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Dissertation

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MASTER OF EDUCATION

in the field of

MATHEMATICS EDUCATION

With the title:

Investigating some grade 9 learners' challenges in Algebra with specific reference to integers.

FACULTY OF EDUCATION

Candidate	:	Mr. Wandile Mangcengeza
Student Number	:	210068868
Supervisor	:	Dr Kwanele Booi

ABSTRACT

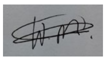
The teaching of integers is essential in the accumulation of mathematics knowledge among learners in primary and early secondary school years. In Grade 9, it is expected that learners develop a sound understanding of integers, which is needed in Further Education and Training as well as tertiary education, for those who opt for pure mathematics or mathematical literacy. To provide a strong mathematical foundation among Grade 9 learners, teachers and learners have to overcome several challenges that affect learners' understanding of fundamental concepts including that of integers. Qualitative research methodology within the interpretivist paradigm was used to address the problem being addressed by this study. The purpose of this study was to explore the challenges experienced by Grade 9 learners in understanding integers in Algebra. The target population for this study consisted of 120 Grade 9 learners. A written test on integers, Algebra, was administered to a sample of 40 learners. The face-to-face interview method was used in collecting data from a purposive sample of 5 learners, upon which the Early Algebraic Conceptual Framework by Kaput (2008) was used to analyses data and interpret results. The trustworthiness of the data and results used in this study was achieved through triangulating instruments. Data from the algebra test, and that from interviews, were analysed using open codes, from which themes were generated, interpreted and discussed based on the reviewed literature and theoretical framework. This study confirmed that Grade 9 learners experienced challenges when learning integers in early algebra. Furthermore, it was found that learners struggle to use symbols to generalise, act on symbols, and follow the appropriate rules. The findings were similar to those made in the Early Algebraic Conceptual Framework. The study concluded that the major challenge faced by Grade 9 learners was in understanding the properties of integers, and this resulted in errors such as incorrect application of rules. The study further concluded that the failure of learners to find relationships between variables and numbers was due to a poor understanding of integers. Moreover, when using distributive property, learners tended to ignore the minus sign preceding the integer they were multiplying with, formed part of the product. These findings of the study underscore the need for teachers to discourage learners from memorising rules, but to understand and be able to apply them correctly when given complex integer problems. The study recommended that an integer baseline assessment need to be administered to Grade 9 learners to create a picture of competence in integers. Early detection of integer knowledge gaps among Grade 9 learners, could help learners who were struggling to become achieve better results. It is further recommended that, with suitable instructional guidance from the educators, learners can formulate their definitions that will help them solve even complex integer problems.

Declaration

I, Wandile Mangcengeza declare that the work presented in this dissertation with a title: “Challenges experienced by Grade 9 learners in understanding integers in early Algebra” is my work and where other sources were used for reference, they were acknowledged according to the Harvard system of referencing.

Name: Mr. Wandile Mangcengeza

at Cape Peninsula University of Technology (South Africa)

Signature:  Date: 21 September 2021

Acknowledgements

"What I will show to you must happen at the time that I have decided. That is a future time, but it will certainly happen, as I have said. It may not happen very soon, but you must be patient. What I show to you will certainly happen at the right time, so wait for it". (Habakkuk 2:3).

I now clearly understand the meaning of this verse. I could not have done it if it was not for the grace of God through his Son, Jesus Christ.

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LIST OF FIGURES

LIST OF EXTRACTS USED IN THE STUDY

TABLE OF CONTENTS

CHAPTER 1. OVERVIEW OF THE STUDY

1.1. Introduction

The underperformance in mathematics by Grade 12 learners in South African schools, and the initiative was taken to redress the situation is well-documented. Current and past studies attribute Grade 12 underperformance in mathematics to various factors. The knowledge of integers is fundamental in the teaching of mathematics at Grade 9, particularly Algebra. However, it would seem that the progress of Grade 9 learners in developing good algebraic skills are hindered by the challenges in manipulating mathematical problems requiring the application of integers (Ncube, 2016; Whitacre, Azuz, Lamb, Bishop, Schappelle & Philipp, 2017). This phenomenon has been observed at the school of practice, hence prompted the researcher to explore challenges experienced by Grade 9 learners in understanding integers in early algebra. Pournara, Hodgen, Sanders and Adler (2016) assert that difficulties in algebra have been linked to a lack of understanding of integers.

The researcher has observed that Grade 9 learners tend to perform badly when given algebraic expressions in which they simplify by multiplying integers. Seng (2013) found that learners in lower grades committed many mistakes and errors when simplifying expressions with negative pre-multiplier in bracket expansion. The author further asserts that learners have difficulty in operating integers and these difficulties appear to be robust. Even those who have completed algebra courses are challenged by problems with negative numbers. A study by Pournara *et al*., 2016 emphasises that on the need for research studies on how lower grade learners' mathematical skills and knowledge develop when they move from simple arithmetic based problems to algebraic expressions.

Learners' poor performance in mathematics has become a source of concern for many stakeholders (Sahat, Tengah and Prahmana, 2018). Many specialists have proposed various solutions to this problem, but the problem persists as it is believed that inadequate basic knowledge of mathematics among learners is the root cause of the problem (Sahat et al., 2018). Despite several attempts to improve learners' understanding of mathematics, algebra remains a major obstacle for many secondary mathematics learners in South Africa (Pournara et al., 2016). One problem learners face when dealing with integers is that they are confused by the signs and operations of the integers, which makes them fail to understand the integers (Sahat et al., 2018). Pournara *et al*. (2016) claim that most Grade 11 learners continue to face challenges when dealing with certain fundamental concepts applied in rudimentary algebra. Literature shows that important aspects of basic algebra are expressions, functions, systems of equations, real numbers, inequalities, exponents, polynomials, radical, and rational expressions (Toth, 2021). According to the CAPS document, these aspects of basic algebra must be taught in early grades, under the topic integers. This reinforces the comments made on the performance of Grade 12 learners in paper one of the National Senior Certificate each year. Paper one consists of equations and inequalities, number patterns and sequences, functions and graphs, financial mathematics, and calculus. South African examiners' reports for three consecutive years (2017, 2018, and 2019) claim that most candidates lacked the fundamental mathematical competencies that could have been acquired in lower grades. They further elaborate that candidates have problems with algebraic skills. This becomes an impediment to candidates answering complex questions correctly (DBE, 2018:140).

Manly and Ginsburg (2010) assert that studies of children and adolescents, showing their difficulty transitioning from arithmetic to algebra and with certain algebraic concepts, should be replicated with adult learners. Adults represent a significantly different population and already have encountered algebra in school (Manly *et al.*, 2010). Algebraic reasoning connects the learning and teachings of arithmetic in elementary grades to functions and calculus in secondary grades (Ministry of education Ontario, 2013). It provides a foundation for the development of abstract mathematical understanding. A study by Cetin (2019), suggests an alternative model that teachers can adopt to solve basic problems that Grade 6 learners face in understanding the concepts and operations associated with integers. . However, little work has been done on challenges experienced by Grade 9 learners in understanding integers in early algebra. As a result, no clear learning, and teaching methods, have been developed to overcome these challenges.

1.3. Rationale for the Study

To improve the performance of Grade 9 learners in mathematics, learners need to understand key concepts that form the basis of Algebra. This is view is supported by Götze (2016) who purports that teaching algebraic concepts in early grades should assist learners to develop mastery of crucial mathematical facets needed in the future. This study intends to explore challenges experienced by Grade 9 learners in understanding integers in early algebra.

1.4. Problem Statement

Mathematics is considered one of the subjects that are critical to the country's economic growth and development (Shay, 2020). This is a global issue and not unique to South Africa. The drop in the number of learners writing Grade 12 mathematics examinations should be of great concern. Shay (2020) notes that good performance in mathematics is essential for university entrance. A decline signals the closing of the doors to prestigious careers such as engineering, architecture, chartered accountancy, medicine, and economics (Siyepu, 2013). This thus requires researchers to focus on concepts key to learning algebra. The proposed study seeks to establish the challenges experienced by Grade 9 learners in understanding integers in early algebra.

1.5. Research Questions

The study answered the following questions:

Main question:

What challenges are experienced by Grade 9 learners in understanding integers in early Algebra?

Sub-questions:

1. How do learners reason when solving integer problems?
2. What are the errors learners make when solving integer problems?

1.6. Purpose of the Study

The main purpose of this study was to establish what challenges are experienced by Grade 9 learners in understanding integers in early Algebra.

The sub-aims of the study were to establish:

1. How learners reason when solving integer problems.
2. The errors learners make when solving integer problems.

1.4. LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

The preliminary literature view was conducted to provide evidence of the inherent challenges faced by learners in basic algebra when dealing with integers. Several studies on the teaching of mathematical concepts on algebra were reviewed to understand issues surround challenges faced by learners in early grades and solutions used. Past and current studies have been reviewed. For example, influential authors such as Kaput (2008), Cooper and Warren (2011), Gilderdale and Kiddle (2011), Blanton, Brizuela, Stephens, Knuth, Isler, Gardiner, Stroud, Fonger and Stylianou (2017, 2018), Deborah (2018) Fuadiah, Suryadi and Turmudi (2017, 2019) have carried out extensive work on the teaching of mathematics and provided the grounding of this study. The works of Kaput (2008) has been highly inspirational across past and current studies which seek to improve the teaching and learning of mathematics particularly in early grades. The basis for the cited works is premised on the notion that quality teaching of mathematics in early grades can lead to the development of higher mathematical skills needed in the tertiary and place of employment. Through these keys studies, the importance of good teaching of algebra at lower grades reverberates. A detailed literature view for this study is presented in Chapter 2. However, an insight into the theoretical framework underpinning this study is provided in the immediate subsection. The literature in this section seeks to introduce the basic concepts of integers and the operations used in the basic algebraic expression. A brief discussion of possible challenges faced by Grade 9 learners in algebraic expressions involving integers is presented and expanded in Chapter 2.

1.4.1 The Conceptual Framework for the Description of Integers

This study is guided by the Early Algebraic conceptual framework developed by Kaput (2008). The conceptual framework illustrates several concepts which are fundamental in the teaching of mathematics to learners in early grades A conceptual framework plays an essential role in a research study when exploring key concepts. It highlights key concepts forming the pillars of the study. The conceptual framework used in this study consists of import concerts derived from the algebraic conceptual framework developed by Kaput (2008). This conceptual framework has been widely used in several studies such as those conducted by Cooper and Warren (2011), Gilderdale and Kiddle (2011), Blanton, et al (2017, 2018), Fuadiah, Suryadi and Turmudi (2017, 2019). The framework provides basic concepts used in the teaching of integers in a basic algebraic expression. It highlights key issues that teachers and learners need to focus on in order to provide a sound foundation in the understanding of mathematics and its application in the real world. Literature shows that learners are exposed to or introduced to several new mathematical concepts during their early stages in the school system (Carolyn, 2018; Deborah, 2018; Pinto, Eder; Cañadas and María, 2019). Some authors emphasise that the teaching of Early Algebra to lower grades should be regarded as an opportunity to encourage learners to think more deeply about the underlying structure of the mathematics that they study (Blanton, et al. 2018; Kaput & Blanton, 2005; Pinnock, 2020). This implies that teachers need to take the teaching of algebra seriously by developing the knowledge of key concepts such as integers, subtraction, addition, positive, negative, odd and even numbers. The early algebraic conceptual framework is based on the concepts such as the integer, operations including; addition and subtraction of integers, multiplication and division of integers. Other key aspects of the conceptual framework include generalising, representing generalisations, justifying generalisations and reasoning with generalisations, each explored in Chapter 2.

1.4.2 The Concept of Integer

Integers are the first numbers that learners are introduced to in both informal and informal learning set up (home and school). Basically, integers include all positive and negative numbers without any decimal or fractional parts. Integers can be manipulated using four basic mathematical operations namely: the positive (+) used to add two or more integers; negative, minus, subtraction (-), for reducing the value of an integer; multiplication (x) used to increase the value of an integer by several times of another integer; and division (÷), used to determine the size or value by which an integer is bigger than the other integer. Learners face challenges when manipulating integers due to several factors. It could be due to a lack of fundamental skills and knowledge in using the operations when manipulating algebraic expressions involving integers and different operations. This was designed to explore the challenges that led to the poor conceptualisation of operations when working with integers among learners in Grade 9. The concept of integers is further elaborated in Chapter 2 of this study.

1.4.3 Generalising and other concepts used in the conceptual framework

Studies on the teaching of algebra to early graders view generalising as an important mental process that should be developed and nurtured in all learners (Blanton, et al., 2018, Pinto, Eder; Cañadas and María, 2019). Literature shows that learners who are provided opportunities to develop generalisation at an early age tend to perform well in mathematics at the later stage of their education and probably workplaces (Warren, 2011; Put, 2008; Pinto, Eder; Cañadas and María, 2019), Based on the successful use of the conceptual framework in several studies, the research found it relevant in the current student. The concept of generalisation is associated with other key concepts such as representing generalisations, justifying generalisations and reasoning with generalisations, which are explored further in Chapter 2.

1.4.4 Challenges Learners Experience with Integers and their Causes

There is a plethora of literature from studies that explored and analysed the teaching of mathematic in general and algebra in particular. There seems to be a consensus from existing studies that the teaching of mathematics is important, but learners face a lot of challenges that need to be addressed (Blanton, et al. 2018; Carolyn, 2018; Deborah, 2018; Pinto, Eder; Cañadas and María, 2019). From the findings and recommendations of these studies, it is important for teachers to establish contextual factors that negatively retards the learning of mathematics and use the right approaches to enhance the development of proper mathematical knowledge among learners in lower grades. This study uses the early algebra conceptual framework to explore the challenges faced by Grade 9 learners in manipulating algebraic expression with integers. In Chapter 2, the researchers present a detailed literature review of the possible challenges that negatively impinge on the ability of learners to develop appropriate skills and knowledge in the use of integers in algebraic expressions.

1.4.5 Models for Teaching Integers

To assist learners to form clear conceptual structures needed in the learning of mathematics, Cetin (2019) encourage teachers to utilise simple number line models, usually called a directional model. A number line refers to the basic model using to understand mathematics in early grades (Pradhan, 2019). Cetin (2019) proposes the use of the 'equality or quantity model' when teaching integer to lower grades. The basis for this model is the marked quantity idea, indicating –n being less than zero. The model explains the concept of integers using opposite ideas such as positive-negative; proton-electron or asset-debt (Cetin, 2019). The model must specify that the addition operation, be used in adding together given quantities while the subtraction operation is used to reduce quantities or merely adding their opposites (Cetin, 2019). When utilised appropriately, learners can develop the minimum required skills to manipulate algebraic expressions involving integers.

The usefulness of the model under discussion is reported in several studies dealing with teaching and learning algebra as it encourages teachers to provide learners with opportunities to solve mathematical problems based on the real world and context. Real-world context learning makes it possible for teachers to provide learners support need in dealing with integers. Findings and recommendations from existing studies can justify developing tools and methodologies for improving the teaching of algebra based on the immediate real-world situation relevant to learners' needs and ways of thinking (Fuadiah, Suryadi and Turmudi, 2019).

1.5. RESEARCH DESIGN AND METHODOLOGY

1.5.1 Research Paradigm, Approach, and Design

The study has employed an interpretive paradigm. Interpretive research was used to examine learners' understanding of basic concepts used to develop rules when learning about integers.

A qualitative approach was used in this study. Qualitative research attempts to obtain an in-depth understanding of the phenomenon, namely, basic concepts of learning integers.

The study employed an exploratory design. An exploratory research design is conducted for a research problem when there are few or no earlier studies to be referred to or relied upon to predict an outcome. The focus is on gaining insight and familiarity for later investigation. The exploratory research design was a useful approach to gain background information about learners' understanding of integers.

1.5.2 Research Methodology

1.5.2.1 Site Selection

The research was conducted at a public high school. The site is situated in the West Coast District of the Western Cape, where the researcher is employed. Geographical accessibility, proximity, and functionality were the factors that influenced the choice of the selected school.

1.5.2.2 Participant Selection

The participants consisted of one Grade 9 Mathematics class, with 40 learners.

1.5.2.3 Data Collection

The study employed document analysis, learners' written tasks (formative assessments), and individual interviews.

In document analysis, the researcher examined the Grade 9 learners' textbooks, workbooks, and online teaching material. The researcher designed a written task that consisted of integer questions in four cognitive levels, to ascertain at what stage learners begin to struggle. The errors learners make in this written task were analysed, and based on such errors, this study predicted the underlying causes of these errors. The errors and the causes of the errors were described as the challenges experienced by the Grade 9 learners in understanding integers.

After the written tasks, the researcher conducted individual interviews, to discuss, with the learners, the errors they had made and the possible causes of the errors.

1.5.2.4 Data Analysis

The researcher employed inductive content analysis to identify themes emerging from the interviews about the errors and the possible causes of the errors. The errors in the written tasks were grouped in the types of errors made.

1.5.2.5 Trustworthiness

To verify the veracity of the findings of the research study, the researcher used triangulation and member checking. The researcher also applied transferability to check if the findings of the study are transferable to other contexts in a similar situation. The researcher also checked confirmability and reliability through a research audit where an outside person reviewed and examined the research process and data analysis to ensure that the results of the study were consistent and could be replicated.

1.5.2.6 The Researcher's Position

The researcher was responsible for the entire process of the research, from developing the research instruments, conducting it, analysing it, to writing the entire thesis. The researcher is not an employee of the school where the research was conducted. This made it easier for the researcher to select any Grade 9 class to avoid bias.

1.5.2.7 Ethical Considerations

Formal permission was obtained from the Department of Basic Education, CPUT, and the Headmaster to conduct this research. The reference for approval from the Western Cape Education Department is **20210514-3011** and the reference from CPUT is **EFEC 3-7/2021**. No one other than the researcher had access to the data. The data collected was not used for any report or published document other than the study to ensure confidentiality.

1.5.2.8 Contribution of the Research Project

The results of this study will benefit learners and teachers since integers are an important part of the algebra curriculum. They will help teachers understand learners' difficulties and find out what teachers should emphasise in the curriculum to improve learners' performance in algebra. Thus, schools that adopt the recommended approach derived from the findings of this study will be able to better engage learners in teaching integers. The study will also help researchers uncover critical areas in learning whole numbers that have been unexplored.

1.6. OUTLINE OF CHAPTERS IN THE DESSERTAION

This dissertion is made of five chapters as outline:

Chapter 1: Introduction and overview of the study

Chapter 2: Literature review and theoretical framework

Chapter 3: Research design and methodology

Chapter 4: Results presentation and discusiion

Chapter 5: Conclusion and recommendations

Have presented the background to the problem, the problem statement, research questions, focus and scpe of the study in this chapter, the next chapter presents literature review and conceptual frame.

CHAPTER 2. LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1 Introduction

The insights into the literature and conceptual framework were provided in chapter 1. In this chapter, a detailed literature review and conceptual framework is presented. The literature review presented is derived from several schoalry sources such as peer reviewed conference papers and peer reviewed journal articles. Google Scholar was used in locating most of the literature sources cited in this study. This chapter mainly focused on two aspects namely: possible causes to challenges experienced by learners in understanding integers a well as models that are used to teach integers and how effective these models are. The choice of literature used in this study was guided by the decline in the number of learners choosing mathematics as a subject when they pass grade 9.

2.2. The Conceptual Framework for the Description of Integers

The conceptual framework used in this study consist of many concept which are briefly described and illustrations provided to make it easy for the researcher justify the selection.

2.2.1 The Concept of Integer

The set of integers includes zero, negative, and positive numbers without any decimal or fractional parts. They are numbers either above (positive) or below (negative) zero.

Adding Integers

Rule: If the signs are the same, add and keep the same sign.

$(+) + (+)$ = add the numbers and the answer is positive.

$(-) + (-)$ = add the numbers and the answer is negative.

Rule: If the signs are different, subtract the numbers and use the sign of the larger number.

$(+) + (-)$ = subtract the numbers and take the sign of the bigger number.

$\boldsymbol{a + -a = 0}$

$\boldsymbol{a - a = 0}$

$(-) + (+)$ = subtract the numbers and take the sign of the bigger number.

Subtracting Integers 'Same/Change/Change (SCC)'

Rule: The sign of the first number stays the same, change subtraction to addition and change the sign of the second number. Once you have applied this rule, follow the rules for adding integers.

$(+) - (+) = (+) + (-)$ SCC, then subtract, take the sign of the bigger number.

$(-) - (-) = (-) + (+)$ SCC, then subtract, take the sign of the bigger number.

$(+) - (-) = (+) + (+)$ SCC, then add, answer is positive.

$(-) - -(+) = (-) + (-)$ SCC, then add, answer is negative.

Multiplying and Dividing Integers

Rule: If the signs are the same, multiply or divide and the answer is always positive.

$(+) \times (+) = +$ $(+)$ divided by $(+) = +$

$(-) \times (-) = +$ $(-)$ divided by $(-) = +$

Rule: If the signs are different, multiply, or divide and the answer is always negative.

$(+) \times (-) = -$ $(+)$ divided by $(-) = -$

$(-) \times (+) = -$ $(-)$ divided by $(+) = -$

Any number multiplied by zero is equal to zero and any number divided by zero is undefined.

This study has adapted the early algebraic conceptual framework developed by Kaput (2008), used in several studies. A study by Asghary, Shahvarani and Medghalchi (2013: 1007) shows that Kaput (2008) pioneered the development of one of the frameworks used in the teaching of algebra in early lower grades. The framework consisted of two key aspects associated with reasoning processes as well as four building blocks of early algebra (Asghary, Shahvarani and Medghalchi, 2013: 1008; Kaput, 2008). According to Pittalis and Zacharias (2019), Kaput's framework regards the use of symbols in generalisation as the most important core aspect of teaching algebra. This is followed by the second core aspect which requires the learner to be active in the use of symbols as guided by rules provided. . The four algebraic strands are presented below.

2.2.2 Generalising

Generalising is regarded as a mental process in which an individual or learner combines several instances into a single form, which is easy to understand (Blanton, et al., 2018, Pinto, Eder; Cañadas and María, 2019). According to Blanton, et al., (2018) and Cooper and Warren, (2011), a learner can learn how to generalise by performing simple computations which lead him/her to realise that several algebraic expressions which involve the addition of positive numbers yield a positive result. . This process is important in assisting learners to realise that dealing with algebraic expressions required compressing several instances following rules that ultimately promote seriously engagement supported with generalisation (Kaput, 2008; Pinto, Eder; Cañadas and María, 2019), When generalisation is supported by good examples, learners easily realise that operation signs preceding algebraic expression with brackets are very important when simplifying the expressions. Learners who can generalise can easily deduce that the sum of several positive numbers remains positive. Studies show that engaging early grade learners in activities that promote generalisation can strengthen the ability of those learners in filtering mathematical information from common characteristics to draw conclusions in the form of generalised (Blanton *et al*., 2018; Pinto, Eder; Cañadas and María, 2019).

2.2.3 Representing Generalisations

The activity of representing mathematical structures and relationships is as important as generalising (Kaput, Blanton & Moreno, 2008). As a socially mediated process whereby one’s thinking about symbol and referent is iteratively transformed, the act of representing, not only gives expression to generalisations learners notice in problem situations, but also shapes the very nature of their understanding of these concepts. Several studies on the teaching of algebra emphasise the practice of generalisation tends to build a stronger understanding in learners that a particular rule can apply to a broad class of instances and this can strengthen a learner views on the importance of generalisation (Morris, 2009; Cooper & Warren, 2011). Based on the positive numbers alluded to in previous sections of this dissertation, learners who can generalise can express in their own words that “the sum of a positive number and a positive number is positive.” With further encouragement, the learner can easily present generalisation in various forms including variable notations.

For instance, Blanton et al., (2018) purport that generalisation can enable a learner to represent the commutative property of addition as $\boldsymbol{a + b = b + a}$; in which a young learner treats, ‘***a***’ and ‘***b***’ as representing the "counting numbers". Subsequently, when a learner's ability to deal with complex situations increases, the view of the existing number domain also expands to real numbers (Blanton et al., 2018; Cooper & Warren, 2011).

2.2.4 Justifying Generalisations

To justify the importance of generalisations, Cooper and Warren (2011) opine that learners need to develop plausible mathematical arguments which use to justify the credibility of the generalisation being pursued. However, some studies argue that the nature of arguments put forward by learners in early grades tends to be immature due to a lack of practical reasoning (Carolyn, 2018; Kaput, 2008; Pinto & Cañadas, 2019). Furthermore, some authors posit that learners who constantly engage in mathematical arguments are likely to develop more sophisticated general forms that are devoid of sound reasoning in most cases, they are asked to solve algebraic problems (Carpenter Bell, Inan, Benson, Sonwalkar, Reinisch & Gallagher, 2003; Schifter, 2009). According to Schifter (2009), there are possibilities that early grade learners to form "representation-based arguments" employing drawings or manipulatives when justifying arithmetic relationships, they would have recognised.

Blanton, Brizuela, Stephens, Knuth, Isler, Gardiner, Stroud, Fonger, and Stylianou (2018) believe that learners can formulate arguments to justify whether the commutative property of addition was reasonable. Such learners tend to construct their arguments in representational forms of cubes which visually illustrate that the sum of a train of cubes remains constant when the train is rotated and takes another (Blanton, et al., 2018:31). For example; a train of 3 blue cubes plus 4 green cubes can be rotated to 4 green cubes and 3 blue cubes. In this case, learners who have grasped the rules of generation will easily argue that it is the same expression that has been simply rotated, therefore the answer remains the same. From this argument, it could be deduced that mathematics teaching in early grades should focus on developing generalisations and justifications so that learners progress smoothly to higher-order mathematical concepts which build on basic algebra.

2.2.5 Reasoning with Generalisations

Existing studies regards reasoned generalisation as a critical mathematical instance that learners who engage in algebraic thinking should be developed (Blanton et al., 2018; Carolyn, 2018). Mathematics teachers must provide such scaffolds to promote reasoning with generalisation to lay a strong foundation for algebraic understanding. Reasoning is an abstract process that requires strong support environmental for smooth mental development for self-reliance. By using a simple example of evens and odds, a learner may be able to develop and use basic generalisations such as “the sum of an even number and an odd number is odd” to justify the sum of any three odd numbers (Carolyn, 2018). A good example, 2+3=5, where 2 is an even number and 3, an odd number. The sum of the two addends is odd. The learner can extend the generalisation when deducing the sum of three odd numbers such as 3+5+7=15. The learner can reason that the first two addends are even because an odd (3) plus an odd (5) is even (8). Even (8) number plus odd (7) is odd (15). The reason is that an even number plus an odd number is an odd number. Sfard (1991) asserts that reasoning in mathematics learning is a cognitive process that signifies an advanced stage in concept formation is supported by sound generalisation in the learners thinking capabilities.

Experience has proved that learners can generalise when they continuously recognise, represent and justify something as being true rather than beliefs (Blanton et al., 2018; Carolyn, 2018). The teaching of algebra to early grades can be negatively affected by the learners' inability to generalise from several instances provided by teachers. Similarly, the inability of the teacher to provide support structures that promoted abstract thinking, a key facet for reasoning may negatively impinge on the ability of learners to reason and generalise.

Based on Kaput’s (2008) formulation of early algebra, Blanton et al., (2016) developed four critical mathematical practices that characterise early algebraic thinking (Pinnock, 2020). These are important as they help to outline what mathematical practices are useful in the development of the different levels of algebraic thinking and will also inform the analysis of the learners’ responses. These are generalising, representing, justifying, and reasoning with mathematical structure and relationships.

Generalising mathematical structures and relationships is of utmost importance as it forces the learner to focus on the relationships that hold specific mathematical structures together, as opposed to focusing on the specifics of particular arithmetic instances. A learner’s ability to generalise by reasoning indicates his/her understanding of the situation, ability to connect it to prior knowledge, and ability to draw conclusions by making a generalised claim.

Representing the mathematical structure or relationships is seen by Kaput (2008) as being of equal importance to generalising. The use of representations, whether conventional or non-conventional, moulds the learner’s understanding of the problem and is proof of the generalisation made. It also shows learners that a specific action can apply to an infinite number of cases and not just one scenario. The act of justifying generalisations involves learners developing mathematical arguments to defend or disprove the validity of a generalisation.

This conceptual framework enables an analysis of learners' algebraic thinking/reasoning when solving integer problems. According to Blanton et al., (2018), the framework demonstrates that learners who are provided with enough support can participate in activities that cover several content areas that promote generalised arithmetic topics addressing, algebraic expressions, simple equations, inequalities, and functional thinking in mathematics

2.2 Challenges which learners experience with integers and causes to these challenges

Pournara *et al*., (2016) assert that error analysis reveals that learners make a variety of errors even in simple algebraic equations. Typical errors include difficulties with negatives, brackets, and a tendency to evaluate expressions rather than leaving them in the required open form (Pournara *et al*., 2016). As indicated earlier, the Department of Basic Education (2019:179), notes that most learners lack fundamental mathematical competencies, which should have been acquired in the lower grades. This becomes an impediment to candidates answering complex questions correctly.
Ipek (2018) claims that learners do not understand addiction and subtraction at a conceptual level because of the intensive use of operational approaches.

An operational approach is a technique by which problems in analysis, especially involving differential equations, are transformed into algebraic problems, usually of solving a polynomial equation. One of the causes of challenges posed by the operational approach is prematurely introducing algebraic addition and subtraction to learners who have not acquired proper knowledge about integers and their characteristics, usually developed at the conceptual level (İpek, 2018). Stephan and Akyüz's (2012:429) state that "learners have similar difficulties, such as conceptualising numbers less than zero; creating negative numbers as mathematical objects and formalising rules for integer arithmetic, particularly the meaning for the opposite of a negative number being a positive number."

Wulandari and Damayantin (2018) assert that when learners do not understand the concept of integers, they experience difficulties in learning the next level of mathematics. Subtraction operations, involving negative integers, make it difficult for learners to solve mathematical problems (Wulandari *and* Damayantin, 2018). Cetin (2019) claims that negative numbers are difficult to understand and conceptualise. There are three major challenges faced by learners when dealing with algebraic expressions requiring the manipulation of integers (Cetin, 2019; Lodge, Kennedy, Lockyer, Arguel & Pachman, 2018; Maghfirah & Mahmudi, 2018). According to Cetin (2019), the use of the minus (–) as an operation symbol and to denote the direction, confuses which lead to the first challenge. Secondly, Cetin (2019), asserts that challenges arise in that learners have to resolve the conceptual contradiction between the quantitative arithmetic meaning of number together with both meanings of quantity and direction. Finally, there seems to be a lack of a practical model that teachers can use to explain and demonstrate the characteristics of the negative number system (Cetin, 2019; Maghfirah & Mahmudi, 2018; Yang & Wu, 2010).

Literature shows that the ease with which learners are able to place and order negative numbers on a number line is overshadowed by the inability to compare the values of the numbers in question (Cetin, 2019; Fuadiah, Suryadi & Turmudi, 2017). A good example involves a situation where learners in early grades are able to make sense of the operations (-3) + (-5) = -8, reasoned from 3+5 = 8 as being equal (Cetin, 2019) Surprisingly, for 3-(-5), learners tend to treat this expression as 3-5. A similar situation is when Grade 5 learners, with limited understanding of negative numbers concept, always confuse the operation 5-7 as 7-5 getting 2 instead of -2, and -5+8 as 5+8 getting 13 instead of 3.. In addition, the teacher needs to reorganise previous work in learners' minds and develop strategies to teach learners to understand new work (Mathaba, 2019). This research suggests why learners make errors because they over-generalise what they have learnt previously.

The worst dilemma faced by learners when dealing with negative numbers is the practical use of '+' and '-' signs, used as operations for the addition and subtraction of integers (Cetin, 2019). Cetin (2019) echoes that formulating negative numbers is problematic, particularly those involving the double utilisation of the negative sign.

Booth, McGinn, Barbieri & Young, (2016) state that teachers should be innovative and clear when exposing learners to algebraic expression and concepts for the first time. Teachers are encouraged to reconsider how they present concepts to learners in early grades and whether it will help them build a better understanding. Booth et al., (2016) are of the view that teachers should give learners to analyse worked examples and to assist them to concentrate on building a meaningful and appropriate conceptual basis supported by relevant mathematical operational skills. This assertion is supported by Fuadiah et al., (2019), who posit that teachers should always use suitable activities to support and encourage the development of suitable concepts to assist all learners in the understanding of the meaning and importance of negative integers in the learning of mathematics.

Cetin (2019) is of the view that operations used when dealing with integers are better understood when taught employing models rather than the direct use of rules. The author further alludes to the importance of allowing learners in early grade to be allowed to develop correct mathematical schema to support clear understanding and formulation of their own simple rules for manipulation with integers (Cetin, 2019). Cetin (2019) argues that the teaching of integers creates the opportunity for learners to develop algebraic reasoning through generalising rules for operating with integers. In so doing, the idea of a variable and can be developed.

Machaba (2017) recommends that to bridge the gap, the concept of a variable should be introduced in the early grades. He further explains that for learners to develop a rationale, rather than an instrumental understanding, they should be encouraged to solve equations through inspection, trial, and improvement, before solving them procedurally. Chaurasia (2016) claims that if teachers want to enhance learners' algebraic reasoning at an elementary stage, it is essential to encourage algebraic thinking in the early years. Learners should be familiarised with algebraic reasoning. This can be done through appropriate tasks and activities related to algebra in elementary school.

Fuadiah *et al*. (2019), claim that the challenges related to low mathematics knowledge and skills were perennial across primary and secondary school learners. Another cause is thought to be related to the learners who are not able to master mathematical terms in depth to meet the expected cognitive levels but just memorise the answers in order to get work done (Lodge, et al., 2018; Fuadiah, Suryadi & Turmudi, 2017). According to Fuadiah et al (2019), the ability of learners to develop quality thinking skills needed in dealing with complex mathematical problems is usually is inhibited by the use of concepts constrained to a single context. This implies that whenever earners are faced with a situation with many concepts, their ability to deal with the situation is seriously retorted.

Pournara et al. (2016) point out that learners refer to the concept of subtraction as 'take away', and this is erroneously applied algebraic terms. This is a clear limited view of the concept of subtraction which tends to arise due to poor exposure of learners to proper mathematical structures linked to subtraction especially, comparison, reduction and 'inverse-of-addition' in number work (Pournara et al. 2016; Cetin, 2019). Mathaba (2019) argues that teachers may fail to ensure that every learner grasps the necessary skills before progressing to new topics. Learners taught using the number line model can easily get confused with which direction to move when adding or subtracting integers, as they think that adding means going forward and subtracting means going backwards (Sahat *et al*., 2018).

2.3 Models for Teaching Integers

2.3.1 Number Line Model

The argument stating that the number line model (directional model) has been highlighted in current literature as being important in assisting learners to form conceptual structure while learning mathematics (Cetin, 2019). Being defined using a metaphor in mathematics, Cetin *ibid* states that number line is used in creating basics in understanding mathematics.

Way back in 2001, Goldin and Shteingold advocated for a "quality or quantity model" for teaching integers using the marked quantity idea (Cetin, 2019). The model depicts –n as less than 0 (zero). According to Cetin (2019), teachers can use this model to explain the concept of "integers" with the "opposite concepts" such as positive-negative, proton-electron or asset-dept. The model can be used to illustrates how the addition operation can be used in adding quantities together while the subtraction operation can be used in subtracting quantities or adding their opposites (Cetin, 2019).

Below is the representation of a number line:

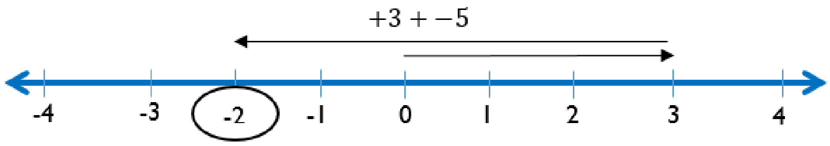


Fig.2.1 Addition of a positive and a negative integer

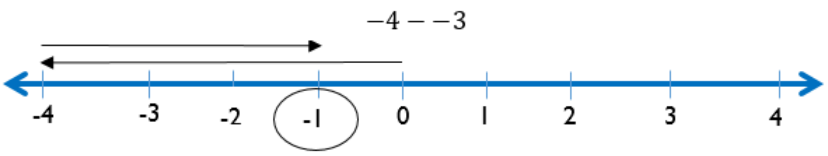


Fig.2.2 Subtraction of a negative and a negative integer.

The number line enables learners to visualise the position of numbers relative to each other. It helps teachers demonstrate calculations and enables students to develop visual strategies (Yilmaz, Akyuz & Stephan, 2019). The number line models are used to help learners to order and compute whole numbers, integers, rational numbers, and real numbers. Yilmaz *et al*. (2019) explain that specifically for integers, the number line is commonly leveraged to visualise “going to the negative”. While the notion of negative can be perplexing for new learners, the number line provides a visual aid by indicating a zero-point, visually separating the positive numbers from the negative ones. As for integers, number line models are also used for rational and real numbers for conceptualising percentages, ratios, proportions, and differences.

2.3.2 Neutralisation Model

The neutralisation method is based on cancelling integers of opposite signs (Verzosa, 2018). The use of red and black rods to represent positive and negative integers respectively, by the ancient Chinese, is one of the earliest examples of the neutralisation method (Merzbach *and* Boyer, 2011). When ten black rods and ten red rods appear together on the counting board, these are considered to add to nothing and are removed from the board. A study by Liebeck (2018) concluded that learners who used the neutralisation model performed slightly better on addition problems. However, the learners have challenges in subtraction problems, especially ones that deal with different signs. She also emphasised that the number line model has the advantage that it supports the understanding of negative numbers. Below is the representation of the use of red and black rods to represent positive and negative integers:

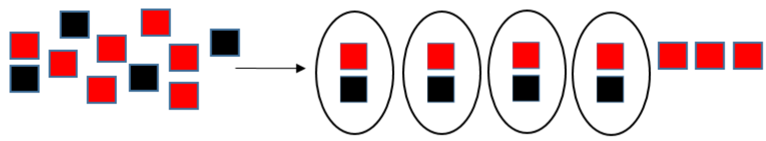


Fig.2.3 containing 7 red and 4 black rods contains 4 pairs of opposite units, and the resultant value of the grouping is 3 red or -3.

Another neutralisation model is proposed in Battista (1983). He used a jar of positive and negative charges as a model that can be used for all four operations, including subtraction of negative numbers. For example, consider the problem, $(+5) - (-3)$. If a jar initially contains 5 positive charges, it is physically impossible to remove 3 negative charges. The solution is to add 3 pairs of positive/negative charges to the jar, which does not induce any change in the collective charge in the jar. After removing the 3 negative charges, then 8 positive charges would remain, answering $+8$. This model was also utilised in the hands-on activity developed by Ponce (2007) which uses a regular deck of playing cards to facilitate class discussion about integers and adding and subtracting integers. Below is a representation of jar of positive and negative charges as a model that can be used add and subtract integers with opposite signs.

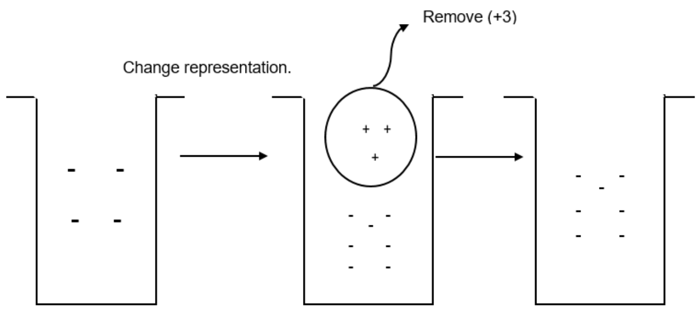
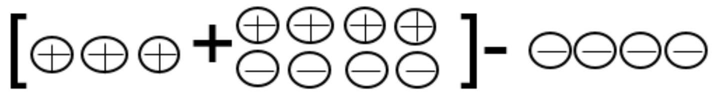


Fig.2.4 use of charges to represent $-4 - (+3) = -7$.

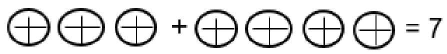
2.3.3 Charged Particle Model

This model was developed by Stanley Cotter, who was a mathematics teacher at Foothill College in 1969. Cotter (1969) claimed to have constructed a model that gives a solid foundation to the mathematical concepts involved and does not violate the scientific evidence.

When using this model to subtract, $3 - (-4)$ for instance, a learner begins with a picture of 3 positive particles, $\oplus \oplus \oplus$. As there are no negatives in the beginning, and the learner needs them there $\ominus\ominus\ominus\ominus$ to 'take away' 4 negatives, the learner must then introduce 4 pairs of positive and negative particles that are equal to 4 0's.



The learner must now take 4 of the negatives away from the 0's that they just introduced and is then left with the original 3 positives and the 4 positives from the remaining parts of the 0's, which are combined (added) to get 7 as the final answer.



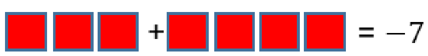
This is a prodigious way to learn it because it shows why $3-(-4)=3+4$.

2.3.4 The Stack or Row Model

According to Yan (2015), the stack model method is an intuitive and creative problem-solving strategy for solving non-routine questions and challenging word problems. He further explains that this method empowers young learners with the higher-order thinking skills needed to solve word problems much earlier than they would usually.

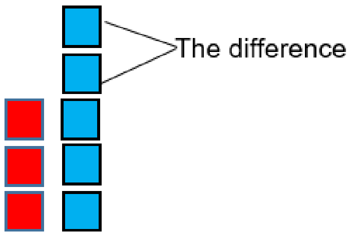
In the stack model, learners use coloured linking and graph paper. A learner must use graph paper and coloured pencils to record problems and results. Learners should also write the problems in standard form and show the results in that particular manner.

Learners should create stacks or rows of numbers with the coloured linking cubes and combine or compare them. If the numbers have the same sign, then the cubes are the same colour, and the learner must combine them to make a stack or row. Hence $-3+-4 = -7$ (all the same colour).

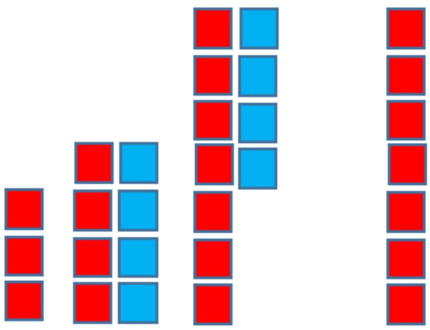


This also applies to $3 + 4 = 7$. This helps learners understand the addition of integers with the same sign.

If the numbers have different signs (colour), for example $-3 + 5 = 2$, a learner should then compare the stacks of different colours and the tallest stack 'wins'. The answer is the amount of difference between the stacks, both are easy to identify and understand.



For subtraction, a learner creates 0's by pairing one of each colour. The learner must then add as many 0's to the first number as needed so that the learner can take away what the problem calls for, $3 - (-4)$.

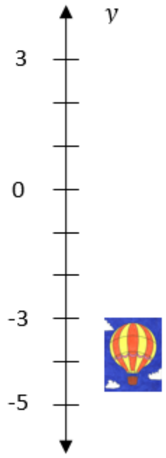


Now the learner should physically take away the indicated amount and see what is left.

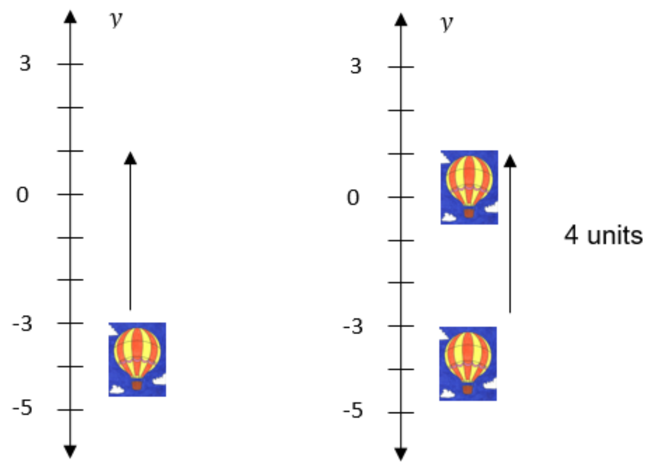
2.3.5 The Hot Air Balloon Model

The hot air balloon model is described by Gilderdale & Kiddle (2011) and is also online and cited verbatim.

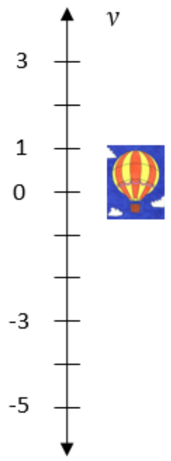
Sandbags (negative integers) and Hot Airbags (positive integers) can be used to illustrate operations with integers. Bags can be put on (added to) the balloon or taken off (subtracted). Here is an example: $-3-(-4)=?$



A learner needs to realise that the balloon starts at –3 and must think of the balloon being 3 metres below sea level or 3 metres below the level of a canyon and the learner takes off 4 sandbags.



Now, the learner should think about what would happen to a balloon if sandbags were removed. The balloon would get lighter. Therefore, the balloon would go up 4 units.



If a learner thinks in terms of a vertical number line, it will start at –3 and end up at 1, so $-3-(-4)=1$. For learners to make the connection between $-3-(-4)$ and $-3+(+4)$, they must present the addition and subtraction questions using the same numbers. For example, the first addition question might be $9+(-5)$ and the first subtraction question would then be $9-(+5)$. The learners see that putting on 5 sandbags produces the same result as taking off 5 hot airbags.

2.4 The Meaning of the Minus Sign

Gallardo and Rojana (1994) and Vlassis (2004; 2008) explain three functions of the minus sign. Vlassis (2004) claims that the term 'negativity' refers to the multiple functions of the minus sign. They referred to these functions as the unary function, the binary function, and the symmetric function (Gallardo*et al.*, 1994; Vlassis, 2004, 2008). The unary function of the minus sign is when a minus sign is attached to a variable or a number to form a negative variable or number (Vlassis, 2004, 2008). In this view, the minus sign is referred to as a 'structural signifier' (Sfard, 2000 cited in Vlassis, 2008). In the binary function, the minus sign is considered as an 'operational signifier' (Sfard, 2000 cited in Vlassis, 2008). This function shows that the learner should subtract.

The last function discussed is the symmetric function. The minus sign is also perceived as an operational signifier but signifies that the learner should take the opposite number or the opposite sum (Vlassis, 2004, 2008). For instance, $7—(—8)$; the minus sign is indicating that 8 should be added (opposite of minus) to 7. It is evident in Vlassis' (2004) study that learners use this function without understanding it, which give rise to errors.

Vlassis (2008) discusses that the challenges that learners experience, when solving sums with negatives, were more to do with how symbols were used, rather than the concept of a negative number. This is caused by the minus sign having to perform several functions. The meaning subtraction (minus) sign used in the arithmetic problems completed by Grade 9 was clear. For example, for the calculation $6—8$, the minus sign shows the operation of subtraction, whether it is taken away, difference, or complete. But despite that, a study conducted by Vlassis' (2004) revealed that a minus sign could have more than one function. She made an example of $4 + \mathrm{n}— 2 = 10$. The intention of -2 could be that 2 must be subtracted from 4, or it could, from an algebraic point of view, mean that one should take the opposite of 2. Vlassis (2004, 2008) observes that learners seem not able to distinguish between the unary and binary function of the minus sign.

One of the effects of not recognising the function of the minus sign is that learners may resort to memorising the rules. In a study done by Gallardo and Rojano (1994) they discovered that most of the learners in their study resorted to inventing rules. These rules give rise to correct and incorrect results.

They explain two cases of errors:

1. Rule of too many signs: this sign $[-a—(b)]$ is to take away, the other $[-a—(—b)]$ is not needed. For example, if learners are asked $-3—(—7)$, they do $—3—7 =—10$.
2. Multiple rules of signs: minus $[—a—(—b)]$ with a minus $[—a—(—b)]$ is plus and plus with minus' $[—a—(—b)]$ is minus. Again, if a learner is asked $—3—(—7)$, they do $3 + (—7) = 3—7 =—4$.

Vlassis' (2008) states that errors made by learners lie in the inadequate use of the minus signs and its change in use from a binary sign to a unary sign. Vlassis (2008) concludes that

The capacity to take account, according to the context, of the unary, binary, and symmetric dimensions and to display considerable flexibility in doing so is important to students' ability to make sense of these numbers which, above all, obey various formal rules (p. 568).

To help learners to differentiate between the two functions of the minus sign, researchers have opted for an alternate notation. Skemp (1964) defined positive numbers as numbers to the right of zero, and negative numbers as numbers to the left of zero. R5 represented 5 and L5 represented -5 (cited in Galbriath, 1974). Ball (1993) was the first person to introduce adding and subtracting negative numbers modelled on a lift that goes up and down floors of a building. She used a circumflex above the numbers instead of the minus sign to indicate negative numbers. For instance, 7 means seven floors below ground (0). The intention of the circumflex is to permit learners to focus on "the idea of a negative number as a number, not as an operation (i.e., subtraction) on a positive number," (Ball, 1993:380).

2.5 Previous Studies in South Africa

Davis (2010) explained that he conducted a study in a South African School in the Western Cape. He considers the question of the constitution of Mathematics, in pedagogic contexts and using the Mathematics and the Science Education Project, as well as being interested in developing the descriptive and analytic resources used to describe the teaching of Mathematics. Earlier studies had not provided a resource that allowed a clearer grasp of the articulation of objects and operations. Davis (2010) focused on the criteria required for the construction of mathematical statements that circulate in pedagogic activity and referred to an exchange between a Grade 10 teacher and his learners about adding two integers. Elucidating how $-8+6$ is calculated, the teacher says, "So if the signs are the same, what do you do? You take the common sign and then you add. If the signs are not the same, what do you do? You subtract." (Davis, 2010:384) This method that this educator used to explain how to calculate $-8+6$ to his learners, was also used by learners in this study.

Davis (2010) discusses that this method for adding two integers is difficult to understand when an integer is the domain of operation for addition. This is because the sign is separated from the integer and is seen as a 'whole number'. When this educator talks about the bigger number, he is ignoring the sips. In simple terms, he is not saying that $6 > -8$, but $8 > 6$.

The method that this teacher has used to calculate $-8+6$ necessitates that the teachers and learners are able to:

1. Separate the sign from the integer, i.e., substituting integers by whole numbers.
2. The recognition of the smaller or bigger 'whole number'.

3. Identifying the sign associated with the bigger 'whole number'.
4. Subtraction of the smaller 'whole number' from the bigger 'whole number'.
5. Attaching the sign of the larger 'whole number' to the calculated difference (Davis, 2010).

Davis (2010) concludes that:

> The regulative criteria required by the procedure indicate that the teacher and learners do not operate directly on the mathematical objects and relations being indexed (integer sums). They operate, instead, on more familiar and intuitive objects and relations ('whole number' sums) (Davis, 2010:385).

Venkat and Adler (2012) considered this data from Davis (2010), in terms of the transformation sequence. They describe the transformation as "steps that produce an interim representation that isnot equivalent to the input object ($-8 + 6$), even though equivalence with the input representation return at the final stage." (Venkat *et al.*, 2012:6) When using a method like the one used by the teacher, described in Davis' (2010) study, there is no mathematical connection between the question and the answer.

2.6 Summary

From the literature review, the researcher has recognised known challenges learners experience with integers. In addition, the researcher has drawn on the work of Runesson, Kullberg & Maunula, (2011), who highlighted four critical features in the learning and teaching of integers. These recommended important features to consider in this study. In the South African context, the work of Davis (2010) directs the possibility of an approach to integers that focuses only on the procedural aspect and avoids (or even negates) a conceptual understanding. His work, together with Kilpatrick, Swafford and Pindell, (2001), is an elaboration of procedural fluency and conceptual understanding and delivers important insights in understanding the interviews conducted in this study.

The next chapter presents detailed account of the research methodology used in this study.

CHAPTER 3. RESEARCH DESIGN AND METHODOLOGY

3.1. Introduction

The foregone chapter, was on literature review and conceptual framework. This chapter discusses the research paradigm that was followed in this study and the research method that was used. Justification of the choice of research paradigm and method is outlined. Furthermore, issues of sampling, data collection, trustworthiness, and ethical considerations are addressed.

3.2 Research Paradigm, Approach, and Design

The study has employed an interpretive paradigm. Interpretive research was used to examine learners' understanding of the basic concepts used to develop rules in the process of learning integers. An interpretive paradigm allows researchers to view the world through the perceptions and experiences of the participants (Thanh & Thanh, 2015). In pursuing the answers for the research, the researcher, who follows an interpretive paradigm, uses those experiences to construct and interpret his understanding from gathered data. Specifically, interpretivism supported scholars, in terms of exploring their world by interpreting the understanding of individuals (Thanh *et al*., 2015). This study seeks to explore the experiences of Grade 9 learners studying at a South African school, to uncover the reality of learning development in Grade 9 learners when they learn integers in early algebra. To support the use of an interpretive paradigm, more characteristics of interpretivism are further clarified.

Interpretive research is more subjective than objective. Willis (2007) argues that the goal of interpretivism is to value subjectivity, and "interpretivists avoid the idea that objective research on human behaviour is possible" (Willis, 2007:110). Following from Willis's points, Smith (1993) believes that interpretivists are 'anti-foundationalists', because "there is no particular right or correct path to knowledge, no special method that automatically leads to intellectual progress," (Smith, 1993:120). Proponents of interpretivism do not accept the existence of universal standards for research, instead, the standards guiding research are "products of a particular group or culture" (Smith, 1993:5).

A qualitative approach was used in this study. Qualitative research attempts to obtain an in-depth understanding of the phenomenon, namely, basic concepts in learning integers. Qualitative research aims to gather an in-depth understanding of human behaviour. It relies on the reasons behind various aspects of behaviour. It investigates the 'why' and 'how' of decision-making, not just 'what', 'where', and 'when'. Hence, the need is for smaller but focused, samples rather than larger random samples. Qualitative research categorises data into patterns as the primary basis for organising and reporting results (Woods, 1996).

The reason for choosing a qualitative approach was that the research occurred in a school and that the learners, as individuals, have a core duty and a responsibility to learn, as well as being a diverse group of pupils with different personalities, ideologies, values, abilities, perspectives, needs, and experiences. According to Bogdan and Bicklen (1982), a qualitative methodology enables researchers to view experiences from the participant's perspectives. Therefore, the method was chosen to enable the researcher to capture each learner's perspective.

The study employed an exploratory design. An exploratory research design is conducted for a research problem when there are few or no earlier studies to be referred to or relied upon to predict an outcome. In this case, very few studies have been done on integers in Grade 9 in the South African context. The focus is on gaining insight and familiarity for later investigation. The exploratory research design was a useful approach to gain background information about learners' understanding of integers.

3.3 Research Methodology

3.3.1 Site Selection

The research was conducted at a public high school. The site is situated in the West Coast District of the Western Cape where the researcher is employed. Geographical accessibility, proximity, and functionality were the factors that influenced the researcher's choice of the school.

3.3.2 Participant Selection

The participants consisted of one Grade 9 Mathematics class with 40 learners, 20 males and 20 females. The reason for choosing the Grade 9 class was that they were the last group in the GET phase and they will be joining the FET phase in the following year, where they will be expected to apply their knowledge of integers more frequently, whether they decide to choose mathematics or mathematical literacy. The researcher would have loved to collect the data from all four of the Grade 9 classes in the particular school where the data was collected from, however, the time was very limited as learners, which were the participants in this study, lost their tuition time due to covid, so the researcher was therefore obligated to work with only 40 Grade 9 learners instead of all of the Grade 9 classes in that particular school.

3.3.3 Data Collection

The study employed document analysis, learners' written tasks (formative assessments) on the "integers" content which were followed up by a test and individual interviews.

In document analysis, the researcher examined the Grade 9 learners' textbooks, workbooks, online teaching material.

The researcher designed a written task that consisted of integer questions in four cognitive levels. The researcher analysed the errors learners make in this written task and based on these errors the researcher made predictions about the underlying causes of these errors. The errors and the causes of the errors were described as the challenges experienced by the Grade 9 learners in understanding integers.

After the written tasks, the researcher conducted individual interviews with 5 learners who performed poorly in the assessment, to discuss with the learners the errors they had made and the possible causes of the errors.

3.3.4 Data Analysis

The marked scripts were arranged in ascending order according to the percentage each learner scored. The median and the average mean were then calculated, the learners who got below the average mean then formed part of those who were interviewed. The researcher employed inductive content analysis to identify themes emerging from the interviews about the errors and the possible causes of the errors. The errors in the written tasks were grouped in the types of errors made.

3.3.5 Trustworthiness

To ascertain the truth of the research study's findings, the researcher employed triangulation and member checking. This requires an understanding of credibility to determine how one knows that one's findings are true and accurate. The researcher also applied transferability to check whether the findings of the study can be applicable to other contexts in a similar situation. The researcher also checked confirmability and dependability by an inquiry audit, using an outside person to review and examine the research process and data analysis to ensure the study findings are consistent and could be replicated.

3.3.6 The Researcher's Position

The researcher was responsible for the whole process of the research, starting from the development of the research instruments, implementation, analysis, and development of the entire thesis. The researcher is not a staff member of the school where the research was done, which made it easy to collect the data from any Grade 9 class to avoid bias.

3.3.7 Ethical Considerations

The learners were allocated numbers from 1-40 to safeguard their anonymity. Formal approval from the Department of Basic Education, CPUT, and the school principal was obtained to conduct this research. The information was not accessed by anyone but the researcher. The data collected was not used for any other reports or published documents other than the study to ensure confidentiality.

A research information sheet and an 'informed consent form were given to all participants and parents in the case of learners younger than 18 years. The learners, or parents of those under 18, signed the 'informed consent form. The anonymity of the school and learners is assured.

3.4 Contribution of the Research Project

The findings of this study will benefit learners and teachers, as integers form a major section of the algebra syllabus in the mathematics curriculum. They will help teachers to understand learners' difficulties and what should be emphasised by teachers in the school curriculum to improve learners' performance in algebra. Thus, schools that apply the recommended approach derived from the results of this study will be able to engage learners better in learning integers. The study will also assist researchers in uncovering critical areas in learning integers, hitherto unexplored.

3.5 Summary

The research methodology used in the study was presented in this chapter. Detailed presentations of the research design and data collection and analysis methods were made. The developed data coolection instruments were there used in data collection, analysed and results presented in Chapter 4.

Chapter 4. Presentation and Discussion of Results

4.1 Introduction

This chapter explains how the data collected was analysed. The process of the data analysis explains and summarises the results that are shown using graphs. The chapter also discusses some errors that learners committed as they were solving integer problems. Lastly, a deeper level of explanation about the learners' responses to the questions is discussed. It should be noted that in this chapter some remarks from learners' scripts were quoted, as they are written on their scripts with no correction in language.

A group of forty Grade 10 learners, twenty males and twenty females, participated in the study. Microsoft Excel was used to analyse the data collected through a test that was out of 20 marks. The content was drawn from topics such as basic algebra and integers. The test was designed by the researcher, with the help of different sources such as textbooks, CAPS documents, and the internet. The following superordinate themes became evident:

- Adding, subtracting, dividing, and multiplying negative numbers.
- Integers with variables.
- Integers with square roots.
- Integers with word problems.

4.2 Responses of Participants from Individual Interviews

The results from the interview done with selected partipants are presented supported with the reults from the test written by students. Extracts from learners work are further provided as evidence of work used.

4.2.1 Learners' Understanding of the Term 'Integers'

From the forty tests received, five learners were selected to get a deeper understanding of what challenges they experienced in understanding integers in early algebra. The five learners were selected using a five-number summary as well as the reasons they provided for some answers. An analysis of the learners' scripts showed that learners do not understand the rules, such as a commutative property of addition, commutative property of multiplication, associative property of addition, associative property of multiplication, distributive property, identity property for addition, identity property for multiplication, and inverse property of addition. Hence interview questions were developed using these properties. The responses will be discussed below, even though learners were interviewed individually, for the sake of simplicity, their responses will be combined for a similar question.

In individual interviews, several participants made utterances that revealed significant disparities in understanding the definition of the term 'integer' such as:

> "*Uhmm…integer is a whole number that can be positive or negative.*"

Another participant reported:

> "*Sir, I think integers are the whole number.*"

Another participant reported:

> "*Errrrr…I don't know sir, sorry (laughing).*"

Yet another participant stated:

> *"An integer is a number that can be written without a fractional component, for example, 21, 4 and 0 are integers."*

The last participant stated:

> "*I think it's numbers that cannot be divided by zero.*"

The purpose of asking the learners to define integers was to assist the researcher to focus on where learners stand academically concerning their conceptual understanding of the topic, as that is the key component of mathematical expertise. Mathematics teachers need to deploy various pedagogical strategies to assist learners to understand mathematical definitions so that learners can have the ability to justify, in a way appropriate to their mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from.

Findings:

Data from individual interviews demonstrated that several factors are pointing to significant challenges experienced by Grade 9 learners in understanding integers in early algebra. Data obtained from a written algebra test and the analysis of the learners' scripts, as well as results obtained from individual interviews, were all designed to answer three pertinent questions of this research. These were:

- What are the challenges experienced by Grade 9 learners in understanding integers in early algebra?
- How do learners reason when solving integer problems?
- What are the errors learners make when solving integer problems?

Results showed that most of the learners from the interviews had an idea of what integers are, even though they failed to answer integer questions in their assessments. This may be because learners were given the rules and definitions to memorise without understanding them. Cetin’s assertion that operations in integers should be presented through models instead of presenting the rules directly is demonstrated as a plausible argument in the results presented in this study. Hence, learners should be enabled to form schemas in their minds that will enable them to create their own rules (Cetin, 2019).

4.2.2 Learners’ Understanding of a Negative Number

Participants were also asked, what it means if a number has a minus sign in front of it, and their responses were as follows:

> “*It’s a negative number.”*

Another participant reported:

> “*It is negative.”*

Another participant reported:

> “*It means that the number is negative.”*

Yet another participant stated:

> *“It means that the number is negative thus we can subtract it if the second number is positive and add it.”*

The last participant stated:

> *“It means it’s negative.”*

The purpose of asking the learners about their understanding of negative numbers was to discover if learners understand what a negative sign means, as it would be difficult for them to perform calculations on negative numbers if they do not know what a negative sign is in front of a number represents. Stephan and Akyüz’s (2012:429) state that “learners have similar difficulties, such as conceptualising numbers less than zero; creating negative numbers as mathematical objects and formalising rules for integer arithmetic, particularly the meaning for the opposite of a negative number being a positive number.”

Findings:

The findings highlighted that it is again evident that learners have an idea of what a negative number in front of another number means and what negative numbers are, but they experienced problems in applying for negative numbers in their assessment. According to Cetin (2019), negative numbers are difficult to understand and conceptualise as they present a complex nature when learners have not grasped the principle of working with negative numbers earlier on in their schooling. This difficulty emanates from 1) the symbol – is used both for the operation and the direction, 2) conceptual contradiction between the quantitative arithmetic meaning of number and both senses of "quantity" and "direction", 3) the lack of a practical model explaining the features of a negative number system.

4.2.3 Learners' Understanding of the Commutative Property of Addition

Significant disparities were shown when learners were asked the following questions:

> *“If you are given the sum $2+5$, does change the order of addends to $5+2$ change the sum? Why? Why not?”*

And their responses were as follows:

> *“Yes, because the signs are positive.”*

Another participant reported:

“It does not change because they just swapped the numbers, and they did not change the sign.”

Another participant reported:

“*Yes, because the sign is positive.”*

Yet another participant stated:

“*No, the answer will be the same because the addition method is used."*

The last participant stated:

“*Yes, because the numbers changed their original positions.”*

Therefore, the study exposed learners’ difficulties in their understanding of the commutative property of addition, as was the case in their performance in the given assessment.

Findings:

The results provided evidence that three out of five (60%) learners did not understand the commutative property of addition, as they thought that changing the order of the numbers will influence the answer. This explains why some of the learners experienced some challenges in the assessment.

4.2.4 Learners' Understanding of the Commutative Property of Multiplication

When the question was posed on the application of knowledge in the commutative property of multiplication during interviews, learners’ responses showed diverse ways of answering questions pertaining to the above principle.

“If you are given the product $(-3) \times 8$*, does changing the order of the factors to* $8 \times (-3)$ *change the product? Why? Why not?”*

Their responses were as follows:

> “*Yes, sir but I don’t know why ke (smiling).”*

Another participant reported:

> *"It does not change because we are still multiplying with 8 and inside the brackets, we still have negative 3."*

Another participant reported:

> “*Yes, because the numbers are different.”*

Yet another participant stated:

> “*No, even though the groupings are different the numbers and signs used are the same, so the answer is still the same which is* -24.*”*

The last participant stated:

> “*Yes, because we had* -3 *before and now we have* 8.*”*

The purpose of this question was to find out whether learners understood the commutative property of multiplication, as they also had difficulty in evaluating it.

Findings:

The results show that even in this question, three out of five (60%) learners did not understand the commutative property of multiplication because they thought that changing the order of the factors changed the product. This explains why some of the learners had some difficulty in the assessment.

4.2.5 Learners' Understanding of the Associative Property of Addition

The participants responded differently when they were asked the following question by the researcher:

> *"If you are given the sum* $(-3+5)+2$*, does changing the grouping of the addends to* $-3+(5+2)$ *change the product? Why? Why not?"*

Participants gave responses such as:

> "*No, signs are the same so the answer will also be the same."*

Another participant reported:

> *"It does not change because* $(-3+5)=2$ *and we will add* 2 *to the answer and it will give us a positive* 4*. If we calculate* $-3+5$ *it will give us a positive* 2 *and we will add that given* 2 *to our answer and it will also give us positive* 4*."*

Another participant reported:

> *"Yes, because the sign that is in the brackets are outside in the brackets in the second sum."*

Yet another participant stated:

> "*No, the numbers are still the same."*

The last participant stated:

> *"Yes, because the number that was inside the bracket is now outside the bracket."*

The purpose of asking this question was to assist the researcher to investigate the learners' understanding of the associative property of addition, as they struggled in their assessment.

Findings:

Results showed that even though some learners understood the question, however, they were not able to explain why they answered ‘yes’ or ‘no’ to the question, this suggests that the learners in this study, did not understand the associative property of addition. The minus sign is also perceived as an operational signifier but signifying that the learner should take the opposite number or the opposite sum (Vlassis, 2004, 2008). For example, $7-(-8)$; the minus sign is indicating that 8 should be added (opposite of minus) to 7. There is evidence in Vlassis’ (2004) study that learners use this function without understanding, which leads to errors.

4.2.6 Learners' Understanding of the Associative Property of Multiplication

On answering the question below, learners gave diverse responses.

> “*If you are given the product* $(2 \times 4) \times 6$*, does changing the grouping of the factors to* $2 \times (4 \times 6)$ *change the product? Why? Why not?”*

Participants gave responses such as:

> “*Yes, the answer will change because brackets are not the same.”*

Another participant reported:

> “*It does not change because the totals are the same and they are both positive.”*

Another participant reported:

> “*Yes, because there is a multiplication sign only."*

Yet another participant stated:

“*No, only signs will change the answer will remain the same.”*

The last participant stated:

“*Yes, the answer will change because the order is not the same anymore.”*

The purpose of asking the question in this instance was to investigate whether learners understood the associative property of multiplication, as results in the assessment manifested learners’ inability to tackle problems associated with the above principle.

Findings:

Results showed that, even though some learners understood the question, they were not able to explain why they answered ‘yes’ or ‘no’ to the question, suggesting that the learners in this study, did not understand the associative property of multiplication.

4.2.7 Learners’ Understanding of Distributive Property

Participants responded as follows, noting significant differences when they were asked the following questions:

“Is multiplying a sum by a number the same as multiplying each addend by that number and adding the two products? Why? Why not?”

Participants gave responses such as:

“*Yes…. but I am not sure why.”*

Another participant reported:

“No, because if there is a number outside the bracket, we will multiply all the sum with that number.”

Another participant reported:

“*I don’t know Sir.”*

Yet another participant stated:

> "*Yes…but I don't know the reason for that, I am sorry sir.*"

The last participant stated:

> "*I am not sure about this one.*"

The researcher developed this question because he wanted to find out if the learners understood the distributive property. After all, they also had difficulty evaluating this property.

Findings:

Results showed that learners did not understand the distributive property. This is a great concern, as this is one of the important properties in mathematics, and learners will need to understand this property to do well in mathematics in the upper grades.

4.2.8 Learners' Understanding of Identity Property for Addition

On learners' understanding of the above-mentioned principle, there were different responses such as:

> **"***What happens to the sum when you add zero to an integer?"*

Participants gave responses such as:

> "*The number will be equal.*"

Another participant reported:

> "*It will remain the same.*"

Another participant reported:

> "*The integer will remain the same.*"

Yet another participant stated:

> "*The number will be equal to itself."*

The last participant stated:

> *"The number will remain the same."*

With this question, the researcher wanted to find out whether the learners understood the identity property of addition.

Findings:

Results showed that learners understood this property, as it was introduced as early as grade 3. However, learners were not able to apply this property when given a complicated problem to solve.

4.2.9 Learners' Understanding of the Inverse Property of Addition

Data collected from interviews relating to the above resulted to utterances made by participants regarding the Inverse Property of Addition were as follows:

> **"***What happens to an integer when you add its additive inverse to it?"*

Participants gave responses such as:

> **"***The answer will be zero".*

Another participant reported:

> **"***The answer will be zero because we minus the same number."*

Another participant reported:

> **"***The integer will remain the same."*

Another participant stated:

“*The answer will be zero as the same numbers are subtracted.”*

The last participant stated:

“*The answer will be the same.”*

In this question, the researcher was investigating if the learners understood the inverse property of addition. Learners need to understand this concept as early as Grade 9 as they will use it more frequently in the upper grades.

Findings:

Results displayed that, even though this property is introduced in the early grades, some learners in Grade 9 still do not understand this property. Mathaba (2019) argues that teachers may fail to ensure that every learner grasps the necessary skills before progressing to new topics. Learners taught using the number line model can easily get confused in respect of which direction to move when adding or subtracting integers, as they think that adding means going forward and subtracting means going backwards (Sahat *et al*., 2018).

4.3 Data Acquired Through a Grade 9 Algebra Assessment.

Results on assessments given to forty learners who participated in the study in this study were entered on Microsoft Excel for further data analysis for a test they wrote whose marks were 20. The content was drawn from topics such as basic algebra and integers. The test was designed by the researcher with the help of different sources such as textbooks, CAPS documents and the internet.

Theme 1

In this question, learners were asked to add, subtract, divide, and multiply integers with negative numbers.

Figure 4.1 below shows the results of this question.

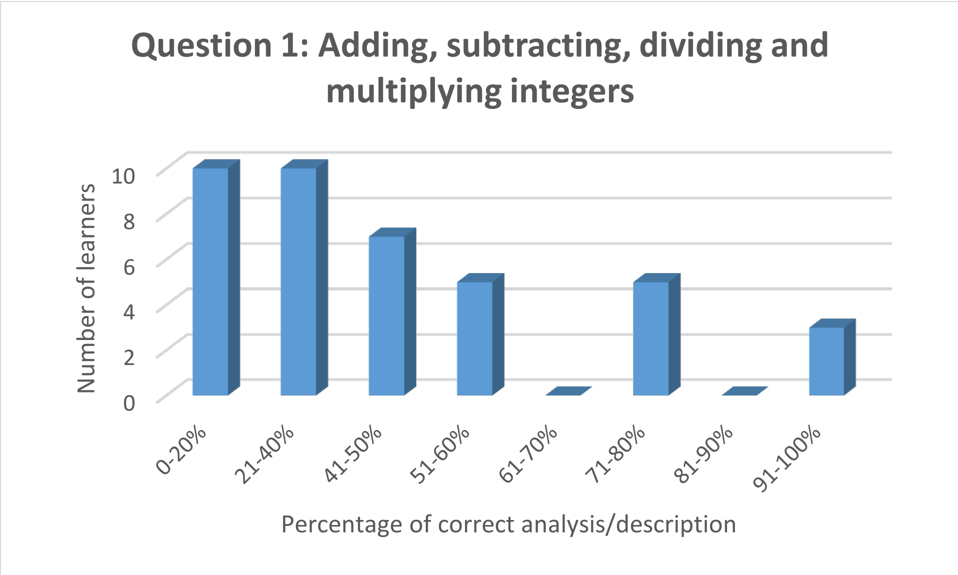


Figure 4.1 Question One Adding, Subtracting, Dividing, and Multiplying Negative Numbers

Few learners were able to answer questions about adding, subtracting, dividing, and multiplying integers. As one would expect that learners in Grade 9 should at least be able to answer questions of this nature, however an analysis of the learners' scripts showed that most of the learners cannot add, subtract, and divide integers with negative numbers, which is a worrying factor as they will need this knowledge for further grades.

Theme 2

In this question, learners were asked to solve integers with variables.

Figure 4.2 below shows the results of this question.

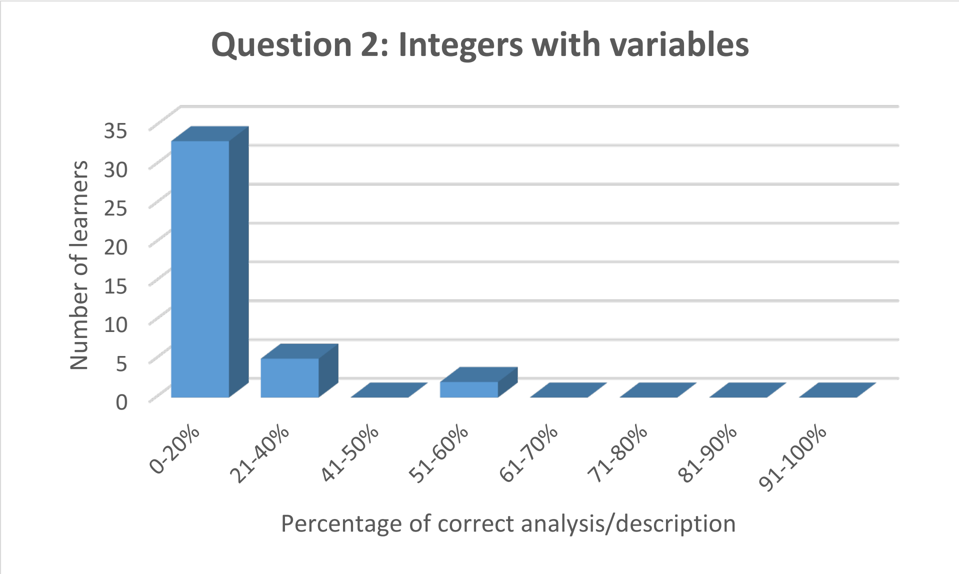


Figure 4.2 Question Two Integers with Variables

An analysis of learners' scripts showed that most learners were not able to answer questions that deal with integers that have variables. They do not understand the fact that even when dealing with integers that have variables, the same rules as those integers that do not have variables with them still apply.

Theme 3
In this question, learners were asked to solve integers with square and cube roots.

Figure 4.3 below shows the results of this question.

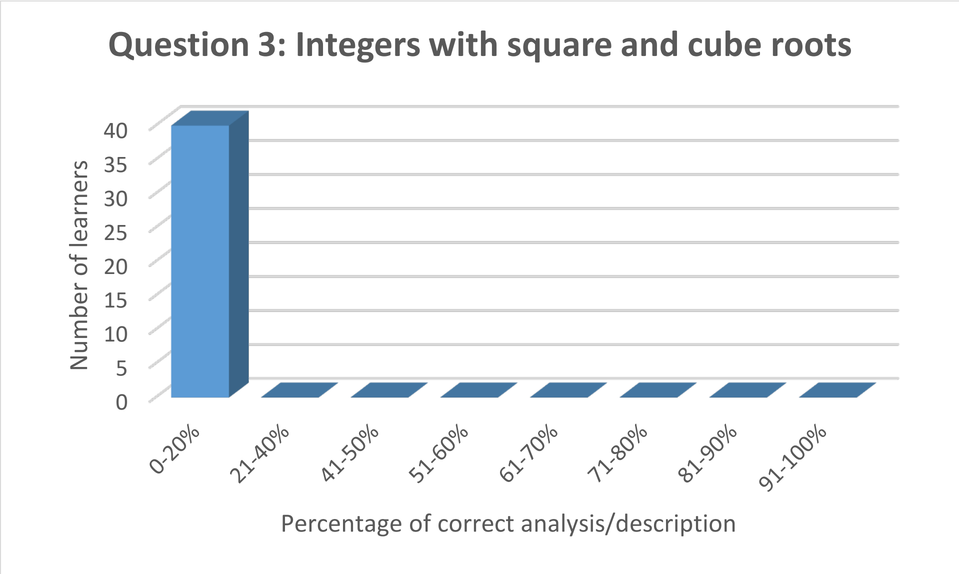


Figure 4.3 Question Three Integers with Square and Cube Roots

An analysis of the learners' scripts showed that learners had serious difficulties in answering integer questions that involve square and cube roots. These results demonstrate an alarming factor as learners in this grade are expected to be able to solve problems of this nature.

Theme 4

In this question, learners were asked to solve integers with easy word problems. Figure 4.4 shows the results for Question Four Integers with Word Problems

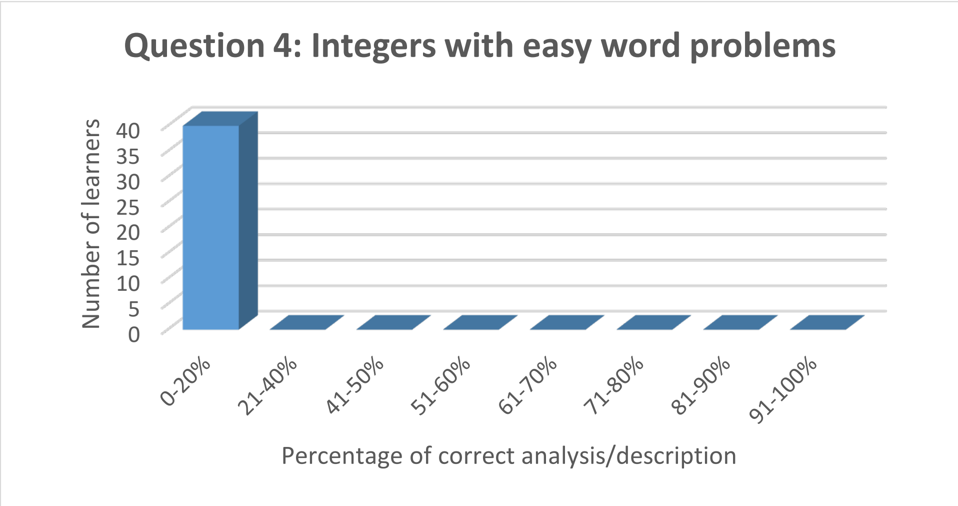


Figure 4.4 below shows the results of this question.

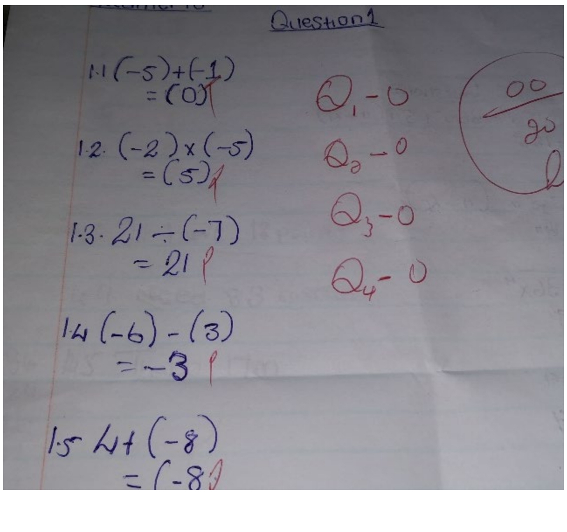
An analysis of the learners' script showed that all 40 learners experienced challenges in solving integers with word problems. This is a great concern as this skill will be needed when they deal with financial mathematics, applications of differential calculus, and other real-life problems in higher grades.

4.4. Discussion Of Results

4.4.1 Adding, Subtracting, Dividing, and Multiplying Negative Numbers

As briefly discussed in the results, very few learners were able to add, subtract, divide, and multiply integers with negative numbers. The analysis of the learners' scripts showed that most of the learners cannot add, subtract, divide, and solve problems of integers with negative numbers. As shown in figure 4.4, about 20 out of 40 (50%) learners experienced difficulties in answering questions of integers with negative numbers. Out of these 20 learners, who scored below 50 per cent, in question one, 10 learners got only 1 mark (20%) for this question, which is a great concern as stated by the assertion made by Pournara *et al*. (2016) that difficulties in algebra have been linked to a lack of understanding of integers. It is evident that most learners in this study were unable to solve simple problems with integers that have negative numbers, this may affect their performance in algebra in the upper grades, Pournara *et al*. (2016) also agrees with this fact. Subtraction operations involving negative integers make it difficult for learners to solve mathematical problems (Wulandari *and* Damayantin, 2018).

Extract: 1 Sample Script that Shows how Participants Approached Question One



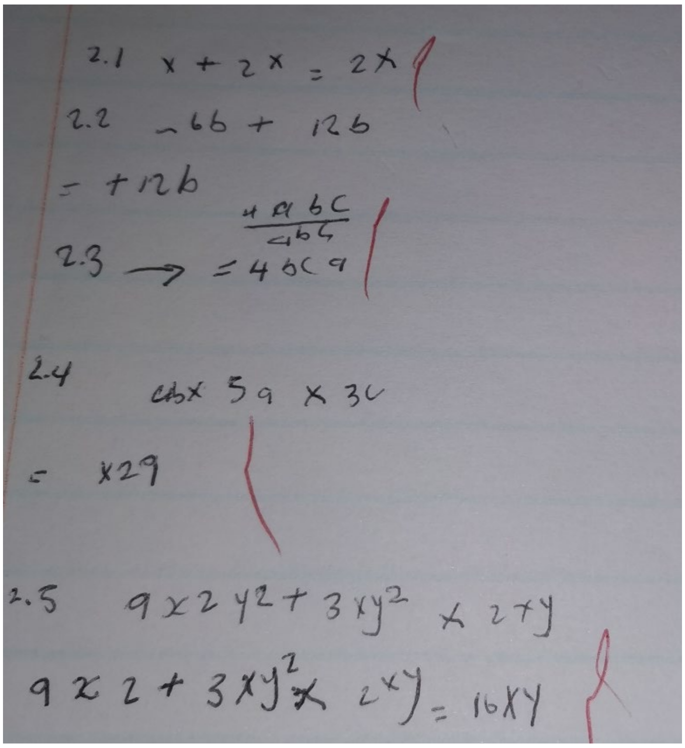
The participants (learners), though in Grade 9, were not able to solve simple questions involving integers with negative numbers.

The early algebraic conceptual framework developed by Kaput (2008) was used in this study regarding all the questions in question one as general questions, where learners need to identify the trend, then generalise. The framework also suggests that it is vital to engage learners in these types of questions because it strengthens their ability to filter mathematical information from common characteristics and draw conclusions in the form of generalised claims. When learners struggle to answer questions of this nature, it makes it more difficult for learners to understand more sophisticated questions.

4.4.2 Integers with Variables

As discussed in the results, when analysing learners' scripts, the majority of learners were not able to answer questions that consist of integers with variables. In figure 1.2, 33 out of 40 (82,5%) learners were not able to answer questions that consisted of integers with variables. The early algebraic framework that was used in this study, classifies questions, such as question two, as "representing generalisations". The framework suggests that if learners fail to generalise problems, they will then find it difficult to represent these generalisations. This explains why 82,5 per cent of learners did not manage to answer questions of this nature because they did not perform well in the previous questions where they had to generalise. As Morris (2009) stated that the practice of representing generalisations builds an understanding that action applies to a broad class of objects, not just a particular instance, thereby reinforcing a learner's view of the generalised nature of a claim.

Extract: 2 Sample Script that Shows how Participants Approached Question Two

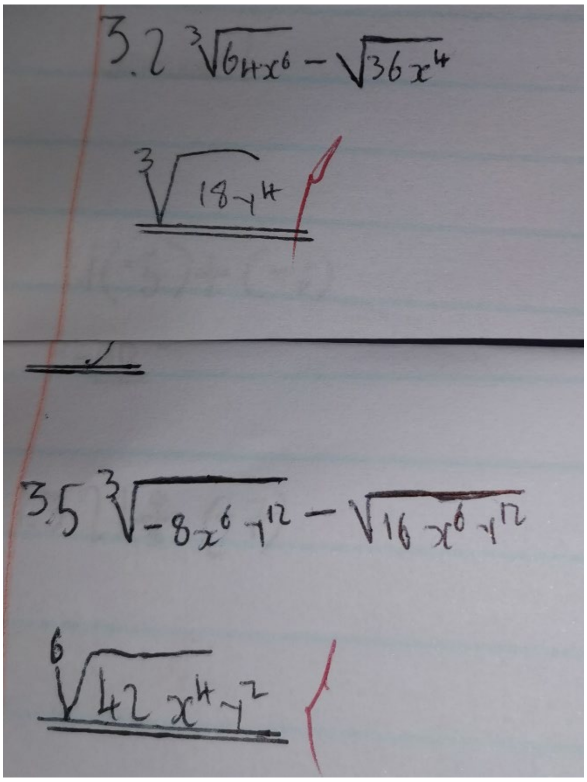


Even though the participant learners are in Grade 9, they were not able to solve questions involving integers with variables. This learner is one of the learners who did not understand that when adding integers with variables, they still need to add the numbers the same way they do with numbers that do not have variables. Such learners failed to follow the rules for adding, subtracting, dividing, and multiplying integers. Ipek (2018) agrees that learners do not understand addition and subtraction at a conceptual level because of the intensive use of operational approaches.

4.4.3 Integers with Square and Cube Roots

In figure 1.3, all 40 (100%) learners struggled to solve problems that consisted of integers with square and cube roots. The early algebraic framework that was used in this study classifies questions, such as question three, as "justifying generalisations". For learners to perform well in questions of this nature, they first need to do well in question one (generalising) and question two (representing generalisations). Learners in this study are experiencing challenges in understanding integers in early algebra.

Extract: 3 Sample Script that Shows how Participants Approached Question Three



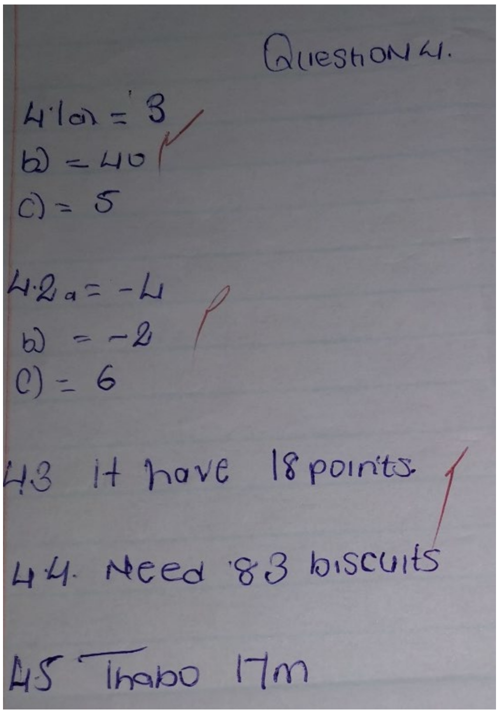
The same results as demonstrated above show that learners were also unable to solve questions involving integers with square and cube roots.

4.4.4 Integers with Word Problems

As briefly discussed in the results, an analysis of the learners' scripts showed that all 40 learners were challenged by problems that consisted of integers with word problems. Figure 1.4 shows that all 40 learners scored between 0 per cent and 20 per cent, such results were: multiplication, associative property of addition, multiplication, distributive property, identity property for addition, multiplication, and inverse property of addition in earlier problems. This has made it difficult for them to solve word problems as they first needed to understand the above rules. Wulandari and

Damayantin (2018) assert that when learners do not understand the concept of integers, they experience difficulties in learning mathematics at the next level.

Extract: 4 Sample Script that Shows how Participants Approached Question Four



Such learners experienced challenges in answering questions that consisted of integers with word problems.

4.5. Summary

One can argue that an analysis of learners' scripts and learner interviews showed that the learners from this study experienced challenges in understanding integers in early algebra. Their greatest challenge was in understanding the properties of integers. The errors that they made consisted of incorrect application of the rules, also they demonstrated a lack of understanding of the relationship between variables and numbers. For instance, they do not understand that the same rule that is used to calculate $2 + 3$ is also the same rule that is used to calculate $2a + 3a$. When using distributive property, they tend to forget that the minus sign that is in front of the number they are multiplying with, also forms part of the sum.

CHAPTER 5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Limitations of the Study

Even though the researcher would have loved to collect the data in different schools situated in different districts, but due to limited time and resources the researcher had to focus only on one school. The researcher would have loved to collect the data from all the Grade 9 classes in that particular school where the data was collected, however, the time was very limited as learners which were participants in this study lost their tuition time due to covid, so the researcher was therefore obligated to work with only 40 Grade 9 learners instead of all grade 9 classes in that particular school. The data (including an analysis of errors that learners made and the interviews which were conducted with the learners) were generated using only one written test. The findings were made mainly using that one written test.

5.2 Conclusions and Recommendations

This study revealed that there are indeed challenges experienced by Grade 9 learners in learning integers in early algebra. Another major finding of the study was that learners struggle to use symbols to generalise, act on symbols and follow rules, which agreed with the early algebraic conceptual framework by Kaput (2008) that was used in this study. It was further concluded that their greatest challenge was in understanding the properties of integers and that lack of understanding resulted in errors such as incorrect application of rules. They also demonstrated a lack of understanding in finding the relationship between variables and numbers. When using distributive property, they tended to forget that the minus sign that is in front of the number they are multiplying with also forms part of the sum.

One-on-one interviews were used as a strategy to engage learners and to collect more data from them. The results obtained from the interviews did not differ much from the results obtained when the learners were given a written task. Learners obtained similar (poor) results when they were given complex integer problems, both in the interviews and in the written task. The majority of learners seemed to have memorised the rules without understanding them, as they could not apply the rules correctly when given a written task.

From the assessment, the researcher analysed and interpreted the answers provided by the learners. The researcher identified five errors as the most common ones in the test. The errors that were identified in no particular order were: incorrect application of rules; misapplication of the distributive property; conjoin of terms; wrong use of signs and misinterpretation of symbolic notation. The most frequent error in this study was due to misapplication of algebraic rules. One can point out that misapplication of rules is caused by the fact that learners are taught to manipulate rules. Due to this, learners lack awareness and understanding of the meaning of expressions. This is due to the fact that at times teachers mainly put emphasis on procedural rules at the expense of conceptual ideas. This inadequate conceptual knowledge leads learners to commit errors. In this study, learners lacked conceptual knowledge and made errors when they were simplifying integers. Learners made errors because they did not have appropriate reasons for what they were doing.

Other learners, overgeneralized correct rules and misapplied them in another situation as a result of explicit declarative knowledge gained from the curriculum. They also misused previously learned procedures and rules in situations where they are not applicable. Learners seemed to have memorised the rules without understanding them, this made it difficult for them to remember the rules well and also to apply them appropriately. There was also an indication that learners had poor arithmetic background. This is due to the fact that learners failed to apply arithmetic knowledge in algebra. Lack of arithmetic skills or failure to link algebra to arithmetic also led learners to make errors.

This study underscores the need for teachers to encourage learners not to memorise rules but to understand them so they can be able to apply them when given complex integer problems. An integer baseline assessment needs to be administered to Grade 9 learners to create a picture of competence in integers. Early detection of integer knowledge gaps among Grade 9 learners could help learners, who are struggling, to become better achievers. It is further recommended that, with suitable instructional guidance from the educators, learners can formulate their definitions that will help them to solve even complex integer problems. Curriculum developers and textbook writers need to look into the early algebraic conceptual framework by Kaput for clues on how to improve learner achievement in mathematics in general and algebra (integers) in particular.

The findings from this study mainly indicated that learners lacked the basics of algebra, and consequently teachers are encouraged assist learners to grasp the following basics of algebra: collecting like and unlike terms; bracket expansion, addition and subtraction of directed algebraic terms. Knowing these basics and more will go a long way in understanding the procedural and conceptual aspects of algebra. Teachers should take the constructivist perspective into consideration and be in a position to create a strong arithmetic background for learners so that the arithmetic background could be applied to algebra.

Teachers are encouraged to use teaching methods that enable learners to gain both procedural and conceptual knowledge. The teaching methods should allow learners to give explanations for their answers. Teachers should listen carefully to learners' explanations and be able to identify learners' misconceptions and find ways of helping learners to understand algebraic concepts. There is need for teachers to create a classroom environment that allows learners to come up with their own conceptions from the procedural and conceptual knowledge taught by the teacher. Learners should also be encouraged to share their successes and problems in algebra in a way trying to clear misconceptions. At times, learners should receive individual attention in order to address the issue of individual differences.

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APPENDICES

APPENDIX A: Consent form

Mr W Mangcengeza

3074 Giyo Giyo Street

Old Crossroads

7755

Tel no: 078 011 7323 12 May 2021

Dear Mr/Mrs/Ms………………………………………………………………………………………………

Request permission to observe your child for my CPUT Masters research project

I am currently affiliated with the Cape Peninsula University of Technology where I am doing my Masters degree specializing in Mathematics Education. My research topic is **Challenges experienced by Grade 9 learners in understanding integers in early Algebra.**

I would like to obtain your permission to examine your child at a time that is convenient for your child during school hours. In addition, I would like your permission to approach your child to complete a questionnaire during the proposed period. My role will be to observe your child's challenges in understanding integers. I will not in any way disrupt his/her other learning processes.

I will require you and the educator, to sign this letter of consent from which gives me your permission to continue with this research. My research plan is to observe your child from 1 June 2021 – 30 June 2021.

All the information obtained from my observation and the questionnaire will be kept strictly confidential and that the above arrangement can be terminated at any time. The research project, when completed, will be available for you to view. Please note that nowhere will you or your child's identity be revealed. Please feel free to contact me if you need any additional information regarding this research.

Yours sincerely

Wandile Mangcengeza

I Mr/Mrs/Ms…………………………………………………… give permission to observe and interview my child.

I Mr/Mrs/Ms…………………………………………………… give permission to interview the educator of my child for your CPUT research project.

APPENDIX B: Ethical Clearance

Cape Peninsula University of Technology

Private Bag X8, Wellington, 7654
Jan van Riebeeck Street, Wellington, 7654
Tel: +27 21 864 5200

P.O. Box 652, Cape Town, 8000
Highbury Road, Mowbray
Tel: +27 21 680 1500

FACULTY OF EDUCATION

On the 27th of July 2021 the Chairperson of the Faculty of Education Ethics Committee of the Cape Peninsula University of Technology granted ethics approval **(EFEC 3-7/2021)** to W Mangcengeza for research activities related to a M. Ed degree.

Title:	Challenges experienced by Grade 9 learners in understanding integers in early Algebra

Comments:

The EFEC unconditionally grants ethical clearance for this study. This clearance is valid until 31st December 2024. Permission is granted to conduct research within the Faculty of Education only. Research activities are restricted to those details in the research project as outlined by the Ethics application. Any changes wrought to the described study must be reported to the Ethics committee immediately.

Date: 4th of August 2021

Dr Candice Livingston

Research coordinator (Wellington) and Chair of the Education Faculty Ethics committee

Faculty of Education

APPENDIX C: WCED Permission

Western Cape Government
Education

Directorate: Research

Audrey.wyngaard@westerncape.gov.za
tel: +27 021 467 9272
Fax: 0865902282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20210514-3011
ENQUIRIES: Dr A T Wyngaard

Mr Wandile Mangcengeza
3074 Giyo Giyo Street
Old Crossroads
Nyanga
7755

Dear Mr Wandile Mangcengeza

RESEARCH PROPOSAL: CHALLENGES EXPERIENCED BY GRADE 9 LEARNERS IN UNDERSTANDING INTEGERS IN EARLY ALGEBRA

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **01 June 2021 till 30 September 2021.**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number.
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:
    The Director: Research Services
    Western Cape Education Department
    Private Bag X9114
    CAPE TOWN
    8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 17 May 2021

APPENDIX D: Instruments 1

The following are interview questions that were asked Grade 9 learners to discover the challenges that they experience in understanding integers:

1. According to your own understanding, what are integers?
2. What does it mean if a number has a minus sign in front of it?

Commutative Property of Addition.

3. If you are given the sum $2 + 5$, does changing the order of addends to $5 + 2$ change the sum? Why? Why not?

Commutative Property of Multiplication.

4. If you are given the product $(-3) \times 8$, does changing the order of the factors to $8 \times (-3)$ change the product? Why? Why not?

Associative Property of Addition

5. If you are given the sum $(-3 + 5) + 2$, does changing the grouping of addends to $-3 + (5 + 2)$ change the sum? Why? Why not?

Associative Property of Multiplication

6. If you are given the product $(2 \times 4) \times 6$, does changing the grouping of the factors to $2 \times (4 \times 6)$ change the product? Why? Why not?

Distributive Property

7. Is multiplying a sum by a number the same as multiplying each addend by that number and then adding the two products? Why? Why not?

Identity Property for Addition

8. What happens to the sum when you add 0 to an integer?

Identity Property for Multiplication

9. What happens to the product when you multiply an integer by 1?

Inverse Property of Addition

10. What happens to an integer when you add its additive inverse to it?

Zero Property of Multiplication

11. What is the product of an integer and 0?

APPENDIX D: Instruments 2

In Appendix, 2 learners were given a test to find out challenges that they experience when learning integers. The questions were arranged in cognitive levels.

Question 1.

Calculate each of the following without using a calculator.

1.1 $(-5) + (-1)$
1.2 $(-2) \times (-5)$
1.3 $21 \div (-7)$
1.4 $(-6) - (3)$
1.5 $4 + (-8)$

Question 2.

Simplify each of the following.

2.1 $x + 2x$
2.2 $-6b + 12b$
2.3 $\frac{4abc}{abc}$
2.4 $4b \times 5a \times 3c$
2.5 $9x^2y^2 \div 3xy^2 \times 2xy$

Question 3.

Simplify each of the following.

3.1 $\frac{-12a^4 + 18a^3 - 36a^2}{-6a^2} + 5a(a-4)$
3.2 $\sqrt[3]{64x^6} - \sqrt{36x^4}$
3.3 $\sqrt{25x^4 - 9x^4}$
3.4 $\frac{6a-12}{3} - \frac{12a-9}{3}$
3.5 $\sqrt[3]{-8x^6y^{12}} - \sqrt{16x^6y^{12}}$

Question 4.

4.1 Give the additive inverse (in other words the number that will make the sum 0) for each of these:

a) 16

b) 40

c) 25

4.2 Give the multiplicative inverse (in other words the number that will multiply to make the given number 1) for each of these:

a) -5

b) -3

c) $\mathbf{7}$

4.3 Solly's teacher has a points system in his classroom. Today, Solly got 3 points for doing his maths homework, then another 5 points for helping his friend. But at break time Solly got into a fight and his teacher took away 10 points. How many points does Solly have at the end of the day?

4.4 Tara bakes 36 biscuits. She promises to give her grandmother 11. Her brother and her eat 12 of them, and she promises to give her mom 24 for work. How many more biscuits does Tara need to bake?

4.5 Johan and Thabo are playing a game of tug of war. Johan pulls Thabo 13m towards himself, and then another 4m towards him, but then Thabo pulls Johan 17m. Johan pulls back another 5m but Thabo regains his strength and pulls Johan 10m towards him. Who will win and by how much?

MEMO

Question 1.

1.1 -6 ✓
1.2 10✓
1.3 -3✓
1.4 -9✓
1.5 -4✓

Question 2.

2.1 $3x$✓
2.2 $6b$✓
2.3 4✓
2.4 $60abc$✓
2.5 $6x^2y$✓

Question 3.

3.1 $= 2a^2 - 3\text{a} + 6 + 5a^2 - 20\text{a}$✓✓ $= 7a^2 - 23a + 6$✓
3.2 $= 4x^2 - 6x^2$✓ $= -2x^2$✓

3.3 $= \sqrt{16x^4}\checkmark = 4x^2\checkmark$

3.4 $= 2a - 4 - 4a - 3\checkmark\checkmark = -2a - 7\checkmark$

3.5 $-2x^2y^4 - 4x^3y^6\checkmark\checkmark$

Question 4.

4.1

a) $-16\checkmark$

b) $-40\checkmark$

c) $-25\checkmark$

4.2

a) $-\frac{1}{5}\checkmark$

b) $-\frac{1}{3}\checkmark$

c) $\frac{1}{\mathbf{7}}\checkmark$

4.3 Solly's points $= 3 + 5 - 10 = -2\checkmark\checkmark$

4.4 Biscuits $= 36 - 11 - 12 - 24 = -11$ Tara needs to bake another 11 biscuits. ✓✓

4.5 Make Johan represent positive numbers and Thabo represent negative numbers: $= +13 + 4 - 17 + 5 - 10 = -5m$ Therefore Thabo wins by $5m$. ✓✓

APPENDIX E: Proof of Editing 1

Edit Report for Wandile Mangcengeza

- Errors in syntax, spacing and punctuation have been corrected.
- I have used the Track Changes facility in Word, so that you can see where I have corrected your document. You may need to turn this facility on, on your computer before making the corrections. Changes will be indicated in red or purple text. When you accept the changes, (by clicking on the blue tick in the task bar) the text will automatically change to black. Accept each change as it appears, if you are in agreement but you would be wise to avoid the 'accept all' option.
- A vertical line will appear in the left-hand margin opposite where I have made any corrections.
- Comments will appear in red or blue in the right-hand margin of the document. Once you have dealt with a comment, right click on it and select the option to delete the comment. Alternatively, overline the comment and select reject (red cross) in the task bar.
- Remember to turn the Track Changes facility off before printing your document.
- You have used a combination of English US and English UK language (MS Word automatically defaults to English US). Formal academic documents in South Africa use English UK. I have corrected this.
- Once all changes have been made, please check the page numbers on your Contents Page.
- Please check where I have reworded your text that I have not inadvertently changed the meaning of what you are trying to say. I have commented with '?' if I was uncertain of what you meant.
- The convention for numbers within your text is as follows: One to ten is written in words, and 11 onwards are denoted by numbers.
- When indicating a percentage, if it is within the text it is written as words as 'per cent', and the symbol (%) is only used within brackets.
- The use of the Oxford comma has been overlooked and duly corrected.
- We do not use personal pronouns (I, we, our), and these have been replaced where necessary with third person pronouns (the researcher).
- There has been a large amount of repeated prose. This leads to redundancy and perhaps the highlighted passages should be rephrased.

APPENDIX E: Proof of Editing 2

Barbara Dupont Editing Services

Cell No: 0846668351
Email: barbs1@skytec.co.za

ABSA Bank - Galleria Branch
Cheque Acc: 4084962935

Bill To: Dr Kwanele Booi

Address: Cape Peninsula University of Technology
Faculty of Education

Phone:

Email: BooiK@cput.ac.za

Invoice No: KB/01.21

Invoice Date: 23/09/2021

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