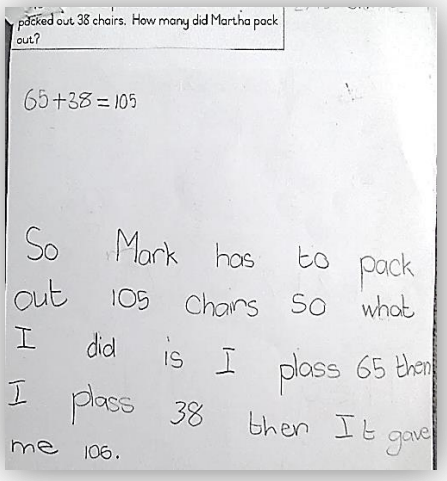


Figure 4.39: Learner 4 (A) Post Q3

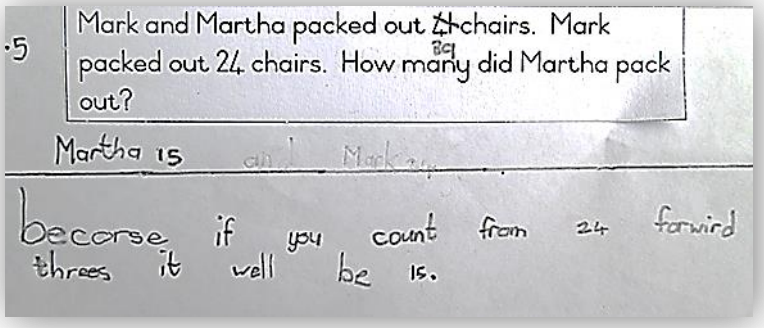


Figure 4.40: Learner 7 (BA) Post Q3

As can be seen in Figure 4.38, Learner 2 (AA) was the only learner to use conceptual place value in her problem-solving strategy. Her strategy and explanation showed an understanding at level 2, incrementing by tens off the decuple: she worked out the difference between 48 and 81. She provided a detailed explanation of her strategy through her writing which justified her thinking: she solved this problem.

Learner 4 (A)'s explanation did not demonstrate conceptual understanding of the problem type because she added the numbers given in the problem (Figure 4.39). This subtraction problem focused on comparison: learners needed to find the difference between the numbers given. Her explanation described how she incorrectly used addition in her strategy.

Learner 7 (BA) did not show a strategy for this problem but stated his solution (Figure 4.40). It appeared that he had solved this problem mentally according to the explanation that he had written. This below average learner used writing that clearly explained how he had solved the problem by using a count-down strategy. This strategy was indicative of stage 3 of the SEAL (Initial number sequence). He used "writing as solving mathematical problems" that made sense and clarified his thinking. This technique displayed a deeper conceptual understanding of the problem type and showed that, despite there being no evidence of a strategy using a number sentence or counting, it was still possible to gauge from his use of this writing task that he understood the context of the problem. His writing indicated the support that "writing to solve mathematical problems" could afford learners.

The other five learners worked at various stages of the SEAL which was suitable for this addition/subtraction problem type, except Learner 3 who displayed no clear strategy. Subsequently, he explained that he guessed this answer during the post-test interview.

Problem 4

1. *After the parent meeting coffee will be served. One pot of coffee makes 7 cups. How many pots of coffee need to be made if each person has one cup?*
2. *After the parent meeting coffee will be served. One pot of coffee makes 7 cups. How many pots of coffee need to be made if each person has one cup?*
3. *After the parent meeting coffee will be served. One pot of coffee makes 5 cups. How many pots of coffee need to be made if each person has one cup?*

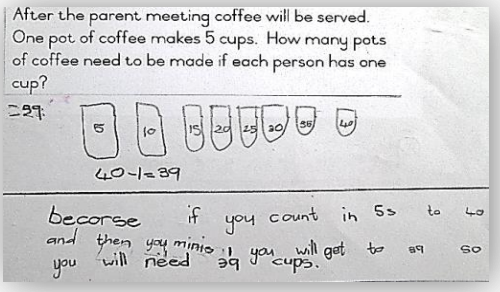


Figure 4.41: Learner 7 (BA) Post Q4

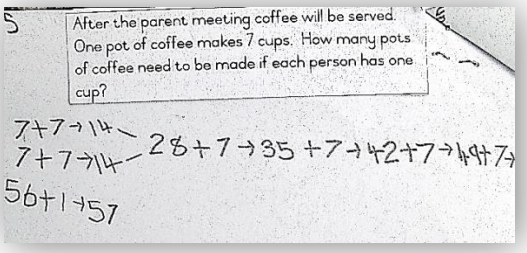


Figure 4.42: Learner 5 (A) Post Q4

In Figure 4.41, Learner 7 (BA) represented his strategy using a drawing, numbers and words that made sense. His strategy reflected figurative composite grouping, level 3 of early multiplication and division. He used repeated addition in such a way where each group is represented as an abstract composite unit (Wright et al., 2006a & 2006b). He wrote an explanation that detailed how he solved the problem. He understood the mathematical

concept required in the problem which was counting in fives. He realised that he needed to subtract one in order to cater for each parent in the problem. This technique is shown by his number sentence where he reached 40 through his counting strategy and then subtracted 1 since there were 39 parents in the problem. Despite being in the below average ability group, this learner was able to link this problem to the previous problems that used the same context. He did not rely on counting by ones and used a strategy that required higher order thinking.

Like Learner 7 (BA), Learner 5 (A) showed an understanding of mathematical concepts previously learned to solve this problem (Figure 4.42). Initially, he used the doubling strategy to a point and incorporated this into a repeated addition sum. He successfully combined two strategies from his prior knowledge which demonstrates a deeper conceptual understanding. This strategy reflected repeated abstract composite grouping, level 4 of early multiplication and division, in the way he used repeated addition as a strategy. Although both learners showed deepened conceptual understanding through their use of mathematical knowledge in their strategies, they did not state the number of pots needed in answer to the question. Both representations showed that they had a clear understanding of how to solve the problem but they failed to provide the solution to the problem.

Problem 5

1. *Xola bakes 4 trays of muffins each for his class party. Each tray can hold 12 muffins. If there are 39 children in his class, will he have enough muffins?*
2. *Xola bakes 3 trays of muffins each for his class party. Each tray can hold 12 muffins. If there are 31 children in his class, will he have enough muffins?*
3. *Xola bakes 2 trays of muffins each for his class party. Each tray can hold 12 muffins. If there are 21 children in his class, will he have enough muffins?*

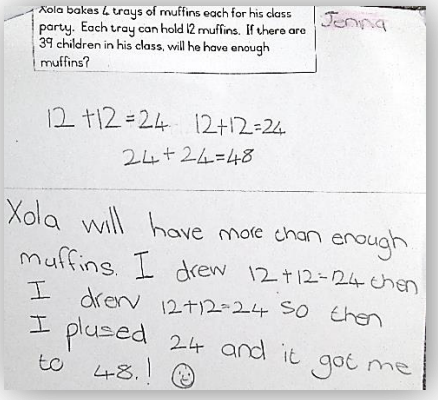


Figure 4.43: Learner 2 (AA) Post Q5

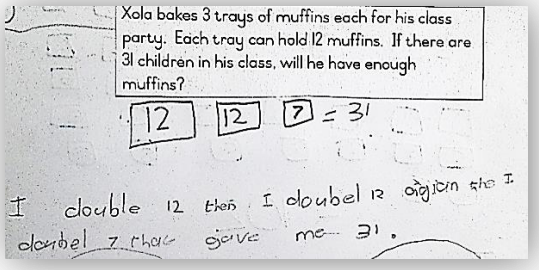


Figure 4.44: Learner 3 (A) Post Q5

As shown in Figure 4.43 above, Learner 2 (AA) wrote an explanation that adequately described the strategy she used to solve the problem. In reading the problem, she was able to see how her prior knowledge of doubling numbers could be used as a strategy which was indicative of perceptual counting in multiples, level 2 of early multiplication and division.

Learner 3 (A) drew two groups of 12 representing two trays with 12 muffins in each tray (Figure 4.44) but the last group in his strategy showed that he did not understand that 12 muffins were in each tray regardless of how many he needed for the class. He changed this number to 7. He added the number of muffins needed for the class of 31 children. The explanation that he wrote showed a mismatch with his strategy. He explained that he doubled each number instead of adding them. He doubled 12, the first two groups, which made 24. Then he explained that he doubled 7 to get to 31 when, in fact, he added 7 to 24. This doubling gave him the exact number of muffins needed for the class of 31 children. His strategy reflected intermediate number sequence, stage 5 of the SEAL, rather than early multiplication and division which was expected according to the problem type. He did not state whether he had enough muffins or not. He had worked out the number of muffins needed. This ability may be related to the fact that he changed the number of the third tray to 7 which meant that he had precisely enough for the class. It is possible that Learner 3 did not interpret the question stated in the problem correctly.

In reflecting on the post-test, it became clear that it was generally the same learners who used “writing to solve mathematical problems” to explain how they solved the problems. Some included more detail than others. The researcher did not specifically ask learners to include written explanations in the post-test in an attempt to see whether the implementation of the different types of writing tasks during the data collection period had an impact on their

ability to solve and explain problems. As evident in many of the examples of learners' strategies during the post-test, it was possible to see more detail in their writing which was apparent in their strategies or explanations. The introduction and implementation of the writing tasks helped learners to think through what they were doing in more detail in order to explain it to others. This type of clarification was encouraged throughout the data collection period when learners solved problems individually and in pairs. The post-test showed many of the learners continued to clarify and explain their thinking in this way. Writing tasks, specifically "writing to solve mathematical problems", supported learners when they solved the problems.

Table 4.2: Summary of LFIN levels and number of learners for each problem in post-test

Problem	LFIN level/stage	Number of learners
1	SEAL Stage 1	1
	SEAL Stage 4	1
	SEAL Stage 5	5
	No clear strategy	1
2	EMD Level 1	3
	EMD Level 3	1
	EMD Level 4	3
	SEAL Stage 5	1
3	SEAL Stage 1	1
	SEAL Stage 3	1
	SEAL Stage 4	1
	SEAL Stage 5	3
	CPV: Incrementing off decuple	1
	No clear strategy	1
4	EMD Level 2	1
	EMD Level 3	3
	EMD Level 4	4
5	EMD Level 1	3
	EMD Level 2	1
	EMD Level 4	2
	SEAL Stage 4	1
	No clear strategy	1

4.6 SECOND SET OF INTERVIEWS

The same learners interviewed after the pre-test were interviewed again after the post-test. The purpose was to ascertain whether the introduction and implementation of different types of writing tasks had an impact on the learners' ability to solve and explain their strategies and solutions to mathematical problems. The interview questions focussed more on the different types of writing tasks and how learners used them within the context of solving problems to understand how writing may or may not give support when solving mathematical problems.

Learners from the above average ability group wrote more detailed explanations in the post-test than learners from the other ability groups. During their interviews, they were able to give more detailed, longer explanations of their strategies. There could possibly have been a link between their use of writing when solving problems and an improved verbal explanation. Below is an excerpt from Learner 1's (AA) interview.

Researcher:	Now if you look at how you went about solving this problem, what type of writing did you use to solve the problem?
Learner 1:	I used numbers.
Researcher:	Ok, mostly numbers, and over here...in your explanation?
Learner 1:	Um, I just said that Anwar has 19, James has 16 and Thandi has 32. I decomposed all the numbers and then I plussed them together. Then I added the numbers together and got 98.
Researcher:	What made you choose working with numbers? Why did you think it would be easier?
Learner 1:	Because, what...if you have to write it, then you must still take time writing the letters so it's faster using numbers.
Researcher:	Ok. So when you are solving a problem, how do you prefer to write down your thinking?
Learner 1:	Um, by mostly using numbers.
Researcher:	Why?
Learner 1:	Because it's easier and quicker.
Researcher:	Oh, is that how you want to solve a problem? Quickly?
Learner 1:	Yes

Learner 1 (AA) Post Interview

Learner 1 (AA) displays a deep understanding of mathematical concepts which enables him to solve problems competently that reflect higher levels of number learning in the LFIN. As with his pre-test and first interview, his strategies showed that he could apply his conceptual understanding to represent his thinking. As shown in the above excerpt of the post-test

interview, Learner 1 (AA) deployed more advanced strategies; giving in-depth explanations of how he solved the problems. The added detail in his writing explanations when solving post-test problems provided further support to his existing knowledge of mathematical concepts.

When Learner 6 (BA) completed the problems in the post-test, she was able to write a detailed explanation of a strategy that she had used. As a result, she could give a verbal explanation in her post-test interview. Below is an excerpt of the interview with Learner 6 (BA) where she describes her strategy as shown in Figure 4.35.

Researcher:	And then you went on to drawing. Why did you choose to draw?
Learner 6:	Because it got me…it made it easier.
Researcher:	Ok, and I liked your explanation you did…a lot of detail on your explanation. Ok. Let's quickly look at the parent meeting. Um, I see you've got numbers here and then you crossed it out. Then you've got something else here and you crossed it out and then you did a drawing. Do you want to explain to me what you did there?
Learner 6:	I did tables with 6 in it. Each person gets 6 at each table and there are 2 people left so I had to put one table with 2 chairs and they sit and have a good time.
Researcher:	So how many tables did you put out for the meeting?
Learner 6:	First I put 6 and then I had to add one more and 2 chairs.
Researcher:	Ok. So how many tables?
Learner 6:	7

Learner 6 (BA) Post Interview

Learner 5 (A) showed an improved understanding of mathematical concepts and problem-solving which was evident through his explanations in his post-test. For all the problems solved during the post-test, he used "writing to solve mathematical problems" to describe his strategy and clarify his thinking.

Other learners from the average and below average ability groups referred to the usefulness of writing explanations of their problem-solving strategies. Most of the selected learners described how their use of writing assisted them in making sense of the problem they were solving. Below is an excerpt from the second interview conducted with Learner 7 (BA). Initially, the learner could not remember or explain his strategy. But, after he read the explanation he had written, he could make sense of the problem and provide the solution. This breakthrough was an indication of how he could verbally explain his thinking: his written

solution and explanation were detailed. His thinking when he solved the problem previously was clearly expressed in his writing as can be seen in Figure 4.41.

Researcher: Let's read. You've got here 40 minus 1 equal 39. Ok. So these pots here, how many pots of coffee do you get altogether?
Learner 7: 40
Researcher: Ok, and you've got this sum here. What does it mean?
Learner 7: I forgot.
Researcher: Um, how many parents are coming to the meeting?
Learner 7: 39
Researcher: Ok. So read your explanation of what you did here.
Learner 7: Because if you count in 5s to 40 and then you minus 1, you will get the…to 39 so…so you will need 39 cups.
Researcher: Good. Ok and how many pots did you draw to be able to make 39 cups of coffee?
Learner 7: 8

Learner 7 (BA) Post Interview

After explaining how they solved the problems in the post-test, learners were given an opportunity to explain how they preferred to write their strategies. Learner 2 (AA) clarified in her interview that using drawing in her writing helped to avoid confusion when she attempted to solve her mathematical problems.

Researcher: Which way do you prefer to solve problems because I see sometimes you, you use sums and sometimes you draw and sometimes you use words. What do you find the easiest for you?
Learner 2: I find the drawings the easiest because then you can see what you, what you doing and sometimes it makes me confused when I use numbers because it's like almost all over and it sometimes makes me confused.
Researcher: Ok. So you prefer to draw.
Learner 2: Yes
Researcher: But I see even when you're drawing, like in the coffee pots, you still using numbers in your drawings.
Learner 2: Yes if I, so then I drew something around it to know that I, I didn't get confused.
Researcher: Ok. So you like to be organised with what you're thinking.

Learner 2: Yes

Learner 2 (AA) Post interview

During the post-test interviews learners were able to provide better verbal explanations when compared to the pre-test interviews. This improvement was due to the fact that most of them used “writing to solve mathematical problems” to solve and explain their thinking. Their post-test explanations contained more detail which assisted them in making sense of their strategies.

4.7 CONCLUSION

Findings presented in this chapter address the research questions of this study. Findings of the pre-test and first set of interviews showed that learners generally solved problems reflecting lower levels and stages of the LFIN. The LFIN is the framework of number learning by Wright, Martland and Stafford (2006), Wright, Martland, Stafford and Stanger (2006) and Wright (2013) used to analyse learners’ problem-solving strategies.

Writing tasks were modelled and implemented in the selected Grade 3 class. Learners were given various writing tasks over eight weeks and the researcher provided scaffolding when needed to support and develop learners’ use of writing.

The post-test and second set of interviews were conducted at the end of the data collection period. The results of the pre-test and post-test were compared to determine whether writing tasks supported learners in solving mathematical problems. The learners’ problem-solving strategies often reflected higher levels and stages of the LFIN when compared to the pre-test. Learners solved problems and explained their thinking behind their solution strategies in more detail during the post-test. Learners were able to provide improved verbal explanations of their strategies during the interviews.

Discussion regarding these findings is presented in the final chapter. Recommendations for further study in the use of writing in mathematics follows as well as reflections on the use of writing to support mathematical problem-solving.

CHAPTER 5
DISCUSSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The purpose of this study was to explore how writing supports Grade 3 learners' mathematical problem-solving abilities. The study employed various writing tasks as promoted by Burns (1995a). A pre-test was conducted with a selected Grade 3 class at the beginning of the data collection period to determine learners' ability to solve mathematical problems. A group of eight learners from the class was selected and interviewed regarding their solutions in the pre-test. The writing tasks were introduced and implemented to support learners in solving mathematical problems. Data were collected through audio recordings of in-class ability group discussions and learners' written pieces. The data collection period concluded with a post-test and interviews of the eight selected learners to gauge the impact of the writing tasks on learners' ability to solve mathematical problems. The research questions that guided the study were as follows:

Research question:
How do various types of writing tasks support Grade 3 learners in solving mathematical problems?

Sub-questions:

1. What support do writing tasks give the development of conceptual understanding?
2. What support do writing tasks give the development of problem-solving strategies?
3. How are writing tasks useful in the Foundation Phase mathematics classroom?
4. What challenges do learners encounter when implementing writing tasks in the Foundation Phase mathematics classroom?

This chapter presents a summary of the research process, followed by a discussion of the findings. How the findings answer the research questions of the study is discussed as well as additional themes that emerged during the data analysis. Implications and recommendations that follow from the study are determined. Reflections on the study include its limitations.

5.2 SUMMARY OF THE RESEARCH PROCESS

In chapter 4 detailed results from the analysis of the data collected in this study were presented. A brief summary of the research process is followed by discussion of the findings.

At the beginning of the data collection period, all learners of the selected Grade 3 class participated in a pre-test. Learners were required to solve five mathematical problems. Most learners had difficulty solving these problems and communicating their thinking in writing. Eight learners were selected to be interviewed regarding the strategies they used when solving the problems in the pre-test. These learners represented three mathematical ability groups in the class. The verbal explanations of the eight learners were restricted: most of them had difficulty explaining their strategies. Probing questions were necessary to assist them during the interviews.

Over a period of eight weeks, various writing tasks (Burns, 1995a; Wilcox and Monroe, 2011) were modelled to all the learners in the class. On these occasions, the purpose of each particular writing task was communicated to enhance learners' understanding of the task expectations and encourage the use of writing to explain their thinking. The writing tasks included writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes, and shared writing. Learners were given various opportunities to complete writing tasks during mathematics lessons.

At the end of the data collection period, a post-test was conducted with all the learners of the selected Grade 3 class. The same procedure was followed as for the pre-test. Learners were not explicitly requested to use "writing to solve mathematical problems" when solving problems in the post-test but a number of them did so, providing varying degrees of detail in the explanation of their strategies. The post-test was used to determine whether the introduction and implementation of writing tasks supported learners to solve mathematical problems and to explain the thinking behind their strategies. The same eight learners were interviewed after the post-test.

5.3 SUMMARY OF THE FINDINGS

Data collected in this study were analysed using the theoretical framework outlined in chapter 2. Data were collected from learners' written work, interviews, field notes and audio-

recordings of ability group discussions. Learners' written work showed the strategies they used when solving the mathematical problems set. These problem-solving strategies were analysed using the work of Wright, Martland and Stafford (2006), Wright, Martland, Stafford and Stanger (2006) and Wright (2013). The Learning Framework In Number (LFIN) provided the stages and levels of number learning as learners develop their mathematical knowledge. These stages and levels were used to describe the strategies learners employed when solving mathematical problems in order to analyse and compare their solution strategies, especially those in the pre-test and post-test.

The findings of the pre-test showed that most learners had difficulty solving mathematical word problems and communicating their thinking through writing. Learners sometimes used strategies inappropriate to the problem types. The strategies usually reflected the lower levels or stages of the aspects of the LFIN, especially from learners representing the average and below average mathematical ability groups.

A sample of eight learners representing the three different mathematical ability groups in the selected class was interviewed about their strategies in the pre-test. Learners had to explain in words the thinking behind their solution strategies. Some learners had difficulty doing so: their verbal explanations sometimes differed from their written strategies. Learners sometimes lacked the mathematical vocabulary to explain what they had done.

After the pre-test and first set of interviews had been conducted, writing tasks were modelled to all the learners of the selected Grade 3 class over an eight week period. Learners were given opportunities to complete writing tasks during mathematics lessons over this period. While data were being collected many learners, especially those from the average and below average ability groups, chose to discuss the problem before tackling a writing task. Some learners chose to write collaboratively more often than others when the opportunity arose. Learners became accustomed to doing the writing tasks and developed more detail in their writing which led to more comprehensive written and verbal explanations of their strategies when solving problems.

The role of language needs to be considered when learners solve mathematical word problems. Many learners, especially from the below average ability group, found it hard to understand the contexts of some of the mathematical problems. This difficulty may have been a result of limited reading abilities amongst these learners.

Learners who wrote detailed explanations when solving the problems of the post-test were able to provide detailed verbal explanations of their strategies and solution processes during the post-test interviews. Their use of writing appeared to help them make sense of their strategies and justify their thinking when solving problems. The introduction and implementation of writing tasks supported learners' mathematical problem-solving abilities.

5.4 DISCUSSION

Through the data analysis of the findings, the research question and sub-questions were addressed. A discussion of the findings is presented in the following section (5.4.1 – 5.4.4).

The work of Wright, Martland and Stafford (2006), Wright, Martland, Stafford and Stanger (2006) and Wright (2013), referred to as the Learning Framework In Number (LFIN), was used to analyse learners' problem-solving strategies. An overview of LFIN was presented in Chapter 2. Although it is primarily a framework for teaching numbers, the LFIN is relevant to this study because it provides stages and levels of development for number learning which helped to analyse learners' strategies. LFIN covers various aspects of number learning such as the Stages of Early Arithmetical Learning (SEAL), conceptual place value knowledge and early multiplication and division (Wright et al., 2006a) which applied to many of the strategies seen in this study. Through analysis, it was possible to pinpoint the exact level at which each learner solved the problem within a particular aspect of the LFIN. This pinpointing enabled comparisons to be made between the levels of problem-solving strategies used in the pre-test and the post-test.

5.4.1 Using writing to develop conceptual understanding

Research sub-question 1 of the study addresses the support writing tasks give to the development of conceptual understanding. When learners' initial and later use of writing was analysed and compared, it was evident they could provide more detail and refer to distinct aspects of mathematical content when they solved problems. Figure 5.1 and Figure 5.2 are examples of Learner 5's (A) solutions to problems in the pre-test and the post-test. Figure 5.1 shows how he was able to solve the problem but did not use mathematical vocabulary to explain his strategy. In Figure 5.2, there is evidence of a detailed explanation using specific mathematical ideas although he arrived at the incorrect answer.

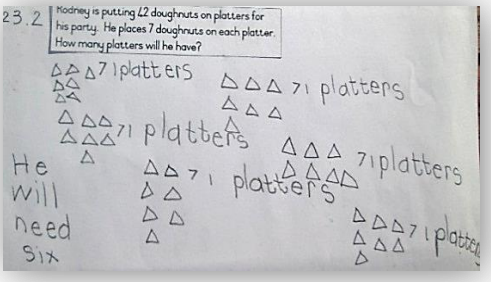


Figure 5.1 Learner 5 (A) Pre Q2

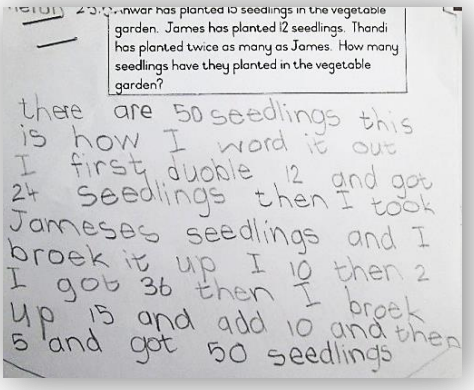


Figure 5.2 Learner 5 (A) Post Q1

In the literature review, conceptual understanding within problem-solving was discussed. O'Donnell (2006:349) states that problem-solving needs to encourage a higher cognitive demand where the mathematical content embedded in the problem may not be obvious to the learner. The problems presented throughout the data collection needed to encourage critical thinking and develop conceptual understanding.

Moreover, Orton and Frobisher (1996:23) suggest that "problem-solving shifts the weight from the acquisition of knowledge and skills to using and applying them". Solving mathematical problems should encourage learners to move beyond the use of procedural knowledge and develop their own conceptual knowledge. As Sfard's (1991:28) theory suggests, learners move between their operational and structural conceptions of mathematical ideas when they solve problems. Miller (1992:354) adds that writing is an active process that promotes students' procedural and conceptual understanding of mathematics. Through writing, learners communicate their understanding of mathematical concepts whenever they solve mathematical problems. Heddens and Speer (2006:84) argue that the opportunity to apply conceptual knowledge is as important as understanding the concepts themselves. It provides more meaning and purpose to the knowledge and skills the learner has acquired. Learners use what they know in order to solve that which is unknown. The learner makes connections with previous knowledge and mathematical problems in order to construct new meaning.

As learners used the writing task, writing to solve mathematical problems, they had opportunities to develop and apply conceptual understanding to the problems. The problems were presented in a way that encouraged learners to connect their existing knowledge to the mathematical content. While learners solved problem 3 during the implementation stage of

data collection, they needed the skill of counting in fours as well as the related multiplication table. (Appendix G lists the differentiated mathematical problems used during the implementation of the task, writing to solve mathematical problems.) Although learners had previously engaged with this mathematical knowledge, they had not done so in that particular day's lesson. If there was a mathematical concept that was required to solve the problem, the same concept or skill needed to be included in the mental mathematics section at the start of the lesson. It appeared that most learners required some level of scaffolding in this regard. Scaffolding will be discussed later in this chapter as one of the themes. Many learners may have had difficulty making the connection between the mathematical content embedded in the problem and their existing knowledge on their own.

Before learners solved problem 5, counting in threes was included into the mental mathematics exercises at the beginning of the lesson. This particular problem dealt with the context of tricycles for which the skill of counting in threes was required. Some learners, generally from the average and below average mathematical ability groups, had difficulty understanding the problem or finding a strategy and solution. This was significant since learners practised the skill of counting in threes before the problem was presented. Most of these learners had difficulty making a connection to their existing knowledge. As can be seen in Figure 5.3, Learner 7 (BA) needed support through written and verbal feedback to make sense of the context of this problem. The written feedback was not sufficient and he required further individual verbal guidance through scaffolding and prompts. Although he could give a verbal explanation of the strategy he had to use, he still arrived at an incorrect solution after he continued working individually. Through scaffolding, Learner 7 began to display conceptual understanding of counting in threes and related it to the mathematical content embedded in the problem. He still did not demonstrate that he fully understood the context of the problem: he circled all the tally marks rather than working with the required number of 65 wheels as stated in the problem.

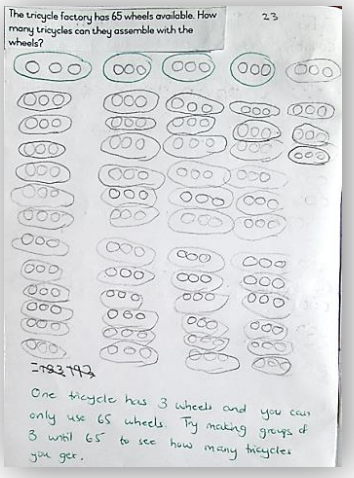


Figure 5.3 Learner 7 (BA) Writing – Problem 5

Based on these findings, it is evident that the use of writing in mathematics supports the development of conceptual understanding. Throughout data collection, learners were encouraged to connect the problem they were solving to a mathematical concept or idea. Learners from the average and below average mathematical ability groups seemed to find this more challenging because they often had difficulty finding the mathematical content embedded in the problems. As the writing intervention progressed, learners were given more opportunities to use writing tasks to explain their thinking. They engaged in writing tasks in a way that encouraged them to think through their strategies and solutions in order to write a suitable explanation of their thinking. Development of their conceptual understanding was particularly evident in the post-test where learners individually wrote more detailed explanations incorporating mathematical ideas.

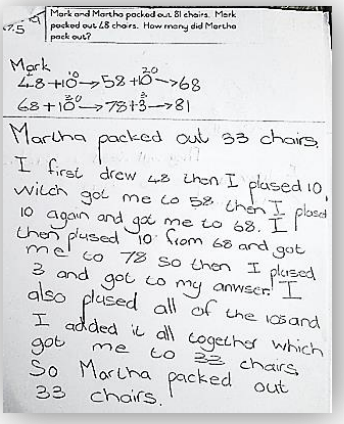


Figure 5.4 Learner 2 (AA) Post Q3

Figure 5.4 shows an example of the development of conceptual understanding. Learner 2 (AA) used incrementing off the decuple as a problem-solving strategy and clarified her thinking in her written explanation. Another example of the development of conceptual understanding is evident in Figure 5.5. This post-test strategy shows how Learner 7 (BA) used figurative composite grouping by adding 5 each time. His explanation showed that he could identify the mathematical content in the problem because he was able to explain his reason for subtracting 1 to reach the correct number of cups. This learner did not solve any problems that reflected higher levels of the LFIN during the pre-test. Figure 5.6 shows how Learner 5 (A) combined doubling and repeated addition in his strategy. He applied his existing knowledge to create a strategy that suited the problem. These examples demonstrate how learners combined and adapted their existing knowledge to the mathematical content in the problems. They developed their conceptual knowledge by restructuring schemas that enhanced their understanding (Skemp, 1987:28).

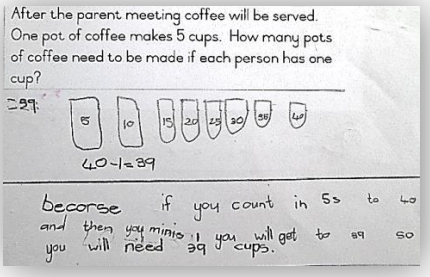


Figure 5.5 Learner 7 (BA) Post Q4

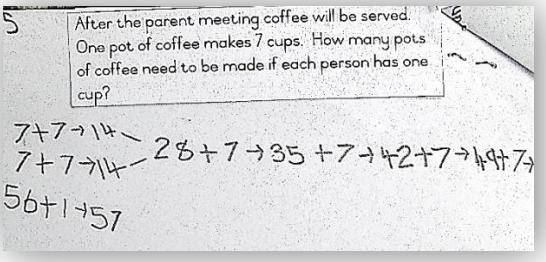


Figure 5.6 Learner 5 (A) Post Q4

These findings show that writing tasks support the development of conceptual understanding. As learners solve and explain mathematical problems, they critically think about the mathematical content in the problem. The majority of problem-solving strategies used by learners in the post-test reflect higher stages and levels of LFIN. These findings suggest that they were able to connect the mathematical content and context of the problem to their existing knowledge. In some instances, learners combined mathematical concepts in their strategies. This showed an improved conceptual understanding: they were connecting concepts to find a solution.

5.4.2 Learners' development of problem-solving strategies

Research sub-question 2 traces the development of problem-solving strategies after writing tasks were implemented. It was evident in the study that learners improved their problem-

solving strategies. The question was whether the lower number range of some mathematical problems had an impact on the levels of problem-solving strategies learners used.

5.4.2.1 Comparison between pre-test and post-test

In the literature review, there was an overview of problem-solving and the use of mathematical problems. It was explained that problem-solving is "a process in which the learner combines previously learned elements of knowledge, rules, techniques, skills and concepts to provide a solution to a situation not encountered before" (Orton, 2004:24). A pre-test and post-test were conducted at the beginning and end of the data collection period. The purpose of doing so was to gauge the levels of problem-solving strategies learners used before and after implementing different types of writing tasks.

Table 5.1: Model for Stages of Early Arithmetic Learning (SEAL) (Wright, Martland, Stafford and Stanger, 2006:9)

Stage 0: Emergent Counting. Cannot count visible items. The child either does not know the number words or cannot coordinate the number words with items.
Stage 1: Perceptual Counting. Can count perceived items but not those in screened (that is concealed) collections. This may involve seeing, hearing or feeling items.
Stage 2: Figurative Counting. Can count the items in a screened collection but counting typically includes what adults might regard as redundant activity. For example, when presented with two screened collections, told how many in each collection, and asked how many counters in all, the child will count from 'one' instead of counting-on.
Stage 3: Initial Number Sequence. Child uses counting-on rather than counting from 'one', to solve addition or missing addend tasks (e.g. 6 + x = 9). The child may use a count-down-from strategy to solve removed items tasks (e.g. 17 − 3 as 16, 15, 14 − answer 14) but not count-down-to strategies to solve missing subtrahend tasks (e.g. 17 − 14 as 16, 15, 14 − answer 3).
Stage 4: Intermediate Number Sequence. The child counts-down-to to solve missing subtrahend tasks (e.g. 17 − 14 as 16, 15, 14 − answer 3). The child can choose the more efficient of count-down-from and count-down-to strategies.
Stage 5: Facile Number Sequence. The child uses a range of what are referred to as non-count-by-ones strategies. These strategies involve procedures other than counting-by-ones but may also involve some counting-by-ones. Thus in additive and subtractive situations, the child uses strategies such as compensation, using a known result, adding to ten, commutativity, subtraction as the inverse of addition, awareness of the 'ten' in a teen number.

Table 5.2: Model for early multiplication and division levels (Wright, Martland, Stafford & Stanger, 2006:14)

Level 1: Initial Grouping. Uses perceptual counting (that is, by ones) to establish the numerosity of a collection of equal groups, to share items into groups of a given size (quotitive sharing) and to share items into a given number of groups (partitive sharing).

Level 2: Perceptual Counting in Multiples. Uses a multiplicative counting strategy to count visible items arranged in equal groups.

Level 3: Figurative Composite Grouping. Uses a multiplicative counting strategy to count items arranged in equal groups in cases where the individual items are not visible.

Level 4: Repeated Abstract Composite Grouping. Counts composite units in repeated addition or subtraction, that is, uses the composite unit a specified number of times.

Level 5: Multiplication and Division as Operations. Can regard both the number in each group and the number of groups as a composite unit. Can immediately recall or quickly derive many of the basic facts for multiplication and division.

As mentioned earlier in this chapter, the stages and levels of the different aspects of LFIN (Tables 5.1 and 5.2) provided clarity and differentiation between the strategies learners used. The results of the analysis are in tabular form below where selected learners' strategies are listed.

Table 5.3 Analysis of pre-test and post-test strategies

LEARNER 1 (AA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Facile number sequence (breaks down into tens and ones and adds separately)	SEAL – Facile number sequence (decomposes)
2	EMD – Initial grouping (drawing shows quotitive sharing)	EMD – Figurative composite grouping (skip counting)
3	SEAL – Facile number sequence (breaks down tens and ones and adds separately – incorrect answer)	SEAL – Facile number sequence
4	EMD – Repeated abstract composite grouping	EMD – Figurative composite grouping (quotitive sharing incorporating skip counting)
5	SEAL – Facile number sequence (breaks down into tens and ones and adds)	EMD – Repeated abstract composite grouping

LEARNER 2 (AA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Initial number sequence	SEAL – Facile number sequence (decomposed, added separately)
2	EMD – Initial grouping (drawing shows quotitive sharing – incorrect answer)	EMD – Initial grouping (quotitive sharing – incorrect answer)
3	SEAL – Perceptual counting	CPV – increment by tens off decuple
4	EMD – Initial grouping (quotitive sharing – incorrect strategy)	EMD – Repeated abstract composite grouping (repeated addition)
5	SEAL – Initial number sequence	EMD – Perceptual counting in multiples

LEARNER 3 (A)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – Facile number sequence (only solved 1 part)
2	EMD – Initial grouping (drawing shows quotitive sharing – adds incorrectly)	EMD – Repeated abstract composite grouping (attempt) (skip counts incorrectly)
3	SEAL – Initial number sequence (incorrect answer)	No clear strategy
4	SEAL – Facile number sequence (incorrect answer)	EMD – Repeated abstract composite grouping (incorrect answer)
5	SEAL – Initial number sequence (incorrect answer)	SEAL – Intermediate number sequence (incorrect strategy)

LEARNER 4 (A)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – intermediate number sequence (incomplete strategy)
2	Combined EMD – initial grouping and SEAL – Perceptual counting	EMD – Repeated abstract composite grouping (repeated addition)
3	SEAL – Facile number sequence	SEAL – Facile number sequence
4	EMD – Perceptual counting (counts all)	EMD – Repeated abstract composite grouping (repeated addition – incorrect answer)
5	Absent	EMD – Initial grouping (quotitive sharing)

LEARNER 5 (A)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – Facile number sequence (decomposed – incorrect answer)
2	EMD – Initial grouping (quotitive sharing)	EMD – Repeated abstract composite grouping
3	SEAL – Combines Perceptual counting and Facile number sequence (incorrect answer)	SEAL – Facile number sequence
4	EMD – Perceptual counting in multiples (incorrect answer)	EMD – Repeated abstract composite grouping (repeated addition)
5	SEAL – Perceptual counting	EMD – Repeated abstract composite grouping (repeated addition)

LEARNER 6 (BA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Initial number sequence	SEAL – Perceptual counting
2	Strategy didn't fit problem type	EMD – Initial grouping (quotitive sharing)
3	SEAL – Intermediate number sequence	SEAL – Perceptual counting (added instead of subtracting)
4	EMD – Initial grouping (partitive sharing – incorrect strategy)	EMD – Perceptual counting in multiples
5	SEAL – Initial number sequence (used strategy incorrectly)	EMD – Initial grouping (quotitive sharing)

LEARNER 7 (BA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – Facile number sequence (didn't complete)
2	Strategy didn't fit problem type	SEAL – Facile number sequence (correct strategy used incorrectly)
3	SEAL – Perceptual counting (incorrect answer)	SEAL – Initial number sequence
4	Absent	EMD – Figurative composite grouping
5	No strategy visible	Strategy didn't fit problem type

LEARNER 8 (BA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting (used strategy incorrectly)	No clear strategy
2	EMD – Initial grouping (quotitive sharing – used strategy incorrectly)	EMD – Initial grouping (quotitive sharing)
3	SEAL – Facile number sequence (counting erased)	SEAL – Intermediate number sequence (incorrect answer)
4	EMD – Initial grouping (number sentence didn't match)	EMD – Figurative composite grouping (skip counting – didn't answer problem)
5	SEAL – perceptual counting (strategy used incorrectly)	EMD – Initial grouping (partitive sharing – used strategy incorrectly, incorrect answer)

Many learners in the selected Grade 3 class were restricted in their use of mathematical problem-solving strategies in the pre-test. Their strategies often reflected lower stages and levels of different aspects of the LFIN. Tallies were frequently used as a strategy in the pre-test and the earlier part of the writing intervention. At this stage learners were not expected to describe their thinking although they had solved mathematical problems prior to this study.

Two learners from the above average ability group already showed strategies that were more advanced during the pre-test when compared to the other learners. When these strategies were compared to those in the post-test, these learners displayed strategies at higher stages and levels where there was evidence of enriched writing to explain their strategies. For example, Learner 2 (AA) usually solved problems in the pre-test at stage 4 of the SEAL (initial number sequence) and level 1 of early multiplication and division (initial grouping) as shown in Figure 5.7 below.

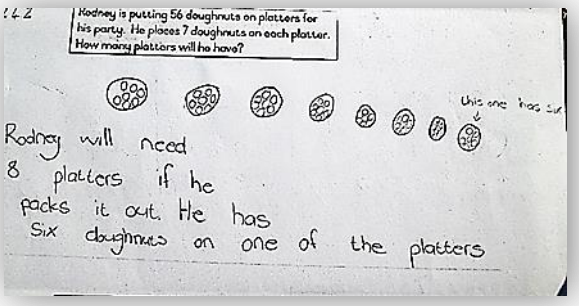


Figure 5.7 Learner 2 (AA) Pre Q2

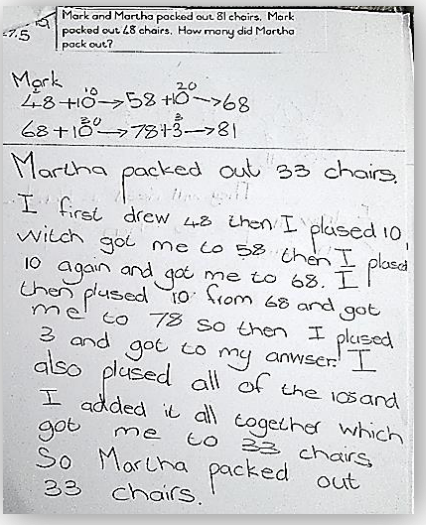


Figure 5.8 Learner 2 (AA) Post Q3

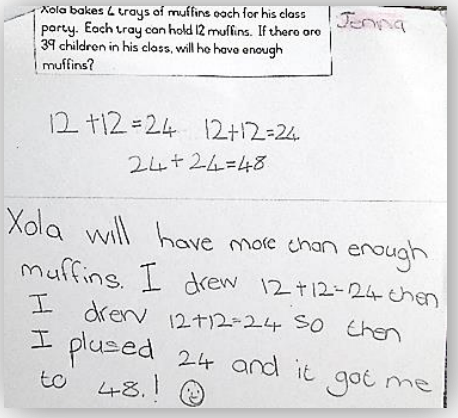


Figure 5.9 Learner 2 (AA) Post Q5

Figure 5.8 and Figure 5.9 show that the same learner solved similar problems at stage 5 of the SEAL (facile number sequence) and levels 2 and 4 of early multiplication and division (perceptual counting in multiples and repeated abstract composite grouping respectively). For one of the problems in the post-test (Figure 5.8), there was evidence of Learner 2's strategy at level 2 of conceptual place value (incrementing by tens off the decuple). This showed that Learner 2 used increasingly sophisticated strategies when the results of the pre-test and the post-test were compared: there was no evidence of strategies of this nature in the pre-test.

Similar results were apparent in strategies used by the average and below average ability groups. There was a marked difference in the strategies Learner 5 (A) used in the post-test when compared to the pre-test. This distinction can be seen in Figure 5.10 and Figure 5.11 below. During the pre-test, his strategies were generally at level 1 of the SEAL (perceptual

counting) and there was evidence of level 2 of early multiplication and division (perceptual counting in multiples). In the post-test he solved problems at the highest stage of the SEAL (facile number sequence) and at level 4 of early multiplication and division (repeated abstract composite grouping).

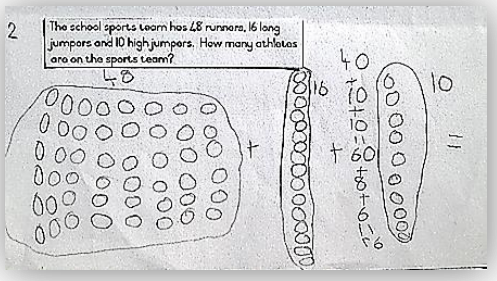


Figure 5.10 Learner 5 (A) Pre Q3

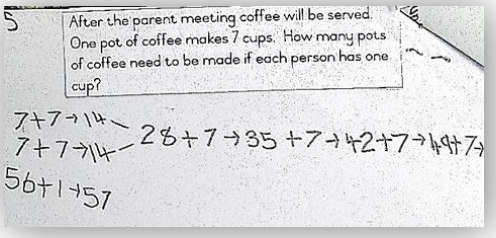


Figure 5.11 Learner 5 (A) Post Q4

Writing improved problem-solving strategies of learners in the below average ability group. Learner 7, for instance, used basic strategies in the pre-test at level 1 of the SEAL (perceptual counting) to solve two of the problems. Figure 5.12 below is an example of the strategies he used. The remaining problems did not have a visible strategy or the strategy used did not match the problem type.

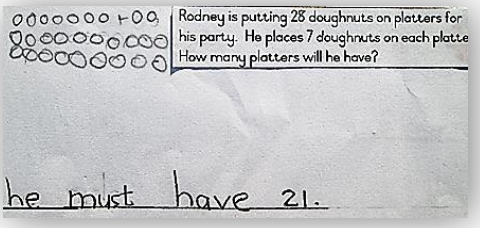


Figure 5.12 Learner 7 (BA) Pre Q2

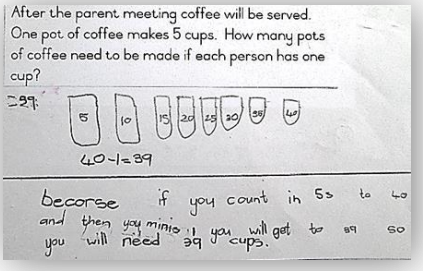
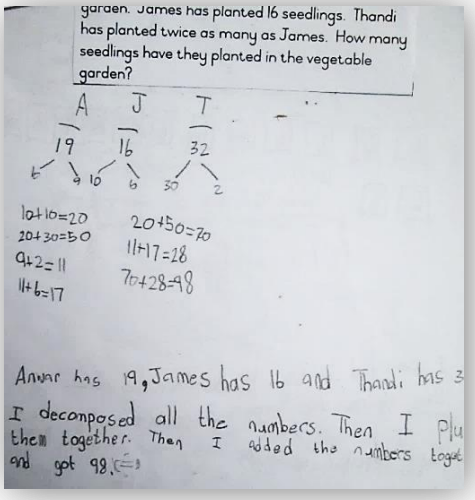


Figure 5.13 Learner 7 (BA) Post Q4

The post-test reflected a significant improvement in the strategies used to solve problems. Use of writing was evident to explain how he solved the problems. The more complex strategies reflected in the post-test were at stage 5 of the SEAL (facile number sequence) and level 3 of early multiplication and division (figurative composite grouping). Learner 7's (BA) strategy and explanation in Figure 5.13 is an example of how he used figurative composite grouping by applying his conceptual knowledge of counting in fives. He realised that he needed to subtract one in order to answer the problem correctly.

The remainder of the selected learners displayed similar tendencies when comparing the strategies used to solve the problems in the pre-test and the post-test. Throughout the data collection period, learners were encouraged to write to explain how they solved mathematical problems. Writing in this way enhanced their problem-solving strategies: they considered their strategies in detail in order to write their explanations. Some learners used mathematical language in their explanations which showed that they were able to link elements of their strategies with particular concepts they had learned previously. For example, terms such as double and decompose were used, which some learners referred to as breaking down (Figure 5.11). This usage was an example of how they used their mathematical knowledge to enhance their strategies when solving problems. This phenomenon related to Sfard's theory of the process and object of a mathematical idea where learners could apply existing mathematical knowledge and vocabulary to the process of problem-solving. As explained in Chapter 2, the process, or operational conception, is the dynamic action where an idea is conceived at a lower level and the object, or structural conception, is conceived at higher levels that underlie relational understanding (Sfard, 1991:16).

Mathematical problem-solving requires applying existing knowledge of mathematical ideas (objects) as well as the conception and development of new ideas (process). Learners' written explanations became more detailed in the post-test, reflecting mathematical knowledge and vocabulary. This observation suggests that concepts taught in mathematical lessons were being connected to problems being solved. Learners engaged in, and used, processes and objects of their mathematical ideas in order to find solutions.


garden. James has planted 16 seedlings. Thandi has planted twice as many as James. How many seedlings have they planted in the vegetable garden?

A J T

19 16 32

10+10=20
20+30=50
9+2=11
11+6=17

20+50=70
11+17=28
70+28=98

Anwar has 19, James has 16 and Thandi has 3
I decomposed all the numbers. Then I plu
them together. Then I added the numbers toget
and got 98.

Figure 5.14 Learner 1 (AA) Post Q1

In this study, learners used writing to solve and explain mathematical problems. When they encountered problems, either individually or corporately, learners appeared to use strategic thinking to determine how to arrive at solutions. Learners drew on their existing mathematical knowledge and applied it to their strategies. At times, mathematical problems required a reconstruction of mathematical knowledge: learners developed further invented strategies by adding to, or combining, existing mathematical ideas (Campbell et al., 1998).

5.4.2.2 Limited use of strategies related to lower number range

This study examines the support that writing tasks give to the development of Grade 3 learners' problem-solving strategies. The mathematical problems used in this study differentiated number ranges according to the three mathematical ability groups present in the participating Grade 3 class. While learners solved mathematical problems, most of the eight selected learners initially used tallies as a strategy. This technique was used by learners representing all three mathematical ability groups. This usage was especially evident during the pre-test and the earlier part of the writing intervention. As learners solved more mathematical problems during the latter part of the data collection, more advanced strategies, less reliant on tallies, were used by the same learners.

Learners from the below average ability group often used tallies in their strategies when compared to the strategies of the above average and average ability groups. They may have used tallies more frequently because their mathematical problems used lower number ranges than the other ability groups. Learners, especially from the above average ability group, seldom used tallies. This fact may have been the result of higher number ranges of their mathematical problems. The use of tallies takes more time. This possibility was confirmed by Learner 1 (AA) when he explained in the post-test interview that he preferred using numbers instead of tallies in his strategies since it was easier and quicker. Learners from the above average ability group were less likely to use limited strategies when compared to learners from the below average ability group.

Although the number ranges of the post-test were not markedly higher than those of the pre-test, learners from all ability groups became less reliant on tallies as a strategy. According to Schoenfeld (2013), learners develop their problem-solving strategies with each problem they solve. They apply familiar knowledge and/or strategies to the problem they had not previously encountered. Each time they approach a problem, they do so with more mathematical knowledge than before. It cannot be assumed that learners used tallies due to

the lower number range. The implementation of writing tasks and the social constructivist approach to this study may have increased development of more advanced strategies when learners were exposed to the strategies of peer learners.

5.4.3 The usefulness of writing in mathematics

Research sub-question 3 describes the usefulness of implementing the writing tasks in the Foundation Phase mathematics classroom, especially in a South African context. Writing tasks used in this study were American based. This section assesses how useful implementation of writing is to support Grade 3 learners' problem-solving abilities.

5.4.3.1 The usefulness of writing in problem-solving

As stated in the CAPS Mathematics curriculum for Foundation Phase, "solving problems in context enables learners to communicate their own thinking orally and in writing through drawings and symbols" (DBE, 2011:9). As explained in chapter 2, the curriculum does not specifically stipulate the use of writing in words when solving mathematical problems but researchers, such as Burns (1995a), promote the use of writing in words. This study sought to determine whether the use of writing, including words, can support learners' mathematical problem-solving strategies. Burns (1995a:13) explains that writing helps learners clarify and define their thinking as well as examine their ideas and reflect on what they have learned in order to deepen and extend their understanding.

According to Morgan (1998:22), writing assists learners in the investigative process, supports reflection and develops problem-solving processes. By engaging with their thought processes, learners deepen their conceptual understanding (Miller, 1991:517). Throughout the data collection period, it was evident the use of writing gave learners opportunities to enhance their mathematical knowledge when they critically engaged with others and developed their thinking. By engaging with others, learners were encouraged to reflect on their strategies and clarify their understanding of mathematical ideas. Writing about problems demonstrated individual learners' understanding, misconceptions and difficulties which may be responded to individually or corporately (Borasi & Rose, 1989:358). Observing the use of writing afforded an opportunity to pinpoint specific misconceptions and address them timeously and appropriately.

There was a marked difference in the learners' written strategies and explanations as well as their verbal explanations of the post-test when compared to the pre-test. The amount of

detail included during the post-test demonstrated that learners could justify their solutions. This demonstration may have been as a result of the writing intervention by which they were able to engage critically in the ability group discussions and learn from the strategies and explanations of other learners. Implementation of the writing tasks had an impact on the development of learners' problem-solving strategies and improved their ability to explain the thinking behind their solution processes. Writing in mathematics can be useful in the South African Foundation Phase classroom.

5.4.3.2 Preferred types of writing tasks

Sub-research question 3 of the study focuses on the usefulness of the writing tasks in the Foundation Phase mathematics classroom, highlighted in section 5.4.3. The theme addressed here discusses preferred types of writing tasks in the participating Grade 3 class which demonstrates the usefulness of only two writing tasks at this level.

At the end of the data collection period, the selected eight Grade 3 learners were interviewed about their strategies when solving mathematical problems. Although the questions focused largely on the strategies and explanations resulting from the post-test, they were asked questions about the types of writing tasks they used as well as the type of writing task they prefer.

Five types of writing tasks were implemented in the selected Grade 3 class. Two of these, namely writing to solve mathematical problems and writing to explain, were more common or popular amongst the sample of eight learners. Three learners mentioned that they preferred writing to solve mathematical problems while two learners chose writing to explain. Each of the remaining three learners chose one of the remaining writing tasks which were writing to record (keeping a journal or log), writing about thinking and learning processes, and shared writing.

In analysing their responses in the second interview in comparison with examples of their writing tasks, learners were more inclined to use two writing tasks: writing to solve mathematical problems and writing to explain. It was noted that, as learners used these writing tasks more frequently, they tended to write longer pieces which included more detail. Learners made more use of mathematical vocabulary in their writing and they showed more evidence of linking the writing task to prior knowledge. Learners were able to construct and develop their knowledge by building on the knowledge they already possessed. As mentioned in chapter 2, learners are required to learn many higher order concepts in

mathematics where it is essential that the learner has already assimilated necessary lower order concepts into his cognitive structure (Skemp, 1989:64). In order to write solutions and explanations, learners needed to build on the concepts and skills they already knew as a means to develop their problem-solving abilities. Writing tasks were a means to support their thinking when they attempted to clarify their strategies.

Learners preferred these writing tasks when the purposes of the writing tasks are taken into account. As learners engaged in writing tasks, similarities appeared between the writing tasks even though the intended purpose of each task was different. In each type of writing task, learners were required to explain their mathematical knowledge at varying levels. In certain writing tasks such as writing to explain and writing to solve mathematical problems, this connection between writing and solving problems was more evident because explaining and clarifying their knowledge was directly linked to the purpose of those particular writing tasks. But, in the other writing tasks, learners were still required to provide an explanation of their conceptual understanding but in different ways.

The purpose of writing to record (keeping a journal or log) is to keep an ongoing record of mathematics lessons (Burns, 1995a:51). The displayed prompts that most learners used gave them opportunities to write about what happened in the lesson as well as what they did or did not understand. They could summarise their mathematical ideas, reflect on their understanding and make observations. By doing so, learners would provide explanations of their mathematical knowledge which serves the same purpose as that of the other writing tasks.

Learners use “writing about thinking and learning processes” to describe mathematics activities in the class (Burns, 1995a:40). When using this writing task, learners provide a description of the activity as well as a reflection on mathematics. They are encouraged to explain their thoughts on the activity they are writing about.

As learners engage in shared writing, they reflect on mathematical concepts through writing a story, for example. This writing task enables learners to review and internalize mathematical concepts and ideas as well as develop mathematical communication (Wilcox & Monroe, 2011:526). Although learners are writing more creatively, their mathematical knowledge is still communicated and explained.

At times, the purpose of each writing task appeared blurred due to the similarities between the different writing tasks. This blurring may have led some learners to write in a different

manner than the writing task required. In Figure 5.15, it is evident that Learner 3 (A) may have confused the purpose of writing about thinking and learning processes with another type of writing task. It became essential to communicate the specific purpose of each writing task repeatedly while encouraging learners to explain their thinking in all of the tasks.

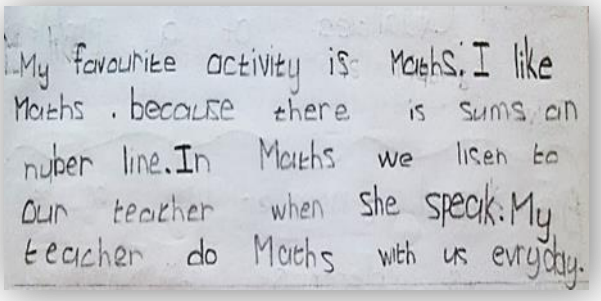


Figure 5.15: Learner 3 (A) Writing-processes

Although each writing task was beneficial in its own right, the findings suggested that implementing these five different writing tasks may not be necessary or appropriate in the Foundation Phase. The three least popular tasks, as indicated by the sample of eight learners in this study, could probably be introduced in the higher grades. Writing to solve mathematical problems and writing to explain may appear to be more relevant to the conceptual understanding at a Grade 3 level because it focuses learners' attention on understanding mathematical ideas. Later in this chapter, recommendations regarding the relevance of specific writing tasks in mathematics as related to the research questions are discussed.

5.4.4 The challenges learners encounter during implementation of writing tasks

In this study, learners encountered various challenges when implementing writing tasks in mathematics lessons. Research sub-question 4 describes these challenges. Various methods were used to help learners overcome such challenges. Learners were given opportunities to work collaboratively on some occasions. Scaffolding was provided in various ways.

5.4.4.1 Comparing individual and collaborative writing

Research sub-question 4 addresses the challenges that learners experienced when they engaged in writing tasks. As a result of these challenges, there were occasions during the writing intervention when learners could work collaboratively. When they did so, they

generally worked in pairs. During the pre-test and post-test, learners solved the mathematical problems individually to determine whether the writing tasks affected their ability to solve mathematical problems. While the writing tasks were implemented in the class during the intervention, learners had opportunities to write individually and collaboratively. On certain occasions learners were encouraged to work with a partner, especially when they were experiencing difficulty reading the problem or the context of the problem was too challenging. All the learners solved problem 1 collaboratively. Since this was the first time they were encouraged to implement "writing to solve mathematical problems", it was hoped that learners would be able to assist each other when they solved the problem and wrote their explanations. Despite solving the problem in pairs, learners required further prompting to guide them in their strategies and writing.

As the data collection progressed, learners were given the option to work in pairs or individually when they solved certain mathematical problems. Some learners preferred to work individually while others seemed more dependent on the support of a partner. Most learners who requested to work collaboratively were from the average and below average mathematical ability groups. These learners often needed varying degrees of scaffolding and support during mathematics lessons.

After learners solved the context-free, routine problem (problem 9 in Appendix G), they were given time to revisit the problem and explain their thinking. A few learners used this time to discuss with their peers what they had done. This kind of discussion among learners occurred more regularly during problem-solving activities as the data collection period progressed. Some learners seemed to be more at ease when sharing their ideas and strategies with peers. Others, however, wanted to know whether their answers were correct before sharing their strategies.

Learners were given opportunities to write collaboratively for the other writing tasks. One of these tasks included writing about thinking and learning processes; where learners wrote a letter to the principal about mathematics lessons. Another collaborative writing task was implementing the use of shared writing. Here, learners wrote a story imagining that they were one centimetre tall to elaborate on their understanding of measurement. In both these instances, learners were engaged in discourse of the mathematical knowledge required to be able to write in a collaborative manner. This provided a meaningful opportunity for learners to participate in discussion where scaffolding of conceptual knowledge was necessary to complete the writing task. Although learners were encouraged to write collaboratively on

these occasions, a few learners chose to write individually. Learners who chose to do so were mostly from the above average ability group.

In reflecting on the learners' development of the use of writing in mathematics throughout the data collection period, it was clear that the more learners were encouraged to incorporate writing into their tasks, the more at ease they were. Some learners, however, required more support through collaborative writing to reach this stage. Based on the findings of this study, it is observed that writing in this way could improve learners' individual writing abilities. This was evident in the post-test where learners solved and explained their problems individually. The majority of the learners wrote longer texts and included more detail than in the pre-test. This improvement was particularly noticeable with learners from the below average ability group and learners who experienced language difficulties. Opportunities to engage in collaborative writing may assist learners when implementing writing tasks and improve learners' ability to write in mathematics individually.

5.4.4.2 Scaffolding using writing tasks

Throughout the data collection period, it was clear that learners encountered challenges when implementing the writing tasks in mathematics lessons. Scaffolding was necessary to address these challenges so that learners were able to use writing tasks in mathematics in Grade 3. In chapter 2, scaffolding was described as the learning activities the teacher or more knowledgeable other (MKO) uses to develop knowledge (Siyepu, 2013:5). In this study, scaffolding occurred in the following ways. Scaffolding was provided through the implementation of writing tasks as a means of breaking up a problem into manageable parts. Peer interaction and collaboration were other forms of scaffolding. Learners were prompted during observation when they solved mathematical problems. Written and verbal feedback were also given to enhance the learners' ability to solve problems.

The writing tasks themselves were used as a form of scaffolding which helped to support and develop learners' ability to solve mathematical problems (Daniels 2001:108). Writing tasks created opportunities for learners to construct and apply mathematical knowledge. The mathematical problems learners solved during this study were linked to the mathematical knowledge they were expected to develop as part of the CAPS Mathematics curriculum prescribed for Grade 3. The problems used learners' number learning within the sphere of addition, subtraction, multiplication and division. By solving these problems, learners made connections with these mathematical concepts. For example, the third problem learners solved as part of the writing intervention required learners to use and apply their knowledge

of counting in fours and/or the related multiplication table (Figure 5.16). Learners' development of mathematical knowledge was scaffolded through this problem. This was the case with other mathematical problems as well as other writing tasks in this study. Burns's (1995a) methodology of using writing in mathematics was introduced and implemented as a tool to scaffold learners' understanding and support learners when they solve mathematical problems.

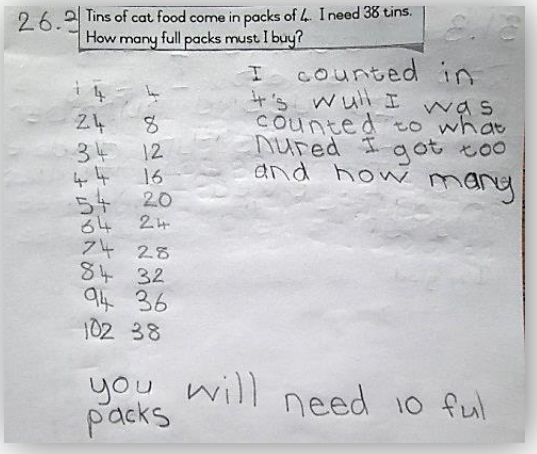


Figure 5.16 Learner 5 (A) Writing – Problem 3

When learners solved the mathematical problems, they were sometimes provided with more manageable steps to find a solution. The problem was broken up into parts so that learners solved one part first before moving on to the next part. Doing so simplified the learners' role in order to solve the problem (Daniels, 2001:107). Below is an excerpt from the field notes taken during research which describes the scaffolding given when learners solved the first problem during the writing intervention. The problem for the average ability group reads as follows:

> *32 birds land on the bird table. There are now 71 birds there. How many birds were already on the table?*
>
> *While moving around the group I realised that the learners did not understand that there were 71 birds in total and they had to find the initial amount. I needed to break down the problem by drawing it on the board and guiding them with prompts and questions to understand that the solution could not be more than 71.*
> *FN 18/3/2015*

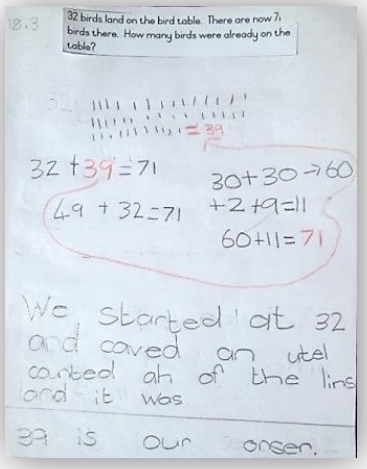


Figure 5.17 Learner 4 (A) Writing-Problem 1

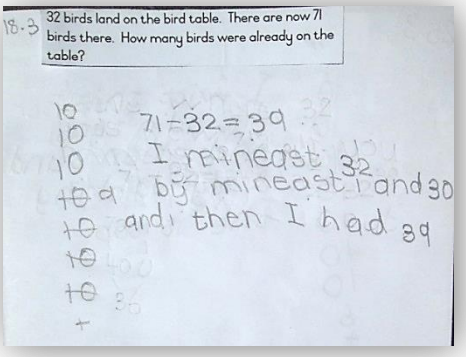


Figure 5.18 Learner 5 (A) Writing-Problem 1

The examples above (Figure 5.17 and Figure 5.18) show how learners from the average ability group were able to solve the problem after scaffolding had occurred. These learners erased the work they had done before scaffolding so no comparison is possible between their strategies before and after discussion. Seeing learners' attempts at various strategies would have helped me to better understand the decisions they made to try alternate strategies. These strategies show that Learner 4 and Learner 5 understood the context of the problem and were able to solve it according to the addition/subtraction problem type. Learner 4 used addition and explained that counting was used to arrive at the solution. Learner 5 used his knowledge of place value to decompose 71 into tens and ones. He explained how he subtracted 32 when he crossed out tens and ones. He displayed knowledge of subtracting through the decade when he crossed out ten and changed it to 9.

The group of average ability learners needed a visual representation of the problem coupled with verbal prompts to understand the context of the problem. Polya (1957:110) explains that relevant elements from formerly acquired mathematical knowledge should be used to solve the present problem. Learners solved similar addition/subtraction problem types in previous mathematics lessons that required them to find the missing addend. By using a visual presentation, it was possible to relate previous problems and mathematical ideas.

Scaffolding occurred in this study through prompts given to learners while they solved mathematical problems (Sperry Smith, 2013:10). One occasion where prompts were required was when learners solved the fifth problem of the writing intervention. As shown in Figure 5.3, Learner 7 (BA) made tallies to represent the number of tricycle wheels in the

problem. But, he did not go beyond drawing these tallies in order to solve the problem. Written feedback was provided in order for him to continue working on this problem the following day. He was uncertain of the feedback given so further scaffolding was necessary. While he was explaining his strategy, his tallies were circled into groups of three by the researcher as a means of scaffolding his understanding of the context of the problem. He was asked what this circling represented. Learner 7 (BA) was able to use the circled tallies to clarify that one group represented one tricycle with three wheels.

When learners engaged in the writing tasks in the lessons, different forms of scaffolding took place to develop learners' understanding of the writing tasks as well as their mathematical knowledge. Scaffolding and prompts were not provided during the pre-test and post-test because this could have negated their purpose which was to determine whether the writing tasks supported learners' mathematical problem-solving abilities.

Based on these findings, it is evident the use of writing in mathematics could be a means of providing scaffolding to overcome some of the difficulties learners encounter when implementing writing tasks. Writing tasks, verbal prompts and the teacher/researcher's written feedback may scaffold learners' conceptual understanding when they engage with mathematical problems.

5.4.4.3 The role of language in problem-solving

One of the challenges observed was the role language plays when learners engage in mathematical problem-solving. As mentioned in chapter 4, some learners either misread or misinterpreted mathematical problems which affected their ability to solve them and explain what they had done. When learners solved the second problem (see Appendix G), it became clear that more time needed to be spent on reading and interpreting the problem in order to select an appropriate strategy and explain it. If learners struggled with the contextual aspect of the problem, they would probably be unable to solve the problem in a meaningful way. As a result, teachers should carefully consider the wording of problems when they are presented to learners. Some learners may become confused by the language in the problem: the context or question in the problem is not always presented in an understandable way.

In other instances it was apparent that, where learners had below average reading and comprehension abilities in language, they experienced difficulty reading and interpreting the mathematical problems coherently. This difficulty was particularly evident in the case of

Learner 7 and Learner 8 who were selected from the below average mathematical ability group. As shown in Figure 5.19, Learner 8 (BA) had difficulty solving the problem about the height of the wall (problem 2), even after the strategies were discussed in the ability group.

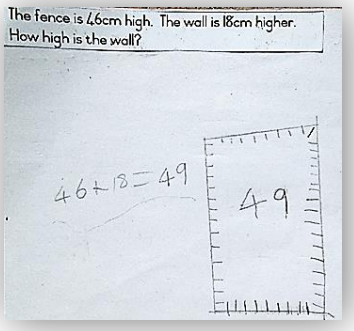


Figure 5.19 Learner 8 (BA) Writing – Problem 2

If learners lack a firm understanding of mathematical vocabulary needed to clarify their mathematical knowledge, their explanations may be limited. Learners may become confused by everyday language that has a more specific mathematical meaning when used in mathematical problems (Luneta, 2013:94). This confusion was evident throughout the data collection period. As mentioned in the literature review, time must be spent on teaching mathematical vocabulary which links different concepts. This linkage enhances learners' conceptual understanding by eliminating errors caused by misunderstanding of mathematics vocabulary (Koshy et al., 2000:177). An increased effort in the development of language and writing across the curriculum could benefit learners and enhance their ability to talk, read and write about what they have done to solve mathematical problems (Clemson & Clemson, 1994:84). Developing learners' language may enable them to express their thinking and justify their solutions in other subject areas, not only mathematics.

5.4.4.4 Strategies according to problem types

Learners encountered challenges concerning which type of strategy to use for which kind of problem. The literature review dealt with problem types and strategies. Distinctions were made between addition/subtraction problem types and multiplication/division problem types.

During the pre-test, the selected learners from the below average ability group did not always use strategies that matched the problem types. For instance, Figure 5.20 shows how Learner 7 (BA) used an addition strategy to solve a multiplication/division problem (problem 2

of the pre-test). When solving problem 5 of the pre-test, the same learner had no visible strategy (Figure 5.21).

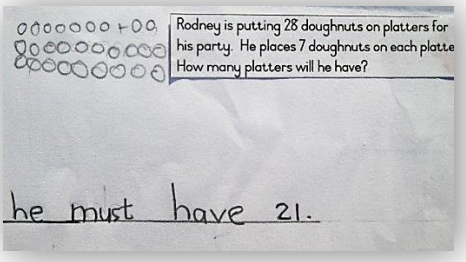


Figure 5.20 Learner 7 (BA) Pre Q2

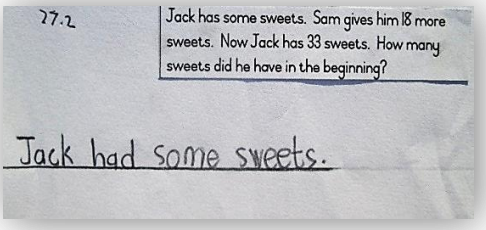


Figure 5.21 Learner 7 (BA) Pre Q5

During the post-test, learners from the above average and average ability groups generally used strategies that related to the problem type. They appropriately chose strategies that reflected the basic operation present in the mathematical problem. Figures 5.22 and 5.21 show two strategies used by learners from the below average group during the post-test. Learner 7 (BA) used multiplication/division appropriately in Figure 5.22 but his solution was incorrect: he stated that 39 cups were needed rather than 8 coffee pots. Although Learner 6 (BA) used tallies in her strategy (Figure 5.23), she did so in groups to show the trays mentioned in the context of the mathematical problem. These strategies reflected the appropriate problem types at Grade 3 level in accordance with the CAPS Mathematics curriculum (DBE, 2011:79). There was more evidence of appropriate strategies occurring in the post-test than in the pre-test.

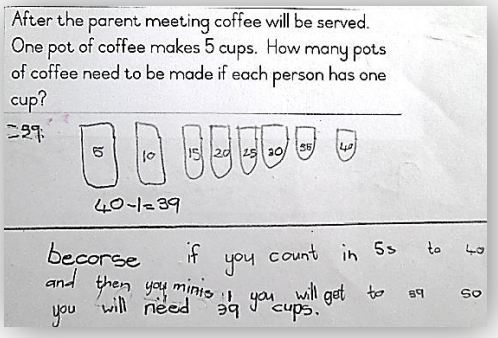


Figure 5.22 Learner 7 (BA) Post Q4

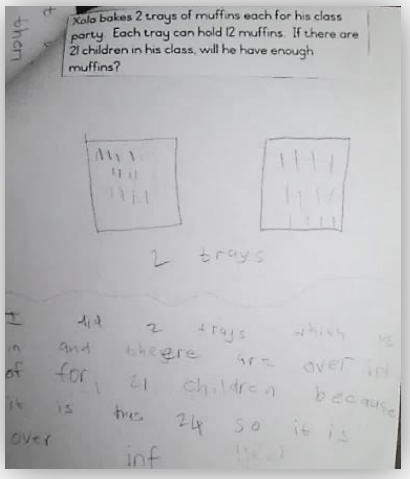


Figure 5.23 Learner 6 (BA) Post Q5

Some learners may have encountered language difficulties that led to inappropriate strategies being used for the different problem types. Another challenge learners may have experienced could have been a lack of deepened conceptual understanding. Learners may have been further challenged if they could not identify the mathematical content embedded in the problems in order to use the appropriate strategies for the problem types. The findings of the post-test suggest that implementation of the writing tasks had an impact on the use of appropriate strategies according to the problem types the learners encountered when compared to the pre-test.

The findings demonstrate the support that writing gives to Grade 3 learners in solving mathematical problems. Consequently, the implementation of writing tasks seems useful in the Foundation Phase mathematics classroom because it could enhance learners' conceptual understanding and problem-solving strategies. Although learners encountered difficulty when implementing the writing tasks, scaffolding and collaborative writing opportunities enabled them to use writing in mathematics successfully as was seen in the results of the post-test.

5.5 SIGNIFICANCE OF THE STUDY

This study is significant for the mathematics classroom, especially in the area of problem-solving. Learners used writing tasks to support their mathematical problem-solving strategies and explain their solutions. Learners actively engaged in the construction of mathematical knowledge and developed conceptual understanding. By working collaboratively, learners were exposed to the problem-solving strategies of others in the ability groups. Exposure to other learners' strategies may have allowed them to reflect on suitable problem-solving strategies and encouraged learners to think critically when they solved and explained problems.

This study is significant for implementation of the current curriculum in South African schools. Foundation Phase learners are expected to communicate their thinking using writing (DBE, 2011:9). This study reveals the need for teachers, pre-service and in-service, to be trained in developing writing skills and implementing such skills. Training enables teachers to model good writing practices by explaining and justifying the solutions for the mathematical problems they encounter.

5.6 LIMITATIONS OF THE STUDY

As mentioned in Chapter one, there were limitations to the study. The researcher for this study was also the teacher of the sample group. As such there is a potential bias to the research process. This bias could have affected the selection of the sample of eight learners and the data analysis process. The validity of the data was ensured, however, by using multiple data collection instruments and audio-recordings of ability group discussions and interviews. The validity of the data helped to secure an objective thesis report.

The sample of the study was small. Eight learners were purposively selected from one Grade 3 class. The small sample limited the study resulting in the inability to generalise the findings to a broader population.

The mathematical problems used in the study were differentiated according to the expected number ranges of the three mathematical ability groups. The contexts, however, were identical for the problems across the ability groups. For some of the mathematical problems, it appeared that the number range was too low and did not present enough of a challenge for learners. This was evident in all the mathematical ability groups. At other times, some of the learners from the above average ability group were not sufficiently challenged by the problem. It appeared that either the context of the problem was too simple or the number range was not suitable. Learners were not adequately encouraged to develop strategies that encouraged a higher cognitive demand: the solution and/or strategy may have been obvious. On other occasions, the context of the problem proved too perplexing for the below average learners. In addition, most learners found aspects of reading and language difficult. The context of the problems may have caused learners to have difficulty identifying the mathematical content within them.

The normal school programme had an impact on data collection envisaged prior to the pre-test. Data could not be collected three times per week as planned. As a result, data collection was shortened to accommodate the assessment programme of the school.

5.7 RECOMMENDATIONS

The purpose of this study was to determine how the use of writing tasks supports learners' mathematical problem-solving strategies. The various writing tasks of Burns (1995a) and Wilcox and Monroe (2011) were used as a writing intervention. The writing tasks were modelled to the learners and implemented in the selected Grade 3 class.

As described earlier in this chapter, there was a distinct difference in the strategies and explanations learners used in the pre-test and the post-test of this study. Learners used "writing to solve mathematical problems" in the post-test without being instructed to do so. Their detailed use of writing allowed them to explain their strategies better during the second interview. This improvement suggests that the use of writing tasks increases learners' ability to describe the thinking behind their solution processes when they engaged in mathematical problem-solving in this study. The use of writing provided the environment for learners to engage with the teacher and peers more openly and critically. They were actively encouraged to reflect on their thinking in order to explain it to others. In addressing two of the research sub-questions of this study, learners improved in their ability to solve and explain mathematical problems which demonstrated the development of their conceptual knowledge.

Writing in mathematics is an essential part of the curriculum in Foundation Phase in South Africa. This study showed the benefits of the use of writing when learners engage in mathematical problem-solving. Although this study used writing tasks initially implemented in the United States, this study proved the usefulness of such tasks in the South African Foundation Phase classroom. Further research is necessary which deals with the use of writing beyond the scope of mathematical problem-solving. Based on the results of this study, it would be fair to assume that other areas of knowledge and skills could benefit from the implementation of writing in the mathematics classroom. Further research needs to be done in the higher grades when learners engage with increasingly complex mathematical concepts.

Previous international research has been conducted where writing explanations in mathematics were part of the content courses for preservice teachers (McCormick, 2010). Research showed it was beneficial to improving conceptual understanding in mathematical problem-solving and developing writing practices. Teachers should model good writing practices by explaining and justifying the solutions for the mathematical problems they encounter. A study conducted by Craig (2011) researched the use of writing as a tool in a first-year university course in South Africa. This study did not rely on pre-service teachers as its sample, unlike the international studies referred to earlier. Similar research should be conducted in education faculties of universities in the South African context so that pre-service teachers are given the knowledge and tools to implement writing in mathematics in their classrooms in future. In this way, mathematics teachers can be equipped to model and implement writing to support learners' mathematical problem-solving abilities. They would be

prepared to deal with any challenges learners may encounter while implementing the writing tasks.

5.8 CONCLUSION

The purpose of this research study was to determine how various types of writing tasks support Grade 3 learners' mathematical problem-solving ability. The writing tasks included writing to solve mathematical problems, writing to record (keeping a journal or log), writing to explain, writing about thinking and learning processes (Burns, 1995a) and shared writing (Wilcox & Monroe, 2011). A sample of eight learners was selected and interviewed regarding their strategies in the pre-test and the post-test. Learners' written pieces produced during the writing intervention, field notes and audio-recordings of ability group discussions formed part of the analysis for this study.

The CAPS Mathematics curriculum for Foundation Phase states that learners should be writing in the mathematics class. This study revealed that writing in mathematics is beneficial to the area of problem-solving within mathematics in accordance with the prescribed curriculum. The writing tasks supported learners in their problem-solving strategies: learners were using more advanced strategies by the end of the data collection period. Selected learners were able to provide better verbal and written explanations of their solutions.

This study showed that two of the writing tasks, namely writing to solve mathematical problems and writing to explain, were valuable tasks that developed the learners' ability to explain their thinking. These two writing tasks should be considered as primary tasks in the mathematics curriculum while the other writing tasks may be secondary. The secondary writing tasks include writing to record (keeping a journal or log), writing about thinking and learning processes and shared writing. These writing tasks did not prove as useful to the sample of learners in this study.

This study concludes that learners who engage in writing in mathematics may be able to reflect critically on their thinking when they construct mathematical knowledge and skills that are essential in the problem-solving process. Teachers, both in-service and preservice, may be encouraged by this study to incorporate writing into their daily mathematics lessons. This incorporation of writing supports learners when they apply mathematical knowledge to problem-solving.

5.9 REFLECTIONS ON THE STUDY

There are a number of elements that have enabled me to develop as a teacher and as a researcher. The opportunity to engage in research of this nature helped me to reflect on and improve my daily teaching practice. Although problem-solving was regularly planned as part of my mathematics lessons, this study made me more attentive to the way I used problem-solving in my daily lessons. Including the writing tasks added a different dimension to my mathematics lessons where I could readily gauge the improvement in the learners' abilities to solve problems.

The learners themselves became increasingly enthusiastic as they continued engaging with the writing tasks during the data collection period. They were more eager to solve mathematical problems than before the writing tasks were introduced. After the data collection was completed, I noticed learners continued using writing to explain their thinking even when I had not prompted them to do so. When I asked them their reason for using writing in mathematics, many of them explained that it helped them to make sense of what they were doing.

Added to this, I found that I became more discerning in my use of scaffolding. This study enabled me to recognise when scaffolding was genuinely needed and when I needed to allow learners to discover the mathematical content on their own. In a sense, I felt more at ease in allowing learners the space to grapple with the context of a mathematical problem that would sometimes take more than one lesson. In other words, I could allow learners to delve deeper into their strategies, taking time to engage in critical thinking and explain their solutions.

During the data collection period, I became increasingly aware of the use of erasers when solving problems. I felt some data may have been lost or incomplete due to learners erasing incorrect strategies. Seeing learners' attempts at various strategies may have helped me to better understand their thinking behind their solutions. These attempts may have given me a better comprehension of their later attempts and the decisions they made to try alternate strategies.

This research study did not follow its original plan. As a teacher-researcher, I was faced with a few challenges in the implementation of the writing tasks and managing the data collection plan. I had originally planned to collect data continuously over a ten week period, excluding two weeks to conduct the pre-test and post-test that stretched over two school terms. My

data collection plan catered for three opportunities per week where learners were either engaged in writing activities or I was modelling the writing tasks to them. However, the daily school programme did not always afford the time for this to occur as planned. At times, the structure and content of certain mathematics lessons required more time to be devoted to content areas that needed attention which meant there was not enough time to comprehensively engage in problem-solving and writing tasks. As a result, I found that some weeks I was able to collect more data than others. Therefore, during certain weeks I was able to collect data almost every day whereas I could only collect data once or twice during other weeks. Added to this, learners' assessments also needed to be completed for the quarterly report cards which meant that I was unable to collect data for a period of two weeks. This delay occurred during the earlier part of the data collection period. I had just introduced and implemented the first writing task, namely writing to solve mathematical problems, and I was concerned that momentum would be lost. This was not the case and the learners were able to continue implementing the writing task and developing their mathematical problem-solving abilities.

Being a teacher-researcher was challenging as I mentioned in the limitations of my study in Chapter one. I had to be continually aware of the tension between the two roles, knowing which role was required more actively at any given time. It was particularly challenging as data were being collected during most mathematics lessons when learners engaged in writing tasks, problem-solving and ability group discussions. I needed to be mindful of when scaffolding was appropriate in my role as a teacher and when I needed to step back in my role as a researcher. As the study progressed, I became more comfortable in my role as researcher and felt more at ease in striking the balance between the two roles during mathematics lessons.

Moreover, I was conscious of the potential bias that could occur as I conducted the study in my own class. As I selected the eight learners as the sample for this study, I had to largely disregard their literacy abilities and focus more on their mathematical abilities. This process was made a little easier in that learners had already been placed in different mathematical ability groups which were separate from their literacy ability groups. This allowed me to ensure that learners selected based on their solutions in the pre-test reflected the three mathematical ability groups.

In future, I would spend more time perfecting the data collection plan as far as possible. I have learned that, as a researcher, I need to be more prepared for, and anticipate, potential pitfalls that may occur.

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APPENDIX A WCED ETHICS CLEARANCE LETTER

Western Cape Government
Education

Directorate: Research

Audrey.wyngaard@westerncape.gov.za
tel: +27 021 467 9272
Fax: 0865902282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20140821-35018
ENQUIRIES: Dr A T Wyngaard

Mrs Belinda Petersen
10 Cinnamon Street
Bardale Village
Kuils River
7580

Dear Mrs Belinda Petersen

RESEARCH PROPOSAL: WRITING AND MATHEMATICAL PROBLEM-SOLVING IN THE FOUNDATION PHASE

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **20 January 2015 till 30 June 2015**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

    The Director: Research Services
    Western Cape Education Department
    Private Bag X9114
    CAPE TOWN
    8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 21 August 2014

APPENDIX B CPUT ETHICS CLEARANCE LETTER

Cape Peninsula University of Technology

APPENDIX II: FACULTY OF EDUCATION ETHICS FOR ORIGINAL RESEARCH

This form is to be completed by the student, member of staff and other researchers intending to undertake research in the Faculty. It is to be completed for any piece of research the aim of which is to make an original contribution to the public body of knowledge.

For students this type of work will also have educational goals and will be linked to gaining credit - it is the type of work that will be the basis for a Masters/Doctoral thesis or any research project for which ethical clearance is deemed necessary:

Name(s) of applicant	Belinda Petersen
Project Title	Writing and mathematical problem-solving in the Foundation Phase
Is this a staff research project?	No
Degree	MEd
Supervisor(s)	Dr Sharon McAuliffe and Prof Cornelis Vermeulen
Funding sources	N/A

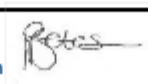
Attached: Information sheet ☐ Consent form X☐ Questionnaire ☐ Other (Specify)

The purpose of this study is to explore writing and how it supports learners as they solve mathematical problems. This study was prompted by the standard of mathematics results in South Africa as determined by its participation in international studies. It discusses the different types of writing in mathematics that could enhance creative and critical thinking as well as encourage reflective thought, thereby deepening conceptual understanding in order to support mathematical problem-solving skills. The theories of Vygotsky, Bruner and Sfard underpin this study.
Permission will be sought from the Western Cape Education Department (WCED), Cape Peninsula University of technology (CPUT), the principal of the school as well as parents of the learners in one Grade 3 class. The participants in the study will be purposively selected. They will be under no obligation to participate. Prior to data collection, the purpose of the research study will be discussed with the learners and their parents. They will be informed of their involvement in the lessons and interviews and their option to withdraw from the study at any stage. The data collection instruments are attached. The researcher's positionality as the teacher will be acknowledged to all parties concerned. This study poses no risk of harm to participants, the participants' community, the researcher, the research community and the University. Parents of the participating learners will read and sign an informed consent form on behalf of the learners granting permission for the participation. Pseudonyms will be used for the school and the learners involved in order to maintain anonymity and confidentiality.

Om completion of this study, the findings will be shared with colleagues in staff development sessions. It may also add to discussions amongst teachers and schools as a tool to improve the standard of mathematical problem-solving. To the best of my knowledge, this study may not raise other ethical issues. As a professional teacher for 11 years, I should be able to deal with any ethical issues if they arise. If there are any concerns, I will discuss the matter with my principal and supervisors.

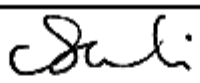
Research Checklist:		Yes	No
1:	Does the study involve participants who are unable to give informed consent? Examples include children, people with learning disabilities, or your own students. Animals?	X	
2:	Will the study require the co-operation of a gatekeeper for initial access to the groups or individuals to be recruited? Examples include students at school, members of self-help groups, residents of nursing homes — anyone who is under the legal care of another.	X	
3:	Will it be necessary for participants to participate in the study without their knowledge and consent at the time — for example, covert observation of people in non-public places?		X
4:	Will the study with the research subject involve discussion of sensitive topics? Examples would include questions on sexual activity or drug use.		X
5:	Will the study involve invasive, intrusive, or potentially harmful procedures of any kind (e.g. drugs, placebos or other substances to be administered to the study participants)?		X
6:	Will the study involve prolonged or repetitive testing on sentient subjects?		X
7:	Will financial inducements (other than reasonable expenses and compensation for time) be offered to participants?		X
8:	Does your research involve environmental studies which could be contentious or use materials or processes that could damage the environment? Particularly the outcome of your research?		X

Signatures:

Researcher/Applicant:	Belinda Petersen 	 Supervisor/Senior investigator (if applicable):
Date:	24 June 2014	Date: 25 June 2014

Please note that in signing this form, supervisors are indicating that they are satisfied that the ethical issues raised by this work have been adequately identified and that the proposal includes appropriate plans for their effective management.

Education Faculty Ethics Committee comments:

EFEC unconditionally grants ethical clearance for the study titled, "**Writing and mathematical problem-solving in the Foundation Phase**". The certificate is valid for 3 years from the date of issue.				
Approved			 **Chairperson: Cina P Mosito, PhD**	**Date:5/8/2014**
Approval Certificate/Reference: EFEC 22-8/2014				

APPENDIX C CONSENT LETTER: PRINCIPAL

10 Cinnamon Street
Bardale Village
Kuils River
7580
28 November 2014
Email: be2buzzy@gmail.com

Dear _____

Request for permission to conduct a research study at your school

I am currently conducting research towards a Masters in Education degree at Cape Peninsula University of Technology (CPUT) in the Faculty of Education and Social Sciences. The title of my research study is: "Writing and mathematical problem-solving in Grade 3".

Permission has been granted by the Western Cape Education Department and CPUT to conduct the study from January to June 2015. The study will include lessons, interviews with learners and evidence of their work. Since these lessons will be used for my research project, I am hoping that you will allow me to audio record them and transcribe these recordings for analysis. In my research study, I will use pseudonyms to ensure confidentiality and anonymity of the participants and the school. The audio recordings are solely for the research purpose of this study.

I request your permission to conduct this research study at your school. If you grant permission, please sign this letter and return it confirming your school's participation in this research study. Once completed, the research study will be available for you to view.

Please feel free to contact me if you need any additional information regarding this research.

Yours sincerely

Belinda Petersen

PERMISSION TO CONDUCT A RESEARCH STUDY

I _____, principal of _____, hereby grant permission to Belinda Petersen to conduct the research study as outlined at this school.

Signature: _____

Date: _____

APPENDIX D CONSENT LETTER: LEARNERS

10 Cinnamon Street
Bardale Village
Kuils River
4 February 2015
Email: be2buzzy@gmail.com

Dear Mr/Mrs .....

Request for permission for your child to participate in a research study

I am currently conducting research for a Masters in Education degree at Cape Peninsula University of Technology (CPUT) in the Faculty of Education and Social Sciences. The title of my research study is: "Writing and mathematical problem-solving in Grade 3".

Permission has been granted by the principal, the Western Cape Education Department (WCED) and CPUT. The research study will be conducted from February to June 2015. The study will include lessons, interviews with learners and evidence of their work. All learners in the class will receive instruction related to the research study as I believe they will all benefit, as shown in similar research studies conducted internationally. However, participation in this study, by using your child's work and comments during lessons and interviews as data in the thesis report, is voluntary. Please be assured that there will be no consequences for giving or withholding permission. You may withdraw consent at any time by contacting me via email or at school (tel: 021-_____). If permission is not given or withdrawn, no work samples, interview comments or researcher reflections regarding your child will be included in the thesis report of this research study.

Since these lessons will be used for my research project, I am hoping that you will allow me to audio record them and transcribe these recordings for analysis. In my research study, I will use pseudonyms to ensure confidentiality and anonymity of your child and the school as an ethical requirement of the WCED and CPUT. The audio recordings are solely for the research purpose of this study.

I request your permission for your child to participate in this study from January to June 2015. If you grant permission, please sign this letter and return it confirming your child's participation in this research study. Once completed, the research study will be available for you to view.

Please feel free to contact me if you need any additional information regarding this research.

Yours sincerely

Belinda Petersen

PERMISSION TO PARTICIPATE IN A RESEARCH STUDY

I _____, parent/guardian of _____, hereby grant permission for his/her participation in the research study towards Belinda Petersen's Masters in Education thesis at Cape Peninsula University of Technology.

Signature: _____ (Parent/Guardian)

Date: _____

APPENDIX E PRE-TEST MATHEMATICAL PROBLEMS

The problems (Fine, Fine & Schimper, 2000; Koll & Mills, 2000; Mathematics Education Primary Programme, 2012; DBE, 2014) used during the pre test are listed below. The problems were differentiated to cater for the mathematical ability groups present in the selected Grade 3 class. The problems below are for the above average ability group (1), the average ability group (2) and the below average ability group (3).

Problem 1

4. A cricket team needs 94 runs to win their match. They already have 47 runs. How many runs do they still need?
5. A cricket team needs 74 runs to win their match. They already have 49 runs. How many runs do they still need?
6. A cricket team needs 34 runs to win their match. They already have 19 runs. How many runs do they still need?

Problem 2

4. Rodney is putting 56 doughnuts on platters for his party. He places 7 doughnuts on each platter. How many platters will he have?
5. Rodney is putting 42 doughnuts on platters for his party. He places 7 doughnuts on each platter. How many platters will he have?
6. Rodney is putting 28 doughnuts on platters for his party. He places 7 doughnuts on each platter. How many platters will he have?

Problem 3

4. The school sports team has 68 runners, 16 long jumpers and 10 high jumpers. How many athletes are on the sports team?
5. The school sports team has 48 runners, 16 long jumpers and 10 high jumpers. How many athletes are on the sports team?
6. The school sports team has 28 runners, 16 long jumpers and 10 high jumpers. How many athletes are on the sports team?

Problem 4

4. There are 17 pins in a box. How many pins will there be in 6 boxes?
5. There are 17 pins in a box. How many pins will there be in 4 boxes?
6. There are 17 pins in a box. How many pins will there be in 2 boxes?

Problem 5

4. Jack has some sweets. Sam gives him 28 more sweets. Now Jack has 73 sweets. How many sweets did he have in the beginning?
5. Jack has some sweets. Sam gives him 18 more sweets. Now Jack has 43 sweets. How many sweets did he have in the beginning?
6. Jack has some sweets. Sam gives him 18 more sweets. Now Jack has 33 sweets. How many sweets did he have in the beginning?

APPENDIX F: POST-TEST MATHEMATICAL PROBLEMS

The problems (Fine, Fine & Schimper, 2000; Koll & Mills, 2000; Mathematics Education Primary Programme, 2012; DBE, 2014) used during the post test are listed below. As with the problems given for the pre test, these problems were differentiated to cater for the mathematical ability groups present in the selected Grade 3 class. Again, the mathematical problems are listed for the above average ability group (1), the average ability group (2) and the below average ability group (3).

Problem 1

4. Anwar has planted 19 seedlings in the vegetable garden. James has planted 16 seedlings. Thandi has planted twice as many as James. How many seedlings have they planted in the vegetable garden?

5. Anwar has planted 15 seedlings in the vegetable garden. James has planted 12 seedlings. Thandi has planted twice as many as James. How many seedlings have they planted in the vegetable garden?

6. Anwar has planted 13 seedlings in the vegetable garden. James has planted 9 seedlings. Thandi has planted twice as many as James. How many seedlings have they planted in the vegetable garden?

Problem 2

4. There will be a parent meeting at school tomorrow evening. 81 parents will be coming. The big tables will be used with six chairs around each. How many tables will need to be set up?

5. There will be a parent meeting at school tomorrow evening. 65 parents will be coming. The big tables will be used with six chairs around each. How many tables will need to be set up?

6. There will be a parent meeting at school tomorrow evening. 39 parents will be coming. The big tables will be used with six chairs around each. How many tables will need to be set up?

Problem 3

4. Mark and Martha packed out 81 chairs. Mark packed out 48 chairs. How many did Martha pack out?

5. Mark and Martha packed out 65 chairs. Mark packed out 38 chairs. How many did Martha pack out?

6. Mark and Martha packed out 39 chairs. Mark packed out 24 chairs. How many did Martha pack out?

Problem 4

4. After the parent meeting coffee will be served. One pot of coffee makes 7 cups. How many pots of coffee need to be made if each person has one cup?

5. After the parent meeting coffee will be served. One pot of coffee makes 7 cups. How many pots of coffee need to be made if each person has one cup?

6. After the parent meeting coffee will be served. One pot of coffee makes 5 cups. How many pots of coffee need to be made if each person has one cup?

Problem 5

4. Xola bakes 4 trays of muffins each for his class party. Each tray can hold 12 muffins. If there are 39 children in his class, will he have enough muffins?

5. Xola bakes 3 trays of muffins each for his class party. Each tray can hold 12 muffins. If there are 31 children in his class, will he have enough muffins?

6. Xola bakes 2 trays of muffins each for his class party. Each tray can hold 12 muffins. If there are 21 children in his class, will he have enough muffins?

APPENDIX G: MATHEMATICAL PROBLEMS

Learners were introduced to various writing tasks during the data collection period. One of the writing tasks they were encouraged to use was writing to solve mathematical problems. As with the pre-test and post-test problems, learners in the selected Grade 3 class were given differentiated mathematical problems according to the mathematical ability groups they belonged to. Below is the list of differentiated mathematical problems (Fine, Fine & Schimper, 2000; Koll & Mills, 2000; Mathematics Education Primary Programme, 2012; DBE, 2014) used for these writing tasks.

Problem 1

1. 32 birds land on the bird table. There are now 91 birds there. How many birds were already on the table?
2. 32 birds land on the bird table. There are now 71 birds there. How many birds were already on the table?
3. 12 birds land on the bird table. There are now 51 birds there. How many birds were already on the table?

Problem 2

1. The fence is 76cm high. The wall is 48cm higher. How high is the wall?
2. The fence is 76cm high. The wall is 18cm higher. How high is the wall?
3. The fence is 46cm high. The wall is 18cm higher. How high is the wall?

Problem 3

1. Tins of cat food come in packs of 4. I need 50 tins. How many full packs must I buy?
2. Tins of cat food come in packs of 4. I need 38 tins. How many full packs must I buy?
3. Tins of cat food come in packs of 4. I need 26 tins. How many full packs must I buy?

Problem 4

1. A medicine bottle holds 95 ml. A teaspoon holds 5 ml. How many teaspoons of medicine in the bottle?

2. A medicine bottle holds 75 ml. A teaspoon holds 5 ml. How many teaspoons of medicine in the bottle?

3. A medicine bottle holds 45 ml. A teaspoon holds 5 ml. How many teaspoons of medicine in the bottle?

Problem 5

1. The tricycle factory has 101 wheels available. How many tricycles can they assemble with the wheels?

2. The tricycle factory has 89 wheels available. How many tricycles can they assemble with the wheels?

3. The tricycle factory has 65 wheels available. How many tricycles can they assemble with the wheels?

Problem 6

1. In the sandpit Ellie jumps 128cm and Ben jumps 95cm. How much further does Ellie jump?

2. In the sandpit Ellie jumps 108cm and Ben jumps 95cm. How much further does Ellie jump?

3. In the sandpit Ellie jumps 98cm and Ben jumps 75cm. How much further does Ellie jump?

Problem 7

1. A car park has 4 floors. 26 cars can fit on each floor. How many cars can fit in the car park? If there are 6 empty spaces on each floor, how many cars are in the car park?

2. A car park has 2 floors. 26 cars can fit on each floor. How many cars can fit in the car park? If there are 6 empty spaces on each floor, how many cars are in the car park?

3. A car park has 2 floors. 18 cars can fit on each floor. How many cars can fit in the car park? If there are 6 empty spaces on each floor, how many cars are in the car park?

Problem 8

1. If one box of eggs contains 12 eggs, how many boxes is 98 eggs?
2. If one box of eggs contains 12 eggs, how many boxes is 78 eggs?
3. If one box of eggs contains 12 eggs, how many boxes is 58 eggs?

Problem 9

1. I have 2 piles of tins. There are 53 tins in one pile and 38 in the other pile. If I put the tins in boxes of 6, how many boxes will I use?
2. I have 2 piles of tins. There are 23 tins in one pile and 38 in the other pile. If I put the tins in boxes of 6, how many boxes will I use?
3. I have 2 piles of tins. There are 23 tins in one pile and 18 in the other pile. If I put the tins in boxes of 6, how many boxes will I use?

Problem 10

1. 29 x 5
2. 19 x 5
3. 14 x 5

Problem 11

1. My aunt has a big nut tree in her garden. When I visit her I always pick a bag of nuts to take home. Mom lets me crack open five nuts each day. How many nuts did I pick if I have enough nuts to last for February?
2. My aunt has a big nut tree in her garden. When I visit her I always pick a bag of nuts to take home. Mom lets me crack open five nuts each day. How many nuts did I pick if I have enough nuts to last for 3 weeks?
3. My aunt has a big nut tree in her garden. When I visit her I always pick a bag of nuts to take home. Mom lets me crack open five nuts each day. How many nuts did I pick if I have enough nuts to last for 2 weeks?

Problem 12

1. Zaid has to subtract 28 from 76. He first subtracted 20 from 70 and got 50. Then he subtracted 8 from 50 and got 42. Then he subtracted 6 from 42 and got 36. Is his answer correct? If it isn't correct, explain what his mistake was.

2. Zaid has to subtract 28 from 56. He first subtracted 20 from 50 and got 30. Then he subtracted 8 from 30 and got 22. Then he subtracted 6 from 22 and got 16. Is his answer correct? If it isn't correct, explain what his mistake was.

3. Zaid has to subtract 28 from 46. He first subtracted 20 from 40 and got 20. Then he subtracted 8 from 20 and got 12. Then he subtracted 6 from 12 and got 6. Is his answer correct? If it isn't correct, explain what his mistake was.

Problem 13

1. John has 30 silkworms. Altogether they laid 120 eggs. How many eggs did each silkworm lay?

2. John has 20 silkworms. Altogether they laid 120 eggs. How many eggs did each silkworm lay?

3. John has 10 silkworms. Altogether they laid 70 eggs. How many eggs did each silkworm lay?

APPENDIX H: INTERVIEW SCHEDULE

1. Explain how you solved this problem.
2. How did you know to solve the problem in this way?
3. What type(s) of writing did you use to solve this problem?
4. What made you choose this/these type(s) of writing?
5. How do you prefer to write down what you are thinking? Why?

APPENDIX I: JOURNAL PROMPTS

Maths journal prompts were displayed as follows:

1. The first thing I did was...
2. First... Next... Then... After that...
3. I figured out _____ by...
4. I noticed...
5. Something that is important to remember is...
6. I thought...
7. I decided...
8. I can show this idea by...
9. I compared...
10. I learned that...
11. Today's lesson helped me to understand...
12. The strategy that helped me to understand this was...

APPENDIX J: ANALYSIS OF PRE-TEST AND POST-TEST

The learners' strategies were analysed and the pre-test and post-test were compared using the LFIN (Wright et al, 2006a & 2006b; Wright, 2013). The analysis is presented in tabular form using the various aspects of the LFIN listed below.

SEAL: Stages of Arithmetical Learning

SNS: Structuring Number Strand

CPV: Conceptual Place Value

EMD: Early Multiplication and Division

Stage 0: Emergent Counting. Cannot count visible items. The child either does not know the number words or cannot coordinate the number words with items.

Stage 1: Perceptual Counting. Can count perceived items but not those in screened (that is concealed) collections. This may involve seeing, hearing or feeling items.

Stage 2: Figurative Counting. Can count the items in a screened collection but counting typically includes what adults might regard as redundant activity. For example, when presented with two screened collections, told how many in each collection, and asked how many counters in all, the child will count from 'one' instead of counting-on.

Stage 3: Initial Number Sequence. Child uses counting-on rather than counting from 'one', to solve addition or missing addend tasks (e.g. 6 + x = 9). The child may use a count-down-from strategy to solve removed items tasks (e.g. 17 – 3 as 16, 15, 14 – answer 14) but not count-down-to strategies to solve missing subtrahend tasks (e.g. 17 – 14 as 16, 15, 14 – answer 3).

Stage 4: Intermediate Number Sequence. The child counts-down-to to solve missing subtrahend tasks (e.g. 17 – 14 as 16, 15, 14 – answer 3). The child can choose the more efficient of count-down-from and count-down-to strategies.

Stage 5: Facile Number Sequence. The child uses a range of what are referred to as non-count-by-ones strategies. These strategies involve procedures other than counting-by-ones but may also involve some counting-by-ones. Thus in additive and subtractive situations, the child uses strategies such as compensation, using a known result, adding to ten, commutativity, subtraction as the inverse of addition, awareness of the 'ten' in a teen number.

Level 1: Initial Grouping. Uses perceptual counting (that is, by ones) to establish the numerosity of a collection of equal groups, to share items into groups of a given size (quotitive sharing) and to share items into a given number of groups (partitive sharing).

Level 2: Perceptual Counting in Multiples. Uses a multiplicative counting strategy to count visible items arranged in equal groups.

Level 3: Figurative Composite Grouping. Uses a multiplicative counting strategy to count items arranged in equal groups in cases where the individual items are not visible.

Level 4: Repeated Abstract Composite Grouping. Counts composite units in repeated addition or subtraction, that is, uses the composite unit a specified number of times.

Level 5: Multiplication and Division as Operations. Can regard both the number in each group and the number of groups as a composite unit. Can immediately recall or quickly derive many of the basic facts for multiplication and division.

LEARNER 1 (AA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Facile number sequence (breaks down into tens and ones and adds separately)	SEAL – Facile number sequence (decomposes)
2	EMD – Initial grouping (drawing shows quotitive sharing)	EMD – Figurative composite grouping (skip counting)
3	SEAL – Facile number sequence (breaks down tens and ones and adds separately – incorrect answer)	SEAL – Facile number sequence
4	EMD – Repeated abstract composite grouping	EMD – Figurative composite grouping (quotitive sharing incorporating skip counting)
5	SEAL – Facile number sequence (breaks down into tens and ones and adds)	EMD – Repeated abstract composite grouping

LEARNER 2 (AA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Initial number sequence	SEAL – Facile number sequence (decomposed, added separately)
2	EMD – Initial grouping (drawing shows quotitive sharing – incorrect answer)	EMD – Initial grouping (quotitive sharing – incorrect answer)
3	SEAL – Perceptual counting	CPV – increment by tens off decuple
4	EMD – Initial grouping (quotitive sharing – incorrect strategy)	EMD – Repeated abstract composite grouping (repeated addition)
5	SEAL – Initial number sequence	EMD – Perceptual counting in multiples

LEARNER 3 (A)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – Facile number sequence (only solved 1 part)
2	EMD – Initial grouping (drawing shows quotitive sharing – adds incorrectly)	EMD – Repeated abstract composite grouping (attempt) (skip counts incorrectly)
3	SEAL – Initial number sequence (incorrect answer)	No clear strategy
4	SEAL – Facile number sequence (incorrect answer)	EMD – Repeated abstract composite grouping (incorrect answer)
5	SEAL – Initial number sequence (incorrect answer)	SEAL – Intermediate number sequence (incorrect strategy)

LEARNER 4 (A)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – intermediate number sequence (incomplete strategy)
2	Combined EMD – initial grouping and SEAL – Perceptual counting	EMD – Repeated abstract composite grouping (repeated addition)
3	SEAL – Facile number sequence	SEAL – Facile number sequence
4	EMD – Perceptual counting (counts all)	EMD – Repeated abstract composite grouping (repeated addition – incorrect answer)
5	Absent	EMD – Initial grouping (quotitive sharing)

LEARNER 5 (A)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – Facile number sequence (decomposed – incorrect answer)
2	EMD – Initial grouping (quotitive sharing)	EMD – Repeated abstract composite grouping
3	SEAL – Combines Perceptual counting and Facile number sequence (incorrect answer)	SEAL – Facile number sequence
4	EMD – Perceptual counting in multiples (incorrect answer)	EMD – Repeated abstract composite grouping (repeated addition)
5	SEAL – Perceptual counting	EMD – Repeated abstract composite grouping (repeated addition)

LEARNER 6 (BA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Initial number sequence	SEAL – Perceptual counting
2	Strategy didn't fit problem type	EMD – Initial grouping (quotitive sharing)
3	SEAL – Intermediate number sequence	SEAL – Perceptual counting (added instead of subtracting)
4	EMD – Initial grouping (partitive sharing – incorrect strategy)	EMD – Perceptual counting in multiples
5	SEAL – Initial number sequence (used strategy incorrectly)	EMD – Initial grouping (quotitive sharing)

LEARNER 7 (BA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting	SEAL – Facile number sequence (didn't complete)
2	Strategy didn't fit problem type	SEAL – Facile number sequence (correct strategy used incorrectly)
3	SEAL – Perceptual counting (incorrect answer)	SEAL – Initial number sequence
4	Absent	EMD – Figurative composite grouping
5	No strategy visible	Strategy didn't fit problem type

LEARNER 8 (BA)		
PROBLEM	**PRE-TEST**	**POST-TEST**
1	SEAL – Perceptual counting (used strategy incorrectly)	No clear strategy
2	EMD – Initial grouping (quotitive sharing – used strategy incorrectly)	EMD – Initial grouping (quotitive sharing)
3	SEAL – Facile number sequence (counting erased)	SEAL – Intermediate number sequence (incorrect answer)
4	EMD – Initial grouping (number sentence didn't match)	EMD – Figurative composite grouping (skip counting – didn't answer problem)
5	SEAL – perceptual counting (strategy used incorrectly)	EMD – Initial grouping (partitive sharing – used strategy incorrectly, incorrect answer)

APPENDIX K: CONFIRMATION OF EDITING

Cape Peninsula
University of Technology

Department of Research

Faculty of Education and Social Sciences

Mowbray Campus

Cape Town University of Technology

To whom it may concern

This is to certify that I have edited the dissertation of Belinda Petersen entitled "Writing and mathematical problem-solving in Grade 3".

Dr M. A. Curr (PhD. U. of London)