Cape Peninsula
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TITLE: THE APPLICATION OF VAN HIELE'S THEORY OF INSTRUCTIONAL DESIGN TO FACILITATE THE LEARNING OF CIRCLE GEOMETRY IN GRADE 11

By

ALI LWANGA

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Supervisor: Dr Williams Chapasuka Ndlovu

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DECLARATION

I, Ali Lwanga, declare that the contents of this thesis represent my own unaided work, and that the thesis/dissertation has not previously been submitted for academic examination towards any qualification. Furthermore, it represents my own opinions and not necessarily those of the Cape Peninsula University of Technology.

A.LWANGA　　　　　　　　　　　　MARCH 2022

Signed　　　　　　　　　　　　**Date**

ABSTRACT

THE APPLICATION OF VAN HIELE'S THEORY OF INSTRUCTIONAL DESIGN TO FACILITATE THE LEARNING OF CIRCLE GEOMETRY IN GRADE 11

This study explores the application of van Hiele's theory of instructional design to facilitate the leaning of circle geometry in Grade 11. Geometry is an essential and compulsory component in secondary school Mathematics. However, learners face problems in understanding geometric concepts; constructing proofs; and in deductive reasoning. The instruction in geometry offered by teachers in most South African schools is inadequate in guiding the learning of geometry. The purpose of this study is the application of van Hiele's theory of instructional desig to facilitate the learning of circle geometry in Grade 11.

This study employed a qualitative approach set within an interpretive paradigm, with a case study design. The emphasis of this study is on exploration, description, explanation, creation of, and testing of instructions based on van Hiele's theory. The methods of collecting data in this study were document analysis, classroom observation, and interviews. The participants in this study were 35 Grade 11 Mathematics learners and one Mathematics teacher.

The findings from document analysis, which largely involved the reading that focused on the Senior Phase (SP) and Further Education and Training (FET) Curriculum and Assessment Policy Statement (CAPS), revealed a disconnect on issues that are covered between SP and FET bands in geometry, curriculum, while content focus for Grade 11 circle geometry requires learners to prove seven theorems and their application, the focus for SP is mainly on the study of space and 2D shapes. Drawing from van Hiele's five phases of learning, classroom observation data showed that the teacher lacked knowledge of van Hiele's theory and phases of learning. Analysing some of the student-written responses revealed that, lessons developed and presented according to van Hiele's phases of learning helped learners to progress through geometric levels of understanding. The findings from the interviews confirmed the findings from classroom observation.

One of the recommendations this study makes is that teachers should embrace the use of van Hiele's theory as a teaching strategy for geometry to ensure that learners understand circle geometry.

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DEDICATION

To my family, friends and colleagues

(Lady "O" Mrs Olivia Lwanga, Katula Adjarn , Ezrah Van Boven and Dr Patrick Bukenya)

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ABBREVIATIONS AND ACRONYMS

NSC	National Senior Certificate
ASSA	Academy of Science of South Africa
CAPS	Curriculum and Assessment Policy Statement
WCED	Western Cape Education Department
LoLT	The language of learning and Teaching
FET	Further Education and Training
SP	Senior Phase
DBE	Department of Basic Education
SA	Republic of South Africa

CHAPTER 1: INTRODUCTION TO THE STUDY

1. INTRODUCTION

This chapter describes the research study presented in this thesis. It includes the origin and background, the rationale, the purpose and significance, the aim and objectives of this study. It also contains a description of each chapter within this study.

1.1 Origin and background of the study

This study explores the application of van Hiele's theory of instructional design to facilitate the leaning of circle geometry in Grade 11. Geometry is a branch of Mathematics concerned with the study of the properties of objects in space. The most common form of geometry is plane geometry, dealing with objects like points, lines, triangles, polygons and circles. Solid geometry deals with objects like spheres and polyhedrons, while spherical geometry deals with objects like spherical triangles and spherical polygons. These forms constitute the two types of geometry: Euclidean geometry and non-Euclidean geometry. While Euclidean geometry focuses on the study of points, lines, planes, angles, triangles, congruence, similarity, solid figures, analytic geometry, and circles, non-Euclidean focuses on hyperbolic, elliptic, and spherical geometry.

The main focus of this study is Euclidean geometry, and in particular circle geometry, because of its significance in developing learners' spatial and visualization capabilities and capacity for deductive reasoning. It also evinces connectivity to every strand in the Mathematics curriculum and several real-life situations. According to De Villiers (1996:1), the only geometry most South Africans know is Euclidean geometry, as they learn it at school. The Euclidean geometry taught in the South African secondary schools concentrates mainly on the following aspects; points, lines, triangles, quadrilaterals, and circle geometry, intending to develop learners' ability to be methodical, to generalize, make conjectures and try to justify or prove conjectures, problem-solving and cognitive skills (South Africa. Department of Basic Education, 2011:8-50). However, The Department of Basic Education's National Senior Certificate diagnostic report (Department of Basic Education, 2016:164) indicates that many candidates struggle with concepts in the curriculum especially those that require deeper conceptual understanding and have difficulty in dealing with complex questions in Euclidean geometry. Several researchers in Mathematics education assert that the instruction in geometry offered by teachers in most South African schools is inadequate in guiding learning of circle geometry (Feza &Webb, 2005:36).

1.2 Background

Geometry is regarded as problematic despite it being an essential and compulsory component in the secondary school Mathematics curriculum in South Africa. According to Alex and Mammen (2016:

2223-2224), learners face problems in understanding geometric concepts and constructing proofs; furthermore, difficulty in deductive reasoning also makes geometry the most dreaded subject in high school Mathematics. Siyepu and Mtonjeni (2014) note that Euclidean geometry remains a challenge in many schools, learners, teachers, curriculum advisors, and education officials in South Africa. Furthermore, Feza and Webb (2005:36-45) assert that the instruction in geometry offered in South African schools is inappropriate. They conclude that traditional teaching strategies do little to explore learners' understanding of geometry.

The challenges learners face during teaching and learning of geometry are increasingly affecting their performance in geometry at the national level in their final examinations. According to the National Senior Certificate (NSC) examination diagnostic report (South Africa (Department of Basic Education, 2021182), the academic performance of many candidates reveals a deficiency in the understading of basic concepts across topics in the curricuclum. The errors made by candidates in answering circle geometry questions originated from a poor understanding of the basics and foundational competencies taught.

1.3 Rationale

The development of theories of learning and teaching, specifically on the teaching of geometry as well as the application of more general theories in pedagogy, has been evident in recent research (Sinclair et al., 2016:692). Several studies, for example, Usiskin (1982), Fuys et al., (1984), Crowley (1987), Mistretta (2000), Feza and Webb (2005), De Villiers (2010), Alex and Mammen (2012, 2016) and Yilmaz and Koparan (2016), identify the Van Hiele's model as an appropriate theory of teaching and learning of geometry. Unfortunately, few researchers have addressed the method and organization of instruction in circle geometry drawing on the Van Hiele's theory. Few studies were conducted researching the impact of the application of the use of van Hiele's theory to facilitate Grade 11 understanding of circle geometry. Crowley (1987:5) identifies the method and organization of geometry instruction as pedagogical areas of concern that should be addressed by Van Hiele's phases of learning, while Sinclair et al. (2016:712) "hope to see increased research interest in the teaching and learning of geometry since it is a topic whose significance has decreased in many countries because of an increased emphasis on number and algebra". Therefore, the need to address the low-performance rate in circle geometry through van Hiele's theory to facilitate mastery of circle geometry in Grade 11 compelled this study.

1.4 Significance of the study

This study is significant in that it gives a comprehensive process of teaching Grade11 circle geometry to alleviate difficulties teachers and learners face during the teaching and learning of geometry. There is a general trend in school for teachers and learners, regarding Euclidean geometry, as one of the most difficult strands in teaching and learning in Mathematics. This study used the van Heile theory of instruction design to describe the processes of teaching circle geometry to Grade 11 and the possible benefits and challenges of using van Hiele theory of instruction design in teaching geometry.

The methods designed and based on van Hiele theory emerging from the process in this study intend to promote the strategy of teaching Grade 11 circle geometry. Therefore, the study has the potential to influence and inform teachers as well as curriculum policies and development on the teaching strategy of circle geometry in a South African context.

1.5 Research question

The main research question is: How does Van Hiele's theory of instructional design facilitate the learning of circle geometry to Grade 11 learners?

The sub-research questions under the main research question are:

1. What are the challenges of using Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?
2. What are the benefits of using Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?

1.6 Aims and Objectives

1.6.1 Aim

The research aims to explore how Van Hiele's theory of instructional design facilitates the learning of circle geometry to Grade 11 learners.

1.6.2 Objectives

The following objectives direct the study, namely,

1. To determine the possible challenges of Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners.
2. To assess the benefits of Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners.

1.7 Structure of thesis

1.7.1 Chapter 1: Introduction

This chapter introduces the study and offers the origin and background, the rationale, the purpose, and the significance of the study. The focus of the study is explained, where the research questions, aim, and objectives are stated. It, finally, contains a description of each chapter of this study.

1.7.2 Chapter 2: Literature review and theoretical framework

This chapter is structured in two major sections. The first section provides a review of some of the previous literature relevant to research on geometry and the framework for teaching and learning geometry and reviews the performance of geometry in South Africa. The second section of this chapter discusses and analyzes the theoretical framework underpinning this study. This section describes the five van Hiele's levels of geometric thinking, their characteristics, and the van Hiele phases of learning. It then provides insight into Grade 11 circle geometry and van Hiele levels, teaching implication of the phases of learning, and the last part of this section examines the criticism of the van Hieles' theory.

1.7.3 Chapter 3: Research design and methodology

This chapter describes the design and methodology of the study. It also discusses the research paradigm, design, and methodology, site selection, sample, data collection techniques, data analysis, researcher's role in the study, and finally discusses trustworthiness and ethical considerations.

1.7.4 Chapter 4: Results and discussion of the findings

This chapter presents the results and discussion of the findings. The results are collected from the document analysis, classroom observation, and in-depth interviews. These results are discussed in relation to the purpose and aims of the study.

1.7.5 Chapter 5: Conclusion and recommendation of the study

This chapter includes a summary of the findings discussed in relation to the research questions of the study. It further presents recommendations, highlights limitations and provides a conclusion of the study. The summary of the findings is discussed in relation to each of the research questions.

The next chapter discusses the literature review on geometry and presents the theoretical framework underpinning the study.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

The focus of this research is the application of van Hiele's theory of instructional design to facilitate the learning of circle geometry in Grade 11. This chapter is structured in two major sections that provide a review of previous literature relevant to research on geometry and the framework for teaching and learning geometry.

The first section of this chapter discusses and examines geometry as an area of concern in teaching and learning. It gives a brief overview of the South African curriculum on circle geometry for Grade 11, the way geometry is taught in the South African context and globally. This section further analyses the trend of learners' performance in Euclidean geometry in the National Senior Certificate for the past four years. More attention is further given to analysis of the performance on circle geometry questions for the 2018 and 2019 as provided for in the Department of Basic Education diagnostic reports. The analysis of results gives insight into the learning outcome if achieved as prescribed in the curriculum document.

The second section of this chapter discusses and analyses the theoretical framework underpinning this study. In this section, van Hieles' theory is presented and discussed as the theoretical framework that underpins this study. The first part of this section describes five van Hiele's levels of geometric thinking and their characteristics, van Hiele's phases of learning as provided by Crowley (1987:5-6), Clements and Battista (1995) and Mason (1998:4-5). It further analyses Grade 11 circle geometry and the van Hiele level and the teaching implication of this theory. The last part examines criticism of van Hieles' theory teaching and learning of geometry and performance in geometry. Although literature presents these themes in a variety of contexts, the primary focus in the study is to explore Grade 11 learners' understanding of circle geometry when van Hiele's theory of instructional design is applied to facilitate learning.

2.2 Circle Geometry for Grade 11

2.2.1 The South African Grade 11 Circle Geometry curriculum

Euclidean geometry is a type of geometry that deals with a logical system. Chern (1990:679) describes Euclidean geometry as one of the great achievements of the human mind. It makes geometry a deductive science and geometrical phenomena as logical conclusions of a system of axioms and postulates. Thus, Euclidean geometry emphasises axiomatic deductive reasoning verified by proofs.

Putten et al. (2010:1) argue that the logic and ability to reason demanded by Euclidean geometry renders its pursuit worthwhile, since these skills are not only essential in all mathematical disciplines, but also in real life.

Despite its immense importance in science, Euclidean geometry was removed as a compulsory component for learners in Grade 10-12 in the revised curriculum of 2006. This meant that Euclidean geometry was optional in these grades and only examined in the optional Paper 3 examining all optional assessment standards in the National Curriculum Statement (NCS). Learners enrolled for Mathematics could choose whether or not to write this examination. This prompted many learners not study the section on Euclidean geometry, it being in an optional paper (Umalusi, 2014:14).

The enrolment for Paper 3 in 2008 was only less than 4% of Grade 12 Mathematics learners (Van Putten et al., 2010). This pitched up debates on the inclusion of Euclidean geometry as part of the compulsory component in the exit examination. Some universities and scholars argued that the removal of Euclidean geometry from the essential curriculum of South Africa created lack of consistency in the study of space and shapes, which diminishes learners' opportunity to work with proofs.

The Academy of Science of South Africa (ASSA) was also concerned about the exclusion of Euclidean geometry as a compulsory component in the curriculum of Grade 11 and 12. In their forum of 2009, Vinjevld commented:

"….I am concerned whether we clearly understand what we have lost. In the past high school Mathematics pupils got most of their problems-solving experience in geometry. Perhaps the loss of Euclidean geometry goes beyond the loss of geometry itself, and there has been a los of something else that now needs to be replaced (ASSA, 2010:30)".

Euclidean geometry, as a core component of the Mathematics curriculum, is made compulsory in the revised South African curriculum known as the Curriculum and Assessment Policy Statement (CAPS) in 2011. One of the aims of re-introduction of Euclidean geometry in the revised curriculum was the need to produce learners who are able to communicate effectively using visual, symbolic or language skills in various modes (Department of Basic Education, 2011: 8-9). However, the addition of Euclidean geometry as a compulsory component examined in Paper 2 raised some concerns from educators and regulatory bodies. For instance, Umalusi was concerned with the increase in the amount of work to be covered that could lead to teachers either omitting certain sub-topics or compromising on the depth at which the work is dealt with (Umalusi, 2014:30). Some teachers did not feel as confident about the strand since it had not been taught for such a long time (Ngirishi & Bansilal, 2019: 82; Ubah & Bansilal, 2019:1).

According to CAPS document, Euclidean geometry for Grade 11 concentrates on working with the geometry of circles deductively. Learners are required to:

- Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other results, concerning tangents and radii of circles;
- Solve circle geometry problems, providing reasons for statements when required; and

- Prove riders (Department of Basic Education, 2011: 14).

The content of Grade 11 Geometry advocates for teaching that involves both "how" and "why" part of Mathematics. It tends to demand insight and involves an understanding of proof in theorems and riders. This type of Mathematics aims at exposing learners to mathematical experiences that give them opportunities to develop their Mathematics reasoning and creative skills. However, Shongwe (2019:100) urges that the weakness in CAPS is that there appears to be lack of explicit focus on argumentation as a heuristic.

The curriculum statement for Grade 11 aims at investigating and proving theorems of the geometry of circles and using theorems and their converses to solve riders. Learners in Grade 11 are required to investigate and prove the following theorems:

- The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
- The perpendicular bisector of a chord passes through the centre of the circle;
- The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
- Angles subtended by a chord of the circle, on the same side of the chord, are equal;
- The opposite angles of a cyclic quadrilateral are supplementary;
- Two tangents drawn to a circle from the same point outside the circle are equal in length; and
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment (Department of Basic Education, 2011: 34).

Research studies have shown the importance of learning proving and proofs in Mathematics. Hanna and de Villiers (2008:330) advances the following reasons for including proof in school curricula as:

- Proof and proving in school curricula have the potential to provide a long-term link with the discipline of proof shared by Mathematicians. For example, proof builds on the learners' use of arithmetic and algebraic symbols to calculate and manipulate symbolism to deduce consequences; and
- Proving provides a way of thinking that deepens mathematical understating and the broader nature of human reasoning.

Proof has six main functions on a learner's ability to reason deductively, from general to the particular. Ndlovu and Mji (2012:181) highlight the six functions of proof as: verification, explanation, systematisation, discovery, communication and intellectual challenge. This suggests that learning proof makes learners able to "show", "demonstrate", "verify", "explain", "discover" and "convince" logically from a general situation to a particular situation. Therefore, learning proof seems to be a reliable approach in helping learners to develop logical thinking skills that are beyond the Mathematics of the classroom.

Despite multiple justifications for the role of proof on learners' deductive reasoning and the goals of teaching proof in geometry (de Villiers, 1996; Dickerson & Doer, 2008; de Villiers, 2010; Ndlovu & Mji , 2012), problems of students' understanding of deductive geometric proof continues to recur (Ndlovu & Mji, 2012:178; Ngirishi & Bansilal, 2019:94). Such recurrence suggests that we revisit our teaching and learning strategy on geometric proofs. Ngirishi and Bansilal (2019) explain poor skills of proofing by learners as rooted from lack of progression from one level to another, as most learners' geometric understanding is limited to the first and second level. In the next section, literature is presented on the teaching and learning of geometry that explains why learners' grapple in understanding geometric proof.

2.2.2 Teaching and learning of geometry

Teaching Euclidean geometry remains a challenge in many South African schools. The re-introduction of Euclidean geometry as a compulsory mathematics content area in the Grades 10, 11 and 12 worried most educators and teacher educators regarding challenges of teaching. Ndlovu (2013:277) confirms that educators had difficulties with Euclidean geometry in the past, and results from Ngirishi and Bansilal's (2019:94) study help to explain why geometry is perceived as a difficult section of Mathematics. This section aims to highlight some of the challenges in teaching and learning of Geometry and difficulties learners' face in learning geometry that results in poor performance circle geometry.

Jones (2002:132) advances that one of the main reasons for learners' difficulties in learning geometry proofs is the lack of coordination of a range of competencies required. This is due to the approach used by some teachers that tends to concentrate on verification and omits exploration of concepts. Teachers are blamed for the explanation that makes it difficult for learners to transit from computational Mathematics to creative Mathematics. "De Villiers (2010:1-2) highlights this as the main reason for failure of traditional geometry teaching. He said that in a South African context, the curriculum is presented at a higher level than that of learners. Therefore, learners generally can neither understand the teacher, and the teacher cannot understand why learners understand geometry".

It is important in terms of pedagogy to know that learners' reason, operate and function at different levels about geometry concepts (Feza & Webb, 2005:45). Mthembu (2007:57) found that the strategy used by teachers in teaching circle geometry emphasises reproduction of demonstrated procedures. This strategy facilitates the process of finishing the curriculum, but less helpful to learners' understanding, thus placing learners at risk of performing poorly in geometry. Atebe and Schäfer (2010:85) recommend that for meaningful teaching, teachers should structure their instruction in ways that discover learners' levels of geometric understanding and ability to reason in a way that matches Van Hiele's levels of geometric thinking to facilitate the instructional design of circle geometry.

Alex and Mammen (2016:2226) note that the teaching and learning of geometry is one of the most disappointing experiences in many schools across nations. This is due to the outdated teaching practices which give few opportunities for learners to discover their potential. Lack of basic content knowledge further exacerbates the problem. Feza and Webb (2005:45) note that poor teaching is due to most teachers relying on knowledge from textbooks only, following algorithms in the textbooks without any clear explanation of the concepts to learners or directing learners' attention to particular facts. This raises the question of whether teachers explore learners' understanding of geometric concepts. Gweshe and Dhlamini (2015:11) recommend that teachers should create a motivating environment in which learners can construct, develop and extend their view for meaningful learning to happen. The environment can be in the form of alternative teaching methods in geometry, such as computer-assisted instruction and an enquiry-based approach, guided by the Van Hiele's phases of instruction.

A review of the literature on this topic by Watan and Sugiman (2018:6-7) found that concepts in Mathematics have a connection between ideas in Mathematics. This is in line with the opinion of mathematical connections in learning Mathematics. Connections support students to understand a concept substantially and help them to improve their knowledge. Teaching instructions that are designed so that learners connect concepts and ideas supports students' understanding of geometrical concepts. Therefore, there is a relationship between teacher instructional practices and students' levels of geometrical thinking and understanding.

In their analysis to explore the extent to which Mathematics tutors facilitate the teaching and learning of geometry at the college of education in Ghana revealed that mathematical tutors exhibit a good conceptual understanding of geometry in facilitating the teaching and learning of Geometry (Armah & Kissi, 2019:9). However, the method used of rote learning, using textbooks to present geometric concepts resulted in the low geometric thinking levels of pre-service teachers. Therefore, the teaching and learning strategy of Mathematics tutors are not structured in a way that supports the development of geometrical thinking and understanding. This analysis confirms findings by Siyepu and Mtonjeni (2014) that teaching of school geometry has proved to be a challenge and became a threat to learners, teachers, curriculum advisors and a number of educational officials in South Africa.

Armah and Kissi (2019), reveal that mathematical teachers exhibit a good conceptual understanding of geometry in facilitating teaching and learning of geometry at van Hiele's level 1 and level 2. This provides opportunities for learners to develop learning of geometric basics. However, at level 3 and level 4 that requires ability to construct proofs and understanding axioms is not structured in a way to support the development of geometric thinking (Armar & Kissi, 2019:9).

According to Sunzuma and Maharaj (2019:1-2), the difficulties teachers and learners face in teaching and learning of geometry is caused by lack of background knowledge, poor reasoning skills in geometry, geometric language, lack of visualizing abilities, teachers' instructional approach and lack of instructional resources. These difficulties pose a challenge to learners' ability to solve geometric problems. Thus, learners are not benefiting from the teaching approach used to teach geometry.

Ponte and Chapman (2006) reported that teachers do not have basic geometrical knowledge and skills for teaching it. Thus, a teacher cannot be effective in the teaching of geometry if they have no skills for teaching it. According to Sunzuma and Maharaj (2019:11), inadequate teaching skills results in avoidance of some geometry topics. Atebe and Shaefer (2009) noted that teachers avoid the teaching of Euclidean geometry in school because of poor mastery of its content and lack of confidence in it. The case of avoiding of geometry topics is evident in the way learner's answers questions in their final exams, using shortcut to mastering the skills in answering questions on Euclidean Geometry (Department of Basic Education, 2019).

Several studies in the last few years, for example, Siyepu (2005), Mateya (2008), Atebe and Schafer (2011) and Ngirishi and Bansilal (2019), conducted on learners' progression from one level of thinking to another revealed that the majority of learners were found to be operating at a lower level and very few learners progress to the next level of thinking. De Villiers (2004) argued the cause of low progression of learners from one level of thinking to another is the language and teacher's way of presentation of material. He maintains that to facilitate progression to the next level, teachers' presentation of materials should be within a certain level close to where learners are, so that the learner will understand what is being taught.

In their investigation into prospective Mathematics teachers' geometry content knowledge in terms of connections made between geometric configuration and geometric principles, Ramatlapana and Berger (2018) show that these teachers encountered difficulties connecting the cognitive processes of visualisation and reasoning. Difficulties they encountered were in terms of identifying and recognising figures, making connections between geometric representation, properties and theorems. These are aspects teachers are expected to teach in Euclidean geometry in Grade 11, yet they have partial knowledge of the relevant circle geometry. These prospective teachers are both learners at university and teachers of geometry in Schools. Ubah and Bansilal (2019:1) attribute the gap some prospective teachers have in Euclidean geometry content to not studying Euclidean geometry in high school, even those who studied it in high school also find it difficult.

Alex and Mammen (2016:2225) argue that teaching begins with a teacher's understanding of what is to be learned and how it is to be taught. Therefore, lack of basic content knowledge has resulted in poor teaching standards. Weak teachers' knowledge emanating from gaps in content knowledge should be addressed in teacher preparation (Ramatlapana & Berger, 2018:172). Alex and Mammen (2018:7) give insight into the quality of learners received by universities for teacher education courses, which reflects the quality of geometry learning in schools. Most Mathematics education students are not familiar with school geometry content to teach it with understanding. Despite intervention programmes by some universities for example, KwaZulu-Natal university designed a series of workshops based on Euclidean geometry to help these learners overcome their fears in Euclidean geometry, many pre-service Mathematics teachers in South Africa are apprehensive about the content of Euclidean geometry (Ubah & Bansilal, 2019:1-2).

Understanding the proof problem is necessary in developing of geometric proof in school. According to Mwadzaangati and Kazima (2019:307), to support learners to understand geometric proofs, three aspects are involved in teaching geometric proofs: defining key Mathematics terms of theorem; initiating activities for introducing the theorem and representing the theorem in a statement to be proved. However, these three aspects of proof development are lacking in the teaching and learning of geometric proofs, thus making learning geometric proof a challenge to many learners as reported by several scholars, such as Battista (2007), Ndlovu and Mji (2012) and de Villiers (2013). They suggest that one way of supporting learners to appreciate the different values of geometric proof development is through the use of a teaching strategy that can enhance students' learning.

Findings by Naidoo and Kapofu (2020:8) confirm the confusion learners experience during the teaching and learning of Euclidean geometry. Their finding reveals that learners experienced Euclidean geometry as difficult and confusing compared to other forms of geometry such as analytical geometry. Learners attributed the confusion in learning Euclidean geometry to a teaching strategy that is non-innovative and concrete manipulative. The Department of Basic Education (2014) draws our attention to challenges learners face in grasping fundamentals of Mathematics since they find it difficult to interact during teaching and learning. Alex and Mammen (2018:7) suggest that educators adopt a combination approach of multiple representations that include visual and verbal representations in geometry to enhance learners' understanding of geometry.

2.2.3 Performance in secondary school geometry

The National Senior Certificate (NSC) examination diagnostic reports for the years 2016, 2017, 2018 and 2019 highlight challenges candidates faced in answering questions in Euclidean geometry that required a deeper conceptual understanding, especially in circle geometry section. The majority of candidates lacked the necessary insight to deal with these questions. These errors were mostly due to a poor understanding of basics and foundation competencies taught in earlier grades (Department of Basic Education, 2016:164; 2018:151). This was also identified by Usiskin (1982:96) as one of the key factors in students' poor performance in geometry.

However, it should be noted that in NSC examinations for 2020 and 2021 candidates' answering of routine questions in Euclidean Geometry shows continuous improvement (Department of Basic Education, 2020:177; 2021:182).

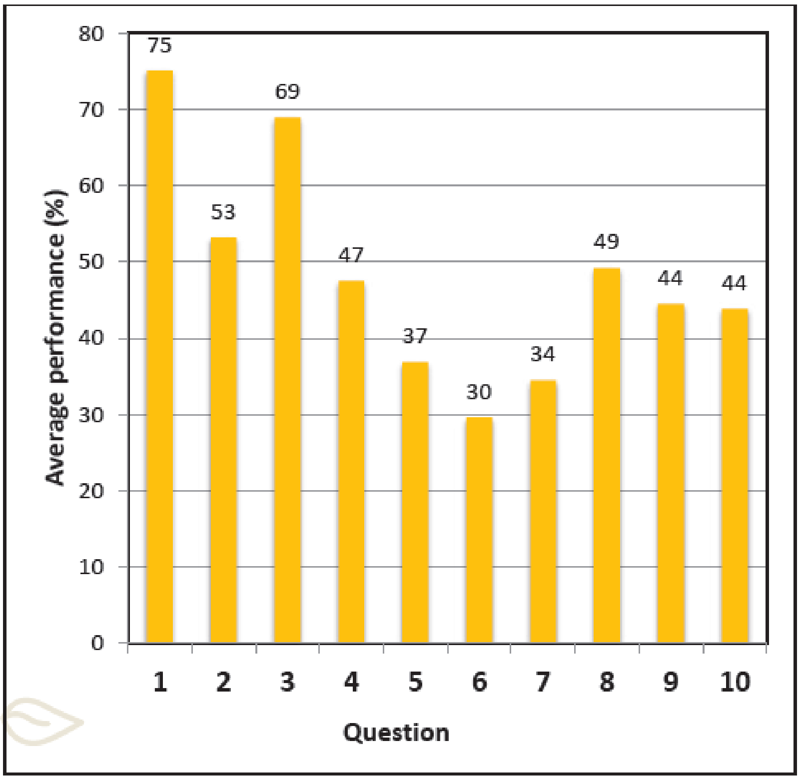
Figure 2.1 below presents the overall achievement rates in Mathematics from 2016 to 2020.

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2016	265 810	135 958	51,1	89 084	33,5
2017	245 103	127 197	51,9	86 096	35,1
2018	233 858	135 638	58,0	86 874	37,1
2019	222 034	121 179	54,6	77 751	35,0
2020	233 315	125 526	53,8	82 964	35,6

Figure 2.1 Overall achievement rates in Mathematics from 2016 to 2020 (DBE, 2021:182)

Despite figure 2.1 revealing an increase in the number of candidates who wrote Mathemathematics examination in 2020 by 11281 in comparion to that of 2019, the performance of learners achieving 30% and above sslightly declined from 54.6 % in 2019 to 53.8% in 2020. The decline in the 2020 Examination showed deficiency in understanding of the basic concepts across some topics in the curriculum, for example, Euclidean geometry, still pose challenges to learners.

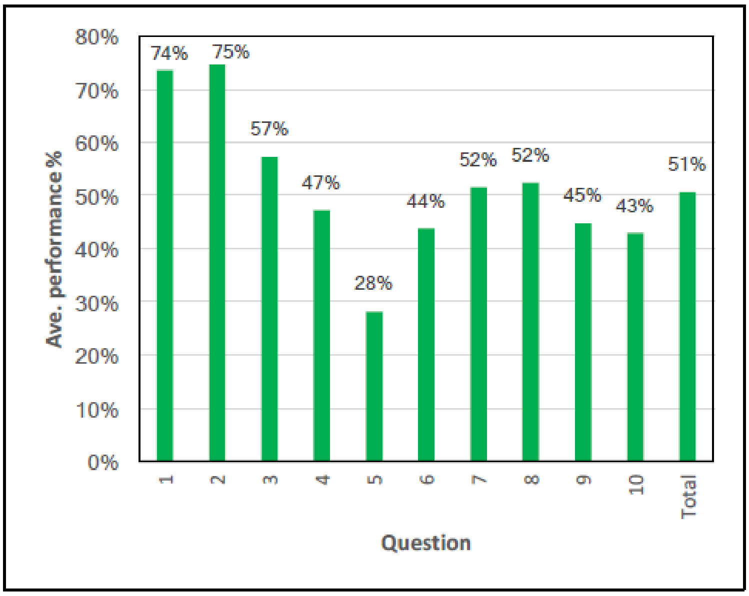
Figures 2.2 and 2.3 below compare the average percentage performance per question for 2019 and 2020, respectively.



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

Figure 2.2 NSC Average percentage performances per question for 2019 in Mathematics paper 2

(DBE, 2020:192)



Q	Topic/s
1	Data Handling
2	Data Handling
3	Analytical Geometry
4	Analytical Geometry
5	Trigonometry
6	Trigonometry
7	Trigonometry
8	Euclidean Geometry
9	Euclidean Geometry
10	Euclidean Geometry

Figure 2.3 NSC Average percentage performances per question for 2020 in Mathematics paper 2

(DBE, 2021:195)

It can be seen in Figures 2.2 and 2.3 that Euclidean Geometry questions still pose some challenges to learners. Few learners can achieve a pass of 50% in Euclidean Geometry questions (questions 8, 9, and 10) compared to other non-Euclidean questions (Questions 1, 2, 3,) where learners achieve a pass of at least 50%.

For example, questions 8 , 9 and 10 that were specifically on circle geometry in 2020 required learners determine and identify angles, to prove theorems and provide reasons for their statements. All the questions had composite diagrams that showed relationship between a cicle and other polygons such as triangles and quadrilatrals. Question 8.1 required learners to determine the size of angles in a circle , with reasons for the angles. In question 9, sub question 9.1 required learners to prove the theorem. Question 9.2 required learners to determine and express angles in terms of a given angle x acyclic quadrilante , giving reasons for their answer and also to prove the midpoint theorem. Question 10 required learners to prove a cyclic quadrilateral, angles and similar trianhles ; giving reasons for their answers.

Figures 2.4 and 2.5 below show the average percentage performance per sub-question for 2019 and 2020.

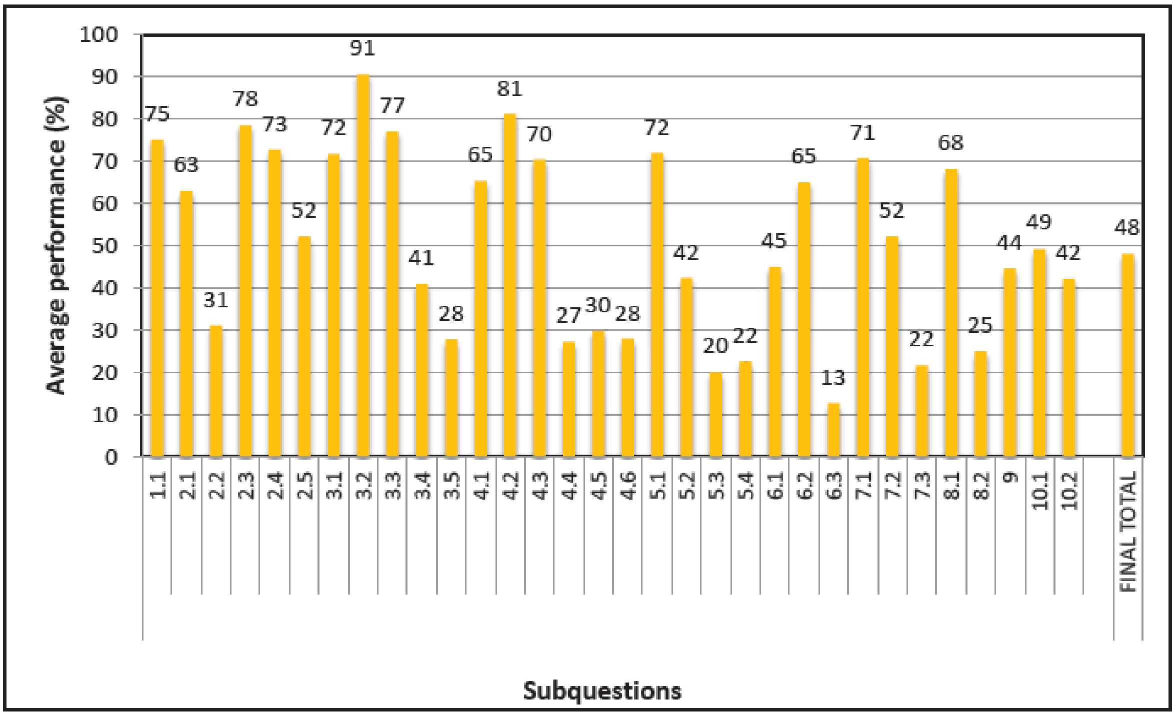


Figure 2.4 Average percentage performance per sub-question for 2019 Mathematics Paper 2 (DBE, 2020:193)

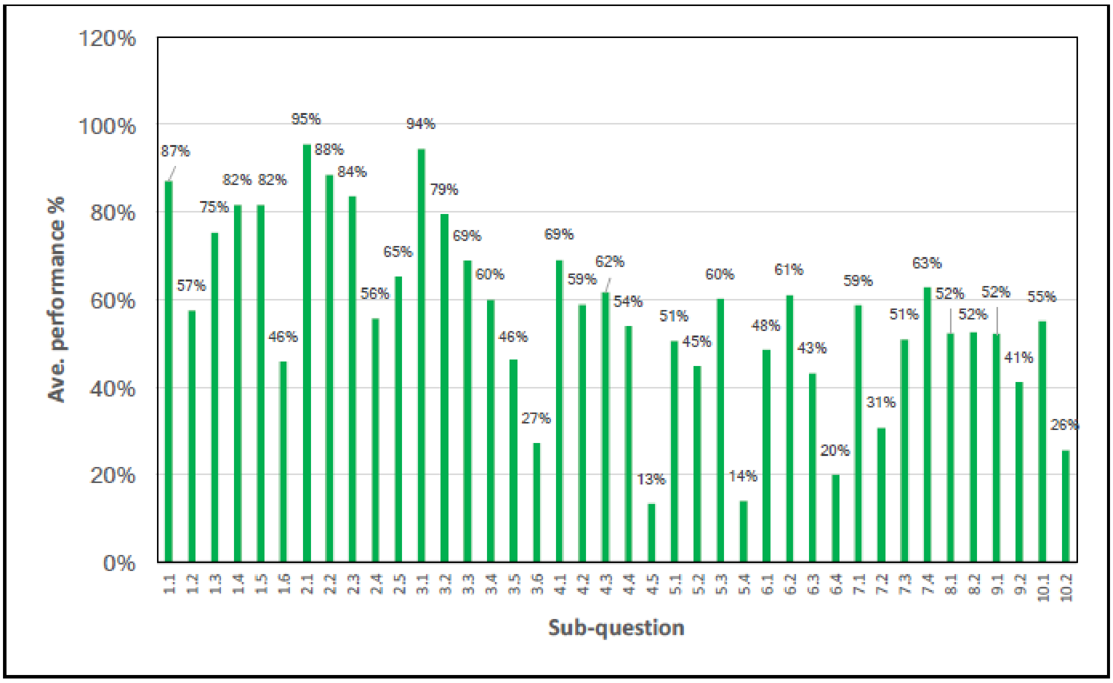


Figure 2.5 Average percentage performance per sub-question for 2020 Mathematics Paper 2 (DBE, 2021:196)

Figures 2.4 and 2.5 show that questions on Euclidean geometry that require learners to prove with reasons still pose challenges to their performance. The majority of the learners failed to provide reason for the statements that they wrote (Department of Basic Education, 2021:203). This is due to difficulties learners face to apply knowledge from one section to another section of work, in interpreting information and substantiating their answers (Department of Basic Education, 2021: 195). For instance, In Q9.2.1 many candidates did not provide a correct or complete reason for their statements (Department of Basic Education, 2021:204).

There are various factors that contribute to learners' poor achievement in Mathematics. According to Marishane et al. (2015:254), the two key factors that contribute to poor achievement in Mathematics are poor subject knowledge and poor teaching competencies of teachers. Performance in Mathematics is determined by a teacher who is grounded in the following three components: Mathematics content, differentiating Mathematics instruction, and modalities of learning Mathematics. However, research reveals that these components are lacking in most Mathematics teachers in South Africa. Bansilal et al. (2014:49) confirm that practising teachers in South Africa struggle with the Mathematics content that they are teaching. Luneta (2015:5) further points out that most Mathematics teachers in South Africa do not have the appropriate skills, content knowledge and pedagogical content knowledge necessary to be effective in a Mathematics classroom.

To close the learner-subject gap to improve achievement in Mathematics, teachers need to be grounded in Mathematics content and pedagogy. This can be achieved through building their capacity in the form of professional development to equip them with the knowledge, skills and attitudes needed for effective teaching and learning (Marishane et al., 2015:254). However Chick and Baker (2005:255) found that some teachers reflect a deep understanding of concepts and strategies for making ideas meaningful for students, but the concepts were not well linked and well supported pedagogically.

The overall performance of learners is aligned with teachers' competencies to teach Mathematics. The NSC diagnostics report (Department of Basic Education, 2016:181) suggests that "teaching of theorems should be done with the relevant understanding. Teachers should refrain from teaching by copying theorems from a textbook". Furthermore, the complexity learners face in disentangling various figures that make up more complex figural arrangement whose properties need to be discerned, renders more concerns on whether learners are exposed to questions in Euclidean Geometry that include theorems and converses (Department of Basic Education, 2019).

According to Naidoo and Kapofu (2020: 2), learners' performance in geometry is influenced by issues such as challenges in completing activities in geometry, teaching and learning resources, interest in geometry and views about learning geometry. In order to alleviate some of these challenges that learners are facing and improve their performance, a suitable pedagogic strategy should be developed. Naidoo and Kapofu (2020:2) highlight that minimal connections developed within learners understanding of mathematical concepts may have an undesirable influence on their performance. Therefore, teachers should enrich their teaching strategies of Euclidean geometry to explore learners' perceptions of Mathematics to improve their performance. Pehkonen and Torner (1998) confirm that perception has an effect on ones' achievement in Mathematics.

The next section examines van Hieles' Theory as a theoretical framework underpinning the research study.

2. 3 Theoretical Framework

Theories of teaching and learning are increasingly becoming one of the key aims of research in Mathematics education. Recent developments in geometry education research have led to a focus on theory of teaching and learning of school geometry. Sinclair et al. (2017: 287) specified van Hiele's model as a reliable theory, specifically on teaching and learning of geometry. Van Hiele's theory continues to be evident in geometry teaching and learning research. This is because it equips teachers with appropriate and active teaching and learning instructions, with the aim of increasing interaction between the teacher and learners. The major role of theories of teaching and learning is that theories act like a lens through which teachers view facts and influence what ones see and what one does not see (Oliver, 1989).

According to Mostafa et al. (2017:93), curriculum theorists believe that teaching and learning methods can be used to achieve goals of education systems. Application of active teaching methods lead to strengthening and development of mental skills in students. Therefore, one of the important and essential actions within education and training is to equip teachers with appropriate and active teaching and learning strategies. Teaching and learning strategies are rooted in theories of learning. Van De Walle (2004:19) asserts that learning theories have been developed through analysis of students as they develop a new understanding. However, it should be remembered that learning theory is not a teaching strategy but rather informs teaching.

According to Ramatlapana and Berger (2018:163), van Hiele's theory has been very popular in South Africa to guide teaching and learning of geometry because of its focus on a learner's level of geometric thinking and the role of geometry instruction in the development of learning. Thus, this study opted for van Hiele's theory as a theoretical framework. The next section discusses Van Hiele's theory in details.

2.3.1 The Van Hiele's theory of Geometric understading

Van Hiele's theory is employed as a theoretical framework underpinning this study. This theory originated in the 1950s but emerged in 1984 from the doctoral work of renowned educators, Pierre Marie van Hiele and his wife, Dina van Hiele-Geldof from University of Utrecht in Netherlands (Crowley, 1987:9). It was influenced by their teaching experience of poor conceptualisation of geometric reasoning to their learners. Thus, they developed five sequential levels of geometrical reasoning to facilitate learners' understanding of geometry. Unfortunately, Dina died shortly after completing her dissertation; her husband amended her work to advance the theory in 1986 by hypothesising five sequential levels of geometrical reasoning as "visualisation", "analysis", "informal deduction", "formal deduction", and "rigour" (Crowley, 1987:9).

Van Hiele was more concerned with the area of pedagogy in the theory; as a result, he developed and proposed five sequential phases of instructional design to address the important area of pedagogical concern such as the method and organisation of instruction, content and material used to promote learners' progression and acquisition of a level (Crowley, 1987:5).

According to Nisawa (2018:62), van Hiele's theory is a process model of understanding comprising:

1. The level of thinking: this describes ways of thinking that can be found in the student's Geometry. This part is mainly concerned with student progression through levels of reasoning during the learning process. The five levels of thinking are Level 1 (recognition), Level 2 (Analysis), Level 3 (informal deduction), Level 4 (Deduction) and Level 5 (Rigour); and
2. The phases of teaching and learning: This part guides teachers how to organise the teaching of geometry to facilitate and promote students to pass from their levels of thinking. The core five phases of learning include: Phase 1(information), Phase 2 (directed orientation), Phase 3 (explication), Phase 4 (free orientation), and Phase 5 (integration).

In his overview of the theory, Ndlovu (2013:227-278) concludes that van Hiele's theory actually gives three stages of cognitive development that:

1. Describe five sequential and discrete levels that student pass through as geometrical thought develops;
2. Discuss the nature or properties of insight into geometric concepts; and
3. Present a guide to the phased development of geometric lessons.

The reasoning behind his conclusion is rooted in the fact that levels are situated, not in the subject matter but the thinking of man. Schoenfeld (1986) noted that van Hiele's theory is an empirical description of relatively stable stages that provides guidance on structuring learners' experiences in geometry. Usiskin (1982:99) confirms Van Hiele's theory to be a reliable theory to explain why many students have trouble learning and performing in geometry and the need for systematic geometry instruction for success in writing proofs. Fuys et al. (1984:6) declares that two major developments of Van Hiele's theory are the role of instruction in the teaching of geometry and the role of instruction in helping learners to move from one level to the next.

According to Alex and Mammen (2016:2226), Van Hiele's theory is primarily directed at improving teaching and understanding of geometry by organising instruction in a way that considers learners' thinking processes while new content is introduced. The model clarifies many of the shortcomings in traditional instruction and offers ways to improve it by focusing on getting learners to the appropriate level to be successful in secondary school geometry (Alex & Mammen, 2016:2227). Furthermore, Luneta (2015:2) asserts that teachers' understanding of these levels enables them to identify the general direction of learners' learning and levels at which they are operating; it also provides teachers with a framework within which to conduct geometric activities by designing these with the assumption of a particular level in mind.

The next section discusses van Hiele's five levels of geometric thinking, as described by Usiskin (1982), Fuys et al. (1984) and Crowley (1987).

2.3.1.1 The Van Hiele's levels of thinking

The five levels and their general characteristics are described by Usiskin (1982), Fuys et al. (1984) and Crowley (1987) as follows:

Level 1 - Recognition: At this level, learners visually recognise figures by their global appearance. They also recognise figures by their shapes, but they do not explicitly identify the properties of these figures. For example, in Figure 2.6, learners can identify a circle and recognise it very easily from other figures because of its circular shape, but cannot refer to the properties of a circle.

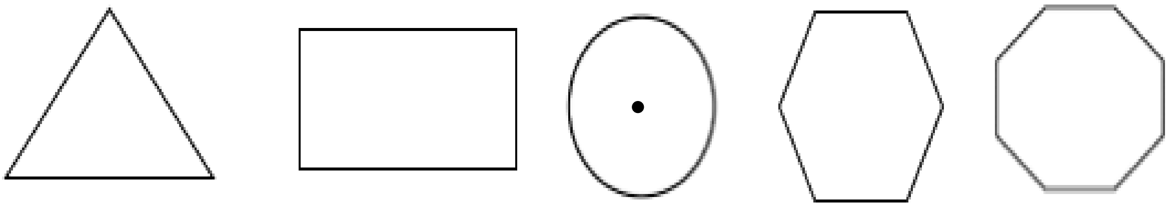


Figure 2.6 Two dimensional shapes

Level 2 - Analysis: At this stage, learners can identify components of a circle such as a radius, diameter, chord, sector, arch, secant, circumference, and semi-circle through observation and experimentation. At this level, the concepts can exist for learners, separate from the situation in which they are developed. For example, learners can recognise parts of a circle as in Figure 2.7. However, at this stage, they can neither recognise nor explain the interrelationships between parts nor define the parts of a circle.

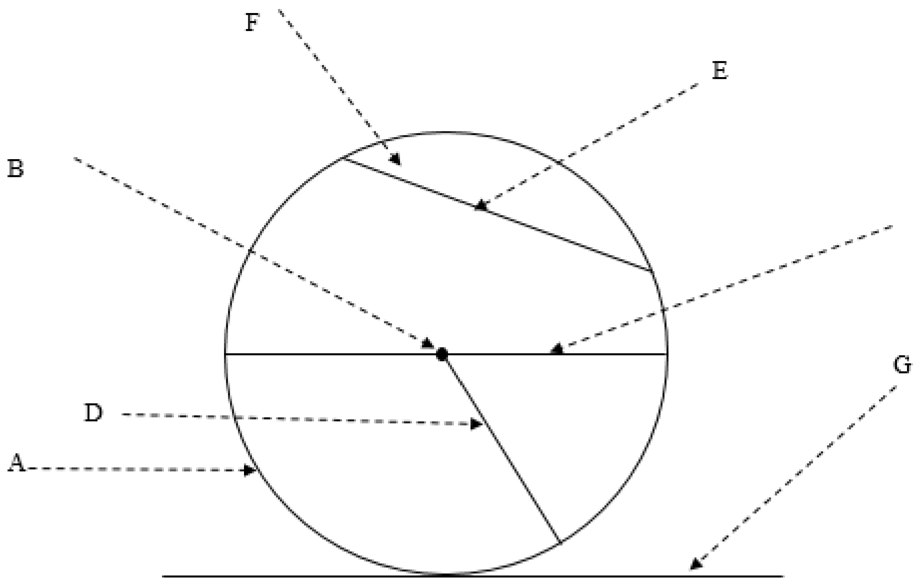


Figure 2.7 Parts of a circle

Level 3 - Ordering: At this level, learners logically order properties of figures by short chains of deductions and understand interrelationships between figures (e.g. class inclusions). At this level, learners describe a class of figures in terms of properties and rules by giving informal arguments. For example, in Figure 2.8, learners can claim that an angle in a semi-circle is a right angle, and in Figure 2.9, that the radii of the same circle are equal. Learners can follow formal proofs for properties of a circle; however, they can neither comprehend the significance of deduction nor the roles of axioms. Therefore, learners do not perceive essential relationships between the properties.

Pierre van Hiele stated, “my experience as a teacher of geometry convinces me that all too often; students have not yet achieved this level of informal deduction. Consequently, they are not successful in their study of the kind of geometry that Euclid created, which involves formal deduction.”

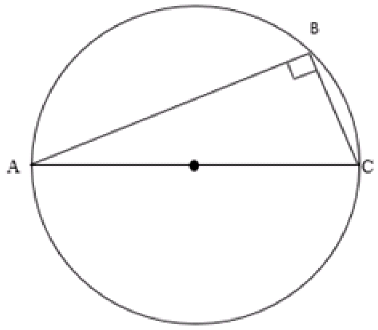


Figure 2.8 Properties angles in a semi-circle

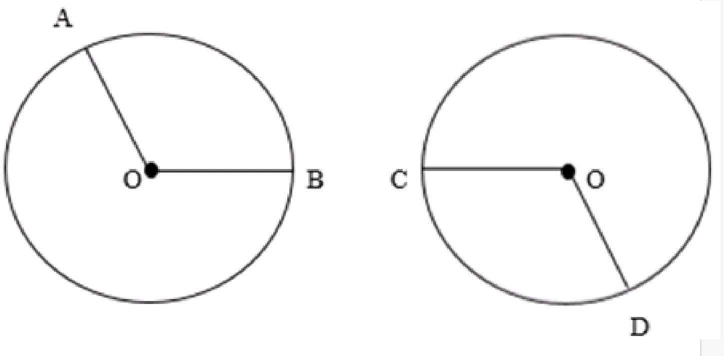


Figure 2.9 Properties of radii in a circle

Level 4 - Deduction: At this level, learners start developing longer sequences of statements and begin to understand the significance of deductions, including the role of axioms, theorems and proofs. At this stage, a learner is able to construct proof and supply reasons for steps in a proof and its converse. Therefore, learners can form long chains of deductions. For example, in Figure 2.10, learners can prove that the angle between the tangent and the chord is equal to the angle in the alternate segment. That is, angle α = angle β.

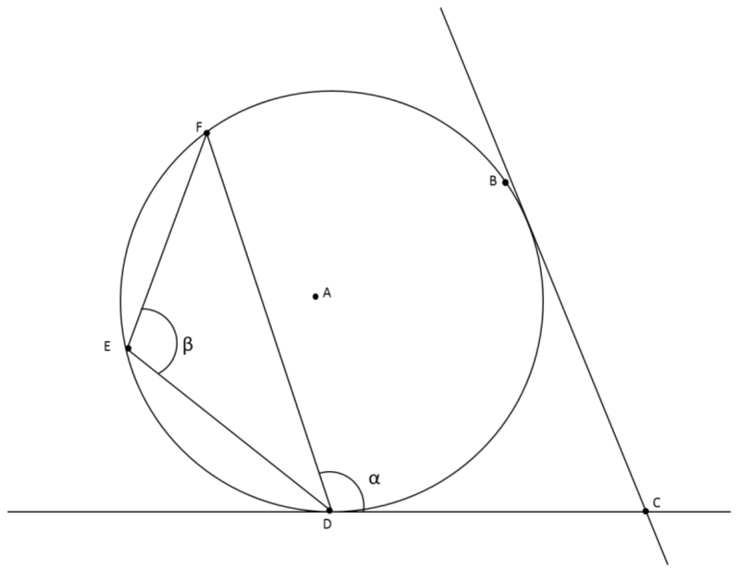


Figure 2.10: Tangent properties

Level 5 - Rigour: At this stage, learners understand the formal aspects of deduction, use of indirect proof, proof by inspection, and non-Euclidean systems. Thus, they can work in a variety of axiomatic systems. However, according to Crowley (1987:11), this was the least developed level in the original work and received little attention from researchers. This is because most high school geometry is taught at Level 4. De Villiers (2010:2) confirms that the first four levels are the most pertinent for secondary school geometry; therefore, this study will not discuss this level since it is not included in the South African curriculum assessment policy statements (CAPS).

2.3.1.2 Characteristics of van Hiele levels

Crowley (1987:4) advances the following five important characteristics of this theory which are significant for teachers when making instructional decisions:

1. **Sequentia**l – learners progress through the levels in order. Each level means an improvement over the reasoning abilities of the previous level. According to Crowley (1987:4), for a learner to function successfully at a particular level, he/she must have acquired strategies of the preceding level. Uziskin (1982:14) and de Villiers (2010:1) confirm that for a learner to understand geometry, s/he must go through the levels in order. Therefore, a learner cannot be at van Hiele's level *n* without having gone through level *n-1*.

2. **Advancement –** learners' progression through the levels depends on the content and methods of instruction received (Crowley, 1987:4). The student can reach a level only if he/she has reached the previous level. Fuys et el (1988:7) asserts that types of instructional experiences can affect learners' progression through levels. De Villiers (2010:1) attributes failure of

traditional geometry curriculum to it being presented at a higher level than those of learners, so methods of thought used to present the curriculum remain inaccessible to the learners. Therefore, methods of instruction can enhance progression or prevent progression between levels.

3. **Intrinsic and extrinsic** – the object at one level becomes the object of a study at the next level. Crowley (1987:4) illustrates this characteristic with an example of a figure that is, at Level 1, students only perceive a figure but the figure is determined by its properties which are at Level 2 where the figure is analysed, and its properties are discovered.

4. **Linguistic** – each level has its linguistic symbols and relations connecting these symbols. Language plays an important role in teaching and learning. As the levels are explained in the previous sections above, it should be noticed that each level has its meaning to a terminology. For instance, the word *proof* has different meaning at each level. At level 2, it means verification; at level 3, it means informal deduction and at level 4, it means formal deduction. In stressing the importance of language, Fuys, Geddes and Tischler note that many failures in teaching geometry result from the language barrier. That is, the teacher using language of a higher level that is understood by learners (Fuys et al., 1988:7).

5. **Mismatch** – learning may not occur if the instruction is not at the level of the learner. Crowley (1987:4) stresses that if the teacher, the instructional materials, content and vocabulary are mismatched, the learner will not be able to follow the learning. Uziskin (1982:14) and De Villiers (2010:1) called this characteristics ***separation*** where two persons reasoning at different levels cannot understand each other. Therefore, learners cannot understand the teacher, and the teacher cannot figure out why they cannot understand.

What is important in terms of Van Hiele's theory, as Crowley (1987), De Villiers (2010) and Mason (1998) note, is for teachers to know that learners should progress through these levels in a specific order without skipping a level. According to Crowley (1987:5), progress through levels depends on the instruction received. Therefore, for meaningful learning to occur, instruction should be organised at the thinking level of the learner. If the instruction is delivered at a higher level than that of the learner, the learner will have difficulty in understanding the content taught. This study demonstrates how to organise instruction on circle geometric activities using Van Hiele's theory of instructional design.

2.3.1.3 Van Hiele's phases of learning

Studies by Crowley (1987), Reed (1996), Mason (1998), Halat (2003), Dongwi (2014), Howse and Howse (2014) and Al-ebous (2016) concur that Van Hiele's phases of learning constitute the most significant instruction guide to design geometric activities for a good model of pedagogy. This is because they include a sequence of activities requiring orientation and discovery, exploration, verbalisation and integration. These phases are intrinsic within a level- enabling progress from one level to the next.

Despite van Hiele being a good model of pedagogy, learners' achievement lies within the direct control of teachers and the curriculum (Senk, 1989). According to Kalyankar (2019), learners' development and achievement proceed under the influence of a teacher, and the teacher must play a vital role in facilitating the progress in the learning process. Howse and Howse (2014) stress that to support learners in their progression through these levels, teachers are required to plan and design their instruction methods strategically according to the five phases of instruction, as proposed by Van Hiele. According to Clements and Battista (1995), each phase describes:

- The goal of learners' learning; and
- The teachers' role in providing instruction that facilitate learning.

The next section discusses the five phases of instruction, as described by Crowley (1987:5-6) , Clements and Battista (1995) and Mason (1998:4-5).

2.3.1.4 Van Hiele Phases of learning

1. *Information*

This is an orientation and discovery phase. Learners are oriented into a new topic and the teacher identifies what they already know about a topic. During this phase, conversation takes place between a teacher and learners regarding the topic of focus; this assists the teacher in discovering learners' prior knowledge of geometric concepts as well as laying a foundation for subsequent learning activities. At the same time, learners start to explore the topic and become familiar with the context of the topic. For instance, the teacher discovers what students think about circles and what level they have attained in circle geometry.

2. *Directed orientation*

This is an exploration phase. The teacher guides learners to uncover connections about the subject matter through structured activities that challenge learners to formally recognise and verbalise their understanding of the topic introduced in the information phase. During this phase, a teacher carefully structures instructions such as construction and measurement to guide learners in gaining an understanding of attributes of the figure and connections between them. For example, for students to explore the size of angles in a semi-circle, they can be instructed to construct a circle, draw a diameter and chords, and measure the sizes of the angles.

3. *Explication*

This is a verbalisation and expression phase. The most prominent feature in this phase is the technical language used by learners to verbalise their understanding of the concepts and connections. During this phase, learners are involved in verbalising explicitly their understanding of the geometric concepts they have observed in acceptable mathematical language. Thus, the role of a teacher is to facilitate dialogue that allows learners to explain their understanding by using appropriate mathematical language. For example, learners discuss with one another and their teachers the various sizes of angles measured from the activity in Phase 2.

4. *Free orientation*

This is a further development of the directed orientation phase where learners develop their way of solving problems and complete geometric tests using connections at their disposal. The major role of a teacher in this phase is to select appropriate geometrical problems that require learners to use connections at their disposal. This allows learners to use their creativity and the teacher to articulate learners' ability to understand related geometric concepts.

5. *Integration*

This phase finalises the teaching process of a particular topic. Learners are in a position to summarise, integrate and build an overview of the content studied. They develop new networks of knowledge and relationships on a topic to reach a new level of geometrical thinking. This new level of thinking replaces the previous level of thinking, and learners form their overview of the topic. In this phase, the role of a teacher is to provide a summary of the main points studied to help learners in the process.

Uziskin (1982:15) believes that cognitive development of learners in geometry can be accelerated by the way instructions are organised. The way instructions are organised and delivered has implications on how they move from one level to the next. The van Hieles gave explanations of how the teacher should operate to lead students from one level to the next by considering the above five phases of instruction.

Atebe and Schäfer (2010:85) recommend educators to structure their instruction in ways that reflect hierarchies of Van Hiele's levels. This is because learners whose instruction experiences are aligned most closely with Van Hiele's phases of learning show a better understanding of geometrical concepts than those whose experiences are not.

2.3.4 Grade 11 circle geometry and van Hiele levels.

Grade 11 circle geometry curriculum is built on geometric proofs and requires proof of the theorem and their application in solving riders (Department of Basic Education, 2011). The theorems deal with the relationship and properties of the component of a circle. According to van Hiele's theory, proof construction and deduction is at Level 4. This is the level of development that they need to be at in order to understand the formal aspects of deduction. Ndlovu (2013:278) confirms that learning levels that require learner s to understand riders and execute proofs about circle geometry integrate level 3 and level 4 of van Hiele's theory.

Investigating and proving theorems in Grade 11 circle geometry occurs at the third and fourth level. At these levels, learners can make connections between the network of statements about properties of circle and relationships between properties of a circle (Level 3) and can make short deductions (level 4). Ndlovu and Mji (2012:181) confirm that an intuitive foundation of proof and deductive reasoning occurs at the third and fourth level of van Hiele's theory. Siyepu (2005:102) found that with learners at van Hiele's level 3 can notice the relationship between properties of a circle, thereby concurring with Ndlovu and Mji. Although this is what is required, Siyepu (2005) in his investigation of a learner's level of thinking in terms of van Hiele's circle geometry, found that most learners enter Grade 11 at van Hiele's level 1 (Siyepu, 2005:59).

2.3.5 Teaching implications of the van Hiele phases

A review of literature on instruction of geometry in school reveals global adoption of the van Hiele phases to be an effective instruction design for teaching and learning of geometry (Howse & House, 2015; Alex & Mammen, 2016; Argaswari, 2018; Watan & Sugiman, 2018; Armah & Kissi, 2019). Therefore, Van Hiele's phases have implications for Mathematics teachers in their instructional process.

According to Erdogan et al. (2009:185), instructions that are designed according to van Hiele models are aimed at developing learners' high-level thinking skills such as implication, association, communication, problem-solving, spatial thinking and creative thinking. According to learning principles and standards (van de Walle, 2004), the major goal of learning Mathematics is to create autonomous opportunities for learner s to apply procedures, concepts and processes. This is what is defined as *learning with understanding* by van de Walle. Thus, learning with understanding is rooted in Van Hiele's theory.

The van Hiele theory places great importance on learner's growth in geometry that takes place in terms of distinguishable levels of thinking. Therefore, in planning geometry lessons, it is important to have these levels in mind (Armah & Kissi, 2018:3).

The van Hiele theory is an important source to understand a teacher's pedagogical content knowledge on geometry teaching (Armar & Kissi, 2018). Pedagogical content knowledge is the knowledge required for teaching Mathematics (Chick & Baker, 2005). Knowledge of teaching Mathematics influences instructional practice and student learning. Therefore, van Hiele's theory categorises students' learning abilities into five hierarchical levels of geometrical thinking and offers a model of instruction proved to be effective on teaching and learning of geometry (Howse & House, 2015; Alex & Mammen, 2016; Argaswari, 2018; Armah & Kissi, 2019).

According to Ndlovu and Mji (2012:181), the major strength of the van Hiele theory is that it emphasises the scaffolding role of teaching and learning that leads the student to progress from one level to the next. Louw and Mbokane (2018:3) explain scaffolding as a process that follows a sequence of 'I do" (teacher demonstrate) by the teacher followed by "we do" (teacher engage with learners) by the learners through a teacher's guidance, and then: "you do" (learner s solve the problems themselves) by the learners alone. Thus, scaffolding is one of the strategies where instruction is based on guided discovery which van Hiele's theory emphasises.

The van Hiele theory strengthens and recognises the role of language in moving through the levels (Ndlovu & Mji, 2012). The levels guide teachers to use descriptive terminologies appropriate to the learners as they progress from one level to the other. For example, teachers should build on learner's language at level 1 and introduce more formal language and terminology as they proceed to the next level. According to Siyepu (2005:18), the phases advocate for gradual transition from the language of learners to the language appropriate to the subject. Thus, learner s can use correct mathematical terminologies at the end of the topic.

2.3.6 Criticism of the van Hiele Theory

Though various studies emphasised the role of van Hiele theory in teaching and learning geometry, the theory is not free from drawbacks and criticisms. There has been various criticism of the van Hiele theory, especially in relation to the nature of progression from one level to another and discreteness of the levels.

2.3.6.1 Is the development of thinking sequential?

According to Sharma (2019:46), van Hiele theory has been criticised for emphasising that the development sequentially takes place. This is because the same learners may possess different van Hiele levels for different geometry concepts and topics simultaneously. This implies that learners cannot be at the same level of geometric understanding in all content strands and topics. Mason (2009:6) maintains that if a learner has done more work with triangles than with quadrilaterals, he or she may think about triangles in a more sophisticated way than he or she would about an unfamiliar figure. Thus, a learner familiarises himself or herself with a certain figure; for example, a circle at Grade 10 enables them to reach the second level surpassing the progress of Grade 11 learner. This is essentially the same argument raised by Kalyankar (2019: 50) that certain topics might be easier to arrive at higher van Hiele levels than others. Therefore, the levels reached by a learner across concepts and topics differ for different concepts and topics.

Burger and Shaughnessy (1986) reported that students show different preferred levels on different tasks, with some oscillating from one level to another on the same task. This raises a criticism whether the development of learning takes place in a sequential manner, as stipulated by the van Hiele theory.

2.3.6.2 Do the levels form a Hierarchy?

Mason (2009:6) highlights that in van Hiele's theory, a learner cannot achieve one level of understanding without having mastered all the previous levels. However, Siyepu (2005: 34) has criticised this notion using his teaching experience arguing that a learner can achieve one level of understanding without having mastered the previous level. He illustrated his argument using an example that it is possible for the learner to prove and apply theorems (at level 4) without knowing short and precise definitions of certain concept (level 2). This criticism related to the initial criticism that learning takes place in a sequential manner, which ought not to be the case. As also noted by Schoenfeld (1986), van Hiele's theory does not give a deterministic view of a fixed progression, but is an empirical description of relatively stable stages. Therefore, a learner may skip a level and still achieve the next level of understanding.

2.3.6.3 Does the van Hiele theory applicable to non-Euclidean geometry?

As mentioned by Sharma (2019), van Hiele theory relies too heavily on the development of concepts of the Euclidean geometry other than for any developmental trajectory for non- Euclidean. As also mentioned by Kalyankar (2019:21), Van Hiele theory best applies to the descriptive geometry of two-dimensional shapes. This limits its application on three-dimensional objects and non-Euclidean objects such as a sphere. Learning three-dimensional objects develops an understanding of the space around us in the real world. According to Sinclair (2008), learning three-dimensional space aid learners to appreciate and understand intuition about the real world.

2.3.6.4 Does the van Hiele theory undertake an approach to the role of language?

The theory undertakes a limited approach to the role of language in the development of geometry concepts (Sharma, 2019:46). According to Sharma, the role of language is restricted in terms of definitions of the geometry concepts. The theory only emphasised the role of language in communicating features and properties of figures. According to Crowley (1987:4), each level has its linguistic symbols and its systems of relations connecting these symbols. Therefore, the language used is only limited to a particular level and its interpretation of the same term, thus limiting verbalisation of the concepts.

The theory lacks depth details regarding the type of reasoning that exists at each level rather a description of characteristics of the thinking and reasoning process (Hourigan & Leavy, 2017: 6).

2.3.6.5 Is there a pre-recognition Level 0?

There is still considerable ambiguity with regards to the existence of a level more basic than Level 1, called *pre-recognition level*. Clements and Battista (1995) proposed the existence of the pre-recognition level to accommodate learners that cannot reason and operate at Level 1. For example, learners may distinguish between a circle and a triangle but may fail to distinguish between a circle and a sphere. This is because learners notice only a subset of the visual characteristics of a shape (circle and triangle) resulting in an inability to distinguish between figures (circle and sphere). Kalyankar (2019: 52) explains that at this level, the object about which students reason is a specific visual or tactile stimuli; the product of this reasoning is a group of figures recognised visually as “the same shape”.

Research by Usiskin (1982) and Senk (1989) found that some learners’ thinking characteristics do not meet the criterion for Level 1. They suggest the existence of Level 0 for learners that do not meet the criterion of Level 1. The flaws of their findings have been recognised by Crowley (2009:5) and Kalyankar (2019: 51) indicating the existence of thinking as more primitive than, and probably prerequisite to, van Hiele Level 1. Therefore, there is a suggestion for an additional Level 0 called pre-recognition level.

Despite all these critics of the van Hiele theory, the theory is considered to correctly, although generally, depict the development of geometry thinking in learners about shapes. However, there is limited literature on criticism of the five sequential phases of learning proposed by van Hiele to address the gap on development of instruction received by learners. Crowley (1987: 5) asserts that instruction developed according to van Hiele’s five phases of learning promotes learners’ acquisition of a level. Therefore, the highlighted criticism is most likely to have a diminutive effect on the findings of this study. This is because the main purpose of this study is to apply van Hiele’s theory of instructional design to facilitate the learning of circle geometry in Grade 11.

2.4 Conclusion

This chapter discussed literature review and theoretical framework. The first section examined and discussed literature relating to geometry as an area of concern in teaching and learning, reviewed the South African secondary school curriculum on circle geometry (SP and FET), the way geometry is taught in the South African context and globally, and the trend of performance in Euclidean geometry in the National Senior Certificate for the past four years (2019, 2018, 2017 and 2016). The second section discussed van Hiele’s theory as a theoretical framework underpinning this study and phases of learning. It further discussed teaching implications and criticism against van Hiele’s theory. The next chapter, Chapter 3 discusses the research design and methodology of this study.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter presents the design and methodology of the study. It discusses the research paradigm, design, and methodology, site selection, sample, data collection techniques, data analysis, researcher's role in the study, trustworthiness and ethical considerations. A summary of this chapter is provided to conclude Chapter 3.

Research design is specific procedures involved in the research process, data collection, data analysis, and report writing (Creswell, 2012:20). This study employed a qualitative case study research backed by the interpretive research paradigm best suited to answer the key research questions below:

- How does the van Hiele's theory of instructional design facilitate the learning of circle geometry?
- What are the challenges of using van Hiele's theory of instruction design to facilitate learning of circle geometry?
- What are the benefits of using van Hiele's theory of instruction design to facilitate the learning of circle geometry?

The research design and methodologies were selected by identifying the purpose and aims of the study as well as the nature of the research question, as stated in Chapter 1. A more detailed discussion of design and methodology is provided in subsequent sections below:

3.2 Interpretive paradigm

The term *paradigm* refers to a set of very general philosophical assumptions about the nature of the world (ontology) and how it can be understood (epistemology) (Maxwell, 2008:224). According to Maxwell (2008), paradigms that are relevant to qualitative research include interpretivism, critical theory, feminism, postmodernism and phenomenology. This study employed the interpretive paradigm that emphasises human interaction with phenomena in their daily lives.

The interpretivist holds the premise that, to understand the world, we should be aware of the fundamental nature of social world at the level of subjective experience. It explains the realm of individual consciousness and subjectivity. According to Günbayi, and Sorm (2018:64), interpretive paradigm puts emphasis on descriptions of what people experience and how it is that they experience what they experience Therefore, based on the purpose of the study, the interpretive paradigm supports van Hiele's theory that frames this study, by acknowledging that learning is more dependent on the instruction received than age or maturation (Crowley, 1987). An interpretive paradigm was used for this research to explore how van Hiele's theory of instructional design facilitates learning of circle geometry to Grade 11 learners.

3.3 Research approach

This study employed a qualitative approach set within an interpretive paradigm, with a case study design. Creswell (2002:16) defines qualitative research as an approach for exploring and developing a detailed understanding of a central phenomenon. The emphasis of this study is on exploration, description, explanation, creation of, and testing of instructions based on van Hiele's phases of learning which require qualitative research. According to Mack et. al (2005), qualitative approach is effective for the study that requires identifying intangible factors, to help interpret and better understand the complex reality of a given situation. Therefore, qualitative approach was chosen because it is one of the most practical ways to achieve the aims and objectives of this study.

3.3.1 Case study

Stake (1995) describes a case study as a study that deals with the particularity and complexity of a single case and coming to understand its activity within the important circumstances around the case. Stake identified a classroom of learners and a teacher, as some examples. In other words, the case is "a bounded system". For example, in this study, we chose to study the teacher and learners, looking broadly at how the teacher applies the van Hiele theory phases of learning, paying particular attention to how circle geometry instructions are designed and facilitated tor Grade 11 learners in a classroom environment.

3.3.2 A qualitative case study

In the context of this study, "a qualitative case study is an approach to research that facilitates exploration of a phenomenon within its context using a variety of data sources" (Baxter & Jack, 2008:544). Yin (2009:18) confirms that a case study has a twofold technical definition. The first part begins with the scope of a case study. The current study is limited to Grade 11 learners' development of understanding when a teacher uses an instructional design based on Van Hiele's theory in the teaching of circle geometry in one secondary school in the Western Cape, South Africa. Yin (2009:18) defines a case study as "an empirical inquiry that investigates an existing phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not evident". It is further defined as relying on "multiple sources of evidence, with data needing to converge in a triangulating fashion" (Yin, 2009:18).

A qualitative case study is a fully compliant research design in geometry studies. Some researchers who employed this design in the study of geometry topics include, Oladosu (2014) to understand and gain insight into the type of meaning secondary school students hold in learning circle geometry.

Similarly, Evbuomwan (2013) investigated difficulties faced by form C students in the learning of transformation geometry in a Lesotho secondary school. This design yielded credible and reliable results in their studies.

3.4 Sample

3.4.1 Site selection

This study was conducted in a public secondary school, in the Metro East Education District, in Cape Town, Western Cape, South Africa. This school is classified as a quintile 1 school under the Western Cape Education Department (WCED). The language of learning and Teaching (LoLT) of the school is English, and the school is located in Cape Town's northern suburb townships. The school follows the South African National Curriculum and Assessment Policy Statement (CAPS) curriculum.

The site was conveniently selected because participants were easily accessible and readily available; the school environment was conducive as I had worked in the school before, which made it easy to engage participants. The reasons to choose convenience sampling link to the criteria suggested by Etikan et al. (2016) that accessibility, availability of participants at a given time, or willingness to participate are practical criteria for convenience sampling. Goodwin and Goodwin (1996:29) noted that selecting a site in which to conduct qualitative study should be deliberate, with emphasis on places likely to yield rich information pertinent to the general topic of interest. According to Yin (1994), a researcher should consider convenience, accessibility and geographical proximity when selecting the site of research. Thus, this site was conveniently selected.

3.4.2 Participants

The participants in this study were Grade 11 learners and Grade 11 Mathematics teacher. The sample size was 35 learners in a class and one teacher for Grade 11 pure Mathematics in this school. Grade 11 learners were enrolled for pure Mathematics. All participants were from the same school, the same classroom and attended to by the same teacher for the entire period of this study. The teacher was adequately qualified to teach Mathematics. His qualification is Bachelor of Education, with five years' teaching experience in Mathematics.

The participants were purposefully selected because they are "information rich" and provided useful information pertinent to the study. According to Creswell (2002: 206), the standard used in choosing participants and the site is being "information rich". Maxwell (2008: 235) highlighted that a purposefully selected sample allows for the examination of cases that are critical for theories subsequently developed. Purposeful sampling suited this study because it examines already developed theory on circle geometry.

3.4.3 Sampling strategy

The sampling strategy was Theory or concept sampling method used to select the Grade 11 pure Mathematics class and a teacher. This sampling method was used because the concept under study is taught in Grade 11 and the teaching of this concept is guided by van Hiele's theory. Creswell (2002:208) explains theory or concept sampling as a purposeful sampling strategy in which the researcher samples individuals or site because they can help the research generate or discover specific concepts within the theory. The participants were chosen on account that the content area of circle geometry explored in this study is taught in Grade 11, and geometry teaching is guided by van Hiele's theory.

3.5 Data collection methods

The data collection methods used to answer research questions included document analysis, classroom observation and in-depth interviews. A matrix was developed to guide logic for deciding on the selection of these data tools. This matrix identified how each of the components of the tool helped to answer the research questions, as indicated in Table 3.1 below.

Table 3.1: Matrix for the selection of data collection methods

Main Research question	Data collection methods
1. How does the Van Hiele's theory of instructional design facilitate learning of circle geometry to Grade 11 learners?	• Document analysis • Classroom observation • In-depth interviews
Research sub-question	
1. What are the possible challenges of using Van Hiele's theory of instructional design to facilitate learning of circle geometry to Grade 11 learners?	• In-depth interviews • Classroom observation
2. To assess the benefits of Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners.	• In-depth interviews • Classroom observation

Source: ***Adopted and modified from the Maxwell (2008:241)***

Data was collected over three weeks for three days each week. During the period of data collection, the country was facing a pandemic called Covid-19. Due to the pandemic, the schools adopted a new attendance modal of classroom rotational on different days to observe social distancing as stipulated in the standard operating procedures. Table 3.2 displays the schedule in the two weeks when Grade 11 learners were attending school during the data collection period.

Table 3.2: Chronological data collection schedule

Day	Mon	Wed	Fri
Week 1 Research Activity	5th October 2020	7th October 2020	9th October 2020
	Initial meeting and introduction	Observation of Teaching activity A	Observation of Teaching activity B
Week 2 Research Activity	12th October 2020	14th October 2020	16th October 2020
	Observation of Teaching activity C	Observation of Teaching activity D	Observation of Teaching activity E
Week 3 Research Activity	19th October 2020	21st October 2020	
	Application Activity F	Interviews	

In the section that follows, each data collection method is discussed in detail, and reasons, why they were chosen, are provided.

3.5.1 Document analysis

"A valuable source of information in qualitative research can be documents" (Creswell, 2012:223). Creswell (2012:223) describes documents as public and private records, such as records in the public domain and personal notes that qualitative researchers obtain about a site or participants in a study. Stake (1995:68) draws our attention to document review in qualitative research through highlighting the importance of document review in data collection. Stake asserts that documents serve as substitutes for records of activities that the research could not observe directly. Curriculum documents and van Hieles' theory phases of learning readings were analysed to obtain data required to answer the research question.

3.5.1.1 Curriculum Documents

Curriculum and Assessment Policy Statements (CAPS) for FET and SP were analysed to ascertain the curriculum of Grade 11, the content specifications for Grade 11 circle geometry, and the link in the geometry curriculum of lower grades to Grade 11. CAPS provides detailed guidelines in respect of the content to be taught in schools. The review of the CAPS provided a detailed analysis of the curriculum statement for circle geometry taught in Grade 11 and in lower Grades. These were used with the van Hiele's phases of learning as mentioned in the theoretical framework to inform the design of the teaching activities

The curriculum documents were also used to investigate the duration of data collection. The information was analyzed to better understand the required duration and term, which guided the researcher while seeking permission from the school principal. This information is indicated in the pacesetter, which supports teachers in the pacing of the curriculum. It indicates the term in which a topic should be covered and outlines the number of weeks required to facilitate a specific topic.

The Mathematics pacesetter for Grade 11 indicates that the topic under study, circle geometry, is taught in Term 3 for duration of three weeks. This information in the pacesetter guided the research on the duration of data collection. The duration of data collection lasted for three consecutive weeks, starting from the second week of Term 3 to the fourth week. As mentioned by Creswell (2012: 283-285), the quality of qualitative research depends on the period of data collection. A long period in the field collecting data results in a good collaboration between the researcher and the participants, and extensive data is collected for a good research report.

The curriculum documents and van Hiele's readings, along with the Grade 11 textbooks were analysed to obtain information on circle geometry used to develop and design teaching activities. Teaching activities were developed and designed based on the curriculum and content classification from the CAPS, guided by phases of learning proposed by van Hiele's theory. Teaching activities were used to collect data during classroom observation, as mentioned in the preceding section on classroom observation.

To determine the curriculum statement and classification of the content on circle geometry for Grade 11, the researcher analysed the topic allocation for Term 3 from the CAPS document. This was used in conjunction with van Hiele's theory readings to design teaching activities to collect data during classroom observation and lesson presentation. Teaching activities were designed using curriculum statement and content classification obtained from the CAPS. This helped the researcher to align the instructions in the teaching activities with the curriculum. The van Hiele readings helped to check the compatibility of the teaching activities that were designed with the curriculum for Grade 11. Teaching activities needed to be clearly and closely matched with both the curriculum and the van Hiele theory, as a means to answer the research questions.

In order to design appropriate teaching activities, chapters on circle geometry were read from the following texts books: Chapter 8, Siyavula: Mathematics (Grade 11), Chapter 9, Platinum Mathematics Grade 11, Pearson (2019-2020). The structure of the text and the focus of the learning were also determined from the textbooks. These textbooks are aligned to Grade 11 curriculum as stipulated by CAPS and were commonly used at the school and easily accessible to the participants. The information from textbooks provided insight on the structure, focus and special learning to design the step-by-step instruction in teaching activities.

3.5.1.2 Teaching activities

Five teaching activities were designed and administered to the participants (**See appendices A1 – A6).** The nine teaching activities were divided into sub-topics, and each sub-topic constituted one investigation. Each investigation constituted proving a given theorem of geometry of circles except the first and last teaching activities. The first teaching *activity A* was about conceptual development that constituted revision of circle geometry concepts and vocabularies. The last application activity was about solving riders. Therefore, each teaching activity was linked to a school lesson. An approximate time for each lesson was allocated according to the suggested time in the CAPS document. Table 3.3 below shows teaching activities and the allocated time.

Table 3.3 Teaching activities and allocated time

Teaching Activity number	Investigation Tittle	Allocated time (minutes)
A	Revise grade 10 work • circle terminologies Axiom: tangent to a circle is perpendicular to the radius, drawn to the point of contact.	60
B	Investigation 1: The line drawn from the centre of a circle perpendicular to a chord bisects the chord	40
	Investigation 2 : The perpendicular bisector of a chord passes through the Centre of the circle	40
C	Investigation 3: The angle subtended by an arc at the Centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the Centre)	40
	Investigation 4: Angle subtended by a chord of the circle, on the same side of the chord, are equal ;	40
D	Investigation 5: The opposite angles of a cyclic quadrilateral are supplementary;	40
E	investigation 6: Two tangents drawn to a circle from the same point outside the circle are equal in length;	40
	Investigation 7: The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	40
F	Use the above theorems and their converses, where they exist, to solve riders.	40

Teaching activities were designed to follow the proposed five van Hiele phases of learning. The activities covered content that relates to intended learning outcomes in the CAPS document. The focus content strand of teaching activities from CAPS was Euclidean geometry, specifically on circle Geometry for Grade 11. The target outcomes addressed by teaching activities are, construct, determine properties of circles and angles, verify properties, make conjectures and prove riders. The content was organized in a way that it is aligned to van Hiele's phases of learning to explore teaching and learning practices that facilitate learners' developmental progression. Therefore, teaching activities were designed with van Hiele's teaching phases as a design framework.

The aims of the teaching activities were for learners to:

- Become familiar with the working domain through discussion and exploration;
- Identify the focus of the topic through a series of teacher-guided tasks. This gave learners the opportunities to exchange views through discussions;
- Become conscious of the new ideas and express these ideas in accepted mathematical language;
- Complete activities in which they are required to find their way in the network of relations; and
- Build an overview of the activities investigated.

The first teaching activity (A) was designed for the conceptual development of the learners. Learners must have a good understanding of circle geometry concepts, vocabularies and axioms studied in earlier grades to investigate and prove theorems. According to van de Walle (2004:28), conceptual understanding is knowledge about relationships of foundational ideas of a topic. As mentioned in van Hiele's theory and phases of learning, each level has its own linguistic symbols and systems of relationships connecting these symbols. Conceptual development teaching activity assisted learners to create new connections with existing ideas in circle geometry. Therefore, learners were able to know what to do and why, in the preceding teaching activities.

Conceptual development is important in developing learners' procedural fluency. Proving theorems requires rules and procedures to carry out the process of proving. Learners could get procedures of proving a theorem once they understand concepts and vocabularies of circle geometry.

The second teaching activity (B) was designed to explore chords and midpoint properties of the circle. An investigative approach was used so that learners could explore chord properties of the circle and form conjectures. The formation of conjectures leads to the derivation of the theorem and its converse. This teaching activity aimed at investigating:

1. A line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.
    - The converse of the theorem: The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord
2. The perpendicular bisector of a chord passes through the centre of the circle.

The third teaching activity (C) was designed to explore properties of angles in circles. It aimed at investigating the following theorem and the converse:

1. The angle at the centre is twice the angle at the circumference subtended by the same arc. In this investigation, learners could deduce the following properties:

- An angle in a semi-circle is a right angle
- The chord that subtends a right angle at the circumference is a diameter

2. The angles subtended by a chord at the circumference of a circle, on the same side of the chord, are equal.
    - The converse of the theorem: if a line segment joining two points subtends equal angles at two other points on the same side of it, the four points are concyclic.

The fourth teaching activity (D) was designed to explore cyclic quadrilaterals properties. It consisted of the following investigation for learners to develop the following conjecture and its converse:

1. The opposite angles of a cyclic quadrilateral are supplementary.
    - The converse of the theorem: if two opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

The fifth teaching activity (E) was designed to explore tangent properties. It consisted of two investigative tasks for the learner to develop the following conjectures:

1. Two tangents drawn to a circle from the same point outside the circle are equal in length
2. The angle between a tangent and a chord drawn to the point of contact is equal to the angles in the alternate segment. In this investigation, the learners develop a converse that:
    - If the line through the endpoint of a chord makes an angle with the chord, equal to an angle in the alternate segment, then the line is a tangent to the circle.

3.5.2 Classroom observation during lesson presentation

To assess the benefits and challenges of designed teaching activities during facilitation, classroom observation was conducted during lesson presentations. According to Creswell (2012:212), observations represent frequently used form of data collection in a school setting. The aim of using classroom observation was to determine whether the teacher delivers instructions on circle geometry as planned, and following van Hiele's phases of learning. The observation allowed the researcher to identify challenges teachers experienced while presenting lessons based on van Hiele's phases of learning. According to Creswell (2014:190), "qualitative observation indicates that the researcher takes field notes on the behaviour and activities of individuals at the research site and record observations".

Although many observational roles exist, such as participant-observer, non-participant observer, changing observational role (Creswell, 2012:214-215), the researcher approached classroom observation as a non-participant; therefore; the major role on was taking field notes while observing. Classroom observation was highly structured on an observation protocol developed in advance to determine which classroom activities should be focussed on, and what information should be collected and recorded on the observation protocol (see Appendix B). (Cohen et al, 2018).

The advantage of being a non-participant observer during classroom observation, while using a pre-developed observation protocol was the comfortability of participants. The researcher was able to observe that learners were comfortable to respond to the teachers' questions freely and the teacher was comfortable to conduct the lessons as were planned (Creswell, 2012). Another advantage was that, the researcher was able to identify some of the challenges the teacher and learners faced and benefits of using van Hiele's phases of learning, while being unobtrusive.

The observation protocol was informed by the description of the five van Hiele phases of learning. According to Serow (2008), the five-phase teaching approach provides a structure on which to base a program of instruction. As can be seen in the observation protocol, the observation was tied to an observable description of each of the phases of learning. The observable description of phases is displayed in Table 3. 4 below.

Table 3.4 Five observable descriptions of the van Hiele phases of instruction

Phase	Observable descriptions of the phase
Information	Discussions take place between teacher and learners that stresses the content to be used
Directed orientation	The teacher guides learners to uncover the connection and to identify the focus of the subject matter through a series of teacher-guided tasks
Explicitation	Learners express new ideas in accepted mathematical language. Teacher's main role to develop technical language with understanding through the exchange of ideas.
Free orientation	Learners can complete activities that require a number of steps in which they are required to find their own way in the network of relations. The teacher selects appropriate geometric activities that require a certain level of thinking for learners to solve them successfully.
Integration	Learners can summarize the new understanding of the concept involved and incorporate the language of the new level by making conjectures. The teacher assists with the correct and appropriate conjectures.

Adapted from Serow (2008)

The field notes recorded mainly focused on the above observable descriptions of five phases and how the teacher sequences the lesson in line with the nature of the classroom interactions, for example, seen at the level of interaction between the teacher and learners, and learner-to-learner conversation. In an attempt to be objective during lesson observation, field notes recorded were both descriptive and reflective in such a way that researcher recorded personal thoughts that relate to the activities and events that emerged during lessons observations.

3.5.3 In-depth interviews

Much of what we cannot observe for ourselves has been or is being observed by others (Stake, 1995:64). Thus, an interview is the main road to discover what others observe. As mentioned by Creswell (2012:217), the interview is one of the most popular methods to collect data in qualitative studies. The researcher believes that the interview method gave room for an in-depth probing that provided better knowledge of the teacher's idea and thinking about processes of instruction design, facilitation and learners' understanding. Therefore, interviewing teacher was critical because the teacher acted as a link between instruction and learners' understanding.

The qualitative interview is described as the process that occurs when the researcher asks one or more participants general, open-ended questions and records their answers (Creswell, 2012). In this study, one teacher was interviewed. The approach to in-depth interviews was one-on-one, where the researcher asked the teacher open-ended questions and recorded his responses in the interview protocol. Open-ended questions allowed the teacher to offer additional information about what he observed while teaching. Jacobs and Furgerson (2012) state one of the goals of qualitative research is to uncover as much information about the participants and their situations as possible. This was done through interviews through open-ended questions.

The in-depth interviews aimed to solicit responses from the teacher on the benefits and challenges of developing step-by-step instructions and implementing them on learners. One of the advantages of in-depth interviewing was that it allowed the teacher to explicitly voice his experiences, challenges, and benefits on delivering circle geometry instructions that were developed according to five phases of van Hiele's theory, unconstrained by any perspectives of the researcher.

The interviews were administered at the end of all teaching activities for 22 minutes during regular school hours. The answers were recorded in the interview protocol (See Appendix C) and audiotaped. Audiotaping of the interview provided a detailed record of the interview and acted as a backup for the interview protocol. It was only the teacher and the interviewer present at the time of interviewing in the room. Therefore, the teacher was free to express himself while answering questions without any interruption.

The one limitation to qualitative interviews, as stated by Creswell (2012), is that it is time-consuming and costly. This was not a deterrent to the researcher since there was enough time to gather data and a suitable venue was found at which to conduct the interviews.

3. 6 Data analysis

Baxter and Jack (2008:554) describe data collection and analysis as a concurrent process in a qualitative study. According to Creswell (2012:236), to have answers to research questions, data analysis requires an understanding of how to make sense of the text and images collected. There are six steps this study commonly used in analyzing data collected, as stated by Creswell (2012: 237):

- Represent preparing and organizing the data for analysis;
- Engaging in an initial exploration of the data through the process of coding it;
- Using the codes to develop a more general picture of the data—descriptions and themes;

- Representing the findings through narratives and visuals;
- Making an interpretation of the meaning of the results by reflecting personally on the impact of the findings and on the literature that might inform the findings; and finally,
- Conducting strategies to validate the accuracy of the findings

This study is a qualitative approach, making use of some of the above given six steps. The steps were not taken in a sequence, instead preceded through them depending on the nature of the data collected. These steps were not all suited to this research study, so some steps were not used to collect data. The problem of the study and the nature of data collected from tools informed the steps that used.

3.6.1 Approach to data analysis

The process followed by this study on data analysis was the "bottom-up" approach. The "bottom-up" approach gives the first major steps in the process of data analysis (Creswell, 2012:237). These steps are explicit of the six steps already mentioned. This approach is inductive in form, going from particular to general. According to Bhattacherjee (2012:113), the vast set of qualitative data acquired through observation, in-depth interviews, or secondary documents are analysed through the inductive technique. Inductive techniques use data to derive the structure of the analysis (Kemparaj & Chana, 2013).To visualise the first major steps in this process, an illustration of the bottom-up approach is given in the following Figure 3.1.

The Qualitative process of Data Analysis

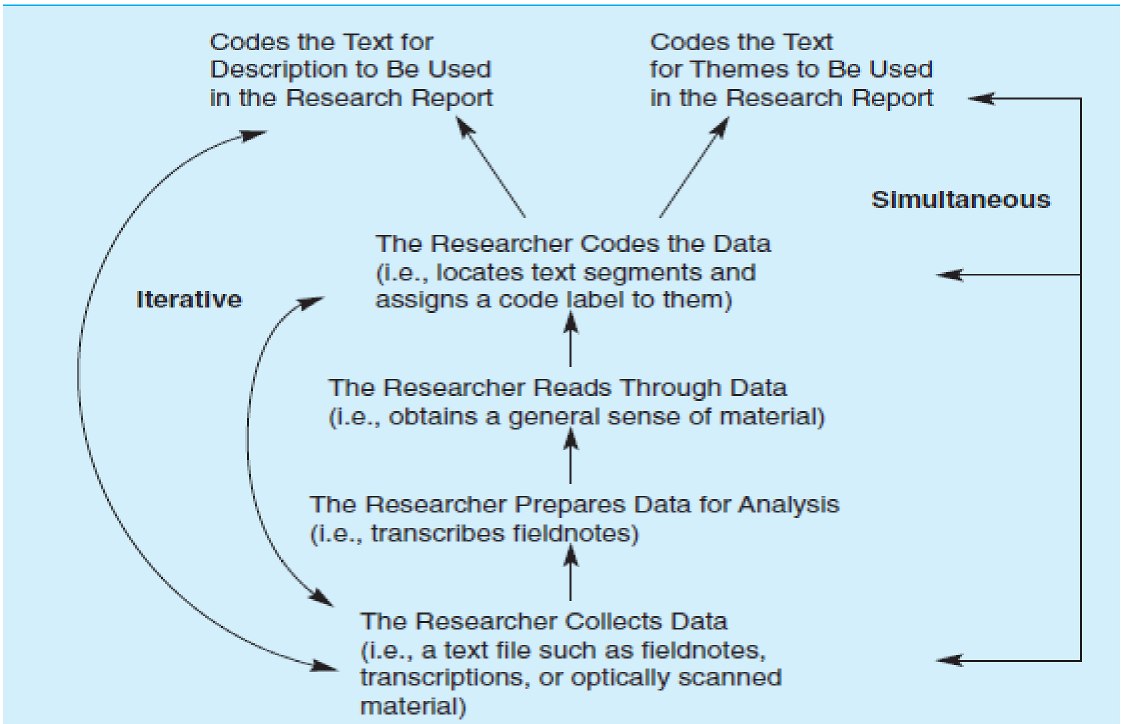


Figure 3.1 The qualitative process of Data Analysis

(Adapted from Creswell, 2012:237)

The concepts in the 'bottom-up' approach informed processes followed to analyse each set of data collected, and was a back and forth between the processes making sense of the data around the steps. The next section provides insight into the steps and processes on how each sent of data was analysed and how they link well in the steps of the "bottom-up" approach.

1. **Preparing and organising the data**

Raw data collected from the field were organised and prepared. This involved typing field notes, transcribing audio recordings and classifying data into different types, depending on the source of data collection. The data were classified into the following types:

- Curriculum, content, and instruction focus;
- All observation protocol; and
- The interviews protocol for teacher.

2. **Engaging in data exploration**

The researcher read and re-read through the data to obtain a general sense of the data, analyse and re-organise to link the data to specific concepts related to the phenomenon of this study. For example, in the CAPS document, key concepts were curriculum content focus for Grade 11 circle geometry, the link between the geometry of lower grades, and instructional design for circle geometry. For classroom observation, the sequence of lessons according to designed teaching activities, development of learners' understanding of concepts, curriculum coverage, and classroom participation, and learners' ability to do deductive reasoning was foregrounded; For interviews, teachers' and learners' experiences during the lesson and the way the lesson impacted their learning were shared. These concepts formed major themes of each set of data.

3. **Representing the findings through narratives**

Qualitative researchers often display their findings visually by using figures or pictures that augment the discussion (Creswell, 2012:253). The major themes were displayed in tables and figures, and constructed a narrative that explained the findings, as presented in chapter 4. The narrative was a detailed discussion that chronologically noted the events across different themes. The findings were interpreted to answer the research questions the study raises.

4. **Reporting the findings**

The reporting of the findings was through the use of a narrative discussion, one of the primary forms of representing and reporting findings in qualitative research (Creswell, 2012:254). The researcher gave a detailed discussion that chronologically notes events across three different methods used to collect data. The report was categorised into three sections, each section provided detailed findings from three data collection methods. The findings were interpreted to answer research questions.

3.7 Validity of the study

There are various tactics for drawing and verifying conclusions to know whether emerging findings are "good". According to Miles et al., (2014:271), the term *good* has many possible definitions: true, reliable, valid, dependable, reasonable, credible, trustworthy, etc. Creswell (2012) adds that there are varied terms that qualitative researchers use to describe the accuracy or credibility of the findings. For this study, the term "trustworthiness" is used to judge the quality of conclusions from the findings.

There are four main overlapping alternatives for assessing the trustworthiness in qualitative research. These are conformability, reliability, credibility, validity and applicability (Miles et.al, 2014:271). Bhattacherjee (2012:110) referred to them as an alternative set of criteria that can be used to judge the rigor of interpretive research.

This study paid attention to the three primary forms of strategy typically used by qualitative researchers: triangulation, member-checking and auditing to ensure trustworthiness in findings (Creswell 2012). A detailed discussion of each of the three strategies and the implementation of the four criteria in each strategy during this study follows.

3.7.1 Methods of ensuring trustworthy

3.7.1.1 Triangulation

As defined by Creswell (2012:259), triangulation is the process of corroborating pieces of evidence from different individuals, types of data, or methods of data collection and in descriptions and themes in qualitative research. Shenton (2004:65) describes triangulation as use of different methods, especially observation, focus groups, or individual interviews, which form major data collection strategies for qualitative research. This study's findings were based on all three sources of data collection methods that complemented each other: observation, in-depth interviews and document analysis.

The researcher examined all data sources and found evidence to support a theme. The use of different methods extended the engagement of the researcher in the field, thus improving the credibility of the findings. According to Bhattacherjee (2012:119), credibility is improved by providing evidence of the researcher's extended engagement in the field. Shenton (2004:66) concurred with Bhattacherjee that results that emerge from different sources and sites may have greater credibility in the eyes of the reader. Furthermore, use of different data sources enhances the accuracy of a study (Creswell, 2012:259).

For the findings to be reliable, the researcher observed similar phenomena at different times. Five teaching activities were observed during the lesson presentation. According to Bhattacherjee (2012:119), interpretive research can be viewed as dependable if the same researcher observing the same or similar phenomenon at different times arrives at similar conclusions. The conclusion the researcher made from observations supported the theme discussed in Chapter 4. This is what Stake (2015: 112) termed "data source triangulation", meaning what the researcher observed and reported carries the same meaning when found under different circumstances. Lessons were observed on different days (circumstances) and after analysing them, results were identical. The underlying issue of dependability of findings, according to Miles et. al (2014) is whether the process of the study is consistent, reasonably stable over time and across research and methods with the research questions. The researcher ensured dependability based on data collected across the full range of appropriate settings, the school, times and participants (teachers and students) over extended engagement in the field, multiple observers' account (five teaching activities were observed).

3.7.1.2 Member checking

Creswell (2012:259) defines member-checking as a scientific process in which the researcher asks someone or more participants in the study to check the accuracy of the account. On the other hand, Stake (1995:115) describes the "process of member checking" as requesting the "actor" to examine rough drafts of writing where actions or words of the actor are featured. The researcher engaged with the supervisor of the study at every stage of the study. At the same time, participants were engaged when no further data was to be collected to check whether the descriptions were complete and realistic, themes were accurate to include, and if the interpretations were fair and representative.

It is notable in Section 3.7 that the researcher asked the teacher to check for accuracy of the transcribed notes. Similarly, in Section 3.6.3, the supervisor validated the teaching activities before being administered in the classroom. According to Shenton (2004:68), member checks are an important provision that can be made to bolster a study's credibility. The checks made were relating to accuracy of the data, verification of tools and data, and credibility of the findings to ascertain whether their participation match what they actually intended in this study. Some of their findings and feedback were worthy of inclusion in the results. Stake (2015:115) urges that member checking validates the findings of the study.

Conformability refers to the extent to which the findings reported in interpretive research can be independently confirmed by others typically, participants (Bhattacherjee, 2012:119). The researcher used participants to check that the findings are the results of the experiences and ideas of the participants, rather than characteristics and preferences of the researcher

3.7.1.3 External audit

Creswell (2012:260) explains the process of conducting an external audit as asking a person outside the project to conduct a thorough review of the study and report back in writing, the strengths and weaknesses of the project. The researcher consulted an external auditor who was a Postdoctoral Fellow student at the University of Stellenbosch, knowledgeable in qualitative research and the nominated supervisor to conduct an audit during the research process and at the conclusion stage. Shenton (2004:68) highlights that credibility of the researcher and investigator depend on background, qualification and experience. Feedback from the supervisor and the postdoctoral fellow student added credibility to this study.

Table 3.5 below summarises criteria employed against the strategy the researcher used to verify findings.

Table 3.5 Summary of methods for ensuring trustworthy

Strategy	Criteria employed
Triangulation	Credibility; Validity and Reliability
Member checking	Credibility; Confirmability and Validity
External audit	Credibility; Confirmability and Validity

3.8 The researcher's position

The researcher was the key instrument in the study. The role of the researcher during data collection is to work with the teacher to develop lessons based on van Hiele's theory and observe and monitor whether presentation has been done as planned, interview the teacher and analyse the documents.

3.9 Ethical considerations

Ethics is defined as conformance to standards of conduct of a given profession or group (Bhaltacherjee, 2012:137). According to Creswell (2012:620), ethical issues in qualitative research include issues such as informing participants of the purpose of the study, refraining from deceptive practices, sharing information with participants, being respectful of the research site, reciprocity, using ethical interview practices, maintaining confidentiality and collaborating with participants. These are no different from what Farrow (2016:94) describes as professional ethics in education. The researcher addressed these ethical issues in two phases: seeking institutional approval and site level consideration. The two phases are discussed in more detail below.

3.9.1 Phase 1: Institutional approval

Phase 1 involved seeking institutional approval. Permission was sought from the Western Cape Education Department (WCED) for approval to conduct research in a public school in the Western Cape and from the Cape Peninsula University of Technology, Faculty of Education Ethics Committee. A detailed description of the study procedures was submitted to both institutions through an application to delineate the approach used in the study. The researcher was provided with an ethical clearance certificate (Appendix D), permitting him to conduct this study. At the same time, the WCED approved the researcher's application and provided an ethical clearance letter (Appendix E) permitting him to conduct research in the respective school.

3.9.2 Phase 2: Site consideration

The second phase involved seeking permission at the site level from the principal of the school first and participating teacher. This was done through a written informed consent letter giving information about the purpose of the study, duration of the study, the role of the researcher and participants, maintaining confidentiality, and assuring participants of anonymity, voluntary participation, and collaboration with participants (Appendix I). The consent letter's content was fully explained to each participant, and they voluntarily agreed to participate in the study.

3.10 Contribution of the study

This study intends to promote teaching approaches designed and based on Van Hiele's theory in circle geometry in a South African context. The study has the potential to influence the development and instruction of circle geometry, and curriculum policies and development in a South African context.

3.11 Conclusion

This chapter described the research design, methodology and data analysis procedures. It further discusses methods used to ensure trustworthiness in this study and anticipated contribution of the study. The findings and results are discussed in the next chapter.

CHAPTER 4: RESULTS AND DISCUSSION OF THE FINDINGS

4.1 Introduction

This chapter presents and discusses findings of the study on the application of van Hiele phases of instructional design to facilitate learning of circle geometry in Grade 11. A qualitative design was adopted, which provided instigation for tools used to collect data with the aim of exploring how van Hiele's theory of instructional design facilitates the learning of circle geometry. The discussion of findings in this chapter is poised to answer the following research questions:

The main research question of this study was formulated as:

- How does the Van Hiele's theory of instructional design facilitate the learning of circle geometry to Grade 11 learners?

Further, the following sub-research questions were asked:

- What are the possible challenges of using Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?
- What are the benefits of using Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?

The findings are organised and presented in three parts based on three sources of data that foreground the study. Part A focuses on findings from the analysis of CAPS documents and van Hiele's phases of instruction reviews. These documents shed some light in the development of geometry instructions, and answering the main research question. Part B relates to findings from classroom observation during the 3 lesson presentations. Part C focuses on the findings from in-depth interviews that were conducted to solicit responses from the teacher.

4.2 Analysis of CAPS documents and van Hielele's phases of learning

4.2.1 Part A: Findings emerging from analysis of caps document and van Hiele phases of learning

The framework I used for document analysis comprised analysing two aspects: First, curriculum content focus for Grade 11 circle geometry and the link between geometry of lower Grades to Grade 11; Second, instructional design focus for circle geometry. To analyse curriculum content focus for Grade 11 circle geometry and the link between geometry of lower Grades to Grade 11, Further Education and Training (FET) and Senior Phase (SP) CAPS documents were used. Further, to analyse instructional design focus for circle geometry, van Hiele's theory and van Hiele's phases of learning were used.

Curriculum and content focus

Curriculum is the planned experience offered to learners under the guidance of the school. One of the elements of the curriculum is content (Ochoma, 2020: 158), the subject matter of instruction (Ochoma, 2020: 159). In the South African context, the focus on the curriculum and content area to be learned in schools is provided for in CAPS. To assess the curriculum and content focus for Grade 11 circle geometry, I examined CAPS FET and SP Mathematics documents. The analysis of CAPS aimed to find the guideline into the curriculum and content of circle geometry to be taught in Grade 11. The findings from this analysis were used to design teaching activities and match it with the van Hiele phases of learning.

The first part of the document analysis investigated the CAPS document for Senior Phase (Grade 7, 8 and 9). The CAPS SP Mathematics policy was analysed to find the correlation between G11 circle geometry and geometry of lower grades, Grades 7, 8 and 9. The findings showed that Geometry done in senior phases focuses mainly on the study of space and 2D shapes, as summarised in Figure 4.1 below.

SPECIFICATION OF CONTENT (PHASE OVERVIEW) SPACE AND SHAPE (GEOMETRY)			
• Progression in geometry in the Senior Phase is achieved primarily by: - investigating new properties of shapes and objects - developing from informal descriptions of geometric figures to more formal definitions and classification of shapes and objects - solving more complex geometric problems using known properties of geometric figures - developing from inductive reasoning to deductive reasoning. • The geometry topics are much more inter-related than in the Intermediate Phase, especially those relating to constructions and geometry of 2D shapes and straight lines, hence care has to be taken regarding sequencing of topics through the terms. • In the Senior Phase, transformation geometry develops from general descriptions of movement in space to more specific descriptions of movement in co-ordinate planes. This lays the foundation for analytic geometry in the FET phase. • Solving problems in geometry to find unknown angles or lengths provides a useful context to practise solving equations.			
TOPICS	**GRADE 7**	**GRADE 8**	**GRADE 9**
3.1 **Geometry of 2D shapes**	**Classifying 2D shapes** • Describe, sort, name and compare triangles according to their sides and angles, focusing on: - equilateral triangles - isosceles triangles - right-angled triangles • Describe, sort, name and compare quadrilaterals in terms of: - length of sides - parallel and perpendicular sides - size of angles (right-angles or not) • Describe and name parts of a circle	**Classifying 2D shapes** • Identify and write clear definitions of triangles in terms of their sides and angles, distinguishing between: - equilateral triangles - isosceles triangles - right-angled triangles • Identify and write clear definitions of quadrilaterals in terms of their sides and angles, distinguishing between: - parallelogram - rectangle - square - rhombus - trapezium - kite	**Classifying 2D shapes** • Revise properties and definitions of triangles in terms of their sides and angles, distinguishing between: - equilateral triangles - isosceles triangles - right-angled triangles • Revise and write clear definitions of quadrilaterals in terms of their sides, angles and diagonals, distinguishing between: - parallelogram - rectangle - square - rhombus - trapezium - kite

Figure 4.1 Specification of content for space and shape

(DBE, 2011:27)

In the senior phase, the geometry curriculum provides learners with geometric skills to construct a wide range of geometric figures, descriptions and classifications of categories of geometric figures and solids in a clear and more precise way and solving a variety of geometric problems drawing on known properties of geometric figures and solids. These skills create foundational competencies for FET phase geometry.

The findings further reveal the following focus in each grade:

In Grade 7, they focus on the following:

- Measuring angles;
- Constructions of geometric figures using compass, ruler and protractor;
- Classifying 2D shapes, and 3D objects;

- Similar and congruent 2D shapes;
- Geometry of straight lines; and
- Transformation of geometric figures and shapes.

The most remarkable results to emerge from analysing the Grade 7 curriculum focus is that in classifying 2D shapes, learners are to describe and name parts of a circle such as radius, circumference, diameter, chord, segments and sectors. Interestingly, this content focus is conceived as foundational to Grade 11 Euclidean Geometry

In Grade 8, the curriculum focuses on:

- Constructions of geometric figures, mostly triangles and quadrilaterals;
- Investigating properties of geometric figures, focusing on angles in triangles and quadrilaterals;
- Classifying 2D shapes, focusing on classifying various types of triangles and quadrilaterals;
- Similar and congruent 2-D shapes;
- Classifying 3D objects, focusing on building models;
- Geometry of straight lines, focusing on angle relationships formed by pairs of perpendicular, parallel lines and transversal; and
- Transformation geometry, focusing on translation, reflection, enlargements and reductions of geometric figures.

Even though the analysis did not show any direct mention of circle concept as seen in Grade 7 curriculum, the geometry studied in Grade 8 introduces core concepts of similarity and congruency of 2-D shapes, which has practical implications when deriving proofs in Euclidean geometry in further grades. The link between Grade 8 geometry and circle geometry for Grade 11 is that, to prove any theorem of the geometry of a circle, we use properties of figures, specifically triangles and geometry of straight lines focusing on angle relationships.

In Grade 9, the curriculum focus on geometry is:

- Construction of geometric figures;
- Investigate properties of geometric figures;
- Classifying 2D shapes focusing on properties and definitions of triangles;
- Similar and congruent triangles;
- Classifying 3D objects, focusing on building models;
- Angle relationships focusing on descriptions of the relationship between angles formed by perpendicular lines, intersecting lines and parallel lines cut by a transversal; and
- Transformation geometry, focusing on translation, reflection, enlargements and reductions of geometric figures.

There was no significant difference between Grade 8 and Grade 9 geometry in terms of curriculum content. Grade 9 geometry mainly focuses on revising and expanding Grade 8 geometry. Overall, SP geometry content focuses on properties, relationships, orientations, positions and transformations of shapes.

The second part of the document analysis investigated CAPS for FET (G10, 11 and G12). CAPS (FET) revealed that the geometry curriculum focuses on Euclidean Geometry and measurement as summarised in Figure 4.2 below.

7. EUCLIDEAN GEOMETRY AND MEASUREMENT					
	(a) Revise basic results established in earlier grades. (b) Investigate line segments joining the mid-points of two sides of a triangle. (c) Properties of special quadrilaterals.		(a) Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles. (b) Solve circle geometry problems, providing reasons for statements when required. (c) Prove riders.		(a) Revise earlier (Grade 9) work on the necessary and sufficient conditions for polygons to be similar. (b) Prove (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem); • that equiangular triangles are similar; • that triangles with sides in proportion are similar; • the Pythagorean Theorem by similar triangles; and • riders.
	Solve problems involving volume and surface area of solids studied in earlier grades as well as spheres, pyramids and cones and combinations of those objects.		Revise Grade 10 work.		

Figure 4.2: Specification of content FET Euclidean Geometry and Measurement (DBE, 2011:14)

The findings revealed the following curriculum focus in each grade:

The curriculum focus for Grade 10 is:

- Revise basic results established in earlier grades regarding straight lines, angles and triangles, especially the similarity and congruence of triangles;
- Investigate line segments joining the mid-point of two sides of a triangle; and
- Properties of special quadrilateral focusing on investigating, making and proving conjectures about properties of the sides, angles and diagonal areas.

In G10, there is no section in the curriculum that speaks to circle geometry. The curriculum emphasises and further expands the senior phase geometry of properties of quadrilateral and straight lines.

In Grade 11, being the main focus of the study, the analysis of CAPS FET revealed that the Euclidean geometry focuses on theorems of the geometry of circle. The curriculum for Grade 11 circle geometry requires learners to investigate and prove the following:

- The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
- The perpendicular bisector of a chord passes through the centre of the circle;
- The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same ac at the circle (on the same side of chord as the centre);
- Angle subtended by a chord of the circle, on the same side of the chord are equal;
- The opposite angles of a cyclic quadrilateral are supplementary;
- Two tangents drawn to a circle from the same point outside the circle are equal in length; and
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternative segment.

The curriculum further requires learners to use the above theorems and their converses, where they exist, to solve riders. However, proofs of these theorems can be asked in examinations but not their converses (SA. DBE, 2011:34). A converse is the opposite of the theorem, therefore in a converse theorem, the logic is reversed. For example, for a theorem that states: *The line drawn from the centre of a circle perpendicular to a chord bisects the chord,* its converse is narrated as, *the line drawn from the centre of a circle to the midpoint of a chord will be perpendicular to the chord*. Although learners apply converse theorems in application questions, they are not required to prove any of the converse theorems. Therefore, teachers need to explain the difference between a theorem and its converse. They should also explain the conditions for which theorems and their converses are applicable. Teachers' focus on the theorems of geometry of circles during instruction should be for learners to be able to investigate, make conjectures and apply them to solve riders.

It seems like in earlier grades, specifically in the senior phase, even though geometry is not called Euclidean geometry, it is clear that some of the concepts covered in these grades are visible in Grade 11 Euclidean geometry. This is because they feature more in circle geometry. As mentioned earlier, investigating and proving circle geometry theorems requires mastery of concepts about properties of triangles and angles relationships. Revising these concepts as stated in curriculum statement creates a baseline to ascertain or understand prior knowledge on learners. As such, teachers need to refrain from using the curriculum as a guideline for preparing lessons only but as an indicator of what is required for learners to focus on during classroom Mathematics-based activities.

In my view, for learners to make logical links between concepts of lower grades to higher grades, specifically circle geometry concepts, teachers need to revise content of G7 on circle geometry rather than only revising G10 work of properties of triangles and angles.

The most intriguing correlation is with the investigations of properties of polygons. In lower grades, learners investigate properties of geometric figures through constructions and in G11, learners investigate theorems of the circle of geometry. Even though the focus in lower grades is on triangles and quadrilaterals, learners get acquitted with concepts of investigation required to investigate G11 circle geometry. In my view, teachers should apply the same skills of using construction to investigate and prove theorems of the circle of geometry.

In Grade 12, the content focuses on the geometry of triangles, specifically proving the midpoint theorem and Pythagorean Theorem. The Euclidean geometry topics and content of Grade 12 are not inter-related with Grade 11 geometry content. The skills and concepts learners acquire in Grade 11 geometry feature more in application to the Grade 12 geometry. The application of the Grade 11 concept also features more in analytical geometry and trigonometry.

The CAPS document policy includes a clarification section with comments clarifying the curriculum statements and examples specific to the four cognitive demand levels. However, no significant association was identified between curriculum statements and the method, and organisation of instruction of theorems, specifically in circle geometry. This tempts teachers not to organise their instructions on geometry of the circle according to van Hiele phases of learning. The method and organisation of instruction in geometry are important areas of pedagogical concern (Crowley, 1987: 5; Sorow, 2008: 445). Therefore, the curriculum content focus became the lens through which to measure and understand how the instruction focus should be viewed on circle geometry.

In the next section, I analyse the van Hiele phases of learning to find the recommended instruction design that focuses on circle geometry.

Instruction design focus

Instruction design is defined as a systematic procedure in which educational and training programmes are developed and composed, aiming at a substantial improvement of learning (Seel.et al.; 2017:1). In other words, it is a process of collecting learning experiences and materials in a way that results in the acquisition and application of knowledge and skills. The focus on instructional design was to find recommendable processes of designing teaching activities on geometry of circles to facilitate teaching and learning of the topic in Grade 11 level. In order to assess appropriate instruction design on geometry of the circles, I analysed van Hiele phases of learning.

The findings from analysing the van Hiele phases of learning revealed five sequential phases of learning, as proposed by the van Heiles to address the method and organisation of instruction on circle geometry. These phases, as outlined in section 2.3.3.2 include; information, directed orientation, explication, free orientation and integration. A number of studies recommend that teachers design their instructions on geometry by employing the van Hiele phases of learning in their classroom based instructional practices (Howse & Hose, 2015; Dongwi, 2014; Alex & Mammen, 2016). The instructions developed and organised according to the sequence of van Hiele phases of learning facilitate learning and acquisition of geometry concepts sequentially. The table below shows the findings on sequence of the five phases, their descriptions and some illustration from the CAPS Mathematics curriculum content.

Table 4.1 Description of the van Hiele phases of learning with some illustration

Phases	Description	Illustration
Information	• Discussion takes place between teachers and learners to familiarise with the content to be learned by engaging in content-based discussions • Teacher identifies learners' prior knowledge for them to be oriented into the topic	• Review the concepts from earlier Grades • Activities that classify the circle geometry concepts established in earlier grades.
Directed orientation	• Teacher guides learners to explore theorems in carefully structured tasks. • Learners explore the topic of study through materials that the teacher has carefully sequenced. • The material should be short tasks designed to elicit specific responses.	• For example, teacher might ask learners to construct circles with different radii.
Explicitation	• Learner describes what they have learned through verbalising their understanding of the concepts of circle geometry • Teacher takes care to develop technical language with understanding through the exchange of ideas	• Learners express and exchange their emerging views about what they have investigated in their own words
Free Orientation	Learners explore and apply the relationships between concepts to solve open ended problems.	• Learners complete complex tasks that requires many steps and can be solved in several ways. • Give converses of the theorem where they exist.
Integration	Learners summarise what they have learned Learner integrate what they have learned to develop a new network of objects and relation.	• Use of the theorem and their converses to solve riders

(Crawley 1987; Mason, 1998; Dongwi, 2014; Serow, 2008)

The van Hiele phases of learning and curriculum focus

At *information phase*, the teacher revises earlier grades' geometry content. The teacher engages in a conversation and provides activities that require learners to make observations. He or she raises questions for learners to become familiar with the topic through exploration and open discussion. The purpose of the information phase is for teachers to understand learners' prior knowledge on Euclidean geometry and circle geometry to help them plan their instruction on the topic. In relation to CAPS/FET Mathematics, revision of Grade 10 Euclidean geometry reflects information phase. It engages learners in investigative activities and conversation to discover vocabularies that are useful in making conjectures and proving those conjectures about the properties of figures. I designed activity one with the aim probing learners' prior knowledge of circle geometry. The findings from teachers' instruction about activity one is discussed in Part B (section 4.3.1) which also relates to the findings from classroom observation during lesson presentation.

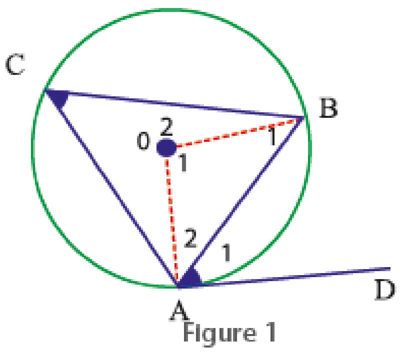
Phase two is a guided investigative phase for learners to engage with concepts in order to develop an understanding of connections between and across concepts. Learners are given opportunities to exchange views about the introduction of formal circle geometry. At this phase, learners are guided with the accepted mathematical language in circle geometry discourse. The phase aims to enable learners explore specific concepts of the topic.

In Phase 3, learners verbally express and exchange their views about connections they have observed during investigation of theorems of geometry of circles. For example, a learner expresses ideas that the line drawn from the centre of a circle perpendicular to a chord bisects the chord. The purpose of this phase is for learner to verbalise what they have investigated using mathematically accepted reasoning in deductions involving circle theorems.

Phase 4 is the application phase. Learners complete complex activities in circle geometry that require a number of steps and to be solved in many ways. Through problem solving, their language develops further through verbalising their justification on every step in the problem solved. For example, investigation 7 (see section 4.3) is a complex activity, learners investigate the angle between the tangent to a circle and the chord drawn from the point of contact as equal to the angle in the alternate segment; this requires them to find their own way(s) in the network of relations. In this phase, learners gain experience in finding their own way of solving the task by orienting themselves in the field of investigation. Figure 4.2 below is an example of a task that requires a number of steps and finding a way in the network of relations. The learners are required to apply and make connection between the concepts such as:

- The shortest distance between a point and a line is the perpendicular distance;
- A radius is always perpendicular to a tangent at the point of contact;
- A chord divides a circle into two segments; and
- Complementary angles add up to 90°.

Given: A, B and C are points on the circle with centre O. DA is a tangent to the circle at A.
Required to prove: In Figure 1, $B\hat{A}D = \hat{C}$ and in Figure 2, reflex $B\hat{A}D = \hat{C}$
Construction: OA and OB



Proof:

Figure 1: RTP: $B\hat{A}D = \hat{C}$

Let $\hat{C} = x$

$\hat{O}_1 = 2x$	$\angle$ at centre = $2\times$ $\angle$s at circumference
$\hat{A}_2 = \hat{B}_1$	$\angle$s opp = sides; OA = OB radii
$\hat{A}_2 = 90° - x$	sum of $\angle$s in $\triangle$AOB
$\therefore B\hat{A}D = x = \hat{C}$	tan $\perp$ radius

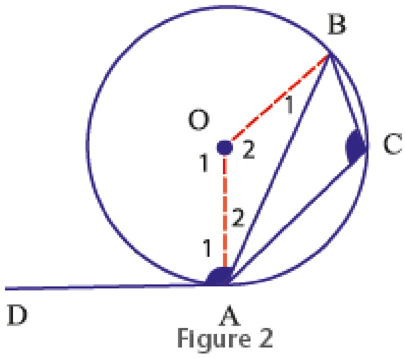


Figure 2: RTP: $B\hat{A}D = \hat{C}$

Let $\hat{C} = x$

$\hat{O}_1 = 2x$	$\angle$ at centre = $2\times$ $\angle$s at circumference
$\hat{O}_2 = 360° - 2x$	$\angle$s around a point
$\hat{A}_2 = \hat{B}_1$	$\angle$s opp = sides; OA = OB radii
$\hat{A}_2 = x - 90°$	sum of $\angle$s in $\triangle$AOB
But $\hat{A}_1 = 90°$	Radius $\perp$ tangent
$\therefore B\hat{A}D = x = \hat{C}$	

Figure 4.3 Investigation of Theorem 7

(Platinum Mathematics Grade 11: 1999: 202)

The purpose of this phase is for learners to solve riders in different ways and develop further their technical language skills in circle geometry. The teacher's role is to select appropriate circle geometry problems (riders) that require application and making connections between circle geometry concepts.

Phase five is an integration phase where learners synthesize and summarise what they have learned to build a coherent argument about the theorem under investigation. The main goal of this phase is for learners to formulate an overview of the new network of knowledge. While the purpose of the instruction is clear to learners, it is necessary for the teacher to assist learners to integrate what they have learnt during this phase, developing a new network of objects and relations.

In circle geometry, learners investigate and prove theorems of the geometry of circles, solve circle geometry problems and provide reasons for statements (DBE, 2011:14). Providing acceptable reasons is attained at phase 5 (Serow, 2008:446). Here a teacher aims at guiding learners to summarise the new understanding of the concepts involved and incorporate technical language to attain a new level of thinking. There are standardised acceptable reasons for theorem statements. Teachers should aim to guide learners to these acceptable reasons and geometry of the circle technical language. Table 4.2

indicates the most acceptable reasons for circle theorems statement using geometry of the circle technical language.

Table 4.2 Acceptable reasons in circle geometry

THEOREM STATEMENT	ACCEPTABLE REASONS
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 × ∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90^0	∠ s in semi-circle **OR** diameter subtends right angle **OR** ∠ in $\frac{1}{2}$ □□
If the angle subtended by a chord at the circumference of the circle is 90^0, then the chord is a diameter.	chord subtends 90^0**OR** converse ∠s in semi-circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp OR converse opp ∠s of cyclic quad

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ OR converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt OR Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem OR ∠ between line and chord

(DBE, 2021: 12)

At the end of phase five, learners attain a new level of geometric thought. As it can be seen, that a five-phase approach involves discussion, inclusion, exploration, application, technical language and problem-solving. Learners need to cycle through the five phases to prove and investigate each theorem of the geometry of circles, as stipulated in the South African curriculum for Grade 11.

4.2.2 Discussion of findings to answer the research question : How does the Van Hiele's theory of instructional design facilitate the learning of circle geometry to Grade 11 learners?

van Hiele's phases of learning provide geometric instructional design to facilitate learning of circle geometry

Instruction design is commonly defined as a systematic procedure in which educational and training program is developed and composed, aiming at a substantial improvement in learning (Seel.et al., 2017:1). Instructional design serves as a frame of reference and regulation of the development of courses and lessons. A well-developed model of instruction design aims at the improvement of learning and influencing learners' motivation and attitudes in such a way that they achieve a deeper understudying of the subject matter.

The starting point for instructional design consists of clarification of what learners should learn. Evidently, what learners should learn in Grade 11 circle geometry is fully outlined in CAPS/FET Mathematics policy document. In practice, what should be learned is framed through models of instruction. The models of instruction frames how the content should be delivered to learners in a manageable way to understand the subject matter to be learned. There is general agreement in literature that instruction designed according to van Hiele phases of learning promotes the acquisition of geometry subject matter (Crawley, 1987; Mason, (1998); Dongwi, 2014; Serow, 2008; Howse & Hose, 2015;

Alex & Mammen, 2016). Thus, van Hiele's theory of instructional design is one of the commonly used models of instruction design to serve as a reference and regulation of development of geometry lessons.

van Hiele's phases of learning provide strategies of instructional design

The effectiveness of instructional design heavily lies on strategies used to design it. Different strategies are applied when designing instructional design, such as organisational strategy, delivery strategy and execution strategies (Seel et al., 2017: 9). In accordance with the basic understanding, instructional design is a complete process that starts from analysing until the point of implementation and evaluation. Therefore, it requires applications of different strategies for instruction design to provide effective learning environment. van Hieles' phases of learning provide strategies of instructional design. Seel et al. (2017) define the three most applied strategies of instruction design as:

- **Organizational strategies** concerned with both the gross and detailed planning of settings of teaching and learning in order to determine how a course of lesson should be arranged and sequenced;
- **Delivery strategies** concerned with decisions on how information can be transmitted to the target group of learners; and
- **Execution strategies** concerned with decisions on methods to assist the learner to deal effectively with instructional materials.

The organisation of van Hiele's phases of learning does provide Mathematics teachers with appropriate experiences and opportunities to effectively discuss and deliver circle geometry content.

van Hiele's phases of learning provide Instructional events that facilitate the learning of circle geometry

The van Hiele's phase of learning provide effective strategies stimulates instructional events. Instructional events are classes of events that occur in a learning situation (Reigelugh, 1983). The van Hiele's phases of learning trigger instructional events through which learners plan, control and monitor their learning. The process of instructional events involves giving instructions that are task-specific and general tasks that provide learners with opportunities to executive them and the teacher to provide support and feedback to the learners. This is what Seel. et al. (2017) termed *cognitive strategy*.

As can be seen in Table 4.1, the van Hiele's phases of learning, as a mode of instructional design, begins with clear teacher direction that describes the strategy through which the teacher and learners engage in conversation and activities that demonstrate the strategy and moves to activities that require student initiatives in the form of problem solving. In the context of the results of this study, it can be seen in the observation protocols, (see Figure 4.4 , 4.5 and 4.6) the teacher planned, controlled and monitored teaching and learning.

The van Hiele's theory of instructional design has been found to be encompassing the nine Gagne's instructional events, as put forward by Reigeluth (1983) and Seel et al. (2017) to create a general framework for preparing and delivering instructional content. Gagne's list of nine instructional events are:

(1) Gaining attention of the students, (2) inform students of the objectives, (3) stimulate recall of prior learning, (4) present the content, (5) provide learning guidance ,(6) elicit performance by practices, (7) provide feedback, (8) assess performance and (9) enhance retention and transfer (Reigeluth, 1983 and Seel et al., 2017)

van Hiele's phases are hardly distinguishable from Gagne's nine events of instruction. The consistency in the instructional actions in the nine events and phases are illustrated in Table 4.3 below:

Table 4.3 Consistency of van Hiele's five phases and Gagne's nine events

van Hiele's Phases of learning	Gagne's instructional event that is consistent with the phase of learning
Phase 1 : Information	Event 1 : Gain attention Event 2: inform learner of the objectives Event 3: Stimulate prerequisite recall
Phase 2 : Directed orientation	Event 4: Present learning material Event 7: Provide feedback
Phase 3: Explication	Event 5 : Provide guidance for learning Event 6 : Elicit performance Event 7 : Provide feedback
Phase 4 : Free Orientation	Event 8 : Assess performance Event 7 : Provide feedback
Phase 5 : Integration	Event 9 : Enhance retention and transfer Event 7 : Provide feedback

(Seel et al. 2017).

The evidence from the events of instruction by van Hiele's phases of learning point to Gagne's nine events of instruction that provide conditions of learning for achieving learning objectives. For instance:

Phase 1: information phase is consistent with Gagne's instructional events 1, 2, and 3. Both Gagne's and van Hiele's emphasis gaining learners' attention as the initial task in any instruction. As explained in Van Hiele's phase 1 speaks to teachers engage with learners in a conversation to gain their attention. The purpose of phase 1 is for learners to become oriented about the topic of study. The instruction action for event 1 is to introduce a stimulus to elicit curiosity to gain the learner's attention so that other instructional events can occur properly. The instruction action for event 2 is to describe the expected performance by communicating the objective to the learners, and the instructional action for event 3 is the recall of concepts and rules before new learning can occur. The instructional actions of the three events are consistent with the purpose of van Hiele's phase 1.

Phase 2: Directed orientation is consistent with instructional events 4. The purpose of phase 4 is for the learners to explore the topic of study. This is done through presenting materials that the teacher has carefully sequenced. This is consistent with the instruction action of event 4, which is to present examples of the concepts and rules.

Phase 3: Explicitation, is consistent with instructional events 5, 6, and 7. The purpose of this phase is for learners to express and exchange their emerging views about what has been learned, and the teacher to guide the learners to accurate and appropriate views. Looking closely at the instructional actions of the three events that are consistent with phase 3, the action for event 5 is to provide guidance for learning so that learners understand critical attributes of the concepts, instructional action for event 6 is to let the learners apply the concepts and rules by asking the learners to perform an overt action, like answering questions verbally, and event 7, is to provide feedback about performance correctness. These three events match instruction actions for phase 3.

Phase 4: Free orientation, is consistent with instructional events 7 and 8. Phase 4 emphasizes the demonstration of the application of concepts by completing activities in which they are required to find their own way to resolve the tasks. The instruction action in event 8 is to determine if the learner obtained the objective and can consistently perform what was intended. One of the ways to examine whether the learner obtained the objective is through looking at the way the learner resolves the tasks given. This is what van Hiele emphasizes in phase 4.

Phase 5: Integration is characterised by the ability of learners to build an overview of what they have studied without presenting anything new. This is in support and consistent with event 9 of enhancing retention and transfer. The teacher aims to help learners in developing expertise and providing a variety of other applications. It can be seen from the figure above that instructional event 7 matches phases 2, 3, 4, and 5 because in all the phases, van Hiele emphasizes instruction actions where the teacher guides and develops learners' views. Thus instruction action of event 7 is homogeneous to the four phases.

Therefore, van Hiele's phases of learning help to design instructions by ensuring that the events in the instructions are planned as catalysts for learning.

van Hiele's phases of learning promotes forms of learning facilitate the learning of circle geometry

The van Hiele's phases of learning promotes the use of cognitive and attitude categories of learning through the creation of learning environment that provide learners with opportunities to learn. Practical learning is realised through specifying objectives in accordance with the basic form of learning, specifying the conditions of a particular learning results and specifying instructional methods necessary to improve the internal process of learning (Seel. et al., 2017: 48).

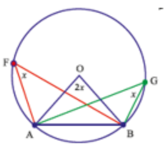
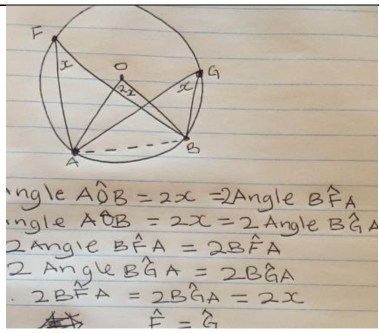
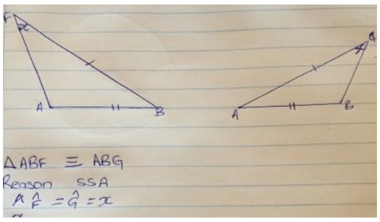
Gagne distinguished five basic forms of human learning as: Cognitive strategies; Verbal information; Intellectual skills; Motor skills and Attitudes. One of the most widely used forms of learning is cognitive strategy because of its emphasis on enhancing learner performance. Cognitive strategy is defined as an internal process through which individuals plan, control and monitor their learning (Seel. et al., 2017: 48). This process is task-specific, general and executive.

Reflecting on the results of this study, it was observed during teaching that learners were discussing, exploring and describe what they learned in their own words, and the teacher provided support and feedback.

Even though the other forms of learning, as described by Gagne, are not explicit in van Hiele's phases of learning as cognitive strategy, some instructional events are implicit in those other forms of learning. For example, See.et. al (2017) define the Attitude form of learning as an internal state, a predisposition, which affects an individual choice of action. The instructional events are to provide learners with approved models, which show positive behaviour, and reinforce this model, and give positive feedback to learners if they execute the desired behaviour. These events correspond with van Hiele's notion that learners should be provided with a wide variety of exploratory geometric experiences such as paper folding, working with grids, collection of shapes. In such events, learners encounter tasks with many steps that can be completed in several ways. Learners gain experience in finding their own way of resolving the tasks. This form of learning links to phase 4 (guided orientation) of van Hiele's phases of learning. In this phase, learners orient themselves in the field of investigation,

For example, during teaching activity (C), to prove that Angle AFB = Angle AGB, learner L007 and L008 approached this proof in two different ways as illustrated in the following Table 4.6:

Table 4.4 Learners' response on proofing theorem in investigation 3.4

Activity	Learner's response L007 written response	Researcher's insights & comments
Application: Prove that the angles subtended by a chord of the circle on the same side of the chord are equal Illustration:  Given A, F, G and B are points on the circle with centre O. Step: Join AO and OB, Required to prove: Angle AFB = Angle AGB.	 L008 written response  	Learner L007 resolved this task using the knowledge gained in 3.4, while learner L008 perceived the two triangles to be congruent. This implies that both learners had to find their own way to resolve the task.

(Source: Primary Data)

In the intellectual skill form of learning, capabilities of learners are observable in using concepts and rules to solve problems, responding to classes of stimuli as distinct from recalling specific examples (Reigeluth, 1999). The instructional events include giving simple and clear rules and concepts, presenting examples of the concepts or rules, asking learners to apply rules or concepts to new examples and confirming the correctness of rules or concept application. The van Hiele's phases of learning provide for these learning capabilities. In phase 2 (Directed orientation), learners explore the topic of study through instructions that the teacher has carefully sequenced. At this stage, learners are given opportunities to exchange views, through discussion. The teacher guides them to the correctness of using the concepts or rules in an accepted mathematical way. Teachers' guidance always happens in Phase 3 (Explicitaion). At phase 5 (integration), learners review and summarise what they have learned with the goal of forming an overview of the new network of objects and relations. The teacher assists learners in this synthesis. The instructional events in the five phases of learning speak to some of the instruction events found in the intellectual skill form of learning.

Verbal information form of learning is described as meaningfully organised sets of information considering the language skills (Reigeluth, 1999). The instructional events in this form of learning indicate the kind of verbal questions to be answered, presenting information in propositional form, providing verbal links to a larger meaningful context, asking for information in the learner's own words, and confirming the correctness of the statement. The van Hiele modal of instruction design stresses the role of language in the development of learners' geometric thinking. In the phases of learning, emphasis is on learners verbalising their understanding associated with words and symbols in their own vocabulary. The main role of the teacher in this phase of learning is to guide learners in a mathematically accepted language. For instance, in phase 1 (Information), learners and teachers engage in conversation and activities about the objects of study. In phase 2 (Directed orientation), students explore the topic of study through tasks teachers design to elicit specific responses that gradually introduce formal language. In phase 3 (Explicitation), students express and exchange their emerging views about the structure that have been observed in accepted technical language. In phase 4 (Free orientation), through problem solving, learners' technical language develops further as they begin to identify cues to assist them. In stage 5 (integration), learners summarise new understandings of concepts involved and incorporate technical language of the new level. The major role of the teacher is to guide learners to mathematically acceptable technical language.

In the case of the motor skill form of learning, Reigeluth (1999) describes it as executing body movements smoothly and in proper sequence. Instructional events for this form of learning are not compatible with the instructional events in van Hiele's phase of learning. Even though instructional events are not linked, to execute instructions given by the teachers requires the combination of learners' muscles and brain to perform a task. Therefore, this form of learning is more linked to procedures requiring practical execution of tasks. Clearly, van Hiele's theory of instructional design constitutes the five forms of human learning.

van Hiele's phases of learning provide Pathway to teaching and learning of geometry

The five van Hiele's phases of learning provide a generic teaching pathway for geometry. This pathway provides a structure on which to base circle geometry instruction. The implication of the five-phase teaching approach is to provide a structure for teachers upon which to base their instructions (Serow, 2008:446). The results of the study from classroom observation analysis of learners' tasks (see table 4.8) indicate that effective learning takes place when learners actively experience the objects of a study in appropriate contexts and when they are involved in discussions and reflections. When teachers effectively follow the phases of learning, effective learning takes place, learners actively participate in learning in appropriate contexts. For instance, in all teaching activities investigated (refer to section 4.3), the teacher provided learners with appropriate instructions and opportunities to investigate and discuss the topic of study.

In conclusion, the findings emerging from Part A reveal that Grade 11 Euclidean geometry focuses only on circle geometry. Learners are required to investigate and prove seven theorems of the geometry of circles. They are further required to use these theorems and their converses,-to solve riders. However, in the CAPS/FET Mathematics policy, there is no specific instruction or direction/pointers on how teachers should proceed with the teaching and learning of circle geometry. The strategy and organisation of instruction on geometry is revealed by the van Hiele's five sequential phases of learning. Geometry instructions developed and delivered according to the van Hilles, modal of instruction promotes understanding. Thus, the five van Hiele's phases of learning address the area of pedagogical concern in geometry.

4.3 Part B: Analysis of classroom observation

Classroom observation field notes were deciphered, dissected and coded meticulously. The information from classroom observation were read and re-read to discover patterns of meaning in the data. The following themes emerged while transcribing the field notes data:

- Learner-Learner interaction;
- Teacher-learner interaction;
- Guided learning; and
- Collaborative learning.

4.3.1 Analysis of classroom observation for teaching activity A

The first teaching activity constituted revising Grade 10 work and investigating circle concepts and terminology (see Appendix A1). The findings of teaching activity A observation are summarised in the following Table 4.5:

Table 4.5 Findings from observation of Teaching Activity A

van Hiele's teaching Phase	What was observed from Teaching Activity A
Information	The teacher illustrated and discussed the following concepts that linked to circle geometry; centre, radius, diameter, sector, chord, arc segment tangent and circumference; and he also explained and gave definitions for the following terminologies: subtend, cyclic quadrilaterals and alternate segments. The teacher discussed with learners how it's best to have construction and measuring instruments to be used in investigating the theorem. He requested learners to bring instruments to be used in the next lesson
Directed orientation	Teacher was leading the discussion without much involvement of learners
Explicitation	Learners were not paying attention during the lesson
Free orientation	Learners were just following what the teacher was writing on the board. Taking notes into their books for the concepts of the circle
Integration	No activity was given to the learners

(Source: Primary Data)

The aim of teaching Activity A was to test the pre-knowledge of the learners about the circle concepts to get acquainted with the working domain of circle geometry theorem. It was observed that the teacher laboured to explain the concepts and new terminologies to be used in the theorem of the geometry of the circle. The exploration of circle geometry concepts and terminologies, and their relationship required learners to construct and measure. At this stage, 3 learners out of 15 learners had instruments to use in constructing and measuring the required angles and lengths. This tempted the teacher to resort to simply demonstrating on the board as learners were taking notes.

It was observed that learners knew the circle, which was the concept under study. Through the conversation between the teacher and learners, learners knew the following concepts: centre, radius, diameter, circumference and tangent, but they could not distinguish between a chord, segment and sector. Even though learners knew the mentioned circle concepts, they struggled to express the concepts in geometrical language. They could express them in their own vocabularies. For example, learner L005 defined diameter as a line from a point of a circle to another point of a circle. This learner illustrated the diameter correctly on the diagram but could not express it in the correct geometric language.

Diameter is expressed as the length of a straight line segment from one point on the circumference to another point on the circumference that passes through the centre of the circle.

Firstly, the inability of learners to construct and measure required angles does indicate that they were unable to find their way in the network of relations. This is because the teacher did not guide learners to uncover the connection between the concepts.

Secondly, the inability of learners to distinguish the terms: chords, segment, and sector confirms what was found from the CAPS document that mention of circle geometry concepts features more in the Grade 7 curriculum only. This creates a vacuum of circle geometry concepts in upper grades that delink student's recalling and understating of these concepts in higher grades specifically Grade 11.

4.3.2 Analysis of classroom observation for Teaching Activity B

Teaching activity B consisted of two investigations, which were designed to discover chord and midpoint properties, (Appendix A2). The aim of investigation 1 was to develop a conjecture that the line drawn from the centre of a circle perpendicular to a chord bisects the chord; investigation 2 was to develop a conjecture that the perpendicular bisector of a chord passes through the centre of the circle.

The findings of teaching activity B are revealed in the Table 4.6 below:

Table 4.6 Findings from observation of Teaching Activity B

van Hiele's teaching Phase	What was observed from Teaching Activity B
Information	There were discussions between learners and teacher to learners as they were executing activities given to them. The learners could ask how to measure, what to do next
Directed orientation	Learners thought assistance from the teacher especially when it came to measurements and the teacher could give help one on one
Explicitation	Teacher could lead the learners onto the conclusion other than waiting for them to express their own idea. Mostly teach was leading. Learners got confused with the statement when the teacher asked them to complete the converse of the statement given. The teacher troubled himself to explain it
Free orientation	The majority of the learners answered correctly the activity 1 in investigation 2.1 it was explicitly clear to them to measure and determine the length of the radius.

	The second activity seemed to be challenging to understand the meaning of the shortest distance. The teacher explained it and left them to do it as homework. But the students seemed confused with how to determine the shortest distance of each chord
Integration	The teacher gave the exact conjectures by reading them and writing them on the board. He told learners to memorise them.

(Source: Primary Data)

In investigation 1, most of the participants in the class could measure accurately the length PR and RQ and could compare both lengths as equal to each other. They also stated clearly that angle PRO = QRO $=90^0$. There were two learners that stated the angle as 85^0as they were struggling with accuracy in measuring the angle. However, in step 7, learners struggled with stating the correct conjecture in correct and acceptable geometric language. The teacher explained that making a conjecture is giving your opinion for generalising your conclusion. Surprisingly, for the converse of the theorem, learners could not complete the statement "the line drawn from the centre of a circle to the midpoint of a chord will be perpendicular to the chord". Instead of stating perpendicular, they could say "half way" not until the teacher illustrated and stated it on the board.

There was a significant positive correlation between investigation 1 and investigation 2. The teacher was clear on the steps that learners were following, and learners could measure accurately lengths RA and RB; they could compare and find them to have the same lengths. They could notice and state the relationship between the two triangles RAT and RTB in their own language. One learner, L005, stated the relationship between the two triangles that they looked the "same". The teacher guided the learner with the correct mathematical language as being "similar triangles".

This teaching activity showed that some learners, for example, learner L001 and L005 follow clearly guided instructions to uncover the connection between concepts, engage with concepts in order to develop an understanding and connection between them. The most remarkable results that emerged from this data were that learners struggled with expressing ideas in accepted mathematical language. For example, expressing relationship between two triangles as "***similar triangles***"; completing the statement that the line drawn from the centre of a circle to the midpoint of a chord will be ***perpendicular*** to the chord. The teacher needs to ensure that learners are guided with accepted technical language.

4.3.4 Analysis of classroom observation for Teaching Activity C

Teaching activity C consisted of two investigations which were designed to discover arcs and angles properties in a circle, (Appendix A3). The aim of investigation 3 was to develop a conjecture that the angle subtended by an arc at the centre of the circle doubles the size of the angle subtended by the same arc at the circle; investigation 4 was to develop a conjecture that the angles subtended by a chord of the circle, on the same side of the chord are equal.

The findings of teaching activity C are revealed in the Table 4.7 below:

Table 4.7 Findings from observation of Teaching Activity C

van Hiele's teaching Phase	What was observed from Teaching Activity C
Information	There were discussions between learner to learner and teacher to learners as they were executing activities given to them. Learners could discuss how to use instruments to measure, and you could hear them talking while assisting each other
Directed orientation	Teacher moved around assisting learners who had questions on some of the instruction especially on step 7 and Step 8 for investigation 4 that required them to draw and measure
Explicitation	Learners used words normally in their own understanding to express relationship between angles and sides. Notably in step 7 of investigation 3, angle AOB is two times ACB
Free orientation	Teacher moved a round assisting learners who had questions on some of the instruction especially on step 7 and Step 8 for investigation 4 that required them to drawing and measuring
Integration	The teacher gave the exact conjecture by reading and writing them on the board. He told learners to memorise them.

(Source: Primary Data)

As classroom observation proceeded with this teaching activity, learners were familiar with using the instruments to measure length and angles, although very few learners displayed small challenges with accuracy. The teacher could move around guiding those who had challenges with drawing and measuring.

There was no significant difference between teaching activity B and teaching activity C in terms of following and executing step by step instructions given by the teacher, verbalising their findings in their own language and the teacher guiding them on acceptable geometrical language use. Learners were able to follow the instructions as stipulated in step by step activities. Two learners found step 7 and step 8 of investigation 4 confusing. However, the teacher moved to them and guided them. At the end of the lesson, I asked the learners what exactly was confusing in both steps, and one learner replied that she could not identify the major arc and segment. When I traced this learner in the first teaching activity A, I found out that she was absent during the first teaching activity on circle geometry concepts, vocabularies and axioms studied in earlier grades to investigate and prove theorems.

4.3.4 Analysis of classroom observation for Teaching Activity D

Teaching activity D consisted of one investigations, which was designed to discover cyclic quadrilateral properties in a circle, (AppendixA4). The aim of investigation 5 was to develop a conjecture that the opposite angles of a cyclic quadrilateral are supplementary.

The findings of teaching activity D are revealed in the Table 4.8 below:

Table 4.8 Findings from observation of Teaching Activity D

van Hiele's teaching Phase	What was observed from Teaching Activity D
Information	Discussions took place between learner to learner and teacher to learners as they were working through step by step activities given to them. Learners got confused with how to measure the angle required in step 3 of the activity. The teacher assisted the learners
Directed orientation	The teacher made expansion of the theorem for learners to discover exterior angles
Explicitation	Learners gave answers according to what they observed after constructing and measuring. At one point after measuring angle D and F the learner said it is equal to 178. The teacher guided them to the required technical language, informing the learners that angles are called supplementary angles.
Free orientation	Most learners could give the correct answer in the activity, but there was one learner who gave wrong answers and when the teacher went to him, he was taking angles on the straight line instead of opposite angles in the quadrilateral
Integration	The teacher gave the exact conjecture by reading it and writing on the board. He told learners to memorise it.

(Source: Primary Data)

All learners were able to construct quadrilateral DEFG in a circle and could easily notice that angle D was opposite to angle F, and angle G was opposite to angle E. Learners found step 2 very easy, perhaps because of their familiarity with constructions and understanding of circle properties. I noticed that one learner raised a question on how to measure the required angles in a quadrilateral enclosed in a circle. However, the teacher guided this learner to the correct way of how to measure and she could come up with the sum of two opposite angles as180^0. Learners came up with various sizes of opposite angles that added up to 180^0 as their constructions was not tailed to a specific measurements. As mentioned in the previous investigations, learners' challenge was to verbalise their findings and express them in acceptable geometric language, which compelled the teacher to guide them with the correct technical geometric language. For example, two angles that add up to 180^0 are called supplementary angles.

This investigation showed that learners were familiar with step-by-step instruction given and guided by the teacher to uncover connections of the subject matter. This suggests that learners can cope well and engage with geometric concepts in order to develop understanding through teacher-guided activities. It further suggests that learners can complete tasks that require a number of steps and can be solved in many says.

4.3.5 Analysis of classroom observation for Teaching Activity E

Teaching activity E consisted of two investigations, which were designed to discover tangent properties in a circle (Appendix A5). The aim of investigation 6 was to develop a conjecture that two tangents drawn to a circle from the same point outside the circle are equal in length; investigation 7 was to develop a conjecture that, the angle between a tangent and a chord drawn to the point of contact is equal to the angles in the alternate segment

The findings of teaching activity E are revealed in the Table 4.9 below:

Table 4.9 Findings from observation of Teaching Activity E

van Hiele's teaching Phase	What was observed from Teaching Activity E
Information	There were discussions between learner to learner and teacher to learners as they were executing activities given to them.
Directed orientation	Teacher walked around as they are working through step by step instructions giving them guidance to discover the angles he was referring to in the segments ,
Explicitation	In investigation 6 the learners were left with little help from the teacher. When I ask him he told he wanted to find out if they can work on their own by following the instruction in the activity Learners helped one another to draw the illustration in investigation 7 and could measure the angles correctly, one

	learner was not measuring accurate angles which gave a laughter from her friend. The teacher went to their desk to help The teacher rushed the learners even before they could complete the whole investigation.
Free orientation	Little time was given to learners to complete the activities, struggling learner could not do the whole questions of the activity in investigation 6.The teacher didn't mind about those learners that were behind with completing the activities
Integration	The teacher gave the exact conjecture by reading them and writing them on the board. He told learners to memorise them.

(Source: Primary Data)

All learners constructed their diagrams accurately, as illustrated in Appendix A5. They noticed and identified the relationship between the lengths RP and RQ as equal lengths. Furthermore, they could measure accurately the angles OPR and OQR as 90^0. Step 3 of investigation 7 seemed challenging to some learners as they could not distinguish between minor and major segments. However, one learner in class went to the board and demonstrated the two segments based on her own understanding of the concepts as **the short arc gives the minor segment and the long arc gives the remaining segment.** This showed that the learner had familiarised himself/herself with content that engages them in a content- based discussion.

The challenge learners faced in Step 3 of investigation 7 was caused by technical Mathematics concepts explored in these investigations (arch and minor segment and major segment). This indicated that teachers need to labour to differentiate between related terms that are homonyms. It further showed that teachers should let the learners verbalize their understanding of the concepts to become more conscious of the new ideas before guiding them to express the concepts in accepted mathematical language.

4.3.6 Analysis of the Application Activities

Application activities consisted of two tasks, which were designed to discover the overall structure of the concepts and where those structures fit in the scheme of theorems of the circle, (appendix A6). The aim of application activities was to use the listed theorems (see section 4.2.1) and their converses, where they exist, to solve riders. However, application activities was not observed in classroom, the analysis was based on the way learners answered the questions. The activities was given to learners on the last day of teaching activity E as an assignment to do that was returned and marked by the teacher. However, the teacher managed only to mark task 1 because other tasks were not returned by the learners.

The analysis of the marked work revealed that out of 15 learners that attempted all questions in task 1, they could determine and express the size of angles in terms of variables as it was required in the questions (Appendix A6). With a few exceptions, the results further showed that learners applied knowledge and skills from previous lessons to solve problems involving complex calculations and higher order reasoning (See Appendix 6A, Task 1).

The following Table 4.10 indicates the findings from analysis of 9 learner's response on Task 1.

Table 4.10 The analysis of learners' response on Application activity 6, Task 1

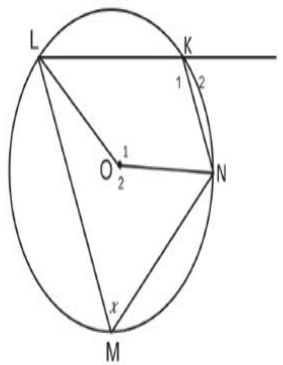
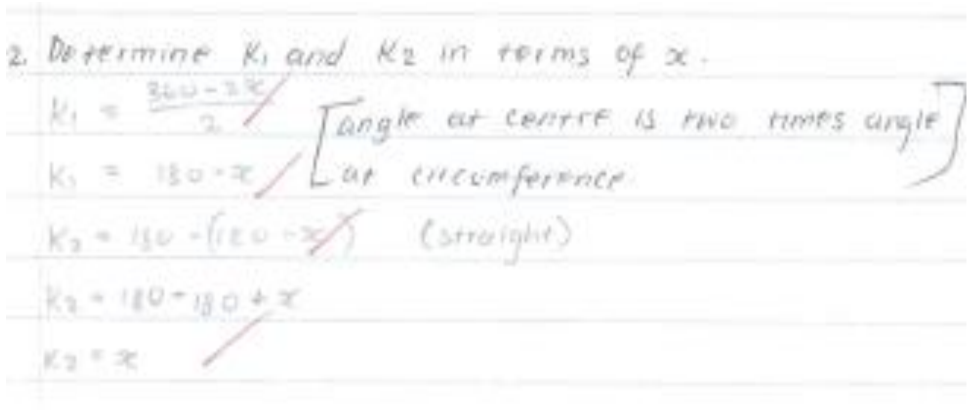
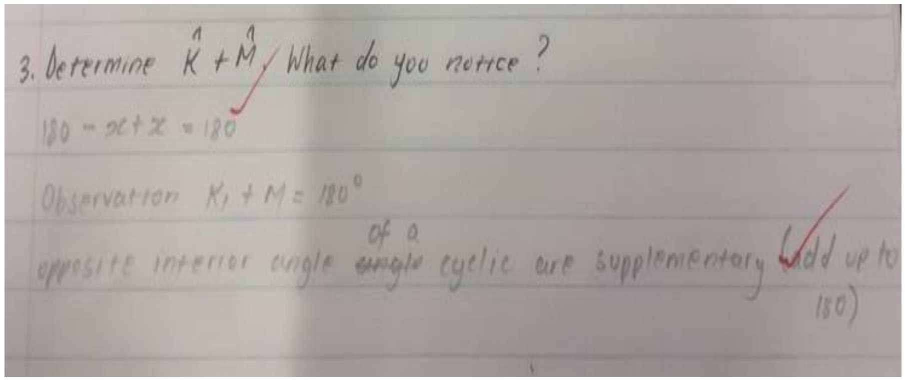
Questions	Number of participant's answers with acceptable reasons N= 9			
	Correct answer with acceptable reason	Correct answer with inappropriate reasons	Wrong answer	Sample size
2.1	9	0	0	9
2.2	7	2	0	9
2.3	9	0	0	9
2.4	9	0	0	9

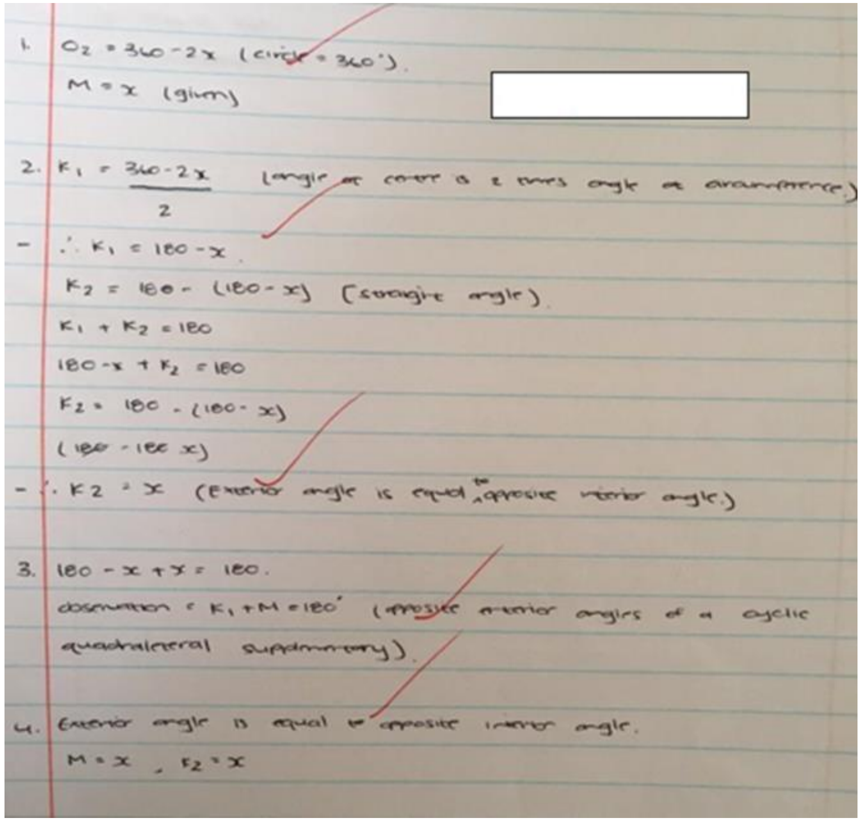
(Source: Primary Data)

It can be seen in Table 4.10 above that learners were able to answer all the questions correctly. The ability of learners to answer questions correctly demonstrates that they knew what to do and why. This is what is called *rational understanding* (van de Walle, 2004). Understanding can be defined as a measure of the quality and quantity of connections that an idea has with the existing idea (van de Walle, 2004). The way learners understand concepts and ideas depends on the existence of appropriate ideas and on the creation of connections. The creation of connections exists a long continuum from instructions. The instruction developed according to five sequences, as mentioned in section 4.2.2, promotes the acquisition of concepts, ideas, expressing these ideas in accepted mathematical language, finding ways in the network of relation, leading to knowing what to do and why.

In task 1, question 2.2, the learner L005 wrote $k_1 = 180^0 - x$, *(angle at center is twice or two times angle at the circumference)* this learner has an understanding of the theorem "the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same ac at the circle (on the same side of a chord as the centre)"; see the learner's answer in question 2.2 in Table 4.11 below.

Table 4.11 Learners' written responses on Application Activity, Task 1

Example of question item	**Learner L005 Response(s)**	**Researcher's insights & comments**
2. O is the centre of the circle below and $\hat{O}_1 = 2x$.  2.1. Determine $\hat{O}_2$ and $\hat{M}$ in terms of x. 2.2. Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of x. 2.3. Determine $\hat{K}_1 + \hat{M}$. What do you notice? 2.4. Write down your observation regarding the measures of $\hat{K}_2$ and $\hat{M}$.	**Learner L005 Response on question 2.2**  2. Determine K_1 and K_2 in terms of x. $K_1 = \frac{360 - 2x}{2}$ [angle at centre is two times angle at circumference] $K_1 = 180 - x$ $K_2 = 180 - (180 - x)$ (straight) $K_2 = 180 - 180 + x$ $K_2 = x$	Learner;s ability to comprehend the concept in the theorem, relation of angles in the circle.
	Learner L005 Response on question 2.3  3. Determine $\hat{K} + \hat{M}$ What do you notice? $180 - x + x = 180$ Observation $K_1 + M = 180°$ opposite interior angle ~~angle~~ of a cyclic are supplementary (add up to 180)	Learners' ability to know what to do and why

	Learner L001 Response	
	 1. $O_2 = 360 - 2x$ (circle = 360°). $M = x$ (given) 2. $K_1 = \frac{360-2x}{2}$ (angle at centre is 2 times angle at circumference) $\therefore K_1 = 180 - x$. $K_2 = 180 - (180 - x)$ (straight angle). $K_1 + K_2 = 180$ $180 - x + K_2 = 180$ $K_2 = 180 - (180 - x)$ $(180 - 180\ x)$ $\therefore K2 = x$ (Exterior angle is equal to opposite interior angle.) 3. $180 - x + x = 180$. observation: $K_1 + M = 180°$ (opposite interior angles of a cyclic quadrilateral supplementary) 4. Exterior angle is equal to opposite interior angle. $M = x$, $K_2 = x$	Learner's ability to apply concepts of the theorem of geometry of the circle, procedures, follow the process and justify. Learner got the skills of carrying out procedures, accurately and appropriately

(Source: Primary Data)

To determine that $k_1 + M$ as equal to 180^0, learner L005 needed to notice that KLMN is a cyclic quadrilateral, to know that k_1 is opposite to M, and to know that opposite interior angles are supplementary in acyclic quadrilateral. The learner L005 has a rational understanding towards the theorem "The opposite angles of a cyclic quadrilateral are supplementary". See the learner's answer on question 2.3 in Table 4.11 above.

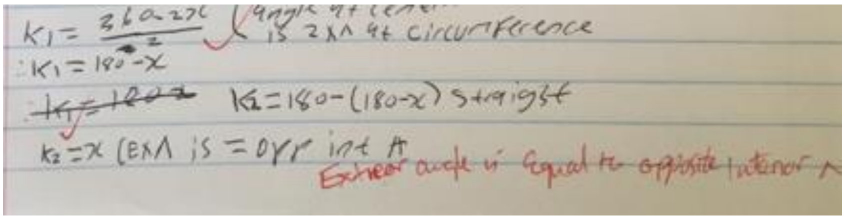
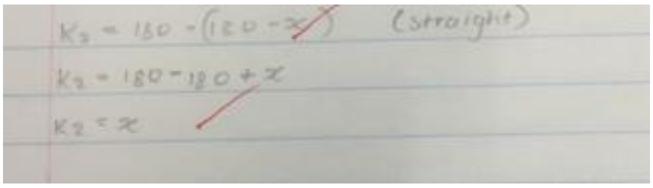
It was found that learners were able to apply procedures, concepts and process when answering questions. For example, task 1 question 2.3 required procedure application and some learners followed procedure to answer that question. For instance, learner L001's responses to the questions in task 1 is given above in Table 4.11. This is consistent to what is referred to as learning with understanding (van de Walle, 2004).

Learner L001 was able to notice that angle $k_1 + M$ are opposite angles of a cyclic quadrilateral and add up to 180^0. Learner L001 to notice that both angles add up to 180^0 had to apply the procedure from question 2.2 of finding the size of angle k_1 as ($k_1 = 180 - x$) and angle $M = x$

Similarly, in question 2.4, learner L001, to see that angle k_2 and M are equal $to\ x$ and angle k_2 is an exterior angle of a cyclic quadrilateral LMNK, confirms that learners were able to conjecture the connections and relationships between concepts. Learners' ability to uncover connection and understanding geometric concepts requires instruction that is designed to promote learners' thinking processes. It is the van Hiele's theory of instruction design that is primarily directed at considering a learner's geometric thinking ability (Alex & Mammen; 2016).

In question 2.3, two learners (L003 & L005) were able to give correct answers with inappropriate reasoning. The procedure they followed was mathematically correct but accompanied it with incorrect reason for the procedure. The learners indicated the reason for $k_2 = 180^0 - (180 - x)$ as straight, see Table 4.12

Table 4.12 Learners' inappropriate written responses on Application Activity, Task 1

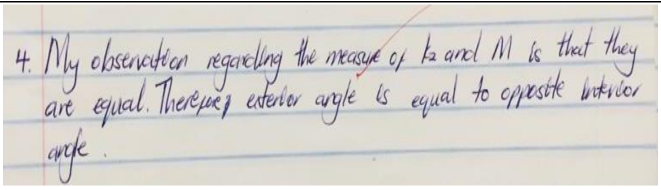
<table>
<tr><th>Example of question item</th><th>Learners' Response on question 2.2</th><th>Researcher's insights & comments</th></tr>
<tr><td rowspan="2">Question 2.2</td><td>Learner L003 Response on question 2.2
</td><td rowspan="2">Learners lacked capacity to give appropriate justification</td></tr>
<tr><td>Learner L005 Response on question 2.2
</td></tr>
</table>

(Source: Primary Data)

This result implies that learners could express their geometric understanding in their own terms. I note that, other than assisting learners in using accurate and appropriate language to give reasons for each step, the teacher's use of technical language and questioning is a crucial factor in directing learners. For example, asking learners how they "know" is important. It is not enough, for example, to be asked the value of an angle; they should be challenged to explain why and think about their explanation. In this way, the teacher develops technical language for the learners.

All learners got question 2.4 correct. This question required learners to observe and make deduction based on own understanding. Learners were required to observe that $\widehat{m} = x$ and $\widehat{k_2} = x$. To identify that $\widehat{k_2} = x$ required procedural fluency, moving part of one number to another. The implication is that they gain experience in finding their own way to resolve the task. For example, learner L004 stated clearly that both angles were equal and stated the sufficient condition why they are equal (see Table 4.13).

Table 4.13 Learners' L004 written response on Application Activity, Task 1, and question 2.4

Example of question item	**Learner L004 Response on question 2.4**	**Researcher's insights & comments**
Question 2.4	 4. My observation regarding the measure of k_2 and M is that they are equal. Therefore exterior angle is equal to opposite interior angle.	Learners had the capacity to observe, make logical thought, reflect and justify

(Source: Primary Data)

The above response shows that learners form an overview of the new network of objects and relations. At this stage, learners have attained a new level of understanding.

4.3.7 Raw data of the observation protocol

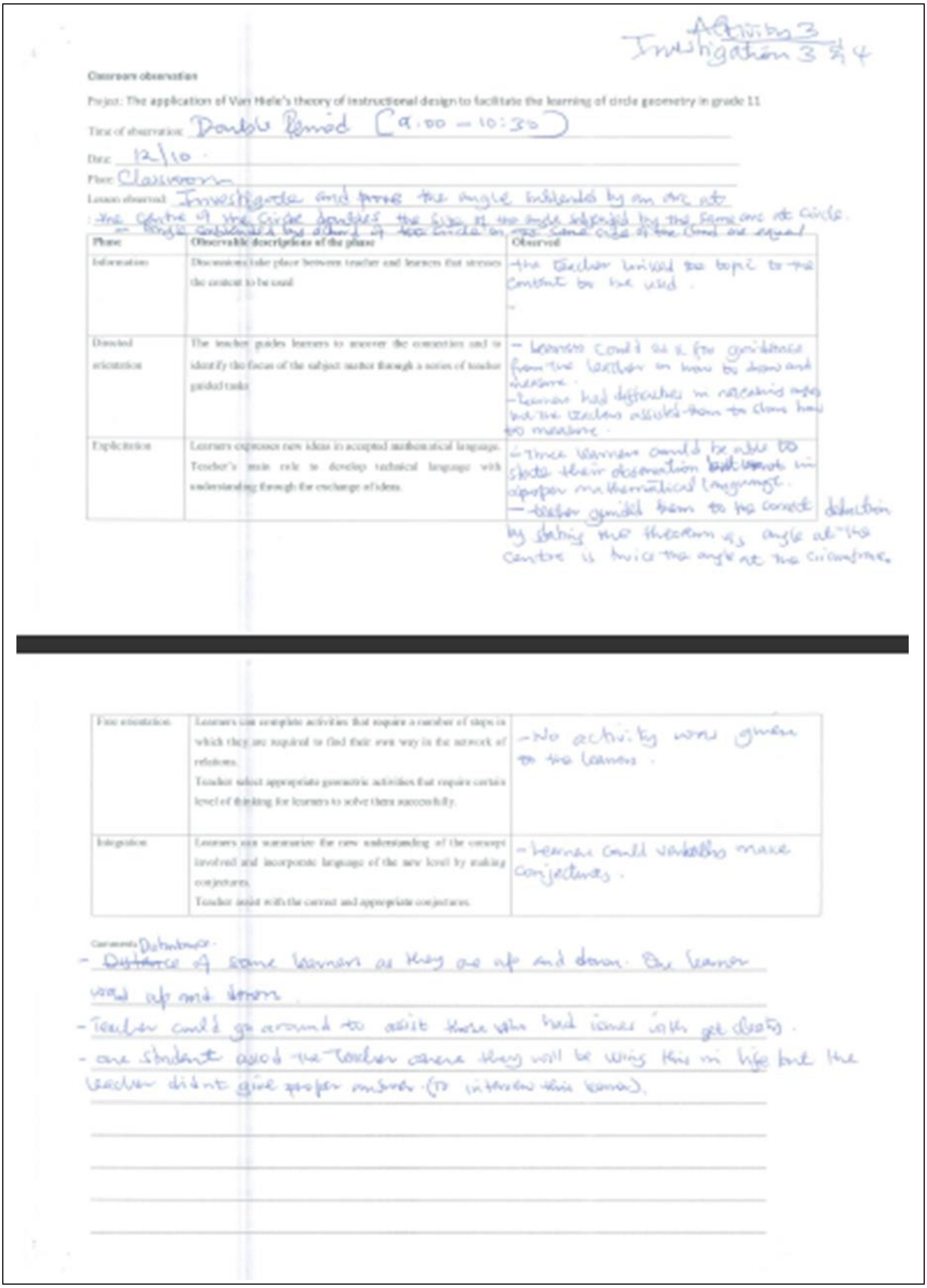
The observation protocol below indicates the researcher's insight on what transpired in the classroom during teaching.

Teaching activity C

It can be seen from the comments made in the observation protocol below, Figure 4.4, that a teacher's instruction on teaching activity C was appealing. Learners could ask the teacher questions and get responses on their questions; the teacher could guide learners the correct deductions when learners verbally state their observations and he could move around classes to keep learners engaged. The tactics the teacher used were: to get learners' attention, confidence and satisfaction through sequencing instruction with a clear step-by-step guide, as provided by van Hiele's phases of learning. His tactics play a central role in motivating learners to learn. As seen in the comments:

- learners asked for guidance on how to draw and measure certain angles, and the teacher provided the guidance they sought (attention, this relates to arousing learners' curiosity and interest);
- learners were able to state their observation (confidence, this relates to learners' success of meaningful tasks); and
- teacher guided learners to the correct deduction of the theorem (satisfaction, this related to build learners' sense of reinforcement and achievements).

The mentioned tactics form part of the major components for motivational learning (See et.al., 2017).


Activity 3
Investigation 3 & 4

Classroom observation

Project: The application of Van Hiele's theory of instructional design to facilitate the learning of circle geometry in grade 11

Time of observation: Double Period (9.00 – 10:30)

Date: 12/10

Place: Classroom

Lesson observed: Investigate and prove the angle subtended by an arc at the centre of the circle doubles the size of the angle subtended by the same arc at circle. — Angle subtended by a chord of the circle on the same side of the chord are equal

Phase	Observable descriptions of the phase	Observed
Information	Discussions take place between teacher and learners that stresses the content to be used	-the teacher linked the topic to the content to be used.
Directed orientation	The teacher guides learners to uncover the connection and to identify the focus of the subject matter through a series of teacher guided tasks	- learners could ask for guidance from the teacher on how to draw and measure. -Learners had difficulties in measuring angles and the teacher assisted them to draw and to measure.
Explicitation	Learners express new ideas in accepted mathematical language. Teacher's main role is to develop technical language with understanding through the exchange of ideas.	- Three learners could be able to state their observation but not in proper mathematical language. — teacher guided them to the correct deduction by stating the theorem e.g. angle at the centre is twice the angle at the circumference.
Free orientation	Learners can complete activities that require a number of steps in which they are required to find their own way in the network of relations. Teacher select appropriate geometric activities that require certain level of thinking for learners to solve them successfully.	-No activity was given to the learners.
Integration	Learners can summarise the new understanding of the concept involved and incorporate language of the new level by making conjectures. Teacher assist with the correct and appropriate conjectures.	- Learners could verbally make conjectures.

Comments: Disturbance.

- Disturbance of some learners as they go up and down. One learner was up and down.
- Teacher could go around to assist those who had issues to get clarity.
- one student asked the teacher where they will be using this in life but the teacher didn't give proper answer. (To interview this learner).

Figure 4.4 Raw data for observation protocol for teaching Activity C

Teaching activity D

It can be seen in Figure 4.5, the observation protocol below, that the discussion that took place between the teacher and learners focused on content. The teacher reviewed the meaning of terminologies in teaching activity D, for example, the meaning of cyclic quadrilateral and supplementary angles. Through this discussion, the teacher guided the learners who had problems to follow step-by-step instructions, by assisting learners on a one-by-one basis. The guidance the teacher gave to learners motivated them to complete step-by-step instructions, exchange ideas through verbalising their understanding and problem-solving.

Activity 4

Classroom observation

Project: The application of Van Hiele's theory of instructional design to facilitate the learning of circle geometry in grade 11

Time of observation: 9 : 00 – 9 : 30 [Single Period]

Date: 14 | 10

Place: Classroom

Lesson observed: The opposite angles of cyclic quadrilateral are supplementary – [illegible]

Phase	Observable descriptions of the phase	Observed
Information	Discussions take place between teacher and learners that stresses the content to be used	– The teacher started by reviewing the meaning of supplementary angles
Directed orientation	The teacher guides learners to uncover the connection and to identify the focus of the subject matter through a series of teacher guided tasks	– There was observable teachers guidance to the learners who were asking him for help.
Explicitation	Learners expresses new ideas in accepted mathematical language. Teacher's main role to develop technical language with understanding through the exchange of ideas.	
Free orientation	Learners can complete activities that require a number of steps in which they are required to find their own way in the network of relations. Teacher select appropriate geometric activities that require certain level of thinking for learners to solve them successfully.	– Learners followed teachers instruction on what they should do. –
Integration	Learners can summarise the new understanding of the concept involved and incorporate language of the new level by making conjectures. Teacher assist with the correct and appropriate conjectures.	The teacher lead the learner to the conclusion of the conjecture.

Comments:

– One learner told the teacher she was confused and the teacher stopped everything to go assist (to interview the learner)

– At one point the teacher could not wait for learners to give their own deductions in their own way of understanding

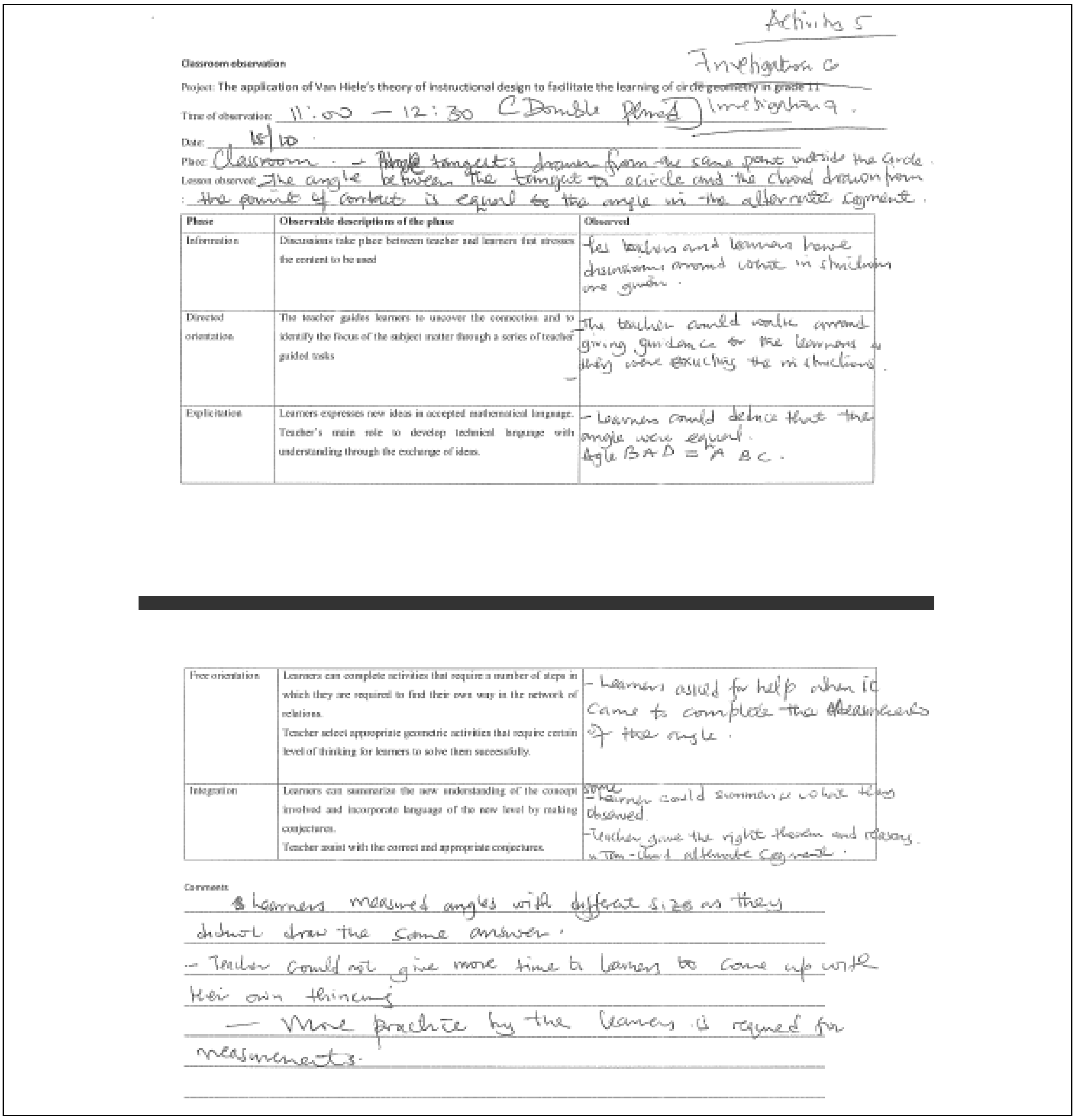
– The teacher expanded the theorem to the exterior opposite which he discussed briefly.

– Learners were given activities to try on their own after 20 minutes of teaching

Figure 4.5 Raw data for observation protocol for teaching Activity D

Teaching activity E

From the observation protocol below (Figure 4.6) for teaching activity E, we can note that discussions took place between learners and the teacher, and the teacher gave guidance to learners on how to follow instructions; learners were able to make a deduction on the sizes of angles that they measure.



Activity 5

Classroom observation

Investigation

Project: The application of Van Hiele's theory of instructional design to facilitate the learning of circle geometry in grade 11

Time of observation: 11:00 – 12:30 (Double Period) Investigating.

Date: 15/10

Place: Classroom. – Angle tangents drawn from the same point outside the circle.

Lesson observed: The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

Phase	Observable descriptions of the phase	Observed
Information	Discussions take place between teacher and learners that stresses the content to be used	The teachers and learners have discussions around what instructions are given.
Directed orientation	The teacher guides learners to uncover the connection and to identify the focus of the subject matter through a series of teacher guided tasks	The teacher could walk around giving guidance to the learners as they were executing the instructions.
Explicitation	Learners expresses new ideas in accepted mathematical language. Teacher's main role to develop technical language with understanding through the exchange of ideas.	– Learners could deduce that the angle were equal. Angle BAD = ABC.
Free orientation	Learners can complete activities that require a number of steps in which they are required to find their own way in the network of relations. Teacher select appropriate geometric activities that require certain level of thinking for learners to solve them successfully.	– Learners asked for help when it came to complete the measurements of the angle.
Integration	Learners can summarize the new understanding of the concept involved and incorporate language of the new level by making conjectures. Teacher assist with the correct and appropriate conjectures.	– Some learners could summarize what they observed. – Teacher gave the right theorem and reasons "tan-chord alternate segment".

Comments:

Learners measured angles with different sizes as they did not draw the same answer.

– Teacher could not give more time to learners to come up with their own thinking.

– More practice by the learners is required for measurements.

Figure 4.6 Raw data for observation protocol for teaching Activity E

4.3.2 Discusion of finding from classroom obersavation in relation to the research sub-question What are the benefits of using Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?

Drawing on the analyses of the observation protocol and teaching activities (section 4.3), the findings highlight the following benefits of using van Hiele's theory of instructional design in facilitating learning of circle geometry:

- **Classroom discussion**
  The approach to teaching using van Hiele phases encourages learning through discussion. As evident in the observation protocols for teaching activities C, D, and E, in their exchange of views and ideas through classroom discussion, the teacher identifies what learners already know and learners become oriented and acquainted with new knowledge. Through discussions, the teacher can identify and coordinate a range of mathematical competencies required to transit from computational Mathematics to creative Mathematics. This helps learners to actively experience the objects of study in appropriate contexts when they engage in discussion. This substantiates previous findings in literature that for meaningful teaching, teachers should structure their instructions in ways that discover learners' ability to reason (Atebe & Schafer, 2010).
- **Collaborative learning**
  Van Hiele's instructional model approach permitted learners to collaborate and develop their Mathematics ideas through helping and assisting one another. As indicated in the observation protocol for teaching activity E, the majority of learners could work collaboratively to execute the steps-by-step instructions given by the teacher. Collaboration learning recognises the importance of the social *milieu* within which learning occurs and its significant influence on what is learned. The teacher's effort to develop a learning community where ideas are discussed and understanding enriched was critical during lesson presentation.
- **Enhancing motivation to learn**
  Increasing a learner's motivation to learn was an important element observed during lesson presentation. It was evident, as seen in the classroom observation protocol for teaching activity C, that in this form of instruction designs, the teacher gave the learners ownership of the process and supported them in developing ownership of problem- solving. Motivation to learn is increased through ownership of learning.

The above-mentioned benefits help teachers to assess students' levels of understanding of geometry and learners to progress through the levels.

4.4.1 Part C: Analysis of interview responses

4.1 Interview data analysis

The purpose of in-depth interviews was to solicit responses from the teacher on the teaching and learning experience, benefits, and challenges of van Hiele's phases of learning in classroom environment. The purpose was to discover and identify the challenges and benefits of van Hiele's theory of instructional design to facilitate learning of circle geometry to Grade 11 learners.

To achieve this, the teacher was asked various questions, as contained in the interview protocol (Appendix C). The teacher's interview session lasted for 22 minutes, and the researcher was able to probe deeply in certain instances to obtain additional information.

4.4.2 Analysis and discussion of teacher's responses from the interview session

Below is the analysis and discussion of the teacher's response on each question, as indicated in the interview protocol. These are presented *verbatim* (unedited) to capture the original essence of their input.

1. **Please describe your experience regarding Mathematics, and what you think about circle geometry in South African curriculum?**

Interviewer: *What do you think about circle geometry in South African curriculum and how do you describe your experience about it?*

Teacher*: From what I have learnt here in South Africa is that some of theorem in their curriculum is not found in other countries curriculum for example the alternate segment only learnt here in South Africa. Also I have a learnt a lot from teaching different grades putting my teaching experience in a better level.*

Interviewer*: Have you ever looked through the CAPS curriculum, especially the section on circle geometry?*

Teacher*: I did look through it from Grade 10; it's user friendly and the books we use are linked to it, user friendly but broad to complete.*

The teacher pointed out that the South African school geometry, as prescribed in the CAPS Mathematics policy document, is broad but user friendly and appropriate per grade.

2. **How would you describe your experience in teaching circle geometry prior to knowing van Hiele's theory?**

Interviewer*: How do you describe your experience in teaching circle geometry before knowing van Hiele and the way using it; do you feel is useful or time wasting?*

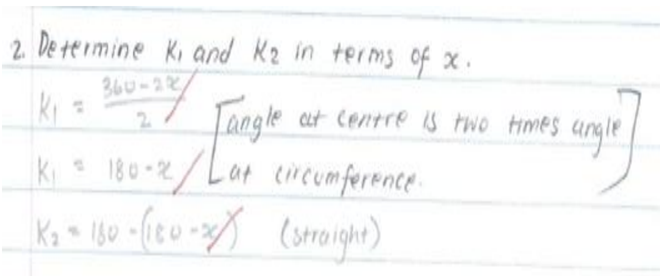
Teacher *: It is very useful, because look now, I will take for example that theorem that states that the angle at the centre is twice the angle at the circumference, so it's like we did an experiment, we experimented it, we saw that it was very true, once the student construct accurately, we did construct , ask them measure and ask them what do you observe? they said this angle at the centre is twice the angle at the circumference, it's very useful, without construction learners struggled to give the observations they make, yah , some don't answer at all,*

Interviewer: *So, in terms of before, do you feel there was something missing in you before knowing the phases?*

Teacher *: Ok, yah, well, look now without doing experiment being told that the exterior angle is equal to the opposite interior angle, without proving it, look, you quickly forget , but when you do it practically, like what they did, they did it practically, now, for them to forget it's not easy, other that telling someone this is that , ah, they can't know why, I had bad experience before with learners, they don't even answer you, eish, you struggle, telling you anything not linking to the statements in the theorem, ,yah, this time learners linked and made statement linking to theorem even though, yah some used own words but the statements were making sense, this teaching is good.*

The teacher's response confirms what I observed during lesson presentation as mentioned in Section 4.3.4. Some learners struggled to verbalise their understanding in a mathematically acceptable language. This finding is consistent to Shongwe (2019) who argues that G11 learners' geometric argumentation is poor. Learners' geometric language expression is judged to be of low quality because they provide either statements according to their own understanding or provide statements that cannot be categorised as a condition under which geometrically acceptable reasons can hold. For example, Application Activity, Task 1, learner L005 could determine k_1 and k_2 in terms x but failed associating the reason for words with their own language.

Table 4.14 Learners' L005 written responses on Application Activity, Task 1, and question 2.4

Example of question item	**Learner L005 Response on question 2.4**	**Researcher's insights & comments**
Question 2.4	 2. Determine K_1 and K_2 in terms of x. $K_1 = \frac{360-2x}{2}$ [angle at centre is two times angle at circumference. $K_1 = 180-x$ $K_2 = 180-(180-x)$ (straight)	Learners use own language, for the reason given for the value of k_1 in terms of x as "two times" and inappropriate statement "straight" for the value of k_2 as the reason.

(Source: Primary Data)

The learner's statement reveals lack of correct use of geometric language. Such inappropriate statements point to the learner's inability to understand use of appropriate vocabulary and geometric symbols.

It is essential that learners talk about their linguistic association for words and symbols. Initially, teachers should leave learners to express their geometric understanding in their own words and the teacher takes care to develop their words into technical language. To address the gap of language in learning geometry, each phase of van Hiele's level of learning has its own language and its own interpretation of the same term, as discussed in Chapter 2, section 2. Discussing and verbalising concepts are important aspects of the information, explication and integration phases of learning (Mason, 1998). Learners should be introduced to standard terminologies and symbols and encouraged to use them precisely. For instance, in the above statement given by the learner, the standard terminology and symbol for the reason should be $\angle$ at centre = $2 \times \angle$ at circumference. Such verbalization requires learners to articulate consciously what might be underdeveloped ideas through conversation with teachers.

3. **Did you know about van Hiele's teaching phases?**

Interviewer: *Did you know about van Hiele's theory before or van Hiele's phases of learning before I came in to say I have this, this is how we are going to do it?*

Teacher: *Ah, no, no, I had no idea what you were talking about, I don't want to lie.*

The above response from the teacher provided evidence that some teachers are neither aware nor understand the van Hiele's phases of learning as a tool to plan and organise instructions on geometry. This is one of the indications of how disturbingly low subject matter acquisition is among learners in geometry, which could attribute to difficulties learners always face in learning geometry. Ndlovu and Mji (2012) assert that Euclidean geometry is a complete disaster because it is badly taught. It is obvious that teachers cannot effectively teach geometry that they themselves do not have knowledge of teaching. This finding revealed that there is a pedagogical gap to geometry teaching. To address the gap of pedagogy in geometry, teachers should plan and organise geometric instruction according to van Hiele's phases of learning. As mentioned in the literature review, van Hieles developed and proposed five sequential phases of learning to address the important area of pedagogy in geometry. Therefore, teachers should be knowledgeable about the five van Hiele's phases of learning and van Hiele's theory to develop and implement van Hiele based materials and instructions in the classroom setting. The need now is for classroom teachers to be prepared and make them aware of the phases of learning through professional development programmes.

4. **How would you describe your experiences of using van Hiele's teaching phase to teach circle geometry?**

The teacher used the following words to describe his teaching experience with van Hiele, ***"helpful", 'make learners to remember the concepts",*** and ***"useful for learner's understanding".*** I probed further to find out how the experience of using van Hiele's phases of learning was useful to his teaching. He replied, *"**it make the learners to remember because if you do something practically the chances of forgetting is very minimal and theory is very easy to forget".***

The teacher's statements concur with previous findings that the five-phase teaching approach provides a structure on which to base geometric instructions and that, van Hiele's theory improves learners' ability to study geometry (Watan & Sugiman, 2018). It was not a surprise to the researcher that the teacher could clearly mention that the experience he received was helpful. During lesson observation and analysing of learners' activities, I noticed learners' ability to understand concepts improved as the teacher navigated through the teaching activities. For example, in the fourth teaching activity (D), learners could develop formal deduction skills by providing a correct conjecture of the theorem. This was as a result of step- by-step instructions given and guided by the teacher for learners to uncover connections between concepts. As mentioned in literature, learners' acquisition and progression through geometric levels of understanding is more dependent on the instruction received.

5. **How would you relate the way lessons were conducted and learners' understanding?**

Interviewer*: How would you relate the way lessons were conducted and learners' understanding?*

Teacher *: Actually, in every learning experience or environment, there are some that understand, I can say three quarters of the them understood and one quarter of them did not, not all can understand at* *one go, there are some were lagging behind so you have to try help them pick up, the conducting of lesson and understanding it links*

The answer by the teacher indicated that this teaching approach provides a structure through which a teacher assesses, identifies and understands learners' geometric levels of thought and progression. The statement made by the teacher: ***" ….so you have to try and help them…"*** suggested that instructions developed and delivered according to van Hiele's phases of learning propose means for identifying a learner's level of geometric understanding and ways to help learners to progress through levels of understanding. Crowley (1987) highlighted that instructions are significant in contributing to a learner's development of geometric thought. For developmet to occur, it is essential to match instruction with the learners' geometric level. Thus, teachers must learn to identify learners' levels of geometric thought. As mentioned by Mason (1998), learners will not understand content that is being taught at a level of thought that is above their level of understanding.

A further probe to find out how the lesson conducted linked to understanding revealed that a consultative learning environment created by following the van Hiele's phases of learning helped learners to understand. However, some learners seemed to be lagging behind such an environment because of the pace of the lesson, specifically for learners that cannot speak out during the lesson. This was observed during lessons presented as some learners were following the step-by-step instruction, as provided for in teaching activities without asking teachers where they could face difficulties.

6. **What experiences impacted your teaching?**

Interviewer*: Is there some experience which you can say in class, this actually impacted your teaching?*

Teacher*: Okay, yah look now, there is a lot of things which impacted my lessons, I will say for example the use of instruments, we last used them long time back, it also give some sort of remembrance, constructing angle, learners' interaction and discussion, asking questions how to bisect lines, and now because of this instruction, gives experience to learners to understand what to do.*

From the above teacher's answers and what was observed during lesson presentation, learners' understanding is better in visually presented concepts than verbally presented one. The phase of learning introduced learners to actively interact with concepts of study through step-by- step instructions. This gives learners the ability to experience concepts and use of instruments to construct, measure and make deductions for learners to own the process of learning. During lesson observation, learners were excited each time they used their instruments.

This finding shares similarities with Alex and Mammen's (2018) findings about learners' understanding of geometry terminology through the lens of van Hiele's theory. Their study found that learners' performance was better in dealing with visually presented terminology items than verbally presented terminology. One of the implications of van Hiele's theory is that effective learning takes place when learners actively experience objects of study in appropriate contexts (Mason, 1999). Embedding construction within a pedagogical framework provide learners with interest in exploring and experiencing objects of study. Therefore, teachers should provide their learners with appropriate experiences and opportunities to discuss them in order to actively interact and develop successive level of concepts understanding.

7. **What challenges did you face when using van Hiele's phases of teaching to prepare and conduct lessons?**

Interviewer*: What challenges did you face when preparing or conducting lessons?*

Teacher*: Okay, when I was conducting the lesson the time, the time was very little, because 50 minutes to teach someone how to construct and measure and explain again , time allocation requires a double period, preparing is also time consuming a bit. Some of them were facing difficulties in using instruments like using a protractor to measure just 60 degrees, which they did long time in grade 8, I was showing them how using instruments was very difficult for them, yah, that took a lot of my time, but it was useful and working for them.*

The teacher highlighted the following challenges he faced during the process of preparing teaching activities:

- Taking time to develop step-by-step instructions, putting into consideration the incorporation of the characteristics of five phases teaching approach. Despite the fact that these activities were designed by the researcher and the teacher, it seemed to be challenging to the teacher as he had no knowledge about phases. He could only follow the processes, as proposed by the researcher. This took some time as I had to explain why a given step should be included in the instruction. I made sure that all descriptions of the van Hiele teaching phases are considered in the step-by-step instruction to learners. I noticed that the teacher was just following the designing of the teaching instruction. A further probe to find out the reason for little input during the development and design of the step by step instructions in the teaching activities revealed that he lacked knowledge of van Hiele's phases of learning. He reminded me that it was I who introduced him to the van Hiele phases of learning and that he did not really know or understand the technicalities of the phases. This implies that teachers' lack of knowledge on van Hiele resulted in taking longer time to develop and design instructions according to the van Hiele's phases of learning.
- He further put forward the following challenges he faced during lesson presentation that, time allocated to each teaching activity was not enough to go through all the steps in the instruction

for learners to provide answers to class activities. However, he acknowledged that the framework used do prepare and deliver the lessons suits the purpose. I noticed during classroom observation that learners were not able to complete some classroom activities due to time limitations. For instance, Teaching for activity E, little time was given to learners to complete the activities, struggling learner could not do all questions of the activity, for example in investigation 6. It was further noticed that, the teacher rushed some of the investigations to complete the steps in a given a period of time. For example, in Teaching Activity B, Investigation 2, the teacher explained the investigation to learners and left it as homework. This signifies that the time allocated for the period was not enough to teach all investigations and for learners to practice. Although there is time constraint, the teacher admitted that the majority of learners could follow and execute the instructions and seemed to obtain answers faster for some steps than expected. The time allocated to each investigation was 40 minutes as indicated in Chapter 3, section 3.6.2. The time allocated to each teaching activity was informed by the prescribed teaching time at the site for each period in line with norms of the duration of the Euclidean geometry topic in CAPS.

8. **What benefits did you get from the lessons?**

Interviewer*: What benefits do you feel you got from lessons you conducted?*

Teacher*: Ah, from my point of view, experimenting is this true or we just reading from books, these authors are they really telling the reality, yah I proved it's the reality.*

Interviewer*: So, you were able to prove your reality from the books?*

Teacher*: Yah, I was able to prove the reality from the books.*

Interviewer*: And in terms of how lessons were prepared and flowing, if someone follows phases in the way they are, how is it beneficial?*

Teacher*: One of the benefits is like it will be very easy for the student to understand and even for the teachers as well.*

Interviewer*: How easy is it?*

Teacher*: Okay because we are taking it in phases, it's very easy for them to understand*

- This is what the teacher answered: ***"one of the benefits is easy for the student to understand and to follow".*** To find out the extent of how easy it is to understand by the learner, a further probe was done from the teacher. The follow up question was "How easy is it to understand?" The teacher had the following to say: ***"because it's taken into phases and the instructions are clear to the learners".***

The answer given by the teacher implies that the phases of learning are seen as a pathway leading to understanding of geometric concepts in circle geometry. This was manifested in the way learners were engaging with concepts through discussion and reflections. When I was observing the lessons, there was class inclusion, learners demonstrated vivid understanding of the step by step instructions, leading them to make conjectures about the relationship between angles, lengths and tangents as required in the investigation activities. The above finding is also in conformity with the findings of Dongwi (2014) which revealed that the phases of learning are seen by the teacher as a good pedagogical tool for planning and presenting lessons.

- The second benefit the teacher highlighted was experiencing teaching of theorems of the geometry of circles as a reality. He was excited about this design to use construction as a way of investigating and exploration the theorem. He said that he experienced what authors mentioned in the book as true. ***"Experiencing as true? Or we are just reading from the book? Are the authors telling the reality? I proved for myself it's the reality and true"***.

  The teacher's statement confirms that too often, geometry is taught in a "mechanical way" by generalising and simply relaying information to the learners. Consider the fact in teaching activity (C), investigation 3, that the angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at any point on the circumference. Frequently, this fact is established by generalising and simply learners are told the information. This way of teaching reduces the level of learners' understanding. Learners should be presented with a wide variety of geometric experiences, in particular, exploratory experiences such as construction of figures, paper folding and collection of shapes.

  Exploratory experiences provide learners with a powerful means, both inductively and deductively, for understanding the concept. Insight into proving the fact in teaching activity C, investigation 3, is obtained from construction and measurement explicit to the five sequential phases of learning. Concomitantly, the groundwork was laid for the formal proof, so learners were able to prove without much difficulty that Angle $A\hat{O}B = 2A\hat{C}B$. Therefore, implicit to van Hiele's phases of learning is the notion that learners should be presented with a wide variety of exploratory geometric experiences and opportunities.

9. **What area in Mathematics should apply similar teaching experience?**

Interviewer*: So, in Mathematics, you know there are a lot of topics, geometry, statistics and others, do you think there is any topic that can come in mind that we can apply the same experience?*

Teacher*: Yes, in Pythagoras theorem, if we apply the same, learners will understanding more.*

The teacher's answer revealed that, similar teaching experiences can be used in geometry of lines, especially during the teaching of Pythagoras theorem. As put forward by Dongwi (2014), the response from teachers pointed to graphs, functions and transformations. This creates avenues for further research to test the use of the van Hiele phases of learning to the mentioned areas.

To sum up, the findings from the teacher's responses show that there is lack of knowledge of van Hiele's theory and phases of learning from some of the teachers; yet the theory is a pedagogical teaching tool that provides a structure on which to base geometric instructions. Some of the benefits of using van Hiele phases of instruction is the ability to provide learning and teaching experiences to the teacher and learners to progress from one to the next level of geometric understanding. However, the van Hiele's phases of learning requires sufficient response time to develop van Hiele-based teaching activities and implement those activities in the classroom setting.

4.5 Conclusion

This chapter presented an analysis and discussed findings from document analysis, classroom observation and in-depth interviews used to provide data for answering the research questions.

The analysis of documents dealt with the CAPS and van Hiele's theory readings. The findings from CAPS revealed the curriculum and content focus for Grade 11 circle geometry. The curriculum for Grade 11 requires learners to prove seven theorems of the geometry of circle and solve riders. The findings from van Hiele's theory reading posit five sequential phases of learning, which are, information, guided orientation, explicitation, free orientation and integration; all these promote learners' acquisition of geometric levels of understanding. Therefore, the five phases of learning provide a pedagogical tool for planning and presenting circle geometry lessons.

The analysis of the teacher's interviews revealed some of the benefits and challenges of using van Hiele's phases of learning. The findings indicate that firstly, some teachers lack knowledge of van Hiele's theory and phases of learning. This challenges the teacher's ability to design, develop and deliver geometric lessons according to van Hiele's phases of learning. Secondly, lessons developed and presented according to van Hiele's phases of learning helps learners to progress through geometric levels of understanding. Research has supported the accuracy of van Hiele's phases of learning as a good pedagogical tool to develop and presents geometric lessons. Drawing on my own lived experience while carrying out this research, it takes time to develop and implement the theory in a classroom environment as careful attention is needed in the planning, design and implementation of the classroom-based activities which is exploratory in nature.

The next chapter presents the summary, recommendation and the conclusion of the study

CHAPTER 5 : SUMMARY, RECOMMENDATIONS, LIMITATION, AND CONCLUSION

5.1 Introduction

This chapter presents a summary of the findings, recommendations, and limitations of the study.

The purpose of the study was to provide a framework on which circle geometry instruction can be structured and taught to Grade 11 learners and explore how van Hiele's phases of learning facilitate the teaching and learning of circle geometry in grade 11. To achieve the purpose of this study, the following research questions were investigated:

- How does the van Hiele's theory of instructional design facilitate the learning of circle geometry to Grade 11 learners?
- What are the challenges of using Van Hiele's theory of instructional design to facilitate learning of circle geometry to Grade 11 learners?
- What are the benefits of using Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?

5.2 Summary of findings

The summary of findings will be discussed according to each of the research questions.

5.2.1 Main Research question

How does the van Hiele's theory of instructional design facilitate the learning of circle geometry to Grade 11 learners?

The findings show that the curriculum content focus area for Grade 11 circle geometry is to investigate and prove the theorems of the geometry of circles. The results from the findings indicated that van Hiele's phases of learning provide a tool to aid the developing and implementing circle geometry activities in the classroom environment through the following ways:

a) instruction design

The van Hiele phases of learning provide teachers with a step-by-step guide on how to develop and deliver circle geometry materials and instructions to learners.

b) Strategies of instruction design

The van Hiele phases of learning provide effective strategies required by teachers to deliver instructions to learners. The strategies that are implicit in the five sequential phases of learning are: organizational, delivery and execution strategies.

c) Instruction events

The van Hiele phases trigger events through which teachers and learners plan, control and monitor teaching and learning. The instruction events triggered by the five van Hiele phases of learning are task-specific and general tasks. These tasks provide teachers with opportunities to assess and provide feedback to learners, and for learners to execute the tasks.

d) Categories of learning

The van Hiele phases promote use of cognitive and attitude categories of learning. These forms of learning provide an internal process through which learners and teachers plan, control, and monitor their learning. Learners take ownership of their learning, and the teacher's role is to guide them to explore the connectivity in the concepts.

c) Pathway to teaching and learning

The van Hiele phases of learning provide a pathway through which the teacher guides the learners' geometric conceptualization by discussion and exploration. Through discussion, learners exchange ideas to develop a network of knowledge leading to the understanding of the concepts.

5.2.2. Specific research questions

5.2.2.1 What are the possible challenges of using van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?

From the findings and analysis of data related to this question, two challenges have been identified. These challenges are that it takes time to plan and design and develop van Hiele based instruction activities that mirrors any given particular classroom context.

Secondly, teachers' lack of knowledge of van Hiele's theory and its effectiveness on teaching and learning hinders their ability to develop and sequence geometric instructions according to the proposed phases of learning by van Hiele. The upshot of this is the possibility that the content covered requires more time to be completed.

5.2.2.2 What are the benefits of using Van Hiele's theory of instructional design to facilitate the learning of circle geometry to Grade 11 learners?

This study has highlighted some of the benefits of using van Hiele's theory of instruction design which act as a vehicle for learners to progress through geometric levels of understanding. These include:

- Learners developing exploratory skills which includes interpreting situations in mathematical terms, making conjectures, exploring various strategies for problem- solving and making generalisation.

- Enhancing motivation to learn: learners' ownership of learning, teaching patterns, the dynamic interaction between learners and teachers, and a good classroom environment produce interest and motivation to learn more effectively.
- Encouraging collaborative learning ; learners develop mathematics ideas through helping and assisting one another during learning.

5.3 Reflection on the study

I have learnt a lot in conducting this research. For example:

- Research can be a guiding tool on classroom teaching and learning;
- Building a good connection with the participants is helpful in encouraging learners to express their thinking when probed with questions about the study;
- There are lessons that can be drawn from the findings such as nature of instruments (teaching activities) and the sequencing of lessons on theorem of the circle of geometry;
- As an educator and practising teacher, I developed a better understanding of van Hiele's theory and the approach to teaching circle geometry;
- I found it challenging during the first two periods of the classroom observation as learners thought I was investigating their teacher's ability to teach.

The list above shows the key areas that I can draw lessons from conducting this research. I found the last point important to me that in my future research, I need to make the purpose of my research clear to the participants.

I found the research project challenging and frustrating at times, for example, the death of my first supervisor, Dr. Siyepu (RIP). His death occurred in the middle of my research project, and it took some time to get a new supervisor. However, the appointed supervisor made wonderful experiences and the research project interesting.

5.4 Limitations of this study

Creswell (2009) acknowledges that all research designs have limitations. Atieno (2009) suggests that one of the main limitations of qualitative research is concern regarding the transferability of findings. There are two main reasons for faulting qualitative research. As discussed by Hamel (1993), one of the reasons is lack of representativity, meaning generalization cannot be made based on sample size; the second reason is lack of rigour in the data collection and analysis based on the bias of the subjectivity of the researcher and that of the respondents. These criticisms is based on the fact that personal experiences and beliefs are biased and subjective.

The findings of this study have implications on the teaching strategy for assisting teachers through the development of a set of instructions that could facilitate the teaching and learning of circle geometry. This study relied exclusively on one teacher and one school, which might have contributed significantly to bias in the design. Hence the findings of the study cannot be generalized.

5.5 Implication of the study

The results of this study provide a guide for classroom instruction design that exposes learners to mathematical reasoning and creative skills. The use of van Hiele's phases of learning, as a pedagogical tool for geometry instruction, provide learners with mathematical experiences to reason and be creative. The emphasis of Mathematics in the FET phase is to expose learners to mathematical experiences that give them many opportunities to develop their mathematical reasoning and creative skills (DBE, 2018: 10). The results of this study give strong support to the notion of developing learners' reasoning and creative skills. What is central in the study is the teaching strategy to assist teachers to guide learners to provide explanations, justifications and prove conjectures. It is within this pedagogical context that this study finds practical significance.

5.6 Recommendations

The purpose of this study was to investigate how van Hiele's theory of instruction designs facilitate learning of circle geometry in Grade 11. This study makes recommendations that positively influence the design and facilitation of circle geometry as a means of transforming the teaching approach which ultimately improves learners' understanding and performance in geometry. The following recommendations are proposed for WCED and DBE, teachers and for future research.

5.6.1 Teachers' professional development

The WCED and DBE should provide adequate and appropriate knowledge-based training to teachers on van Hiele theory and phases of learning through teachers' professional development workshops. The professional development of teachers is more important than it has ever been in history. Teachers need to keep abreast of the emerging knowledge within their subject area through a knowledge base for education (Guskey, 2000). Therefore, teachers must get acquainted with the van Hiele's models. Professional development of teachers is, therefore, crucial in ensuring that teachers fulfil the requirement of being well-grounded in knowledge and skills related to their discipline.

5.6.2 Teaching pedagogy

Mathematics teachers should shift from traditional approaches of teaching circle geometry to what van Hiele's theory proposes. The findings of this study show that successful teaching of Grade 11 circle geometry design of instruction needs to draw on the five van Hiele's phases of learning. To date, many learners encounter difficulties in school geometry, especially when it comes to answering complex questions in Euclidean geometry. This can largely be alluded to the teachers' use of traditional approaches. Teachers should encourage learners to use construction as a strategy for the investigation of theorems in order to establish connections and new conjectures.

5.6.3 Recommendations for Further research

The review of literature revealed that little research has been conducted on teachers’ knowledge of van Hiele’s phase of learning as a pedagogical tool for geometry. As such, there is a need for future research and open debates within the broader Mathematics education community. Further research may be conducted on teachers’ knowledge of the van Hiele’s theory and phases of learning as a pedagogical tool for geometry to act as a contributor to teaching and learning. For this specific study, the sample size of one teacher was sufficient in terms of time, depth, and objectives of the study. However, for conducting a more in-depth study on teachers' knowledge about the van Hiele’s theory and phases of learning, a larger sample size of teachers in different schools is recommended.

It would be interesting to further investigate the application of van Hiele’s theory and phases of learning on other mathematical content areas other than geometry. As the results of the investigation of this study seem to agree with previous studies carried out on geometry, further research studies that can explore the application of van Hiele’s theory on other mathematical content area is needed.

5.7 Conclusion

This chapter provided a summary of the findings by focusing on how research questions were answered. Data suggests that van Hiele’s theory of instruction design facilitates the learning of circle geometry in grade 11. It is therefore important that teachers embrace van Hiele’s theory as a teaching strategy for geometry to ensure that learners understand the concepts of the theorem of the geometry of the circle. Although some teachers are not knowledgeable on the van Hiele theory and its phases of learning, continued professional development opportunities are suggested as a means of improving their teaching capability and making them aware of the van Hiele theory and van Hiele phases of learning.

Further, it is imperative that teachers and all role-players understand the need to design instructions that are aimed at promoting understanding in Mathematics.

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APPENDICES

APPENDIX A1: TEACHING ACTIVITY A:

Week 1: Day 1

Duration: 60 minutes

Day	Activity	Van Hiele level	What to observe
Monday	Revise Grade 10 work • circle terminologies • Axiom: tangent to circle tangent to a circle is perpendicular to the radius, drawn to the point of contact.	• Level 2 • Level 3	Analysis of the identification of Components of a circle Short chain deduction.

Step 1: To draw a circle and explore the following circle concepts and terminology.

Exploring circle concepts:

- Centre , Radius , Diameter
- Sector . Arc, Chord ‘ Segment
- Tangent , and Circumference

Terminologies:

- Subtend
- Cyclic quadrilaterals
- Alternate segments.

Step 2:

- To draw a circle with centre O.
- Draw a tangent PQ to the circle and join the point of tangency T to the centre O
- With a protractor measure angle STO.
- Make a conjecture about the tangent relationship between the tangent and the radius drawn at the point of contact with the circle.

APPENDIX A2: TEACHING ACTIVITY B:

INVESTIGATION: 1

Week 1: Day 2

Duration: 40 minutes

Day	Activity
2	Investigate and prove: the line drawn from the centre of a circle perpendicular to a chord bisects the chord.

Investigation

Step 1: construct a circle with centre O

Step 2: select two centres on the circle. Label them P and Q

Step 3: Draw a line from the centre O to bisect the chord PQ at R, the centre point on the chord.

Step 4: With your protractor, measure angle PRO and QRO.

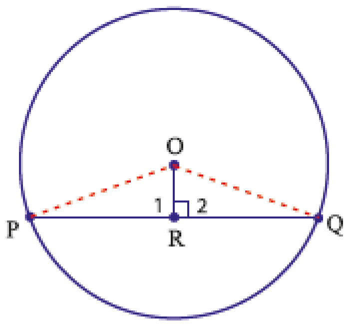
Step 5: With a ruler measure length PR and RQ

Step 6: compare length PR and RQ

Step 7: make a conjecture.

Proof that triangle OPR is similar to triangle OQR

Illustration of the activity

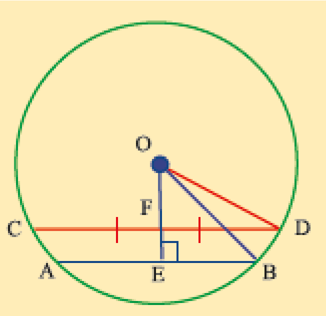


Converse of the theorem:

Complete the statement: the line drawn from the centre of a circle to the midpoint of a chord will be …………………………………………..to the chord.

Activity 1:

In the diagram below OE ⊥ AB , CD = FD, OE = 63cm, FE – 3cm, AB = (2x -18)cm and CD = 2xcm. Determine the length of the radius.



APPENDIX A 2: TEACHING ACTIVITY B:

INVESTIGATION: 2

Week 1: Day 2

Duration: 40 minutes.

Day	Activity
	Investigate and prove: the perpendicular bisector of a chord passes through the centre of the circle.

Investigation 3

Step 1: construct a circle

Step 2: select two points on the circle and draw a chord AB

Step 3: Bisect the chord AB at point T such that AT = TB

Step 4 Construct a line QT such Q is at the circumference of the circle angle QTA = QPB = 90^0

Step 5: Construct line RA and RB where R is any point on QT

Step 6: measure length RA and RB. How is the measure RA compare with the measure RB?

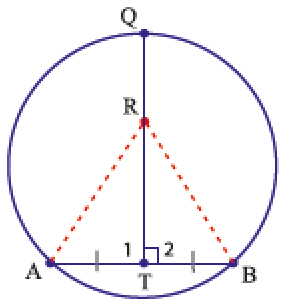
Step 6: What is the relationship between triangle RAT and RTB?

Step 7: show that in triangle RTA and RTB, RA = RB

Step 9: Make the following conjectures: what is the relationship between:

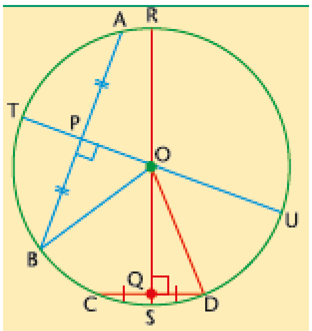
1. Equidistant point from the chord to the line bisecting the chord?
2. The centre which is equidistant to all points on the circumference?

Illustration:



Application: Activity:

A circle with radius 50 units has chords AB =60 units, and CD =28 units. TU and RS are the perpendicular bisectors of AB and CD respectively. Determine the shortest distance of each chord from the centre



APPENDIX A3: TEACHING ACTIVITY: C
INVESTIGATION: 3

Week 2: Day 3

Duration: 40 minutes.

Day	Activity
	Investigate and prove: the angle subtended by an arc at the centre of a circle doubles the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre).

Investigation 3:

Step 1: construct a circle with centre O

Step 2: Select two points on the circle label them A and B to form arc AB.

Step 3: Draw line AO and OB such that arc AB subtend an angle AOB at the centre.

Step 4: Select a point on the circle label it C such that C is on the major arc.

Step 5: Join Point C to points A and B such that arc AB subtend angle ACB at the circumference.

Step 6: With a protractor, measure angles AOB and ACB? How does the measure of angle AOB compare with angle ACB?

Step 7: Similarly draw line CO extend it through O to the circumference to make point Q.

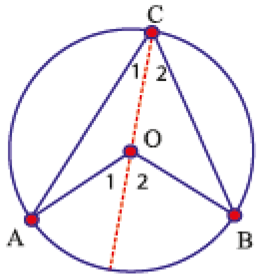
Step 8: With a protractor, measure the following angles AOQ and BOQ, ACO and BCO

Step 9: How does the measure of angle AOQ compare with angle ACO, and BOQ compare with BCO

Step 10: How does the measure of angle AOB compare with the sum of AOQ and BOQ

Step 11: what is the relationship between angle AOB and ACB.

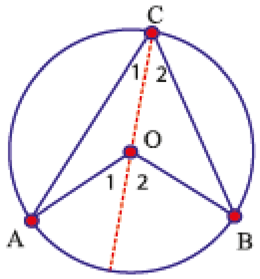
Step 12: Make a conjecture: what is the relationship between the angle subtended by an arc at the centre of a circle and the angle subtended by the same arc at the circumference.



Application

Prove that the angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at any point on the circumference of the circle.

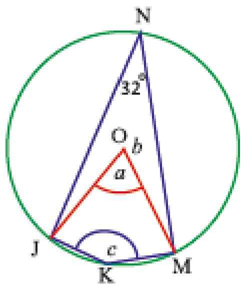
Illustration:



Required: Prove that angle AOB = 2 ACB

Activity:

O is the centre of the circle in the figure below. Determine the value of *a* and *b*



APPENDIX A3: TEACHING ACTIVITY: C
INVESTIGATION: 4

Week 2: Day 3

Duration: 40 minutes.

Day	Activity
	Investigate and prove: angles subtended by a chord of the circle, on the same side of the chord are equal.

Investigation 4

Step 1: Construct a circle with centre O

Step 2: Select two points on the circle label them A and B to form arc AB. Join A to B to form chord AB

Step 3: Select a point on the circle label it F such that F is on the major arc,(in the major segment)

Step 5: Join Point F to points A and B such that chord AB subtend angle AFB.

Step 6: With a protractor, measure angles AFB?

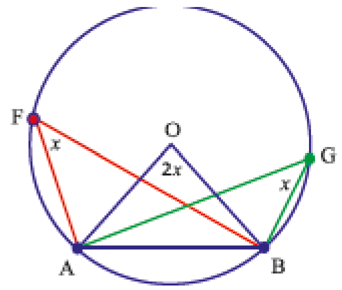
Step 7: Similarly select another point on the circle and label it G such that G is on the major arc (in the major) segment, Join G to points A and B such that chord AB subtend angle AGB.

Step 8: With a protractor, measure the angle AG B.

Step 9: How does the measure of angle AFB compare with angle AGB

Step 10: Make a conjecture: what is the relationship between the angles subtended by a chord of a circle at the same side of the chord?

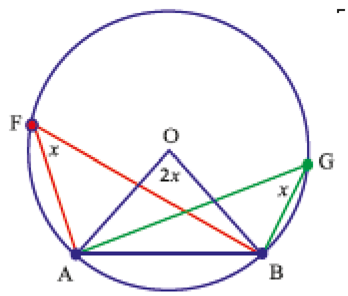
Illustration:



Application:

Prove that the angles subtended by a chord of the circle on the same side of the chord are equal

Illustration:



Given A, F , G and B are points on the circle with centre O.

Step: Join AO and OB,

Required to prove: Angle AFB = Angle AGB.

APPENDIX A4: TEACHING ACTIVITY: D
INVESTIGATION: 5

Week 2: Day 3

Duration: 40 minutes

Day	Activity
	Investigate and prove : the opposite angles of a cyclic quadrilateral are supplementary

Investigation 5:

Step 1: construct a circle with centre O

Step 2: Select four points on the circle label them D, E, F and G. Join these points such that DEFG is a quadrilateral.

Step 3: With your protractor, measure the size of following angles:

D and F

G and E

Step 5: Compare the sum of the measure for angles

D and F

G and E

Step 6: What is the relationship between the sums of opposite angles in a cyclic quadrilateral?

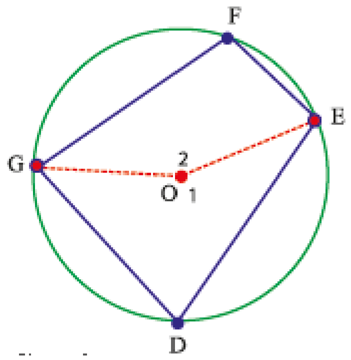
Step 7: Similarly draw a line GO and OE.

Step 8: Measure angle GOE to the side of F and to the side of D

Step 9: How does the measure of GOE to the side of F compare with GOE to the side of D?

Step 10: Make a conjecture about the opposite angles of a cyclic quadrilateral.

Illustration



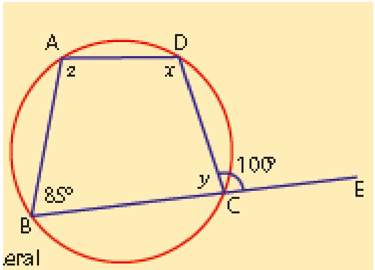
Application:

Prove that the opposite angles of a cyclic quadrilateral are supplementary

Required to prove D + F = 180 and DEF + DGF =180.

Practice Activity:

A, B, C and D lie on the circle and BC is produced to E. Angle B = 85 and DCE = 100. Determine, with reasons the size of *x*, *y* and *z.*



APPENDIX A5: TEACHING ACTIVITY: E

INVESTIGATION: 6

Week 2: Day 3

Duration: 40 minutes.

Day	Activity
	Investigate and prove: two tangents drawn to a circle from the same point outside the circle are equal in length.

Investigation 6

Step 1: Construct a circle with centre O

Step 2: Select two point outside of the circle label it R

Step 3: Draw two tangents to the circle from point P such that P and Q are the respective points of tangency for the two lines RP and RQ.

Step 5: Measure the length RP and RQ

Step 6: What is the relationship between the lengths RP and RQ?

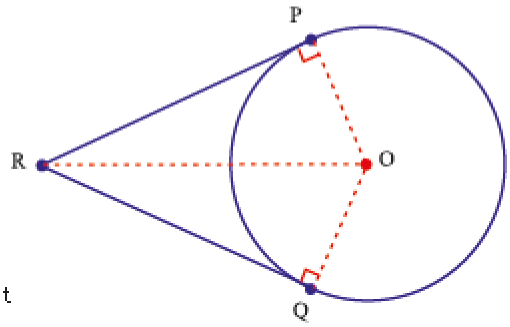
Step 7: Construct line PO , QO and RO to the centre O

Step 8: Measure angle OPR and OQR

Step 9: What is the relationship between the two angles?

Step 10: Make a conjecture about the length of two tangents drawn from the same point outside circle.

Illustration:



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Application:

Prove that two tangents drawn to a circle from the same point outside the circle are equal in length.

Required to prove: RP = RQ

APPENDIX A 5: TEACHING ACTIVITY: E

INVESTIGATION: 7

Week 2: Day 3

Duration: 40 minutes.

Day	Activity
	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

Investigation 7

Step 1: Construct a circle with centre O

Step 2: Select two points on the circle and draw a chord AB

Step 3: Select other two points C and D in the minor and major segments such chord AB subtends angle ACB and BAD in the respective segments.

Step 4: Draw a tangent AD to the circle such that point A is a points of tangency

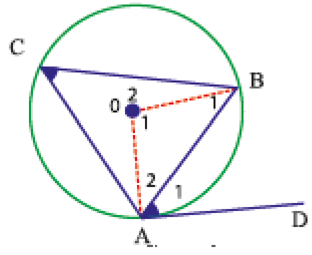
Step 5: Measure the following angles; DAB ; ACB ;

Step 6: What is the relationship between the following angles:

1. DAB and ACB?

Step 7: Make a conjecture about the angle between the tangent to a circle and the chord drawn from the point of contact to the angle in the alternate segment.

Illustration



Application

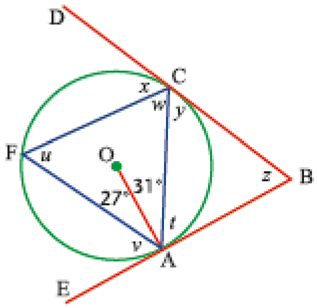
Prove the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle subtended by the chord in the alternate segment.

Required to prove : angle BAD = C

Activity:

O is the centre of the circle in the figure below. Determine the value of *u, v, t, w, x, y,* and *z*

1



APPENDIX A6: ACTIVITY 6: APPLICATION

TASK 1

Week 3: Day 1

Duration: 40 minutes.

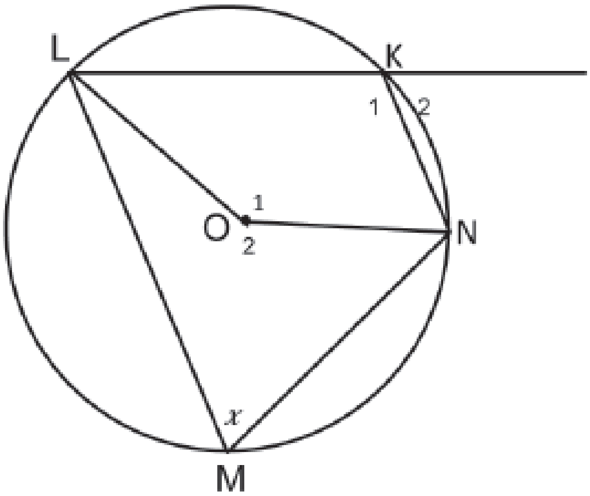
Day	Activity
	Use the above theorem and their converses to solve riders.

Examples: Adopted from CAPS page 34

Example:

1. *AB* and *CD* are two chords of a circle with centre O. *M* is on *AB* and *N* is on *CD* such that $OM \perp AB$ and $ON \perp CD$. Also, *AB* = 50mm, *OM* = 40mm and *ON* = 20mm. Determine the radius of the circle and the length of *CD*.

2. *O* is the centre of the circle below and $\hat{O}_1 = 2x$.



2.1. Determine $\hat{O}_2$ and $\hat{M}$ in terms of x.

2.2. Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of x.

2.3. Determine $\hat{K}_1 + \hat{M}$. What do you notice?

2.4. Write down your observation regarding the measures of $\hat{K}_2$ and $\hat{M}$.

APPENDIX A6: ACTIVITY 6: APPLICATION

TASK 2

Week 3: Day 1

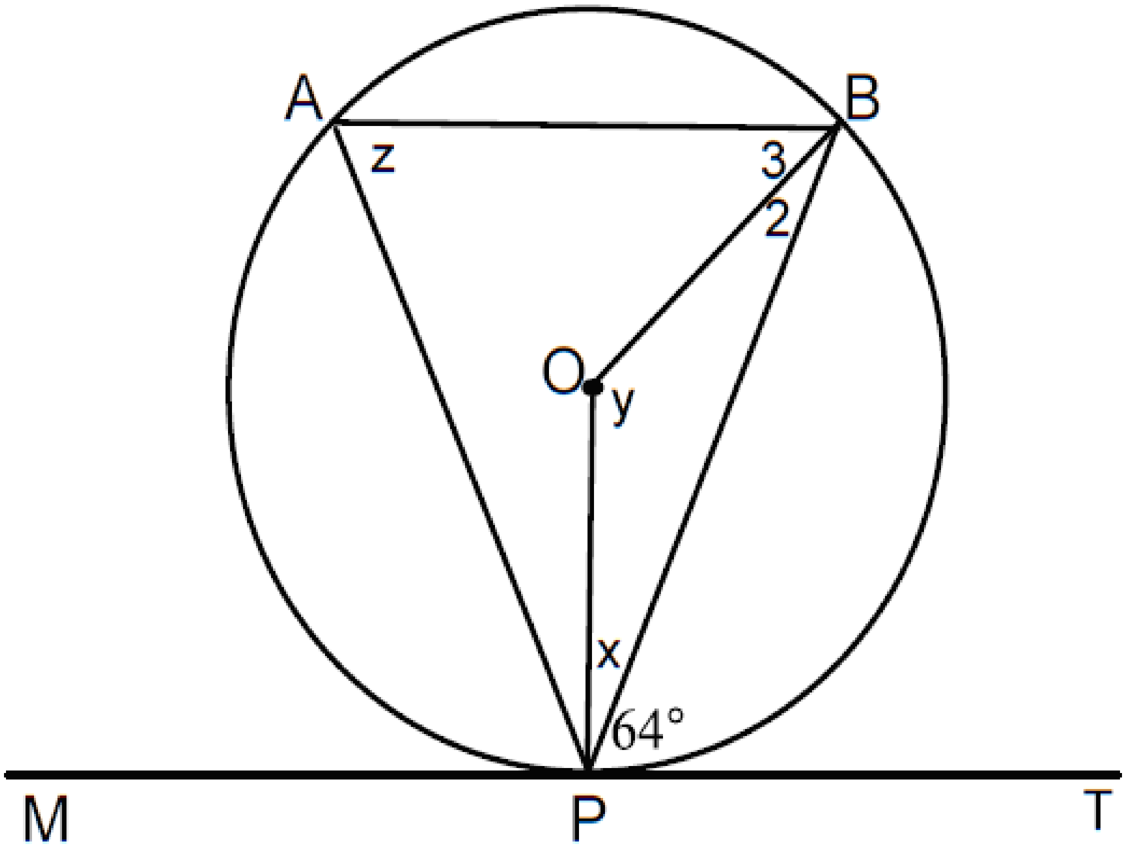
Duration: 40 minutes.

Day	Activity
	Use the above theorem and their converses to solve riders

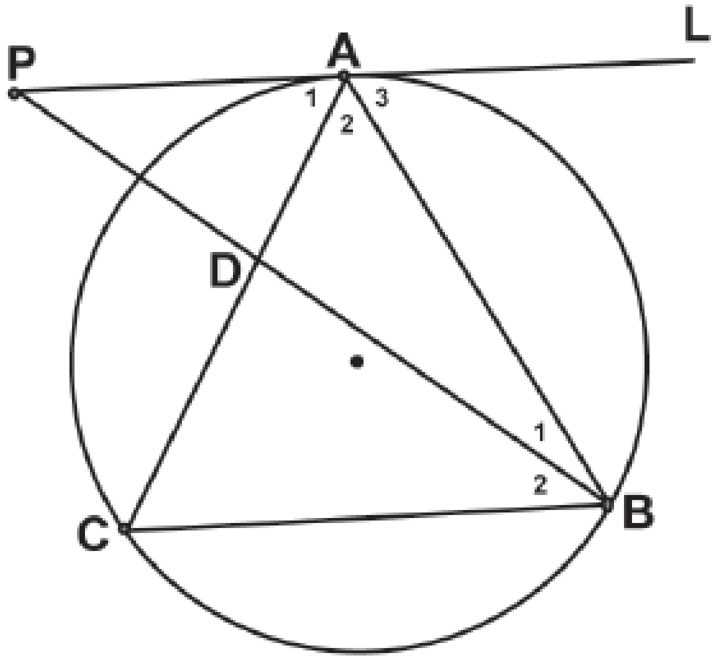
Teacher's instruction:

Examples: Adopted from CAPS page 35

3. O is the centre of the circle above and MPT is a tangent. Also, $OP \perp MT$ Determine, with reasons, x, y and z.



4. Given: $AB = AC$, $AP \parallel BC$ and $\hat{A}_2 = \hat{B}_2$.



Prove that:

4.1 *PAL* is a tangent to circle *ABC*;

4.2 *AB* is a tangent to circle *ADP*.

APPENDIX B: OBSERVATION PROTOCOL

Classroom observation

Project: The application of Van Hiele's theory of instructional design to facilitate the learning of circle geometry in Grade 11.

Time of observation: ____

Date: ____

Place: ____

Lesson observed: ____

Phase	**Observable descriptions of the phase**	**Observed**
Information	Discussions take place between teacher and learners that stresses the content to be used	
Directed orientation	The teacher guides learners to uncover the connection and identify the focus of the subject matter through a series of teacher guided tasks	
Explicitation	Learners express new ideas in accepted mathematical language. Teacher's main role is to develop technical language with understanding through the exchange of ideas.	
Free orientation	Learners can complete activities that require a number of steps in which they are required to find their own way in the network of relations. Teacher selects appropriate geometric activities that require certain level of thinking for learners to solve them successfully.	

Integration	Learners can summarize the new understanding of the concept involved and incorporate language of the new level by making conjectures. Teacher assists with the correct and appropriate conjectures.	

Comments

APPENDIX C: INTERVIEW PROTOCOL

INTRODUCTION

Project: The application of Van Hiele's theory of instructional design to facilitate the learning of circle geometry in Grade 11

Time of Interview: ____

Date: ____

Place: ____

Interviewer: ____

Interviewee: ____

Position of Interviewee: ____

I want to thank you for taking the time to meet with me today. My name is ___________________________ and I would like to talk to you about learning experienced during the lesson and how it impacted your understanding of circle geometry. Specifically, one of the components of evaluation and assessing effectiveness of Van Hiele instruction is design to guide learning in order to capture lessons that can be used in future interventions.

The interview will take less than fifteen minutes. I will be recording the session because I do not want to miss any of your comments. Although I will be taking some notes during the session, I cannot possibly write fast enough to get it all down. Because I am recording, please be sure to speak up so that I do not miss your comments.

All responses will be kept confidential. This means that your interview responses will only be shared with research team members and we will ensure that any information we include in this report does not identify you as the respondent.

Remember, you do not have to talk about anything you do not want to and you may end the interview at any time.

Are there any questions on what I have just explained?

____

Are you willing to participate in this interview?

__________________	__________________	______________
Interviewee	Interviewer	Date Questions:

QUESTIONS:

1. Please describe your experience regarding Mathematics and what do you think about circle geometry in South African curriculum?

____

____

2 How would you describe your experience in teaching circle geometry?

____

____

3. Did you know about van Hiele's teaching phases?

____

____

4. How would you describe your experiences of using van Hiele's teaching phase to teach circle geometry?

____

____

5. How would you relate the way lessons were conducted and learners' understanding?

____

____

6. What experiences impacted your teaching?

____

____

7. What challenges did you face using van Hiele's phases of teaching to prepare and conduct the lessons?

8. What benefits did you get from the lessons?

____

____

9. What area in Mathematics should apply similar teaching experience?

____

____

CLOSING:

Is there anything more you would like to add regarding the lesson?

____

____

Thank you for your cooperation and participation in this interview. I would like to assure you once again that responses will be kept confidential and anonymous.

Thank you for your time.

APPENDIX D: SCHOOL INFORMATION LETTER

The Principal

4th February 2020

Dear Sir

<u>REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN YOUR SCHOOL FOR MY CPUT RESEARCH PROJECT</u>

I am currently affiliated with Cape Peninsula University of Technology where I am doing my Master in Education: Mathematics Education.

My research topic is:

THE APPLICATION OF VAN HIELE'S THEORY OF INSTRUCTIONAL DESIGN TO FACILITATE THE LEARNING OF CIRCLE GEOMETRY IN GRADE 11

I would like to obtain your permission to carry out my research at this School. I would also request your permission to approach the Grade 11 Mathematics teacher to seek his permission to participate in this study.

My role will be to observe a Mathematics lesson in Grade 11 classroom and interviewing the teacher. I will not, in any way, disrupt other learning processes and school activities

My research period will be for 3 weeks where possible in Term 3. I will work closely with your teacher and learners every day for 60 minutes.

All the information obtained from my observation and the interview will be kept strictly confidential and the above arrangement can be terminated at any time. The research project, when completed, will be available for you to view. Please note that nowhere will you, your school, teacher or learners' identity be revealed in my findings.

I will require you and the educator, to sign this letter of consent to give me your permission to continue with this research.

Please feel free to contact me if you need any additional information regarding this research study.

Yours sincerely

____

(Ali Lwanga)

0619492792

lwngali@gmail.com

I Mr. ____give permission to conduct research at this School for your CPUT Research project.

____

(Principal)

Mr. ____give permission to observe and interview my Grade 11 Mathematics class for your CPUT Research project.

____

(Teacher)

APPENDIX E: CPUT ETHICAL CLEARANCE CERTIFICATE

Cape Peninsula University of Technology

Cape Peninsula University of Technology

Private Bag X8, Wellington, 7654
Jan van Riebeeck Street, Wellington, 7654
Tel: +27 21 864 5200

P.O. Box 652, Cape Town, 8000
Highbury Road, Mowbray
Tel: +27 21 680 1500

FACULTY OF EDUCATION

On the **14 July 2020** the Chairperson of the Education Ethics Committee of the Cape Peninsula University of Technology granted ethics approval **EFEC 5-5/2018** to **A Lwanga** for research activities related to the degree **Masters in Education** the Cape Peninsula University of Technology.

Title:	THE APPLICATION OF VAN HIELE'S THEORY OF INSTRUCTIONAL DESIGN TO FACILITATE THE LEARNING OF CIRCLE GEOMETRY IN GRADE 11

Comments:

Permission is granted to conduct research within the Cape Peninsula University of Technology only. Research activities are restricted to those details in the research project. Ethical clearance for this project is only valid until 31st of December 2023.

CLivingston

Date: 14 July 2020

Dr Candice Livingston

Research coordinator (Wellington) and Chair of the Education Faculty Ethics committee

Faculty of Education

APPENDIX F: WCED ETHICAL CLEARANCE LETTER

Western Cape Government
Education

Directorate: Research

Audrey.wyngaard@westerncape.gov.za
tel: +27 021 467 9272
Fax: 0865902282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20180418–1372
ENQUIRIES: Dr A T Wyngaard

Mr Ali Lwanga
33 Hazel Street
Avonbood
Elsies River
4970

Dear Mr Ali Lwanga

RESEARCH PROPOSAL: THE APPLICATION OF VAN HIELE'S THEORY OF INSTRUCTIONAL DESIGN TO FACILITATE THE LEARNING OF CIRCLE GEOMETRY IN GRADE 11

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **19 July 2018 till 31 August 2018**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

    The Director: Research Services
    Western Cape Education Department
    Private Bag X9114
    CAPE TOWN
    8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 23 April 2018

Lower Parliament Street, Cape Town, 8001   Private Bag X9114, Cape Town, 8000
tel: +27 21 467 9272   fax: 0865902282   Employment and salary enquiries: 0861 92 33 22