



Cape Peninsula
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**An evaluation of a connectivist learning approach with college students in
South Africa**

by

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ABSTRACT

The patterns of interaction formed by students and their learning practices are elements that distinguish a student-centred approach from a teacher-centred approach. In a connectivist learning approach, the learning interactions are emergent and dynamic. This study validates the four connectivist attributes of students (autonomy, connectedness, diversity and openness) during the learning of mathematics in higher education. There is a need to improve the design of teaching and learning processes of mathematics in this sector to increase student engagement in the subject. There are two research questions that led towards the aim of this study. Firstly, what patterns of learning interactions appear from learning of mathematics with TVET college students? Secondly, how do these patterns inform about students' learning practices? The objectives were to explore the interaction patterns and explain the learning practices of based on their interactions with their peers.

The learning intervention conducted in this study was designed using the Analysis, Design, Development, Implementation and Evaluation (ADDIE) model of instructional design. The Frameworks for an Integrated Methodology (FraIM) was employed as the research approach. Observation and interviewing methods were conducted to gather data about the interaction patterns and their impact on academic achievements of students. Four patterns of interaction have been identified through this study. The attributes of connectivism of students in each pattern vary from one pattern to another. Therefore, the interactivity and the learning practices of students also vary with patterns. This study, therefore, recommends further studies that validate a connectivist learning approach through a dual theoretical lens in order to refine the findings of the current study.

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Table of Contents

DECLARATION	i
ABSTRACT	ii
ACKNOWLEDGEMENTS	iii
Chapter 1	1
1.1 Introduction	1
1.2 Background to the study.....	2
1.3 Statement of the research problem	4
1.4 Research questions, objectives and methods.....	4
1.5 Research aim	5
1.6 Rationale / motivation	6
1.7 Research layout.....	6
Chapter 2 Literature review	7
2.1 Introduction	7
2.2 Background to the literature review.....	7
2.3 Learning perspectives of connectivism.....	14
2.4 Literature on patterns of learning interactions	16
2.4.2 Review of empirical studies and relevant research methods.....	19
2.5 Literature review on learning mathematics	22
2.5.1 Literature review on learning mathematics through groups.....	22
2.5.2 Studies on interaction during learning of mathematics	23
2.6 Attributes of connectivism in learning design	25
2.7 Conclusion: Gaps in literature.....	27
2.8 Research questions	27
2.9 Research framework	28
2.10 Summary of chapter 2.....	29
Chapter 3.....	31
Research Design	31
3.1 Pragmatic research philosophy.....	31
3.2 Research method	32
3.3 Structure and process of the learning Intervention	33
3.4 Research Data collection.....	36
3.5 Data Analysis	38
3.6 Interpretive evaluation	40

3.7 Trustworthiness of data	41
Chapter 4.....	43
Results.....	43
4.1 Introduction	43
4.2 Data analysis Procedure.....	44
4.3 Data Presentation	45
4.4 Interview data presentation and analysis.....	57
4.5 Trustworthiness of data analysis	65
4.6 Discussion: Validation of connectivist attributes.....	66
4.7 Conclusion.....	76
Chapter 5.....	80
Discussions, contributions and recommendations.....	80
5.1 Introduction	80
5.2 Summary of the findings	80
5.3 Discussion	83
5.4 Research limitations.....	89
5.5 Contribution of the study.....	90
5.6 Recommendations	91
References	94

List of Tables

Table 1: Research questions, sub-questions and objectives	5
Table 2: Observation data- Emerging codes and categories on patterns of interaction.....	50
Table 3: Levels of interaction of sampled students and learning sessions.....	56
Table 4: Peer assessment results on each sampled participant	57
Table 5: Interview data codes emerging from reasons on when and how participants interacted.....	65
Table 6: Emerging categories based on codes about interaction and learning processes.....	66

List of figures

Figure 1: Principles of complexity theory (Adapted from Cohen, Manion and Morrison (2007b))....	12
Figure 2: Interaction patterns in Blended-learning (adapted from (Shu & Gu, 2018))	17
Figure 3: Communication networks (Adapted from Baran (1963)).....	18
Figure 4: The research conceptual framework	29
Figure 5: Processes of the ADDIE model on the learning intervention	34
Figure 6: Research data collection process.....	36
Figure 7: Structure of chapter 4 data analysis	43
Figure 8: Conceptual model of research data analysis	44
Figure 9: The pattern of interaction formed on the first learning sessions.....	46
Figure 10: The pattern of interaction formed on the second learning sessions	48
Figure 11: Learning interactions from the third session.....	49
Figure 12: Multidimensional space for evaluating connectivists' learning interaction dynamics.....	77

List of appendices

Appendix I: Ethical Clearance.....	102
Appendix 2: Student activity.....	104

GLOSSARY

Terms/Acronyms/Abbreviations	Definition/Explanation
SNA	Social network analysis
TVET	Technical vocational education and training colleges
FraIM	Frameworks for an Integrated Methodologies

Chapter 1

1.1 Introduction

This research is about interaction patterns and learning practices that emerged during the learning of mathematics by vocational college students in Cape Town. The development of these patterns and practices show a significant shift from individual construction of knowledge to collective efforts through peer connections. Knowledge therefore lies in networks of both human and non-human artefacts. The definition of learning is described as the ability to traverse connections within these networks (Siemens, 2005b). The learning theory of Connectivism promotes the students' personal learning networks. In these learning networks, students collaborate (collectively) and cooperate (towards individual gain) to create and distribute their knowledge (Bessenyei, 2008). The evolution of mathematics learning in the 21st century equips students to construct mathematical knowledge both individually and collectively, in communities of practice (Roth, 2016b). There is an understanding that teaching and learning is shifting away from a teacher-centred to student-centred approach. The learning dynamics are self-organized through a connectivist approach (Siemens, 2005a). There are four attributes of a connectivist teaching and learning approach that are attributed to these learning dynamics. These four are autonomy, connectedness, diversity and openness (Downes, 2012:9).

This thesis argues that the four attributes of connectivism are also portrayed during a student-centred mathematics learning intervention. I assessed these attributes by recording the frequency and durations of interactivity of students with their peers. As a unique pathway, I first established the patterns developed by students during peer interactions. Four patterns of interactions that emerged include paired students, educator-monitored small groups (subnetworks), peer-dependent small groups (decentralised networks) and whole class connected discussions (community of practice). Students interacted in a self-organised manner. Participation of students in these patterns is useful for interpretation of their inherent connectivist attribute(s). Hence, the current study submits that certain patterns of interaction have specific attributes of connectivism.

It is necessary to note that although this submission is made, the conception of social networking of students defines the centre of the argument. However, divergent views may be coined due to complexity within the learning as a process. Moreover, learning is a social process that requires social interaction by students. This study therefore neutralizes diverging perceptions in literature about learning of mathematics in traditional classrooms. On one hand, researchers and teaching practitioners regard teaching and learning of mathematics as a teacher centred process (Meijer, Bulte & Pilot, 2013 in van den Akker, Bannan, Kelly, Nieveen & Plomp, 2013 & Bannister, 2016) On the other hand, recent developing arguments hold that learning of mathematics through social interactions yields notable academic achievements (Brunetto, Parolini, & Verani, 2018; Helm, 2017). Although not

explicitly stated, both arguments address affording students some attributes of autonomy, connectedness, diversity and openness. The current study in its connectivist framework, therefore, advances that perspectives into complexity principles are crucial in establishing the participation of mathematics students. The study establishes that it is incomplete to characterize the interaction behaviour of students without some implied understanding of emergent behavioural changes.

1.2 Background to the study

Connectivist learning has been largely studied in online and blended learning in higher education. However, research is increasing its focus on connectivism in various sectors of education. More so, researchers are shifting knowledge on connectivism from online learning platforms to traditional classes (Rice, 2018b). Earlier, Lee (2000) reported a gap between practices of students before and within the digital age. It is now almost two decades into this digital era, and students are becoming more reliant on technology than before. Downes (2010) states that improvements in technology are reshaping the way people think and learn. For example; in mathematics teaching practice I find that a boundary line can be drawn to distinguish performance of a student with and without a technological device as a learning affordance. It is notable that students in this digital age also rely on the use of social and interactive technology. Recently, researchers found the need to position a connectivist learning approach in a traditional classroom environment in order to define the role of the educator and also the practices of students (Smidt & Thornton, 2017; Utecht & Keller, 2019).

This current study positioned connectivist learning approach in vocational college mathematics classrooms. In this sector of education little research has been conducted with focus on a connectivist learning approach. Ngoveni (2018) has expressed concern that in South African vocational colleges poor mathematics results of students is attributed to both students' personal characteristics and the state of the learning environment. Downes (2010) cited autonomy, connectedness, diversity and openness as attributes in a connectivist learning approach. These attributes are implied in both behavioural practices of students and their learning environment. In order to gain further understanding into Ngoveni's (2018) findings and recommendations, I conducted this study focused on the practices of individual students that characterize their learning connections. Hence, the study is premised on insights drawn from personality and determination theories indicating autonomy, connectedness, diversity and openness (Tschofen & Mackness, 2012).

In practice, vocational mathematics continues to be perceived as an isolated academic subject associated with failure and dissatisfaction (Dalby & Noyes, 2015b; Ngoveni, 2018; Wedege, 2013). Students seem to concentrate more on administrative matters than deep learning in this area. As a result, peer-to-peer interaction towards learning is minimal. While Downes (2012) argued that

knowledge in a classroom is static and knowledge in a network is dynamic, Thoms and Eryilmaz (2014) believed learning as a social process facilitate the sharing of knowledge. Therefore, the rationale of the connectivists is to shift knowledge from a static to a dynamic state. By using the concept of systems dynamics, Williams, Karousou, and Mackness (2011) attribute the nature of a learning process as a shift from general systems to complex systems.

It is stipulated that the learning theory of connectivism establishes on principles of chaos, complexity and network theories (Siemens, 2005a). Therefore, I referred to implications of the trio in my analytical perspectives into the practices of the students. Studies have been conducted in education to describe the learning process and its dynamics using implications of the three (chaos, complexity and network) theoretical lenses. The studies addressed these elements independent of each other. However, these studies seldom address the practices of individual students. More so, a connectivism theoretical framework has not been incorporated in such studies. In recent theoretical studies of connectivism theory and also in empirical studies of the connectivist teaching and learning approach, there are no implications of chaos and complexity principles that are explicitly expressed. The principles of network theory are implied through formation of learning networks and peer connections.

While Siemens (2005a) highlighted that principles of chaos and complexity characterize a connectivist learning approach, Kernick (2004) suggested that there is a contested relationship between the two principles. However, Kernick states that chaos theory is a quantitative study of dynamic non-linear systems, while complexity is a qualitative study of non-linear systems. The key elements to note during the learning dynamics are unpredictability, diversity, connectedness, self-organisation and continual feedback into the systems. This school of thought cautions against methodological practices considered when studying a learning process that is dynamic. Therefore, the learning process is explained by experiences based on a given learning network.

In my literature review, there is a body of knowledge investigating the impact of principles of connectivism learning in both higher education and in high schools. However, it is further suggested that more knowledge can be acquired about connectivism through replicating studies. The studies will be replicated in different learning environments (physical and virtual learning spaces), with various actors (instructors, students and administrators) and also in various areas of learning (Bannister, 2016; Barnett, McPherson, & Sandieson, 2013; Smidt & Thornton, 2017). The trends in research studies on connectivism learning practices deliberate on student academic achievement under the four principles of connectivism (openness, diversity, autonomy and connectedness). Earlier, Williams et al. (2011) indicate that the four principles result in an emergent learning phenomenon. In

this conception, the role of students is redefined and distinguished from being consumers of knowledge to producers of knowledge. Mixed reactions expressed both positive and negative consequences from learning through connectivism in mathematics (Mohamed et al., 2018).

1.3 Statement of the research problem

This section presents a personal, practical and an academic problem. I feel that mathematics educators in vocational education and training colleges in the Western Cape Province in South Africa are dominantly practicing a teacher-centred approach. The practical problem is that a shift from a teacher-centred approach to a student-centred approach is recommended. However, there is little informed in practice about when and how teachers adopt this shift. There is need for an intervention framework to promote and maintain peer interactions during a student-centred approach. Therefore, it is critical to understand the learning interaction dynamics.

The academic problem is in exploring the interaction attributes of students that inform about their learning practices in mathematics. The connectivists' perspectives maintain that interactivity of students is based on four principles of connectivism (autonomy, connectedness, diversity and openness) (Bannister, 2016; Downes, 2010). Several studies confirmed these four attributes were conducted with online students. To date, a few studies have been conducted using a connectivist theoretical framework on practices of students in classroom environment (Utecht & Keller, 2019). The intellectual puzzle lies in establishing the extent to which the four attributes of connectivism manifest and impact on learners' mathematics learning practices in a Technical college environment.

1.4 Research questions, objectives and methods

In order to achieve the connectivists' perspective I contend that there is need for an empirical study to explore the dynamics in peer (or group) interactions. Cobb, Stephan, McClain, and Gravemeijer (2011) indicated that interacting agents are explained through both a social and a psychological perspective. A psychological perspective entails an analysis of interactions to determine the attributes of an individual agent (student). A social perspective yields a holistic analysis into the structural attributes of a learning community (group or network). For this current study I investigated the structure of the entire group of students and the characteristic interactive features that arose during learning. Therefore, my guiding question is explorative: **What patterns of learning interactions appear from a connectivist learning approach of mathematics with TVET college students?** The objective of this question is to explore the patterns that emerge in order to identify the group interactivity and the structural dynamics. I established these patterns by observing the interacting students. I later transferred my observations into sociograms.

In my second research question I seek to gain an understanding towards students' learning practices in their patterns of interaction. The question is: **How do these patterns inform about students' learning practices?** The objective of this question has been to explain the level of learning interaction practices of students with their peers in a given pattern. I conducted interviews with students and peer assessments to achieve this study objective. I finally drew some conclusions informed by Cobb, Stephan, McClain & Gravemeijer's (2001) interpretive framework. I collated themes from narratives and observations to validate the attributes of autonomy, connectedness, diversity and openness in students. This has been my analysis of the individuals and group's patterns of interaction.

Table 1: Research questions, sub-questions and objectives

Research aim	To explore the interaction practices of students towards their academic achievement in vocational education mathematical literacy.	
Rationale	To validate the connectivism design principles during a mathematical literacy learning intervention.	
Research questions	Objectives	Research method(s)
1. What patterns of learning interactions appear from a connectivist learning approach of mathematics by TVET college students?	To identify the patterns of interaction of students during a learning intervention designed on principles of connectivism.	Observation Sociograms Numeric (Social network analysis)
2. How do these patterns inform about students' learning practices?	To determine the extent to which the types of patterns explain the learning practices of students.	Narrative (interviews and content analysis on chat messages) Artefacts analysis (assessments)

1.5 Research aim

The aim of this research is to explore the interaction practices and experiences of students towards their academic achievement in mathematical literacy in vocational education. South Africa is a nation with vocational education institutions whose students have diverse racial, cultural and socioeconomic backgrounds. These factors may impact on students' interactions in their learning environments. Although the investigations into the impact of these factors will not be part of this research, my study informs on the nature of patterns of students' interactions and their practices when a connectivist learning approach is used to teach mathematics in vocational colleges in South Africa. The

instructional designers for both classroom and online spaces in South Africa will also build their work upon the findings of my study.

1.6 Rationale / motivation

The rationale of this study is to validate the principles of connectivism theory as being paramount to the design and development of a mathematics learning intervention. Plomp, Nieveen, Bannan, Akker, and Kelly (2013) state that design research aims either to develop a research-based solution in educational practice or to validate the theories about the teaching and learning processes. The purpose of this study is to evaluate a mathematics learning process under the principles of the learning theory of connectivism. These principles define the nature of the learning design and the attributes of individuals. The four principles will be explained in the literature review section and are openness, diversity, connectivity (interactivity) and autonomy.

1.7 Research layout

In chapter 2, I will describe the learning theory of connectivism that entails learning through distributed networks. I unpack the theory by revealing its three constituencies and their implications in learning. These three are chaos, complexity and network theories in the application of connectivism theory in research and in practice. In chapter 3, I reveal the research philosophy (pragmatism) that underlies my study and also my research approach (Frameworks for an Integrated Methodology (FraIM)). I also explain how I conducted my learning intervention and reveal the aspects of connectivism that underpin the learning process. The methods employed to gather data about interactions between students and the interaction patterns that emerged will be explained in chapter 3. In chapter 4, I present the results of my study, which are the four patterns of interaction that emerged and the students' practices (interaction and learning) within these patterns. In addition, I validate the four connectivist attributes based on the interaction patterns and practices of students. Finally, in chapter 5 I provide the limitations of my study and show the contribution of this study maintains that connectivist attributes characterize the learning behaviour of mathematics students. More so, it adds that manifestation and impact of these attributes towards learning is conditional. Therefore, guidelines are recommended for teaching and learning practices and further studies.

Chapter 2 Literature review

2.1 Introduction

In this chapter I present a review of literature that constitutes the formulation of the purpose of this study and its problem statement. I begin the chapter by explaining the learning theory of connectivism, its inherent theories and the findings from studies on dynamics experienced during a connectivist learning process (both in mathematics and in other learning areas). Literature on both theoretical and empirical studies about processes of interaction of students has been the main premise of this review chapter. Extensive arguments are presented on findings from previous empirical studies on interaction patterns of students. Accordingly, the themes that characterize students' learning interactions are developed from literature. The research gaps were identified to establish the relevance of the study, hence formulation of the two research questions:

1. What patterns of interaction appear from a connectivist learning of mathematics with TVET college students?
2. How do these patterns inform about students' learning practices?

I consulted a range of studies conducted on students learning interactions, learning practices in mathematics and on teaching and learning developments in vocational education mathematics. I followed the posts and discussions on blogsites of pioneers of the theory of connectivism as my primary stage of identifying the gurus in connectivism. Then I conducted my literature survey through Google Scholar, a free scholarly search engine. I accessed some articles from online textbooks. Furthermore, I did a bibliography analysis to identify the most cited and contributing authors in areas related to my research. Studies into applications and implications of connectivism are conducted in various philosophical perspectives. I will present the philosophical positions of various researchers and their studies from literature and identify gaps to be addressed for a study underpinned by the theory of connectivism.

2.2 Background to the literature review

Connectivism is described as a learning theory of the digital age that denounces the limitations of behaviourism, cognitivism and constructivism (Downes, 2006; Siemens, 2005a). Several studies conducted show that connectivism theory is used on digital platforms such as MOOCs and other social networking platforms. However, there is an increase in the number of studies in which this theory is employed in classroom environments. A few of these studies are on mathematics at different levels of education. In my literature review I provide the overview of mathematics as a compulsory teaching and learning subject in South African Technical Vocational Education and Training (TVET) colleges. Mixed comments have been posted regarding the design, teaching and learning of this learning area. Little has been published in connectivist perspectives towards the teaching and learning of

mathematics, specifically in South African TVET sector. In my review of literature, I discuss the findings and gaps that inform my study.

2.2.1 Connectivism introduced

The learning theory of connectivism is mostly used in Massive Open Online Learning Communities (MOOCs). These are open, non-controlled learning environments (Wang et al., 2016). Researchers extended applications and principles of connectivism into classroom environments and practices (Rice, 2018a & b). Various research methods have been applied in these studies. A social network analysis has also been used to explain the learning processes and interactions in these open learning methods (AlDahdouh, Osorio, & Caires, 2015). Some enthusiasts of learning theory of connectivism established a research framework guided by principles of this theory in various disciplines (Wang, Chen, & Anderson, 2014). It is reported that in teaching practice, college instructors confirm that principles of connectivism (autonomy, diversity, connectedness and openness) have a significant positive impact on effective learning of Science, Technology, Engineering and Mathematics (STEM) subjects (Smidt & Thornton, 2017). Little has been said about these principles from students' perspectives and experiences within a classroom environment. Hence, the rationale of my study is validating these four principles as experienced in the classroom.

There are mixed reactions in literature about the applications and implications of the theory of connectivism. There are authors who express criticism of learning theory of connectivism (Kop & Hill, 2008; Morbitzer, 2013; Wiley, 2014). They suggest connectivism is incomplete, hence it does not qualify to be considered a learning theory, but a phenomenon. Furthermore, they hold that connectivism does not offer any difference from the traditional learning theories. Kashefi, Ismail, Mirzaei, Tak, Wan Obeng and Ching (2017) report that in this 21st century, the proportion of mathematics educators that teach through behaviourist and constructivist approaches are more than those that use a connectivist approach. I do not consider their finding as a criticism from the educators on the use of connectivism theory, but I take it as an indicator for a need to increase the amount of research into connectivism towards the teaching and learning of mathematics.

2.2.2 Vocational mathematics education

Vocational education is for students who transitioning from high school education to the workplace or to higher education. Learning under vocational education is distinguished from academic learning in that all subjects are workplace oriented. Vocational education mathematical literacy is for real life and workplace oriented practices (Dalby & Noyes, 2015a). Therefore, in South African Technical Vocational Education and Training (TVET) colleges, mathematical literacy has been introduced as a compulsory subject area. Every student is expected to be equipped with numeracy and quantitative skills and become a competent participant when dealing with everyday life and workplace related

contexts. However, there are challenges within the teaching and learning of both mathematics and mathematical literacy. Haara, Bolstad, and Jenssen (2017) cite design and implementation of both teaching and learning practice and material as an obstacle. More so, Ngoveni (2018) reports that the challenge around the poor performance of students in mathematics in South African TVET colleges is a combination of the personal status of students and issues pertaining to the learning environment. I find that both reports are quite general and so broad that one cannot differentiate the cited challenges and classify them as unique from other learning subjects.

Literature shows that since the introduction of vocational education in South Africa in 2006, research has been aimed at addressing these challenges in the learning of mathematics and mathematical literacy (Buytenhuys, 2006; Frith, 2009; & Graven, 2008). Mohd Rameli et al. (2014) indicated that the poor performance in vocational education mathematics correlates with students' anxiety. Young and Young (2016) have investigated and remain inconclusive on whether mathematics anxiety in students may be defined according to student diversity. Hence, I find it worthy incorporating a connectivist learning design towards mathematics. The key aspects to guide my study and contribute towards the learning of vocational college mathematics are the four connectivist attributes (autonomy, connectedness, diversity and openness). The learning and teaching will not centre on individuals but is distributed across a network of students. For this reason, my research explores the interaction patterns and practices of students during learning of mathematics under connectivism.

2.2.3 Connectivism theory

The proponents of this theory, Siemens (2005a) and Downes (2006) posit that the learning theory of connectivism establish on principles of chaos, network and complexity theories. It is advanced as a theory of the digital age. The pioneers of the theory distinguish it from traditional theories (behaviourism, cognitivism and constructivism) claiming that it considers the significance and impact of technology in the field of education. The two pioneers of connectivism theory contributed principles that underpins current research work and guiding practices during teaching and learning.

According to Siemens (2005a) connectivism is based on the understanding that decisions and the process of making these decisions is not rigid, but changes according to the underlying conditions. He adds that it is crucial to have the ability to recognize changes in the basis we make our decisions on. The following are eight principles of connectivism learning according to Siemens (2005a)

- Learning and knowledge rests in a diversity of opinions.
- Learning is a process of connecting specialized nodes or information sources.
- Learning may reside in non-human appliances.
- Capacity to know more is more critical than what is currently known

- Nurturing and maintaining connections is needed to facilitate continual learning.
- Ability to see connections between fields, ideas, and concepts is a core skill.
- Currency (accurate, up-to-date knowledge) is the intent of all connectivist learning activities.
- Decision-making is itself a learning process. Choosing what to learn and the meaning of incoming information is seen through the lens of a shifting reality.

On the other hand, Downes (2006) postulates that connectivism is a thesis of knowledge that is distributed across the network. Hence, he defines learning as a process of traversing these networks. According to him connectivism is informed by network or communities. Siemens (2005b) indicates that his second principles (among the eight above) concurs with the concept of networks. The four dimensions of a community posited by Downes that are important to consider in a connectivist learning approach are:

- autonomy
- connectedness
- diversity
- openness

My discussion in this literature review will be premised on the implications of these principles during teaching. Since the birth of connectivism as a learning theory, there are mixed views, both acknowledging and denouncing it as a new and unique learning theory. However, studies in literature have shifted focus from arguing about the veracity of the theory but advance it as a lens with which to explore, explain and develop various processes and aspects of learning models. Connectivists are of the opinion that technological changes in today's world continually reshape our everyday life process. In the education sector the teaching and learning practices also respond to this dynamic world. Siemens and Downes iterate that connectivism addresses teaching and learning in a changing world of technology. However, I find that little is put across both in research and in practice to distinguish a connectivist approach from other instructional and learning theories. Hence, prior to a review of previous studies, I present an overview of the theoretical principles (chaos, complexity and networked) embedded in connectivism.

2.2.4 Theoretical underpinnings

Connectivism learning theory is premised on principles of chaos, complexity and network theory. I feel that researchers and the general populace give limited reference to these principles while distinguishing connectivism from other learning theories. I provide separate insights into these principles to enhance a good understanding of learning theory of connectivism. Quite imperative is the implications of these principles in education.

a. Implications of principles of chaos theory

Due to continual technological advancement, Lee (2000) alerts education practitioners that emergence and chaotic learning is a new learning style. Therefore, it is recommended to identify and address gaps between learning styles inside and outside the classroom. Reigeluth (2008) points out that principles of chaos theory are crucial to understand the changing learning styles. More so, the concepts of chaos theory have been argued to contribute towards the understanding of designing of a learning process and environment (Weichhart, 2013). Although these studies were not presented in the framework of connectivism, there is substantive knowledge that distinguishes a connectivist approach from other instructional and learning theorists. For example, Akmansoy and Kartal's (2014) empirical study approves the 'butterfly effect' in the education system. This is a metaphor of chaos theory that translate to imply that minor changes in the system yield disproportionately massive and unknown consequences. In an analytical perspective Kernick (2004) suggests that chaos is a quantitative approach into the nonlinear systems, while complexity is a qualitative approach. This implies principles between the two may overlap. However, studies into the complexity plane focus on the interaction dynamics in a learning process.

b. Implications of principles of complexity theory

Complexity theorists describe processes in a learning system with a holistic approach more than a dissection approach which analyses properties of an individual agent (Kernick, 2004). To be concise, the overarching principles of complexity include unpredictability, non-linearity and self-organization. (Cohen, Manion, & Morrison, 2007b) present a detailed structure of complexity theory in a diagram below.

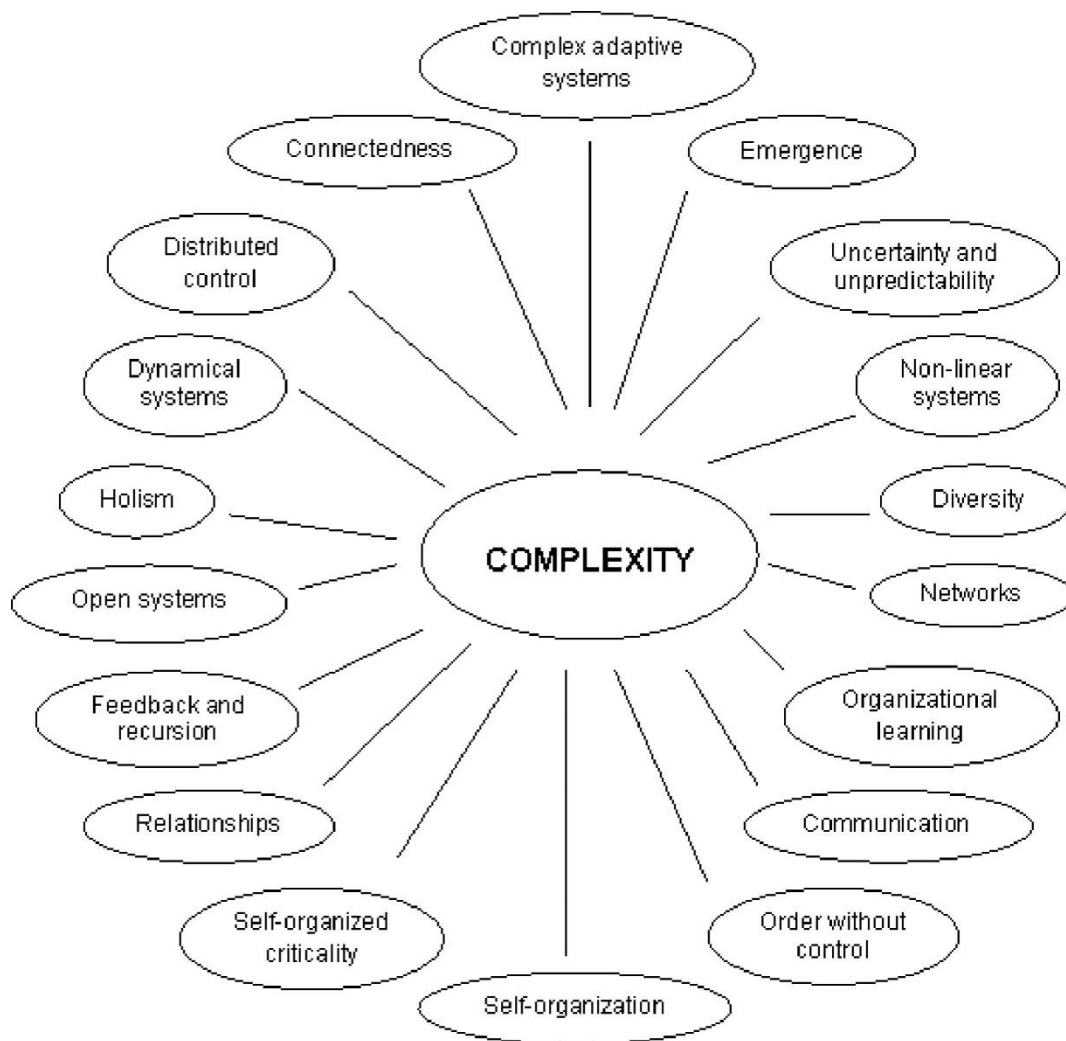


Figure 1: Principles of complexity theory (Adapted from Cohen, Manion and Morrison (2007b))

Since my study is fully underpinned by the complexity theory, among all the components of complexity provided in the Figure 1, I will focus mainly on the four (autonomy, connectedness, diversity and openness. Downes (2012:71) described these four as dimensions in the context of students as participants learning in a network. While Siemens (2005b) educate that learning is the formation of networks I will address my first research question to patterns of interaction with network diagrams. By taking into consideration that networking is a complex process, I will note down all the emergent behaviours in students during their interactions and describe them according to the components of complexity theory listed above.

In the literature, I find that interaction dynamics of students are better explained using agency and connectedness (Steenbeek, Vondel, & Geert, 2017). There is limited reference to other components of complexity in studies that are informed by a connectivism research framework. For example there are studies that approve the use of connectivism in classroom environment (Klinger, 2011; Bannister, 2016; Utecht & Keller, 2019) and also those disputing it as theory (Clarà & Barberà, 2014). In fact, the

principles of complexity are neither mentioned nor acknowledged. However, the proponents of the learning theory of connectivism indicate that learning process is distinguishable from a linear systems approach to a complexity approach. They argued that in linear systems, interacting variables are ordered, have predictable behaviour that is explained through the concept of cause-and-effect relationship. However, since complexity constitute connectivism, therefore emergent and unpredictable outcomes may be experienced.

The connectivists advocate that learning systems have some complexity and chaotic attributes. That implies the causes and effects of a learning process are not considered as linearly related, but as fuzzy and unpredictable. The continuing exponential development of knowledge due to technological advances implies there is need to adopt a non-linear, complex pedagogical approach. Therefore, complexity attributes will exist in neural (cognitive), internal (conceptual) and external (social level) interactions (Siemens, 2005b). However, in my study I will focus on external (social) interactions only. The literature relates the learning process mainly to a complex system and a chaotic system (Akmansoy & Kartal, 2014; Bloom, 2013; Lee, 2000). Koopmans & Stamovlasis (2016) describes a learning system as a group of interacting individuals with focus on learning and interaction trajectories of an individual. Moreover, Buchenroth-martin, Dimartino and Martin (2017) add that the size of the group influences the complexity of the learning due to diversity of the students. Complex and chaotic systems do not have a predictable cause and effect relation. Cobb et al. (2011) reveal that the assumption of deriving the instructional practices from theory is quite often in the academic discourse. However, they suggest a pragmatic approach towards understanding the learning of students. In my study I will envisage a pragmatic view to describe and understand the practices of students while learning and forming their interaction patterns. Due to the complexity implications to my study, at this stage I do not posit an assumption into the outcome of the rationale of my study (to validate students' connectivist attributes).

c. Implications of principles of network theory

Siemens (2005b) posited that the learning process is the creation of networks. He adds that the epitome of connectivism is concepts of Personal Learning Networks and Community of Practice. Therefore, some network diagrams will be my tools for addressing my research question into the patterns of interaction of students. It is confirmed that network theory perspective has offered improved representation and analysis of relationships between various agents (people, concepts and various artefacts) in educational interactions (Carvalho & Goodyear, 2018). From a philosophical standing, Downes (2012) states that: "connectivism is the thesis that knowledge is distributed across a network of connections. Hence learning consist of the ability to construct and traverse these networks". In that view, there is sufficient evidence that a connectivist learning approach to some

significant extent implies social learning. However, the concept of networks distinguishes a connectivist approach from many other learning approaches that can be described from a social learning perspective. Central to a connectivist learning network, Downes reveals that there are four attributes that describes the state of the students involved and also the entire learning process as a system. In the following sections of my literature review I will elaborate on the meaning and implications of each of these attributes in a learning context.

2.3 Learning perspectives of connectivism

Studies about the connectivist learning approach investigate the ways students acquire and distribute knowledge through connecting with each other. There are a few studies that investigate practices and characteristics of interaction among the students. Rank (2018) has guided educators towards the implementation of a connectivist approach. Although she does not focus on the four attributes of a connectivist learning network, she advances that connectivist students make choices, collaborate in groups, hold different views and are able to make decisions. This respectively translates to embedded autonomy, connectedness, diversity and openness amongst students. Downes (2012) indicates, learning in a connectivist platform implies the ability to construct learning networks and traverse them in order to establish knowledge. The current study finds it worth to validate the position of four connectivist attributes of interaction. A connectivist learning approach implies a constructivist conception in that students connect with each other to construct knowledge and establish the learning networks.

Bannister (2016) conducted a qualitative interpretive study that examines the four principles, coded as characteristics of a connectivist learning. The purpose of his study was to investigate how educators experience and interpret these four connectivist characteristics in a business course. I adopt his recommendation for future studies. He suggests that findings from his study may not be transferable to a mathematics learning context. He confirms that the four principles exist in adult learning. Nonetheless, I feel that further to his study, a student centric data collection approach may be more relevant than his educator- informed approach. For a clearer understanding of a connectivist learning approach, Downes (2012) distinguishes the role of the educator from that of a student. He clarifies that in a learning network the educator demonstrates, while the students practice learning. Hence, I subscribe that research into the impact of specific practices in learning is incomplete without perspectives of the one practicing learning.

Siemens (2008) highlights that there is a 'growing disconnect' between the classroom experiences and those from other platforms of learning. That implies research into connectivist learning interactions is mostly done for online learning platforms. While Rice (2018a) points out that connectivism has not been widely researched under traditional classrooms, it is worth noting that

characteristics of students interacting in both online and classroom environments are important for understanding the learning process. I reviewed the study by c that scrutinize and compare the interaction characteristics between online and classroom students. Both their study and my current study aim to contribute understanding of the interaction of students. However, in Shu and Gu (2018), a comparison between the online and classroom learning interactions was done using a social network analysis and thematic analysis only. Their comparison dwells more on group interaction attributes than individual attributes. I feel that, my study would add a further understanding of attributes of individuals through both a social network analysis and an interpretive analysis towards connectivist attributes of students (autonomy, connectedness, diversity and openness).

The three measures of social network analysis (SNA) used in Shu and Gu (2018) study include connectivity, reciprocity and transitivity. Connectivity refers to the sharing of information between members in a group, reciprocity refers to the active participation of every member in a group and transitivity refers to the extent to which every member is connected to all other members. I suggest further study will add an understanding of the interaction of students through unpacking and validating the other three attributes autonomy, diversity and openness.

The four connectivist attributes are acknowledged for effectively addressing the learning processes in the current digital age (Foroughi, 2015). These attributes prompt students to connect with peers and work collectively. It is almost a decade since connectivist learning approach has been commended for improving performance of students in mathematics (Klinger, 2011). Kashefi et al. (2017) argue that more mathematics teachers prefer a behaviourist approach to a connectivist learning approach. He highlights that in a behaviourist approach, students exhibit passive behaviour traits, and they need a stimulus to learn. However, in a connectivist approach (the latest approach) the students are active, self-driven, self-organised, create personal learning networks through peer connections (Downes, 2010). I am of the view that more knowledge about a connectivist learning approach need to be established on the teaching and learning of mathematics. The extent to which students show these connectivist attributes will be important to approve or disapprove Kashefi et al's claims.

For example, Bannister (2016) confirmed through perceptions of educators that college students display connectivist attributes in business courses. He acknowledges that his findings form the basis of further discussions about connectivist attributes of college students. While he recommends perspectives of students as agents of learning to be considered in future studies, my study's research problem stems from the need to fill the gap between characterizing learning practices during learning of college mathematics. Hence a connectivist's theoretical lens in its totality of implications of complexity, chaos and networking will be relevant in my study.

The proponents of learning theory of connectivism also state that the theory is appropriate for describing learning under complex circumstances (Siemens, 2005a). Firstly, in this digital age information is so abundant that individuals and groups can easily access each other through connecting to information sources (Wang, Anderson, & Chen, 2018). The teaching and learning processes has become distributed rather than centrally controlled. Secondly, learning is now considered to be a nonlinear process where the cause –effect relationship cannot be assumed to suggest the student performance. It is characterised as an unpredictable process whereby multiple internal and external human behavioural factors significantly contribute towards learning (Jörg, 2016). Moreover, Roth (2016a) advances that, specific to the learning of mathematics, there is a dynamic perspective of distributed agency that stem out of recent learning theories. The term distributed agency refers to: “the situation (coordinated or uncoordinated) where the actions or operations of a range of different individuals often with different motivations, interests and in different places combine to create an outcome they all wanted” (Online Oxford Encyclopedia, n.d.).

With reference to my research problem statement towards evaluating the interaction dynamics of mathematics students, I contend that the connectivist lens should be accompanied with a pragmatic methodological approach. Therefore, various approaches may be applied to evaluate a learning process. In the next section I present various interaction patterns of students from literature and researcher’s perspectives through which these patterns were established.

2.4 Literature on patterns of learning interactions

Literature presents various theoretical lens to determine the statics and dynamics that entail the students learning interactions. For example, the dynamical systems approach adds understanding into the complexities within dynamics of students’ learning groups (Forsyth, 2014). Meanwhile, Cesar (1998) reports that an interactionist perspective confirms that cognitive levels improve during learning of mathematics through social interaction in a classroom. This contrasts a structuralist’s perspectives by focusing on individual attributes of interaction. Furthermore, a positioning theory perspective has been effectively employed to understand the position and role of individuals through conversions (Davies & Harre, 1990). According to the limited scope of my study, these three perspectives may be too broad to address an explorative objective. But a connectivist perspective continues to be welcomed as a theory of the digital age that expresses the behaviour of students and their learning patterns (Wang et al., 2018). Some empirical studies represent these interaction patterns of students through social network diagrams. A social network analysis is the quantitative method used to describe the interactivity of students.

There is further understanding of these interaction patterns through an interpretive analysis of the visual displays of student networks. Therefore, my objective to identify the patterns of interaction that emerge when conducting a connectivist teaching and learning will be guided by concepts of network diagrams. A network diagram is defined as nodes connected with edges to form a community.

2.4.1 Types of interaction patterns

After reviewing literature on learning interactions, I noted various presentations and descriptions of the patterns established. I have found four patterns that represent a variety or a combination of learning patterns. These four are community network, centralized patterns, decentralized patterns and distributed patterns.

Community network

The Internet Society (2018) describes a community network as “a local group of people working together, combining their resources, organizing their efforts, and connecting themselves to close connectivity and cultural gaps with telecommunications infrastructure”. Mesch (2015) adds that community networks also involve people in face-to-face interactions and participations. In my study on interaction dynamics of students I relate the attributes of a community network to whole class collaborated discussions. Shu and Gu (2018) used diagrams from a social network analysis to distinguish interaction patterns of online students from classroom students. As I said in the previous section, their diagrams (**Figure 2**) below show densely connected and distributed peer-connections in online learning more than in classroom learning. Additional to density and distribution of these peer-connections, a variety of lessons can be drawn out through an interpretative analysis on students’ interactional behaviour.

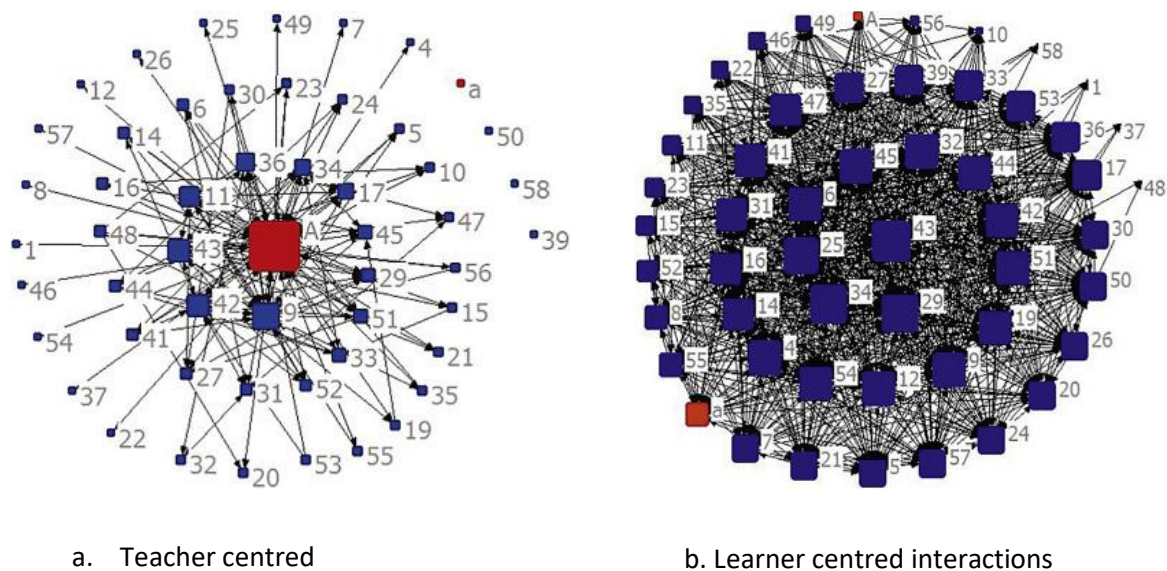


Figure 2: Learning interaction patterns (adapted from Shu & Gu, 2018)

In classroom interactions (**Figure 2a**), the highest number of connections that are directed towards the teacher may imply limited connectedness between the students. Furthermore, learning might be centred on, and controlled by the teacher, hence students may have little autonomy to determine their own learning pace and styles. Multiple characterizations may also be drawn out with reference to openness and diversity of students in these two patterns. Although Shu and Gu (2018) applied both a SNA and a thematic analysis to describe interactions of their students, I challenge them and propose that a thematic analysis should be specific to the context of a learning area. For example, to address my research problem in a mathematical context, an understanding into these interaction patterns and practices may be informed by reference to a mathematics based interpretative framework. I will use Cobb, Stephan, McClain and Gravemeijer's (2001) interpretative framework for mathematics students that will be explained in chapter 3 (section 3.6)

To understand learning behaviour through patterns formed during interaction, Downes (2011) likens the state of interactions to Baran's (1963) communication network diagrams. Baran introduces three types of communication networks as centralized, decentralized and distributed networks (Figure 3). I find these patterns relevant to my study, because in my practical encounters and also in various studies about teaching practices, it is evident that learning interactions can be related to these patterns. Comparable to teaching and learning conceptions, I equate a centralized interaction pattern to a teacher centred teaching approach.

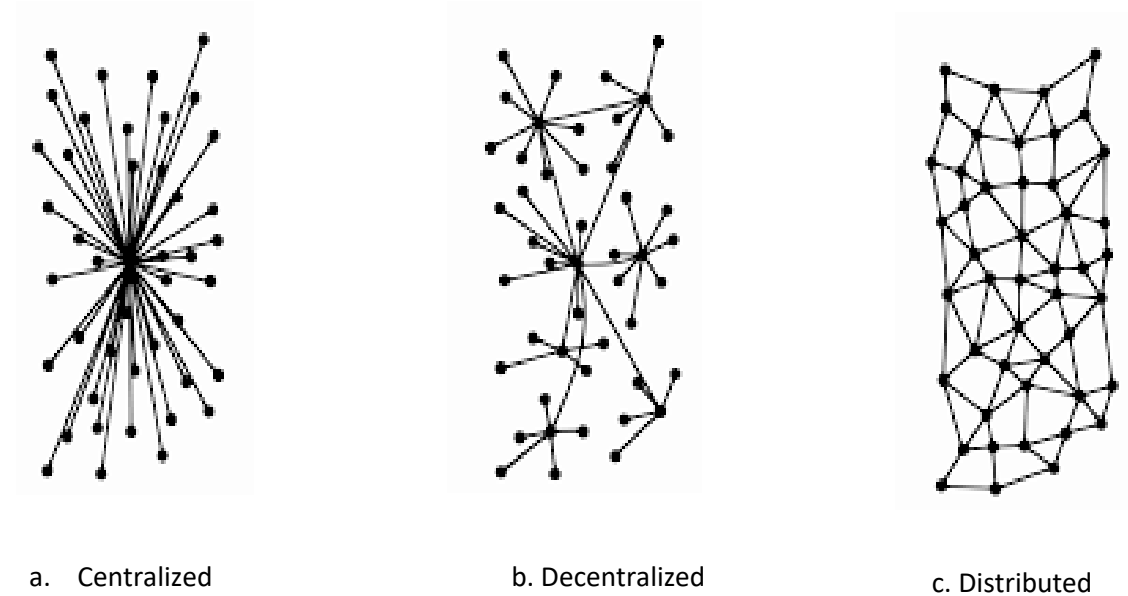


Figure 3: Communication networks (Adapted from Baran (1963))

Centralized interaction pattern

Figure 3a indicates that in a centralized pattern most (or all) members depend on the same individual. This is a vulnerable pattern that may fail to sustain itself in the absence of the central member.

Edwards (2015) and Helm (2017) denounce a teacher centred approach as limiting to students' intrinsic motivation and learning performance. In a learning environment, if the interaction of students develop into a centralized pattern, I feel that students do not possess all the four attributes of connectivism. It becomes necessary to re-design a learning approach for students to be autonomous, connected with peers, diverse and open minded. Hence, this supports the second research question of my study to investigate the impact of a learning pattern on practices of students and their academic achievement.

Decentralized interaction pattern

When the network does not have complete reliance on a single point, Baran (1963) termed it a decentralized network. In comparison to a connectivist learning system, although students connect and learn from peers Downes (2010) advocates that the role of the educator is to manage these connections. That implies a transition from a teacher centred approach to a student centred approach, promoting instructional support amongst peers. This results in the transformation of a centralized network to a decentralized network. However, the educator will still set the pace and have control of the learning process. For example, the educational reform movement in the United States of America recommends whole class communication in a mathematics class, but De Freitas (2012) finds that the educator has a regulatory role in a mathematics discipline. During the analysis of learning interactions, Duque, Gómez-Pérez, Nieto-Reyes and Bravo (2015) acknowledged that groups of students are either homogeneous or heterogeneous groups, subsequently educators need to handle such groups with flexibility. Homogeneous groups are described as comprised of students with similar skills and learning practices, while in a heterogeneous group students have diverse skills.

Distributed interaction pattern

The third type of a communication network that Baran (1963) introduced is a mesh Figure 3, that is characterized by high connectivity. This type of network mitigates the connectivity disadvantage when an active, transmitting node is eliminated as in both a centralized and decentralized networks. Mohamed *et al.* (2018) highlight that, if learning exists in networks, students excel in capacity and performance through the new learning theory of connectivism. This explains the idea that when peers are well connected, and self-directed, effective learning processes may be recorded. While Roth (2016a) affirms that recent learning theories invoke a distributed agency in learning of mathematics, it is indisputable to interpret his argument and represent it by connections between the peers.

2.4.2 Review of empirical studies and relevant research methods

De Laat *et al.* (2005) investigate the nature of interaction patterns of students in a networked community. Their findings contribute a foundational premise to my study. We have the same research objective to explore patterns and analyse the structure of these patterns. Furthermore, I will also

apply a social network analysis as part of my research method to understand these patterns. However, their study differs from mine in that they are informed by a networked learning theory while I am guided by a connectivist theoretical framework. Therefore, with regards to interaction patterns, I will further validate whether connections of students are constructed through principles of a connectivist approach. Downes (2012) refers to these principles as attributes of a successful network. From educators' perspectives, besides relying on students' narrated accounts of the origin of pattern, knowledge of students driving personalities may be relevant to sustain these interactions. Sometimes, herding or information cascading can influence students' interactions. This is described by Easley and Kleinberg (2010) as interaction influenced by others' decisions, without personal choices and decisions. This is the reason I focus on four principles of the learning theory of connectivism.

Furthermore, I reviewed the Grunspan, Wiggins and Goodreau (2014) study on the interaction of students and their learning behaviour within classroom environments. They recommended a correlation of measures of interaction and the performance of students to be a future study that enhances an understanding of cooperative and collaborative learning processes. The recommendation has inspired other studies already conducted in various learning disciplines for example Buchenroth-martin, Dimartino and Martin (2017). Researchers formulated various research questions that coined different methodological approaches. A social network analysis has been used to visualize and analyse student to student interaction. In these studies, authors focused on studying interaction and the learning performance. Little attention has been given to the nature and influence of the learning design under which students interact. Hence my study will consider the visual patterns from a social network analysis on interaction of students to validate the four principles of a connectivist learning design and evaluate the learning process.

In addition to the use of social network analysis, some statistical tests have been conducted to establish the relationship between the students' interaction measures and their learning performance. The measures of SNA have been correlated to student performance. However, Maxwell (2004) noted that the causal explanation between events and processes could be driven from a variable oriented approach (variance theory) or the process oriented approach. He argued that the variance theory deals with correlations between the variables. Literature indicates that studies on interaction of students usually correlate SNA measures of individuals and their performance to explain the students learning behaviour. A process oriented approach explains causation based on processes involved. My study will consider student interaction and student performance as dynamic processes, not events, hence a process based approach will be adopted by using a framework of integrated methodological approach (Plowright, 2011). The indicators of learning performance in my study will be achievement scores and their creativity and participation. It is because Cobb *et al.* (2011) identify

participation of students as an act that frames students' mathematical reasoning. It has also been classified as a social perspective in an interpretive framework used to analyse the individual mathematical reasoning and communal mathematical development.

I also reviewed a case study by Ergün and Usluel (2016). Their study explores the use of social network analysis on describing the interaction attributes of the students. Earlier, Downes (2012) stated that a social network analysis is an essential method for describing the connectivist dynamics within a learning network. These connectivist dynamics according to Downes are autonomy, connectedness, diversity and openness. The presence of the four in any particular learning network have been suggested to indicate that students share and develop new knowledge. The study into the learning interactions by Ergün and Usluel (2016) provides sufficient knowledge into the use of a social network analysis of both a face-to-face (classroom) platform and an online learning platform. Their data analysis communicated related findings between interaction behaviour of students in two separate learning platforms (online and classroom). Therefore, in my study on face-to-face learning interactions I will also employ a social network analysis before establishing the student practices and connectivist attributes.

Another literature review conducted is on an empirical study by Hernandez-Garcia *et al.* (2016). They also declared that their study was partly inspired by the recommended futures studies in the primer study by Grunspan *et al.* (2014). Hernandez-Garcia *et al.* (2016) identify that learning facilitators have difficulty in keeping track of student interactions and progress in online and classroom environments. To resolve the problem, they used Gephi software to visualise and measure the interaction of students with one another. Furthermore, they correlated the academic performance and interaction measures of the students. Statistical tests and analysis of variance (ANOVA) were conducted to compare the results from different groups. The data analysis has been applied quite extensively. However, determinants of academic performance have not been provided in their study. Contrary to their objectives and analytical approach, my study will additionally consider the nature of interaction patterns and involve student narratives aimed towards understanding students' learning behaviour within a connectivist learning design. This gap identified for my study may be closed if design of learning activities is specified as structured from an acquisitionist or a participationist framework (Sfard & Cobb, 2014).

In learning mathematics, Sfard and Cobb (2014) posit that an acquisitionist framework is when the learning is viewed as the acquisition of the pre-set content structures and procedures. While participationist framework comprises learning that occurs under formal and informal learning situations. The descriptions of these two frameworks can be attributed to Bernstein's (1999) vertical and horizontal developments of a person. For this reason, my study's second question particularly

focuses on academic performance. The indicators of knowledge production and not consumption of knowledge will characterise the measure of my students' performance.

Findings from Buchenroth-martin et al. (2017) indicate that student interaction networks are dynamic. Even though their study differs from my study on didactic discipline and the sample size, their study aim is on measuring the interactions. It is noticeable from literature that a number of studies also have the same objective to measure and understand the student interaction using a social network analysis as their methodology (Bolíbar, 2016; Simon, Finger, Krackhardt, Siewiorek, & Smailagic, 2015; Wang et al., 2018). However, it seems there is a need to determine and associate the individual, as well as the whole group interactivity with the extent of production of knowledge as promoted in the digital age. Buchenroth-martin et al. (2017) referred to some students with a high degree of interactivity as wanderers and those with low degrees of interactivity as settlers. To evaluate learning many questions can be raised on one considered a 'wanderer' or a 'settler'. It has not been made clear whether these two labels translate to good or poor learning performance. This suggests the necessity of student narratives through interviews. Hence, my second research question will address this inquiry.

2.5 Literature review on learning mathematics

This section presents the literature regarding learning mathematics in groups, and interactions during learning of mathematics. I also review to analyse the extent to which the learning practices of students show the four connectivist attributes.

2.5.1 Literature review on learning mathematics through groups

Literature indicates that both a cooperative and collaborative learning approach have been broadly used in evaluating learning of mathematics in various stages of education (Johnson, Johnson, & Smith, 2014). These studies use group work as the term that specifies the way students participate. Positive learning results are associated with group work in either the classroom environment, online platforms or in web based learning. In as much as students are benefiting from learning through these groups, it has been evidenced that educators also improve their understanding into the learning of mathematics in groups (Sofroniou & Poutos, 2016). However, in spite of the fact that my study is informed by connectivist research framework as a theory that embeds networks during teaching and learning, I find more gaps from literature with regards to the effect of working in groups. It has been cautioned that diversity of members in a group results in loss of motivation, for example when there is an imbalance in group member contribution towards the task. The lack of connection between the active and passive students results in decomposition of the group. Members opt out of the group in order to establish new connections with outside members (Hütter & Diehl, 2011). This evidences that grouping students based on any criteria could limit student agency as opposed to the effects of

exercising their own choices and decisions during learning (Rajala, Martin, & Kumpulainen, 2016). Therefore, I perceive that something better than grouping could be considered when encouraging autonomous learning behaviour. Hence, relevant studies may advance the knowledge on restructuring the dynamics within the groups formed by peers.

2.5.2 Studies on interaction during learning of mathematics

The literature indicates that both vertical and horizontal conceptions of learning are relevant in the area of mathematics (Bernstein, 1999). Vertical development is a hierarchical construction, with the assimilation and accommodation of new knowledge, and happens mostly within formal education. A horizontal development occurs when one develops mathematical skills from everyday contexts and applications through interactions with community (Wake, 2014). My study will focus on the horizontal approach, evaluating student learning through connections. Literature in mathematics education reveals that social interaction promotes effective learning (Cesar, 1998). Although Crawford (1999) notes that learning differences of individuals may have a negative impact on mathematics learning, interaction of peers have been recommended to improve learning. For example, Ng (2018) highlights that promoting peer interactions as a differentiated teaching approach to mathematics students from diverse backgrounds improves their confidence levels and academic achievements. In this current study these interactions between peers from different patterns of interaction would be represented in order to understand the manner in which students collaborate. This is the reason for validating diversity of students' on the learning practices. Therefore, the use of a connectivist perspective will make my study a unique approach.

Roth (2016a) notes when a mathematics student encounters problem solving difficulties, they construct better understanding through connecting with other peers. This observation contributes to the development of my study's second objective to seek an understanding of connectivist interaction patterns of students and their learning achievement. Roth and Maheux (2015) state that emergent learning is associated with learning of mathematics. As unpacked earlier in this chapter, connectivism constitutes implications of complexity which is also explained through emergent processes and outcomes. Therefore, it will be important to perceive both interaction and learning practices of students and note any emergent themes. Mitchell *et al.* (2016) describe emergent learning as a non-linear, unexpected, novel and newly created learning process, managed not by consensus but by principles of self-organisation. In their study, they add that while students can construct knowledge from visible conceptions, there are some invisible conceptions from phenomenological emergence that add to their knowledge construction.

The contribution by Roth (2016b) is that it is not uncommon to have mathematics students misinterpret the prescribed learning path, but successfully achieve some relevant knowledge. My

interpretations based on Roth and Maheux (2015) is that a teacher- centred learning approach limits the creativity and innovativeness of students. Hence, autonomous learning promotes negotiated learning in mathematical literacy. Therefore, the role of the mathematical literacy teacher should be clarified when implementing a connectivist learning approach. The learning conceptions of visible and the invisible learning will then effectively co-exist.

Furthermore, Roth (2016a) coins the phenomena, *growing–making* mathematics that results from distributed agency. The phenomena argue for shifting learning perspectives from transitive to intransitive learning. He gives a comparative discussion between the classical theories of knowing and learning and the recent theoretical advances on learning mathematics in a distributed agency. The purpose of Roth’s study is to emphasise the notion that students do not construct and hold on to mathematical knowledge, but they indirectly come with and continue to gain knowledge. This notion may be relevant in learning mathematics in vocational education and training. There is a need to master both abstract knowledge and apply that knowledge in future workplaces. My study will not assess the students’ competency levels in mathematics. Both research questions determine level of autonomy, connectedness, diversity and openness of students as they engage in their learning subject. Since my study intends to determine the effect of the student interaction patterns towards learning, the growing making notion may imply that a student- to –student interaction may additionally result in a significant learning. Rajala *et al.* (2016) describe such an effect as a distributed agency towards learning. So the phrase ‘growing together’ coined by Roth’s concept of growing –making mathematics informs the objectives of my study and the desire to couple the nature of interaction of students and their learning behaviour.

In the study of factors that result in poor performance of TVET college mathematics students, Ngoveni (2018) identifies a range of factors ranging from student level factors to college level factors. Multiple elements are listed in each category of these factors. However, from an analytical point of view, I submit that all these factors contributing to poor performance points to diversity among the students and instructional strategies employed. To mention a few factors listed under student level, anxiety around the subject, background for special needs in the subject and differences in levels of practicing and preparedness for the subject. All these factors display differences attributed to intellectual state of the students. Appropriate instructional means would be relevant in order to create a learning atmosphere that suits all individuals. For instance, in vocational colleges students, a student centred approach may promote social interactions, and improve performance (Cesar, 1998). According to the theory of connectivism, principles of autonomy, connectedness, diversity and openness of students may be sufficiently considered to characterize the learning environment (both online or classroom) (Downes, 2010).

2.6 Attributes of connectivism in learning design

In this current study I am going to look at the four attributes that describes a learning process under the theory of connectivism. These four are autonomy, connectedness, diversity and openness (Downes, 2012). The four attributes describe both the learning environment and the students involved.

2.6.1 Autonomy

The concepts of choice, control and independence have been identified as descriptors of student autonomy in a connectivist environment (Tschofen & Mackness, 2012). Students choose their own connections and information without external control by traditional instructional processes. Dron (2007) cited in Tschofen and Mackness (2012) notes that further to autonomy of choice and control, psychological autonomy of students should be recognised. Ryan and Deci (2002) described psychological autonomy as the source of one's behaviour through his interests and integrated values. A psychological perspective in the interpretive framework by Cobb et al. (2011) on the participation of mathematics students shows that intellectual autonomy of students is key towards learning. They described it as a way of participating in a learning community during understanding and acquiring social norms, socio-mathematical norms and acceptable mathematical practices.

Meijer, Bulte, and Pilot (2013) claim that the nature of science subjects (mathematics, chemistry and physics) limits the autonomy of students. They argue that these subjects have a content based curriculum, hence the learning pace and sequencing of the content will be dominated by a top – down teaching approach rather than a distributed teaching and learning approach. Earlier, Cobb et al. (2011) advised educators and facilitators to support the students' autonomy. It has been suggested that mathematical argumentation should not always be directed to the textbook or teacher. Therefore, students should learn a wide variety of mathematical contributions and judgements that are insightful, efficient and acceptable.

2.6.2 Openness

During my literature review I find the principle of openness worth mentioning from both a network and a personality trait perspective. From a network perspective, the principle of openness is promoted to allow people a free entrance into and exit from a learning network. Downes (2010) states that people should not be barred from a learning system. He adds that the flow of ideas and artefacts must be free. Many studies conducted on interaction of students, shows some decrease in the number of participants attending an open learning course. The quantitative data analysis performed on determining the interactivity of the remaining participants does not highlight factors that resulted in other participants dropping out (Buchenroth-martin et al., 2017; Grunspan et al., 2014; Mitchell et al., 2016).

In psychology, one's personality is rated on the basis of the so called big five factors namely extraversion, agreeableness, neuroticism, openness to experience and conscientiousness (Costa & McCrae (1992) in DeYoung, Peterson & Higgins (2005)). However, due to the scope of this study, only the openness trait had been extracted for its match with a connectivist learning approach. According Schretlen et al. (2011) openness as a personality trait characterizes an individual's level of intelligence and divergent thinking. In addition, Fraser-Thill (2019) describes a low level openness person as routine, predictable and naïve to novelty, while a person high in openness is a flexible, curious and adventurous. These implications of openness may open a broad field of research for the understanding of a connectivist learning approach. Although literature contains these references, no empirical study has yet been premised on a connectivist framework which has evaluated openness from a psychological perspective.

2.6.3 Diversity

The aspect of diversity is a broad term that refers to properties of students relative to each other. It also refers to disparities within and across the confluence of the micro, meso and macro spheres of interactions in a learning environment (Bolíbar, 2016). In education, the principle of diversity is attributable to states of all agents (people, environments, tools and content) involved in the teaching and learning process. Studies conducted on interaction of students, report on diversity of participation of students. This is the aspect that my study focused on in order to evaluate the design of a connectivist learning approach. In connectivism terms Downes (2010a) refers diversity to the state of a designed learning system. Davidson (2017) indicates that diversity of students is among the determinants of their Personal Learning Environments. Therefore, all that is experienced by students should be considered in order to understand their level of diversity.

Tschofen and Mackness (2012) suggests that the strength of connectivism's diversity principle is in the potential to support self-motivation and competence of students. However, they note that due to the complex nature of connective learning, diversity may be limited from the outset due to a need of high competence levels and communication skills. Studies on interaction and participation of students investigated the student competence based mostly on age and gender as elements of student diversity.

Little has been reported on the principle of diversity in the participation of students during the learning of mathematics. Roos (2015) conducted an inclusive mathematics intervention that revealed diversity among students, and now advocates for a differentiated teaching approach. In the data analysis of this study, interactivity of individuals will be measured, and their participation during learning will be explained. Jung, Lee, and Karsten (2012) discovers the significant difference in

diversity of responses between the contributions from extravert and introvert students. Quality and quantity of student contributions may need further investigation to contribute towards the diversity of students' intellectual capabilities in mathematical learning. This study's question of interaction patterns of students seeks to identify the extent of diversity in college mathematics students and how it attributes to their learning behaviour.

2.7 Conclusion: Gaps in literature

In the background of this literature review I present aspects of connectivism. These are the theoretical principles of chaos, complexity and network theories. I focused on the implications of these principles in a learning process. It is understood that connectivist learning is distinguished from other traditional theories through perspectives and implications of these principles. However, it was rare to find literature on connectivist learning (both theoretical and empirical studies) acknowledging the implications of chaos, complexity and network theory. Instead, a significant number of studies put forward the idea that the learning process is a manifestation of chaos principles, complex dynamical systems and networked systems. I am of the view that there is an intellectual gap for researchers on connectivist learning approach to address. That is, a more detailed theoretical connotation of chaos, complexity and network principles should be outlined through empirical studies.

The non-linearity property of a learning process should be advanced regardless of one's philosophical position and research strategy. This need to be contained in the aim, conclusion and the recommendations of a connectivist study. Meanwhile, related insights are portrayed through empirical studies underpinned by social interaction. It is evidenced that effective learning of mathematics is achievable through social interactions (Price, n.d.). In networked learning environments students' connections with peers promote personalized and differentiated learning styles. However, although students' general learning performance has been informed, there is little analysis characterizing individuals behavioural and learning practices during their social interactions. I suppose a connectivist theoretical framework may be inculcated in studies since it has wide analytical framework. There is need to explain the complexities that are constituted within such learning processes. The connectivist studies in the classroom environment highlight presence of the four principles, autonomy, connectedness, diversity and openness (Bannister, 2016). The extent and conditions to which these four connectivist principles still need to be established.

2.8 Research questions

The level at which students either experience or portray the four principles of connectivism needs to be investigated. Either contextual or situation based perspectives would be substantive towards adding knowledge about connectivist learning practices. Various studies have been conducted about student connections. Further interpretive analysis has to be established from a connectivist

perspective on whether students connect and interact under the four principles of connectivism. In order to contribute further knowledge of these connectivist practices, I hypothesize that the four principles are situationally experienced. Hence, they may be identified based on the nature of interactions and the levels of participation of students in these patterns. Thus, my first research question is:

RQ1. What patterns of learning interactions appear from a connectivist learning of mathematics with TVET college students?

This will establish whether college students interact in static patterns or in dynamic patterns. In the learning of mathematics mixed perceptions are rebutting the interactivity, hence the extent to which they display connectivism principles (Bannister, 2016). In addition to exploring the patterns of interaction of peers, I extend my view in explaining their learning practices and their learning achievements. My second question therefore, is:

RQ2. How do these patterns inform about students' learning practices?

My study rationale is to contribute knowledge towards research into designing and developing a model for a connectivist teaching and learning practice.

2.9 Research framework

To address my two research questions, my study is premised on the connectivist concept of networks. Since learning is a non-linear process, all the dynamics that are experienced in a learning process are not predictable although they can be controlled (limited or influenced). In this current study, the attention will be given to student interactions with their peers and their learning performance. The desire is to validate the four principles of learning theory of connectivism namely student autonomy, diversity, interactivity and openness. The connectivism theory will be my theoretical framework. I will be mindful that the learning dynamics are attributable to principles of chaos, complexity and network theories. The Analysis, Design, Development, Implementation and Evaluation model of instructional design will underpin the design of the learning process. The proponents of learning theory of connectivism urge that a social network analysis may be used to understand connectivist learning (Siemens, 2005a). Hence, my analytical framework will include numeric data, student narratives and visual representations of peer interactions. I will evaluate the learning process with a focus on students' actions during learning and also their reactions towards their peers. The diagram below shows the conceptual framework for my research. Connectivism is the overlying theory that guides my research study. In the primary stage, I have designed a learning intervention in which students will learn through connecting peers. The patterns and interaction practices and learning practices of students will be recorded and interpreted according to Cobb et al. (2001) interpretive framework.

The lens of my interpretation will consider implications of complexity, chaos and network since the three are the constituting theories of connectivism.

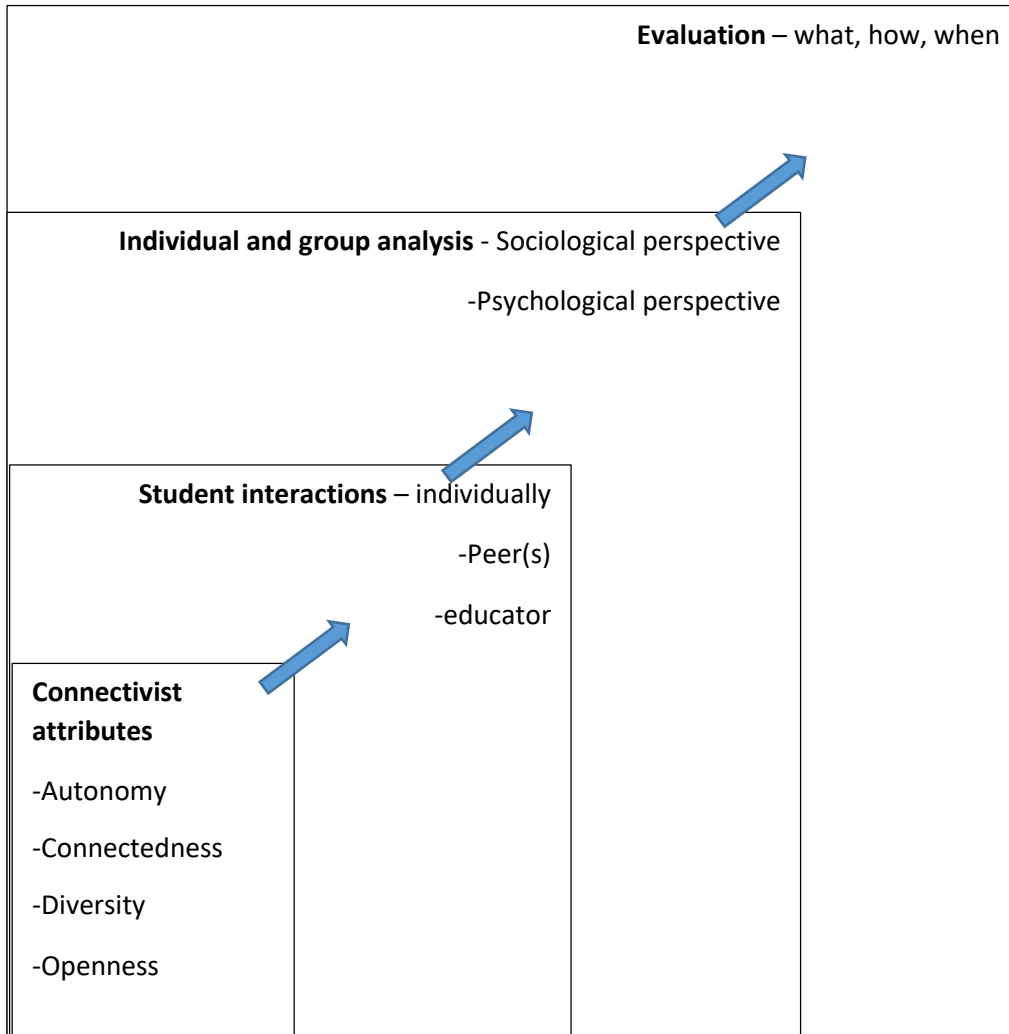


Figure 4: The research conceptual framework

2.10 Summary of chapter 2

From reviewing the literature, I find that interactions of students are not fully described as dynamic processes. They are presented as permanent, fixed structures. The literature is short of informing the dynamics that entail the interactivity of students. I propose that addressing this shortfall through the lens of connectivism will contribute significant knowledge into practice and research on both connectivism and learning interactions. In this chapter I have unpacked all the underlying components of connectivism that have seldom been used in literature to characterize learning interaction across the various learning areas. I have identified gaps that will be addressed through my study by validating connectivist attributes of interacting students. In chapter 3, I provide the steps and procedure of my learning intervention that I conducted with students and my data collection. The methods adopted in

chapter 3 shall endeavour to cover gaps identified in literature, and thus address the two research questions of my study.

Chapter 3

Research Design

This chapter explains how I approached data collection and analysis in order to address the two research questions of my study. I start by unpacking pragmatism and its relevance as the underpinning philosophy of my research. This study is based on the learning and interaction practices of college mathematics students. I conducted a teaching and learning intervention informed by guidelines of a connectivist teaching and learning approach. Stages and processes of this learning intervention are presented in this chapter. Furthermore, I introduce and explain Plowright's (2011) Frameworks for an Integrated methodology (FraIM) that informed processes of my data collection and analysis. In addition, I outline the procedures employed during a purposive sampling and a peer assessment. I end this chapter with an interpretive framework that substantiates the process of validating the principles of connectivism in students. In the concluding part of this chapter, I indicate how the trustworthiness of my data is ensured.

3.1 Pragmatic research philosophy

As mentioned in the previous chapter, my study is informed by two research questions: What patterns of interaction emerge during a connectivist learning of mathematics with vocational college students? And, how do these patterns inform about the students' academic achievements? The first question led me to explore the interaction patterns of students, while the second question yielded some explanation that addresses the interaction practices and academic achievements in students' interaction patterns. I therefore adopted a pragmatic philosophical approach in this study. Plowright (2016) describes Charles Sanders Peirce's pragmatist philosophy as a logical way aimed at understanding a concept in terms of the practical outcomes. While traditional research approaches such as correlational studies, surveys and experiments are predominantly used in education research, Van der Akker (1999), as cited in Plomp et al. (2013) refutes that these approaches provide useful prescriptions in education. Hence, the philosophy of pragmatism is widely considered in education researches. For example, Adeleye (2017) hints that pragmatic philosophical implications explain the interplay of the educators' actions between a teacher-centred approach and a student-centred approach.

Koopmans and Stamovlasis (2016) argue that an understanding of a learning process is underpinned by a plethora of complexities. In this current study, I describe the learning behaviour of an individual based on the interaction attributes of the group. In my pragmatic approach I considered Ostrow, Wang and Heffernan's (2017) concept of data *flexibility*. They argue that various processes can be considered when handling data collected and analysed during teaching and learning. Therefore, I used

observations, social network analysis, interviews and peer assessments as various methods to collect and analyse data.

3.2 Research method

I used the Framework for an Integrated Methodology (FraIM) as my research method. Plowright (2011) proposed FraIM as a model that is not restricted to principles of quantitative and qualitative research concepts. Instead, Plowright's model supports the convenient and logical integration of various epistemological elements and principles in a research process towards an effective study of a given phenomenon. He adds that procedures for collecting and analysing data are neither restricted to mathematical nor statistical analyses but can include pragmatic processes that conveniently address both research aims and questions. Hence, I have employed social network analysis to evaluate the interactivity of individuals and the entire group of voluntary students who participated in this study. Under this research method, quantitative data are regarded as numerical data and the qualitative data narrative data. Plowright (2011:17) explains that numerical data refer to numbers obtained from counting or measuring, and also that the narrative data can be words, still and moving images and audio texts. I used three methods of collecting data that are recommended when using FraIM, and these are observation, asking questions and peer assessments. I will unpack the use of these three methods in later sections of this chapter (sections 3.4.1, 3.4.4 Interviews)

I observed students during their learning interactions in order to gather data for my first research question. During observations I collected numerical data by counting the number of students in a particular group. Furthermore, I counted the number of times a certain individual interacted with other students. Out of the recorded data, I used an excel spreadsheet and Gephi software to generate socio-grams that would illustrate patterns formed by interacting students. Bastian, Heymann and Jacomy (2009) present Gephi as an open source software for graph and network analysis. This software is commended for its network multiple features for exploring and manipulating interaction patterns to enhance readability and interpretation of network data. The socio-grams (network diagrams) became the artefacts in my study. I further analysed and interpreted data using the numerical measures generated in Gephi. This enhanced some understanding of individual and group interaction practices. In order to address my second research question: How the patterns of learning interaction inform about students' practices impact on their learning achievements?

I obtained narrative data through asking students some questions (through face-to-face interviews) about their interactive behaviour. They provided their perspectives on individual learning practices. While Plowright (2011:120) advises that through FraIM, numerical data and narrative data can be integrated without being restricted to mathematical or statistical analysis, in order to validate the

four attributes of connectivism, I evaluated interactivity of students through written descriptions and explanations drawn from social network analyses. In order to collect all the data for my study, I first designed a learning intervention for voluntary adult students (all above the age of 18 years) who gave me their consent to be regarded as my research participants. Out of 42 students who volunteered to take part in this intervention, on the first session only 24 students were present. The attendance increased to 36 on the second learning session and on the last session the figures dropped to 28. There were no restrictions imposed on students about entering or leaving the learning intervention.

3.3 Structure and process of the learning Intervention

I conducted a learning intervention during the time when students were preparing for their end of year examinations. Therefore, the content we covered had been taught earlier during the year. The intervention was conducted for three days, with one-hour of in class contact learning sessions per day. The instructional design principles of Analysis, Design, Development, Implementation and Evaluation (ADDIE) model informed the process of this intervention. According to Nichols Hess and Greer (2016) instructional design is:

“the science and art of creating detailed specifications for the development, evaluation, and maintenance of situations which facilitate learning and performance”

The definition is a collective of all aspects of learning that are incorporated in order to achieve effective teaching and learning. This affirms the relevance of a pragmatic philosophical position of my study. It is noticeable that Nichols Hess and Greer (2016) contend that the ADDIE model of instructional design can be used to achieve a variety of instructional interactions in face to face interactions as much as it is in online learning. Since my study is underpinned by a connectivist theoretical framework, I expected a greater variety of instructional methods to be shared among the students during their interactive learning than the input students would receive from the co-educator who volunteered to facilitate in this learning intervention. The voluntary educator conducted most of the facilitation role and also attended to the concerns of the individuals while I focussed on evaluating and re-designing the learning material. I also focussed on observing and recording the data on connections among the students in order to gather information for my first research question. Below is a diagram (Figure 5) that presents the ADDIE model as an iterative process of conducting a learning intervention. All five processes of the ADDIE model are outlined below the diagram, expanding on the stages of my learning intervention and the manner in which principles of connectivism were inculcated to ensure that I promoted a student centred approach.

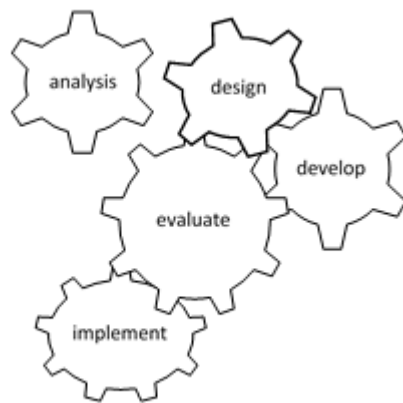


Figure 5: Processes of the ADDIE model on the learning intervention

Analysis

Students chose the learning outcomes they wanted to be covered. This ensured students to be open minded, have autonomy and exercise diversity as attributes of connectivism. During an evaluation of a model that measures effective learning during a connectivist learning Mohamed et al. (2018) indicate that autonomy, diversity and openness of students contributed to high achievements in students. The chosen outcomes as stated in the DHET (2013) vocational mathematical literacy curriculum are: 1. Perform calculations correctly to solve problems regarding tariffs systems and financial interest in workplace contexts and other areas of responsibility, including national/global issues. 2. Draw graphs to determine the break-even points in a business environment and 3. Draw up a projected plan/budget/cash flow forecast for a business based on expected income and expenditure. Students indicated their competency levels on each outcome in oral discussions before the first learning session of the intervention, thus personally defining the extent of the support they needed. The previous teaching and learning approaches were also discussed in order to introduce the connectivist learning approach. My intention was to equip the students with a range of perspectives to differentiate between a teacher-centred and a student-centred learning approach. This guided the students solve their mathematical problems by engaging their peers more than the educators and the design of the intervention became more socially and dialogically oriented.

Design

While the desired learning design in my study was social and dialogical, this intervention promoted student interactions. I prepared the learning material for a problem based learning (PBL) strategy. This promoted collaboration among the students, and also reduced the number of students that worked in isolation. Smidt and Thornton (2017) affirmed that PBL contributes to both personal and social development of students. The PBL questions were designed based on Anderson and Krathwohl (2001) Revised Bloom's Taxonomy. Three cognitive domains of the taxonomy were chosen to intrigue higher

order problem solving skills and these three are applying, analysing and evaluating. Furthermore, the numbering of desks was designed for anonymous student reference. Some desk positions were changed in order to avoid students' obvious comfort zones. All students were mandated to maintain their desk number for the entire period of the intervention.

Development

Widyastuti (2019) in his development of the mathematical learning material emphasizes that learning material and resources should be in accordance with the needs of the students. Therefore, prior to this intervention I consulted the students to suggest the sequencing of the learning sessions. In addition, this process of developing the intervention involved preparation of students to be aware of the manner they had to participate. I emphasized this along with the four attributes of connectivism that are validated by this study. Views of students were also requested in the development of the learning material. I compiled the worksheets that were used in this intervention. These were handed out on the day of learning. Students used their own extra resources and their preferred peers to do their work as a way of promoting diversity of learning material and encouraging students to extend their connections with the learning content through open sources. We explained the four principles of a connectivist interaction (i.e. autonomy, connectedness, diversity and openness) to students. Therefore, during the implementation of the learning process, the educator and the students were both aware of when and how the learning interactions were to be adopted.

Implementation

A voluntary mathematics educator assisted students by moderating the learning interactions. We arranged students' desks, spaced out in rows and columns for easy movement of students during peer interactions. We handed out the learning materials every day at the beginning of the intervention session. Students embraced a self-directed learning approach. They set their own targets to solve mathematical problems based on the learning outcomes mentioned above during the analysis stage of my intervention. During the learning intervention students were mostly self-organised. They worked according to both their own learning pace and space. The interactivity of the outstanding students who showed great confidence were tracked and compared with the interaction practices of students who seemed to have little confidence and level of interactions. The intervention lasted for three days in order to ensure manageability of captured data using video for analysis after the intervention. Too many learning sessions would imply a high volume of observation data to be analysed establishing the patterns of learning interactions.

Evaluation

Mckenney and Reeves (2014) indicate that the goal to evaluate an intervention is either to keep driving the development of the intervention or to inform participants of new knowledge. For this

study, I only focused on the process of the intervention and not achievement of the learning outcomes after the intervention. That means only the interactivity of students with peers was considered. That would allow me to use video footage and track the patterns of interactions in a manageable manner. Therefore, I used a video camera to record all the movements of students while the students engaged with each other. Furthermore, I took down the notes about the frequency and duration of engagements. After the third learning session, I took all the notes and the recorded video footages for analysis. The steps of this data analysis are in the following section.

3.4 Research Data collection.

Data were collected over a period of three days. This was to ensure all the three learning sessions were accounted for in my analysis and results. Each research question was approached with a data collection method different from another research question. Numeric data were collected towards addressing the first research question. The frequency in which particular students engaged with each other had to be captured, before representing peer connections with network diagrams generated in Gephi. During the learning intervention I focussed on collecting data while the voluntary educator facilitated the learning process. On a few occasions I attended to students who required clarity on the questions. Narrative data were gathered for addressing the second research question. The entire process of my data collection is outlined in the figure below. The diagram shows specific methods of collecting data used to address each research question. Moreover, this diagrammatic representation displays the manner in which data collected between the two questions address the purpose of the study.

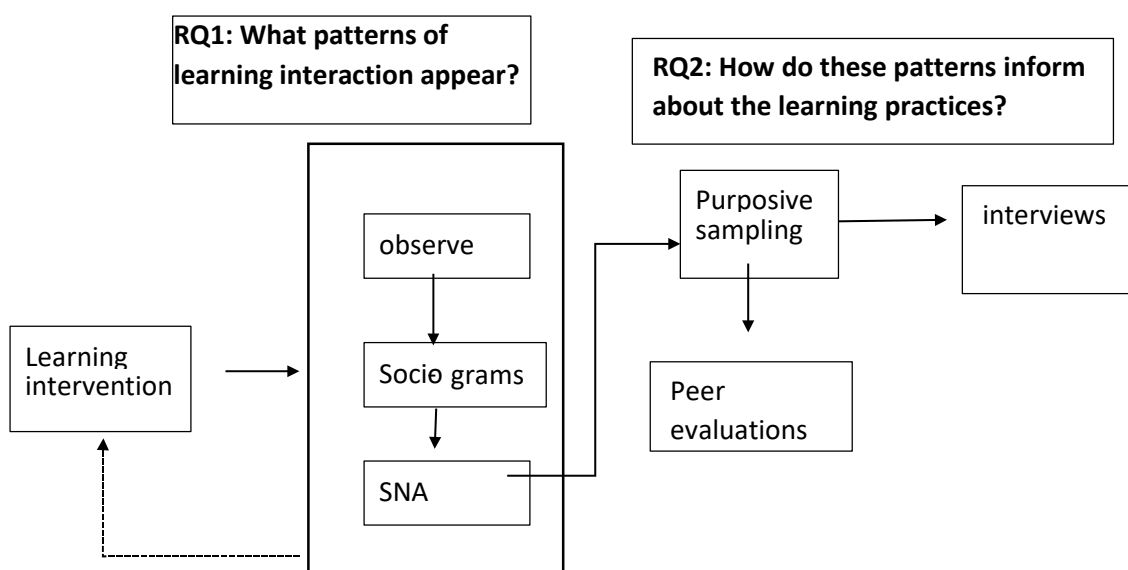


Figure 6: Research data collection process

3.4.1 Observations

Mäkitalo (2016) urges that observing participants is an effective method for tracking their autonomy and independence during the learning interaction. Therefore, the observation process was my primary stage of research data collection, in order to identify and track the patterns developed by students and their connectivism attributes in these patterns. I addressed my first research question through the observed data. I observed the students as they interacted with peers. I observed the learning interactions through an observer-as-participant. Plowright (2011:67) emphasizes that although the observer and the participants interact during an observer-as-participant approach, the two stay detached from each other. The advantage to this approach would be that there is limited interference in participants from the researcher. In all the learning sessions of the intervention, students were mostly facilitated by the voluntary educator. Since my study is guided by a connectivist theoretical framework, as a researcher I ensured that I did not have significant influence on the interaction process of students. I used a video camera to capture all the learning sessions. I also recorded notes on the interaction dynamics for further use in my data analysis. I used this data further to inform the next data collection stage of interviews to generate diagrams showing patterns of learning interactions.

3.4.2 Purposive sampling

A purposive sampling was conducted before interviewing the students. Six participants were sampled based on both their positions in the generated socio-grams and their sociometric measures. I used the criterion of extreme measures. That is students with lowest and highest value of in-degree, out-degree and betweenness were used. According to Agneessens, Borgatti and Everett (2017) degree centrality (in-degree and out degree) reflect the influence one had while interacting with peers in his/her surrounding circle. Therefore, I considered those who were least surrounded and others who were most connected with peers. In these connections the measure of in-degree records the communications a particular individual received and the out-degree records the amount of communications a particular individual sends out to various peers. Furthermore, betweenness measures the extent to which an individual brokered between different patterns of interactions (Freeman, 1977). This measure has been quite significant to the aims of my study as it shows interacting students with significant impact towards unifying the whole group discussions, especially when a particular learning discussion spreads across the whole class.

3.4.3 Peer evaluations

In order to develop interview questions specific to the unique attributes of the sampled students, I conducted a peer assessment. This process yielded insights on a sampled participant's inputs and contributions during interacting with peers. The entire group applied their perceptions to evaluate the

six sampled students. The evaluations were based either on actual (known) or suggested experiences from interacting with or observing a given participant. I engaged the whole group of twenty six students to evaluate the six sampled students. While Ostrow et al. (2017) hint that there is a unique perspective of understanding through manipulating or combining data, I requested peers to evaluate the sampled participants using nominal data. The evaluation form contained closed questions for peers to evaluate one's performance per learning session as poor, average, good or very good (Appendix I). In the event that they did not interact directly with the sampled student, I advised them not to give any comment, unless they wanted to apply their personal view based on the interactivity of the sampled student. In addition to drafting of interview questions, the general performance of a sampled learner contributed insights towards addressing my second research question that investigates interaction practices and academic achievements.

3.4.4 Interviews

In studies of social network analysis during learning interactions, Bolibar (2016) indicates that collection and analysis of narrative data is more precise when the interviews are preceded and guided by the observation method. In the current study I conducted the interviews on all the six purposively sampled students. I prepared four open-ended questions. Two questions required students to outline in detail, their interaction practices and also their recommendations for future design of a connectivist interaction based learning. The other two questions required students to outline their learning process while interacting with other peers and also to recommend future considerations for effective learning during connectivist learning. Cohen et al. (2007a:170) highlight that no predetermined responses should be framed by interview questions. Therefore, in addition to the four questions stated above, I posed some individual specific follow up questions to gain a deeper understanding into students' interaction and learning practices. These follow up questions were informed by an individual's position in generated patterns of interaction, values in the measures of social network analysis, peer assessment results and also their responses from the four main questions. On average, each interviewee responded to a total of 23 questions. I conducted these interviews in the same venue where learning took place. I used the venue in order to assist students to recall their sitting positions and their movements during the learning intervention.

3.5 Data Analysis

An integrated data analysis that uses observed data and interview data has been recommended towards gaining reliable insights on how students negotiate their mathematical identities (Harper, 2019). I analysed my data in three stages. Firstly, I analysed my observation data per learning session to address the first research question. Secondly, the interview data was analysed to address my second question. And finally, I conducted a methodological triangulation using data collected and

analysis through observations and interview processes. Cohen et al. (2007a) reveal that triangulation is a method that enhance the validation process in a study. In the case of my study, the purpose is to validate the attributes of connectivism from the learning interactions of students, based on four principles of connectivism autonomy, connectedness, diversity and openness.

3.5.1 Data analysis Procedure

My data analysis has been guided by the concepts of design research, network analysis and interpretive evaluation and validation. The participants interacted in a learning intervention designed through the ADDIE processes of instructional design. The evaluation process of my intervention was conducted at the same time as the data collection process for my study. During the time of stimulation and regulation of learning interactions, I observed the students' practices in these interactions. After establishing the interaction patterns I conducted some peer assessment and interviews on purposively sampled students. Using Gephi I generated the interaction patterns of students based on trends observed from peer interactions. The premise of analysing these patterns addressed my first research question, and the analysis of the peer assessments together with the interview transcripts addressed my second research question. Finally, I synthesized findings from my two research questions to accomplish my research purpose. The research purpose is to evaluate a connectivist learning approach through validating its four principles, autonomy, connectedness, openness and diversity.

Patterns of Interaction

I used data from observations to come up with socio-grams (diagram representations) of the patterns of interactions. I presented these patterns per learning session. After the learning intervention I identified all the interaction patterns that emerged within and across the learning sessions.

Diagrammatic presentations

Since my first research question investigates the patterns of interactions that emerge through student interactions, network diagrams generated in Gephi were presented in the form of network diagrams. These diagrams only presented the structural nature of interaction patterns after every learning session. Students were seated in five rows (A, B, C, D and E), with nine students in each row. The pseudonyms were used on students according to their seating positions for example A1 for the first desk in row A. In the next chapter of data analysis these patterns will be presented, compared and contrasted. However, the interpretations of individuals and the entire group were advanced through the use of numerical SNA measures.

Social Network Analysis

Bolíbar (2016) argues that a social network analysis is a research methodology that clarifies an understanding into the interaction of community members. However, according to the aims of this study (to explore the patterns of interaction), I have selected a few measures of SNA to add some

understanding to the diagrams generated by Gephi. I focussed more on a narrative analysis than the numerical analysis of the learning interactions. My first research question, pertaining to the exploration of patterns of learning interactions, used diagrams to display patterns as they emerged across the three learning sessions. I first captured data from observations in an Excel spreadsheet, and then exported it into Gephi (an open software) (Bastian et al., 2009). The software generated diagrams of interaction in the form of network diagrams

Interactivity versus learning practices

My second research question investigates how interaction practices influence the learning achievements of participants during mathematics learning. I addressed this question from a narrative perspective. Buchenroth-martin et al. (2017) applied a correlational analysis in their study to address a related objective. However, since I was guided by the FraIM approach, I applied a narrative analysis. I adopted this particular approach because I only started with 42 students, a smaller sample size than their study with 115 students. Hence, a pragmatic perspective was quite convenient for my study.

Analysis of peer evaluations

In order to address the second sub-question of my study, I started with an analysis of ratings provided by peers on the six sampled students. I put together the results of assessment per candidate, evaluated in four categories. These four categories are poor, average, good and very good. The six purposively sampled students were rated by 26 students available on the day of evaluations. I presented the findings in form of a visual diagram that displayed clusters of students who rated a particular candidate. A diagrammatic form of clusters added a sense of connectivity by which a particular student might be perceived. I further drew up the interview questions per particular individual based on ratings awarded by peers.

Analysis of interview data

The interview scripts were transcribed using Google docs. From these transcriptions, themes were drawn. The interview questions required information about two categories; interaction practices and academic achievement. Each category was comprised of four themes. I equated all themes to the four attributes of a connectivist learning approach. These are namely autonomy, connectedness, openness and diversity. The four underpinning principles guide the evaluation of a connectivist approach in this study.

3.6 Interpretive evaluation

My research aim was to evaluate the four attributes of a connectivist learning approach. I evaluated these four attributes (autonomy, connectedness, diversity and openness) by filtering through the findings of my two research questions. The first question looks at interaction patterns and the second question investigates interaction practices and their impact on students' learning achievements. I

adapted Cobb et al.'s (2001) Interpretive framework for my validation process. The framework reveals that interaction of students during learning of mathematics is identified through sociological and psychological perspectives.

3.7 Trustworthiness of data

Houghton, Casey, Shaw, and Murphy, (2013) urge that trustworthiness of a study lies in generalizability and/or transferability of its findings. In my numeric and narrative study, the findings in this study may not be generalised since my analysis was not restricted to quantitative (mathematical and statistical) principles. However, my research aims in this study are informed by a narrative and artefact analysis, and the findings can be transferred into various academic fields of study and learning platforms. Therefore, in this study I ensured that transferability as an aspect of trustworthiness would be ensured through credibility, dependability and confirmability.

Dependability

To ensure that the findings of my study can be relied upon, I implemented my data analyses using both excel spreadsheets and Gephi software for mapping out the connections between and among the peers. The moments when the learning interactions captured on a video got messy and repeated, the diagrammatic representations provide a simplified trace of all the interaction dynamics that transpired. Rice (2018a) suggests that dependability of the results may be threatened if the method of data analysis is not clear in a connectivist study. Therefore, I went through my video footage and my observations to ascertain that all interactions were noted.

Credibility

According to Houghton et al. (2013), credibility refers to the extent to which the study and its findings can be believed in terms of its aims. I enhanced credibility of my findings through a methodological triangulation. In social science studies Cohen et al. (2007a) describes triangulation as a multi-perspective way to acquire the richness and complexity of human behaviour. They add that methodological triangulation involves using different research methods and applying either normative or interpretive techniques. Since in my data analysis I used both narrative and artefact analysis, ensured credibility of my findings through an interpretive evaluation on observed and interviewed findings is ensured. Both themes from interviews and observed patterns confirms presence of autonomy, connectedness, diversity and openness of students.

Confirmability

While Houghton et al. (2013) suggest that methods of dependability address the confirmability of the findings, I further ensured that Cobb et al.'s (2001) interpretive framework was used. Earlier, this framework affirmed the participation of students during the learning of mathematics. The

perspectives associated with analysing the individual student and the group of students reveal the attributes of connectivism that are being validated in this study.

Summary

In this chapter 3, I presented pragmatism as my research philosophy. I explained FraIM as my research method. Before explaining my data collection methods, I presented the instructional design model of ADDIE and its processes that I followed to teach 45 voluntary students. Three methods of data collection that I explained in this chapter include observations, interviews and peer assessments. Before employing my peer assessments, I indicated that I conducted a purposive sampling on my students based on three measures of social network analysis. These three are in-degree, out-degree and betweenness. I explained them in this chapter. For my data analysis I indicated that I used Gephi, a free social network analysis software to generate the patterns of interaction of students. I described the Cobb et al. (2001) interpretive framework that I used on themes from my interview data. In chapter 4 I will present the results of my data analysis.

Chapter 4

Results

4.1 Introduction

This chapter presents findings that address my two research questions. The first question is: What patterns of interactions emerge during a connectivist learning approach with vocational college mathematics students? The second is: How do students' practices within these patterns impact on academic achievement? The purpose of this study is to evaluate the feasibility of a connectivist learning approach during mathematics learning in vocational education and training colleges. My data analysis is informed by two approaches. I first instituted an inductive data analysis approach based on patterns observed from student interaction. This is in response to my first research question. With regard to my second research question, I applied a deductive approach from interview themes based on individuals' interaction with their peers as well as their academic achievements. The diagram below outlines the structure of this chapter and the stages of my data analysis and findings.

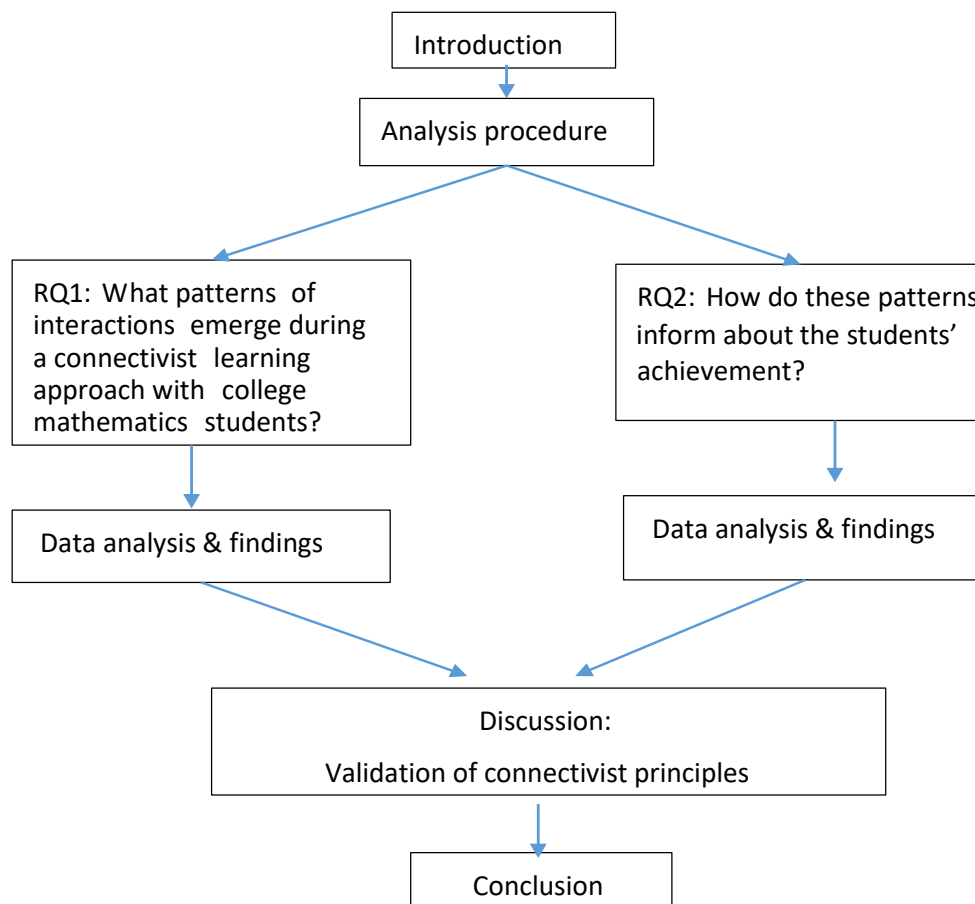


Figure 7: Structure of chapter 4 data analysis

I have structured this chapter by starting with a layout of the procedure of my data analysis. Then I have presented and analysed the data on each research question. Within the analysis of my data I

have evaluated a connectivist learning approach through validating the four principles autonomy, connectedness, openness and diversity. Student to student interactions were analysed to establish emergent patterns. My findings on patterns of interaction (first research question) shows that students participated pairs, subnetworks, decentralized patterns and collaborate groups. Interestingly, each of these structures is characterised by varying levels of autonomy, connectedness, openness and diversity of students. In my second research question, I find that student narratives yield themes about students' interaction practices and also academic performances. The students' responses explored the way they interact with their peers rather than explaining the possible relationship between interaction practices and academic achievements. I employed a methodological triangulation in order to achieve trustworthiness of my validation study.

4.2 Data analysis Procedure

I generated the network diagrams based on trends observed from peer interactions. The premise of analysing these networks addresses my first research question, and the analysis of the peer assessments, together with the interview transcripts, addresses my second research question. Finally, I synthesized findings from my two research questions to accomplish my research purpose. The research purpose is to evaluate a connectivist learning approach through validating its four principles, - autonomy, connectedness, openness and diversity. I sequenced my data analysis as represented in Figure 8 below:

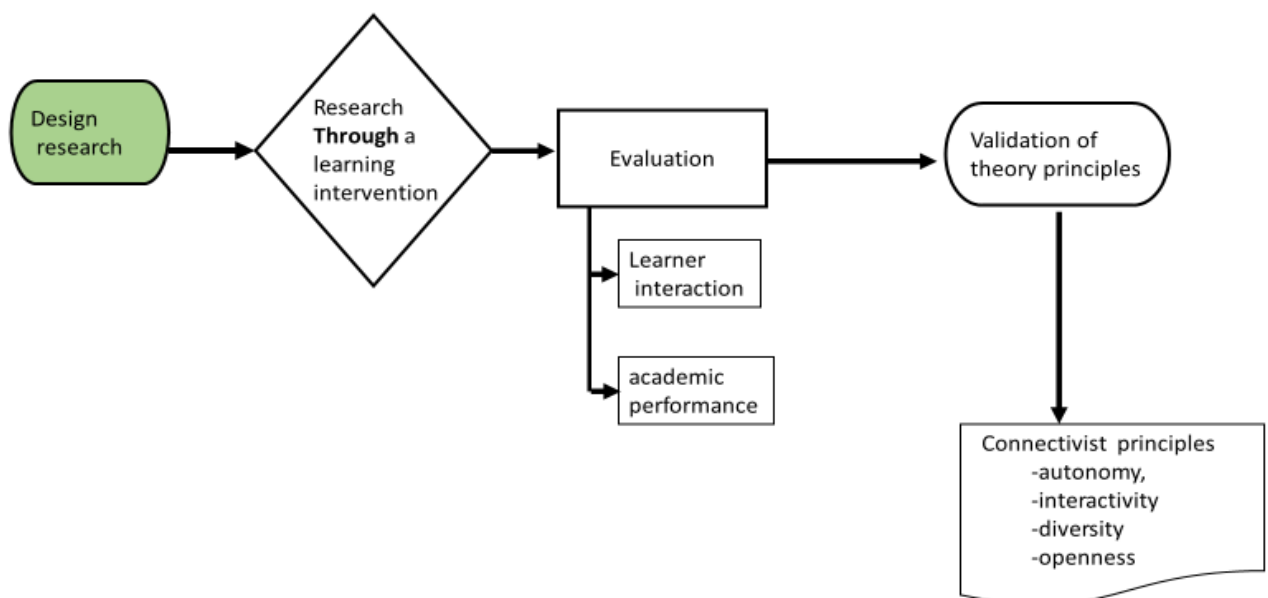


Figure 8: Conceptual model of research data analysis

I used two instruments to collect my data, an observation instrument and an interview instrument. The observation data translated into network diagrams. I extracted the structure of interaction patterns that emerged from these network diagram and applied a social network analysis to explore

the interactivity of individuals and the group. My data analysis has been informed by the conceptual model derived from the literature survey (*Figure 3*). The codes of my data analysis are established by individual practices.

After the observation instrument, I addressed my second research question using an interview instrument. During interview process I gathered data asking the four same questions to all the six sampled participants. Thereafter, I asked various, unique follow up questions about the individual's learning performance. I synthesized the codes that emerged from observational instrument and my interview instrument as a data triangulation step. My interviews responses were narrative, hence codes of my data analysis made up four categories on my connectivist approach: autonomy (choice), connectedness (interactivity), openness (of the learning process and the individual personality) and diversity (of the learning process and the participants).

4.3 Data Presentation

In this section I will present the results from my research questions. Subsequently, I will introduce the discussion of these results to show how my findings address the aim of my study. In my results presentation, I will start with diagrams of patterns developed by students during the interaction phase, followed by the themes provided from interviewing the students. Four themes emerged to explain the innovations that drive students to participate in given patterns of interaction. There were also four themes that explain how interaction of students and their academic achievement can be linked during mathematics learning.

4.3.1 Patterns of interaction: Research question 1

I noted four patterns of interaction that address my first research question: What patterns of interaction emerge during a connectivist designed mathematics learning intervention with college students? These patterns are isolated pairs, educator-controlled separate groups (subnetworks), peer- dominated separate groups (decentralized patterns) and whole class-joined interaction (community of practice). Although these patterns display patterns of peer interactions, I also noted that while some students maintained interactions with the same peers, others interacted with a variety of their peers, participating in various patterns during and across the learning sessions. Furthermore, each learning session was characterized by differing interaction patterns. These patterns are presented using network diagrams generated by Gephi software for social network analysis.

Observed interactions on the first learning session.

Participants interacted with peers positioned in their immediate surroundings. The edges between the nodes (participants) in the network diagram indicate that participants connected with each other.

Most edges appear indistinct. These faint edges imply that connections between respective peers were quite rare and that discussions only lasted for a short period of time. Only a few lengthy, often repeated connections appeared bold in the network diagram (Figure 9). These were recorded between students D7 & D8, D1 & B1 and C2 & C3.

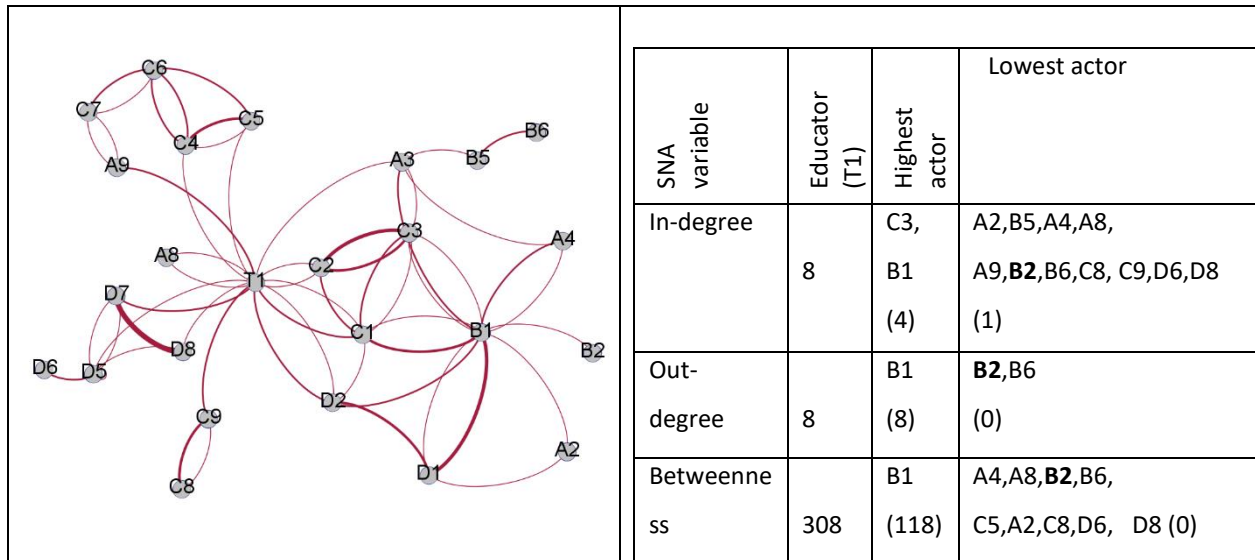


Figure 9: The pattern of interaction formed on the first learning sessions

Participants C8 and C9 worked without contacting other participants in this entire learning session. The only external consultation they did was with educator T1. The difference in boldness between the two edges connecting the two participants implies that one of the two participants was more actively engaged in consulting with her/his peer.

In this first learning session, two characteristic practices relevant to my research objective, emerged. The objective is to describe the patterns emerging during the learning interactions. Firstly, I observed that the educator (T1) maintained a central role in facilitating the learning process. Although the students interacted with their peers, the educator supported all students in all groups. Secondly, the students interacted to form three separate learning groups without the educator(s) influence (Figure 9). These groups were comprised of a large group and two smaller groups. The largest group consisted of 12 participants, while two smaller groups had five and four students respectively. There were no rules and principles set to hinder or guide the students' level of interaction. The educator responded to the requests of these separate groups, as well as to questions posed by students. At times, he also posed questions to stimulate further interaction among the students.

The social network analysis measure of out-degree recorded highest on participant B1. This measure implies that B1 connected and sent communications to the highest number of participants in the group. She appeared to be the most interactive student who influenced activity of members in the largest group. She engaged with an equal number of participants as the educator, T1. Furthermore,

B1 recorded the highest measure of betweenness. This measure implies that in her group she had a significant role in handling and moderating discussions between and among various students. Meanwhile, in contrast, no interactivity was recorded from participant A8. He behaved similarly to participants C8 and C9 who did not interact with other participants outside of their own paired connection. This study's academic problem lies in evaluating and identifying the attributes that resulted in the different interaction behaviour among these students (for example B1, A8, C8 and C9). I will later unpack this evaluation in the next section with my discussion and contribution of my study.

Facilitation of learning and stimulation of peer interactions

The educator, T1 switched his connections with members in all groups of learning. Although he seldom engaged with active students, I noted that he still monitored student progress across the three groups. His supportive role was central to all groups formed in this entire network. The students needed the input of the educator to ascertain their learning progress. This was noticed with students who requested assistance by raising hands. The educator recorded the highest measure of SNA - betweenness. This implies that he played the most significant role in facilitating the learning process in this session. Student B1 had the highest student role in facilitating and stimulating her peer group (with 12 members). T1's level of controlling the entire group almost doubled that. According to the network diagram (Figure 9), members in the two smaller subnetworks were so connected that they equally influenced each other.

Observed interactions on the second learning session

The comparison of interaction practices shows differences between the first and second learning sessions. I noted that participants interacted more with peers in the second session than in the first session. More so, although the peer interactions appeared more connected than in the previous session, students also worked as individuals. The educator, however, reduced his supportive role in order to encourage the students to work under the four principles of autonomy, connectedness, diversity and openness. I further noted that some participants on this day did not interact with peers. More than half of the participants were dormant. They interacted with either only one or two participants.

On the first day, participants maintained interactions with those seated next to them. A few movements were made by students to connect with other new students, seated some distance from them, but the majority of interactions were made between participants close to each other. I participated as an additional facilitator on this day in order to activate further interactions among the

group. I noted that participants like B1 and C3 who were most active and highly interactive on the first day, on this day were less connected. Participants D3 and E2 who were not present on the previous day became the most active participants.

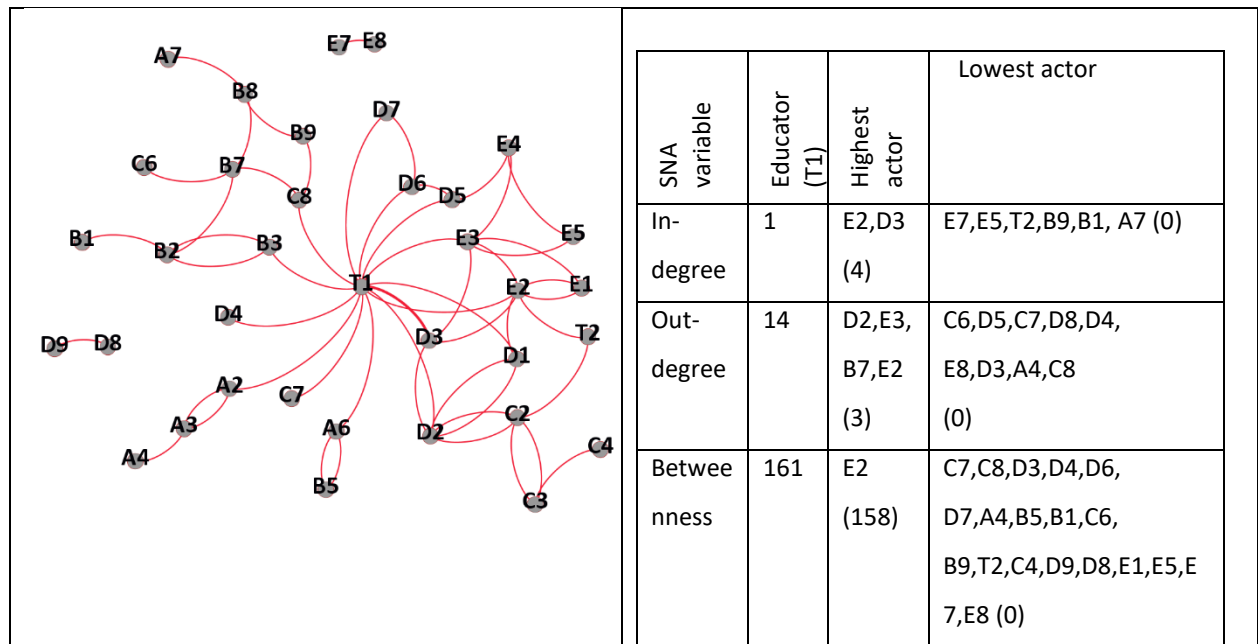


Figure 10: The pattern of interaction formed during the second learning sessions

Fourteen participants formed a large community that assisted each other to solve problems. They connected each other as per their seating plan. In their vicinity, they worked to share views and solutions around the given problems. They formed their own learning community. The members in this community were so participative that they discussed each other's efforts. I participated as the second educator, (T2). The two educators, T1 and T2, supported participants. While T1 assisted participants in this community as well as others who did not belong to this community, T2 was called to support those who only belonged to the active group.

The educator maintained the intention to have students learn through interacting with peers. The three measures of SNA shows that T1 interacted with fourteen participants in this session, with the result that many students received his input. This meant that he was checking on student performance and also facilitating their learning (out-degree of 14). Similar to the first learning session, the educator had the largest number of connections with participants. The nature of the connections with T1 shows that he connected with participants through responding to their requests (in-degree), and also through follow-up monitoring of individual's performance (out-degree). But, on the second day the educator has less requests from the students, and he made more follow-ups than responses.

In this second learning session, among the students who did not form part of the larger working community, some paired up with their peers to work in isolation. Three pairs were noted throughout

the learning session. There are also others who worked as individuals. The students who worked in isolation were neither influenced to change their interaction behaviour nor interact with others.

Observed interactions on the third learning session.

Students worked in interconnected subgroups. Discussions that started in one group spread into other subgroups. Students shared their methods and solutions across the whole class. The message was effectively spread from one corner of the classroom to another corner because the students were more connected in this session than in previous sessions. In this session students had more repeated and longer interactions with peers, compared to their connections from the first two sessions. Therefore, the edges of the network diagrams in the first and second sessions were less bold than the edges of the network diagram generated for this learning session.

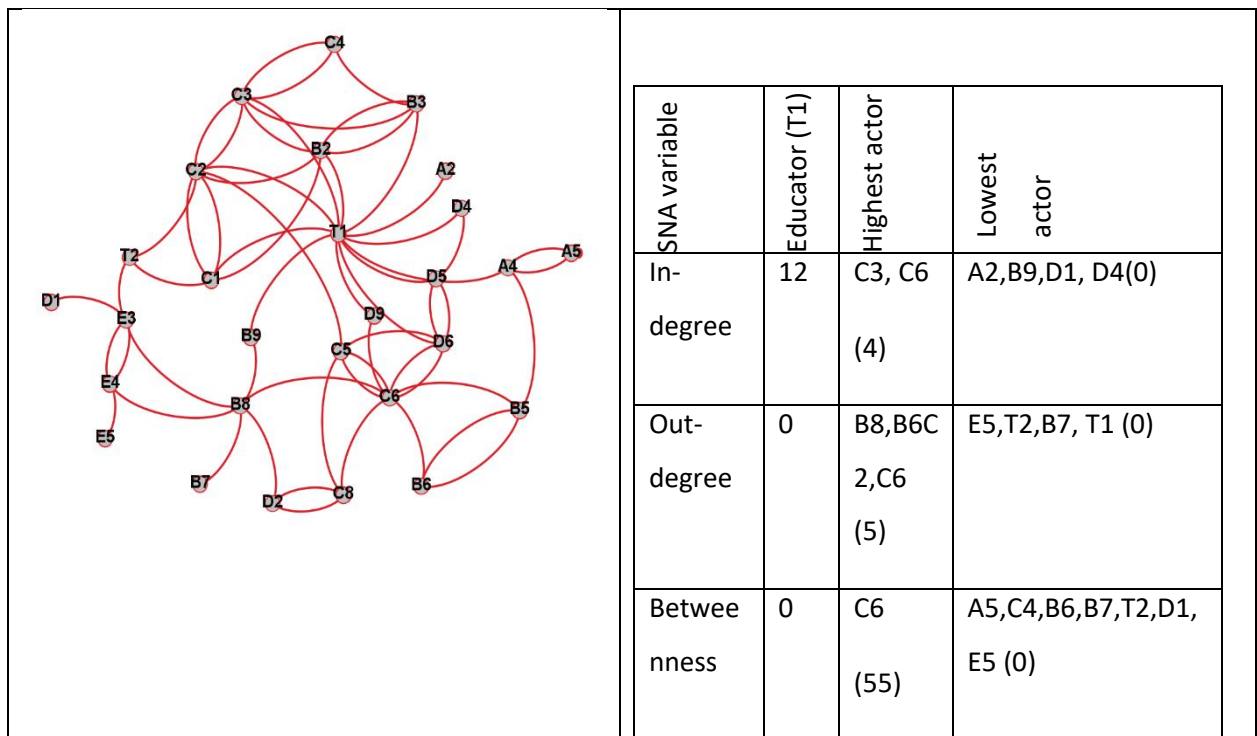


Figure 11: Learning interactions from the third session

In the first learning session, T1 interconnected the separate subgroups of students, but his role was reduced in this third session since students became more connected than in the previous sessions. According to the recorded measures of social network analysis, T1 has a betweenness value of 0. That affirms that his teaching support could not be regarded as more influential on one group, compared to another. This might be due to the fact that connections among the students spanned across the classroom and no distinct groups were formed by students. Although he facilitated some learning discussions when students invited him, he did not instigate a teaching role on his own. Instead, student

C6 had the highest values of out-degree and betweenness in this learning session. This implies that he was the most influential participant in the whole class.

The less active participants in this session were quite few compared to the first two sessions. The students had repeated interactions with participants in their neighbouring seating positions. Only one student, A2 was not open to interact with other students on this day. She only preferred to work alone and only engaged with T1 for any learning support. The students exhibited improved participation amongst themselves. Even though I was available to assist students on this day, only two participants approached me for explanations. This provided evidence of Improved peer trust.

4.3.2 Interpretation of learning interactions: Emerging patterns of interactions

I used an inductive approach with my observation instrument to come up with categories that define the types of patterns formed by students. The observation notes (appendix), diagrams of interactions generated using Gephi software and the measures of social network analysis informed my definitions of the developed interaction patterns. The following categories emerged from the three days learning interactions. Pairs, subnetworks, decentralized pattern and community of practice. These categories emerged as the types of interactions from one learning session to another. While pairs and subnetworks appeared only once in these learning sessions, the decentralized and the community of practice were repeatedly experienced.

In the table below I present the constructs from my observation data and the codes that characterized the interaction patterns as they were generated from Gephi. The table provides the multiples features that describe a particular connection between and among the students. In the table it is easy to differentiate these patterns based on outstanding elements (key features) unique to a particular grouping of students.

Table 2: Observation data- Emerging codes and categories on patterns of interaction

Multiple features observed (description)	Key features	Identified/emerging pattern
Distributed connections, peers equally work together, sharing of ideas is a norm	<ul style="list-style-type: none"> • large group • peers well connected 	Whole class connected (Community of practice)
Random connections, can disconnect anytime, connect occasionally, connect to rely on the educator/certain peer	<ul style="list-style-type: none"> • teacher controlled • peer dominated 	Peer dominated learning group (decentralized pattern)
Smaller or larger numbers, same peers connected, not	<ul style="list-style-type: none"> • separated group • peers well connected 	Educator- monitored separate group (Subnetworks)

connected to other groups, some peers contact the teacher		
Have close friendship, need no other peer,	<ul style="list-style-type: none"> • two peers • or small group all members connect 	Isolated pairs

a. Isolated pairs

Some participants engaged with only one peer and worked in pairs for the entire learning session. On a few occasions they engaged the educator, however they consistently assisted each other without comparing their efforts to other students. When discussions from other students became loud and uncontrollable, members in these pairs remained focused on their position. The connection between these pairs was independent of other learning dynamics that transpired in the classroom. For instance, these pairs were not permanent across the learning sessions. Instead, participants either paired up with different members or participated in a group with more than two members. While other participants engaged educators, these paired connections managed their own learning pace. For example, the pairs of participants D8 and D9, A6 and B5, and also E7 and E8 maintained their paired connections. They decided on the methods and order of solving the given problems.

Perspectives on isolated pairs from chaos, complexity and network

This pattern of interaction did not continue to appear in all learning sessions. Students involved in this pattern did not maintain this isolated state of participation. Downes (2012) refers to such an occurrence as 'network death'. The members either cease to interact or display homogeneous interactive attributes. Hence they end up abandoning their pair mainly due to lack of diversity. That suggests the reason for members extending their connections to other new peers in subsequent learning sessions. Notwithstanding, the principles of connectedness and unpredictability are confirmed in this pattern. This is so because students in these pairs maintained a single peer for the entire session. However, the dissolution of their pairs explains that duration of their connections cannot be predicted. This is a complexity principle of emergence and unpredictability.

b. Educator- monitored separate groups (Subnetworks)

Self-organized groups formed but were continually engaging with and monitored by the educator. These groups had interaction contained amongst group members. This was notable in the first two learning sessions. Students initiated their interaction with those positioned in their vicinity. Later on, they spread their movements to interact with participants positioned across the hall. An increasingly evident property of self -organization was portrayed as subnetworks developed while students

repeatedly interacted with the same peers. Although these group members interchangeably connected and supported each other, I noted that they contacted the educator on regular basis. Hence, the high frequency of educator engagements by these group members illustrates that the educator moderated the dialogues amongst the students. During the outbreak of lengthy dialogues and moments of unresolved learning debates, the educator intervened by clarifying the matters of concern. For example; varying solutions obtained by students failed to disapprove methods and solutions of other students. Therefore, the role of the educator was quite pivotal in confirming students' work progress. In addition, the role of the educator in intervening in these subnetworks, was to stimulate interactions of students when discussions seemed either too lengthy or counter-productive. Importantly, I noted that members in these groups did not maintain connections with the same members for the entire learning sessions. In the third learning session they all belonged to a large group of interacting peers.

Perspectives on subnetworks from chaos, complexity and network

Learning through subnetworks evidences the presence of many principles of complexity theory. In Figure 1 (chapter 2), among the listed components of complexity, the principle of self-organisation is partly confirmed in these subnetworks. On one hand, the principle is confirmed when students find their own peers to connect and work with. On the other hand, the presence of self-organisation is negated when the educator constantly intervenes to moderate the students' discussions. Minimal self-organisation is reflected in established learning subnetworks due to the high educator interventions while facilitating learning among and across the various learning groups. Socio-numerically the educator recorded the highest measures of betweenness in Figure 9 and Figure 10.

Furthermore, subnetworks attest to other principles of complexity theory such as a dynamic nature of the pattern. This was through the increase in size of the groups as we moved from one learning session to another. The fact that students interchangeably connected and supported each other in a subnetwork while solving their mathematical problems, is an indication of the principle of feedback and recursion. Cohen et al. (2007b) state that this principle of complexity is crucial for maintaining connectedness and interaction of students.

c. Peer dominated learning group (decentralised pattern)

The learning interactions appeared characterized by random and temporary groups that lasted for short periods of time and dissolved immediately. Such interactional dynamics occurred in all the learning sessions. The factors that resulted in these groupings include eventful circumstances that drew students to gather around an individual for a group explanation. Immediately thereafter, students would resort back to their normal learning positions and peers. I recorded examples of such interactional practices on various occasions. Firstly, I noted that an individual would attract and

dominate their peers after claiming to have a unique, easy mathematical method and acceptable solution to a given problem. Students would gather around such individuals for a group demonstration. Secondly, I also experienced that these demonstrations were done by either the educator or students. Thirdly, after the educator's announcement of one's excellent effort, he referred students to gather around him or her for explanations. The most participative students appeared to be the centres of attraction of these random and temporary groups.

While the teacher was the most consulted person, the structure of observed network diagrams and their measures of degree added that the most connected students recorded highest in-degree and out-degree when compared to their peer groups. Therefore, in the absence of the educator, the centrally connected student in these groups controlled their neighbouring participants. On day 2 for example, B1 was central, and had the greatest number of connections. That shows she had some influence on her peers' interactivity. And on day 3, E2, E3 and B7 were central in their peers' network. Therefore, the teacher's role of explaining work and responding to concerns was performed by those who had multiple connections. Thus it can be ascertained that a decentralized learning network existed in these interaction patterns.

The educator T1 facilitated the learning even though separate networks of interaction developed, as evidenced by the social network analysis. I found that learning discussions in decentralized patterns were centred and dominated by specific individuals.

Perspectives on decentralised patterns from chaos, complexity and network

From a complexity perspective, the decentralized patterns incorporated aspects of emergence and a distributed control. These two attributes are unique when compared to other learning patterns developed during this intervention. The emergent principle is identifiable through the random nature by which these decentralized patterns were experienced. As observed and stated above, they lasted for a short while and disintegrated as students sought to benefit from a group discussion where one student demonstrated to many peers. This was an eventual experience when students encountered extremely challenging mathematical problems. The distributed control principle is also explained through the same occurrence. Instead of all the students waiting to receive assistance from the educator, the stronger students demonstrated and guided their peers when needed. In these instances, when peers gathered in decentralized form, only one centrally placed student communicated, while others listened. Therefore, student openness could be attributed to the dominant student in the group, more than his or her peers surrounding her/him for assistance.

d. Whole class connections (Community of practice)

As students became familiar with one another, they relied increasingly on their peers more than their educators and I experienced that learning connections became more evenly distributed across the whole group. Even though I noticed small groups of peer interactions, these were not separated from each other nor the rest of the class. Since the premise of this study was intent on activating learning interactions and promoting learning connections among the students, I discovered that out of my three observation instruments, observations recorded on the third day of my intervention displayed the most characteristics of a connectivist learning approach. Students seemed to participate with adequate principles of connectivism. In this whole class interaction, I further perceived that peer-interactions were more frequent than in those experienced in other patterns of interaction. I ranked students with good levels of self-motivation, more stimulated by their peers than the educator in their learning interactions. The students developed a more interactive classroom community.

The entire group developed into a community in which members shared responsibilities towards accomplishing the work. Collaborated efforts were displayed as students worked through challenging questions. For example; when one individual moved to consult a peer seated away from his/her immediate surrounding, fellow students seemed anxious to share the feedback. Similarly, when one participant engaged with the educator, fellow peers showed interest in getting the educator's input. There was good evidence that, in those instances where students worked together, they first confirmed each other's solutions before they moved on to solve subsequent problems. That indicated collaborative effort amongst students. With all these experienced dynamics, I find that autonomy, diversity, openness and connectedness as principles of connectivism characterized the formation and sustainability of these communities of practice.

Students seemed to be quite selective of and decisive in the questions to answer through their collaborated efforts. They continued to work from easy to challenging questions. From the beginning of the intervention I noticed that students interacted in groups. However, on the third day, peers extended their connections to members in other groups. Then students spread their learning interactions to various members of the entire class. With this, peer connections developed into one big group. There was an improved movement of students to engage peers seated both in close range and further afield. Some students showed high excitement and argued in a competitive manner. I noted that the interactions were driven by desire of students to compare and contrast each other's effort. As the two educators available for this intervention, educator T1 and I (T2) did not intend to guide and lead the teaching and learning process but expected students to be independent participants. Therefore, we believed that when students are given the opportunity to interact under

the four attributes of a connectivist, they may establish a well-connected and distributed learning mathematics classroom environment.

Perspectives from, chaos, complexity and network

The dynamics of interactions experienced through a whole class community interaction evidenced the aspects of dynamical and complex systems. In *Figure 11* it is evidenced that on the third learning session, all the separate patterns experienced in the two precedent sessions finally merged into a massive community. The learning dynamics in this pattern indicated that students were open minded. They were more self-informed in their interaction practices compared to other learning patterns. There was increased movement and communication among the students within the classroom and students were more open to interact with others. The state of the learning discussions looked unruly, but manageable. This indicated that students had a significant level of self-organization, an implication of connectivist learning. They alternated between an orderly and chaotic learning atmosphere. The limited inputs from the educator promoted a good sense of self-organization among the students.

4.3.3 Students' academic practices: Research question 2

My second research question addresses how these patterns inform about the students' academic practices. The following section presents students' perspectives towards their learning practices while participating in patterns experienced. Students established four interaction patterns described earlier which are isolated pairs, separated educator monitored groups (subnetworks), individual dominated groups (decentralized) and whole class group (community). The next section is an outline of the purposive sampling procedure and results of peer assessments that I facilitated.

Purposive sampling

The six participants were sampled based on their measures of interaction from a social network analysis. I was guided by the network diagrams and the measures of social network analysis generated from Gephi software. The sampling of participants was based on participants recording highest and lowest values of out-degree, in-degree and betweenness. I noticed that some participants, for example B1 and A8, participated once out of the three conducted learning sessions. However, because of their exceptional levels of participation while interacting with others, I decided to interview them to get a deeper understanding of their interaction practices and their academic achievements. The following table show the participants that were sampled based on their level of participation. I chose these levels based on highest and lowest measures of social network analysis.

Table 3: Levels of interaction of sampled students and learning sessions

Student	In-degree	Out-degree	Betweenness	Learning sessions
B1		✓ (highest)	✓ (highest)	1 st
B8		✓ (highest)		3 rd
A8	✓ (lowest)			1 st
C4			✓ (lowest)	2 nd & 3 rd
E2	✓ (highest)			2 nd
C3	✓ (highest)			1 st & 3 rd

Peer assessments

I first identified the extent to which each participant achieved academically. To accomplish this, I considered both personal perception of each individual and other participants' opinions. Therefore, I first requested the entire group to rate the performance of a specific student on a structured assessment template. All the purposively sampled participants were peer assessed by the entire group. Secondly, I conducted interviews with the six participants. The findings from the peer assessments provided the premise for a set of follow up interview questions for each specific participant, as well as the means by which to focus the whole exercise towards addressing the impact of the patterns of interaction on student academic achievement.

During the peer assessments, the group used their own perceptions of the performance of sampled participants. Therefore, they were informed by the extent to which they interacted with or observed practices of the sampled participants. The recording sheet was structured. Four categories were provided for respondents to use, these were namely low, average, good or very good performance. When all forms were completed, I did a physical count of the number of responses in each of the four performance categories per participant. Below is the table showing the respondents per performance level that rated each sampled student.

Table 4: Peer assessment results on each sampled participant

Student	Student E2	Student A8	Student B1	Student B8	Student C3	Student C4
performance						
Very good						
Good						
Average						
low						

The participants did not receive low performance ratings from their peers in the conducted intervention. All the assessed participants were mostly rated as good achievers. Since the peers applied their perceptions to evaluate the performance of a particular individual, this reveals that participants in a connectivist learning environment have informed reasons to interact. So the four principles of connectivism namely: autonomy, connectedness, openness and diversity may be related to the ratings of achievements. Therefore, these results formed the basis of the unique follow up interview questions on specific sampled participants.

4.4 Interview data presentation and analysis

When I conducted the interviews, I found that students' responses informatively addressed two categories. The two categories refer to interaction practices and academic achievements.

a) Perceptions about interaction practice

Four themes emerged with regards to the students' interaction practices. The four themes are: concerns over personal and shared space, desire for academic excellence, need for alternative methods and trustfulness (personal and peers). Views of the students indicate that their interaction practices are both psychologically (internal) and socially (external) driven. This, therefore, had an impact on the extent to which they decided to interact.

- i) Concerns over personal and shared space
- Slow students preferred to maintain their pace to avoid failing to understand given work. Students showed mixed feelings when approached by peers to initiate a learning process. Other students accepted connections from any peer at any time. However, others, especially those who struggled to master learning felt disturbed and not interested in learning from their peers. They cited that they were disturbed due to differences in their learning pace. Hence, the students who struggled with work, distanced themselves from connecting with other peers, but trusted only the educator's supportive efforts. Students who found material challenging or difficult did not find it easy to work with classmates to find solutions. They preferred not to engage with those perceived to have a better understanding on the material.

Researcher: ... was it not possible just to ask questions when you needed help?

A8: yeah maybe others found it easy to ask anyone but I feel that I am a slow student, so I decided to focus on my work at my own pace putting my own effort

Researcher: what else do you think made other people not ask you questions?

A8: maybe because I was working alone or maybe I was not friendly to them so they also decided not to come to me. but I think it's because I was not interested in working with any other people so to them maybe thought that A8 is not interested in sharing the answers and ideas

While low achieving students felt that their minimal interaction is restricted to the subject experts, the high achieving students also indicate that they interacted to assist the low achieving students. Therefore, the limited interaction in some students was driven by their feelings.

- ii) Desire for academic excellence
- Well-connected students, during the problem based connectivist learning approach, cited a healthy desire to compete for excellence with each other. This formed part of their motivation. Firstly, they worked under pressure to avoid being humiliated by poor academic achievements. Secondly, higher achievers noted that while many students in general classify mathematics as a challenging learning area, they get curious about the best way of achieving good results in this subject. Out of curiosity, students increase their learning connections with various students, comparing and sharing their best practices. For example, B1 hinted that by working under pressure, students may achieve consistency by committing themselves to effective learning behaviour.

Researcher: so from your own view do you think of anything else that can be done towards good achievement

***B1:** I think as we always do this at the class activities daily I think student must always be told that whatever they are doing it is very important for their end-of-year marks and it will be considered for the final marks so they must always be put on pressure and themselves also see how much they understand.*

iii) Need for alternative methods and solutions

The students with a high measure of degree of interaction (both in-degree and out-degree) expressed that reasons for connecting with many peers included the desire to search for alternative methods. When they lacked a good understanding of the question, even though their problem solving methods and solutions were acceptable, they further engaged their peers. Therefore, although they were certain of obtaining acceptable solutions, they still desired to learn from peers' alternative methods. For example, student C4 cited reasons he seek to adopt either short or easy methods in order to master his work.

***Researcher:** so how did you get to know that you were wrong and they were right*

***C4:** ... I was working like comparing methods from two groups and answers from different groups*

***Researcher:** so ... how sure were you that this group is got the right answer and the other one has got the wrong answer?*

***C4:** actually I was interested in the easy way the ways that were easy to understand that's ...*

On the other hand, the students who were closed to experience expressed little intention of learning from peers. They showed some interest in a traditional classroom learning approach whereby the educator closely controls the students' pace and effort. They worked either individually or by consulting the teacher. For example: A8 suggested that students should be advised to work more as individuals than in peer connections. However, the idea has been opposed in other students' views.

***Researcher:** what do you think must be done more to encourage students to work alone as you think is the right thing to do in maths?*

***A8:** I think so! It is going to be difficult because in maths even myself I always look for my friend and we work together but in this intervention I decided I need to do my work alone. so students may not be encouraged to work alone but I think they need to be advised sometimes that they need to test themselves and stop relying on other people that is when they will see for*

themselves that they need time to put effort on their own when they get used to that I think they will not always take maths as a difficult subject.

However, participants felt that they had minimal confidence when exposed to a new learning environment. In order to master the learning content, participants would continue to work alone. However, the more they encountered uncertainty and a desire to review their progress, they engaged with either their peers or the educators. For example, E2 highlighted that before joining his peers, he worked alone while observing the manner in which other classmates interacted with the learning content and also other students:

Researcher: I want you to tell me how you worked with other people. How you benefited.

E2: The first time I came to class I sat alone. I observed how others were working. I managed to see that others were discussing...

This evidences that participants relate their understanding of the process to the way their peers respond to the learning material. They do this in order to position and familiarize themselves in the learning environment. Their decisions to work either as individuals or connecting with particular peers was informed after observing others' learning practice.

Furthermore, students trust their perceptions about the competency levels of their peers in order to connect with each other. Participants highlight that they position their own competency levels either above or below those they choose to work with. For instance, B1's narratives confirm that she chose a peer she perceived as the top achiever, while C4 indicated that he initiated connections with peers he believed needed to be assisted. Therefore, it is affirmed that before students connect with others in a connectivist environment, they first position their learning potential and relate themselves to competency levels of other peers.

Researcher: My first question is about the first day we had this learning intervention. How did you find yourself interacting with others?

B1: First day I was scared; I did not know what to do. Was even scared of talking to others. But there was only one person I started talking to. And that person is Naeem. I did not know anyone, but as of now I know everyone, so now I can work with anyone.

Researcher: That's good! Tell me was there anything that brought you closer to Naeem?

B1: Yes, he was the brilliant one. He could answer all the questions during class discussions.

The participants expressed their preferences for working independently and also their ability to interpreted one another's problem solving ability before interacting with each other.

iv) Trustfulness (personal and peer)

The students expressed that their participation with specific peers were driven by personal impressions. These impressions are based on the individual's performance and their fellow students' performance. Students expressed that they engaged with peers based on their frequency of participation and the quality of learning contributions during open learning dialogues. Implied in this practice is that the seldom participating students received fewer peer connections than the high participating students. Similarly, those students who did not always contribute acceptable solutions received fewer peer connections than their counterparts who always contributed appropriate and relevant problem solving methods and solutions. Mixed sentiments were expressed with regards to observed participatory effects in various students. On one hand, highly interactive students attracted other peers who were open to experience new and alternative methods. But, students who were closed to experiencing a variety of working methods and solutions, withdrew from highly interactive peers to avoid disturbances. They preferred to work in isolation, either individually or in pairs. Therefore, aesthetic sensitivity and active imagination as dimensions of openness in this study are not clear indicators of either one's desire to interact with peers or his/her academic achievement level.

b) Perceptions about academic practices

In my second research question I address how practices of students in their patterns of interactions impact on their academic practices. My data findings from the first research question provide insights into student to student interactions, whilst the second question delves into the cognitive engagement of these students during their interactions. These are self-orientation, decision on effective learning, self-evaluations and adoption of academic practices

i) Self-orientation

The elements outlining the recognition of personal working space provided the orientation of student to student interaction. However, a few elements describe each student's cognitive levels. I find that students highlighted a range of their cognitive orientation before the intervention. The variety of cognitive attributes recorded include slow-students, good background, not struggling and good mathematics student.

ii) Ways students decide on effective learning during peer interactions

The students made their own decision to interact and they interacted with peers of their choices. Students advanced reasons for participating in specific group sizes and attributes (Table 5). Some felt that they had better control of their learning pace while working in pairs as opposed to large groups. They cited minimal disturbances when participating in smaller groups than in larger groups and it assisted in the accomplishment of tasks with good

understanding when interactions were in smaller groups. Meanwhile, larger groups were considered most time efficient for solving mathematical problems. For example, B1 highlighted that by interacting with many peers, responsibility was shared collaboratively.

Researcher: *on what basis did you choose one another and ...the reasons that put you together?*

B1: *actually we understood each other like a class. Sometimes not everyone will understand in the same way so we actually get together because we could understand each other that if this one is a slow student he could be accommodated by the rest. So that's how we got along.*

In addition to making their own choices of group members and sizes, the students expressed appreciation of each other's efforts. Regardless of varying cognitive levels, students supported each other. Moreover, students acknowledged that differences in cognitive views opened opinions and discussions, which in turn improved their level of achievements.

Researcher: *how can you describe the way you interacted with other students*

B1: *I think we worked well as a group to put our point of view like debating on the main subject or the problem that we were solving everyone had their own point-of-view like the ways to solve the problems*

iii) Monitoring of individual learning path whilst working alone or interacting with peers
Students described their learning practice of connecting with others as self-controlled. While they relied on their peers to measure their progress, the role of the mathematics educator remained crucial in allowing students to ascertain their progress. Both individuals and large groups showed that they needed the educator to moderate their group discussions. Furthermore, they indicated that the educator was also expected to demonstrate solutions to more complex mathematical problems. In a decentralized learning system, learning is centred on high achievers (subject expert). The diverse opinions amongst students indicate that individuals preferred to manage their interaction. For example; A8 posited that working individually promoted self-judgement about one's competency level in mathematics, whereas B1 made the point that students should be flexible within different working positions, for example, working as an individual or as part of the group. She opined that collaborative skills are important not only when learning as a group but also when working as individuals.

Researcher: *Do you think it should be encouraged or not for people to interact with peers in learning mathematics*

***B1:** I think it's both. Because some people may always want to rely on others... so I think to some extent we can advise people to work together but we can also advise people to work on their own, like everyone always wants to get good results and pass on their own*

Students have diverse views towards developing competency when learning mathematics. The role of the educator can be suggested as important in mining high performance levels through the simulation of student interactions. Some individuals believe that the educator must control their learning pace, and that learning under pressure from the educator promotes students to become highly committed to their work.

Researcher: *so are you saying students must be driven by the pressure from the teacher?*

***B1:** yes I say so! I think if there is no pressure from the teacher they will not be able to work on their own ... so I think the teacher must always put the pressure and some strategies to make sure that students put their efforts*

iv) Perceptions on academic practices

Students described their achievements based on their level of confidence obtained from participating in a particular manner. The contributions shared with peers in given learning opportunities are among the ways that displayed an individual's level of learning. However, evaluations based on this factor differ between introvert and extrovert students. Responses from students indicate that introvert students had minimal interactions with peers but analysed the learning content well and showed a positive self-evaluation.

However, the active participating students contended that through interaction one could discover and stimulate his/her own competency level.

Researcher: *what are your views about having people interacting together?*

***B1:** I think it's nice because it helps one another to see what others think. Then you can also see how you think then it can help you to boost up your self-confidence. You can also find out that you know something that another student doesn't know and you feel like at least there is something I know in maths*

Learning interactions benefited students through successive sharing of various learning practices, and interpersonal skills. Participants expressed that they differed in terms of personalities and comprehensiveness. The educator held a crucial, catalytic role in controlling interactions in such diverse environment. According to B8, her peers started their community after debating each other's ideas. She indicated that each participant had either a different way of problem solving or a different solution to a problem. Therefore, they engaged the educator to moderate their discussions, thus

guiding participants towards rightful problem solving techniques and solutions. B8 further expressed the following:

Researcher: ...the first question is, how can you describe the way you interacted with other students?

B8: I think we worked well as a group sir, to put all other thoughts and views together, people we were like debating on the problem that we were solving because everyone had their own point of view, own way to solve the problem

In summary: second research question findings

Interview themes emerged under two categories. These categories are perceptions of interaction practices and academic achievement. The two categories inform each other conceptually, however student practices and perceptions could not establish a distinct causal relationship between the two. Interaction practices are driven by four main factors (themes): concerns over personal and shared space, desire for academic excellence, need of alternative methods and solutions and trust. When these factors are examined with reference to observed interaction trends and practices from observed diagrams of interaction patterns, I find a good level of coherence between the observed patterns and the narrated patterns. In the category of academic achievement, themes are revealed as self-orientation, ways students decide on effective learning, monitoring of individual learning paths and confirmation of academic achievement.

Although no causal relationship could be established between the students' practices and their academic achievements, important insights could be drawn from the two aspects. A different perspective emerged to characterize the dynamics involved as these themes could not be sufficient to bring about the causal effect of the interaction practices on academic performance. It appears indisputable that an individual's level of academic achievement is suggested by the individual's practices of and extent of interaction.

Table 5: Interview data codes emerging from reasons on when and how participants interacted

	Group size	Reasons
B1	Alone	-To test myself, avoid disruptions, target questions of interest to me
	Large group	-get ideas, helping others, to be sure that I am doing the right thing, to quickly get done with the work, comparing answers
A8	Alone	-avoid disruptions, only need the teacher, others don't explain clear, I also struggle so can't help others, I am slow student, they were not my friends
C3	Alone	-because my peers were not present yet, I thought I was right, to avoid being complacent about work
	Large group	- am flexible to work with anyone, get ideas, builds self-confidence, competition with one another, help others
B8	Small group	-motivated, trust solutions, some do not want to help, others have complicated methods
	Large group	-quick to find out that answers are right, quick to agree, show others that I am better
C4	Small group	-have enough time to share answers, to support each other, not to waste time going far, understand teachers when explaining to us, we can switch the language to our own
	Large group	-working as a group finishes work quick, inspired by others, learn various techniques
E2	Large group	-I help they help me, they were relying on me, I can work with anyone

4.5 Trustworthiness of data analysis

Themes from my observation instrument and my interview instrument, with all the six participants, constitute the table of summary below, which presents themes extracted from the four principles of connectivism and are derived from student perceptions. It is evident that each principle can describe attributes of students in a particular pattern of interaction. I noted that, while participants responded towards their interaction behaviour, they also expressed their performance levels. They highlighted their motives for participating in a particular manner (Table 6).

Table 6: Emerging categories based on codes about interaction and learning processes

	Autonomy	Connectedness	Openness	Diversity
<p>Pair (work alone or in pairs)</p> <p>Cases: A8,B1,C3</p>	<p>Good opportunity to make own choice, and decision on pace and methods to use</p>	<p>Very limited connections between the pair, can change peer to seek alternative inputs from other peers</p>	<p>Can try new procedures on my own, open to the peer but not others, avoid disturbances, sees self as different</p>	<p>Avoid others because of disruptions</p> <p>Lack trust to other people trust in self-discovery</p>
<p>Peer dominated Separate groups (decentralized)</p> <p>Cases: B8, C4</p>	<p>Controlled actions, momentarily active or passive</p>	<p>Less sharing among peers</p> <p>Limited self-reliance</p>	<p>-students rely on one trusted centre, students gather to acquire knowledge with little contribution</p>	<p>Communication is most centred on one person, therefore dissolve soon after assistance, no continual assistance</p>
<p>Teacher-monitored (sub-networks)</p> <p>Cases: A8</p>	<p>Freedom of participation, contributing,</p>	<p>Peers connected, depend on the teacher to determine correct or incorrectness</p>	<p>Enough opportunity to Sharing, but still Teacher dependent</p>	<p>limited opportunity for accepting other inputs as they always engage the teacher, desire academic excellence</p>
<p>Whole class/large group (community of practice)</p> <p>Cases: B1,B8,C4,E8</p>	<p>Variety of choices-peers and problem solving methods. Peers exposed to alternative methods, have self- assessment against others performance</p>	<p>Unlimited connections, active connections among the students</p>	<p>Helpful to each other</p> <p>Group achievement, some people contribute, some do not contribute, easy to learn from others, we all pass</p>	<p>Helpful to each other, variety of views competition to excel, desire for achieve quite high</p>

4.6 Discussion: Validation of connectivist attributes

I adopted a multi-dimensional perspective in validating the four attributes of a connectivist learning approach. I looked at both group and individual interaction practices, levels of participation and the academic achievement of individuals. This process of validation was informed by findings from both research questions. The gathered findings are four observed patterns of student interactions (research question 1) and findings on practices and academic achievements of students (research question 2). The four patterns are isolated pairs, separate educator-controlled groups (subnetworks), separate peer-controlled groups (decentralized patterns) and whole class-joined interaction (community of practice). The four categories of academic achievement were structured as low, average, good and very good (

Table 4). These four are all based on students as individuals (personality), structure (group) and performers. In the following sections, patterns of interaction and students' academic achievements are explored. These form the basis of achieving the purpose of my study.

4.6.1 Autonomy

In my validation of the principle of autonomy, I premised findings on the three pillars of decision making, choice making and independence of students (Downes, 2012). My observation instrument and my interview instrument confirmed only two of these three pillars, being decision making and choice making. Although students worked under minimum supervision of the educator, they depended on the input of the educator and their peers when they encountered challenging mathematical problems. More so, students depended on the educator during moments when they reached an impasse in their learning interactions. The educator was invited to moderate learning disputes forward from which students could not move without clarification. I found that both instruments agreed on features that students displayed to interpret their level of choice and decision making.

Choice making

In connectivist dynamics, Downes (2012) described the choice of students with reference to learning outcomes, objectives and networks in which to participate. According to my observation instrument, on the first day, students did not make choices on their sitting positions and peers with whom to work. In the second session, students selectively positioned themselves next to peers they were comfortable interacting with. Although many students appeared to engage with peers from the previous day's learning session, it has been evidenced that some students connected with new peers of their choice. Two themes showed students made their own interaction choices. In their perception of interaction, the non-interacting students, for example A8, substantiated her choice for not interacting with others. She cited this was based on differences she had to other students in understanding the work. So the interview instrument provided reasons explaining decisions that informed students' choices.

Decision making

Lerma and Kreinovich (2015) argue that, in mathematics, if the same questions are given to students, then students would have equal chances for effective learning. However, their argument suggested limited levels of students in making their own choices and decisions to answer their own questions. Students having limited opportunities to decide their own learning path, indicates of a teacher centred learning approach. In contrary, through my connectivist learning intervention, students solved given problems they felt mattered most to them. The intention was to focus on more challenging questions rather than easy questions. Hence, students applied their own discretion in differentiating between

easier and more difficult questions to answer. In fact, students answered different questions at different times, based on their emotions. Moreover, according to my interview instrument, there are themes that emerged to confirm students interacted and learned through their personal decisions. These themes are desire for academic excellence, self-orientation and the search for alternative methods and solutions.

In order to address the two research questions, I synthesised findings from my two research instruments. In terms of patterns of interaction, I found that students experienced high autonomy levels when participating in pairs and in large groups. The other two patterns, educator controlled and peer dominated patterns, gave students little opportunity to make learning choices and decisions about the level difficulty of the given questions, because they are always guided by a particular member. On the other hand, the students in the former two patterns indicated that they had enough opportunity to rate themselves.

Student independence and decision making in a pair

This is the only pattern in which peers relied on their own efforts. They did not engage with members from other groups. Instead, they confirmed their working progress through consulting extra activities and with various resources. This provided them with the opportunity to develop a deeper understanding of the context and concepts relevant to a given problem. In the event of encountering more challenging questions, they decided on attempting all the other problems and revisit the most difficult at a later stage. Therefore, they worked at a slow pace. These pair students supported their decision to work independently from others, indicating that it gave them a good reflection of their ability. For example, A8 acknowledges herself as a slow student who needed more time for self-discovery. She believed in exhausting all possible ways of problem solving from her own perspectives of understanding the work. She perceived peers participating in other large groups as evidence of surface learning.

Autonomy during learning in a large group discussion

Choice and decision making are the characteristic elements of the autonomy exercised by students participating in a large group or in whole class discussions. According to my interview instrument, this is evidenced by examples of multiple dialogues students engaged in. Students in a large group expressed that they learnt from alternate solutions given by peers. They trusted both solutions and methods shared by various peers, hence they worked independently of the educator, but dependent on each other. While working through their own choices, students involved in a community of practice, controlled their learning pace. During whole class learning interactions, the aspect of learning independence is more attributable to the entire group than to an individual. Students indicated that since individuals had minimal independence and worked collaboratively, they had enough opportunity

to choose easily understandable explanations. The learning path was therefore developed through deciding on when to collaborate and by comparing learning contributions from other peers.

According to my observation instrument, there is educator presence both in small and large group learning interactions. Small groups contained 4 or 5 students while large groups comprised of more than 5 students. In large groups, students engaged the educators less often than their counterparts in small groups. That explains why students became more autonomous when participating in large numbers than when participating in small groups. It is important to note that learning interactions were developed and maintained in a self-organized state. I also found that the two aspects of making decisions and choices during interaction with other peers was emphasized, but not with regards to the inclusion of the educator. Students raised this in their perceptions towards building their academic achievement. Very good student achievement has been recorded for individuals who contribute to a community of practice. A competitive attitude may be the driving factor that encourages students to strive to perform very well. A student's achievement could result from being inspired by his/her peers.

4.6.2 Connectedness

Both the observation instrument and my interview instrument display that connectedness of students has been the most evidenced principle in this study. Kernick (2004) explained connectedness as more than mere connections between the individuals, but adds that it entails continued feedback, and flow of information between interacting members. Students were involved in active discussions in which everyone contributed. The three patterns of subnetwork, decentralized network and community of practice (large group) appeared with connected students in all learning sessions. The observation instrument displays these patterns continuing to appear across the learning session, regardless of some members dropping out and other new members joining the intervention. However, the connectedness among the students in these three patterns varied across the patterns. With some students being more active than others. Moreover, students did not maintain their participation levels while changing from one pattern to another. While some students increased their participation by connecting new peers, other students became less participatory than they were, either in their preceding learning sessions or interaction patterns.

Although students extended their learning connections to different patterns from one learning session to another, I noted that students maintained connections with certain peers. The study therefore maintains that student connectedness is the most recognised attribute of a connectivist teaching and learning approach in classroom mathematics. However, each pattern has unique attributes that describe the interaction practices of its members. In order to respond to my second research question,

peer assessment results did not yield conclusive differences between students who participated in various interaction patterns. Therefore, the interview instrument details themes that distinguished the connectivity and the learning performance of the students according to patterns.

Connections between students while monitored by the educator

In this study I refer to such a pattern of interaction as a subnetwork. While students in this pattern are well connected to each other, they also engaged the teacher on regular basis. Students acknowledged that more learning support came from their peers than the educator. For example; in my interview instrument, responses from B8 expressed that working closely with peers in a subnetwork provided ample time to share and analyse individuals' methods and solutions to given problems. In spite of the fact that students in these subnetworks have frequent engagement with their teacher, they got opportunities to identify various options of deriving a solution to a given mathematical problem. For example; both individuals B8 and C4 highlighted that by participating in a small group (in the form of a subnetwork), mathematics students built academic trust among group members.

Furthermore, the element of motivating one another characterizes members of a subnetwork. Both internal and external motivation has been cited by members in a subnetwork. On one hand, students connected through a subnetwork indicated that they became self-motivated after sharing an acceptable mathematical solution with peers. On the other hand, external motivation in a subnetwork has been cited in instances when one was assisted by other peers. Consequently, the students in a subnetwork had a good rating of achievement due to their collaborated working efforts. More so, because peers in a subnetwork engaged the educator on regular basis, students worked with confidence. Therefore, the learning practices of members in a subnetwork differed from practices of students in other patterns.

Connections among students in peer- dominated patterns

A decentralized learning pattern has a group member who dominates the learning dialogue. This individual had the highest competency level and he/she supported the entire group members. My observation instruments recorded, from the three learning sessions, displayed connections among students where some individuals were more connected than others. For example; in the first session, the educator was placed centrally in relation to all separate groupings that developed. Additionally, in the first learning session, B1 was the most centrally connected member. Subsequently, E2 and C6 were the most connected and central group members in the second and third learning sessions respectively. Although the educator, T1, was the most connected in the entire class, according to my observations instruments I noted that entire group appeared to have multiple centres with some well -connected students. From the first day of the intervention to the third day, students reduced levels of reliance

on the educator. This supports that students improved peer connection and teaching and learning support was increasingly distributed among students.

Although the interview instrument provided remarkable understanding of connectivity in group dynamics, I noted that a social network analysis drew out some inferences specific to individual students. Three measures of social network analysis, in-degree, out-degree and betweenness evaluated the role of certain individuals in these learning connections. Students with high scores have been the most connected. High in-degree showed that some fellow peers connected with them by questioning while a high out degree shows how much the connected student communicated out to the surrounding peers. Betweenness describes the measure of how one controlled his/her fellow peers during learning. In instances when students experienced challenging problems, they always directed their questions to the good students. While students started participating in this learning intervention by consulting the educator very often for assistance, only A8 expressed that she did not consult others due to lack of trust in their solutions. Unlike in a closely connected pattern, the students in a decentralized pattern have average learning achievement. This concurs with the implications of interactive learning in which students empower each other through connections. It is undisputable that in these decentralized patterns, the members relied upon by the rest are those who recorded good and very good ratings from peer assessments.

Connectivity among students in whole class discussions (community of practice)

Peer assessment results suggests that students in this learning pattern have been rated as very good mathematical achievers. Perceptions of peers inter-connected in this pattern indicates that community of practice is a way of promoting a distributed learning support. Various attributes have been cited as the basis that maintain connections between and among peers.

4.6.3 Diversity

My observation instrument shows that various interaction patterns were formed by students during and across the learning sessions. Students expressed their diversity through continued changes in peer connections. In addition to the contribution of the observation tool, a social network analysis and my interview process affirm student diversity in learning practices. The existence of four interaction patterns indicate that students supported each other in different ways. Student narratives also submit unique motives for interacting or not interacting within a learning environment. Reasons provided for connecting other students revealed the desire to improve their understanding regardless of any physical or intellectual differences. Since the learning intervention was designed to be student oriented, educator presence was minimal enough that students engaged with their peers for learning.

While a small number of students expressed their dissatisfaction at working as a group, others indicated their preference to work by connecting with many peers. Therefore, different learning patterns resulted from various learning preferences chosen by students. Students with minimal peer-to-peer interaction worked in pairs or as individuals and students with a high degree of interactivity were found in small groups with a high frequency of consulting the educator, as well as in large groups. Students interacting in these two patterns appeared quite innovative and supportive of each other. Analysis of highly interactive learners indicates good academic achievement experienced by those working in small groups and very good academic achievement for those participating in large groups.

Implications of diversity on learning patterns

Various patterns of interaction were developed during peer connections. Diversity of students is evidenced in spite of the fact that students interacted interchangeably to develop learning patterns in various forms, namely pairs, subnetworks, decentralized networks and community of practice. The presence of diversity as an aspect of the formation of learning patterns was noticed in all conducted learning sessions. As interactions between both students with low and high measures of cooperation were established, interactive behaviour among the students could not be defined based purely on participation, but also reasons for interacting. The reasons for formation of learning groups highlight the need to achieve effective learning. This affirms that diversity is an aspect worth noting amongst students. Furthermore, the reasons for interacting with various peers reflected the socio-psychological attributes of an individual. For example, among the reasons for participating in small groups, it can be argued that learning discussions are more manageable and less disruptive than in other learning groups. However, in contrast, some students suggested that large group interactions promote effective collaborative efforts, leading to better learning achievements.

Implications of diversity on learning achievements

According to peer evaluations, the students who participated in large groups had high ratings. Through my interview instrument, I learned that a diversity of students created an active learning dialogue, which resulted in deep learning or understandings of the material. Meanwhile, students in small groups had some remarkable learning achievements, but could be regarded as less successful than their counterparts who learned through large group collaboration. This potentially suggests that when students collaborate in large groups they encounter more diverse learning opportunities. As a result, they learn more effectively. In contrast, the participants who worked in pairs or as individuals may only encounter limited alternatives to problem solving. Therefore, students with little connections have been regarded as average achieving students.

Implications of diversity in dynamic interaction practices

While addressing my first research question, evidence indicated that students did not display constant participatory behaviour across all learning sessions. They interchanged with their peers and also the extent to which they participated. While some students dropped out of the learning intervention, other students joined the group in the middle of the assessments. However, participating students managed to establish their connections. For example, while D8 and D9 formed a pair on the second learning session, but in the first session D9 was absent and D8 connected with other participants. More so, on the third session when D8 was absent, D9 worked by connecting with T1 and C6. Therefore, the formation of a pair as a form of interaction pattern was incidental. An understanding may be drawn from the explanation that mathematics students in a connectivist learning framework may engage with each other in various patterns and sub-patterns. This further indicates that students were so diverse that they could establish multiple personal connections. Hence, we may confirm their diverse capabilities of learning through various connections, within different learning groups.

The other structures that were established in these patterns include subnetworks and decentralized patterns. Both patterns referred to small groups which, while structurally related, differed from each other based on the roles played by each participant. In addition, the way students controlled their learning space explains the extent to which diversity was reflected. In a subnetwork, students had equal opportunities to contribute and express themselves, while in a decentralized pattern, one student had the highest out-degree. This identifies that his/her main role was to determine the learning pace of other participants. To summarize, in a subnetwork all members shared their diverse personae, but in a decentralized pattern the students resembled a homogeneous pattern that relied on the most vocal individual to guide them.

When participants interacted in a large group, such as the pattern formed in the third learning session, there is evidence of students accommodating one another for the sake of interacting and learning together. In such large groups, students collaborated to achieve solutions to given mathematical problems. Every student took part in solving the problems and shared their contribution with all classmates. Students appreciated everyone's effort and accepted or corrected each other. This level of cooperation was attained after students became used to one another and were more willing to discuss their work and guide one another in a way that could encourage further constructive engagements. So the level of peer moderation and facilitation was quite significant that the educator's role could interfere with students' self-directed learning.

Diversity based on individual traits

The measures of social network analysis distinguished the individuals' interactivity. Although all students learnt through connections, some students were more participative than others. The lowest

and highest measures of centrality (in-degree and out-degree) and betweenness showed that there was a wide difference in student attributes. From the first learning session, the most interactive student (B1), was so engaged that she interacted with both the highly interactive participants and also the least interactive students. Therefore, diversity of interacting students may be reported through the individual measures of out-degree and in-degree. When participants with the most and the least measures of centrality interact, it implies there was no selective interaction between the outspoken and reserved participants.

The interview data has further contributed to the understanding of the presence of diversity and its impact on learning interactions and academic achievements in this study. The themes drawn out from students' transcripts indicate students supporting, testing, competing, trusting and encouraging each other. It can be explained that the students' intentions to engage peers according to these criteria, was an indication that students had varied attributes themselves, but the embedded aim was to achieve uniform goals. That means they first appreciated their diverse attributes before they worked towards achieving a holistic learning success.

As far as the academic achievements were concerned, connections also appeared between good and very good achievers and the low academic achieving students. However, there was a significant drop in interactivity of both high and low achievers in these patterns. For example, participant A8 participated in the first learning session and pulled out of the intervention for the next two sessions. He explained that because he was a slow student, he could not cope participating without a one –on –one engagement with the educator. He expressed that he prefers learning when guided by an educator. Hence, connecting with other participants was not conducive to his learning process. As a result, he did not take part in the second and third sessions. Similarly, B1 was the most interactive student in the first learning session. however, she became least active in the second session when E2 joined and dominated discussions.

4.6.4 Openness

The structural attributes of the learning patterns formed and the personal traits of students are the focal descriptors of openness in this study. Most students joined from the first learning session to participate in the learning intervention. However, a few joined later while others dropped out of the intervention. This indicates the presence of openness as a characteristic property of the entire learning process. However, this study explores further to validate the property of openness through a deep analysis into the properties of each learning pattern. In particular, the openness has been well experienced in small groupings that had frequent consultations with the educator and also in large groups when students worked as a class by engaging the educator less often. Academic achievements

of students who participated in these two learning patterns were more favourable than for those in other patterns.

Openness in a small group network pattern

While small groups were established, the students did not always maintain working with same peers from one learning session to another. The absence of certain students from subsequent learning sessions was one of the factors that caused some students to group with different peers. In so doing, students proved their tolerance to interact with and learn from different classmates. This evidence explores the openness of students, not only in connecting with new peers but also stirring up their learning discussions to sustain their connections. Due to the presence of open learning discussions, students had to self-regulate learning. This opportunity enabled them to achieve a recognized, good academic record. Although the educator appeared to support these small groups, the students continued to support each other.

Further peer to peer supportive teaching and learning practice have been noted in large groups. While teaching and learning support was noted as quite minimal from the educators, the dense connections among the students indicated that students were so open that they supported each other. The aim of achieving good academic records for all is among the various reasons cited for strengthened support in these large group interactions. More so, the students further expressed that an additional aim was competitive, to prove their unique and most efficient method to solve a given problem. Therefore, students with the most potential opened up to be approached by various peers. Their desire was to attract connections with peers. Since students expressed their autonomy and diversity during large group participation, some were of the view that various learning options shared by peers supported a very good learning practice.

Therefore, in this study I found that mathematics students can be open to each other based on the way they interact. Out of four experienced patterns of students' interaction, students are least participative when led by one person, either by an educator or any peer who seems to dominate the learning discussions. When students learn through participating individually, in pairs or in a decentralized learning pattern, students' participation becomes so limited that they have little opportunity for critical thinking and deep learning. This can be compared to their moments in, or their counterparts' participation in two unlimited patterns. These two unlimited patterns are subnetworks and large groups. The students had more opportunity to present their efforts, scrutinize them as a group and improve each other's learning practice. Hence these patterns are more likely to produce either good or very good academic achievers.

4.7 Conclusion

I came up with four major findings while evaluating a connectivist learning approach. The practices and perceptions of students indicated that the four attributes of connectivism (autonomy, connectedness, diversity and openness) are integral to the learning of mathematics. These attributes define the extent to which students interact in a specific pattern. In addition to that, it is evident that the presence of these attributes in students yields good academic achievements. This study's findings affirm that learning is a non-linear, complex process. Themes from my interviews reveal a reciprocal influence between academic achievements and the interaction practices of students. That is, at a given moment it is either one's performance which influences one's interaction practice or interaction practices which influence the student's academic achievements. For example: desire for academic excellence and trustfulness are two interview themes that yielded diverse perceptions of student interactions. To validate each attribute, I focused on patterns of interaction and levels of learning achievement. My findings report where effective learning and positive academic achievements were perceived. The figure below shows a multidimensional space as a format for presenting and explaining interaction practices of students with respect to their inherent connectivist attributes.

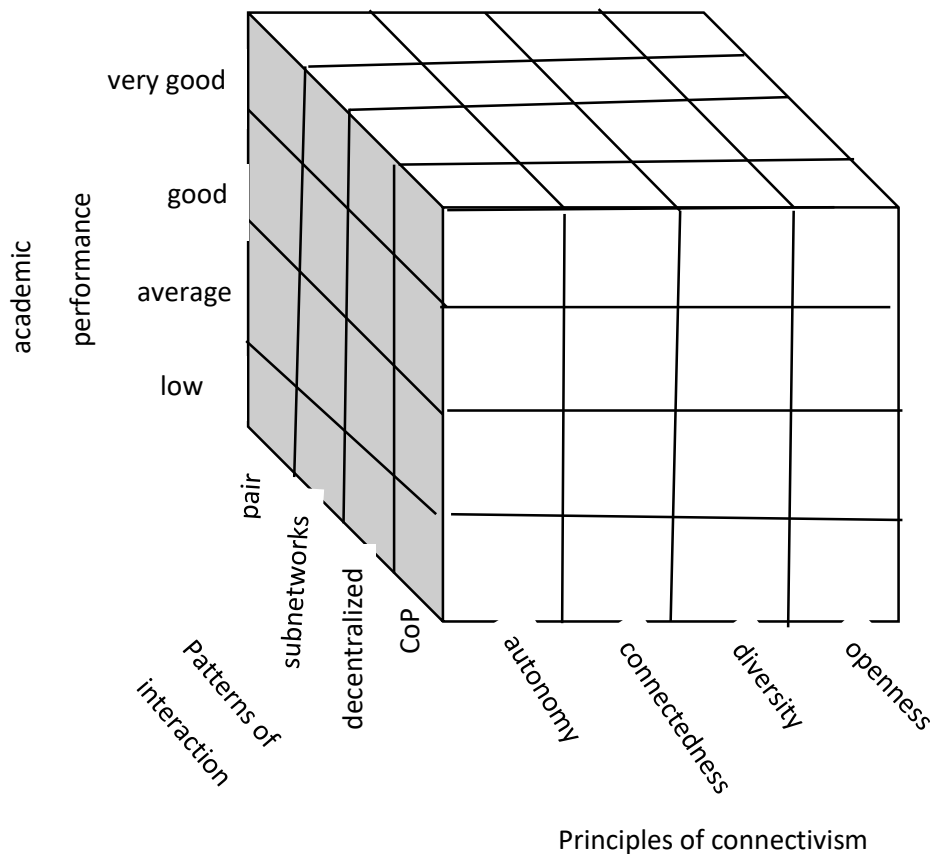


Figure 12: Multidimensional space for evaluating learning interaction dynamics

The four learning patterns identified through this study are pairs, subnetworks, decentralized patterns and community of practices. There are notable social and psychological elements that distinguish students from one pattern to another. From a connectivist’s perspective, these patterns translate to unique levels of autonomy, connectedness, diversity and openness of students. Furthermore, these four patterns are distinguished by the number of participants in a pattern and their interactivity behaviour. This study contributes to the basis for understanding interaction dynamics of connectivist students. Therefore, I confirm connectivist attributes of students with the following four major findings in my study.

Students’ choice on interaction yield effective learning

This study finds two peer interaction patterns in which students exercise a high level of independence, with many opportunities for making their own learning choices and decisions. The two are namely pair and community of practice. The students who participate in these two patterns have a good, perceived academic performance. However, the practices of students that result in this academic record differ between the two patterns. In a pair, students ensure total control of their learning by not engaging either with other peers or the educator. These students contained their learning discussion between themselves. They solved given problems through several attempts. Their efforts

were independent from other students. They worked more thoroughly than their peers who worked in larger numbers. Hence, they achieved deep learning through self –discovery efforts. Peers in a pair highlighted that their decisions and choices of methods of solving mathematical problems were based on their cognitive and assimilation capacity.

According to students who participated in a community of practice, their autonomous behaviour promoted peer support that yielded a positive impact on the good learning practice of students. The group members expressed that their decisions were not only individual, but also a result of collective efforts with other peers in their community. The collective efforts were reached through the students' ability to organize themselves with minimal interference from the educator. Unlike in a pair where decisions to choose a specific method were mostly individual choices, peers in a community of practice depended on each other, but progressed without close supervision from the educator. The subnetwork and the decentralized patterns are the other two established patterns of interaction but seem to have low levels of student autonomy. Learning progress in these two patterns seemed quite dependent on input from either the educator or the dominant student in the group.

Variety of views promote learning interaction

I found that the attribute of diversity has been noticeable through connections and dialogical learning among the students. When the educators promoted learning interactions, I found that majority of the students connected increasingly with their peers. Among the four patterns that were established by students in this study, students in subnetworks and in community of practices attest to a significantly diverse learning experience. This has been experienced through interchanging their learning dialogues. Peers in these two patterns engaged various peers, unlike students in a pair or a decentralized state who relied on input from a single participant. The practices of students that characterized the diversity of students included the desire to support each other, compete with each other and also search for alternative learning explanations. Students interacting in a large group, were the ones who expressed highest attributes of working from various inputs from other students.

The little guidance provided by the educator during problem solving in mathematics evidenced that students continued to interact with peers in a way to discover solutions through questioning and challenging their peers. In a connectivist learning approach, although the educator intervened to calm and facilitate prolonged learning dialogues, it was a common practice that students managed their diverse mathematical views by agreeing on an approvable solution. I evidenced that students had different methods and disputable procedures for problem solving. Therefore, the individual's desire to master methods of other peers resulted in dialogues for teaching and learning from each other.

Interactivity and connectedness

I found that mathematics students connected with peers quite often but showed little consistency. The manner in which students developed various patterns of interaction with new peers in every learning session was a good indication that they had the capability to connect peers and work. This attribute was quite significant in a community of practice where the highest number of connected peers were recorded in a group. Through this study I contend that when students portray high connectedness there is evidence that students display all the other attributes of a connectivism learning approach. For example; the diversity and autonomy of participants as attributes would be attainable due to connections between and among the students. The good learning performance of students that resulted from their connectedness was related to various student practices. The students indicated that, through connections, they developed each other's confidence to solve mathematical problems. When educators withdrew their teaching support, vocational students connected peers and performed remarkably well. This implies that the students in a densely connected group performed much better than when in participating in smaller groups.

Attribute of learning through discussing with peers.

Finally, I discovered that students enjoyed learning from and with peers. Although they cited aspects of personal gains such as competing with peers and also some desire of their own academic excellence, students offered open support to their peers. Across the learning sessions, students connected with new peers in the interest of learning. In so doing, students continued their learning discussions displaying an openness to assist each other. Students engaged with each other interchangeably, both within and across the learning sessions. They showed good interpersonal skills in connecting with new peers. This can be attributed a personality trait of enthusiasm for learning. Since students self-positioned themselves to work with particular peers, I found that some mathematics students who participated in a decentralized pattern were less active compared to their counterparts in other interaction groups. It is because they relied on total guidance from the educator or the most dominating student in the group. Therefore, they had little exposure to innovative thinking. Instead, they preferred working through a traditional, teacher centred approach with the teacher instructing students about their chosen subject/topic (passive learning).

In Chapter 5, I will discuss these findings from both a connectivist conceptual framework and the claims from relevant literature. I will further provide the contribution of the study together with limitations of my study.

Chapter 5

Discussions, contributions and recommendations

5.1 Introduction

In this chapter I present a summary of my research followed by a discussion of my findings. The study was informed by two research questions: What patterns of learning interactions appear from a connectivist learning approach of mathematics with TVET college students? And, how do students' practices in these patterns inform their academic achievements? The purpose of the study is to explore the connectivist learning approach by validating the presence of the four connectivist attributes – autonomy, connectedness, diversity and openness. Section 2 is a summary of the entire thesis, section 3 contains the discussion of the study that reflect on methodological processes and the relationship of the study to previous studies. In section 4, I provide the limitations of the study followed by the contributions of the study in section 5. These contributions are focussed in scientific discourse and classroom practice. Finally, in section 6 there are recommendations into practice of a connectivist learning approach and for further research.

5.2 Summary of the findings

The first research question is underpinned by an objective to explore the patterns that are formed during peer interactions. The objective of the second research question was to explain the level of academic achievements of students based on their practices during learning interactions. The two objectives can be linked to promote a connectivist learning approach during learning of mathematics in vocational colleges. During my data collection and analysis, I focussed on students' attributes that describe peer to peer interaction more than the attributes that describe the whole class' interactions and their changes. Wang et al. (2018) iterate that the structure of patterns of relationship between the students result from interactivity of an individuals. Therefore, I focussed on interaction dynamics that are displayed by individual students while connecting their peers. However, I noted that individuals' interaction practices influenced the formation and decomposition of groups of students. Hence, I validated the four connectivist attributes based on interaction dynamics across the groups

There were four patterns established by students during peer interactions. The four patterns are isolated pairs, educator-controlled separate groups (subnetworks), peer-controlled separate groups (decentralised) and a whole class interaction (community of practice). The establishment of these patterns is not a new phenomenon in research and also in the practice of teaching and learning (Wang et al., 2018). Although Rice (2018b) neither identify nor characterize the connectivist learning networks, she confirmed that they yield positive impact towards achievement of students. Patterns related to the ones obtained in my study were identified by Wang et al. (2018) However, my study

differs from previous studies that did not intend to validate connectivist attributes from these patterns. I find that some of the four attributes may be found in students who constitute a particular pattern of interaction and not found in other patterns. In my second research question I found mixed perceptions from students about the interdependence between the academic achievements and the interaction behaviour of a particular interaction pattern.

My second research question has been addressed by themes in two categories. Four themes that emerged explored the interaction practices of students within and across the patterns of interactions. The other four themes that emerged reflect an interchangeable impact between the level of interaction and the level of academic achievement. Students gave mixed views on the how their academic achievement and their levels of interaction can be interrelated. Mohamed et al. (2018) support that interactivity of students in a connectivist environment yields good academic achievement. However, my study informs that academic achievements also influence one's level of interaction. I validated the four attributes of connectivism from both perspectives of relating interaction and academic achievement. Themes that inform about student practices on interaction include territorial and workspace maintenance, desire for academic excellence, need for alternative methods and trustfulness. More so, themes that emerged on academic achievement include personal cognitive-orientation, decisiveness towards effective learning, monitoring of individual's learning path and capacity to confirm achieved learning goals.

The extent to which students showed attributes of connectivism was noticeable while they interacted with other peers. Where the student did not interact with peers, he or she is likely labelled to be not an open minded student, not a diverse student, a student with no connectedness with other peers. But the attribute of autonomy may not be evaluated on such students. However, such students who opt to work in isolation have been identified to be independent from other students' inputs and decisions. I find that to validate the four connectivist attributes from peer interactions both individual and group practices should be considered. According to my observation and interview instruments, I find that a certain connectivist attribute may be implied through the peer-interactions. Firstly, autonomy of students has been noticeable in pairs and during whole class combined discussions. In these two patterns students had active interactions with their peers. However, this was unlikely in a decentralized pattern where students gathered around an individual without making learning contributions. Secondly, the attribute of connectedness of students has been experienced in pairs, subnetworks and whole class discussion patterns. Students in these patterns showed that they need connections with peers in order to initiate and maintain a good learning achievement. Therefore, out of all the four patterns, it is only the decentralised learning pattern where peers are connected, but students experience little or no interactivity. Although peer connections promoted learning, other

students expressed mixed feelings about learning through these connections. For example, participant B1 cited that she switched from working in a subnetwork to work individually because she wanted to avoid disruptions from peer discussions. Her sentiment could suggest that students applied their personal decision on when to connect and interact with others.

Diversity of students is the third connectivist attribute evidenced by formation of various interaction patterns. Similarly, previous studies confirm that interactivity influence diversity of academic achievements during a connectivist learning approach (Mohamed et al., 2018). The number of students in these peer-interaction patterns changed as students re-joined different groups. Downes (2012) indicates that formation of multiple connections of students is an indication of diversity of students and search for novel ideas. During interviews with students they indicated that they continue to connect peers in order to explore new and alternative methods and solutions to solve their mathematical problems. Hence, students displayed their diverse attributes during learning of mathematics. Among all four interaction patterns in this study, I found that students connected in a pair and in a decentralised pattern have little interactional diversity. In a pair, homogeneity of students implies brief learning discussions with few diverging views. More so, in a decentralised pattern, an individual dominates by demonstrating to other members gathered around him or her. Hence learning is transmitted from one active person to passive students. Therefore, students end up with a single method of problem solving. Meanwhile, peer interactions in subnetworks and in whole class discussions were distributed among all the students. These discussions promoted equal opportunities for peers to express and share diverse learning ideas. Hence, in this study diversity of students has been validated based on multiple contributions shared from various students in a learning community.

I validated the connectivist attribute of openness from two perspectives. Firstly, as a personality trait and, secondly, as group attribute. In regard to individual or personality traits, the student narratives revealed that connections with peers were intended to acquire alternative problem-solving methods. This indicates that students were open to experience novel learning methods (regardless they have or the have not some adequate knowledge to solve a given problem) to solve a given problem. While a connectivist learning design entails learning should be distributed across the network of connections, I find that some students preferred to work individually. They indicate their preference to a teacher monitored and controlled learning pace. They displayed a personality closed to experience diverse and novel ideas from peers. But are accustomed to routine, and traditional ways of learning from the teacher. Therefore, the patterns in which peers expressed open attributes are subnetworks and whole class discussion. In these two patterns students were active participants. They learnt through multiple consultations with peers. Unlike in a decentralised pattern, passive students gathered to learn while dominated by an individual.

While addressing openness of this intervention through the perspective of group interactivity, I found that the total number of students per learning session changed from one learning session to another. It is an aspect of openness of a given community to have members who exit at any moment of interaction (Downes, 2011). However, peers managed to connect with new peers in each subsequent learning session. Moreover, I found that further openness emphasized use of voluntary participants, with some deciding to drop out of learning in the middle of the learning intervention. Others also joined the intervention in different days.

5.3 Discussion

Several lessons can be drawn from this study about educational research and practice. I discuss these lessons based on the following methodological, substantive and scientific reflections.

5.3.1 Methodological reflection

I conducted my study as a design experiment. While, pioneers of the learning theory of connectivism advance that the theory of connectivism integrates on chaos, complexity and network theories, I was of the view that the feasibility of a connectivist learning approach is informed by many factors. These factors are referred in literature as emergent, they are unpredictable, either noticed or unnoticed during interaction of students (Koopmans & Stamovlasis, 2016). Therefore, I grounded my study on a pragmatic philosophical view. In so doing, I designed the learning intervention to be facilitated by a voluntary educator. When required, the educator stimulated and mitigated levels of interactions among peers. These educator practices contradicted the setting of a connectivist learning which is considered to be a student-centred approach. Thus my intervention design process deviated from the tenets of a connectivist open and autonomous learning environment. As a corrective measure my data collection and data analysis excluded actions of the educator. Hence, I focussed on student-to-student interaction practices.

On implications of pragmatism on teaching and learning, Adeleye (2017) recommends that a pragmatic view on educators' actions promote both the traditional (teacher -centred) and facilitative (student -centred) practices. However, in the present study the educator(s) exercised the facilitating role only. During implementation and evaluation stages of my learning intervention I focussed on promoting and analysing data on four connectivist attributes interaction of students (autonomy, connectedness, diversity and openness). Therefore, my findings are neutral on whether the educator, in his practices honoured or overstepped into the students' proximal zone of development.

I employed a Framework for an integrated methodology (FraIM) (Plowright, 2011) as my research strategy to collect and analyse data. The FraIM entails a shift from using terms quantitative and qualitative to numeric and narrative data. I collected data using observation, interviews (asking

questions) and peer assessment methods. I opted for a research strategy that is not bound to a specific set of rules in order to address complexities within the dynamics (both sociological and psychological) in the learning interactions. During application of the FRAIM, I was content that my findings from the observations and interviews were sufficient to address the two research questions of my study. However, I did not collect and use answers written by students during my data analysis. I suppose, although academic achievements were reflected through peer assessment, a more conclusive reflection could be reached by analysing mathematical solutions written by the students during problem solving.

I conducted my intervention guided by the processes of the ADDIE model of instructional design. My learning intervention covered the content previously taught in the topic of financial mathematics, with applications to determine and analyse financial investment scenarios. However, the required skills of problem-solving required students to integrate concepts in number patterns (numerical and geometrical sequences). I believe that the interactivity of the students and the role of the educator may vary when the intervention would be conducted at a different time, topic and environment. I participated as a co-facilitator, my involvement with students was so limited that I recorded the interaction data as an observer-participant. Plowright (2011) indicates that in an observer-as-participant approach participants are aware of being observed although the researcher interacts with participants and stay detached them. The voluntary educator had more contact time with students, therefore he might have observed and learnt more practices of students than myself. But I did not engage him for data collection.

The entire observation process for collecting the interaction data was recorded on a video footage for a repeated analysis of all the learning interaction dynamics. The representation of the patterns of interaction in diagrammatic form (Figure 8, Figure 9 & Figure 10) in chapter 4 together with a social network analysis added a good understanding on interactivity of both individual student and entire group of students. Grunspan, Wiggins and Goodreau (2014) affirmed that the use of SNA in classrooms promotes an informative analysis about relational attributes of students. However, my study has been more descriptive therefore in addition to use of diagrams, I distinguished the interaction practices among the students using three measures of social network analysis (in-degree, out-degree and betweenness). Ultimately, the SNA supported a good understanding on interactional behaviour of students in this present study.

The observation and interview methods provided my data analysis with methodological triangulation. Mäkitalo (2016) argues that an understanding of observed practices is achieved through interviewing the interacting students. I employed the two methods to address my first research question and my

second research question respectively. I find that the elements that described or characterized the interactions of students were affirmed through interview responses. These elements addressed both questions of my study. For example, students B1 and E2 were seemed connected to several peers. Both students cited they compared their efforts and other students' efforts, hence they desired academic excellence. These two claimed to be good mathematics in the subject and were open to interact with various students. Therefore, data gathered through both instruments (observation and interview) explore the interaction behaviour of students, hence validates the four connectivist attributes in students.

I contend that my research findings have a substantive reflection on the needs of the current research purpose. However, there is a notable concern, a gap necessarily to be addressed with regards to my second research question. The research objective derived from this question describe the academic achievements based on classroom environment interaction practices of students. Cerezo, Sánchez-Santillán, Paule-Ruiz and Núñez (2016) confirmed a positive correlation between interactivity of students and their academic achievements. In this study I do not contest their finding. However, my study adds that, according to students' perspectives their level of interactivity is a result of a student's competency level or the learning performance. The basis of this proposition is backed by the postulation that learning process is a complex process without definite causes and effects (Kelly, 2017). Furthermore, Stamovlasis (2016) posit that students under free interactions develop nonlinear dynamical interactions. My study adds that the purpose of validating the connectivist attributes (on levels of autonomy, connectedness, diversity and openness) is achievable. The analysis of student narratives established that student interaction practices were performed without external influence. and their intentions during interaction with peers.

5.3.2 Relationship to previous research

In this section I discuss my study's relationship with regards to findings on patterns of interactions and the findings that subscribe to the validation of four connectivist attributes. There are patterns of interaction that I found. These were found in previous studies, but they were experienced and presented in different ways from my experiences in this study. The four connectivist attributes were confirmed in studies, but little have been put forward in the context of learning of mathematics

Patterns of interaction

In terms of exploring the patterns that appear during learning interaction during connectivist learning of mathematics, I find four patterns of peer interactions. These are in line with findings from researchers in various fields. These fields include college classroom interactions (Elliott, Gamino, & Jenkins, 2016; and Webb & Engar, 2016), mathematics learning (Harper, 2019) and in learning interactions in various learning areas (Grunspan et al., 2014). Firstly, I observed that interaction

dynamics within mathematics students, although seemingly unpredictable, develop from individual to a whole class learning community as students become familiar to each other. In the classroom community, students expressed high participation in multiple connections. Students displayed little intent to compete with peers, but clearly showed collaborated learning. My finding is in harmony with Elliott et al. (2016) who claim that college mathematics students participate as a community with a superordinate goal to succeed in learning. In this study students echoed a similar intent for of a common goal of academic support on each other.

In my study, pairs appeared to be significant connections between students. These connections grew from pairs to bigger groups as students got used to each other. This finding is counter to Webb and Engar's (2016) claim that a classroom community degenerate into subgroups and pairs. In contrast, I noted that students increased their peer connections over time. This differentiates interactions experienced during traditional, educator-centred classroom environments from a connectivist learning community. Therefore, when connectivist learning is promoted, the pairs gradually open up to become larger subgroups. Students continue to connect new peers while searching for further mastery of their learning content through alternative methods and solutions. This finding agrees with Buchenroth-martin et al. (2017) who found that interactions from small groups expanded into one massive learning network.

Furthermore, I found that it is worth noting the emergence of random decentralized learning patterns that unfold as dynamics in peer-interactions culminate in a messy learning community. Adamson and Walker (2011) described random and messy interactions as collaboration associated with complexity, unpredictability and management dilemmas. During these random emerging interactions in my study, the voluntary educator and elite individuals simultaneously facilitated the learning process. Recently, Harper (2019) indicated that the random decentralised interactions emerge as a result of challenging mathematical problems that requires critical thinking. While the learning process and students' positioning looked a bit messy, such emergent, decentralized interactive processes have been attributed to the complexity of education with possibilities of self-strengthening and self-propagating (Jörg, 2016) Therefore, my finding on decentralized interactions broadly concurs in theory and in practice with implications of collaborated learning of mathematics.

On subnetworks as a pattern of interaction in which students interacted but frequently engaged the educator, I found that students had a good level of participating and completing their task with confidence. However, they seemed to rely on the educator while their connections with the entire class remained limited. Nevertheless, they maintained close connections in their small groups. Buchenroth-martin et al. (2017) confirmed that small group interactions would maintain their

membership if students no members from other groups intervene their learning discussions to members in other groups. Therefore, heterogeneity of students expand the learning community and views of its members. in this study these subnetworks ended up joining into a whole class discussion.

Findings on connectivist attributes

Openness

Zou, Chen and Su (2018) differentiate openness in a traditional classroom from that of an online learning platforms. They pointed out that students can experience unlimited learning through openness of a MOOC than they can experience in traditional classroom. I concur with their argument in spite of the fact that the students who did not form part of my other learning sessions missed out on the learning proceedings for the day. I found that student practices that confirmed openness of individuals include engaging in frequent learning dialogues, connecting with peers throughout the learning session, exchanging peer connections within and across the learning sessions and participating in lengthy dialogues with peers. Downes (2010) confirmed that these attributes of students expressed their ability and willingness to share learning, hence their openness. Additionally, these findings portray connotations of openness in personality traits promoted by DeYoung, Peterson and Higgins (2005) and Schretlen, Der Hulst, Pearlson and Gordon (2011)

Diversity

In a learning environment diversity is a complex component ascribed to various elements that characterize the students and various environmental conditions (Downes, 2010; Tschofen & Mackness, 2012). In this study I considered two perspectives on diversity to narrow down the scope of my study on this attribute. Firstly, through observations I looked into the interactional practices of students. I focussed on the extent to which a student connect various peers through learning dialogues. Prior studies correspond with my findings on diversity by highlighting that students express diversity through interest in participating in various group sizes, either inside or outside the classrooms (Roos, 2018). In my study, development of four patterns of interaction confirms student diversity. Secondly, by interviewing students I considered diversity of students based on their learning practices. I found that learning of mathematics under a connectivist design approach promotes diverse learning practices. The notable practices include constant changing of peers, participation in multiple peer interaction patterns and changes in frequency of participation within these patterns. This finding is in harmony with Davidson's (2017) argument that high levels of student diversity is evidenced through communicating and collaborating with peers to establish their personal learning networks.

According to my interview instrument, diverging and overlapping views narrated by students reveals their diversity. The themes that emerged with respect to diversity of students include the desire for alternative methods and solutions and self-appreciation and recognition of others' efforts. Students

interacted in their learning practice with diverse goals. On one hand, the desire for academic excellence was considered achievable through interacting with peers, and on the other hand, it was perceived attainable through individual efforts under the educators' guidance. The mixed views reflect the diverse nature of students.

Autonomy

I found that autonomy of students during learning of mathematics is recognizable through the individuals' activity and also the group environment he or she inhabits. The individual activities that I noted include ability to work as an individual with minimal educator support, ability to communicate with peers during interactions and resistance to uptake peer's contributions without first making own work efforts. This finding concurs with Rajala et al. (2016) who expressed that autonomy of students is determined through their participation in class discussions and also when they insist on working as individuals without being influenced. Yackel and Cobb (1996) referred to autonomy of students as social autonomy, participation autonomy and intellectual autonomy. In my study I found that the three characterizations of autonomy can be related to different patterns of interactions established by students. Students in a community of practice (whole class interaction pattern) and in pairs (paired interaction pattern) registered both social and participating autonomy. They participated in lengthy discussions without influenced or controlled by the educator or other students. They seldom invited the educator's input, which implies that they mostly solved problems as they preferred (Steenbeek et al., 2017). Snyder (2006) found that mathematics students enjoy working in groups where they have opportunities for asking and contributing knowledge to peers. I evidenced this in whole class interactions.

On the other hand, I noted that a few students repeatedly invited the educator for learning assistance on regular basis. These students were in subnetworks and decentralised learning patterns, who relied most on either the educator or a dominant and good student. This finding is in line with findings from an empirical study by Harper (2019). She posited that where collaborated learning was not promoted, the educator controlled the learning pace and space, hence students worked as individuals with less interest to be active but remained passive students.

Connectedness

Utecht and Keller (2019) revealed that peer-to peer interactions as a property of a connectivist learning approach in a physical space is quite important on promoting sharing of learning information. In all the learning sessions of my study students worked through engaging peers. I found that only a handful worked individually in the second learning session. The students who worked through peer connections showed interest to support each other. The connections were established both with students in close positions and also others who were seated at a distance away from one's siting

position in the classroom. Peers from positions far apart moved around to interact with excitement. This finding strongly agrees to Brunetto et al. (2018) who posit that when given the opportunity, mathematics students are happy to work in groups. Furthermore, they indicate that peer interactions improve teaching and learning of mathematics. In the present study, while mathematics students were afforded opportunities to interact with peers, I noted that various patterns of peer interactions develop to confirm that learning connections can emerge without educators intentionally group the students. These patterns appeared in all sessions of the intervention. Students' narratives confirmed that students resorted to connect with peers in order to achieve their learning.

5.4 Research limitations

My research purpose was limited to the validation of four connectivist attributes (autonomy, connectedness, diversity and openness) of students. Although learning theory of connectivism is a constituency of chaos theory, complexity theory and network theory I did not consider these three theories as multiple lens of my study. However, my study referenced to some implications of these theories in order to clarify the phenomena experienced by students and the researcher. The complexity theory fitted in my study in describing the interactional practices students in their experienced patterns of interaction. The analysis into the attributes of the entire group was beyond the scope of my study. I limited my social network analysis to the use of three relational measures (in-degree, out-degree and betweenness) in order to quantifying and differentiate interactivity between the individual students. A structural analysis into the combined behavioural characteristics of the entire group was beyond the scope of study. (that is looking at other SNA measures the structural properties of the entire group.

Cohen et al. (2007a) indicate that findings from a quantitative study can be generalised and findings from the qualitative study can be transferred to other studies. Although I used a SNA (regarded by as a quantitative method of data analysis the findings of my study cannot be generalised in spite of the fact that I used a small sample size (24 students). However, the study can be replicated in various areas of learning, learning platforms and also mathematics topics. As I took part in facilitating the learning process, my presence in the room may have influenced behavioural changes with the students. However, the findings of the study do not take into account possibilities of educator-impact on the level of students' four attributes. I conducted my research and data analysis on pragmatic procedures where I employed a social network analysis to differentiate between lowest and the highest participating students. However, I did not premise my interpretations on the computations of this SNA. Since my study was based on a learning intervention designed for revision purposes, the students' responses may differ from other stages of teaching the topic.

5.5 Contribution of the study

The purpose of this mixed method study was to validate four attributes of a connectivist learning approach in college mathematics students. These four attributes are autonomy, connectedness, diversity and openness. Prior studies that have investigated these attributes, confirmed the presence of these attributes when students participate in business management subjects (Bannister, 2016). Bannister's findings were premised on perceptions of the educators. It was confirmed that students exhibit all four attributes, however recommendations were advanced for a similar study into science, technology engineering and mathematics (STEM) subjects. The four attributes of students were positively correlated with successful learning practice (Smidt & Thornton, 2017). The present study established that during learning of mathematics the four attributes were conditionally experienced. It yielded unique findings to identify the connectivist attributes of students based on patterns formed during peer-connections. This mean that researchers and teaching practitioners may improve their conduct based on the findings in this study. This will improve a student centred teaching and learning of mathematics. To be specific, the research into interaction patterns in classroom environments would not be premised on a 'one size fit all' basis since my study put forward that students interact in various forms that either imply or may be implied by their academic achievements. As the connectivist attributes of students vary from one pattern to another their learning practices and their academic achievements also vary. However, the variations are not predictable.

5.5.1 Research contributions

The current study shows that patterns formed during learning interactions are good indicators of four attributes of connectivism. These patterns are not static but dynamic in nature. They shape and reshape within and across the learning sessions. It is therefore highlighted that the study into connectivist dynamics (attributes) during learning interactions requires a dynamic approach. According to Koopmans and Stamovlasis (2016), a dynamical system in education refers to a group of members interacting that shows deviations in its properties from time to time. And these deviations are not predictable. Although Downes (2010) as one of the pioneers of learning theory of connectivism alluded that the presence of connectivist attributes implies development of new knowledge, this study does not confirm the postulation in this mathematical learning since no analysis was made into the contributions of the students.

According to the four patterns of interaction experienced, this study put forward that interaction and learning practices of students per pattern are intertwined with various connectivist attributes. That implies it is not sufficient to separate the connectivist attributes from each other when describing the interaction dynamics of students. This suggests the relevance of implications of a complexity theoretical lens on analysis of learning interaction dynamics. Hence, this study confirms that there are

complexities that premise during the formation, existence and decomposition of the learning interaction patterns.

The presence of one connectivist attribute may suggest that other attributes are also present in a given learning situation. Hence, inferences from a particular attribute should be regarded as indicators of various attributes of connectivism. For example, an analyses into the diversity of students based on multiple connections between peers yield related interpretation on connectedness and autonomy of students. This is when the both interaction diagrams and interview themes are used during the process of analysing student interaction practices. Therefore, further to perceptions of the educators reported in Bannister (2016), the current study forwards that student narratives may be more relevant towards interpreting and understanding the interactivity of students. Therefore, a good understanding of a connectivist learning approach is achievable by considering experiences and perceptions of students (through both observations and interviews) more than relying on researchers' or educators' perspectives. observations and interviews.

5.5.2 Practical contribution

In practice, this study aware mathematics educators that when facilitating in a student centred lessons, interactions are dynamic. Hence, the teaching instructions may depend on the manner in which students connect each other in class. For example, when students are not open to engage the educator, the educator may promote decentralized interaction patterns, whereby students receive and share various methods and mathematical solutions through other students. of students. It is important for educators to recognise that peer interactions are characterized by connectivist attributes. The educators therefore need to understand the necessity of self-forming learning groups during teaching and learning of mathematics. The facilitation role of the educator should not interfere with the manifestation of the connectivist attributes in students. While Smidt and Thornton (2017) advance the practice of a connectivist learning approach in STEM subjects, this current study reveals that during a connectivist approach educators input may distract students who prefers to learn through their peers. learning practice. On the one hand, educators limit the students' connectivist attributes in smaller groups, and on the other hand, educators facilitative dialogical learning. When students, are a bit passive educators could identify and connect with the influential students and encourage them to stimulate learning in several.

5.6 Recommendations

Based on the findings of my study, my recommendations are in three categories. These are recommendations for future study, recommendations on practice and recommendations on policy.

5.6.1 Policy recommendations

Each student should be tasked to make a contribution. The fact that these patterns are formed in an emergent manner and quickly decompose, it may be recommended that students who participate in such emergent interactions may be tasked to share their discoveries with the entire class to encourage active participation in class. Hütter and Diehl (2011) highlight that cascading and free-riding yields negative learning. Free-rider is the term that refers to participants who switch groups without making learning contributions. Cobb et al. (2011) suggested that students with such an acquisitionist approach should embrace a participationist approach for an effective learning of mathematics. That implies in mathematics students should have adequate time to interact with the learning content.

5.6.2 Recommendations for practice

After finding that more than one connectivist attributes may characterize students in a particular interaction pattern, I recommend educators to take notice of the patterns that develop while teaching different topics in mathematics. Since these patterns have been confirmed to be dynamic, it will be helpful for educators to plan their teaching and learning approach under changing situations. I recommend educators to replicate their teaching approach with either same or different student groups in order to identify and address the attributes or elements related to these attributes that hinder the participation of students. With regards to my research finding about the nonlinearity between the interactivity of students and the academic achievement, I recommend that instructional designers develop the learning material that promote both interaction between the students, hence ensure students will have a good understanding from learning through the designed material.

5.6.3 Recommendations for future research

There are three recommendations for further research based on my findings. My main finding from validating four connectivist attributes in mathematics students is that the describing elements on one attribute imply the presence of another or other attributes. Therefore, I recommend that studies into the interaction practices of students or connectivism theory should be addressed in dual theoretical or conceptual lens. This will ensure a good understanding of the characteristics and effects of the connectivist attribute(s) under study. More so, my second finding is that the patterns formed during interaction of students are dynamic. They expand or decompose due to the underlying connectivist attributes that characterize members of the pattern. Smidt and Thornton (2017) noted that patterns expand into one massive network. However, the factors that determine such changes were not informed. Therefore, it is a gap to be filled through research into the dynamical state of the interaction pattern of students. Moreover, related to this finding, I recommend that future research should investigate how the connectivist attributes of students may be manipulated to ensure active participation of students a distributed learning across the whole classroom group (or network). This

will yield the *'how'* and the *'how not'* to manipulate patterns of interaction towards creating and maintaining a learning culture in students. While Wang et al. (2018) recommended that analysis into the patterns formed during connectivist learning be connected with behavioural analysis, I further recommend research into this area. The third finding in my study shows that there may be some debatable causal relationship between the academic achievements and the interactivity of students.

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Appendix I: Ethical Clearance



University of Technology

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Roeland Street, Vredehoek, Cape Town 8001.

Office of the Research Ethics Committee	Faculty of Informatics and Design
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At a meeting of the Faculty Research Ethics Committee, ethics approval was granted to

Mr Wellington Nyadenga student number 210277831 for research activities related to the MTech: Design degree at the Faculty of Informatics and Design, Cape Peninsula University of Technology.

Title of dissertation/thesis:	An evaluation of a connectivist learning approach with college students in South Africa
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Comments

Research activities are restricted to those detailed in the research proposal.

 Signed: Faculty Research Ethics Committee	<u>14/5/2017</u> Date
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COLLEGE OF CAPE TOWN
In s p i ri ng minds

I, Petrus Johannes Kaubseh in my capacity as Campus Manager City Camp at Ellington adoo - cr- give consent in principle to allow a student at the Cape Peninsula University of Technology, to collect data in this company as part of his/her M Tech Design research. The student has explained to me the nature of his research and the nature of the data to be collected.

This consent in no way commits any individual staff member to participate in the research, and it is expected that the student will get explicit consent from any participants. I reserve the right to withdraw this permission at some future time.

In addition, the company's name may or may not be used as indicated below. (Tick as appropriate.)

	Thesis	Conference paper	Journal article	Research poster
Yes				
No				

PJ Kaubseh
 <<name>>
 PJ Kaubseh

28/03/2017

<<i date>>

Appendix 2: Student activity

- 1.1 Mr Nxexe realised that when a loved one died, there is always need for instant cash flow of money. He studied various life cover policies from different companies. The one that caught his attention is given in the table below. Mr Nxexe family consists of his wife and two children.

The following table illustrates the pay-out value details in rands of the life cover policy.

Life cover	Pay-out value				
	Main member	Partner	Child aged 0-5	Child aged 6-12	Child aged 13 plus
Year 1	150 000,00	100 000,00	80 000,00	60 000,00	50 000,00
Year 2	D	105 000,00	84 000,00	63 000,00	C
Year 3	165 375,00	110 250,00	A	66 150,00	55125,00
Year 4	173 643,75	115 762,50	97 240,50	B	57881,25
Year 10	232 699,23	155 132,82	124 106,26	93 079,69	77 566.41
Year 20	379 042,53	252 695,02	202 156,02	151 617,01	126347,51

NOTE: compulsory annual increases: **annual benefit increase 5%**

- 1.1.1 Calculate the following missing values:

- (a) Pay-out value of a child aged 0-5 years in the 3rd year (A)
- (b) Pay-out value of a child aged 6-12 years in the 4th year (B)
- (c) Pay-out value of a child aged 13 plus years in the 2nd year (C)
- (d) Pay-out value of a main member in the 2nd year (D)

- 1.1.2 Give ONE advantage and ONE disadvantage of paying money into a life cover plan.

- 1.2 Mr Nxexe, a 45-year-old manager, studied the tax tables for the 2016–2017 financial year to determine the tax that he had to pay for the financial year. He ignored the medical and pension deductions. Study the tax table below and answer the questions that follow.

Taxable annual income	Rates of tax
R0 –R188 000	18% of each R1
R188 001–R293 600	R33 840 + 26% of the amount above R188 000
R293 601–R406 400	R61 296 + 31% of the amount above R284 100
R406 401–R550 100	R96 264 + 36% of the amount above R406 400
R550 101–R701 300	R147 996 + 39% of the amount above R550 100
R701 301 and above	R206 964 + 41% of the amount above R701 300
Tax rebates	
Primary rebate	R13 500
Secondary rebate (for persons 65 years and older)	R7 407
Tertiary rebate (for persons 75 years and older)	R2 466

[Source: SARS pocket guide, 2016/2017]

Mr Nxexe has a taxable monthly income of R36 240,96. His monthly PAYE is R7761.75. is his tax calculated correctly? If not, what should it be? Show all your calculations

- 1.3 Mr Nxexe started teaching on 1st January 2010. He contributed to the government Employees Pension Fund (GEPF) from 1st January 2010. He considered taking an early retirement on 31st December 2018. Use the information below to calculate his retirement pay-out packages.

Retiring with less than 10 years of service	Retiring with more than 10 years of service
You receive a once-off lump sum(gratuity)	Your benefits consist of two parts: A once-off lump sum(gratuity) A monthly pension called annuity

The gratuity and annuity are calculated using the following formulae:

Gratuity = $6,72\% \times \text{final salary per year} \times \text{years of pensionable service}$

- 1.3.1 Calculate his pensionable years of service if he retire 31st December 2018.
- 1.3.2 Use the appropriate formula to calculate his gratuity, rounded off to the nearest rand if there has not been any increase in his monthly salary since 2016.