

Decentralized discrete optimal quadratic automatic generation control of interconnected power systems based on the functional minimization method

by

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Thesis submitted in fulfilment of the requirements for the degree

Doctor of Engineering: Electrical Engineering

in the Faculty of Engineering & the Built Environment

at the Cape Peninsula University of Technology

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Bellville

7th December 2023

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ABSTRACT

The distribution of electrical power over a large geographic area, the recent integration of distributed resources, the adoption of novel concepts like smart grids, and the digitisation of the power systems leads will undoubtedly increase the size of Multi-Area Interconnected Power Systems (MAIPS). Multi-area interconnected power systems are significantly complicated to control. Therefore, designing the Automatic Generation Control (AGC) is more complex and challenging. In order to control interconnected power systems, a robust AGC approach based on a Linear Quadratic Regulator (LQR) is being considered as a solution to these AGC problems. The LQR control approach provides wide stability margins, strong rejection of nonlinearities, reliability, simplicity, and good disturbance rejection. Despite the difficulty in determining the optimal state and controlling weighting matrices, the AGC design based on the LQR technique is not commonly employed.

This study focuses on creating Automatic Generation control strategies based on Linear Quadratic Regulator and Functional Minimization Methods (FMM), and developing software programs and algorithms to address the AGC problems in MAIPS. In addition, such a complex system requires an analysis of the step load disturbances, modelling uncertainties, and nonlinearities of the MAIPS using decomposition and decentralisation approaches. According to the literature review, most AGC controllers are designed in continuous time and implemented in discrete time. The literature review findings show that the functional minimisation method (FMM) is suitable for constructing weighting matrices. The research work designed and implemented LQR-based AGC for the interconnected power systems in the discrete-time domain.

To address the difficulties in AGC design, MAIPS has developed two AGC strategies, namely Discrete Centralized Optimal Quadratic Automatic Generation Control (COQAGC) and Discrete Decentralized Optimal Quadratic Automatic Generation Control (DOQAGC). DOQAGC is formulated for AGC problems in MAIPS based on optimal control theory and overlapping decomposition technique. The state and control weighting matrices are carefully selected for both developed control techniques using the functional minimisation process. At the same time, the overall system is divided into various subsystems via overlapping decomposition. The developed control methods and algorithms are tested in the MATLAB/SIMULINK environment with step load perturbations, modelling uncertainties, and generation rate limits (GRC),

The selection of the states and control weighting matrices for both controllers is further streamlined and systematized by FMM. However, the DOQAGC's dynamic responses are greatly impacted by the fact that it disregards the effects of subsystem interconnections. The

simulation findings show that DOQAGC is more reliable than COQAGC.

Keywords:

Multi-area Power Systems, Automatic Generation control, Decentralized Discrete Optimal Control, Functional Minimization Method, Linear Quadratic Regulator, Optimal Theory, and Overlapping Decomposition.

ACKNOWLEDGEMENTS

I wish to thank:

- The members of my family, Father Yagoub Ibrahim, Mother Fatima Adam, Brothers Adam and Ahmed, Sister Maryom for their encouragement, patience, and understanding during the research journey.
- Late Professor Raynitchka Tzoneva for her valuable guidance, supervision, support and suggestions since the beginning of the study when she suggested the project title. She is always remembered; may God has mercy on her.
- Thanks to Associate Professor Senthil Krishnamurthy for his supervision, excellent guidance, and support. I am grateful for his keen involvement and, this thesis would not have been possible without his suggestions, dedication, patient and his mentorship on the project.
- The NRF, ESKOM TESP and the centre of Substation Automation and Energy Management Systems (CSAEMS) at Cape Peninsula University for their support in different ways.

DEDICATION

To all my family members all, my friends Abdulaziz, Safa, Ahmed and Hajer and all the members of the Centre for Substation Automation and Energy Management Systems (CSAEMS), CPUT

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GLOSSARY

Abbreviation/Acronym	Definition/ Explanation
1 st ODEs	First Ordinary Differential Equations
2DOF-ID	Two Degree of Freedom Integral Derivative
3DOF-ID	Three Degree of Freedom- Integral Derivative Control
3DOF-PID	Three Degree of Freedom-Proportional Integral Derivative control
ACE	Area Control Error
AGC	Automatic Generation Control
AIC	Area interchange control
ANFIS	Adaptive Artificial Neural Fuzzy Inference System
ANN	Artificial Neural Network
ARC	Algebraic Riccati Equation
AVR	Automatic Voltage Regulator
A wind farm	is a group of wind turbines in the same location used to produce electricity.
BBO	Biogeography-Based Optimization
BD	Boilers Dynamics
BFOA	Bacterial Foraging Optimization Algorithms
CDO	Collective Decision Optimization Algorithms
CMPC	Centralized Model Predictive Control
OQAGC	Centralized Discrete Optimal Quadratic Automatic Generation Control
COQAGC	Centralized Discrete Optimal Quadratic Automatic Generation Control
DADRC	Decentralized Active Disturbance Rejection Control
DETC	Decentralized Event Triggered Control
DISCOs	Distribution Companies
DLFC	Decentralized Load Frequency Control
DLQR	Distributed Linear Quadratic Regulator

DMPC	Distributed Model Predictive Control
DOISMCLFC	Decentralized Optimal Integral Sliding Mode Control based Load Frequency Control
DOQAGC	Decentralized Optimal Quadratic Automatic Generation Control
DPM	DISCO participation matrix
DRRT	Disturbance Rejection Response Time
DSA	Dragonfly Search Algorithm
Eigenstructure assignment	A technique is used for assigning the eigenvalues and eigenvectors of a closed-loop linear system via a feedback control law.
ESO	Extended State Observer
$e_{ss}(\infty)$	Steady State Error
FA	Firefly Algorithm
FAIPS	Four area interconnected power systems
FFOPIFOPD	Fractional order proportional integral-fractional order proportional derivative
FGSC	Fuzzy Gain Scheduling Controller
FMM	Functional Minimization Method
FOPI-FOPD	Fractional Order Proportional Integral-Factional Order Proportional Derivative Control
FSFC	Full State Feedback Control
GAs	Genetic Algorithms
GENCOs	Generation Companies
GDB	Governor Dead Band
GOA	Grasshopper Optimization Algorithms
GRC	Generation Rate constraints
GWO	Gray Wolf Optimization
HCPSO	Hybrid Particle Swarm Optimization
hIFA-PS	Hybrid Firefly Optimization Algorithm and Pattern Search technique.
H_{∞}	H-infinity
H_{∞} -SOF	H_{∞} -Static Output Feedback Control

HBF-PSO	Hybrid Bacteria Foraging oriented Particle Swarm Optimization
HYGWO-SCA	Hybridized Gray Wolf Optimization and Sine Cosine Algorithm
I	Integral Control
IACE	Integral of Area Control Errors
IEC61499	Is an International Standard for Modelling Distributed Control Systems
IEC61850	Is an International Standard Defining Communication protocols for Intelligent Electronic Devices at Electrical Substations.
I/O	Input/output
ID	Integral Derivative Control
ILMI	Iterative Linea Matrix Inequalities
ITAPS	Identical Two Area Power System
IPA	Interior Point Algorithm
IPA-TID	Interior Point Algorithm-based Tilt Integral Derivative Control
ISE	Integral Square Error
ITAE	Integral Time Absolute Error
LFC	load Frequency Control
LMI	linear Matrix Inequality
LQR	Linear Quadratic Regulator
LQR-LFC	LQR based Load Frequency Control
MAIPS	Multi-area Interconnected Power Systems
MATLAB	MATrix LABoratory Software
MPC	Model Predictive Control
MSPGs	Multi-Source Power Generators
NAIPS	N Multi-Area Interconnected Power Systems
NRA	Newton-Raphson Algorithm
OAGC	Optimal Automatic Generation Control
ODLQR	Optimal Decentralized Linear Quadratic regulator control
ODLQRLFC	ODLQR Load Frequency Control

OFC	Output Feedback Control
Pattern search	Numerical optimization methods that do not require gradients
PI	Proportional Integral Control
PID	Proportional Integral Derivative control
Peak OS	Peak Overshoot
PS	Pattern Search Technique
PSO	Particle Swarm Optimization
PUS	Peak Undershoot
PVS	Photovoltaic Systems
Photovoltaic	Photovoltaic convert light into electricity using semiconducting materials.
Q	State Weighting Matrix
QOHS-PID)	Quasi Oppositional Harmony Search Algorithm –Proportional Integral Derivative Control.
R	Control Weighting Matrix
RCGA	Real Coded Genetic Algorithm
RDLFC	Robust Decentralized Load Frequency Control
RES	Renewable Energy Sources
RGM	Relative Gain Matrix
RTDS	Real Time Digital Simulator
SARE	Sub Algebraic Riccati Equation
SAS-SCA	Sine augmented scaled sine cosine algorithm
SCA	Sine Cosine Algorithm SCA
SIMULINK	A graphical programming environment built on MATLAB that allows the modeling, simulation, and analysis of dynamical systems.
SMC	Sliding Mode Control
SLPs	Step Load Perturbations
SSV	Structured Singular Value
ST	Settling Time

STM	Super-Twisting Method
RDE	Riccatti Difference Equation
TAIPS	Two Area Interconnected Power systems
TD-TI	Tilt-Derivative Tilt-Integral Control
TID	Tilt Integral Derivative Control
TLBO-based PID	Teaching–Learning-based Optimization based PID
TPBVP	Two Point Boundary Value Problem
TRANCOs	Transmission Companies
u	Control Signals
VIU	Vertically Integrated Utility
VSC	Variable Structure Control
VSC/SMC	Variable Structure Control based on Sliding Mode Control
WCA	Water Cycle Algorithm

CHAPTER 1

INTRODUCTION

1.1. Background and problem statement

This section discusses the background and problem statements.

1.1.1. Automatic Generation Control (AGC) for interconnected power systems

The fundamental problem with automatic generation control in interconnected power systems is the imbalance between generated power, load demand, and system losses (Arya et al., 2012). This has been one of the most essential control concerns in the design and operation of electric power systems for decades. As an ancillary service, AGC plays a vital role in ensuring the necessary standard of electrical power system reliability. To successfully operate interconnected power networks, total power generation must be matched with total load demand and associated power system losses (Kumar, & Kothari, 2005). The power system operating point changes as demand deviates from its nominal value by an unpredictable moderate amount of load, and the power system may encounter deviations in nominal frequency and scheduled tie line power exchanges, which may impact system operation (Kumar, & Kothari, 2005). The two primary goals of AGC are to regulate the frequency to the specified nominal value and to maintain the interchange power between control areas at the scheduled values by measuring a control error signal known as the area control error (ACE), which is a linear combination of tie line power and frequency deviations (Kundur, 1994; Tungadio et al., 2018; Bevrani, 2014).

Various control strategies have been developed over the years to solve AGC problems. Initially, traditional linear control approaches were developed to address AGC problems (Tan & Xu, 2009, Delassi, Arif & Mokrani, 2015). Classical linear control methods are designed based on linear control theory (superposition and homogeneity). When superposition and homogeneity are satisfied, the controller may operate under all operational conditions and achieve global stability. However, while linear controllers are simple and easy to design and implement, they cannot handle parameter fluctuations or model uncertainty (Alhelou et al, 2018). As a result, traditional linear techniques are unsuitable for complex interconnected power systems (Ibraheem & Kumar, 2013). The above-mentioned linear control challenges prompted the development of soft computing methods to control systems, such as the Adaptive Neuro-Fuzzy Interference System (ANFIS), which have been utilized to deal with nonlinearities and load disturbances (Khuntia & Panda, 2012, Prakasha & Sinhab, 2014). It combines a neural network's learning capability with the adaptability of fuzzy logic. However,

it has difficulty with control system analysis associated with constraints.

The literature presents robust control systems for different operating conditions, plant model uncertainties, and load disturbances. (Tan & Xu, 2009, Pandey, Mohanty, & Kishor, 2013). The majority of most of these approaches include the Riccati equation approach (Wang, Zhou & Wen, 1993), H-infinity robust control (Wang & Sznajder, 1994), Robust pole assignment approach (Azzam, 1999), Iterative Linear Matrix Inequalities (ILMI) (Bevrani, Mitani & Tsuji., 2004), Active Disturbance Rejection Control, (ADRC) (Dong, Zhang & Gao, 2012), Kharitonov's theorem, Decentralized Neuro-optimal control approach (Ansarian et al., 2006), the combined Relative Gain Matrix (RGM) and Structured Singular Value (SSV) (Filha et al, 2016; Tan & XU, 2009) and Optimal decentralized linear quadratic regulator load frequency control (ODLQRLFC) (Abdelhamid & El-Gaafary, 2018). The first eight robust control strategies above are appropriate for multivariable systems, and they provide the optimum performance for a power system with uncertainties and load disturbances and do not require prior information about unpredictable inputs. They are, nevertheless, impractical for interconnected power systems with large dimensions and frequent parameter fluctuations (Ullah et al., 2021). Furthermore, variable structure control is effective and robust in dealing with modeling uncertainties, parameter fluctuations, nonlinearities, and load disturbances (Chan & Hsu, 1981; Das et al., 1991; Vrdoljak, Perić & Petrovi, 2010). On the other hand, the control law depends on the power system model and is associated with chattering phenomena, which are undesirable characteristics for large-scale interconnected power systems.

Compared to the previously mentioned robust control methods, the ODLQRLF scheme has large stability margins, inherent robustness, and reliability regarding nonlinearities for complex multi-area interconnected power systems (easing the control design and implementation of complex interconnected power systems (Tungadio, Ramesh & Mukwanga, 2018). As a result, in this thesis, discrete optimal decentralized automatic generation based on LQR will be developed and applied. In addition to the challenges mentioned above, the modern interconnected power system is growing in size and complexity as a result of the increased integration of intermittent renewable energy resources such as photovoltaic (PV) and wind turbines, as well as smart grid and unsafe communication layers across the physical structure of the power network (Vlahakis, Dritsas, & Halikias, 2019). Another finding from the literature is that measurements of tie-line power and frequency are implemented and delivered as discrete signals to the system control center via communication channels, even though the system is developed based on continuous time modeling (Shang-Guan et al., 2020); this, however, shows that the AGC controller operates in the discrete-time domain, but the power system operates in the continuous time domain (Bevrani, 2014). As a result, a

flexible and robust discrete-mode AGC scheme is required, capable of facing the new challenges of complex interconnected power systems and considering the sampling period characteristics of both the AGC controller and the interconnected power system during the design phase (Shang-Guan et al., 2020).

The research problems are stated as follows:

Problem 1: Understanding the design of discrete decentralized optimal Automatic Generation Control (AGC) based on Linear Quadratic Regulator (LQR) for complex interconnected power systems

Various AGC control solutions have been developed in the literature to handle the AGC problem in large-scale interconnected power systems. Initially, multivariable control techniques were utilized to create centralized AGC controllers (single controllers) that dealt with all information supplied by a group of sensors or actuators (Bensenouci & Abdel Ghany, 2005). Due to the centralized AGC controller's limited communication and computation capabilities, as well as the inherent structure of large-scale systems (Tan, 2011; Sabri, Kamoun, & Lakrami, 2019), various types of decentralized control methods have been proposed in the literature (Yang, Ding, & Yu, 2002; Bensenouci & Abdel Ghany, 2005; Abdelhamid & El-Gaafary, 2018; Alhelou et al., 2023). The design and execution of these control techniques are simple, and they segregate the complex multi-area power system into subsystems, each with its local controller.

Tan (2011), for example, proposed a decentralized load frequency controller (LFC) to assess the stability of multi-area power systems while accounting for the system's fundamental structure. Based on the architecture of Active disturbance rejection control (ADRC), Dong, Zhang, and Gao (2012) suggested a robust decentralized load frequency for a three-area interconnected power system with non-reheat, reheat, and hydropower plants in the face of system uncertainties and step load disturbances (SLPs). Linear Matrix Inequality (LMI) and Lyapunov theory are utilized by Xiaohua, Jiawang, and Yang (2015) to propose a robust overlapping decentralized control approach based on dynamic output feedback for a four-area power system with a radial structure. Yang, Liu, & Hill, 2021 introduced a decentralized event-triggered control (DETC) technique for asymptotically stabilizing the closed-loop frequency in multi-area power systems. Alhelou et al., 2023 recently investigated decentralized optimized integral sliding mode-based load frequency control for interconnected multi-area power systems to deal with frequency regulations with varying load demand, modeling uncertainties, Step load perturbations, and communication time delay.

Many of these control solutions greatly simplify the complexity of an interconnected power

system; however, they are created in the continuous time domain and implemented in the discretized time domain. As a result, the closed-loop performance is poor, which may cause the system to become unstable. Furthermore, these methodologies do not consider modeling the interconnected power system as a hybrid model comprising traditional generators and renewable energy sources. Recently, the focus has been on robust optimal AGC for large-scale interconnected power systems based on Linear Quadratic Regulator (LQR). Vlahakis, Dritsas, & Halikias, 2019 developed a distributed Linear quadratic regulator (LQR) with six identical control areas and three interconnections for large-scale interconnected power systems. Abdelhamid and El-Gaafary (2018) suggested Optimal Decentralized LQR Control to increase AGC interconnected power system stability. Panwar et al. (2022) used hybrid bacteria foraging oriented particle swarm optimization (HBF-PSO) with linear quadratic regulator (LQR) design for interconnected power system hydro turbines to test the robustness of the designed controller against step load disturbances, nonlinearities and parametric variations. It was proved that this current control theory-based control method can achieve good performance in terms of large stability margins, robustness, and reliability in terms of nonlinearities in complex multi-area interconnected power systems (Tungadio, Ramesh & Mukwanga, 2018).

As a result, decentralized optimum AGC control based on LQR is required for large-scale multi-area interconnected power systems to provide robustness against load disturbances, model uncertainties, and physical constraints. This thesis addresses this by developing and comparing optimal discrete centralized and discrete decentralized automatic generation controllers based on optimal control theory, linear Lagrange multipliers, sampling time, and state and control weighting matrices.

Problem 2: Understanding the Selection of State and Control Weighting Matrix for Discrete Optimal LQR Automatic Generation Control based on Area Control Errors, Integral of Area Control Errors, and Control Signals

Despite the advantages of optimal AGC-based LQR, LQR's performance depends on selecting the state cost weighing matrix (Q) and control cost weighting matrix (R) to obtain the best control efforts and closed-loop performance (Sharma et al., 2016; Arya, Kumar, & Gupta, 2017); Panwar et al., 2022). State and control signals are penalized when R and Q value increases (Rangi, Jain, & Arya, 2022). Choosing a large number for the R matrix shows that the designers intended to keep the system stable using less energy (cheap control technique). However, choosing a small number for R implies that the designers are not adjusting the regulator output (cheap control strategy). A larger Q value means settling the system with the least state variation possible, whereas a smaller Q value implies less

concern about state deviations.

Several ways for determining state and control weighting matrices have been presented to solve these challenges. The Q and R identity matrices of appropriate dimensions were employed in earlier LQR research. Because of this decision, LQR's control efforts were limited; thus, LQR provided limited action to obtain the best AGC performance (Sharma et al., 2016; Arya, Kumar, & Gupta, 2017; Sharma, 2019). The weighting matrices were selected based on the participation factor analysis, as mentioned by Prakash et al. (2022), in which states with the most significant participation in relevant modes were given higher weights. Panwar et al. (2019) calculated the weighting matrices Q and R for LQR using hybrid bacteria foraging-oriented particle swarm optimization (HBF-PSO) for hydro system automatic generation control. In another study, weight matrices were chosen based on the controllability and observability indices, and the power system's state matrix was transformed into a diagonal matrix form using the Eigenvalues and Eigenvectors decomposition approach (Ibraheem & Kumar, 2013). However, because of the extensive participation of many states, most of these methods are not direct and unsuitable for large-scale power systems. Furthermore, the complicated power system structure makes it impossible to gather accurate information on all system states.

The functional minimization method optimizes the functional cost function to create the best weighting matrices for interconnected power systems (Pathak, Nasiruddin & Bhatti. 2015). It provides more realistic responses and is easily can be extended to the class of large dynamical systems, such as power systems coupled to interaction signals. Furthermore, because it uses a portion of the state variables to construct weighting matrices, it eliminates the need for an observer to estimate the power system states. Therefore, the selection of weighting matrices and the design of an Optimal AGC-based LQR need to be investigated to determine whether the closed-loop performance of optimal based will improve by using the functional minimization method.

1.1.2. Increasing size and complexity of interconnected Power systems

Electricity was initially introduced in the late 1800s. A few neighboring consumers were served through a single-area power system. The long-distance transmission was unnecessary in those days because generation and distribution were both localized. Electrical networks grew in size and geography as electricity demand increased. The isolated single-area systems were linked by a high-voltage transmission connection known as a tie line (Arya, Kumar, & Gupta, 2012; Wood, Wollenberg & Sheble, 2014). This concept is known as multi-control areas, and tie lines exchange contractual energy between control areas (Jaleeli et al., 1992). As the number of control areas increases, the electrical power system increases in size and complexity for power distribution over a large geographic area (Lunze, 1992) new structure environment known as deregulated market operation instead of a monopoly entity known as a vertically integrated utility (VIU). The deregulated market has been divided into four independent competitive entities to break the market monopoly: distribution companies (DISCOs), transmission companies (TRANSCOs), generating companies (GENCOs), and autonomous operators known as independent system operators (ISOs) (Ibraheem et al., 2012, Shankar, Chatterjee, & Bhushan, 2016, Arya, & Kumar, 2016). DISCOs and GENCOs' principal mission is to foster competition and enable consumers to receive power at a reduced cost. On the other hand, ISO assures that all power transactions between DISCOs and GENCOs are secure and reliable. It also provides several extra functions, including frequency adjustment through automatic generation control (AGC) (Rangi, Jain, & Arya, 2022). These changes introduce substantial power system control and operation uncertainties, particularly in solving the AGC problem (Bevrani, Mitani, & Tsuji 2003).

Integrating renewable energy resources (RES) into electrical systems has increased electricity generation towards meeting the global electricity demand. It is predicted to double between 2000 and 2030, with a 2.4 percent yearly growth rate (Alam et al., 2016). The integration of new distributed resources (wind farms and PVs) into the primary grid has created new problems for modern power system operation, including issues such as stability, uncertainty, and low inertia (Pandey, Mohanty, & Kishor, 2013; Yan et al., 2015; Namara et al., 2016). As a result, the interconnected power systems are growing in size and complexity, generating a quick response and high frequency and tie-line deviations. Thus, the multi-area interconnected power systems introduce more difficulties and challenges to AGC control design (Lunze, 1992).

Problem 3: Understanding how to use the overlapping decomposition approach to decompose a large interconnected power system into discrete decentralized subsystems.

Another challenge, according to the literature, is decomposing large-scale interconnected

power systems into subsystems. An overlapping decomposition method and the inclusion principle are necessary (Chu, Chta & Siljak, 2009; Ahmadi, Aldeen & Abdolmaleki, 2014). The inclusion principle is a mathematical expansion principle that enlarges the state space model of a large-scale interconnected power system into subsystems (Chen & Stankovi, 2005). This allows the designer to recognize and differentiate between different subsystems and interaction signals. Contraction principles are then used in the subsystems to implement control laws in the original system.

Problem 4: Examine the Optimal AGC' based LQR performance under various system conditions for step load perturbations, sensitivity analysis, model uncertainties, and generation rate constraints (GRC).

The previous investigations addressed step load perturbations, modelling uncertainties, nonlinearities, and robustness. These findings indicated that the power system's complexity resulted in complex AGC control challenges related with load perturbation rejection problems caused by generation rate constraints (GRC), parameter fluctuations, modelling uncertainty, and communication channel time delays. Thus, designing and executing an effective AGC-based LQR technique to address load perturbations, nonlinearities, and modelling errors associated with large-scale multi-area interconnected power systems is crucial in real-time practical implementations.

1.2. Hypotheses and Research Questions

The following hypotheses have been developed in response to the problems raised in section 1.1.

Hypothesis 1:

- a. Most studies describe interconnected power systems in the continuous time domain; however, the discrete controller is often implemented on embedded computers in communication channels, reducing closed-loop performance and making the power system unstable. The power system implementation can be enhanced by discretizing and designing the power system modeling and the controller using the Euler discretization method.
- b. The design of centralized and decentralized discrete optimum quadratic AGC controllers using optimal control theory, Lagrange multipliers, multivariable control theory, and functional minimization can be successfully used to improve multi-area interconnected power systems.

Hypothesis 2:

The most widely utilized state and control weight matrices selection approaches, such as identity matrices, controllability and observability indices, and hybrid bacteria foraging oriented PSO methods, are insufficient for optimal AGC-based LQR in large scale interconnected power systems. To accurately select the state and control weighting matrices, the Functional Minimization Method, which is derived from the partial derivative of the function of the cost function is applied.

Hypothesis 3:

The use of an overlapping decomposition method and inclusion principles to divide a large-scale interconnected power system into independent subsystems simplifies design obstacles and minimizes computational time.

Hypothesis 4:

The development of centralized and decentralized control methods based on functional minimization method, overlappingdecomposition, and decentralization approaches coupled with the requirement of AGC problems can be extremely beneficial in achieving robust performance against step load perturbations, modelling uncertainties, and nonlinearities such as generation rate constraints (GRC), which are the most important aspects of large-scale systems.

The research questions listed below are used to guide the investigation and answer the research hypotheses:

1. How are continuous state space equations built as a combination of first order differential equations obtained from the transfer functions of various cases of multi-area interconnected power systems?
2. Using Shannon's theorem and Euler's discretization criterion, how are discrete state space equations created for various cases of multi-area interconnected power systems?
3. How are discrete centralized and discrete decentralized optimal quadratic AGC controller problems formulated utilizing optimal and suboptimal quadratic cost functions with Lagrange multipliers, subject to discrete state space equations and discrete sub-state space equations?
4. a) What are the design requirements for selecting state and control weighting matrices with minimal state variables using the Functional Minimization Method?

b) What influence do the state and control weighting matrices based on the Functional minimization approach have on the proposed discrete Centralized Optimal Quadratic

Automatic Generation Control (COQAGC) and the discrete Decentralized Optimal Quadratic Automatic Generation Control (DOQAGC)? and,

- c) Can using the Functional Minimization Method (FMM) with few variables aid in creating the proposed controllers without requiring an observer to estimate the variables??
5. What improvements do discrete centralized and discrete decentralized optimal quadratic automatic generation (AGC) controllers provide to multi-area power systems in terms of robust performance against step load perturbations, modeling uncertainties, and nonlinearities such as generation rate constraints (GRC) when compared to other control methods from the literature for frequency and tie-line deviations?
 6. What impact does the overlapping decomposition approach have on the design of discrete decentralized optimal quadratic automatic generation (DOQAGC) and multi-area interconnected power systems in terms of AGC design issues and computation difficulties?
 7. What software environment is utilized to build and test control algorithms for a multi-area interconnected power system?

1.3. Hypothesis testing: Chapter overview and findings

The dissertation contains six chapters that test the four primary hypotheses given in section 1.2. Figure 1.1 depicts an overview of this thesis.

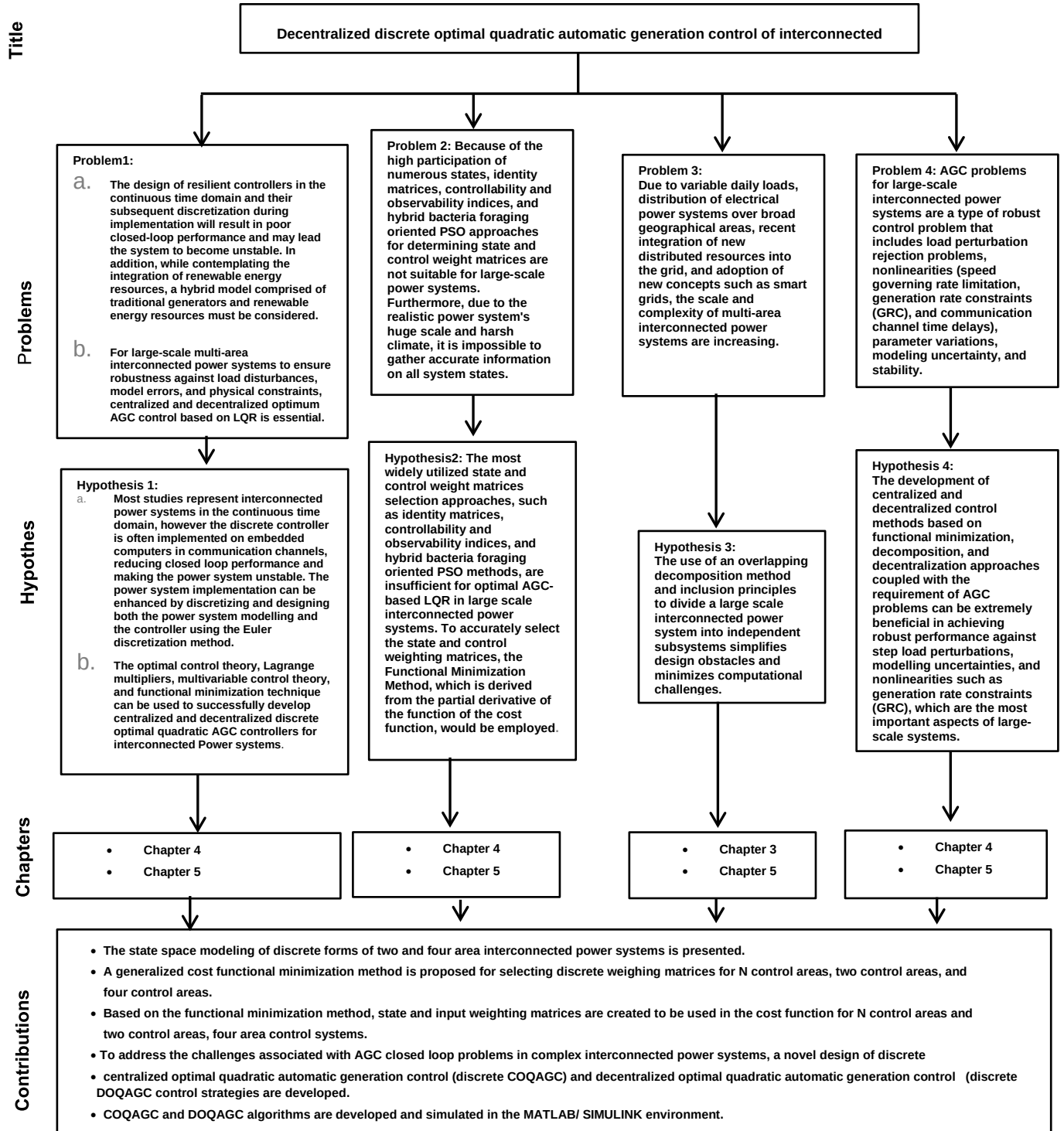


Figure 1.1. Thesis overview

1.4 Research design

1.4.1 Research Aim and objectives

AIM

The project aims to develop discrete centralized and decentralized optimum quadratic automatic generation optimal controllers (COQAGC and DOQAGC) for an interconnected power system.

Objectives

- i. Conduct a literature review of existing control approaches for AGC control problems in multi-area interconnected power systems, including methods, algorithms, and software programs.
- ii. Determine the multi-area power system's continuous and discrete state space equations.
- iii. Develop a functional minimization approach based on partial differentials for calculating the state and control weighting matrices for discrete centralized and discrete decentralized AGC proposed controllers.
- iv. Formulate the optimization problem for discrete centralized and discrete decentralized AGC proposed controllers using the optimal quadratic cost function and Lagrange's multipliers.
- v. Develop/Utilize an overlapping decomposition approach for decomposing the multi-area interconnected power systems into discrete subsystems.
- vi. Develop a decentralized discrete DOQAGC and simulate the closed-loop system in MATLAB/SIMULINK for set-point disturbance rejections, robustness against modelling uncertainties, and GRC.
- vii. Develop a centralized discrete COQAGC and simulate the closed-loop system in MATLAB/SIMULINK for set-point disturbance rejections, robustness against modelling errors, and GRC.
- viii. Compare the set-point disturbance rejections, robustness against modelling uncertainties, and GRC of the decentralized discrete DOQAGC with the centralized discrete COQAGC in the MATLAB/SIMULINK software environment.

1.4.2 Research Methodology

The following steps summarize the detailed methodology of the developed controllers.

Step 1: Obtain and present the first-order differential equations for the discrete optimal AGC model of multi-area power systems in state space form.

Step 2: Input the model parameters and data for power system simulation.

Step 3: Compute the augmented continuous time matrices and

Step 4: Obtain the discrete-time and augmented matrices obtained in step 3 using Euler's discretization criteria.

Step 5: Initialize the steady Riccati matrix.

Step 6: Calculate the discrete optimal feedback gain matrix and the discrete optimal control law.

Step 7: Initialize the state variables and solve the closed-loop closed loop iteratively using the control law derived in step 6.

Step 8: Using MATLAB, simulate the multi-area interconnected power system with COQAGC and DOQAGC.

Step 9: Calculate the dynamic response to step load perturbations (SLPs).

Step 10: Repeat step 9 to evaluate the dynamic responses for robustness, GRC, parameters, and modelling uncertainties.

1.5 Research Delimitations

- The proposed research focuses on the design and implementation of centralized and decentralized automatic generation control controllers using a functional minimization method and software tools such as MATLAB/Simulink, and optimal control theory.
- Multi-area interconnected power systems (MAIPS) are used as a case study, with a thermal power plant with two identical control areas, a four-area interconnected power system with non-reheat, reheat, hydro, and gas turbines, and a multi-source two-area power system with non-reheat and hydro plants in control area 1 and a wind turbine plant in control area 2.
- For large scale systems, such as interconnected power systems, the decomposition method that divides overall systems into subsystems is limited to

overlapping decomposition.

- The optimum control theory and Lagrange's multipliers are used to develop discrete centralized and discrete decentralized optimal quadratic automatic generation controllers.
- The control methods are developed in MATLAB/SIMULINK. To demonstrate the feasibility of the proposed control approaches, simulations and control implementation are carried out.
- A comparative performance investigation of the proposed controllers on test cases of interconnected power system dynamics with/without perturbations was undertaken using software created in the MATLAB/SIMULINK Environment.
- This study will not cover economic dispatch, emergency control, fault control, automated voltage regulator (AVR) control, coordination control, and distributed energy resources except the wind turbine.

1.6 Assumptions

The following assumptions are established to correspond with the thesis objectives:

- For simplicity, the models of interconnected power systems are considered to be linear based on feedback Linearization and thus, the performed studies on frequency and tie line dynamic responses of multi-area interconnected power systems are generated based upon a linearized model analysis. However, the multi-area power systems are highly nonlinear and associated with physical constraints. For simplicity, the models of interconnected power systems are assumed to be linear based on feedback Linearization. Thus the completed studies on frequency and tie line power dynamic responses of multi-area interconnected power systems are developed based on a linearized model analysis.
- To demonstrate the usefulness of the suggested controllers, it is usual practice to evaluate the impacts of physical restrictions such as Governor dead band (GDB), Boilers dynamics (BD), GRC, and Time delays owing to communications channels in the dynamics of multi-area interconnected power systems. Considering all physical constraints in frequency control synthesis and analysis is challenging and ineffective (Bevrani, 2004). The generation rate limitations (GRC) and power system uncertainties (sensitivity analysis) are considered for practical evaluation
- For simplicity, the power demands of interconnected power systems are assumed

to be constant, and each control area is assumed to have only one generating unit with a participation factor of one. As a result, the tie-line power deviations are the same. However, the author is aware that power consumption varies according to consumer loads in industrial power systems.

- In a multi-area interconnected power system, the maximum number of tie-lines is given as (Krishnamurthy, 2013):

$$n = \frac{(m - 1)m}{2}$$

where

m denotes the total number of the areas

n denotes number of tie-lines

- The tie-line resistance is neglected (zero) between the control areas of multi-area interconnected power systems because of the high value of the inductance and the current I_{12} flows from the control area 1 to the control area 2.
- The nature of the equations characterizing the interconnected power systems to be controlled enables for the design of the controlled by centralized and decentralized automatic generation control (AGC) methodologies.

1.7 Research contributions

The following are the main contributions of this thesis:

- To evaluate and analyse existing advanced control methods and approaches utilized in modelling the multi-area interconnected power system (MAIPS) for automatic generation control problems (AGC).
- Convert the transfer functions of a two-area and four-area power systems into first-order differential equations and construct continuous state space equations.
- To develop discrete state space equations, use Shannon's theorem and Euler's discretization criterion for state space equations of two and four interconnected power systems, as well as state space equations of two and four subsystems.
- To develop an efficient cost function minimization using squared area control errors, squared the integral of area control areas, and squared control energy expenditure for two, four, and N control areas.
- Decompose two, four, or N multi-area interconnected power systems using overlap-

ping decompositions for decentralized AGC control.

- To develop state and control weighting matrices (Q and R) for centralized and decentralized controllers using a functional minimization method.
- Formulate centralized and decentralized control problems that are subject to discrete state space equations and discrete sub-state space equations utilizing optimal and suboptimal quadratic cost functions with Lagrange multipliers.
- The goal of this project is to develop discrete centralized and decentralized optimum quadratic automatic generation optimal controllers (COQAGC and DOQAGC) for a two-region power system with identical non-reheat in each area. Non-reheat, reheat, hydro, and gas plants comprise a four-area power system, while reheat, hydro, and wind turbine power plants comprise a two-area multi-source power system.
- To develop software and conduct closed-loop system simulations of the designed controllers in MATLAB/SIMULINK for set-point disturbance rejections, robustness against modelling errors, and GRC in various power system scenarios.
- To developing an interactive program based on Lagrange's multipliers and a discrete centralized control method to solve AGC problems iteratively in closed-loop power systems, which will be used to evaluate the dynamical performances of identical two-area and four-area interconnected power systems.
- To provide a software program for measuring the dynamical performances of identical two-area and four-area interconnected power system problems using overlapping decomposition, Lagrange's multipliers, and discrete decentralized control methods.
- Comparison of the relative dynamic performance of developed discrete optimal AGC controllers with controllers from the literature to validate them and find the best control approach.

1.8 Chapter break down

This thesis consists of six chapters arranged as follows:

Chapter 1 introduces the thesis's problem statement and background. The study aim and objectives, problem statement, hypotheses, research delimitation, and assumptions.

Chapter 2 surveys the literature on single and multi-area interconnected power system modelling concepts. It investigates the distinctive properties of several advanced control methods, such as centralized and decentralized control methods. It concludes with a discussion of the constraints associated with multi-area interconnected power systems that affect AGC performance. Moreover, soft

computing and intelligent control methods, robust control methods, linear quadratic regulator (LQR) control methods, and other control methods are reviewed.

Chapter 3 describes the decomposition of identical two-area interconnected power systems into two subsystems is described. First, the transfer functions of the identical two-area power system are translated into first-order differential equations, which are then combined to produce a state space model of the identical two-area thermal power system with non-reheat turbines. The concepts of overlapping decomposition, disjoint decomposition, inclusion principles, and decentralization are stated. Finally, the overlapping decomposition approach is applied to decompose the same two-area thermal system into two independent subsystems

Chapter 4 introduces optimal control principles and design as well as describes the developed functional minimization approach for a power system with N control areas, which was used to develop a centralized discrete COQAGC controller for two area interconnected power systems with identical non-reheat turbines and four area interconnected power systems (FAIPS) with non-reheat, reheat, hydro, and gas turbines. For the AGC problem, continuous and discrete state space models are proposed. In the MATLAB environment, various realistic parameters of interconnected power systems are used to test the centralized discrete COQAGC control algorithm. For the following experiments, the Functional minimization approach and centralized discrete COQAGC algorithms were developed and simulated:

Systems test 1: On TAIPS with identical non-reheat turbines

- Case 1.1: COQAGC as a regulator with no disturbances based on a functional minimization approach.
- Case 1.2: COQAGC as a regulator with disturbances based on a functional minimization approach.
- Case 2.1: COQAGC with area control errors (ACEs) with no disturbances based on a functional minimization method.
- Case 2.2: COQAGC with area control errors (ACEs) with disturbances based on a functional minimization method

Systems test 2: On FAIPS with non-reheat, reheat, hydro and gas turbines

- Case 2.1: The initial values of the frequency deviations in four areas are set at 0.0 Hz for each area in centralized discrete COQAGC controllers with 0.01 pu step load perturbations.
- Case 2.2: COQAGC with concurrent step load perturbations set to 0.01pu and

initial frequency deviation values set to 0.0 Hz for each area.

- Case 2.3: Sensitivity Analysis of optimal COQAGC controller.
- Case 2.4: Physical Constraints study considering GRC of optimal COQAGC controller.

Chapter 5 provides an introduction to the decentralized discrete DOQAGC for AGC problems. An optimization problem design technique is given and used for identical two-area interconnected power system (TAIPS) with non-reheat turbines and four-area interconnected power systems (FAIPS) with non-reheat, reheat, hydro, and gas turbines. For a four-area interconnected power system, two identical subsystems and four subsystems including non-reheat, reheat, hydro, and gas turbines in each control area, continuous and discrete state space equations are developed and demonstrated. A large, interconnected power system is decomposed into subsystems using the overlapping decomposition method, and their weighting matrices are constructed using the sub-functional minimization methodology. The DOQAGC control algorithms are developed using optimal control theory for the AGC problem and tested using two types of MAIPS models: a) two area interconnected power system (TAIPS) with identical non-reheat turbines and b) four area interconnected power system with reheat, non-reheat, hydro and gas power plants.

Sub-functional minimization techniques, centralized control algorithms, and decentralized control algorithms were developed and simulated for the following tests:

Test system 1: On two identical sub-systems:

- Case1.1: DOQAGC with 0.01 pu input step load perturbation at $t = 0$ s.
- Case 1.2: DOQAGC with 0.01pu concurrent input step load perturbations.
- Case 1.3: Comparison of DOQAGC and COQAGC controller performance in cases 1.1 and 1.2.

Test system 2: Four-area interconnected power system comprising non-reheat, reheat, hydro, and gas turbines.

- Case2.1: Suboptimal DOQAGC controllers with input step load perturbations set to 0.01 pu at $t=0$ s.
- Case 2.2: DOQAGC with 0.01pu concurrent input step load perturbations.
- Case 2.3: Comparison of the performance of DOQAGC versus the performance of COQAGC in the above cases 2.1 and 2.2.
- Case 2.4: Sensitivity Analysis of optimal DOQAGC controller.

Test system 3: DOQAGC and COQAGC performance robustness and physical

constraints investigation on four areas interconnected power

Case 3.1: Sensitivity analysis of four sub-optimal DOQAGC controllers and the performance of an OQAGC controller.

Case 3.2: Constraints considering the GRC on Four Suboptimal DOQAGC and COQAGC Controllers

Chapter 6 The thesis conclusion is presented. The chapter discusses the applications of the thesis findings, future work, and publications by the author.

1.9 Conclusion

This chapter introduces the problem statement and examines the research background. The research purpose, a problem statement, objectives, assumptions, thesis outline and deliverables were stated.

The following chapter discusses the literature in order to gain a better understanding of the problem. The literature was especially reviewed for papers that propose unique control methods and algorithms for Multi-Area Interconnected Power Systems. The review chapter went over optimal control optimization methodologies as well as large-scale system design requirements including decompositions and decentralizations.

Chapter 2

Literature Review

2.1. Introduction

Automatic Generation Control (AGC) is a very important control method for maintaining the active power balance within the desired limits in the multi-area power system (Shankar et al. 2017). The main objectives of AGC control are to maintain the frequency deviations at a nominal value, keep the tie-line power changes between areas at a scheduled value, and ensure that the frequency variations are returned to zero (Alrifai et al. 2011, Esmail, Tzoneva & Krishnamurthy. 2017, Tungadio et al. 2018). The variation of the daily load makes this control problem challenging. Besides, the increasing size of the interconnected power system and the integration of the new distributed resources i.e., wind farms and, PV, into the main grid makes this control even more complex and challenging.

Several researchers have studied the modelling and control of interconnected power systems. In 1956, Cohn introduced bias tie-line control in interconnected power system in his research work, the frequency of a power system was controlled via the governor of the synchronous machine using flywheel mechanism (Cohn, 1956). This technique was later found to be insufficient. Therefore, the idea of supplementary control was added to the governor with help of the area control error (ACE) (Kumar and Kothari. 2005). Because of their simplicity of design and implementation, classical control methods such as Integral (I), Proportional Integral (PI), and Proportional Integral Derivative (PID) controllers are commonly used as supplementary control in the literature. The design of these controllers is based on the power system model and linear control theory. Hence, the major drawback was the degradation of the controlled system performance because of the nonlinearities of the system and the system operating points (Delassi, Arif & Mokrani, 2015). In addition, power outages may occur as oscillations propagate in interconnected power systems because of the size and complexity of modern power systems (Tan & Xu 2009).

Many studies have been conducted about AGC in interconnected power systems over last few decades. The early studies can be found in the work conducted by Chritie and Bose (1996); Kumar, Ng & Sheble (1997); Bakken and Grande (1998); and Meliopoulos et al (1999). Further, Bevrani et. al., (2005) modified the conventional AGC system to consider the effect of disturbances on the dynamic of response of the system. Tyagi and Srivastava (2006) used a state space model of multiple power system to design decentralized state feedback controller using eigenstructure assignment. The most of mentioned studies are done on thermal plants which don't represent the real scenario of AGC problem in large multi- area interconnected power systems (Pandey, Mohanty, & Kishor, 2013). As result,

some recent studies have investigated the combination of multiple source generators such as thermal, hydro, nuclear, gas and renewable power plants within a single control area. For examples, Ram and Jha (2010) and Rahaman et al (2015) explored two control areas with thermal power plant in area 1 and hydro-thermal plants in area 2. In another study, the dynamic of the two control areas consisting of thermal and gas power plants have been reported in Hota and Mohanty (2016). Further, Shree and Kamaraj (2016) investigated the performance of AGC in three control areas consisting of thermal and hydro plants taking in consideration the Generation Rate Constraint (GRC).

Recently, soft computing and intelligent control techniques such as fuzzy logic and artificial neural network were combined as hybrid methods to deal with uncertainties, nonlinearities, and complexities of the interconnected power systems. For instance, Rajesh, Dash & Rajagopal, 2019 proposed the hybrid Firefly Optimization Algorithm and Pattern Search technique (hFA-PS) to tune the parameters of fuzzy aided PID controller for five areas thermal power. This technique combines the global exploration abilities of firefly algorithm (FA) with the local search capabilities of Pattern Search technique (PS). The physical constraints, generation rate constraints (GRC) and governor dead band (GDB) were considered in the study. In comparison with optimized I/PID controllers, the hFA-PS fuzzy-tuned PID controller has a better dynamic performance. Sahu, Prusty, & Panda, 2020 designed a robust type-II fuzzy PID controller that minimizes changes to frequency deviations, as well as changes to tie line power flow, using a hybridized Gray Wolf Optimization and Sine Cosine algorithm (hyGWO-SCA). Based on the Sensitivity Analysis results, the proposed hyGWO-SCA tuned type-II fuzzy controller is superior to the original Gray Wolf Optimization (GWO,) Sine Cosine algorithm (SCA) and particle swarm optimization (PSO) algorithms.

In another study, the novel sine augmented scaled sine cosine (SAS-SCA) algorithm was introduced by Khadanga, Kumar & Panda, 2021, for multi-area hybrid distributed power system. SAS-SCA algorithm is used for tuning tilt integral derivative (TID) controllers for frequency regulation of renewable energy systems, comprised of different renewable energy sources. A SASSCA-based TID controller is found to be more effective than a conventional SCA-based PID, SAS-SCA-based PID and SCA-based TID controllers. Further, in 2022, Dey et al. presented a detailed model that considers stator currents, field voltages, damper winding voltages, and grid equations for single and multi-area power systems. Area interchange control (AIC), integral (I), integral derivative (ID), and proportional integral derivative (PID) controllers have been used to mitigate frequency oscillations and tie-line power deviations. To tune the controllers, Salp swarm algorithms (SSA), Grasshopper optimization algorithms (GOA), and Collective decision optimization algorithms (CDO).

Based on the results of low, medium, and heavy load perturbations, it was found that the proposed method with SSA gave the best-controlled coefficients.

Yin & Wu, 2022, proposed a mode-decomposition memory reinforcement network strategy based on the divide-and-conquer strategy for reducing power system uncertainties and fluctuations. This strategy uses empirical mode decomposition, proportional-integral derivatives, long short-term memory networks, and reinforcement learning algorithms. According to two case studies, the mode-decomposition memory reinforcement network has a higher control effect and lower generation cost than the other four strategies. This system reduces frequency deviation and generation cost reasonably in four area interconnected power system.

Robust control methods also received attention in the literature. For example, Arya, 2019 proposed a hybrid fractional order proportional integral-fractional order proportional derivative (FFOPIFOPD) controller for isolated and interconnected multi-area power systems. A sensitivity analysis and physical constraints suggest that the controller is robust to large variations in system parameters and random load patterns. Zhu et al, 2021, proposed a sliding mode control strategy based on the combination of interval observer and disturbance reconstruction technology for hybrid energy storage system. The results demonstrated that the proposed control strategy suppressed frequency fluctuations and tie-line variations caused by load disturbances and ensured power system stability and reliability.

In terms of control structure, the centralized control of the load frequency has many disadvantages, so researchers have suggested the optimal decentralized Linear quadratic regulator (ODLQR) control (Abdul-Hamid and Al Jaafari, 2018; Hakimuddin, Khosla, & Garg, 2020) as well as overlapping decentralized control (Chen & Stankovic, 2005) and distributed control (Vlahakis, Dritas & Halikas, 2019) to improve performance over centralized control structures

As above literature stated, various control strategies were successfully adopted to improve AGC control design objectives in multi-area interconnected power systems (MAIPS). However, the system uncertainties, nonlinearities associated with AGC system, such as governor dead band, boilers dynamics, generation rate constraints and time delays due communications channels, systems made the power system to be controlled too large and the problem to resolved is too complex. Because in most studies, the nonlinearities associated with power systems are not considered. In addition, the integration of renewable energy sources into power systems increases the size, and complexity of the interconnected power system. According to Bakule (2008), difficulties mainly arise from dimensions, limitations of information structure, uncertainties, and time delays. As a result, relevant advanced control techniques, such as those shown in Figure 2.1, have been used for this

chapter's review.

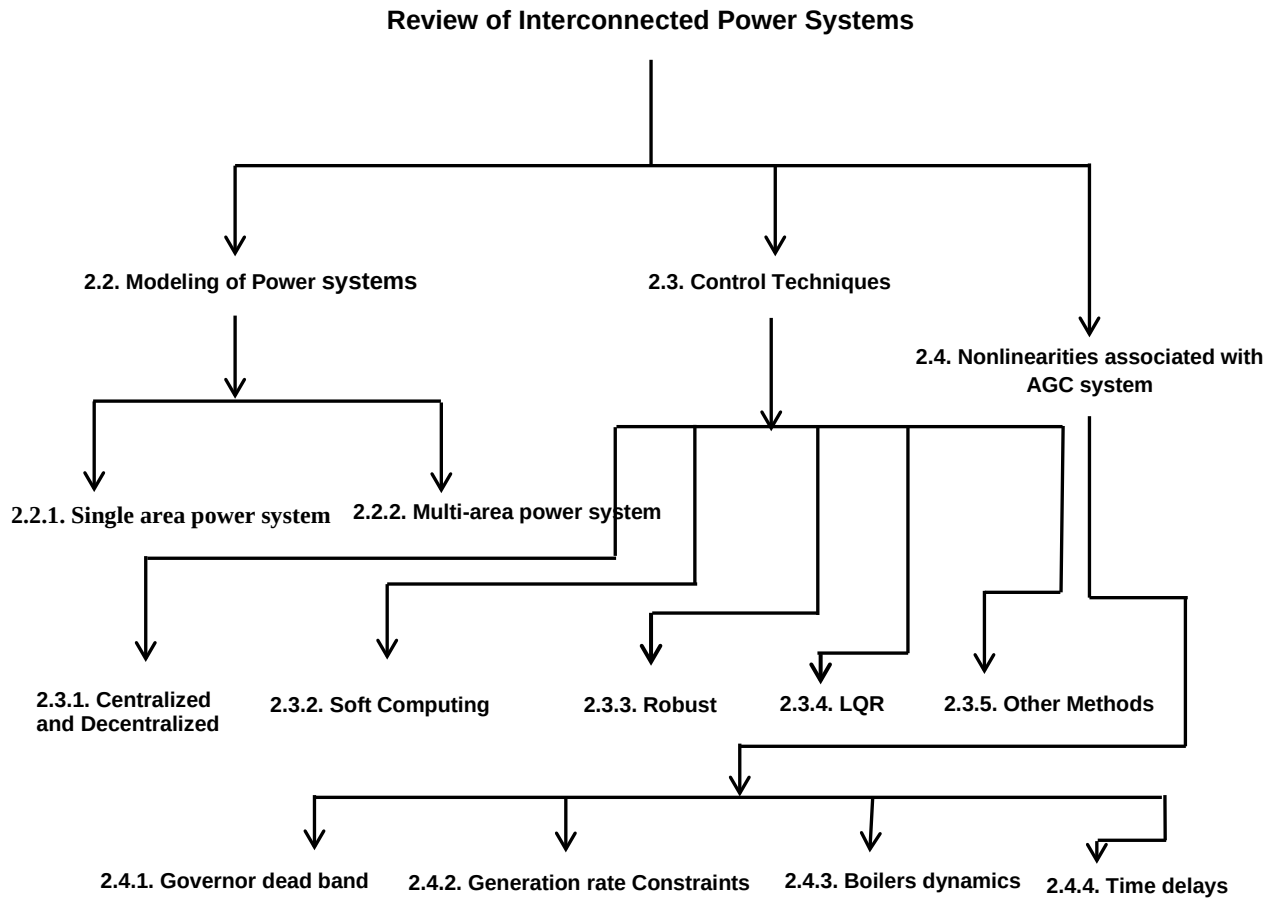


Figure 2-1. Modeling and Control review associated with the AGC control problems in MAIPS.

This chapter reviews the modeling of interconnected power systems, as well as some advanced control methods for solving AGC control issues. This review divides the AGC problem into a modeling and control system design as shown in Figure 2.1. This chapter is organized as follows: in section 2.2 a modelling of power systems is presented. Section 2.3 describes the AGC control techniques that are mostly investigated in the literature. Section 2.4 discusses the nonlinearities that associated with AGC system model. A comparison of different control techniques and strategies is discussed in section 2.5, and some concluding remarks are presented in section 2.6.

2.2. Modelling of Power Systems

2.2.1 Single Area Power System Model Having One Thermal Unit in a Control Area

The frequency regulation of interconnected power systems depends on active power balance. When the active power demand of the power network changes, a deviation in frequency occurs throughout the entire power system. Therefore, the frequency can measure the imbalance between active power, generation, and load. The first studies on AGC concern the design and implementation of AGC controllers for an isolated power system (Wood, Wollenberg & Sheble, 2014). The main purpose of AGC in a single power system is to maintain the frequency deviation and restore it to its nominal value (Bevrani & Hiyama, 2011, Bevrani, 2014). In a single power system, real power is controlled by controlling the prime mover's mechanical output. The prime mover can be a hydro turbine, steam turbine, gas turbine or any other type of turbine, depending on the type of the generation. In a steam or hydro turbines, the mechanical power is controlled by the opening or closing of the valves regulating the input of steam or water flow into turbine (Paramr, 2013). In the case of a generating units, supplying a load to an isolated single area power system, depending on the droop characteristics of the governor, and the frequency characteristics of the system load, the load produces frequency deviations of a certain magnitude. Whenever the load changes, the supplementary control loop restores the frequency to its nominal value, or the frequency error will be forced to zero by adjusting the speed reference set point. This action is achieved by adding an integral control to the governor, as shown Figure 2.2. On the other hand, the purpose of the primary loop is adjusting the turbine power in response to a change in frequency. The speed governor continuously measures the turbine speed and adjusts the inlet valve to change the mechanical power output to compensate for load changes and restore frequency to the nominal value (Wood, Wollenberg & Sheble, 2014). The command ΔP_c is entered into a hydraulic amplifier that causes a position change ΔP_v of the steam control valve or control gate in the case of a hydro plant.

In an interconnected power system, the control area concept is used for analysing and designing the AGC control system. A control area usually consists of a group of generators and loads that are connected by high voltage transmission line called tie-line (Shayeghi & Shayanfar, 2009). The dynamics of large electrical power systems are highly time-varying and nonlinear. In the presence of step load perturbations, a simple linearized model is usually used for Automatic generation control design and analysis. The transfer functions of interconnected power systems below are adopted from Bevrani & Hiyama, 2011, Parmar, Majhi & Kothar, 2012, Wood, Wollenberg & Sheble, 2014.

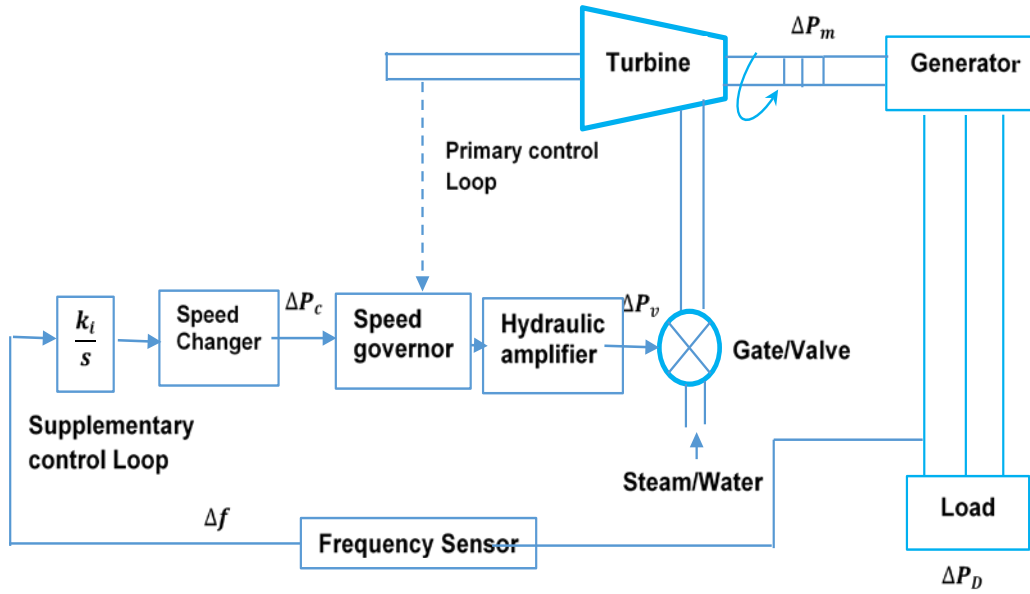


Figure 2.2. Diagram of a synchronous generator with basic control loops (Bevrani & Hiyama, 2011, Paramr, 2013).

Figure 2.3 shows the linear model of an isolated power system with a primary load Frequency Control (LFC) loop. In this section below we will also present the equations associated with this model. Increases in ΔP_g and decreases in frequency deviation Δf lead to an increase in governor command ΔP_g . We can therefore write for a small increment.

$$\Delta P_g(t) = \Delta P_c(t) - \frac{1}{R} \Delta f(t) \quad (2.1)$$

where, ΔP_g denotes small change in governor command, ΔP_c denotes small change in reference power setting, and R represents the speed drop characteristics (Hertz per pu MW). An R represents the ratio of a frequency change (Δf) to a change in output power (ΔP_g) or valve/gate change

$$R = \frac{\Delta f}{\Delta P_g} \quad (2.2)$$

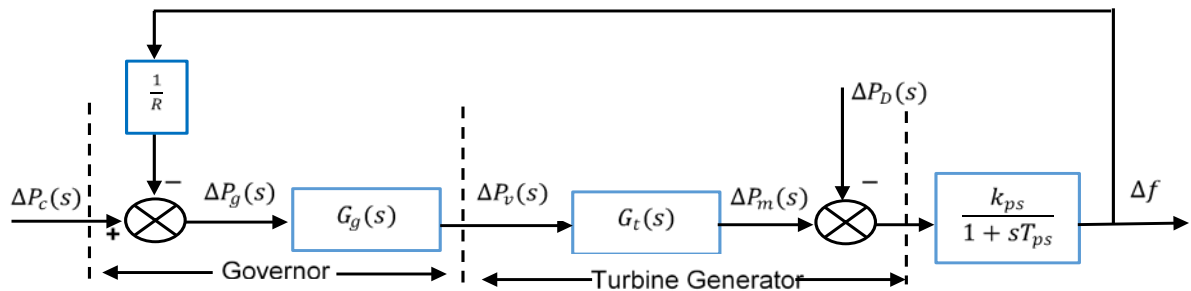


Figure 2.3. Linear Model of primary loop (Paramar, 2013).

Laplace transformation of the equation 2.1 yields

$$\Delta P_g(s) = \Delta P_c(s) - \frac{1}{R} \Delta f(s) \quad (2.3)$$

The transfer function of the speed governor is provided by:

$$G_g(s) = \frac{\Delta P_v(s)}{\Delta P_g(s)} \quad (2.4)$$

The transfer function of the speed governor system $G_g(s)$ depends on the type of governing systems used in the generating unit. Steam turbine speed governing systems have the following transfer function (Kothari & Nagrath, 2011, Elgerd,1983):

$$G_g(s) = \frac{1}{1 + sT_G} \quad (2.5)$$

where

T_G denotes the speed governor time constant.

The increase in turbine power $\Delta P_m(t)$ depends entirely on the increase in the power of the control valve $\Delta P_v(t)$ and the response characteristics of the turbine. Turbine transfer function is written as follows

$$G_t(s) = \frac{\Delta P_m(s)}{\Delta P_v(s)} \quad (2.6)$$

There are a variety of turbine types, and each has its own transfer function. The transfer function $G_t(s)$ of a non-reheat steam turbine is as follows (Kothari & Nagrath, 2011, Elgerd,1983):

$$G_t(s) = \frac{1}{1 + T_T s} \quad (2.7)$$

where, T_T represents the steam turbine time constant

The increase in generator power $\Delta P_m(t) = \Delta P_D(t)$ depends entirely on the changes $\Delta P_D(t)$ in the load fed from the generator, which adjusts the output so that it satisfies the demand variables. Since these adjustments are essentially instantaneous in comparison with slow changes in turbine output, we can set

$$\Delta P_m(t) = \Delta P_D(t) \quad (2.8)$$

Power system model (Generation–load dynamics) is governed by the transfer function (Bevrani and Hiyama, 2011):

$$G_p(s) = \frac{K_{ps}}{1 + sT_{ps}} \quad (2.9)$$

where

$K_{ps} = \frac{1}{D}$ represents the power system gain, Hz/pu MW

H represents the equivalent inertia constant, MW-s/MVA

$T_{ps} = \frac{2H}{fD}$ represents the power system time constant, sec

and, $D = \frac{\partial P_L}{\partial f} \cdot \frac{1}{P_{rt}}$ represents the equivalent system damping coefficient, pu MW/Hz

The frequency $\Delta f(s)$ can now be expressed as

$$\Delta f(s) = G_p(s)[\Delta P_m(s) - \Delta P_D(s)] \quad (2.10)$$

Figure 2.3. shows a block diagram of a single area power system without supplementary control based on the above equations. It is now crucial that the frequency deviations settle to zero at the steady state error. “To accomplish this supplementary control loop must be closed and the speed changer is manipulated in accordance with some suitable controller action”. The signal fed into the controller is called as area control area (ACE)” (parmar, 2013). The ACE in an isolated power system can be expressed follows as:

$$ACE = \beta \Delta f \quad (2.11)$$

where, β denotes the frequency bias coefficient, pu MW/Hz

The speed change can be commanded by a control signal ΔP_c obtained by a suitable control action on the area control error signal

$$\Delta P_c = K(s)ACE \quad (2.12)$$

where $K(s)$ denotes the controller transfer function

For instance, if the controller is an integral controller $k(s) = -\frac{K_i}{s}$ then ΔP_c may be defined as

$$\Delta P_c = K_i \int \beta \Delta f dt \quad (2.13)$$

Where K_i denotes the integral gain.

Figure 2-4 shows the block diagram single system with supplementary control.

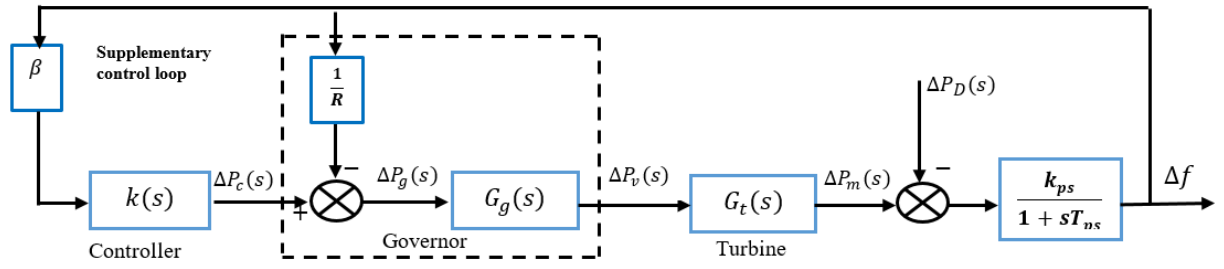


Figure 2.4. Linearized Model with Supplementary AGC Loop.

2.2.2. Review Discussion on Single Area Power System Model Having one Thermal Unit in a Control Area

Models of subsystems of a single power system with loads as inputs are described using mathematical equations. The concepts of droop characteristics and frequency characteristics of the system load are discussed. Moreover, the concept of supplementary control and primary control actions as well as the concept of area control area error are explained. The idea of adding an integral controller to achieve zero frequency deviations at state steady is discussed.

2.2.3. Multi-area power system model having more than one renewable/non-renewable units in a control area

In a single power system, control of tie-line power is not a control task, and the LFC objective is limited to restore the system frequency to specified nominal value. The control area concept is used to generalize the described model for interconnected power systems (Bevrani, 2014). Typically, a control area comprises a group of generators and loads, where all generators react to changes in load or speed-changer settings simultaneously. Interconnected power systems consist of control areas that are connected by high voltage transmission lines called tie-lines. Usually, the change of the measured frequency in each control area is used to determine the mismatch power in the entire interconnected power system. It is the LFC's responsibility to control the exchange of power between its area and other areas as well as the local frequency in multi-area interconnected power systems (Kundur, 1994, Bevrani & Hiyama, 2011, Bevrani, 2014). Because of this, the LFC model in Figure 2.3 must be modified to include the tie-line power signal. In this regard, Figure 2.5 illustrates a power system with N-control areas. According to Bevrani, 2014, the tie-line power change ΔP_{tie12} between areas 1 and 2 is as follows:

$$\Delta P_{tie,12} = \frac{2\pi}{s} T_{12} [\Delta f_1 - \Delta f_2] \quad (2.14)$$

Similarly, the tie-line power change ΔP_{tie13} between areas 1 and 3 is given by:

$$\Delta P_{tie,13} = \frac{2\pi}{s} T_{13} [\Delta f_1 - \Delta f_3] \quad (2.15)$$

Similarly, the tie-line power change $\Delta P_{tie,14}$ between areas 1 and 4 is given by:

$$\Delta P_{tie,14} = \frac{2\pi}{s} T_{14} [\Delta f_1 - \Delta f_4] \quad (2.16)$$

and, the tie-line power change $\Delta P_{tie,15}$ between areas 1 and 5 is given by:

$$\Delta P_{tie,15} = \frac{2\pi}{s} T_{15} [\Delta f_1 - \Delta f_5] \quad (2.17)$$

Adding equations (2.14, 2.15, 2.16 and 2.17), the total the tie-line power change $\Delta P_{tie,1}$ between areas 1 and the other three areas is calculated as follows:

$$\Delta P_{tie,i} = \Delta P_{tie,12} + \Delta P_{tie,13} + \Delta P_{tie,14} + \Delta P_{tie,15} = \frac{2\pi}{s} \left[\sum_{j=2,3,4,5} T_{1j} \Delta f_1 - \sum_{j=2,3,4,5} T_{1j} \Delta f_j \right] \quad (2.18)$$

Similarly, for N control areas shown in Figure 2.5 **Error! Reference source not found.**, the total tie-line power exchange between areas 1 and other control area i system (Bevrani & Hiyama 2011) is obtained as:

$$\Delta P_{tie,i} = \sum_{\substack{j=1 \\ j \neq i}}^N \Delta P_{tie,ij} = \frac{2\pi}{s} \left[\sum_{\substack{j=1 \\ j \neq i}}^N T_{ij} \Delta f_i - \sum_{\substack{j=1 \\ j \neq i}}^N T_{ij} \Delta f_j \right] \quad (2.19)$$

where $\Delta P_{tie,i}$ is the tie-line power exchange of area i and T_{ij} is the torque coefficient between areas i and j , N is the number of the areas, Δf_i is the frequency deviation in the area number i , Δf_j is the frequency deviation in the area number j . The tie-line power exchange with neighbouring areas can be maintained at scheduled values by considering the supplementary controller in the presence of tie-line signal. Figure 2.6 represents control area i connected to the other areas through the tie line, where, B_i is a frequency bias factor in area i , AEC_i is the area control error in area i , R_i is the speed drop in area i , ΔP_{ci} is the power input reference, ΔP_{gi} is the governor input power in area i , ΔP_{vi} is the change in governor valve position in area i , $\Delta P_{tie,i}$ is the tie-line power flow changes in area i , ΔP_{mi} is the mechanical power change in area i , ΔP_{Di} is the load power(pu) in area i , T_{psi} is the power system time constant in area i , Δf_i is the frequency deviation in area i , Δf_j is the frequency deviation in area j , Δf_k is the frequency deviation in area k , T_{ij} is the synchronizing torque coefficient between the area i and area j , T_{ik} is the synchronizing torque coefficient between the area i and area k and T_{ji} is the synchronizing torque coefficient between the area j and

area i , i, j and k are three different control areas. Beside the frequency regulation, the control area error (in area i is feed to the supplementary feedback control to schedule the net interchange power with the neighbouring areas at the scheduled values. ACE_i is the sum of the tie-line flow and frequency changes. Therefore, ACE is calculated as follows:

$$ACE_i = \Delta P_{tie,i} + B_i \Delta f_i \quad (2.20)$$

where B_i represents the frequency bias factor of i th control area and its value is obtained as:

$$B_i = \frac{1}{R_i} + D_i \quad (2.21)$$

where R_i is calculated as in equation (2.1), D_i is the damping coefficient in the area number i .

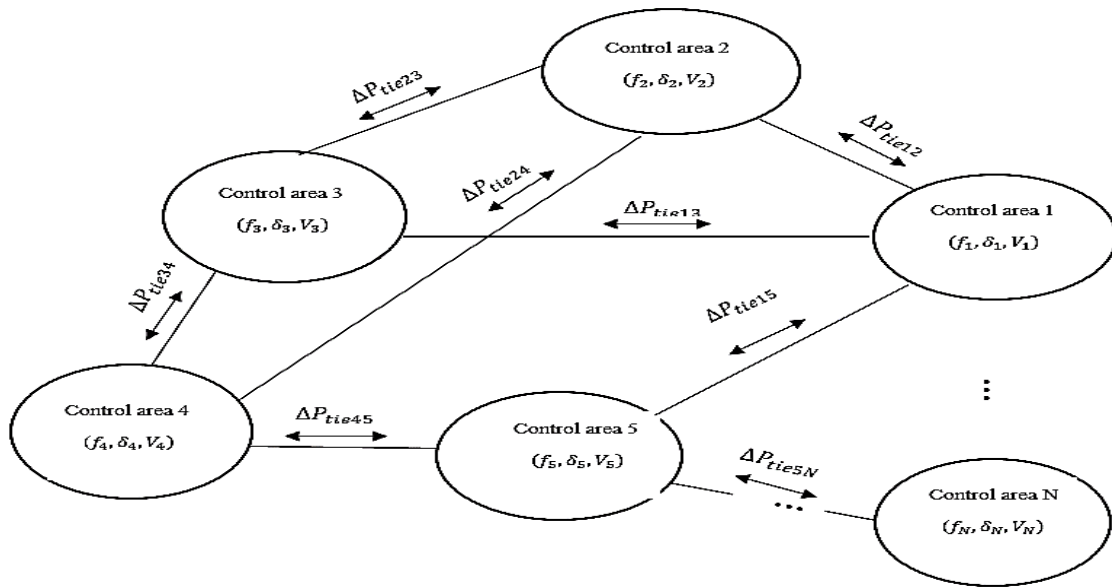


Figure 2.5. N control areas of large-scale interconnected power system (Ma et al, 2014, Bevrani,2014).

A block diagram of Figure 2.6 illustrates how supplementary control is implemented with the ACE signal. Input signals are step load perturbations and tie-line power errors between areas, while output signals are frequency deviations and area control errors. Each tie-line power follows and frequency is monitored by its own control area in the area control centre. Based on the calculation and allocation of ACE , the controller $K(s)$ acts. Lastly, the corresponding control signal on an i th control area will change the speed. Thus, the supplementary control shown in Figure 2.6 should be able to meet the basic LFC objectives and maintain the area frequency and tie line interchange as scheduled.

general structure of DPM for large power system with m DISCOs, and n GENCOs can be expressed as:

$$DMP = \begin{bmatrix} cpf_{11} & cpf_{12} & \cdots & cpf_{1(m-1)} & cpf_{1m} \\ cpf_{21} & cpf_{22} & \cdots & cpf_{2(m-1)} & cpf_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ cpf_{(n-1)1} & cpf_{(n-1)2} & \cdots & cpf_{(n-1)(m-1)} & cpf_{(n-1)m} \\ cpf_{n1} & cpf_{n2} & \cdots & cpf_{n(m-1)} & cpf_{nm} \end{bmatrix} \quad (2.23)$$

where, Cpf_{ij} ($i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$) represent the contract participation factors that correspond to the fraction of the total load power contracted by DISCO j from GENCO i . The sum of all the entries in a column in this matrix is unity i.e.

$$\sum_{j=1}^n cpf_{ji} = 1 \quad (2.24)$$

The model of deregulated power systems is same as the traditional AGC model architecture except in the later structure, additional signals are added to the former structure. These additional signals are used to capture the possible various contracts between control areas such as the contracted load demands between the DISCOs and other DISCOs and GENCOs, local contacts within a single area and the bilateral contracts, as well as the load demand tracking between control areas. They also can adjust the scheduled flow over tie-line. The general AGC model structure for the control area- i in deregulated environment is shown in Figure 2.7 (Bevrani, Mitani & Bevrani, 2005). This model consists of the traditional AGC model plus the new information's signals that capture the various possible contracts between DISCO i and other DISCOs and GENCOs. These input signals include the total of the local contracted demands and local area disturbances (V_{Li}), the interface effects between different control areas (V_{Ii}), the scheduled tie-line power exchange (V_{Si}) and the bilateral signals vector (V_{Bi}) as shown in Figure 2.7. The V_{Li} and V_{Ii} can be expressed mathematically respectively as follows:

$$V_{Li} = \Delta P_{Li} + \Delta P_{di}, \quad V_{Ii} = \sum_{\substack{j=1 \\ j \neq i}}^N T_{ij} \Delta f_j \quad (2.25)$$

where, ΔP_{Li} is the local contracted demand of area i , N is the number of control areas, ΔP_{di} is disturbances and uncontracted demand in area i , Δf_j is the frequency deviation in control area number j and T_{ij} is the torque coefficient between the area number i and area number j .

where, α_{ni} is an ACE_i participation factor, V_{Si} is given as in equation (2.26). It should be noted that in a deregulation environment, both DPM and apfs are rigorously used. The DPM matrix can be chosen based on market economics, but no approach has been identified for choosing apfs (Ram and Jha, 2010), which may lead to unacceptable modelling and hence the control implementation. The bilateral signals vector contains all the bilateral contracts between other DISCOs and GENCO of the area i which is calculated as follows:

$$V_{Bi} = [V_{Bi-1} \quad V_{Bi-2} \quad V_{Bi-3} \quad \cdots \quad V_{Bi-n}] \quad (2.29)$$

where,

$$\begin{aligned} V_{Bi-1} &= \sum_{j=1}^N cpf_{1j} \Delta P_{Li} \\ V_{Bi-2} &= \sum_{j=1}^N cpf_{2j} \Delta P_{Li} \\ &\vdots \quad \quad \quad \vdots \\ V_{Bi-n} &= \sum_{j=1}^N cpf_{nj} \Delta P_{Li} \end{aligned} \quad (2.30)$$

The most important issue in deregulated environment is that the steady state error between the generation of each GENCOs and the contracted demands of DISCO should be zero at the steady state. Therefore, based on the DMP entries, the desired total power generation of GENCO i is given as follows:

$$\Delta P_{d-t-i} = \sum_{j=1}^N cpf_{ij} \Delta P_{Lj} \quad (2.31)$$

where, ΔP_{Lj} is the contracted demands in area j , ΔP_{d-t-i} is the desired total power generation of GENCO i and cpf_{ij} is the contracted participation factors between the area i and area j .

2.2.4 Review discussion on multi-area area power system model having more than one renewable/non-renewable units in each control area

Mathematical Modelling of single area power system, multi-area interconnected power systems (MAIPS) and deregulated multi-area interconnected power systems (DMAIPS) of the AGC design problem are presented. The total the tie-line power change ΔP_{tiei} between areas i and the other areas is explained. Considering the deregulation environment, the concept of disco participation matrix is discussed. The obtained mathematical models of MAIPS will be converted in chapters 3, 4 and 5 into continuous and discrete state space equations.

2.3. Control Techniques for Multi-Area Interconnected Power Systems

This section reviews some of advanced control methods that were widely investigated in the literature for LFC control problems. These include, centralized and decentralized control methods, soft computing and intelligent control methods, Linear quadratic regulator control methods, Robust control methods and other types of control methods that involved in LFC control problem. The layout for the review carried out on AGC control techniques is shown in Figure 2.8.

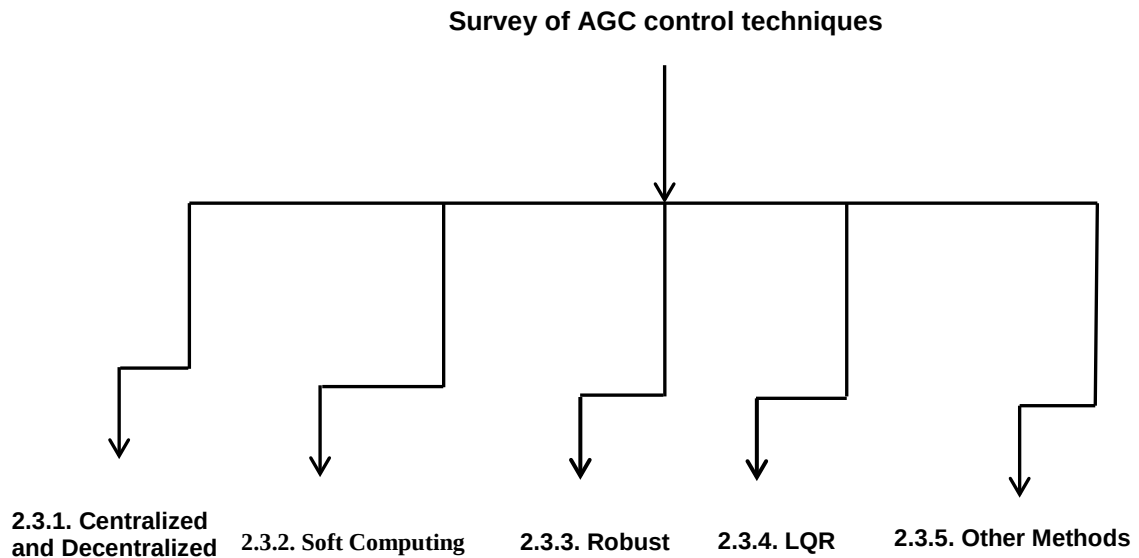


Figure 2.8. Illustration of review on AGC control techniques.

2.3.1. Centralized and Decentralized Controllers

During the last decades, many advanced control structures and algorithms have been proposed and applied to Multi-Area Interconnected Power Systems (MAIPS). However, few of these schemes are completely acceptable. Therefore, three control design strategies have been explored in literature, namely centralized, decentralized, and distributed control

systems. The distributed control systems framework is out of the scope of this work. Centralized load frequency control can be defined as a global control operator that gathers information from all system states and responds accordingly. In addition, it can be designed and analysed based on linear and nonlinear control theories. According to Feliachi (1987) and Pathak et al, 2015, centralized AGC controllers, and a geographically distributed interconnected power systems do not have similar information structures because some state variables aren't physically accessible. So, in order to implement centralized control schemes, a complex data acquisition system is required. In addition, because of system redundancy, the system becomes more complex and the control system uses an extensive communication system. For these reasons, it is necessary to implement state estimation methods to eliminate errors resulting from poor data transmission. For all these reasons, the cost of hardware increases due to (i) back-up control if the communication link fails, (ii) delay for data transmission and (iii) delay in data, in other words, as the system's dimension increased, it is too complex and difficult to implement and tune the centralized AGC schemes (Tyagi and Srivastava, 2006), due to the required stability, robustness, inherent computational complexity, as well as communication bandwidth limitations (Pathak et al, 2015).

Because of the above reasons, decentralized control framework has been proposed to many large multivariable systems, including MAIPS. The decentralized scheme is easy to be implemented in practice, and reduces the complexity that is associated with the centralized control scheme (Rerkpreedapong and Feliache 2002). Thus, several decentralized control strategies were introduced in the literature to effectively deal with the difficulties of centralized AGC schemes. For instance, Ikeda, Siljak, & White, 1981 proposed overlapping control for linear power systems, Feliachi, 1987 presented the feasibility analysis and design of optimal decentralized load frequency control for multi-area interconnected electric power systems, Bakule, 2008, reviewed the decentralization, decomposition, and robustness methods in decentralized control approach.

Recently, Ansaria, Abbasi, and Firouzia 2022 presented a decentralized linear matrix inequality (LMI) event-triggered integral sliding mode control to deal with disturbances and model uncertainties for interconnected power systems. In all decentralized control methods mentioned above, the decentralized control unit is a feedback controller that comprises independent control units, each of which receives the measured output x_i ($i = 1, 2, \dots, N$), and affects the control input u_i ($i = 1, 2, \dots, N$) and it only controls its own subsystem as shown in Figure 2.9. The control laws are independent from each other (Lunze, 1992, Vu, 2022).

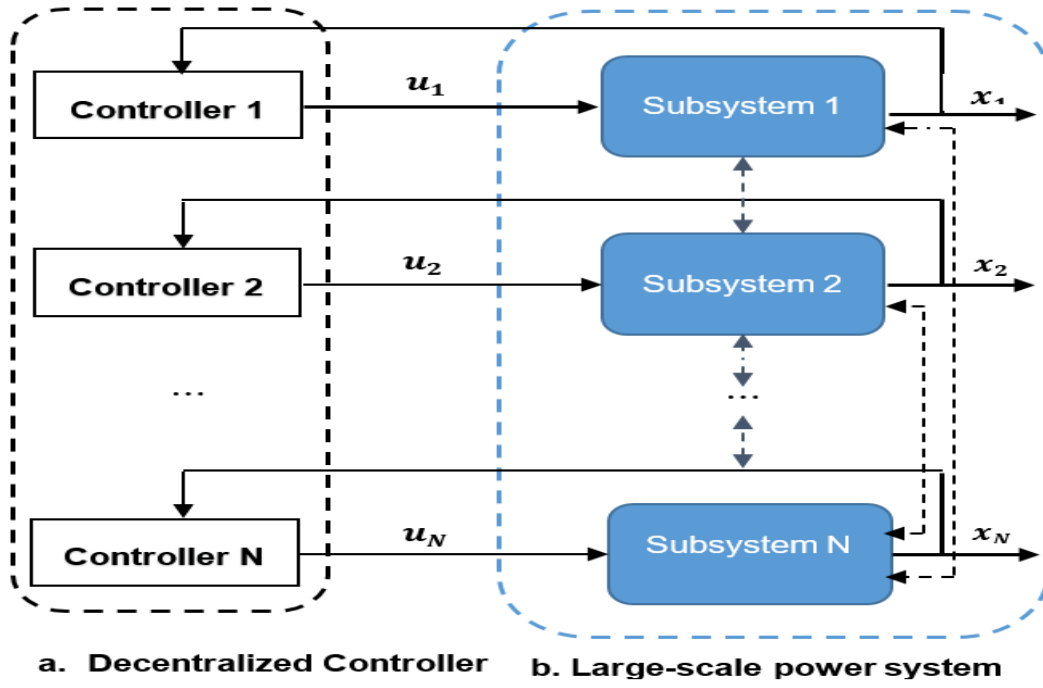


Figure 2.9. Structure of Decentralized Control System (Vu, 2022).

The large-scale multi-area interconnected power system is divided into subsystems using one of the decomposition methods. The main decomposition systems include disjoint subsystems, overlapping subsystems, symmetric composite systems, multi-time scale systems and hierarchically structured systems. The standard linear control theory is applied to design a local controller for each subsystem after large systems are decomposed into subsystems. However, the dynamical performance may be degraded in case the interactions effects are not considered between adjacent subsystems, which assumes there are no tie-line power flow exchanges, and may result in poor system performance (Hao, Tan & Ali, 2014). The decentralized control performance is often improved by taking the amount of interactions dynamics in consideration.

As an examples of reviewed papers, Hasan & Hussein, 2014 presented and designed decentralized a AGC based on PID, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) for an interconnected power system. The proposed controllers were tested on identical two-area interconnected power system consisting of thermal power plants considering AC tie-line and Asynchronous tie-line. At 1% step load perturbations (SLPs), the dynamic response analysis revealed that PSO had a better dynamic response result compared to the PID and GA controller for both types of area interconnection. To achieve robustness against disturbances rejection, model uncertainties and improve MAIPS stability, Abdelhamid & El-Gaafary, 2018, proposed Optimal Decentralized LQR Control for Multi-Area nterconnected power system. In this technique, the multi-area power system is divided into four subsystems using decentralized input control methodology. Then, standard linear control

theory is applied to design local controller for each subsystem. Some controlled subsystems may have poles close to the imaginary axis of the complex plane, which may push the aggregated overall system into the region of instability. Therefore, the proposed controller is applied at subsystem level to shift undesired poles of the subsystem one by one to predetermined stable region which in turn shift the slow poles of the overall system and reduces the effect of the interaction between the interconnected subsystems. One disadvantages of this procedure is that it has been applied as many times as the number of undesired poles until all desired poles have been moved to the region of the stability. The dynamic response revealed the effectiveness of the proposed method in terms of steady state error of frequency deviations and robustness against the model uncertainties compared to the work of other researchers. Recently, to reduce the effects of load disturbances in power system, Hakimuddin, Khosla & Garg, 2020 designed and implemented proportional integral derivative (PID) structured centralized and decentralized AGC controllers using Genetic algorithms (GAs) for realistic two-area interconnected power system with multi sources power plants in each area. The GAs optimized the integral square error (ISE) cost function to tune the parameters of the proposed controllers. At 1% step load perturbation, the decentralized controller has better disturbance rejection when compared to centralized control method.

2.3.2. Review Discussion of Centralized and Decentralized Controllers

In summary, Decentralization, decomposition, and robustness are effective tools for dealing with difficulties associated with large systems, such as high dimensionality, information

structure constraints, uncertainty, and time delay because of communication channels.

2.3.3. Soft Computing Techniques in Multi-Area Interconnected Power Systems

Several researchers have investigated the control of interconnected power systems using soft computing/intelligent techniques such as Newton-Raphson algorithm (NRA), Genetic Algorithms (GA), and Bacterial Foraging Optimization Algorithms (BFOA), as well as artificial intelligent control techniques. Dande et al., (2001) was one of the first researchers who used trajectory insensitivities to obtain optimal parameters of the integral control using gradient type Newton-Raphson algorithm. Rerkpreedapong, Hasanovic & Feliachi (2003) used genetic algorithms and Linear Matrix Inequalities (LMI) to develop robust Proportional Integral (PI) Automatic Generation Control (AGC) against load disturbances. GA has been used for optimizing the integral control gains and frequency bias factors in a three - area power system (Demiroren and Zeynelgil 2007). AGC control based on Adaptive Artificial Neural Fuzzy Inference System (ANFIS) and Particle Swarm Optimization (PSO) has been

proposed for optimal gain scheduling (Hosseini and Etemadi 2008). In this study, generation rate constraint is considered in two control areas composed of steam turbines and the ANFIS is used to estimate the dead band. The simulation results demonstrated that the control system provides good dynamic response for both control areas frequency deviations and tie-line power error. In addition, adaptive control gains can be updated in real - time according to the variations in the power demands. Despite, good performance was obtained; PSO took a long time to compute the optimal gains, which is impractical for real time applications. Bacterial foraging technique is applied to optimize simultaneously the gains of non-integer controller and speed regulation parameter R in multiple-area thermal power systems (Debbarma, Daikia & Sinha, 2013). In these studies, it is important to note that all the authors studied AGC problem without considering the combination of multiple power sources such as thermal, hydro, nuclear and renewable power sources in one control area which did not reflect the real scenario case (Hota and Mohanty 2016).

In the last decade, many efforts were made to combine multiple power sources in one control area. Ram and Jha (2010) examined conventional and artificial intelligent control techniques such as fuzzy logic, neural network and neuro-fuzzy using genetic algorithms against load disturbances in hydro-thermal power system. Moreover, work by Shree and Kamaraj (2016) has proposed Hybrid Neuro Fuzzy approach for automatic control problem. The research has focused on the robustness of ANFIS control against large load changes and disturbances in presence of model uncertainties and the system nonlinearities. In this approach, three area hydro-thermal power systems with Generation Rate Constraints (GRC) under various operating conditions and different contracted scenarios are tested for the controller evaluation. Advantage of this approach over Hybrid Particle Swarm Optimization (HCP SO), Real Coded Genetic Algorithm (RCGA), and Artificial Neural Network (ANN) techniques lies on its simple structure, easy implementation, and high robustness capabilities. This method requires computational power and is not efficient in industry. It needs a bigger training set to enhance the efficiency of the controller. This in turn increases the calculation time. Since in the real time the calculation is carried over the sampling period. Thus, this control technique is not useful for real time power systems.

In another study, Rahman, Saikia & Sinha, (2015) have used Biogeography-Based Optimisation (BBO) technique to optimize simultaneously the three degree of freedom proportional integral derivative (3DOF-PID) controller gains as well as the electric governor parameters for two-area power system (thermal-hydro). In this study, Integral of Squared Error (ISE) is taken as a cost function. The physical constraints, GRC and dead band are considered in the study and the system dynamics was evaluated considering 1% step load perturbation. The results reveal that the three degree of freedom integral derivative (3DOF-

ID) controller has better performance than I, integral derivative (ID), and two degree of freedom integral derivative (2DOF-ID) controllers in terms of settling time, peak deviation, and magnitude of oscillation. In addition, the 3DOF-ID's parameters optimized by BBO are found to be robust against load changes and inertia constant. The weakness of this approach is that the frequency bias coefficient (B) and governor speed regulation parameter (R) are selected based on trial- and-error guidelines.

Later, Quasi Oppositional Harmony Search Algorithm-Proportional Integral Derivative (QOHS-PID) was proposed and applied by Shiva and Mukherje (2016) to a five-area power system. In this study each area consists of multiple combinations of generation units, such as reheat thermal, hydro and gas generating units. The physical constraints nonlinearities such as GRC and governor dead band time delay are considered, imposed, and tested for five area power system. In addition, Integral of Squares Error (ISE) cost function was used by QOHS algorithm to optimize the PID controller parameters. The simulation results demonstrated that QOHS-PID control stabilized the frequency deviations, scheduled tie line power flow deviations to their nominal values in presence of the physical constraints. Although good performance was observed, the communications delays are not considered and QOHS-PID practical implementation is not yet demonstrated.

Recently, several researchers proposed the cascaded controlled approach to improve interconnected power system performance and disturbance rejection with high-load perturbations and nonlinearities. The Cascade control approach is well known approach applied in many industrial applications where the system is controlled by using two loops, namely primary and secondary loops Celik, 2021, Barakat et al., 2022). The objective of the primary loop is to suppress the effects of disturbances. This idea is extended to the problem of load frequency control (LFC) in interconnected power systems. To test two-area thermal power system with/without governor dead band (GDB) nonlinearity and three-area hydrothermal power with generation rate constraints, (Celik, 2021) proposed a cascade fractional order proportional integral-fractional order proportional derivative (FOPI–FOPD) for LFC controller of electric power-generating systems. FOPI-FOPD consists of fractional order PI and fractional order PD controllers connected in series where the orders of integrator (k) and differentiator (l) is fractional. The Dragonfly search algorithm (DSA) is used to minimize the integral time absolute error (ITAE) cost function and concurrently tune the gains and fractional order parameters of the proposed control system. DSA is a mathematical model and computer simulation of static and dynamic swarming behaviours of dragonflies in nature. The results revealed that the optimized FOPI–FOPD controller based on DSA managed to reduce settling time, undershoot and overshoot of frequency and tie-line power oscillations. For the same purpose, Barakat et al., 2022, proposed a tilt-derivative tilt-integral (TD-TI)

cascade controller based on a water cycle algorithm (WCA) for two areas of the interconnected power system. The dynamic response reveals that the proposed technique is superior in terms of performance than teaching–learning-based optimization (TLBO)-based PID) and the Interior Point Algorithm (IPA)-based tilt integral derivative (TID) controller (IPA/TID). In general, all these methods need to be tested on systems with high dimensions' states.

2.3.4. Review Discussion on Soft Computing Techniques in MAIPS

The optimization techniques such as Newton Raphson method, the combined Genetic Algorithms and ILMI, BFAO, PSO all are shown robustness performance against the load changes in multi-area intercommoned power system control but AGC problem has been studied without considering all the nonlinearities or the effects of the multiple sources area not studied in a control area which doesn't reflect the real scenario. This encourages some researchers to propose soft computing techniques such as ANN, Fuzzy logic, Hybrid neuro-fuzzy control as well as the optimization techniques such as Biogeography based Optimization and QOHS Algorithm for the multiple sources in a single area. The best soft computing technique for large load changes is the Hybrid neuro-fuzzy because of its simple structure, easy implementation, and high robustness but it requires large number of neurons, large training data and large training time which made it not to be practical for real time power systems. The optimization methods BBO and QPHS are robust again large load changes and transient performances and the simultaneous input profiles, but the BBO still relies on trial and error method while the QOHS is only tested on the simulations.

Finally, a cascaded FOPI–FOPD cascade controller based on the Dragonfly search algorithm (DSA) and TD-TI cascade controller based on a water cycle algorithm (WCA) improved the dynamic performances. But these methods need to be tested on power systems with high dimensions' states.

2.3.5. Robust control Techniques in Multi-Area Interconnected Power Systems

Robust based automatic generation Control approaches also have received attention by researchers for their stability and robustness characteristics against plant uncertainties, parameters variation, and load disturbances in interconnected power systems. Since then, there have been some of works and applications explored in the literature. In these approaches, the frequency regulation, tracking the load variations, and maintaining tie-line power exchanges with specified values in presence of physical constraints and plant uncertainties are the main objectives of these studies. The control design problems on above approaches can be classified into two main control problems: the first one is the robust

control problem including H_∞ control and multi-objective control problem via mixed H_2/H_∞ control technique. The second one is the μ synthesis and analysis theory. The following section provides a brief review of these approaches.

The early robust load frequency control problem was addressed by Feliachi (1996) where AGC problem was formulated as H_∞ control problem to guarantee the frequency regulation tracking in presence of step perturbations. However, applying this procedure yields a controller that has a higher order which may need to be minimized for real-time and complex systems. In Addition, Bevrani et al., (2003] used μ synthesis and analysis theory to design a robust AGC and guarantee stability in presence of multiplicative uncertainty and load disturbances for four control areas. The μ synthesis approach has proven to be a useful technique for minimizing the effect of disturbances and achieving acceptable frequency regulation in presence of uncertainties and load variation. Later, Bevrani et al (2005) proposed mixed H_2/H_∞ technique to design decentralized robust AGC in interconnected power system. The AGC problem is formulated as multi-objective optimal control problem where several different performance and robustness objectives can be met. In this technique, H_2 control minimizes the linear quadratic cost of tracking error and the control input while H_∞ control ensures the stability of the closed loop system in the existence of control constraints and uncertainties. Iterative linear matrix inequality method is used to solve this convex optimization problem, however, the design problem for optimal robust controllers based on H_∞ techniques results in a non-convex optimization problem, which suffers from computational intractability and conservatism (Bhatt et al., 2010).

Robust H_∞ -Static Output Feedback Control (H_∞ -SOF) to Multi-area interconnect power system is considered in the literature (Bevrani, 2014). It uses realization of state-space equation (2,32) for a linear time invariant system $G(s)$ and closed-loop configuration shown in Figure 2.10 to formulate the H_∞ -SOF problem.

$$\begin{aligned}\dot{x} &= Ax + B_1w + B_2u \\ z &= C_1x + D_{12} \\ y &= C_2x\end{aligned}\tag{2.32}$$

where x represents the state variable vector, w represents the disturbance and other external input vector, and y represents the measured output vector.

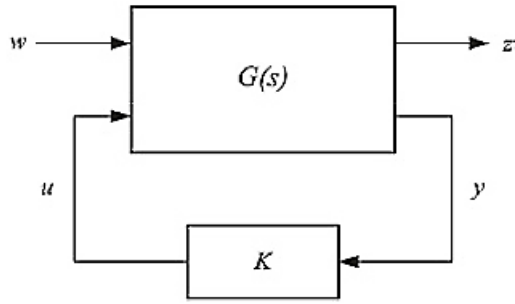


Figure 2.10. Feedback system via H_∞ control (Bevrani, 2014).

The task of control design is to find the static output feedback law $u = ky$ (matrix k) as shown in **Error! Reference source not found.**, which enables the resulted closed-loop system to be internally stable, and the H_∞ -norm from w to z is smaller than γ as follows:

$$\|T_{zw}(s)\|_\infty < \gamma \quad (2.33)$$

where γ small positive number. where γ small positive number. There should be a symmetric matrix $X > 0$ if the matrices (A, B_2, C_2) is stabilizable and detectable and this makes s matrix K will be a dynamic and hence this condition will occur

$$\begin{bmatrix} A_{cl}^T X + X A_{cl} & X B_{cl} & C_{cl}^T \\ B_{cl}^T X & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \quad (2.34)$$

where,

$$\begin{aligned} A_{cl} &= A + B_2 K C_2, & B_{cl} &= B_1 \\ C_{cl} &= C_1 + D_{12} K C_2, & D_{cl} &= 0 \end{aligned}$$

To ensure the stability of the power system with the communication time delay, Azimi, Afzalian & Ghaderi, 2021 proposed a H_∞ decentralized static output feedback controller. Each area is controlled by a diesel generator, wind turbine, photovoltaic panel, and battery storage. Using the modified event triggering method, the network control system model is extended. Wirtinger's inequality is used to derive the least conservative fitting conditions for linear matrix inequality. Results showed that the proposed method improved H_∞ control's stability and dynamic performance. Recently, to decrease the effects of chattering, oscillations and nonlinearities, Guo, 2022 developed adaptive high order sliding mode control (HOSMC) for two area control and three area control power systems. The proposed method showed robustness against parameters variation. In addition, it attenuates chattering and

oscillations more than super-twisting method (STM). For the SMC to ensure the best dynamic performance, the state equations must be estimated by an observer in order to maintain best dynamic performance. Generally, Robust control techniques are not practical for power systems with high dimensions and extensive parameters (Ullah et al, 2021).

2.3.6. Review Discussion on Robust Control Techniques in MAIPS

In this section, the literature review of robust control methods such as H_∞ control, AGC robust control based on μ synthesis and analysis and mixed H_2/H_∞ control in interconnected power system have presented and advantages and drawbacks of each controller have been discussed. In this review, the mixed H_2/H_∞ decentralizes the design and overcome the problems that associated with H_∞ control. However, according to Ullah et al, 2021), the robust control techniques are techniques are not practical for power systems with high dimensions and extensive parameters.

2.3.7. Linear Quadratic Regulators (LQR) in in Multi-Area Interconnected Power Systems

Most of the discussed control design methods used in the AGC problem have not been ideal or sufficient for large and complex power systems (Ibrahim and Kumar, 2013). In recent studies, modern optimal control theory has been investigated in the literature as one of the control strategies to deal with the complexity of interconnected power systems (Alrifai & Hassan ,2011 (Tungadio, Ramesh & Mukwanga, 2018), Vlahakis, Dritsas, and Halikias 2019). The first optimal AGC control was introduced by (Fosha, and Elgerd 1970; Elgerd and Fosha 1970) for two-area interconnected power systems. In recent studies, the Linear Quadratic Regulator (LQR) has received attention for optimal AGC design in large and complex interconnected power systems. This is mainly because of its large stability margins in its stabilized solutions (Vlahakis, Dritsas, and Halikias 2019). Optimal AGC based on LQR can handle the robustness, reliability, and nonlinearities associated with power system control (Tungadio et al. 2018). For instance, a distributed centralized LQR design is proposed for a class of large-scale multi-area power systems to guarantee the overall stability of the network and disturbance rejections of power-load step variations (Vlahakis, Dritsas, and Halikias 2019). The authors (Tungadio, Ramesh & Mukwanga, 2018) used optimal control theory to control the active power balance of two microgrids linked by two AC tie-lines. In another study, Yang, Zhang & Zomaya, 2020 combined the Lyapunov energy function, optimization design theory, and an iterative linear matrix to design optimal AGC control for two and five area interconnected power systems respectively. The design of optimal quadratic control aims to determine the optimal control law that can transfer the system from its initial state to the final state such that a given cost function is minimized (Ko et al. 2004). Optimal control methods provide optimum system control and regulation of all

dynamic states of the controlled systems. For optimal feedback control to be feasible, all state variables must be observable (Alhelou et al, 2018).

In the design of the optimal quadratic control, the most important first step is choice of the state and control weighting matrices Q and R . The weighting matrices Q and R play a vital role in determining the amount of state error, energy expenditure, and system performance (Ibrahim and Kumar 2013). The authors used a trial-and-error method based on the characteristics of the system state and controller (Bimarta et al. 2018), Bryson's rule in which the matrices Q and R are taken as diagonal matrices with diagonal entries (Chitu, 2013) for selecting the weighting matrices. Besides, Q and R were also selected based on the desired natural frequency and damping ratio of the closed-loop system for LQR based linear controllers (He, Wang & Lee, 2000). Authors (Das et al. 2013) proposed using global optimization techniques, i.e., real coded genetic algorithm to optimally find the weighing matrices that are associated with LQR based PID control design. In recent studies, three approaches i.e. the controllability and observability indices of the system, minimization of the cost function value, and engineering judgement have been systematically developed for selecting state weighting matrices (Pathak, Nasiruddin, & Bhatti, 2015). In these approaches, optimal LQR AGC is designed based on the controllability and observability indices of the system, minimization of the cost function value and engineering judgement for two-area interconnected power systems. In the middle method, the optimal quadratic AGC control is designed to transfer the system state from an arbitrary initial state to the final state in an infinite amount of time so that the cost function is minimized. The weighting matrix Q was defined for the dynamic system under study by considering the excursion of area control errors, the excursion of the integral of area control errors and the excursion of the control vector about the steady state. This method was recently applied to a realistic model of centralized optimal AGC in a real-time-time environment (Pathak, Nasiruddin, & Bhatti, 2015). In these approaches, optimal LQR AGC is designed for two areas of control in the existence of communication delays. The results revealed that the cost functional minimization method provides more realistic responses, and it can be easily extended to the class of large dynamical systems i.e., power systems coupled with interaction signals. The discussed technique, on the other hand, has the advantage of using part of the state variables to build the state weighting matrix, and it is connected with a computational burden as the dimensions and scale of the system increases. Based on the literature review, two studies have only been conducted for the functional minimization method for two-area control system with an identical non-reheat turbines. There is no research related to hydro and gas generators, as well as the combination of these two generators and other types of generators.

In optimal control theory, the LQR controller design can either be formulated as full state

feedback control (FSFC) or output feedback control (OFC) strategies (Parmar, Majhi and Kothari, 2012). Usually, the signals of FSFC control are generated by linear combination of all states of the system. On the other hand, the signals of OFC strategy are generated by linear combination of states to be measured. Measuring all states of a system is difficult and expensive in practice. Moreover, all the states variables of the system are required to be monitored during the implementation of FSFC strategy in the closed-loop. Usually, a smaller of states are used by OFC when compared to FSFC strategy. As result, it reduces cost, complexities, and number of sensors. Therefore, the centralized discrete decentralized optimal quadratic automatic generation control (DOQAGC) based on Optimal control theory and functional cost minimization are developed for interconnected power systems in this thesis. Since, functional minimization approach provides a simple and straightforward systematic approach, considers partially known state variables, and enables the cost function to optimize the state feedback gains matrix using the well-known linear quadratic theory, state and input weighting matrices are constructed using it. Further, due to high precision, smaller controller size and adaptable and less noise, of the digital control methods (Shayeghi, Shayanfar & Jalili, 2009, Ullah et al, 2021), the state space models are converted from continuous forms to discrete forms. Moreover, DOQAGC compared with existing control approaches such as continuous optimal automatic generation controllers (OAGC). Table 2.1 summarizes the contribution, implementation, and capabilities between the reviewed studies and discrete DOQAGC controllers.

Table 2.1. DOQAGC versus OAGCs controllers in the literature based on functional minimization.

Contribution/Implementation/ Novelty	Criteria	OAGC (Hasan et al. 2013)	OAGC (Pathak et al, 2015)	DOQAGC (Esmail, 2023)
Contributions	What type of control theory is used?	Linear Quadratic Optimal control theory	Linear Quadratic Optimal control theory	Linear Quadratic Optimal control theory
	Continuous and discrete?	Continuous	Continuous	Discrete
	The complex of the system Single or multi-area	A two-area interconnected power system with identical plants consisting of	A two-area interconnected power system with identical plants	Four area control of Non-reheat, Reheat, Hydro and generators
Implementation	Simulation or real-time use case scenario (MATLAB or PLC (literature aspect) or any other simulation platforms)	Simulation based study	Real-time Implementation	Simulation based study (MATLAB)
Novelty	What type of control algorithm is used?	• Full state feedback control • Functional Minimization Method for two identical control areas	• Global state feedback controller • Functional Minimization Method for two identical control areas	• Centralized full state feedback controller • Functional Minimization Method for N non-identical control areas

2.3.8. Review Discussion on LQR Control Techniques in MAIPS

The centralized control technique and Decentralized Linear Quadratic Regulator (DLQR) in large and complex interconnected power systems are demonstrated and compared. LQR controller is robust against nonlinearities associated with power systems, reliable, and has large stability margins in its stable solutions as well as good disturbance rejection performance. Different methods of selecting state and control weighting matrices Q and R , are discussed. For systems with a short dimension, controllability and observability indices work better since they depend on Eigenvalues and Eigenvectors in the state space model. On the other hand, the functional cost minimization method provides more realistic responses, and it can be easily extended to the class of large dynamical systems coupled with interaction signals. Functional minimization is an easy and systematic approach, and it uses only a few variables in the system, such as area control errors, integral of area control errors, and input variables for control. In the end, the functional minimization approach will be successful in optimizing the discrete centralized and decentralized optimal quadratic controllers (COQAGC and DOQAGC) for interconnected power systems under functional minimization approach.

2.3.9. Other control Techniques in Multi-Area Interconnected Power Systems

Beside the methods above, variable structure control based on sliding mode control (VSC/SMC), Distributed Model Predictive Control (DMPC) and Decentralized Active Disturbance Rejection Control (DADRC) methods have been proposed for interconnected power systems. The layout for the review carried out on other AGC control techniques is shown in Figure 2-11.

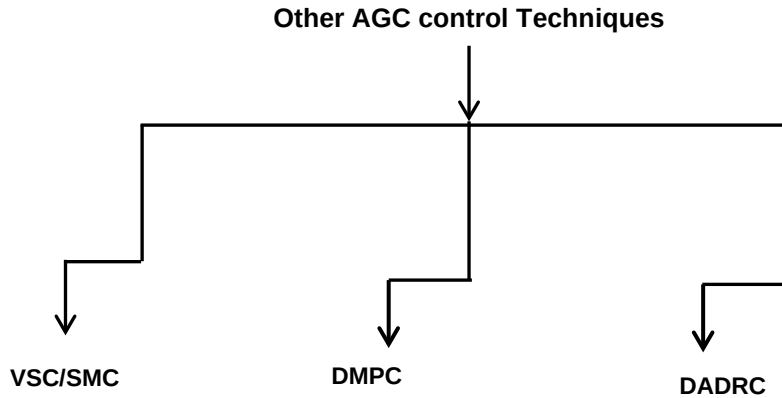


Figure 2.11. Illustration of review on other AGC control techniques.

Sekhar and Vaisakh (2013) presented VSC/SMC Controllers for AGC in interconnected power systems to achieve high-performance and robustness of AGC. This approach combines the features of both variable structure control and sliding mode control. The results reveal that the proposed controllers are quite insensitive to parameter variations and speed governor dead band with or without generation rate constraints.

In another study, DADRC has been designed to enhance AGC control performance in multi-area power systems (Hao, Tan & Ali, 2014). This control is often improved the dynamic performance by taking the amount of interactions dynamic in consideration.

A distributed control systems framework has been proposed for large-size power systems (Ma et al, 2017). In this framework, the large system is decomposed to several subsystems, where each controller locally controls its own subsystem. In addition, the controllers exchange information between them via communication bus. Distributed control systems were found to be suitable for large-size systems if the proper nonlinear control has been chosen. This approach doesn't need the complete model of the power system and disturbances, as an Extended State Observer (ESO) is used for estimating unknown generalized disturbances. The approach achieves a good disturbance rejection performance, and its simple structure can be easily implemented in practice. However, DADRC is associated with worse performance when anti- GRC is not optimized. To overcome the limitations of the above control method, in 2016, Ma et al., (2016) proposed a new

Distributed Model Predictive Control (DMPC) for a large-scale power system in presence of external disturbances and generation rate constraints (GRC) and load reference set-point within boundary. The main idea was to design local DMPC for each subsystem, in which controllers from different areas exchange information between themselves via communication bus. The simulation result shows that this method is effective for large-scale power systems in comparison to traditional model predictive control (MPC) and centralized model predictive control (CMPC). However, DMPC will not be practical for real time applications for the following reasons: the control design is model based approach which needs accurate information of the plant; the closed loop stability performance is based on and dependent of constraints; the optimization step size has an impact on the online optimization time; the on-line optimization needs expensive computational power.

2.3.10. Review Discussion on Other Control Techniques MAIPS

This section has demonstrated the feasibility of the other control methods such as VSC/SMC, DMPC and DADRC as well as the distributed control systems in interconnected power systems. The distributed control systems and DMPC are competitive and suitable for large systems, but DMPC is not practical for real-time systems because of its model-based design. On the hand, distributed control has good disturbance rejection performance, we can easily implement its simple structure in practice but because of the communication between control areas this method is beyond this thesis.

2.3.11. Summary of Reviewed Control Techniques

This section summarizes the earlier discussion on advanced control techniques for LFC problem in MAIPS, named with centralized and decentralized control methods, soft computing and intelligent control methods, robust control methods, Linear Quadratic Regulator method and other control techniques. The summary considers the capabilities and drawbacks of the reviewed control techniques as shown in Table 2.2.

Table 2.2 Summary of various control methods' capabilities and drawbacks in AGC problem.

Control Method	Capabilities of the method	Drawbacks of the method
Conventional linear control (P,PI,PD and PID)	Simple design and implementation.	-takes more time and gives large frequency deviation. -fail to provide best control performance over a wide range of operating conditions.
Newton-Raphson	Bilateral contracts simulation, achieves optimum integral control gains and frequency bias.	Simulation study only and not tested on real time power system
GA+ LMI	Robust against load disturbances, low control order.	Simulation study not tested on real time power system.

ANFIS based on PSO	-Provides good dynamic response for the both control areas frequency deviations and tie-line power error. -Adaptive control gains can be updated in real – time.	PSO took long time to compute the optimal gains, which is impractical for real time applications.
ANFIS based on ANN	Simple structure, easy implementation and high robustness capabilities.	Bigger training set is needed in order to enhance the efficiency of the controller which in turn increases the calculation time which impractical in real time.
BBO	-Robust against load changes and inertia constant - Very good dynamic responses terms of settling time, peak deviation, and magnitude of oscillation.	The parameters B and R were selected based on trial-and-error guidelines.
QOHS-PID	Promising optimization method for complex multi-units power systems in	Simulation study only, communications delays were not considered in the study.
H^∞ control	Robust against disturbances for large scale power systems	yields a higher order control
μ synthesis control	Effective for disturbances rejection.	Has Low frequency regulation in presence of uncertainties and load variation.
H_2/H_∞	Has good tracking error and stability performances and robust for bilateral contracts.	Requires static feedback synthesis and LMI for robust control design.
VSC/SMC	Insensitive to parameter variations, nonlinearities and physical constraints as well it requires low computational cost.	Chattering effects was not considered which makes its implementation impractical.
DMPC	-Formulates the AGC problem as a tracking control Problem of large-scale system in presence of both external disturbances and constraints, bounds. -Improves the closed-loop performance.	Model based approach; the closed loop stability performance is dependent of constraints and the on-line optimization needs expensive computational power.
DADRC	Has simple structure and implementation, good disturbances rejection.	The performance was worsening when anti-GRC is not optimized.

2.4. Nonlinearities Associated with AGC system model

For practical implementation, some physical constraints/ nonlinearities must be considered in the dynamic modelling of interconnected power systems. They include the dead band associated with the governor, the generation rate constraint (GRC) because of the limitation of the thermal and mechanical movement, boiler dynamics, and time delays associated with the communication as shown in Figure 2.12. The following describes important physical constraints affecting power system performance.

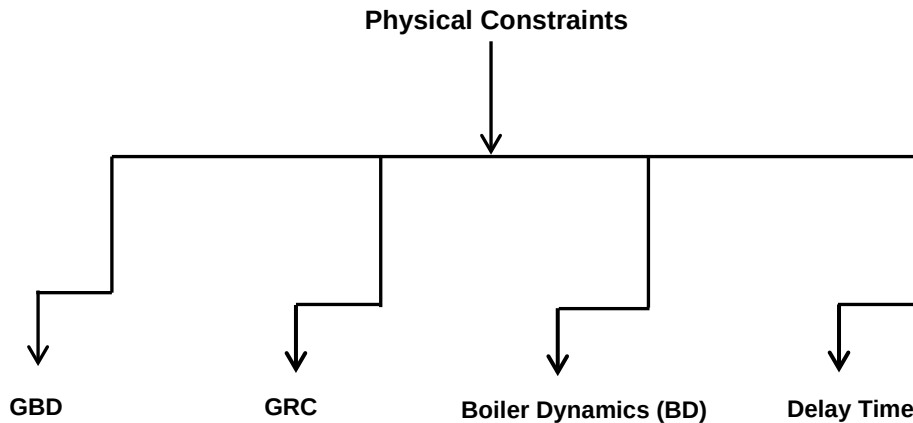


Figure 2.12. Physical constraints associated with AGC system model.

2.4.1. Governor Dead Band (GDB)

Dead band is defined as the total amount of a continued speed change within which there is no change in valve position. Steam turbine dead band is due to the backlash in the linkage connecting the servo piston to the camshaft. Much of this appears to occur in the rack and pinion used to rotate the camshaft that operates the control valves. Due to the governor dead band, an increase/decrease in speed can occur before the position of the valve changes. The speed governor dead band has a great effect on the dynamic performance of electric energy system. The backlash non-linearity tends to produce a continuous sinusoidal oscillation with a natural period of about 2s seconds. In the present work backlash non-linearity of 0.05% for the thermal system and 0.02% for hydro system is considered (Mohanty, Panda, & Hota, 2014).

2.4.2. Generation Rate Constrains (GRC)

Generation Rate Constraints in a power system, power generation can change only at a specified maximum rate known as Generation Rate Constraint (GRC) (Golpîra, Bevrani, & Golpîra, 2011). In past studies, a GRC of 3% per min is considered for thermal units (Mohanty, Panda, & Hota, 2014, Sahu et al, 2013) while others have a GRC between 5% and 10% per minute (Parmar, Majhi and Kothari, 2012). The GRCs considered for hydro unit

are 270% per minute for raising generation and 360% per minute for lowering generation (Khuntia, & Panda, 2012).

2.4.3. Boiler Dynamics (BD)

Boiler dynamics configuration is incorporated in thermal plants to generate steam under pressure. Changes in the steam flow and deviations in pressure are sensed and the corresponding action is initiated by the turbine control valves and boiler control (Mohnty et al, 2014).

2.4.4. Time Delays

The time delay in the AGC is due to the delays in the communication channels, filters, and signal processing units. Time delays pose significant challenge to AGC study especially in interconnected power systems. Their absence could endanger security, reliability, and stability of power system (Naidu et al, 2014). They can degrade the dynamic performance of the system and even turn system instability (Bevrani & Hiyama, 2007). A time delay of 50ms is considered in ΔPC signals of the realistic AGC system under study (Sahu et al, 2015).

Table 2.3. summarizes the typical generation rates for some types of the generators

Table 2.3. Generators units versus generation rates constraints (Bevrani & Hiyama , 2011).

Generator unit	Percentage capacity
Diesel engines	40%/min
Industrial Gas Turbine cycle (IGTC)	20%/min
Combined Gas Turbine (CGT)	5-10%/min
Steam Turbine Plants	1-5%/min
Nuclear plants	1-5%/min
Hard-coal-fired	2-4%/min
Lignite-fired-plant	1-2%/min
hydro unit	270- 360%/min
thermal units	3-10%/ min

In real-time control systems design these nonlinearities need to be considered because the dead-band increases the steady-state error of the frequency which in return affects the performance of the AGC control. On the other hand, time delays on communication channels between the control centre and operating stations, are becoming a significant challenge in AGC control performance as the time delays get bigger.

2.5. Discussion

Regarding the AGC control the strengths and weaknesses of the control methods are summarized as follows: conventional control design was simple but takes more time and gives large frequency deviation (Ram and Jha 2010,). "Fixed gain controllers are designed at nominal operating conditions and fail to provide best control performance over a wide range of operating conditions" (Zou et al, 2015). Both intelligent control methods and optimization algorithms are called soft computing methods (Pandey, Mohanty, & Kishor, 2013). Artificial intelligent control methods such as fuzzy logic control, artificial neural network, reinforcement learning and ANFIS are model independent methods, but they are time consuming, not easy to design and not efficient in the industry (Shree and Kamaraj, 2016). On the other hand, the optimization algorithms such as genetic algorithm (GA) (Demiroren and Zeynelgil, 2007), Hybrid Particle Swarm Optimization (PSO) (Bhatt et al, 2010), Bacterial Foraging (BF) algorithm (Debbarma et al, 2013), and Differential Evolution (DE) algorithm (Hota and Mohanty, 2016) have shown dynamical improvement, but are not tested to a more realistic power system model, considering time-delay, governor dead band and GRC, all-together, as imposed in real-time power system under large scale power systems. These limitations of the mentioned optimization algorithms are addressed by Shiva and Mukherje (2016) using quasi-oppositional harmony search algorithm. However, the communication delays are still not solved.

Robust control design such as H_∞ , μ synthesis and mixed H_2/H_∞ schemes guaranteed closed loop robust stability from disturbances and achieved acceptable frequency regulation in presence of uncertainties and load variation. However, they were restricted to constraints uncertainties and in the most cases provided conservative solution with higher order control. On the other hand, the advanced control designs such variable structure with sliding mode control and distributed model predictive control are model based schemes which need accurate information of the plant, so they are not practical for large size power systems. Finally, on the other hand, the decentralized AGC optimal control based on LQR received attention and it may be a solution for large, interconnected power systems.

2.6. Chapter Conclusions

The continuous increase in size and complexity of interconnected power system makes it challenging to design a controller and analyse the performance of the entire power system. Most of the control methods significantly reviewed simplifies the complexity of an interconnected power system. However, because most of these control methods are developed in the continuous time domain and implemented in the discrete-time domain, the closed-loop performance is poor, and the system may become unstable. In addition, these methodologies do not consider modelling the interconnected power system as a hybrid model consisting of traditional generators and renewable energy sources. Furthermore, in most studies, the control methods failed to achieve satisfactory practical performance because the effects of physical constraints such as GRC, Boiler dynamics, dead band, time delays, and sensitivity analysis were not explicitly considered in the controller design. As a result, the discrete decentralized optimal automatic generation control based on a quadratic linear regulator (LQR) will be a good control choice for large, complex, interconnected power systems. The LQR control method has design performance characteristics that solve the control design challenges posed by increasing the size of large, interconnected power systems due to integrating distributed resources on the electrical power grid.

The literature review in Chapters 1 and 2 shows that the LQR control method is robust against nonlinearities associated with power systems, reliable, and has large stability margins in its stable solutions as well as good disturbance rejection performance in comparison to other robust control methods from the literature and this can turn into the benefit by easing the control design and implementation of complex interconnected power systems (Tungadio, Ramesh & Mukwanga, 2018). However, for large, interconnected power systems, weighting matrices (Q and R) selection strategies, such as the identity matrix approach, trial and error approach, the participation factor analysis, hybrid bacteria foraging oriented particle swarm optimization (PSO), and eigenvalues and eigenvectors decomposition method, are not suited for large-scale power systems since it is impossible to gather accurate information on all system states as the power system size increases (Arya, Kumar, and Gupta, 2017; Panwar et al., 2022). However, to address these shortcomings, the functional minimization method (FMM) is a suitable approach for constructing weighting matrices that is simple and systematic as the size of the interconnected power system grows and will be beneficial by taking a few state variables into account.

This chapter discussed advanced control techniques and methods for solving load frequency control (LFC) problems in multi-area interconnected power systems (MAIPS). A functional minimization method (FMM) will help design optimal discrete centralized and decentralized

optimal quadratic controllers for large, interconnected power systems. Finally, the overlapping decomposition method will be used to divide the system into subsystems, and the optimal LQR method will be used to generate discrete decentralized suboptimal quadratic automatic generation control (DOQAGC) controllers for the subsystems.

Chapter 3

Modelling of Interconnected Power Systems and Decomposition Techniques

3.1. Introduction

This chapter discusses the modeling of the interconnected power system. The control of an interconnected power system as a large complex system with high dimension, model uncertainties, and information structure constraints is becoming a challenge using multivariable theory alone. Therefore, some structures i.e. unstructured model, the input/output-oriented model, and the interaction-oriented model as well as the control strategies i.e. Centralization and decentralization are reviewed to simplify the modeling, control, and analyzing the complexity of interconnected power systems. Decomposition, decentralization, and inclusion principles are discussed to assist in developing the decomposed model of interconnected power systems.

The chapter is structured as follows: In section 3.2, a dynamics of interconnected power systems is presented. In section 3.3, large system modeling structures and decomposition methods are presented. In section 3.4, the decomposed subsystems models of two area interconnected power system are derived. Section 3.5 presents the summary arising from the analyses and the significance of obtained over all model and decomposed subsystems.

3.2. Dynamics of interconnected power systems

Interconnected power systems are large scale systems with strong coupling, wide geographical distribution and overlapping interactions among different subsystems. Their dynamics are associated with a large number of inputs, outputs and long state order as well as nonlinearities (Lunze 1992). The state-space model of an interconnected power system is obtained from the transfer functions of the generator (Governor and Turbine) and the transfer function equation describing the frequency of the power system. **Figure 3.1. Model of two area interconnected power system with identical non-reheat thermal turbine** Each an area is represented with power system transfer function model, turbine, governing systems with speed drop R (Kundur,1994). The notations T_{ps1} and T_{ps2} are time constants for the power system in area 1 and area 2 respectively and T_{12} is the tie-line synchronizing torque coefficient. The tie-line exchange power x_1 is positive value when power transfer is increased from area 1 to area 2. Vice versa it has a negative value when power transfer is decreased from area 1 and area 2. The control signals $u_i = \Delta P_{refi}$ are the set-points of the power generated by i^{th} area at nominal frequency.

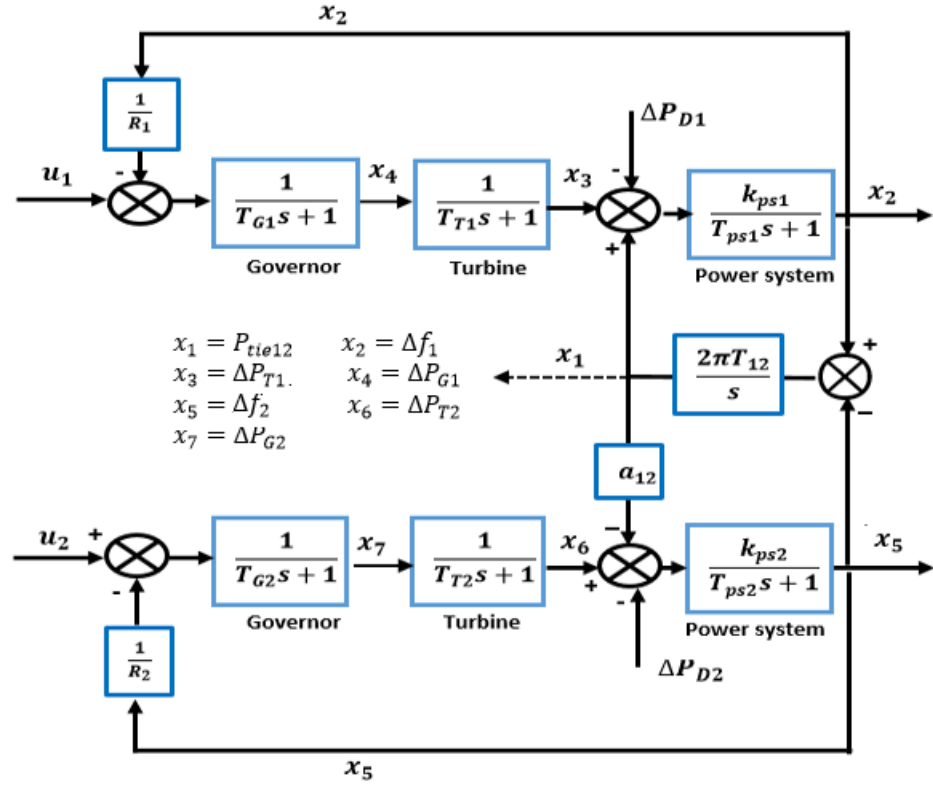


Figure 3.1. Model of two area interconnected power system with identical non-reheat thermal turbine (Wood, Wollenberg & Sheble, 2014).

The concept of converting the transfer functions of the interconnected power system to a state-space model has been established and discussed by Rakhshani et al (2016) and Deepak & Abraham (2015). Therefore, to model interconnected power systems in state-space representation it is necessary to first derive the first order differential equations by describing each individual subsystem.

Since the most important subsystems of the interconnected power system are the Governor, Turbine and the power system as well as the tie-line model, the development of the differential equations of these subsystems is done for each area starting with the Tie-line. Then the state space model for the overall structure of the interconnected power systems is developed and is discussed in the next section.

3.2.1. State-space representation of the interconnected power system

To obtain the state-space model that describes the overall behavior of the interconnected power systems, the principle of conversion of the transfer functions in Laplace domain to first order differential equations (ODEs) is applied to each subsystem as discussed and presented in (Deepak and Abraham,2015). To explain this concept, the two thermal interconnected power systems are considered as shown in Figure 3.1. Model of two area interconnected power system with identical non-reheat thermal turbine. The state variables are chosen as the tie-line deviation in the tie-line flow ($x_1 = \Delta P_{tie12}$), deviations in frequencies ($x_2 = \Delta f_1, x_5 = \Delta f_2$) in area1 and area2 respectively, the deviation in the power output of turbine1 in area1 ($x_3 = \Delta P_{T1}$) and the deviation in the power output of turbine 2 in area 2 ($x_6 = \Delta P_{T2}$), the deviation in the power output of governor1 in area1 ($x_4 = \Delta P_{G1}$) and the deviation in the power output of governor 2 in area 2 ($x_7 = \Delta P_{G2}$). First Ordinary differential equations (ODEs) for the different variables in the two-area interconnected power system are obtained as follows (Rakhshani et al, 2016):

$$\dot{x}_1 = 2\pi T_{12}x_2 - 2\pi T_{12}x_5 \quad (3.1)$$

$$\dot{x}_2 = -\frac{k_{ps1}}{T_{p1}}x_1 - \frac{1}{T_{ps1}}x_2 + \frac{k_{ps1}}{T_{ps1}}x_3 - \frac{k_{ps1}}{T_{ps1}}\Delta P_{D1} \quad (3.3)$$

$$\dot{x}_3 = -\frac{1}{T_{T1}}x_3 + \frac{1}{T_{T1}}x_4 \quad (3.2)$$

$$\dot{x}_4 = -\frac{1}{T_{G1}R_1}x_2 - \frac{1}{T_{G1}}x_4 + \frac{1}{T_{G1}}u_1 \quad (3.4)$$

$$\dot{x}_5 = -\frac{a_{12}k_{ps2}}{T_{ps2}}x_1 - \frac{1}{T_{ps2}}x_5 + \frac{k_{ps2}}{T_{ps2}}x_6 - \frac{k_{ps2}}{T_{ps2}}\Delta P_{D2} \quad (3.5)$$

$$\dot{x}_6 = -\frac{1}{T_{T2}}x_6 + \frac{1}{T_{T2}}x_7 \quad (3.6)$$

$$\dot{x}_7 = -\frac{1}{T_{G2}R_2}x_5 - \frac{1}{T_{G2}}x_7 + \frac{1}{T_{G2}}u_2 \quad (3.7)$$

Therefore, in matrix form the above first order differential equations can be presented as:

$$\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} &= \begin{bmatrix} 0 & 2\pi T_{12} & 0 & 0 & -2\pi T_{12} & 0 & 0 \\ -\frac{k_{ps1}}{T_{p1}} & -\frac{1}{T_{ps1}} & \frac{k_{ps1}}{T_{p1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 \\ -\frac{a_{12} K_{ps2}}{T_{ps2}} & 0 & 0 & 0 & -\frac{1}{T_{ps2}} & \frac{K_{ps2}}{T_{ps2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{R_2 T_{G2}} & 0 & \frac{1}{T_{G2}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{G1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{G2}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\frac{K_{ps1}}{T_{ps1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{K_{ps2}}{T_{ps2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta P_{D1} \\ \Delta P_{D2} \end{bmatrix}
\end{aligned} \tag{3.8}$$

where x , u and p are the n dimensional state, two dimensional control and two dimensional load disturbance vector respectively, whereas A , B and Γ are the respective matrices with appropriate dimensions. Since, A , B and Γ are introduced in Equation 3.26 in section 3.3.1 at page 10). The outputs of the model are $x_1 = \Delta P_{tie12}$, $x_2 = \Delta f_1$, $x_5 = \Delta f_2$ and which is represented by

$$y(t) = C_i x_i(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \tag{3.9}$$

The vectors x , u and p for two-area interconnected power systems with one thermal generation unit in each area are defined as follows:

$$\begin{aligned}
x^T &= [\Delta P_{tie12} \ \Delta f_1 \ \Delta P_{T1} \ \Delta P_{G1} \ \Delta f_2 \ \Delta P_{T2} \ \Delta P_{G2}]^T \\
u^T &= [u_1 \ u_2]^T \\
p^T &= [\Delta P_{D1} \ \Delta P_{D2}]^T
\end{aligned} \tag{3.10}$$

In next section, the linear model of the Tie-line between Two-area interconnected power systems are discussed.

3.2.2. Two –area Interconnected Power System with Tie-line

The principles of load flow equations to develop a linear model of a tie-line for two area interconnected power system with reference to Khezri et al (2019) and Deepak & Abraham (2015) is shown in Figure 3.2. Tie line Model between two-area interconnected Power System (Deepak & Abraham, 2015). The method used to calculate the tie-line power for two area interconnected power system assumes that the tie-line resistance is negligible because of the high value of the inductance jX_{12} . It is also assumed that the current I_{12} flows from the control area 1 to the control area 2 and resistance of the tie-line transmission is zero. The P_{r1} and P_{r2} are the rated power capacities of the control areas 1 and 2 respectively, S_{tie12} is the apparent tie-line power injected in bus 1, S_{tie21} is the tie-line apparent power injected in bus 2, V_1 and V_2 are the voltage magnitudes in bus1 and bus 2 respectively, δ_1 and δ_2 are the voltages angles at the bus 1 and bus 2 respectively. The combination of the reactive power (Q) and true power (P) is called apparent power (S). It is the product of a circuit's voltage and current without reference to phase angle and is measured with Volt-Amps (VA) unit.

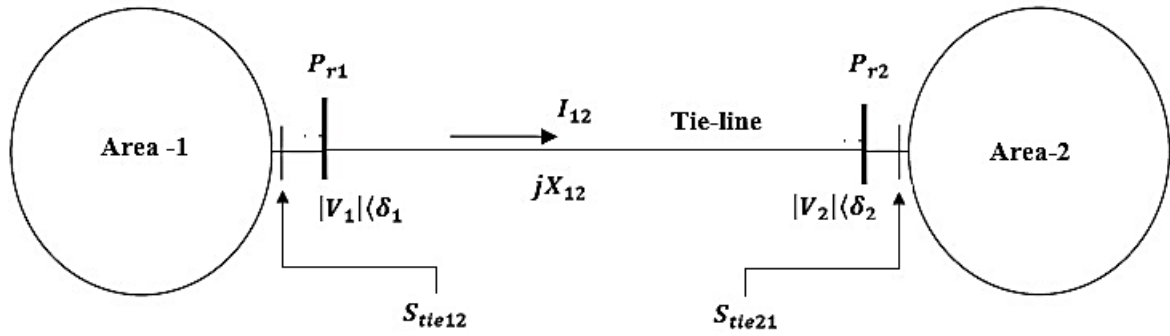


Figure 3.2. Tie line Model between two-area interconnected Power System (Deepak & Abraham, 2015).

The apparent power S_{tie12} is injected in the Tie-line at the bus 1 using load flow formula as follows

$$S_{tie12} = P_{tie12}^0 + jQ_{tie12}^0 = V_1 \times \hat{I}_{12} \quad (3.11)$$

where $\hat{I}_{12}$ is the complex current flow from area-1 to area-2 via high transmission bus, P_{tie12}^0 and Q_{tie12}^0 are real power and the reactive power from area-1 to area-2 respectively, V_1 is the voltage at bus 1. Similarly, the apparent power conjugate $\hat{S}_{tie12}$ is represented by the following complex equation:

$$\hat{S}_{tie12} = P_{tie12}^0 - jQ_{tie12}^0 = \hat{V}_1 \times I_{12} \quad (3.12)$$

where $\hat{V}_1$ is the complex voltage in area-1. By using the ohm law and considering the

voltages potential difference over the reactance, the current I_{12} which flows from area -1 to area-2 can be described as follows:

$$I_{12} = \frac{V_1 - V_2}{jX_{12}} = \frac{|V_1| \angle \delta_1^0 - |V_2| \angle \delta_2^0}{jX_{12}} \quad (3.13)$$

Substituting Equation (3.13) into Equation (3.12) and changing the angle of the real part, the conjugate apparent power yields

$$\begin{aligned} \hat{S}_{tie12} &= P_{tie12}^0 - jQ_{tie12}^0 \\ &= \frac{|V_1| \times |V_2|}{X_{12}} \sin(\delta_1^0 - \delta_2^0) - j \left[\frac{|V_1|^2 - |V_1| \times |V_2|}{X_{12}} \right] \cos(\delta_2^0 - \delta_1^0) \end{aligned} \quad (3.14)$$

As result, the real power and reactive power that are injected at the bus can be described by Equations (3.15) and (3.16) respectively.

$$P_{tie12}^0 = \frac{|V_1| \times |V_2|}{X_{12}} \sin(\delta_1^0 - \delta_2^0) \quad (3.15)$$

$$Q_{tie12}^0 = \left[\frac{|V_1|^2 - |V_1| \times |V_2|}{X_{12}} \right] \cos(\delta_2^0 - \delta_1^0) \quad (3.16)$$

Assumption: power lossless in the tie-lines is same at bus 1 and bus 2 if the line resistance component is neglected.

Now let's assume there is load disturbances ΔP_{L1} and ΔP_{L2} in the area-1 and the area-2 respectively. Introducing the load disturbances in one control area or in both control areas, the angles δ_1 and δ_2 , as well as the scheduled tie-line real power flow P_{tie12} will deviate from their nominal values. As result. Equation (3.15) can be written as:

$$P_{tie12} = \frac{|V_1| \times |V_2|}{X_{12}} \sin(\delta_1 - \delta_2) \quad (3.17)$$

Where, $\delta_1 = \delta_1^0 + \Delta\delta_1$, $\delta_2 = \delta_2^0 + \Delta\delta_2$ and $P_{tie12} = P_{tie12}^0 + \Delta P_{tie12}$. Therefore, the Equation (3.17) can be written as follows:

$$P_{tie12}^0 + \Delta P_{tie12} = \frac{|V_1| \times |V_2|}{X_{12}} \sin((\delta_1^0 - \delta_2^0) + (\Delta\delta_1 - \Delta\delta_2)) \quad (3.18)$$

$$= \frac{|V_1| \times |V_2|}{X_{12}} \sin(\delta_1^0 - \delta_2^0) \cos(\Delta\delta_1 - \Delta\delta_2) + \cos(\delta_1^0 - \delta_2^0) \sin(\Delta\delta_1 - \Delta\delta_2)$$

Assuming that $\Delta\delta_1 - \Delta\delta_2$ is very small, this leads $\cos(\Delta\delta_1 - \Delta\delta_2) \cong 1$ and $\sin(\Delta\delta_1 - \Delta\delta_2) \cong (\Delta\delta_1 - \Delta\delta_2)$. Substituting these values into Equation (3.18), the real power is obtained as follows:

$$\Delta P_{tie12} = \frac{|V_1| \times |V_2|}{X_{12}} \cos(\delta_1^0 - \delta_2^0) [\Delta\delta_1 - \Delta\delta_2] \quad (3.19)$$

Taking in the consideration the rated power capacity P_{r1} at the bus 1, the change tie-line power flow in per unit can be given as follows:

$$\Delta P_{tie12}(\text{pu}) = T_{12} [\Delta\delta_1 - \Delta\delta_2] \quad (3.20)$$

where

$$T_{12} = \frac{|V_1| \times |V_2|}{X_{12}} \cos(\delta_1^0 - \delta_2^0) \quad (3.21)$$

Given the relationship between area power angle and the frequency, Equation (3.20) can be written as (Bravani, 2005):

$$\Delta P_{tie12}(\text{pu}) = 2\pi T_{12} [\int \Delta f_1 - \int \Delta f_2] \quad (3.22)$$

Taking the Laplace transform of Equation (3.22), we can obtain $\Delta P_{tie12}(s)$ as:

$$\Delta P_{tie12}(s) = \frac{2\pi T_{12}}{s} [\Delta f_1 - \Delta f_2] \quad (3.23)$$

Where Δf_1 and Δf_2 are frequency deviations for areas 1 and 2 respectively, and T_{12} is the synchronizing torque coefficient. Equation (3.23) is actually referring to state-space representation Equation (3.1) and in the form of the block diagram.

3.2.3. Review summary on two area interconnected power system and the motivation for introducing decomposition method for large, interconnected power systems.

A detailed derivation of the over all state-space representation and Tie-line model of two area interconnected power system has been developed and discussed in sections 3.2.1 and 3.2.2. The concept of converting the transfer functions of the subsystems to first order differential

equations adopted in order obtain the overall state space model of the system. This approach is also applicable to interconnected power systems with any number of control areas or generating units. But this increases the dimension of the system state space equations in which made the modelling, analysing and controlling the system to be a big challenge. Therefore, in the next section presents the modelling structures of large-scale systems and its decomposition methods and decentralization control strategy are discussed in detail.

3.3 Decomposition Methods for Large Complex interconnected system with Multiple Area

The basic control design problem is to find a suitable control law $u(t)$ for a large-scale interconnected system represented by a set of mathematical equations to meet the design requirements (Khorsand, 2010; Drouin & Abou-Kandil,1991). In order to solve this design problem, the large-scale interconnected systems with high dimensionality, uncertainties and constraints are modelled based on one of the three structures, namely unstructured model, input/output-oriented model and interaction-oriented model (Lunze,1992) as shown in Figure 3.3. The difference between these model structures depend on the internal complexity of dynamics and size of the system. Unstructured model corresponds to centralized control strategy which is used for multivariable systems and theory. It particularly doesn't account for the subsystems of the complex system. Rather it considers the whole system in the design as demonstrated in Figure 3.3. Structure of the models of interconnected systems: (a) unstructured model; (b) I/O oriented model; (c) interaction-oriented model (Lunze,1992). Figure 3.3. Structure of the models of interconnected systems: (a) unstructured model; (b) I/O oriented model; (c) interaction-oriented model (Lunze,1992). 3.3(a) and its state-space model is described as(Lunze,1992):

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) & x(0) &= x_0 \\ y(t) &= Cx(t) + Du(t)\end{aligned}\tag{3.24}$$

where x is the n -dimensional state vector of overall system, the matrices A , B , C and D are the state, input and out matrices with appropriate dimensions.

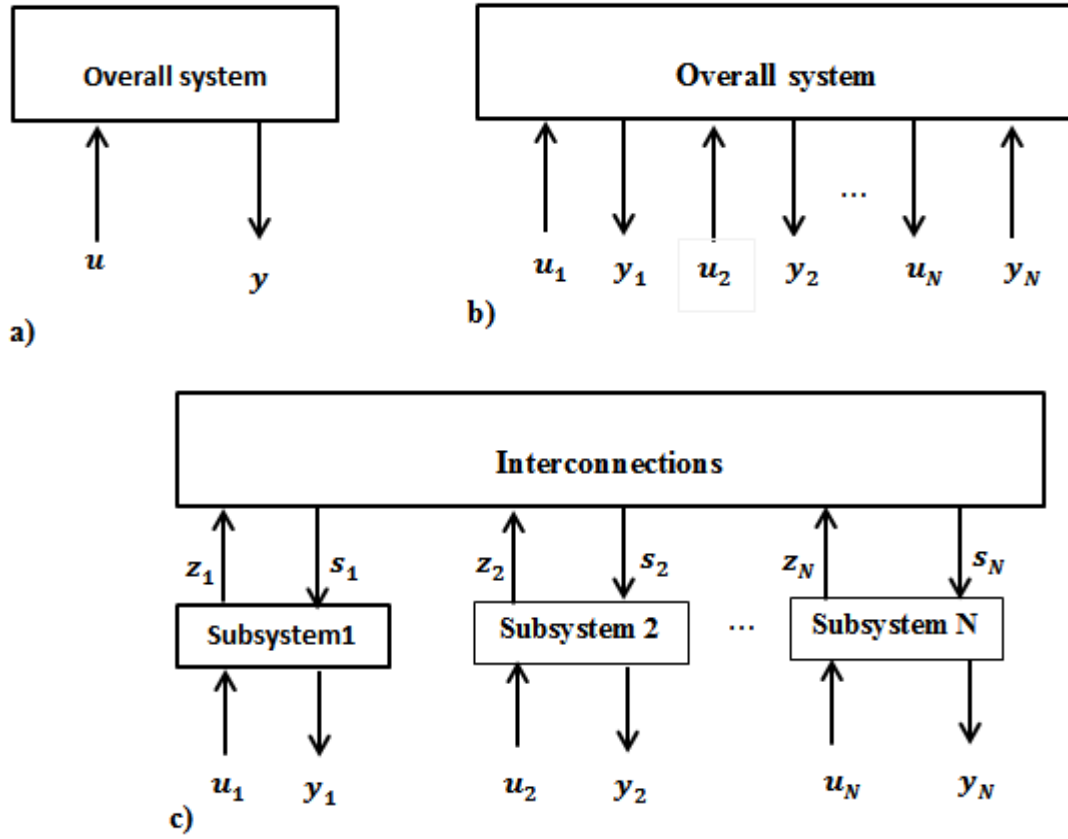


Figure 3.3. Structure of the models of interconnected systems: (a) unstructured model; (b) I/O oriented model; (c) interaction-oriented model (Lunze,1992).

Input/output-oriented model is used for decentralized control design as shown in Figure 3.3. Structure of the models of interconnected systems: (a) unstructured model; (b) I/O oriented model; (c) interaction-oriented model (Lunze,1992). 3.3(b). The system input and output vectors u_i and y_i ($i = 1, 2, \dots, N$) are partitioned into sub-vectors $u = (u_1, u_2, \dots, u_N)'$ and $y = (y_1, y_2, \dots, y_N)'$ respectively in the considered overall system. Its state space model is described as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^N B_i u_i(t) & x(0) &= x_0 \\ y_i(t) &= C_i x(t) + \sum_{j=1}^N D_{ij} u_j(t); & i &= 1, 2, \dots, N \end{aligned} \quad (3.25)$$

The matrices B, C and D from Equation (3.24) are partitioned into sub-matrices to obtain the

sub-matrices B_i , C_i and D_{ij} from Equation (3.25) as follows:

$$B = (B_1 \quad B_2 \quad \cdots \quad B_N)$$

$$C = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_N \end{pmatrix}, D = \begin{pmatrix} D_{11} & D_{12} & D_{13} & \cdots & D_{1N} \\ D_{21} & D_{22} & D_{23} & \cdots & D_{2N} \\ D_{31} & D_{32} & D_{33} & \cdots & D_{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ D_{N1} & D_{N2} & D_{N3} & \cdots & D_{NN} \end{pmatrix} \quad (3.26)$$

From Figure 3.3. Structure of the models of interconnected systems: (a) unstructured model; (b) I/O oriented model; (c) interaction-oriented model (Lunze,1992). (b) 3.3(b) and Equations (3.24 to 3.26), we can observe that the I/O-oriented model has advantage of showing the structure of the inputs and outputs but it doesn't show the interaction signals between the subsystems.

In the interaction-oriented model, the large-scale interconnected systems are decomposed into number of different subsystems that interact with each other through coupling signals. Each subsystem contains its separate inputs and outputs, and local control laws. Also, its dynamics are affected by both local corresponding inputs and interactions with neighboring subsystems.

The practical applications, where the interaction-oriented model has been used include modeling of large-scale interconnected power systems, traffic system, economic systems, and distributed control systems. In those applications, the interaction-oriented model is received attention as the best modeling structure for describing the behavior of the interconnected power systems. Next section presents the interaction-oriented model.

3.3.1. Interaction-oriented model

The linearized interaction-oriented model of an interconnected power system is described in Equation (3.27) as discussed and represented in the literature (Lunze, 1992, Siljak, 2007, Tlili & Braiek, 2009, Deepak&Abraham, 2015. This model captures the subsets models of the different subsystems and their interaction signals where the interactions between the different subsystems are exchanged through the input coupling signals $s_i(t)$ and output coupling signals $z_i(t)$. Equations (3.1 to 3.7) are written in interaction-oriented model shown in Equation(3.27) such that the matrix A has the structure as given in Equation (3.30).

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \Gamma_i s_i(t) \quad x_i(0) = x_{i0}$$

$$y_i(t) = C_i x_i(t) \quad (3.27)$$

$$z_i(t) = C'_{zi} x_i(t) \quad i = 1, 2, \dots, N$$

where x_i , u_i and s_i are the n dimensional state, control and load disturbances vectors whereas A_i , B_i and Γ_i are the system, input and disturbance matrices with appropriate dimensions.

In the interaction-oriented model, the relationship between the interconnections input s and interconnection output z can be described as follows:

$$s_i = L z_i \quad (3.28)$$

Where the vectors s_i and z_i of dimension m_s or r_z respectively and L is matrix with suitable dimension. Neglecting the interactions ($s_i(t) = 0$) between subsystems, Equation (3.26) yields the model of an isolated subsystem

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u(t) x_i(0) = x_{i0} \\ y_i(t) &= C_i x_i(t) \end{aligned} \quad (3.29)$$

According to Lunze (1992), the matrix A_i must be constructed with the following specific interaction structure:

$$A_i = \begin{bmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n} \\ e_1 & A_2 & 0 & \cdots & 0 \\ e_2 & 0 & A_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_n & 0 & 0 & \cdots & A_n \end{bmatrix} \quad (3.30)$$

So that $a_{1i} = \pi T_{1i} C_{zi}$ ($i = 2, \dots, n$) holds. Where n is the number of the states, e_1 is the ratio between the power system gain and the time constant which is related to the interconnection signals and C_{zi} is the interconnection signal output. Usually, this matrix is constructed at the designer's choice. For the N -area control interconnected power system, the tie-line power deviation is usually chosen as the first state vector derivative. Then we choose the frequency, turbine power, and the rest of the powers in the first area, Then the same is done for the second area and this is continued in the same way until the area number N is reached. Finally, all the differential equations will be combined to obtain the matrix as shown

in Equation (3.8) and Equation (3.30).

Several methods have been recommended to analyze and control the complications of the structures mentioned above. The two main methodologies that are utilized to deal with the complexity of the large-scale interconnected system are decomposition and decentralization methods. These approaches decompose the overall complex problem into simple separate problems in order to find independent solutions for each of them (Šiljak, 1991). In decomposition process, the large system is partitioned into a number of subsystems of lower dimensions. Overlapping decompositions (Šiljak, 1991), Nested epsilon decompositions (Sezer & Šiljak, 1991), and Balanced BBD decompositions (Zečević & Šiljak, 1994) are most decomposition methods applied for large-scale systems and parallel computations (Chen&Standkovic,2005). Overlapping decomposition has been used effectively by researchers in many fields as an important decomposition method in order to decompose the systems with overlapping structures into subsystems of lower dimensions. These include economic systems, automated highway systems, electrical systems, and mechanical systems among others (Chen & Stankovie, 2005). On the other hand, decentralization structure is used for information exchanging and decision making process. Using decentralization, assists to implement separate local control units on the given complex dynamical systems. Next two sections present the disjoint and overlapping decompositions methods respectively.

3.3.1.1. Disjoint Decomposition

One direct decomposition methodology to decompose the overall system completely into separate decentralized subsystems due to weak interactions among the subsystems is called disjoint decomposition method. In this method, the state space equation of overall system is first transformed to input-output decentralized system and then the transformed into state space model is partitioned completely into disjoint subsystems (Bakule and Lunze, 1988). Decentralization is motivated as an input-output decentralized system can be designed using local feedbacks for independent subsystems and the influence of interconnections on global system performance can be tested ((Bakule and Lunze, 1988).

In this methodology, overall state space model Equation (3.24) is first transformed into Luenberger's canonical form using

$$\tilde{x} = Q^{-1}x \quad (3.31)$$

with

$$Q = (b_1 \quad b_1 A \quad \dots \quad b_1 A^{\alpha_1-1} \quad b_2 \quad b_2 A \quad \dots \quad b_2 A^{\alpha_2-1} \quad \dots \quad b_m A^{\alpha_m-1}) \quad (3.32)$$

where the vectors b_i are the m columns of the matrix B and Q is invertible transformation matrix with appropriate scalars $\alpha_1, \dots, \alpha_m$.

As result, the transformed state space model is described by:

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}u \quad \tilde{x}(0) = \tilde{x}_0 \\ y &= \tilde{C}\tilde{x} \end{aligned} \quad (3.33)$$

with

$$\begin{aligned} \tilde{A} &= Q^{-1}AQ \\ \tilde{B} &= Q^{-1}B = \text{diag}[b_1, \dots, b_m] \\ \tilde{C} &= CQ \end{aligned} \quad (3.34)$$

Such that the overall transformed system partitioned into the following subsystems

$$\begin{aligned} \dot{x}_i &= A_{ii}x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N A_{ij}x_j(t) + B_{ii}u_i \\ y_i &= \sum_{\substack{j=1 \\ j \neq i}}^N C_{ij}x_i(t) \end{aligned} \quad (3.35)$$

Where the matrices $\tilde{A}, \tilde{B}$ and $\tilde{C}$ are partitioned into the subsystem's matrices A_{ii}, A_{ij}, B_{ii} and C_{ij} according to the partition of the state x , the control input u and the control output y .

3.3.1.2. Overlapping Decomposition

One reason that Overlapping decomposition is popular is that it accepts the mathematical inclusion principle. This basically expands the original state space model of the large-scale interconnected system so that the overlapping subsystems appear as disjoint. Then, traditional techniques for the design of decentralized controllers are used for the expanded system and the resulting controller is contracted for implementation on the original system

(Trave et al,1989). Block diagram of overlapping decomposition is demonstrated in Figure 2.4.

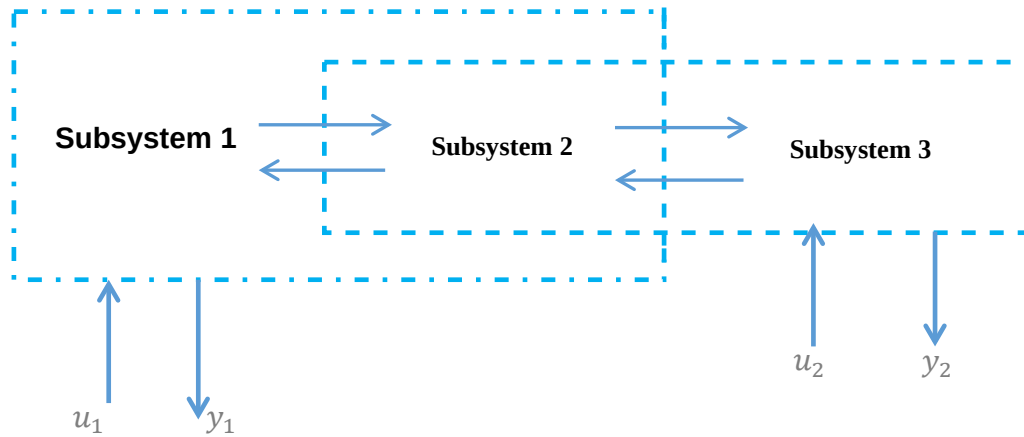


Figure 3.4. Overlapping Decomposition.

Overlapping decomposition is a conceptual technique of decomposing the model of the large –scale interconnected systems into overlapping subsystems to share a common information as shown in **Error! Reference source not found. 3.4** (Ikeda and Sijak,1980). It forms a way of reducing computational cost and formulating control laws that can be implemented on the original system (Iljak,1993). Because the high dimensionality problem makes the implementation of the online control is not possible. Thus, a solution to a time control problem is usually found by computational steps not exceeding a certain limit (Lunze,1991). Therefore, these problems should be broken down into smaller problems to reduce the computational cost. Overlapping decomposition has four concepts for it to be implemented.

- i. The first one includes the expansion of the original system based on the inclusion principle.
- ii. The second concept is the application of the overall decomposition onto the expanded systems to obtain disjoint subsystems.

The third and fourth concepts include the controller design for each disjoint subsystem and stability checking for the entire large interconnected system when the inclusion conditions are met (Chen & Stankovoić,2005, Lunze,1992). In this way, the subsystems can be distributed, and the control laws normally are implemented in the original system after appropriate contraction.

To illustrate the concept of overlapping decomposition, consider the interaction-oriented model as represented in Equation (3.27) with matrix A is given in Equation (3.29). Before we proceed let's assume the interaction-oriented model is the original model of the system and it is abbreviated with notation S and the expanded model is abbreviated with

notation $\check{S}$. To expand the original interaction-oriented-model, a linear transformation to relate the original system S and the expanded system $\check{S}$ under a certain conditions is used. Hence, the original model S can be expanded to $n-1$ overlapping subsystems (Lunze,1992):

$$\check{S}: \check{x}_i(t) = \tilde{A}_{ii}\check{x}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^n \tilde{A}_{ij}\check{x}_j + \tilde{B}_i\check{u}_i(t) \quad (3.36)$$

$$y_i(t) = \tilde{C}_i\check{x}_i(t)$$

where $\check{x}_i \in R^{\tilde{n}}$ is an $\tilde{n}$ state vector of the expanded system; $\check{u}_i \in R^{\tilde{m}}$ is the input vector in the expanded system space; $\check{y}_i \in R^{\tilde{l}}$ is the output vector in the expanded system space and $\tilde{A}_{ii}$, $\tilde{B}_i$ and $\tilde{C}_i$ are $\tilde{n} \times \tilde{n}$, $\tilde{n} \times \tilde{m}$ and $\tilde{l} \times \tilde{n}$ constant matrices.

The original system from Equation (3.27) and Equation (3.28) are related by Equation (3.36) into

$$T = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}, T^+ = \begin{bmatrix} N-1 & 0 & N-1 & 0 & \dots & N-1 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.37)$$

Therefore, the expanded system shown in Equation (3.36) has been obtained using a linear transformation as follows:

$$\check{x}_i = T x_i \quad (3.38)$$

where T is a $\tilde{n} \times n$ constant matrix with full column rank.

Definition 1 (Chen & Stankovic, 2005; lunze,1992): the system $\check{S}$ includes the original system S if there exist a matrix T such that for any initial condition x_0 of the original system S , the choice:

$$\check{x}_0 = T x_0 \quad (3.39)$$

of the initial state $\check{x}_0$ of $\check{S}$ implies that:

$$x_0(t, x_0, u) = T^+ x_0(t, x_0, u) \forall t \geq 0 \quad (3.40)$$

For any constant input $u(t)$, where T^+ is pseudoinverse of a rectangular matrix and it satisfies the condition $T^+T = I$.

Definition 2: Inclusion principles (Lunze, 1992):

If the system $\check{S}$ includes S , then $\check{S}$ is said to be expansion of S and S is called a contraction of $\check{S}$.

Now under the transformation given in Equation (3.37), the matrices of the expanded system $\check{S}$ can be found using the following matrices:

$$\begin{aligned} \tilde{A}_{ii} &= TA_iT^+ + M_A \\ \tilde{B}_i &= TB + N_B \\ \tilde{C}_i &= CT^+ + L_C \end{aligned} \quad (3.41)$$

Where, M_A, N_B and L_C are constant complementary matrices of proper dimension.

Theorem 1 (Lunze,1992): The system in Equations (3.36) and (3.41) is expanded form of the original system given in Equation (3.27). If there exists a full rank matrix T so that the following conditions are satisfied:

$$\begin{aligned} T^+M_A^i T &= 0, T^+M_A^{i-1}N_B = 0 \\ L_C M_A^{i-1}T &= 0, L_C M_A^{i-1}N_B = 0 \end{aligned} \quad (3.42)$$

In practice, for many complex subsystems, the matrices in Equation (3.41) are obtained using two-steps procedure as proposed by Hen and Stankovic (2005) so that the pair-wise subsystems can be appeared as disjoint. The idea is to use a linear transformation and an appropriate choice of complementary matrices so that the interconnection terms can be eliminated. Then, an adequate permutation transformation can be applied, and the desire expanded system can be achieved. This approach is explained as follows:

In the first step, the matrices T , T^+ and M_A are selected as shown in Equation (3.43)

$$T = \text{blockdiag}(I_{11}, [I_{22}, I_{22}]^T, \dots, [I_{n-1n-1}, I_{n-1n-1}]^T, I_{nn})$$

$$T^+ = \text{blockdiag}(I_{11} 0.5 \times [I_{22}, I_{22}], \dots, 0.5 \times [I_{n-1n-1}, I_{n-1n-1}], I_{nn})$$

$$M_A = 0.5 \times \begin{bmatrix} A_{11} & -A_{11} & A_{12} & -A_{12} & \cdots & -A_{1n} & A_{1n} \\ -A_{11} & A_{11} & A_{12} & -A_{12} & \cdots & -A_{1n} & A_{1n} \\ -A_{21} & A_{21} & A_{22} & -A_{22} & \cdots & 0 & 0 \\ -A_{21} & A_{21} & -A_{22} & A_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & -A_{n1} & 0 & 0 & \cdots & A_{nn} & -A_{nn} \\ A_{n1} & -A_{n1} & 0 & 0 & \cdots & -A_{22} & A_{nn} \end{bmatrix} \quad (3.43)$$

Then, the expanded system state matrix $\tilde{A}_{ii}$ is obtained as follows:

$$\tilde{A}_{ii} = \begin{bmatrix} A_{11} & 0 & A_{12} & 0 & \cdots & -A_{1n} & A_{1n} \\ 0 & A_{11} & A_{12} & 0 & \cdots & -A_{1n} & A_{1n} \\ 0 & A_{21} & A_{22} & 0 & \cdots & 0 & 0 \\ 0 & A_{21} & 0 & A_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & 0 & 0 & 0 & \cdots & A_{nn} & 0 \\ A_{n1} & 0 & 0 & 0 & \cdots & 0 & A_{nn} \end{bmatrix} \quad (3.44)$$

In the second step, the permutation matrix is selected as follows:

$$P_A = P_{A12} P_{A23} \cdots P_{A(2n-1)(2n)} = \prod_{k=1}^{2n-1} P_{Ak(k+1)} = \begin{bmatrix} 0 & \cdots & 0 & I_{11} \\ I_{11} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & I_{nn} & 0 \end{bmatrix} \quad (3.45)$$

Resulting in

$$\tilde{A}_P = P_A^{-1} \tilde{A}_{ii} P_A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} & 0 \\ A_{21} & A_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & A_{nn} & A_{n1} \\ 0 & A_{12} & \cdots & A_{1n} & A_{11} \end{bmatrix} \quad (3.46)$$

3.3.2. Summary and Recommendation

A review of modeling structures and decomposition methods in the field of the large-scale interconnected power systems have been carried out in this section. Interaction oriented

model received attention for the large system because it presents the interconnection of the signals and the inputs and the outputs for the system. Two methods of decompositions, namely disjoint and overlapping decomposition methods discussed for the decentralization of the large-scale interconnected power system. Disjoint decomposition is direct and suitable for interconnected power systems with low dimensions' states while overlapping decomposition is reasonable for the large-scale interconnected power system's with higher dimensions' states. Even though there are numerous decomposition methods exist, thinking of direct and more easy decomposition method is still relevant for two area control systems. Next section adopts the decomposition methods described above in section 3.4 to solve the two-area interconnected power system with tie-line with two identical non-reheat thermal generator system on each area.

3.4 Decomposed Models for two Area Control System

Based on Equation (3.36), the above model presented in Equation (3.8) can be written into two interconnected overlapping subsystems (Alrifai et al,2011):

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{ps1}} & \frac{k_{ps1}}{T_{p1}} & 0 \\ 0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} \\ -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{G1}} \end{bmatrix} u_1 + \begin{bmatrix} -\frac{K_{ps1}}{T_{ps1}} \\ 0 \\ 0 \end{bmatrix} \Delta P_{D1} + \begin{bmatrix} 2\pi T_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (3.47)$$

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{ps2}} & \frac{k_{ps2}}{T_{p2}} & 0 \\ 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} \\ -\frac{1}{R_2 T_{G2}} & 0 & -\frac{1}{T_{G2}} \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{G2}} \end{bmatrix} u_u + \begin{bmatrix} -\frac{K_{ps2}}{T_{ps2}} \\ 0 \\ 0 \end{bmatrix} \Delta P_{D2} + \begin{bmatrix} -2\pi T_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \end{bmatrix} \quad (3.48)$$

The interconnection signals are $\Delta P_{tie12} = x_1$ and $a_{12}\Delta P_{tie12} = a_{12}x_1$. Hence, the interconnection variables are given by the variables Z_1 and Z_2

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -\frac{k_{ps1}}{T_{p1}} & 0 \\ 0 & -\frac{K_{ps2}}{T_{ps2}} \end{bmatrix} \begin{bmatrix} x_1 \\ a_{12}x_1 \end{bmatrix} \quad (3.49)$$

where $X_1 = [x_2 \ x_3 \ x_4]^T$ is the state vector of the first subsystem, $X_2 = [x_5 \ x_6 \ x_7]^T$ is the state vector of the second subsystem and Z_1 and Z_2 are the interconnection signal.

Objectives for the decentralized AGC control design(Šiljak,2007):

Zero steady state errors for the frequency and tie-line power exchange

The local control law should be linear, constant, and independent of the area load disturbances

The decentralized AGC should improve the transient response and increase stability margins

of the whole interconnected power system.

To achieve these objectives, the equations of the two subsystems are expanded with an integral of dynamic area control error (ACE) which adds additional state variables (Calovic, 1972), Šiljak (2007) and Alrifai et al (2011).

$$u_i(t) = u_i(0) + \int_0^t AEC_i(t)dt \quad (3.50)$$

Where $u_i(t)$ and $u_i(0)$ are the control actions at time t and the initial time t_0 , and $i = 1, 2$. The area control error for area number i is defined as:

$$AEC_i(t) = \Delta P_{tiei} + \beta_i \Delta f_i, \quad i = 1, 2 \quad (3.51)$$

The β_i and Δf_i are the frequency bias factor and frequency deviation in the i th area, $\Delta P_{tie1} = 2\pi T_{12}x_2$ and $\Delta P_{tie} = -2\pi T_{12}x_5$ the equation for the automatic area control is rewritten as

$$AEC_i(t) = \Delta P_{tiei}(t) + d_i^T x_i(t) \quad (3.52)$$

where $d_i^T \in \mathbb{R}^{n_i}$. Secondly, to achieve the objective (3), the area participation factor (α_i) is considered in the design process so that the desired area control law $u_i(t)$ can be obtained by imposing the equality constraints on the area control vector $\tilde{u}_i(t)$ in Equation (3.35) as follows

$$\tilde{u}_i(t) = \alpha_i u_i(t) \quad (3.53)$$

Decentralized system model- standard form, including the interconnections can be written as

$$\dot{\tilde{x}}_i(t) = \tilde{A}_i \tilde{x}_i(t) + \tilde{B}_i \alpha_i u_i(t) + C_i z_i \quad i = 1, 2$$

$$z_i(t) = \sum_{\substack{j=1 \\ j \neq i}}^N L_{ij} \tilde{x}_j = \sum_{\substack{j=1 \\ j \neq i}}^2 L_{ij} \tilde{x}_j \quad (3.54)$$

$$\dot{z}_i = A_{z_i} z_i(t)$$

where, the structure of the state vectors to be done for the system in Equation (3.54) are:

$$\tilde{x}_1 = \begin{bmatrix} \Delta f_1 \\ \Delta P_{T_1} \\ \Delta P_{G_1} \\ IACE_1 \\ \Delta P_{tie1} \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ IACE_1 \\ x_1 \end{bmatrix} \quad \tilde{x}_2 = \begin{bmatrix} \Delta f_2 \\ \Delta P_{T_2} \\ \Delta P_{G_2} \\ IACE_2 \\ \Delta P_{tie1}(t) \end{bmatrix} = \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ IACE_2 \\ a_{12}x_1 \end{bmatrix} \quad (3.55)$$

Equations (3.47) and (3.48) do not contain the interconnections $z_i = [z_1 z_2]^T$ given by the first column of the matrix A in Equation (3.47). The matrix C_i have to be determined on the basis of the Equation (3.46) using the vectors from Equation (3.55). From Equation (3.49):

$$z_1 = -\frac{k_{ps1}}{T_{ps1}} x_1 = C_1 x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{k_{ps1}}{T_{ps1}} \end{bmatrix} x_1 \quad (3.56)$$

Similarly, from Equation (3.49)

$$z_2 = -\frac{k_{ps2}}{T_{ps2}} a_{12} x_1 = C_2 a_{12} x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{k_{ps2}}{T_{ps2}} \end{bmatrix} a_{12} x_1 \quad (3.57)$$

In the same way the matrices L_{12} and L_{21} can be determined considering the interconnection signals z_1 and z_2 between the two subsystems as follows:

$$z_1 = L_{12} X_2 = [1 \ 0 \ 0 \ 0 \ 0] X_2 = [1 \ 0 \ 0 \ 0 \ 0] \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ IACE_2 \\ x_1 \end{bmatrix} \quad (3.58)$$

and

$$z_2 = L_{21} X_1 = [1 \ 0 \ 0 \ 0 \ 0] X_1 = [1 \ 0 \ 0 \ 0 \ 0] \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ IACE_1 \\ x_1 \end{bmatrix} \quad (3.59)$$

or

$$\begin{aligned}
z_1 &= L_{12}X_2 = z_1 = L_{12}\tilde{x}_2 \\
z_2 &= L_{21}X_1 = z_2 = L_{21}\tilde{x}_1
\end{aligned} \tag{3.60}$$

Now we can determine the equation $\dot{z}_i = A_{z_i}z_i$ by considering Equation (3.59). The derivatives of the interconnection signals z_1 and z_2 can be found as:

$$\begin{aligned}
\dot{z}_1 &= -\frac{k_{ps1}}{T_{ps1}}\dot{x}_1 \\
\dot{z}_2 &= -\frac{k_{ps2}}{T_{ps2}}a_{12}\dot{x}_1
\end{aligned} \tag{3.61}$$

The decentralized subsystem models that will be used for the design of the controllers will be

$$\begin{aligned}
\begin{bmatrix} \dot{v}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \Delta\dot{P}_{tie1} \end{bmatrix} &= \begin{bmatrix} 0 & d_1^T & 0 & 0 & 1 \\ -\frac{1}{T_{ps1}} & \frac{k_{ps1}}{T_{p1}} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} & 0 & 0 \\ -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 \\ 0 & 2\pi T_{12} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ -\frac{K_{ps1}}{T_{ps1}} \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_{D1} \\
&+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2\pi T_{12} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}
\end{aligned} \tag{3.62}$$

and

$$\begin{aligned}
\begin{bmatrix} \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{v}_1 \\ \Delta \dot{P}_{ue2} \end{bmatrix} &= \begin{bmatrix} -\frac{1}{T_{ps2}} & \frac{k_{ps2}}{T_{p2}} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} & 0 & 0 \\ -\frac{1}{R_2 T_{G2}} & 0 & -\frac{1}{T_{G2}} & 0 & 0 \\ d_2^T & 0 & 0 & 0 & -1 \\ -2\pi T_{12} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_2 + \begin{bmatrix} -\frac{K_{ps2}}{T_{ps2}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_{D2} \\
&+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -2\pi T_{12} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_1 \\ x_2 \end{bmatrix}
\end{aligned} \tag{3.63}$$

3.4.1. Summary and Recommendation

Overlapping decomposition has been developed for two area control system with non-reheat power system in each control area. The inclusion principles have been used to accomplish the Overlapping decomposition. The resulting decomposed system consists of a single subsystem of the original system in each area. The AGC control problem has not been solved using the general Functional minimization method. Therefore, simulation results for decentralized discrete Optimal Quadratic AGC (DOAQGC) control are represented in Chapter 5 (section 5.5).

3.5 Conclusions

This chapter presented the concepts of decomposition, inclusion principles and decentralization methods which enables us to deal with the complexity of large-scale power systems. The complex interconnected power systems are characterized by system states with high dimensions, system model uncertainties, nonlinearities and information constraints. To deal with these control design challenges, the overall system is decomposed into subsystems via the decomposition methodology and inclusion principles. Decentralization facilitates the design of local feedback control laws for independent subsystems and allows us to test the impacts of interconnections.

The advantages of using decomposition, inclusion principles and decentralization for large-scale interconnected power systems provides procedure on how to break the overall design problem into sub-problems, for ease of modeling and control implementation, and ability to analyze the overall system performance which includes stability, robustness, and steady-state performances. The overall system is controlled by several independent controllers; all together are known as decentralized controller. For implementation and testing purpose, the

overall state-space model of the two-area interconnected power system is derived and decomposed into two continuous and discrete disjoint subsystems. Overlapping decomposition method is formulated in Chapter 5, Section 5.2 and it utilized in sub-sections 5.3 and 5.4 respectively to decompose two area interconnected power (TAIPS) with identical non-reheat turbines and four area interconnected power system (FAIPS) with non-reheat, reheat, hydro and gas turbines into two and four subsystems.

Chapter 4

Centralized Discrete Quadratic Optimal AGC Control Systems for Interconnected Power Systems

4.1 Introduction

This chapter discusses a systematic design of Centralized Discrete Optimal Quadratic Automatic Generation Control (COQAGC) for Multi-area interconnected power systems. The design of a Centralized Discrete COQAGC controller is based on the control theory of the Linear Quadratic Regulator (LQR) and the Functional Minimization Method (FMM). LQR is a study of optimal control problems which explains how to use a linear dynamic system and quadratic cost function to design an optimal controller in the form of state feedback control law $u(k)$. (Luenberger, 1979). On the other hand, a Functional Minimization Method (FMM) is a mathematics approach to selecting the state and control-weighting matrices of the cost function. During the design process, the discrete cost function is defined in terms of area control errors, integral area control errors, and control signal variables which are subject to physical constraints of state and control weighting matrices which are formed from partial derivatives of the cost functional function. Due to the design process and the capabilities of the stability analysis, the quadratic structure has been widely chosen in several studies among other cost function structures (Naudo, 2002).

First: For comparison purposes, two types of Centralized discrete optimal AGC controllers are designed for two control area power systems as follows:

1. Centralized discrete COQAGC regulator is designed with the aim of frequency regulation without considering area control errors.
2. The centralized discrete COQAGC controller is designed with the aim of returning the frequency deviations and tie-line power deviation to zero at the steady state.

Second: The Centralized Discrete COQAGC controller is designed with the aim of returning the frequency deviations and tie-line power deviation to zero at the steady-state for four area interconnected power systems with non-reheat, reheat, hydro and gas turbines in each area.

Equations (4.1 - 4.18) represent the procedures for designing a centralized discrete COQAGC regulator that solves the state regulation problem while Equations (4.19 - 4.23) represent the steps and procedures for designing a Discrete Centralized OQAGC controller that solves the steady-state problem.

To test the feasibility of the developed methods, the centralized optimal AGC controllers are designed for two identical non-reheat thermal turbines. The performance of power system dynamics has been studied with/without disturbances and area control errors.

The main contributions of this chapter are as follows:

- o The design of a centralized Discrete COQAGC-Regulator is based on the control theory of the linear Quadratic Regulator developed for two area interconnected power systems.
- o Discrete centralized optimal quadratic automatic generation control (COQAGC) is designed based on optimal control theory framework and functional minimization approach for N interconnected power systems with load disturbances.
- o The discrete cost function requirements are defined in terms of area control errors, integral area control errors and control energy expenditure so that the function of the criterion can be minimized. This in return ensures the improvement of the power system dynamics in terms of the stability and the system's steady state performance
- o Performance of the COQAGC on power system dynamics has been studied with/without disturbances and area control errors by considering two identical control areas and four area interconnected power systems.

The rest of the chapter is organized as follows: in section 4.2 the Optimal Control Design and Principles are explained. The formulation of state regulation and steady-state problem are discussed in sections 4.2.1 and 4.2.2 respectively. In section 4.3, the general formulation of functional minimization for N control areas is presented. In section 4.4 selection of state and control weighing matrices for a two-area control power system are determined. Section 4.5 presents the parameters of the two-area control power system and the system modelling and simulation results. In section 4.6, the design of a centralized discrete COQAGC for four area interconnected power system is formulated and designed. Section 4.7 presents the sensitivity analysis of the centralized discrete COQAGC for FAIPS. Finally, sections 4.8 and 4.9 present multi-source power systems with renewable energy sources and the conclusion of the chapter respectively. The layout for discussion carried out on discrete centralized COQAGCs controllers is shown in Figure 4.1.

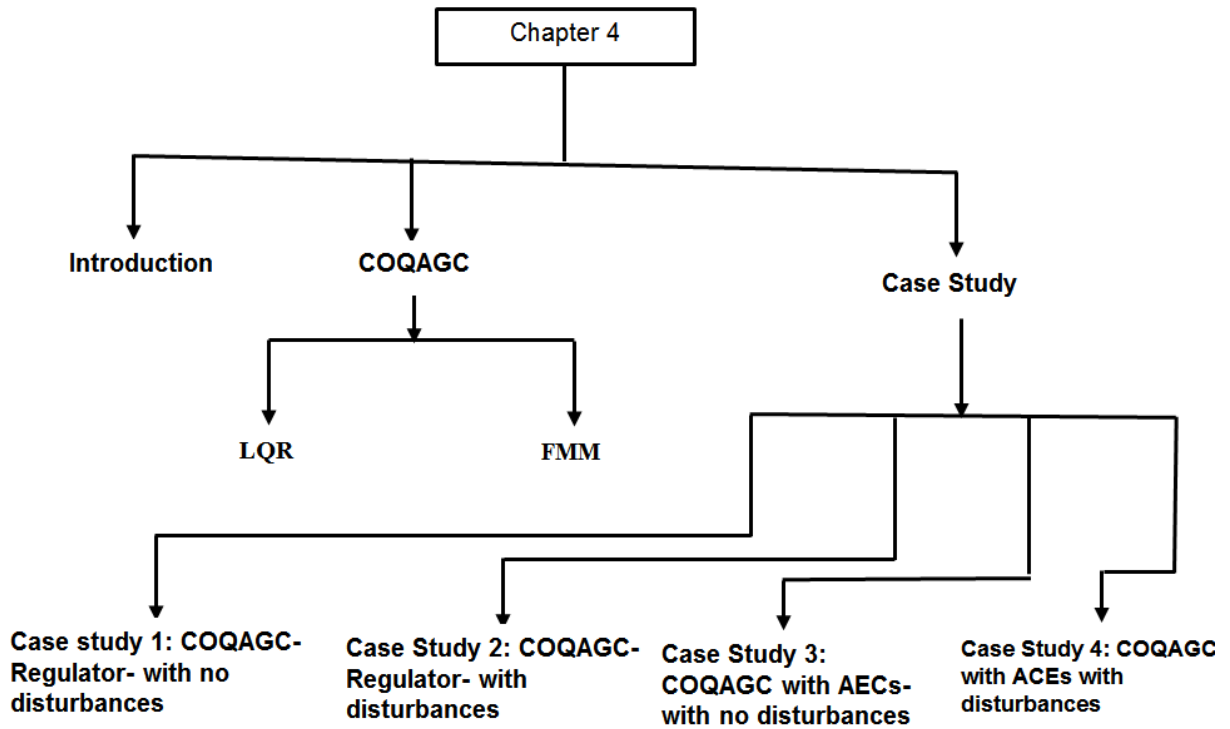


Figure 4.1. Structure diagram of flow of the chapter contents

4.2. Optimal Control Design and Principles

The control system technique using augmented cost function is either minimized or maximized subject to equality or inequality dynamic and static constraints are called optimal control problem. The objective of optimal control problems is to find whether the optimal control laws will cause the closed-loop system to meet certain constraints (Naidu,2002). The mathematical model of the system, specification of the cost functional boundary condition, physical constraints of state variables and control signals are required to formulate the optimal control problem. The optimal control problem can be either formulated and analyzed in the continuous-time domain or in the discrete-time domain.

One of the best analytical approaches for solving the optimal control problem is a quadratic optimal control technique (Ogata,1994). In this technique, the cost function that is used to design the control system is defined as a quadratic function subject to physical constraints. The advantages of the quadratic optimal method are that it makes the design easy and it ensures the system is asymptotically stable. Asymptotic stability is one of the stability concepts in Lyapunov's theory which ensures that the equilibrium point of dynamic systems is stable and also the trajectories of system states starting from any initial conditions converge to the origin as time approaches infinity (Slotine & li,1991).

Linear Quadratic Regulator (LQR) is the most popular closed-loop optimal control scheme

system with a quadratic cost function. The objectives of the LQR control problem can be formulated to address the state regulations, output regulation at steady-state and output tracking with a reference to set-point. The choice to choose among the three controller types is dependent on the design requirements or specifications. In a state regulation type control system, the LQR control objective is to design control laws for states of the system described by a discrete state-space model from non-zero initial states to zero states. In this case, the closed-loop control system has no reference input, and the sudden changes or disturbances are considered input signals to the feedback control system.

The LQR method is found to be useful for the automatic generation control problems of power systems subjected to sudden load changes (Vlahakis, Dritsas, & Halikias, 2019). One of the challenges of this method is that there is no clear way to select the states and input weighing matrices of the cost function. On the other hand, the steady-state regulation LQR controllers keep the output constant at the steady state when the time interval is infinity and zero reference input is applied to the system (Naidu, 2002). The sudden load changes are considered as the input disturbances to the closed-loop system. This case is very useful for regulating the frequency and tie-line deviations at steady-state. However, it requires integral control augmentation so that the zero frequency and tie-line power deviations at steady-state are achieved. LQR for output tracking keeps the state or the output near the desired trajectory. The most important objective of LQR control is to achieve zero steady-state error. It can be observed that the reference is zero while the tracking control system steady-state error is zero.

The next section provides the design principles for Optimal Quadratic Regulators.

4.2.1. Design of Centralized Optimal Quadratic Regulators

In the state's space regulation problem, LQR regulators are designed to minimize quadratic cost functions using Lagrangian and Hamiltonian functions. These methods are based on Lagrange multipliers. The control design objective is to drive the states of the closed system from initial states values to zero (Ogata, 1994). Thus, the optimal control law is given as:

$$u(k) = -Lx(k), \quad k = 1, 2, \dots, \infty \quad (4.1)$$

where $L \in \mathbb{R}^{r \times n}$ is time-varying matrix which becomes constant when $k = \infty$ and $x(k) \in \mathbb{R}^{n \times 1}$ is n state vector. It can be noticed that there is linear relationship between the optimal control function and the state variables vector. Therefore, to guarantee the existence of the optimal control system the states of the control system must be available to the feedback control law

or observer must be used to estimate un-measurable states.

In order to obtain the sequence of the optimal control action $u(0), u(1), u(2), \dots, u(k_{f-1})$, the following quadratic cost function is minimized.

$$\begin{aligned} \min J(x(k_0), u(k_0), k_0) \\ = \frac{1}{2} x^T(k_f) F x(k_f) + \frac{1}{2} \sum_{k=k_0}^{k=k_{f-1}} x^T(k) Q_k x(k) + u^T(k) R_k u(k) \end{aligned} \quad (4.2)$$

Subject to the equality constraint: discrete linear control system.

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) + \Gamma w(k) \\ x(k_0) &= x_0, \quad x(k_f) = x_{k_f} \end{aligned} \quad (4.3)$$

where F and $Q_k \in \mathbb{R}^{n \times n}$ are symmetric positive semi-definite matrices, and $R_k \in \mathbb{R}^{r \times r}$ is a symmetric positive semi-definite matrix, $k = k_0, k_1, \dots, k_{f-1}, k_f$, $x(k)$ is n th order state vector, $u(k)$ is r th order control vector and A_k and B_k are the time-varying matrices of $n \times n$ and $n \times r$ dimensions, $x(k_0)$ and $x(k_f)$ are the initial and final state conditions respectively, $w(k)$ is p th order, and $\Gamma \in \mathbb{R}^{n \times r}$ are input load changes vector and a load disturbance matrix respectively.

The F, Q_k and R_k weighing matrices are responsible about the performance of the quadratic cost function and dynamics response of the system. The derivation of the optimal control further is described without consideration of the external disturbances.

A set of Lagrange multiplier's $(\lambda(1), \lambda(2), \lambda(3), \dots, \lambda(k_f))$ are used to adjoin the cost functional in Equation (4.2) and the equality constraint given in Equation (4.3) so that the following augmented cost functional is minimized (Ogata, 1994).

$$\begin{aligned} J_a = \frac{1}{2} x^T(k_f) F x(k_f) + \frac{1}{2} \sum_{k=k_0}^{k=k_{f-1}} [x^T(k) Q_k x(k) + u^T(k) R_k u(k)] + \lambda(k \\ + 1) [A_k x(k) + B_k u(k) - x(k+1)] \end{aligned} \quad (4.4)$$

where $\lambda(k+1)$ is the Lagrange multiplier. From the definition of the augmented cost functional, the Lagrangian and Hamiltonian functional are defined as follows:

$$\begin{aligned}
L(x(k), u(k), x(k+1), \lambda(k+1)) \\
= \frac{1}{2}x^T(k)Q_kx(k) + \frac{1}{2}u^T(k)R_ku(k) + \lambda^T(k \\
+ 1)[A_kx(k) + B_ku(k) - x(k+1)]
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
H(x(k), u(k), x(k+1), \lambda(k+1)) \\
= \frac{1}{2}x^T(k)Q_kx(k) + \frac{1}{2}u^T(k)R_ku(k) + \lambda^T(k \\
+ 1)[A_kx(k) + B_ku(k)]
\end{aligned} \tag{4.6}$$

where the Lagrangian functional in Equation (4.5) and Hamiltonian functional in Equation (4.6) are related by

$$L(x(k), u(k), x(k+1), \lambda(k+1)) = H(x(k), u(k), \lambda(k+1)) - \lambda(k+1)x(k+1) \tag{4.7}$$

The necessary conditions that minimize the augmented cost function J_a are obtained by taking the partial derivatives of the Hamiltonian functional with respect to the state $x(k)$, co-state $\lambda(k+1)$ and the transforming control $u(k)$ respectively as follows:

$$x(k) = \frac{\partial H(x(k-1), u(k-1), \lambda(k))}{\partial \lambda(k)} \tag{4.8}$$

where at the next time interval $(k+1)$, the Equation (4.8) can be written as

$$x(k+1) = \frac{\partial H(x(k), u(k), \lambda(k+1))}{\partial \lambda(k+1)} = x(k+1) = A_kx(k) + B_ku(k) \tag{4.9}$$

$$\lambda(k) = \frac{\partial H(u(k), x(k), \lambda(k+1))}{\partial x(k)} = \lambda(k) = Q_kx(k) + A_k^T \lambda(k+1) \tag{4.10}$$

$$0 = \frac{\partial H(u(k), x(k), \lambda(k+1))}{\partial u(k)} = R_ku(k) + B_k^T \lambda(k+1) \tag{4.11}$$

Then, the optimal open-loop $u(k)$ is obtained from Equation (4.11) by:

$$u(k) = -R_k^{-1}B_k^T \lambda(k+1) \tag{4.12}$$

Thus, by substituting the optimal open-loop control law into the state Equation (4.9), the Hamiltonian system or the State and Co-state system is obtained as follows:

$$\begin{bmatrix} x(k+1) \\ \lambda(k) \end{bmatrix} = \begin{bmatrix} A_k & -E_k \\ Q_k & A_k^T \end{bmatrix} \begin{bmatrix} x(k) \\ \lambda(k+1) \end{bmatrix} \quad (4.13)$$

where $E_k = B_k R_k^{-1} B_k^T$. The block diagram of the State and Co-state system is shown in Figure 4.2.

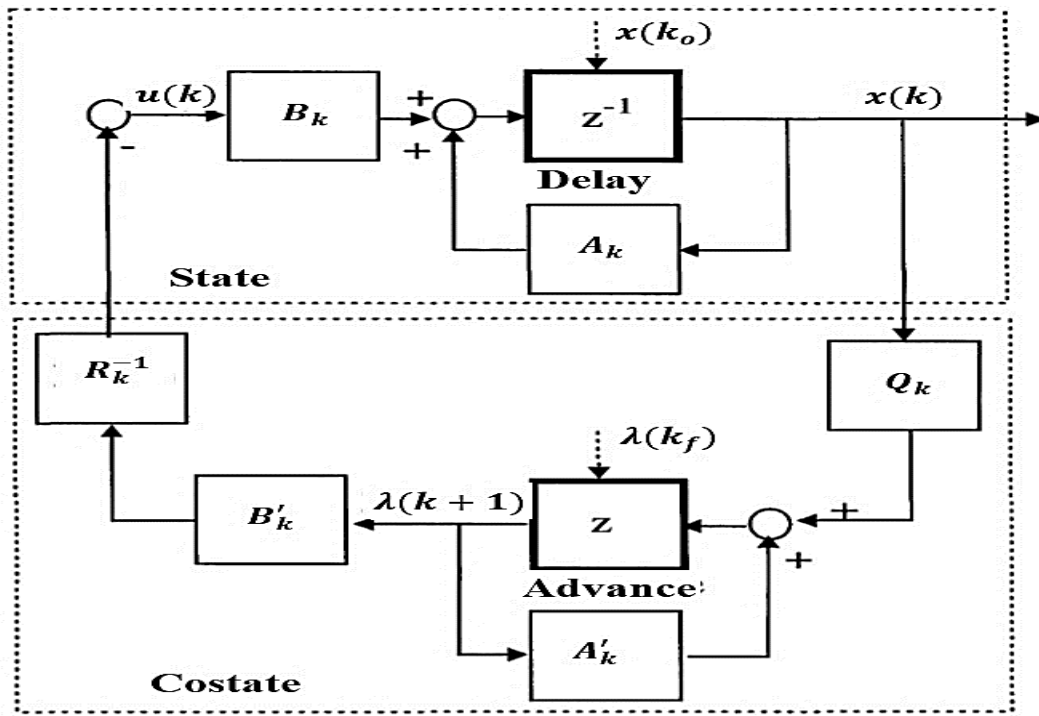


Figure 4- 2. Open-loop State and Co-state System (Naidu,2002).

The State and Co-state system optimization problem is called Two Point Boundary Value Problem (TPBVP). The TPBVP problem can be solved using the boundary conditions $x(k_0)$ and $x(k_f)$. Since, the final term is considered in the augmented cost function, then the final condition will be fixed, and it can be expressed with:

$$\lambda(k_f) = F(k_f)x(k_f) = P_{k_f}x(k_f) \quad (4.14)$$

where P_{k_f} is the value of the Riccati matrix at the final time moment k_f

Because of the coupling nature associated with the solution of the TPBVP, the state $x(k)$ is solved forward starting from its initial conditions $x(0)$ while the co-state $\lambda(k)$ is solved

backward starting from its final condition $\lambda(k_f)$. On the other hand, the closed-loop optimal control law $u(k)$ can be determined from the solution of the real symmetric matrix P_k . This matrix is solved backward starting from the final condition Equation (4.14) as a solution of the Matrix Riccati Difference Equation (RDE) (Tran et al, 2018) which is given by the following equation:

$$P_k = A_k^T P_{k+1} [I + E_k P_{k+1}]^{-1} A_k + Q_k \quad (4.15)$$

Then, the feedback gain matrix L_k is determined using P_k as follows:

$$L_k = R_k^{-1} B^T A_k^{-T} [P_k - Q_k] \quad (4.16)$$

By substituting Equation (4.16) into Equation (4.1), the closed-loop optimal control law is obtained as:

$$u(k) = -R_k^{-1} B^T A_k^{-T} [P_k - Q_k] x(k) \quad (4.17)$$

where L_k is Kalman gain, A^{-T} is the inverse of the matrix A^T and P_k is the positive semi definite Riccati matrix. Substituting the optimal feedback control law $u(k)$ from Equation (4.17) to the original discrete-time state space system (4.3), the optimal closed-loop system is expressed as:

$$x(k+1) = [A_k - B_k (R_k^{-1} B^T A_k^{-T} [P_k - Q_k])] x(k) + \Gamma w(k) \quad (4.18)$$

Equations (4.1 - 4.18) are used as steps and procedures for designing a Centralized Quadratic Optimal AGC controller for the interconnected power system as a solution of the state regulation problem. For this case, the centralized discrete-time model of the interconnected power system is considered without area control errors.

4.2.2. Design of Centralized Discrete Optimal Quadratic Automatic Generation Controller (COQAGC)

Voltage and frequency are the main variables of the power systems which determine the stability and quality of the power supply. The active power imbalance requires proper control of the electric power generation. Automatic Generation Control (AGC) is a very important control method for maintaining the active power balance within the desired limits in the multi-area power system (Shankar et al. 2017). The main objectives of AGC control are to maintain the frequency deviations at nominal value, keep the tie-line power changes between areas at a scheduled value, and to ensure that the frequency variations are returned to zero (easing the control design and implementation of complex interconnected power systems (Tungadio, Ramesh & Mukwanga, 2018); Alrifai et al. 2011). AGC adjusts the outputs of the generators based on the load disturbances. It controls the local area frequency and the tie-line power between different control areas as shown in Figure 4.3. For explanation purpose, Figure 4.3 is recalled here from Chapter 2, Figure 2.5. N control areas of large-scale interconnected power system (Ma et al, 2014, Bevrani,2014).

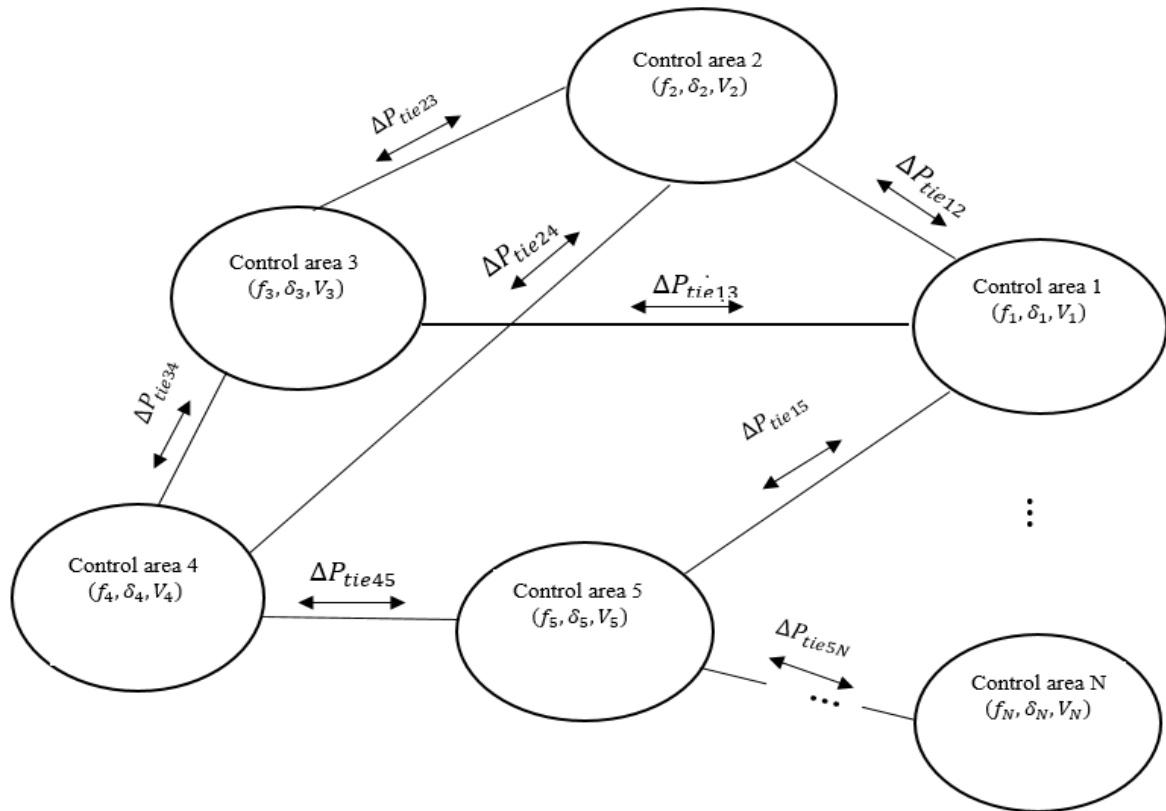


Figure 4.3. N control areas of large-scale interconnected power system (Bevrani, 2014).

where f_i is the area frequency, V_i is the area voltage and δ_i is the area angle ($i = 1, 2, 3, \dots, N$). As the power system grows in size and complexity, its control becomes challenging using AGC classical control methods as can be depicted in Figure 4.3. In recent studies, the Linear

Quadratic Regulator (LQR) received attention for optimal AGC design in large and complex interconnected power systems. This is mainly because of its large stability margins in stabilized solutions (Vlahakis, Dritsas, & Halikias, 2019). In addition to that, Optimal AGC based on LQR can handle the robustness, reliability, and nonlinearities associated with power system control (Tingadio et al. 2019). The designing methodologies developed for such optimal control method based on two methods: 1) LQR control theory and 2) Functional minimization method. Functional minimization approach is a mathematics approach that has been used to choose the state and control cost-weighting matrices (Q_k and R_k). In this research work, the optimal AGC controller developed based on these two design methodologies are called Centralized Discrete Optimal Quadratic Automatic Generation Control (OQAGC).

For the COQAGC control design the control system can be optimized using optimal theorem and Lagrangian multipliers. The closed-loop discrete optimal quadratic control system with load disturbance signal $w(k)$ is shown in Figure 4.4, where $y(k)$ is the output signal.

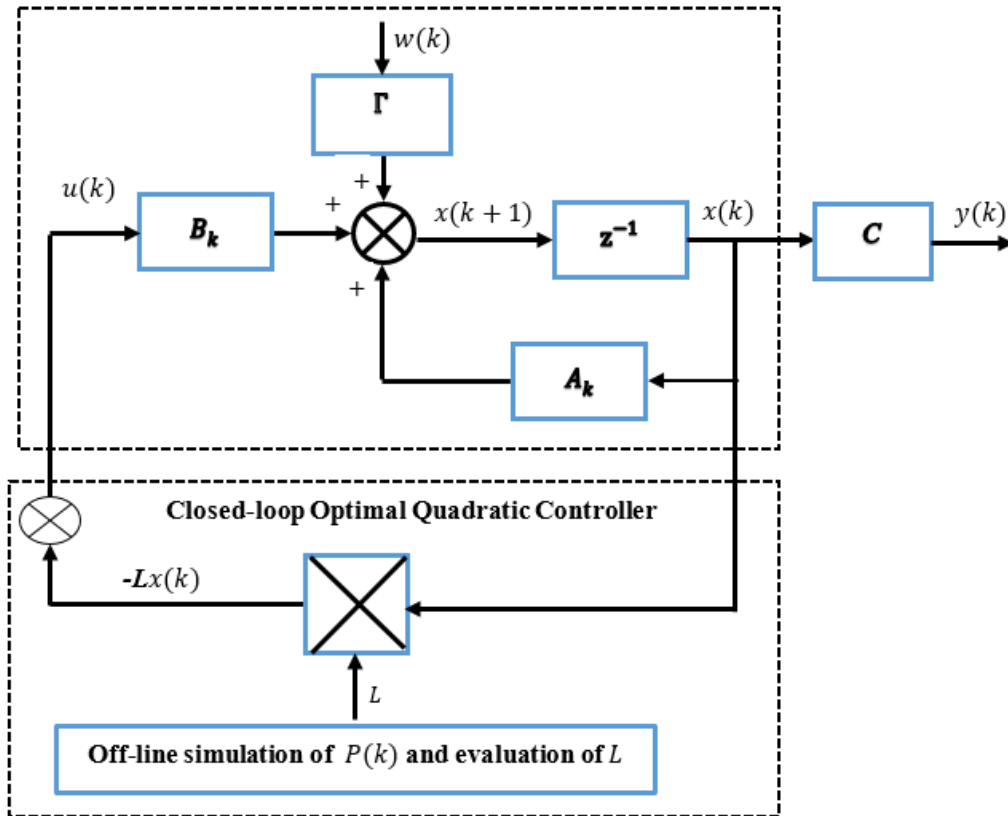


Figure 4.4. Closed-loop for an optimal control system with disturbances (Naidu,2002).

Consider the steady-state optimization problem of an interconnected power system consisting of an N control area with a cost function from Equation (4.2) can be redefined as

the follows:

$$\min J = \frac{1}{2} \sum_{k=k_0}^{\infty} x^T(k) Q_k x(k) + u^T(k) R_k u(k) \quad (4.19)$$

Subject to the equality constraint (the centralized system model) as given in Equation (4.3). In similar way as in the case of the optimal regular control presented in section 4.2.1, the Lagrange or Hamiltonian functions and the analysis of the optimal control theory can be used to obtain the necessary conditions for the existence of the minimum cost function and the Steady-state Riccati Equation Matrix P_k . Thus, the steady-state Riccati Equation Matrix can be defined as

$$P = Q_k + A_k^T \left\{ P - P [B_k^T P B_k + R_k]^{-1} B_k^T P \right\} A_k \quad (4.20)$$

where at the steady-state, $P_{k+1} = P_k = P$. The optimal feedback control becomes

$$u(k) = -Lx(k) \quad (4.21)$$

where the optimal feedback gain matrix K is given by the formula,

$$L = [R_k + B_k^T P B_k]^{-1} B_k^T P A_k \quad (4.22)$$

Substituting, the optimal feedback control law from Equation (4.21) in the discrete centralized model of the interconnected power system, the optimal closed-loop system with disturbances is given as follows:

$$x(k+1) = [A_k - B_k ([R_k + B_k^T P B_k]^{-1} B_k^T P A_k)] x(k) + \Gamma w(k) \quad (4.23)$$

where $w(k)$ is the vector of load disturbance set points. In this design process, the centralized COQAGC control provides a steady-state solution and a constant steady-state feedback gain matrix L as the final time instant k_f converges to infinity (Ogata, 1994).

Therefore, the term $x^T(k_f) F x(k_f)$ approaches to zero due to $k_f = \infty$, and the cost function converges to a constant value:

$$J(k) = x^T(k) P x(k) \quad (4.24)$$

In control systems (continuous and Discrete), Eigenvalues has been used to analyze the

stability and speed of the closed-loop control systems. Eigenvalues can be defined as the scalar values of the characteristic equation of the system or the closed-loop (Yang et al, 2005). In the discrete closed-loop control systems, the absolute values of all the eigenvalues of the closed-loop matrix $[A_k - B_k[R_k + B_k^T P B_k]^{-1} B_k^T P A_k]$ should be less than 1 in order for the closed-loop control system to be stable and fast. Usually, the speed of the system is determined by the smallest absolute value of eigenvalue among the other Eigenvalues. In the case of a continuous-time system, the real parts of all the eigenvalues must be negative for stability and the smaller a negative eigenvalue is, the faster the closed-loop system. For any state space equation, the eigenvalues can be determined by finding the determinant of the following equation:

$$\text{Det}[\lambda s - A] = 0 \quad (4.25)$$

Where A is the open-loop or closed-loop matrix, I is identity matrix, and λ is the Eigen value.

To illustrate the impact of Eigenvalues, consider the SIMULINK block diagram for a single-area power system with a generator, non-reheat turbine, and governor shown in Figure 4.5.

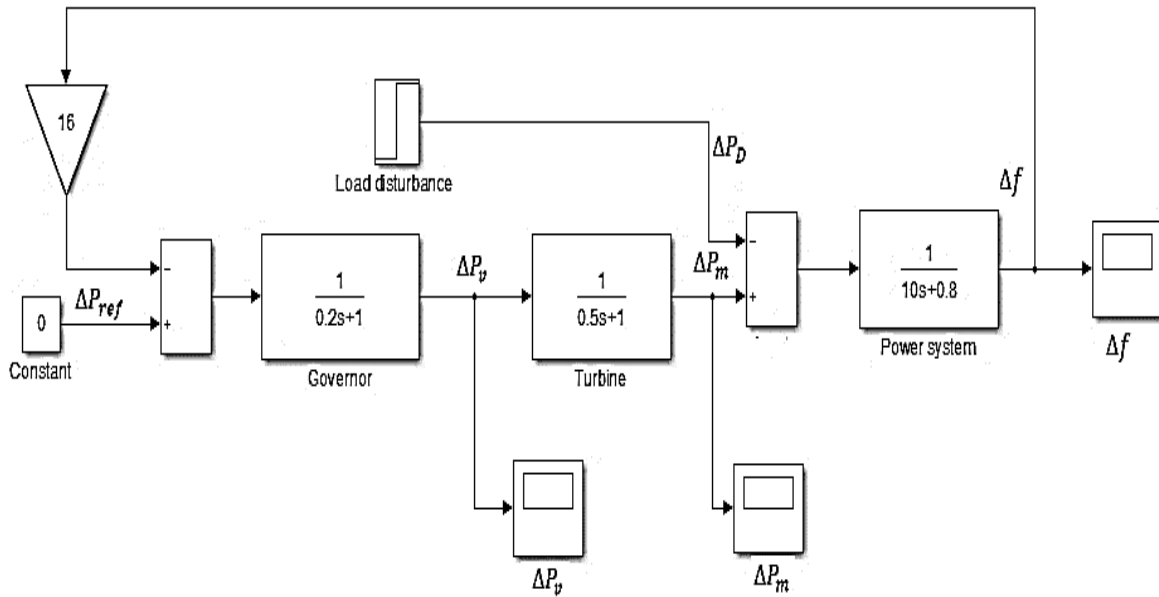


Figure 4.5. Single area power system with non-reheat turbine (kundur, 1994).

The parameters for transfer functions of the power system, the turbine and governor shown in Figure 4.5 were adopted from the work conducted by Pillai et al, 2020 with $[T_t = 0.5 \text{ sec}, R = 0.05 \text{ pu}, T_g = 2 \text{ sec}, H = 2 \text{ kg}, D = 0.8, f = 60 \text{ Hz}$ and $S = 1000 \text{ MVA}]$. Where T_t is the Turbine time constant, R is Speed regulation, T_g is the Governor time constant, H is the Inertia constant, D is the frequency sensitive load coefficient, f is the nominal frequency and S is the base power.

Assuming, $x_1(t) = \Delta f$ is the frequency deviation, $x_2(t) = \Delta P_m$ is the turbine power output, $x_3(t) = \Delta P_v$ is the governor power output, $u(t) = \Delta P_{ref}$ is the control input reference and ΔP_D is the load disturbance, where the first order differential equations of the three subsystems (power system, Turbine and Governor) are found as:

$$\begin{aligned}\dot{x}_1 &= -0.08x_1 + 0.1x_2 - 0.1\Delta P_D \\ \dot{x}_2 &= -2x_2 + 2x_3 \\ \dot{x}_3 &= -100x_1 - 5x_3 + 5u\end{aligned}\tag{4.26}$$

From which the matrices A, B, Γ are found as

$$A = \begin{bmatrix} -0.08 & 0.1 & 0 \\ 0 & -2 & 2 \\ -100 & 0 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \text{ and } \Gamma = \begin{bmatrix} -0.1 \\ 0 \\ 0 \end{bmatrix}\tag{4.27}$$

The Eigenvalues analysis of the single area power system shows complex conjugate eigenvalues of $-0.5968 \pm j1.7825$ and Eigenvalue of -5.8863 located on the left half plane of the complex plane as shown in Figure 4.6. The real parts being negative indicates that the single area power system is stable.

Simulation results shown in Figure 4.7 shows Eigenvalues analysis of the single area power system subjected to step load disturbance $\Delta P_D = 1\text{pu}$. The time response of the frequency deviation, turbine power out and governor power output associated with oscillations due to the imaginary part of the conjugate eigenvalues. The time response for frequency deviation is varied from 0Hz to -0.06 Hz .

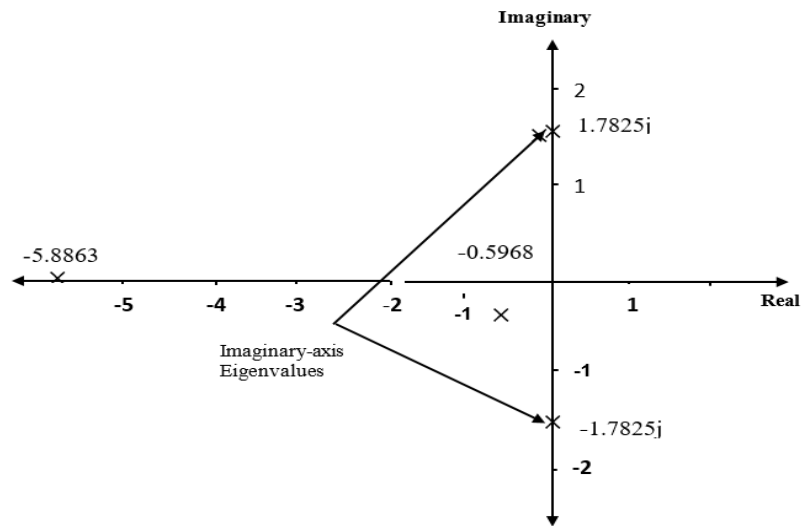


Figure 4.6. Single area power system eigenvalues' locations on the complex plane.

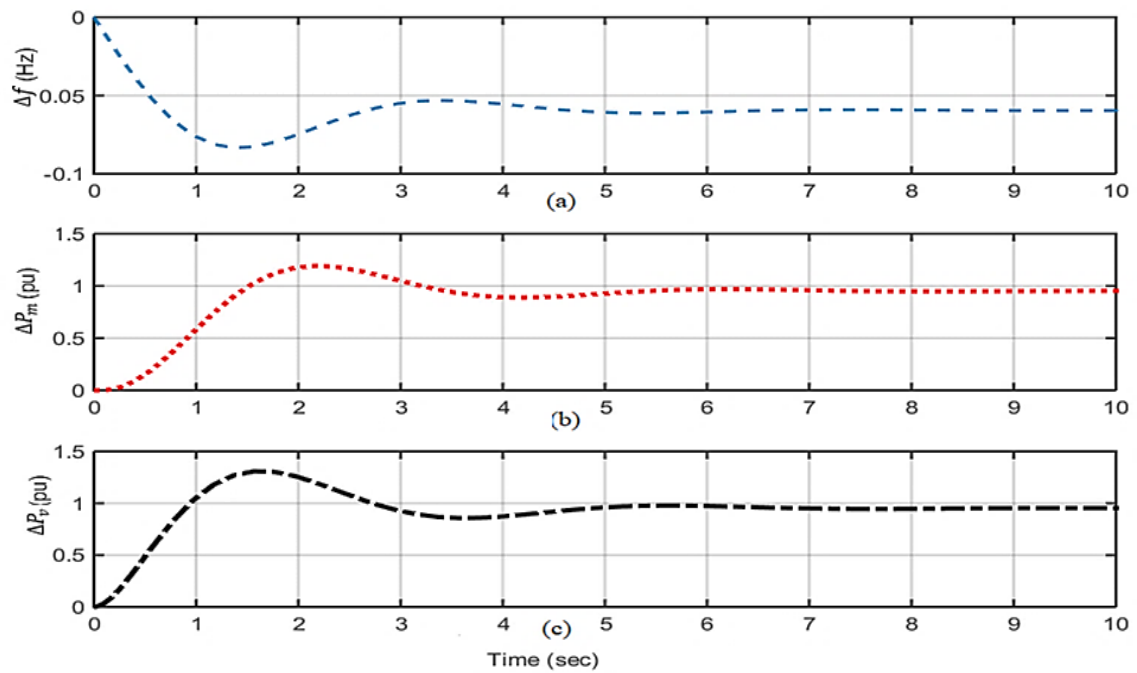


Figure 4.7: Dynamic responses of the single area power system with 1 pu step load perturbation (a) frequency deviation, (b) Turbine power deviation and (c) Governor power.

4.3 Functional Minimization Method for Centralized Discrete COQAGC

Functional minimization approach is mathematics approach for selecting the state and control-weighting matrices (Q_k and R_k) (Pathak et al, 2015). In this approach, the design requirements are transformed to the cost function so that ACEs, IAECs, of all areas and the control vector $u(k)$ from the steady state are minimized. The steady-state values of ACEs are to be zero, while the steady-state values of IAECs, and the control vector are to be constant. The three basic steps in functional minimization approach to systematically form the state and input weighing matrices are consisting of:

1. Define the cost function in terms of the Area Control Errors (ACE), Integral of Area Control Errors (IAEC), and the total control efforts as given in Equation (4.28).
2. Apply concept of the partial derivatives with respect to each state and each control effort.
3. Combine first partial derivatives of all states and the partial derivatives of the control efforts to form the state and the input weighting matrices (Q_k and R_{ki}) respectively.

The main design requirements of functional minimization approach for interconnected power systems are as follows:

- Deviation of ACEs from their steady-state values are minimized; the steady-state values of ACEs are zero.
- Deviations of the $\sum AEC_i$ of all areas from their steady-state values are minimized; the summation of the steady-state values of $\sum AEC_i$ must be a constant.
- Deviations of the control vector $u(k)$ from its steady-state value is minimized; steady state value of the control vector must be a constant

Consider the cost function that satisfies the above design requirements for N areas ($i = 1, 2, \dots, N$) as shown in Figure 4.8. Each non-reheat thermal generation, power system and load is considered as an area.

$$\begin{aligned}
 J = & \frac{1}{2} \sum_{k=k_0}^{\infty} \{ (AEC_1)^2 + (AEC_2)^2 + \dots + (AEC_N)^2 \} \\
 & + \left\{ \left(\sum_{k=k_0}^{\infty} AEC_1 \right)^2 + \left(\sum_{k=k_0}^{\infty} AEC_2 \right)^2 + \dots + \left(\sum_{k=k_0}^{\infty} AEC_N \right)^2 \right\} \\
 & + \rho \{ u_1^2 + u_2^2 + \dots + u_n^2 \}
 \end{aligned} \tag{4.28}$$

The sums in Equation (4.28) represent integration in the discrete time domain, where $ACE_1, ACE_2, \dots, ACE_N$ and $\sum ACE_1, \sum ACE_2, \dots, \sum ACE_N$ are the area control errors and integral of the area control errors respectively, $u_1, u_2, \dots, u_n$ are the control vector signals for the entire interconnected power system and ρ is AGC participation factor used to consider the variety of generation dynamics and their sharing rate in the AGC control action (Bevrani & Hiyama, 2011).

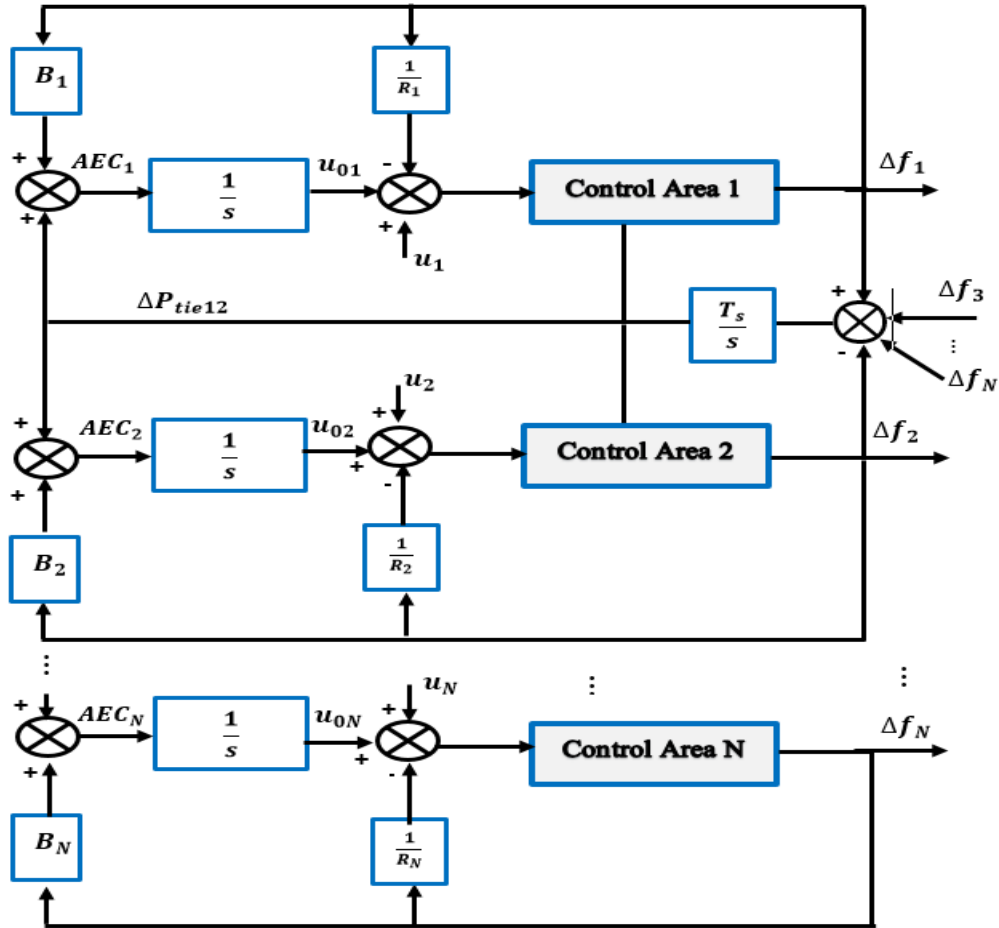


Figure 4.8. N control area power systems with non-reheat turbine in each area.

In each control area, the sum of the participation factors should be equal 1. In this chapter, the ρ is selected to be 1 since we have only one generator in each control area.

The area control errors for the N control areas can be defined as

$$ACE_1 = \beta_1 \Delta f_1 + \Delta P_{tie1}$$

$$ACE_2 = \beta_2 \Delta f_2 + a_{12} \Delta P_{tie2}$$

$$ACE_3 = \beta_3 \Delta f_3 + a_{13} \Delta P_{tie3} \quad (4.29)$$

⋮

$$ACE_N = \beta_N \Delta f_N + a_{1N} \Delta P_{tieN}$$

where $\Delta f_1, \Delta f_2, \Delta f_N$ are the frequency deviations for area1, area 2, and area N respectively, the ΔP_{tie1} and ΔP_{tieN} are the Tie line power deviations from area 1 to area 2 and from area 1 to area N and $a_{12} = -1$ is a constant coefficient that changes the sign of the tie-line power towards area2.

Similarly, the integral of the area control errors for the Ncontrol areas can be defined by

$$\begin{aligned} IACE_1 &= \sum_{k=k_0}^{\infty} (\beta_1 \Delta f_1 + \Delta P_{tie1}) \\ IACE_2 &= \sum_{k=k_0}^{\infty} \beta_2 \Delta f_2 + a_{12} \Delta P_{tie2} \\ IACE_3 &= \sum_{k=k_0}^{\infty} \beta_3 \Delta f_3 + a_{13} \Delta P_{tie3} \\ &\vdots \\ IACE_N &= \sum_{k=k_0}^{\infty} (\beta_N \Delta f_N + a_{1N} \Delta P_{tieN}) \end{aligned} \quad (4.30)$$

where $IACE_1, IACE_2, IACE_3, \dots, IACE_N$ are the integrals of $ACE_1, ACE_2, \dots, ACE_N$.

Substituting Equations (4.29 - 4.30) to Equation (4.28), the cost functional expression becomes:

$$\begin{aligned} J &= \frac{1}{2} \sum_{k=k_0}^{\infty} \{ (\beta_1 \Delta f_1 + \Delta P_{tie1})^2 + (\beta_2 \Delta f_2 + a_{12} \Delta P_{tie2})^2 + \dots \\ &\quad + (\beta_N \Delta f_N + a_{1N} \Delta P_{tieN})^2 \} \\ &\quad + \{ (IACE_1)^2 + (IACE_2)^2 + \dots + (IACE_N)^2 \} \\ &\quad + \rho \{ u_1^2 + u_2^2 + \dots + u_N^2 \} \end{aligned} \quad (4.31)$$

with the cost function

$$f(.) = \frac{1}{2} \{ (\beta_1 \Delta f_1 + \Delta P_{tie1})^2 + (\beta_2 \Delta f_2 + a_{12} \Delta P_{tie2})^2 + \dots + (\Delta f_N + a_{1N} \Delta P_{tieN})^2 \} \\ + \{ (IACE_1)^2 + (IACE_2)^2 + \dots + (IACE_N)^2 \} + \rho \{ u_1^2 + u_2^2 + \dots + u_N^2 \} \quad (4.32)$$

We assume $\Delta P_{tie1} = \Delta P_{tie2} = \dots = \Delta P_{tieN}$ and u is the control vector for the N control areas. Now, the cost functional $f(.)$ can be used systematically to formulate the weighting matrices Q_k and R_k . First, write the ACEs and IACEs in terms of the state variables and substitute them into the cost functional in Equation (4.32). Hence, the cost function becomes a function of the state variables and control signals. The aim is to represent the cost function Equation (4.31) by a standard quadratic functional using the matrices Q_k and R_k for the state and control variables. It can be noted that the length of the state's vector depends on the types and the numbers of the power generators installed in each control area as well as the configuration of the Tie-lines. For instance, if we have N control areas and Tie-line power is selected as the first state in an N control area which contains non-reheat thermal power systems and with one generator in each control area. Then, the states vector $x^T = [x_1 \ x_2 \ x_3 \ \dots \ x_N]$ for the N control areas is given by (refer to equation number (3.10) section 3.2.1):

$$x^T = [P_{tie1} \ \Delta f_1 \ \Delta P_{T1} \ \Delta P_{G1} \ \Delta f_2 \ \Delta P_{T2} \ \Delta P_{G2} \ \Delta f_3 \ \Delta P_{T3} \ \Delta P_{G3} \ \dots \ \Delta f_N \ \Delta P_{TN} \ \Delta P_{GN} \ AEC_1 \ AEC_2 \ AEC_3 \ \dots \ AEC_N]$$

Secondly, we take the first partial derivatives of the functional $f(.)$ with respect to all states and the control signals, the Q_k and R_k matrices can be constructed. In this regard, the partial derivatives with respect to frequency and the Tie-line states can be obtained in a form of first Ordinary Differential Equations (ODEs) as follows:

$$\begin{aligned} \frac{df}{d\Delta f_1} &= \beta_1^2 \Delta f_1 + \Delta P_{tie1} \\ \frac{df}{d\Delta f_2} &= \beta_2^2 \Delta f_2 + a_{12} \Delta P_{tie2} \\ &\vdots \\ \frac{df}{d\Delta f_N} &= \beta_N^2 \Delta f_N + a_{1N} \Delta P_{tieN} \end{aligned} \quad (4.33)$$

and

$$\begin{aligned} \frac{df}{d\Delta P_{tie1}} &= \beta_1 \Delta f_1 + \Delta P_{tie1} \\ \frac{df}{d\Delta P_{tie21}} &= \beta_2 \Delta f_2 + a_{12} \Delta P_{tie2} \end{aligned}$$

$$\vdots \quad (4.34)$$

$$\frac{df}{d\Delta P_{tieN}} = \beta_N \Delta f_N + a_{1N} \Delta P_{tieN}$$

In the same way, taking the partial derivatives with respect to the Integral of the Area Control Errors (IACEs) states, the following first-order ODEs are obtained:

$$\begin{aligned} \frac{df}{dIACE_1} &= IACE_1 \\ \frac{df}{dIACE_2} &= IACE_2 \\ &\vdots \\ \frac{df}{dIACE_n} &= IACE_N \end{aligned} \quad (4.35)$$

Solving the above complex problem, in this chapter the assumption is made that power demands on the interconnected power systems are the same, therefore tie-line power deviations are the same (however, the author do aware that in industrial power systems the power demand vary with respect depends on the customers loading), and the tie-line deviation is considered to be the first state variable in the augmented state vector. Also, all Ncontrol areas have identical non-reheat thermal power systems to simplify the state weighting matrix development process. Then, from Equations (4.33- 4.35) the state weighing matrix Q_k with dimensions $n + N$ by $n + N$ is constructed as follows:

$$Q_k = \begin{bmatrix} (N + a_{12}^2) & \beta_1 & 0 & 0 & (N + a_{12})\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \beta_1 & \beta_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ a_{12}\beta_2 & 0 & 0 & 0 & \beta_2^2 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & 0 & 0 \\ \beta_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & \beta_N^2 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Proof: The following steps illustrate the construction details of the matrix Q_k :

1. Rewrite and extend the functional $f(ACE, IACE, u)$ into the form:

$$\begin{aligned}
f(.) = \frac{1}{2} \{ & [(\beta_1 \Delta f_1)^2 + 2\beta_1 \Delta f_1 \Delta P_{tie1} + \Delta P_{tie1}^2] \\
& + [(\beta_2 \Delta f_2)^2 + 2a_{12}\beta_2 \Delta f_2 \Delta P_{tie2} + (a_{12} \Delta P_{tie2})^2] + \dots \\
& + [(\beta_N \Delta f_N)^2 + 2a_{1N}\beta_N \Delta f_N \Delta P_{tieN} + (a_{1N} \Delta P_{tieN})^2] \} \\
& + \{(IACE_1)^2 + (IACE_2)^2 + \dots + (IACE_N)^2\} + \rho\{u_1^2 + u_2^2 + \dots + u_N^2\}
\end{aligned} \tag{4.36}$$

2. Define the state variables i.e. $x_1 = \Delta P_{tie1} = \Delta P_{tie2} = \dots = \Delta P_{tieN}$, $x_2 = \Delta f_1$, $x_3 = \Delta P_{T1}$, $x_4 = \Delta P_{G1}$, $x_5 = \Delta f_2$, $x_6 = \Delta P_{T2}$, $x_7 = \Delta P_{G2}, \dots, x_{3N-1} = \Delta f_N$, $x_{3N} = \Delta P_{TN}$, $x_{3N+1} = \Delta P_{GN}$, $x_{3N+2} = IACE_1$, $x_{3N+3} = IACE_2 \dots x_{4N+1} = IACE_N$. Then substitute them into Equation(4.32) in which becomes:

$$\begin{aligned}
f(ACE, IACE, u) = \frac{1}{2} \{ & [(\beta_1 x_2)^2 + 2\beta_1 x_2 x_1 + x_1^2] \\
& + [(\beta_2 x_5)^2 + 2a_{12}\beta_2 x_5 x_1 + (a_{12} x_1)^2] + \dots \\
& + [(\beta_N x_{3N-1})^2 + 2a_{1N}\beta_N x_{3N-1} x_1 + a_{1N}^2 x_1^2] \} +
\end{aligned} \tag{4.37}$$

$$+ \{(x_{3N+2})^2 + (x_{3N+3})^2 + \dots + (x_{4N+1})^2\} +$$

$$\rho\{u_1^2 + u_2^2 + \dots + u_N^2\}$$

3. Partially differentiating the functional $f(.)$ in Equation (4.32) with respect to the state variables $x_1 \ x_2 \ x_3 \ \dots \ x_N$, the following first order differential equations are obtained:

$$\frac{1}{2} \frac{\partial f}{\partial x_1} = \beta_1 x_2 + x_1 + a_{12}\beta_2 x_5 + a_{12}^2 x_1 + \dots + a_{1N}\beta_N x_{3N-1} + a_{1N}^2 x_1 \tag{4.38}$$

$$= (1 + a_{12}^2 + \dots + a_{1N}^2)x_1 + \beta_1 x_2 + a_{12}\beta_2 x_5 + \dots + a_{1N}\beta_N x_{3N-1}$$

$$\frac{1}{2} \frac{\partial f}{\partial x_2} = \beta_1^2 x_2 + \beta_1 x_1 + 0 + \dots + 0 \tag{4.39}$$

$$\frac{1}{2} \frac{\partial f}{\partial x_3} = 0 + 0 + \dots + 0 \tag{4.40}$$

$$\frac{1}{2} \frac{\partial f}{\partial x_4} = 0 + 0 + \dots + 0 \tag{4.41}$$

$$\frac{1}{2} \frac{\partial f}{\partial x_5} = a_{12}\beta_2 x_1 + \beta_2^2 x_5 + 0 + \dots + 0 \tag{4.42}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \tag{4.43}$$

$$\frac{1}{2} \frac{\partial f}{\partial x_{3N-1}} = \beta_2 x_1 + \beta_2^2 x_5 + 0 + \dots + 0 \quad (4.44)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_{n+1}} = x_{n+1} + 0 + 0 + \dots + 0 \quad (4.45)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_{n+2}} = 0 + x_{n+2} + 0 + \dots + 0 \quad (4.46)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_{n+3}} = 0 + 0 + x_{n+3} + \dots + 0 \quad (4.47)$$

$$\vdots \quad \vdots \quad \ddots \quad \vdots$$

$$\frac{1}{2} \frac{\partial f}{\partial x_{n+i}} = 0 + 0 + 0 + \dots + 0 + 0 + x_{n+i} \quad (4.48)$$

Where the n is the order of states in all control areas, $i = 1, 2, 3, \dots, \infty$, the element $n + a_{12}^2$ is obtained by considering the same tie-lines power between all control areas.

Similarly minimization concept can be applied for analyzing control energy expenditure where the first-order partial derivatives with respect to control input signals is obtained as follows:

$$\begin{aligned} \frac{df}{du_1} &= u_1 + 0 \\ \frac{df}{du_2} &= 0 + u_2 + 0 \\ \frac{df}{du_3} &= 0 + 0 + u_3 + 0 \end{aligned} \quad (4.49)$$

$$\vdots \quad \vdots \quad \ddots$$

$$\frac{df}{du_N} = 0 + 0 + \dots + 0 + 0 + 0 + u_N$$

Then, from the Equation (4.34) the control weighing matrix R_k can be constructed as given below:

$$R_k = \begin{bmatrix} 1 & \dots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In conclusion, the concept of functional minimization makes choosing the states and control cost-weighting matrices (Q_k and R_k) easy and systematic. This results in effective and robust cost functional design using area control error AEC_i , the integral of area error control $IAEC_i$ and the control signals u_i in interconnected area powers systems. The matrices Q_k and R_k , in addition to the power system matrices A, B, C and Γ as developed in chapter 3, section 3.2 are used to design Optimal Centralized Discrete Quadratic Automatic Generation Controllers(COQAGCs) as given in Equations [(4.20) – (4.23)].

4.4 Case Study: Design of a Centralized Discrete COQAGC for Two Area Power System

The procedures that are described in sections 4.2 and 4.3 are used to design Centralized Discrete optimal quadratic controllers for the two-control area power system with non-reheat Turbine. Section 4.4.1 presents the continuous time model of two area control power system with and without area control errors. The functional minimization method for two area power system is discussed and developed in section 4.4.2, and control design steps based on the functional minimization method is discussed in section 4.4.3.

4.4.1. Dynamics of Two Area Power System

As derived in section 3.2, the state, control input and disturbance matrices for two area power system ($N = 2$) with identical non-reheat power system in each area is given in Equation(4.50):

$$A = \begin{bmatrix} 0 & 2\pi T_{12} & 0 & 0 & -2\pi T_{12} & 0 & 0 \\ -\frac{k_{ps1}}{T_{p1}} & -\frac{1}{T_{ps1}} & \frac{k_{ps1}}{T_{p1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 \\ -\frac{a_{12} K_{ps2}}{T_{ps2}} & 0 & 0 & 0 & -\frac{1}{T_{ps2}} & \frac{K_{ps2}}{T_{ps2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{R_2 T_{G2}} & 0 & \frac{1}{T_{G2}} \end{bmatrix}, \quad (4.50)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{G1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{G2}} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{G1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{G2}} \end{bmatrix}$$

we explained in previous section 3.2 the above matrices can't be used for regulating the frequency while maintaining tie-line power exchanges and frequency deviation at the same time. Because the area frequency regulation is the only signals considered in AGC feedback control (Bevrani, 2014). In order to meet the basic objectives of AGC and regulate both the area frequency and tie-line power interchanges at the same time, area control errors(ACEs)

must be considered in AGC feedback as shown in Figure 4.9. where T_s is the tie-line coefficient and $1/s$ is an integrator, u_{01} and u_{02} are the integrals of the ACE_1 and ACE_2 respectively and u_1 and u_2 are the state feedback control signals to area 1 and area 2 respectively.

ACE is a linear combination of frequency and tie-line power exchanges of AGC to maintain the tie-line power. The ACEs signals are computed and feed to the integrator block to ensure zero steady-state deviations (frequency and tie-line power deviations). This process increases the dimensions of the interconnected power systems by adding the derivatives of the integral of the area control errors to the model of the interconnected system as shown in Figure 4.11, section 4.5.

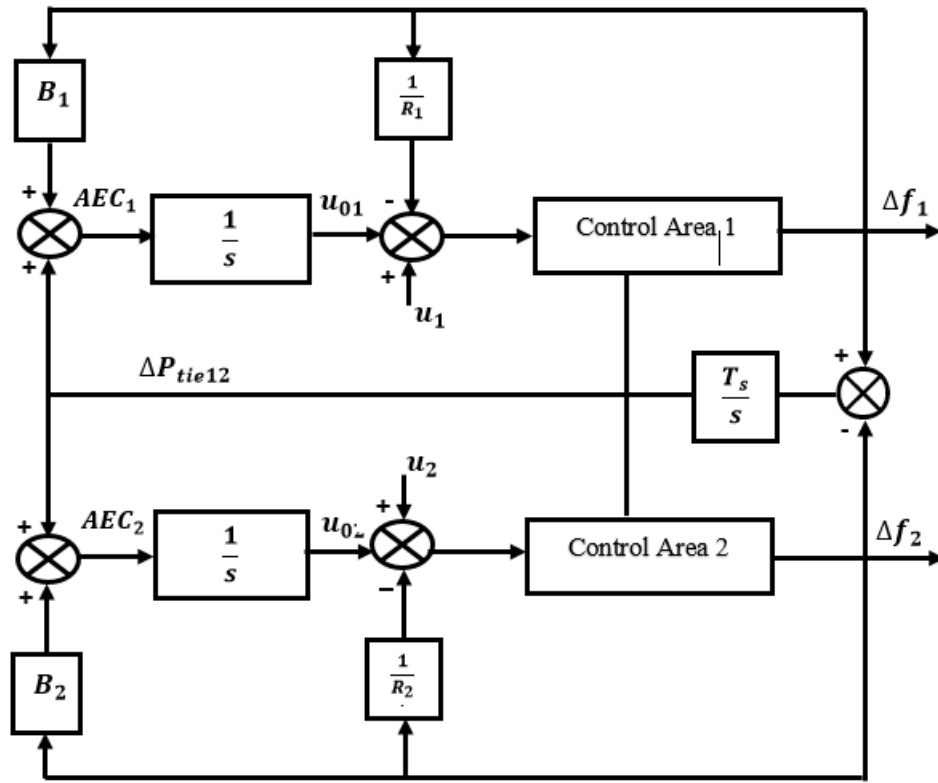


Figure 4.9. Two area power system with area control errors.

Therefore, the continuous time augmented matrices of two control area power system with identical non-reheat turbine with area control errors in each area given in Equation (4.51).

$$A_a = \begin{bmatrix} 0 & 2\pi T_{12} & 0 & 0 & -2\pi T_{12} & 0 & 0 & 0 & 0 \\ -\frac{k_{ps1}}{T_{p1}} & -\frac{1}{T_{ps1}} & \frac{k_{ps1}}{T_{p1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 & 0 & 0 \\ -\frac{a_{12} K_{ps2}}{T_{ps2}} & 0 & 0 & 0 & -\frac{1}{T_{ps2}} & \frac{K_{ps2}}{T_{ps2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{R_2 T_{G2}} & 0 & \frac{1}{T_{G2}} & 0 & 0 \\ 1 & \beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.51)$$

$$B_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{G1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{G2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Gamma_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{G1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{G2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

It is noted that Equation (4.50) and Equation (3.51) provide state, control input and disturbance matrices for two area power system without and with ACEs respectively.

4.4.2. Functional Minimization Method for Two Area Power System with ACEs

As explained earlier, in the case of optimal AGC control design, for any given design requirements, the cost function as well as the state and control weighting matrices are formulated and chosen respectively based on the functional minimization method. The objective of the functional minimization method is to identify the most controllable and observable states of the system and ignore the poorest controllable and observable ones (Ibraheem & Kumar, 2013). For interconnected power systems, the main variables of the cost function consist of the area control errors (ACEs), integral of area control errors (IACEs), and control signals (u). In case of two area power system with area control errors as shown in Figure 4.9, the cost function from Equation (4.31) is of the form

$$J = \frac{1}{2} \sum_{k=k_0}^{\infty} ((AEC_1)^2 + (AEC_2)^2) + \left\{ \left(\sum_{k=k_0}^{\infty} AEC_1 \right)^2 + \left(\sum_{k=k_0}^{\infty} AEC_2 \right)^2 \right\} + \{u_1^2 + u_2^2\} \quad (4.52)$$

From Equations (4.29 - 4.30), the area control errors (ACE_1 and ACE_2) and the integral of area control errors ($IACE_1$ and $IACE_2$) of the two control areas are defined as follows:

$$\begin{aligned} ACE_1 &= \beta_1 \Delta f_1 + \Delta P_{tie12} \\ ACE_2 &= \beta_2 \Delta f_2 + a_{12} \Delta P_{tie12} \\ IACE_1 &= \sum_{k=k_0}^{\infty} (\beta_1 \Delta f_1 + \Delta P_{tie12}) \\ IACE_2 &= \sum_{k=k_0}^{\infty} (\beta_2 \Delta f_2 + a_{12} \Delta P_{tie12}) \end{aligned} \quad (4.53)$$

$$(4.54)$$

Substituting ACE_1 and ACE_2 , $IACE_1$ and $IACE_2$ from Equations (4.53 and 4.54) to Equation (4.52), the cost function can be given by:

$$J = \frac{1}{2} \sum_{k=k_0}^{\infty} ((\beta_1 \Delta f_1 + \Delta P_{tie12})^2 + (\beta_2 \Delta f_2 + a_{12} \Delta P_{tie12})^2 + \{(IACE_1)^2 + (IACE_2)^2\} + \{u_1^2 + u_2^2\}) \quad (4.55)$$

β_1 and β_2 are the frequency bias for both areas respectively, ΔP_{tie12} is the tie-line power deviation and $a_{12} = -1$, is the constant coefficient which changes the sign of the tie-line power towards area 2. The vectors x , u and w for the two-area interconnected power system with one thermal generation unit with non-reheat turbine in each area are defined as follows:

$$\begin{aligned} x^T &= [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9]^T \\ x^T &= [\Delta P_{tie} \ \Delta f_1 \ \Delta P_{T1} \ \Delta P_{G1} \ \Delta f_2 \ \Delta P_{T2} \ \Delta P_{G2} \ IACE_1 \ IACE_2]^T \\ u^T &= [u_1 \ u_2]^T \\ w^T &= [\Delta P_{D1} \ \Delta P_{D2}]^T \end{aligned} \quad (4.56)$$

where Equation x^T refers to SIMULINK block diagram Figure 4.11, section 4.5, x_8 and x_9 are the integral of the Area control errors of the area 1 and area 2 respectively. It can be observed that the function $f(ACE, IACE, u)$ at the moment k of the functional in Equation (4.55)

can be presented in the form of Equation (4.32), and by taking the partial derivatives of the obtained function with respect to state variables as in Equations (4.33-4.35) and with respect to the control input signals as in Equation (4.36), then, the matrices Q_k and R_k can be derived based on minimization approach of the function as follows:

1. Rewrite and extend the functional $f(ACE, IACE, u)$ from Equation (4.55) into the form:

$$f(ACE, IACE, u) = \frac{1}{2} \{ [(\beta_1 \Delta f_1)^2 + 2\beta_1 \Delta f_1 \Delta P_{tie1} + \Delta P_{tie1}^2] + [(\beta_2 \Delta f_2)^2 + 2a_{12}\beta_2 \Delta f_2 \Delta P_{tie2} + (a_{12} \Delta P_{tie2})^2] \} + \{ (IACE_1)^2 + (IACE_2)^2 \} + \rho \{ u_1^2 + u_2^2 \} \quad (4.57)$$

2. Define the state variables i.e. $x_1 = \Delta P_{tie1}$, $x_2 = \Delta f_1$, $x_3 = \Delta P_{T1}$, $x_4 = \Delta P_{G1}$, $x_5 = \Delta f_2$, $x_6 = \Delta P_{T2}$, $x_7 = \Delta P_{G2}$, $x_8 = IACE_1$, $x_9 = IACE_2$, and $u = [u_1 u_2]^T$ and choose the constant coefficient $\rho = 1$. Then, substitute them into Equation (4.57) which becomes:

$$f(ACE, IACE, u) = \frac{1}{2} \{ [(\beta_1 x_2)^2 + 2\beta_1 x_2 x_1 + x_1^2] + [(\beta_2 x_5)^2 + 2a_{12}\beta_2 x_5 x_1 + (a_{12} x_1)^2] \} + \{ (x_8)^2 + (x_9)^2 \} + \rho \{ u_1^2 + u_2^2 \} \quad (4.58)$$

3. Partially differentiating the function $f(ACE, IACE, u)$ in Equation (4.58) with respect to the state variables $x_1 \ x_2 \ x_3 \ \dots \ x_9$, the following first order differential equations are obtained:

$$\frac{1}{2} \frac{\partial f}{\partial x_1} = \beta_1 x_2 + x_1 + a_{12}\beta_2 x_5 + a_{12}^2 x_1 = (1 + a_{12}^2) x_1 + \beta_1 x_2 + a_{12}\beta_2 x_5 \quad (4.59)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_2} = \beta_2 x_1 + \beta_1^2 x_2 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \quad (4.60)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_3} = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \quad (4.61)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_4} = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \quad (4.62)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_5} = a_{12}\beta_2 x_1 + 0 + 0 + 0 + \beta_2^2 x_5 + 0 + 0 + 0 + 0 \quad (4.63)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_6} = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \quad (4.64)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_7} = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \quad (4.65)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_8} = 0 + 0 + 0 + 0 + 0 + 0 + 0 + x_8 + 0 \quad (4.66)$$

$$\frac{1}{2} \frac{\partial f}{\partial x_9} = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + x_9 \quad (4.67)$$

Similar minimization concept can be applied to control energy expenditure where the first-order partial derivatives with respect to control input signals can be obtained as follows:

$$\frac{df}{du_1} = u_1 + 0$$

$$\frac{df}{du_2} = 0 + u_2 + 0 \quad (4.68)$$

Then, from the above state partial derivative Equations (4.59 to 4.67) and from the control partial derivatives from Equation (4.68) the state weighing matrix Q_k and the control weighing matrix R_k can be constructed as given in Equation (4.69) below.

$$Q_k = \begin{bmatrix} (1 + a_{12}^2) & \beta_1 & 0 & 0 & a_{12}\beta_2 & 0 & 0 & 0 & 0 \\ \beta_1 & \beta_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{12}\beta_2 & 0 & 0 & 0 & \beta_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4.69)$$

The matrices of the power system and of the Q_k and R_k are used further for the calculation of the matrix of the Optimal Quadratic AGC controller, following the Equations (4.20 – 4.23).

4.4.3. Centralized Discrete COQAGC Control Design

In sections 4.2.1 and 4.2.2, the design of discrete optimal quadratic AGC controllers and functional minimization method were discussed. The basics of the design procedure are to design an optimal feedback gains L so that the cost function of the system is minimized. Therefore, the optimized quadratic AGC controllers can be developed in two steps: systematic selection of weighting matrices (Q_k and R_k) and design of optimal quadratic control law $u(k)$ that minimizes the closed-loop system under investigation. In designing the control law $u(k)$, the backward iteration process is performed on the matrix Riccati equation in order to find the gain feedback matrix L which in return minimizes the cost function. As a result, the minimized cost function will transfer the system state from an arbitrary initial

conditions to the origin as the time interval converges to infinity (Pathak et al, 2015). In interconnected power systems, the basic objectives of optimal quadratic AGC controllers are to ensure that the power systems operate at nominal operating condition subject to step load perturbations (SLPs) (Ibraheem and Kumar, 2013). Over all flow chart of the centralized discrete COQAGC controller algorithm and design progress is given in Figure 4.10. This algorithm can be applied either to the case without area control errors or with area control errors. The following steps provide the centralized discrete COQAGC control process:

Step 1. Define State and control variables

Step 2. Formulate Matrix A, matrix B and Matrix Γ (system modelling: continuous and discrete)

Step 3. Cost function definition based on the design requirements.

Step 4. Formulate State and control weighing matrices (R_k and Q_k) based on Functional Minimization Method (FMM).

Step 5. Initialize state vector, calculate matrix E and substitute matrix $P(k + 1)$ to matrix F .

Step 6. Iteration process in MATLAB simulation and computation environment (find the solution of discrete Riccati equation based on Equation (4.20))

Step 7. Evaluation process to calculate the optimal feedback gain matrix L using Equation (4.22)

Step 8. Iteration process to solve steady state optimal closed-loop problem

Step 9. End of computation process.

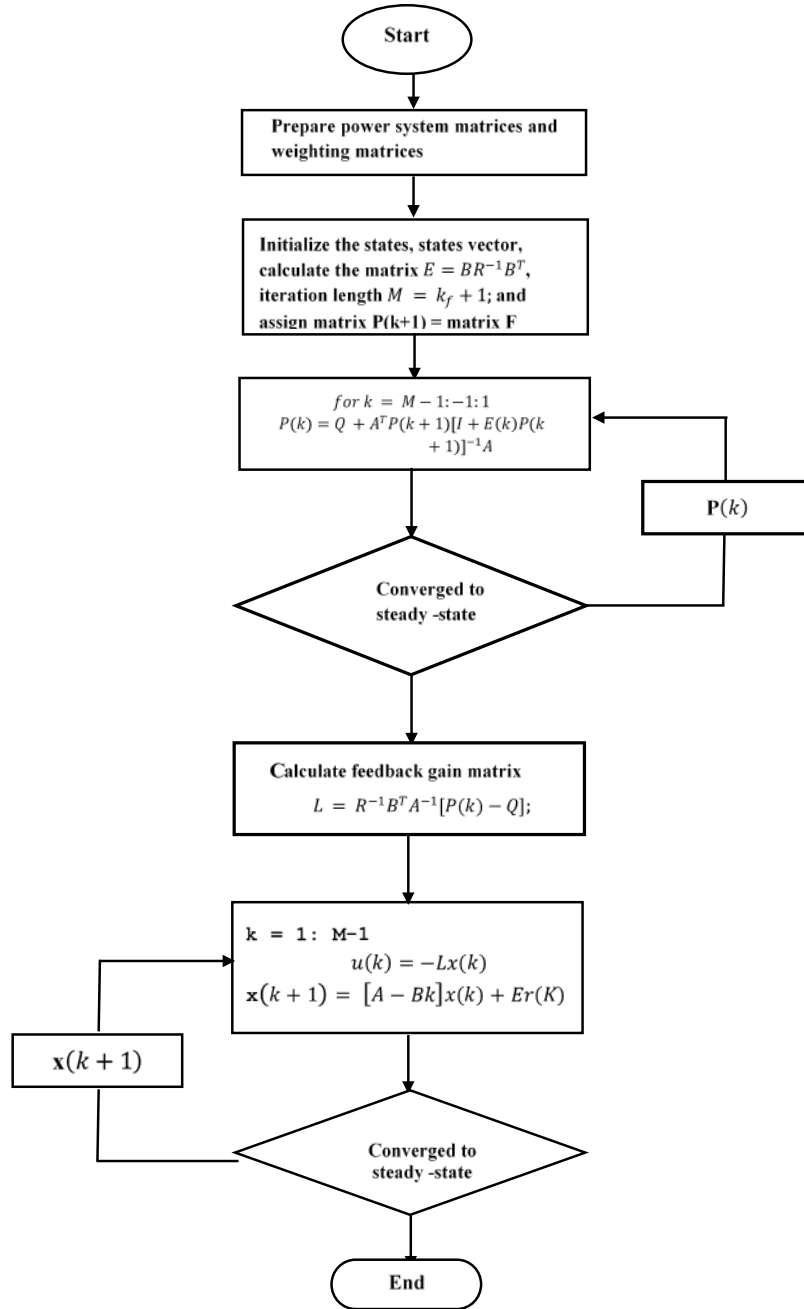


Figure 4.10. Flow chart of centralized discrete COQAGC for interconnected power systems (Esmail & Krishnamurthy, 2023).

The simulation results are shown in the next section, where the centralized discrete LQR regulators and centralized discrete COQAGC controllers are implemented to the two-area control system.

4.5 Simulation Results and Discussions

In this section, the two types of the developed optimal quadratic controllers are tested using MATLAB/SIMULINK simulation, namely 1) centralized discrete COQAGC regulator without ACEs and 2) centralized discrete COQAGC controller with ACEs for a two-area power system. To demonstrate the feasibility of both optimal quadratic control techniques, they are implemented in a discrete-time form with a sampling of 314 *ms* based on the Shannon theorem (Boukas & AL-Sunni, 2011). The simulation study investigated the robustness of the developed controller performance and efficiency with and without disturbances.

Dynamics of electrical power systems are nonlinear in nature. However, for Automatic generation control the linearized model can be used to design the discrete optimal quadratic AGC controllers. The parameters of the two-area thermal power system are given in Table 4.1. The block diagram of the centralized optimal quadratic closed-loop system of two area interconnected power system with identical non-reheat turbine in each area control is depicted in Figure 4.11.

Table 4.1. Parameters of the thermal power system with non-reheat turbine in each area (Hakimuddin et al, 2018).

Parameter	Symbol	Unit	Area 1 ($i = 1$)	Area 2 ($i = 2$)
Rate Power	P_R	MW	2000	2000
Power system time constant	T_{ps}	sec	20	20
Turbine time constant	T_t	sec	0.3	0.3
Power system gain	K_{ps}	Hz/pu, MW	120	120
Tie-line coefficient	T_{12}	MW/Hz	0.0867	0.0867
Governor time constant	T_G	sec	0.08	0.08
Frequency dropping	R_i	Hz/pu, MW	2.4	2.4
Frequency Bias constant	B_i		0.425	0.425
Damping coefficient	D_i	N/m.sec	0.0833	0.0833
$\delta_1 - \delta_2$	$\Delta\delta$	Degree	30°	30°
Maximum tie-line power	$P_{tie-max}$	MW	200	200

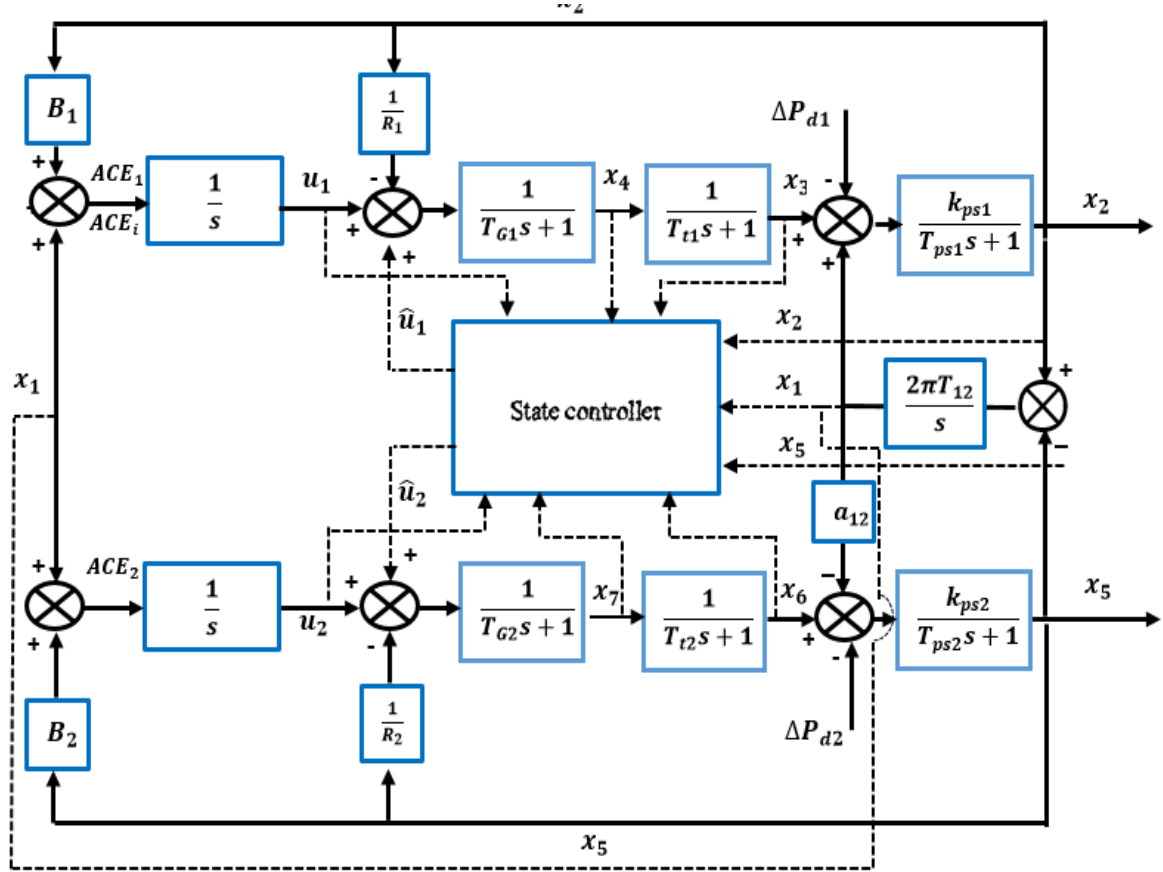


Figure 4.11. Two area closed loop control system with state feedback control.

4.5.1. Calculation of the centralised optimal quadratic regulator and performance investigation of the closed-loop system using MATLAB/SIMULINK Simulation

The simulation results of two area control power systems were studied in this section and the closed-loop performance of a centralized COQAGC regulator with no area control errors. Initially, the continuous and discrete models of the power system were analysed using one step Euler discretization procedure for converting continuous-time state-space models to discrete ones (Kögel & Findeisen, 2015). The cost function is minimized for the design of the centralized discrete COQAGC regulator and state and control weighing matrices (Q and R) were estimated based on FMM. Also, the feedback gain matrix L is obtained using MATLAB environment. Then, the performance of a centralized discrete COQAGC regulator is examined with and without disturbances as discussed in section 4.5.1.2.

4.5.1.1. Calculation of the continuous and discrete matrices of the state space model of the two-area power system with no area control error

The state matrix A, the input matrix B and disturbance matrix Γ from Equation (4.50), section 4.4.1, the maximum power generation of 2000 MW on each area with identical non-reheat thermal plants presented in Table 4.1 as given in Equation (4.70) below.

$$A = \begin{bmatrix} 0 & 0.545 & 0 & 0 & -0.545 & 0 & 0 \\ -6.024 & -0.050 & 6.0024 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.333 & 3.333 & 0 & 0 & 0 \\ 0 & -5.208 & 0 & -12.500 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & -0.050 & 6.024 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.333 & 3.333 \\ 0 & 0 & 0 & 0 & -5.208 & 0 & 12.500 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 12.500 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 12.500 \end{bmatrix} \text{ and } \Gamma = \begin{bmatrix} 0 & 0 \\ -6.024 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -6.024 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (4.70)$$

and the output matrix C is,

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (4.71)$$

Therefore, one step Euler discretization procedure (Kögel &Findeisen, 2015) and sampling time are used to convert the continuous-time matrices A and B from Equation (4.44) to the discrete-time matrices A_k and B_k as represented in Equation (4.46) Euler procedure is a direct and easy approach for providing approximated discrete state space equations (Kögel &Findeisen, 2015). In this procedure, the continuous-time and discrete-time matrices are related with the formulas:

$$A_k = (I + T \times A)$$

$$B_k = (T \times B) \quad (4.72)$$

where I is an identity matrix and T is the sampling period. The turbine's time constant $T_{t1} = T_{t2} = 0.3$ sec is chosen for each area which gives the bandwidth frequency of 0.333 rad/sec. Thus, as explained earlier, the sampling period T is found to be 0.0314 sec based on the Shannon theorem (Boukas &AL-Sunni, 2011). As result, the discrete matrices A_k and B_k are obtained as given in Equation (4.73).

The discrete matrices A_k and B_k were used in the next section in simulating the centralized discrete COQAGC regulator. These matrices were involved in the calculation of the discrete Riccati matrix and in the simulation results of the closed-loop system. In the next section, a calculation of the discrete Riccati matrix, the discrete regulator, and simulation results of the

closed-loop system is discussed.

$$A_k = \begin{bmatrix} 1.0000 & 0.0171 & 0 & 0 & -0.0171 & 0 & 0 \\ -0.1892 & 0.9984 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8953 & 0.1047 & 0 & 0 & 0 \\ 0 & -0.1635 & 0 & 0.6075 & 0 & 0 & 0 \\ 0.1892 & 0 & 0 & 0 & 0.9984 & 0.1884 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8953 & 0.1047 \\ 0 & 0 & 0 & 0 & -0.1635 & 0 & 1.3925 \end{bmatrix}, B_k = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.3925 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.3925 \end{bmatrix} \quad (4.73)$$

4.5.1.2. Calculation of the discrete Riccati matrix, the discrete regulator and simulation results of the closed loop system

In the case of Centralized Optimal Quadratic Regulators, the terminal weighing matrix F , the state weighing matrix Q_k and the input weighing matrix R_k are chosen to be diagonal matrices and their numerical values were found by trial and error method as follows:

$$F = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12 \end{bmatrix}, \quad Q_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_k = \begin{bmatrix} 60 & 0 \\ 0 & 20 \end{bmatrix}$$

Settings: the initial tie-line power is set as 150 MW (0.15pu) and the initial values of the frequencies deviations in the two areas are set as 50 Hz for each area. The maximum power generation on each area is 2000 MW using identical non-reheat thermal generators. No area control errors and loads references are considered in this case. The simulation is run for 600 iterations with $k_0 = 0$, and $k_f = 600$. The final terminal matrix in Equation (4.14) is used to solve the difference Riccati equation matrix in Equation (4.15) backwards starting from k_f to k_0 . The P_k matrix was calculated as

$$P_k = \begin{bmatrix} 1068.6 & 173.1 & 149.2 & 29.8 & 214.1 & 449.2 & 115.9 \\ 173.1 & 066.5 & 3.5 & 15.6 & 12.3987 & 74.9 & 27.2 \\ 149.2 & 73.5 & 121.9 & 29.8 & 28.10 & 77.7 & 06.3 \\ 29.8 & 15.6 & 29.8 & 9.2 & 6.3 & 17.1 & 6.7 \\ 214.1 & 28.1 & 28.4 & 6.3 & 113.2 & 137 & -56.7 \\ 149.2 & 74.9 & 77.7 & 17.1 & 137 & 246.8 & 46.8 \\ 115.9 & 27.2 & 30.2 & 6.7 & -56.7 & 46.8 & 26.43 \end{bmatrix}$$

where for examples, $P_{11} = 1068.63.1$, $P_{12} = P_{21} = 173.1$, $P_{32} = 15.6$ and $P_{77} = 26.43$. For

illustration purpose, Figure 4.12 shows the behavior of 4 Riccati coefficients out of 49 coefficients for the state regulation problem. It can be observed that all Riccati coefficients are converged to zero at $k_f = 600$.

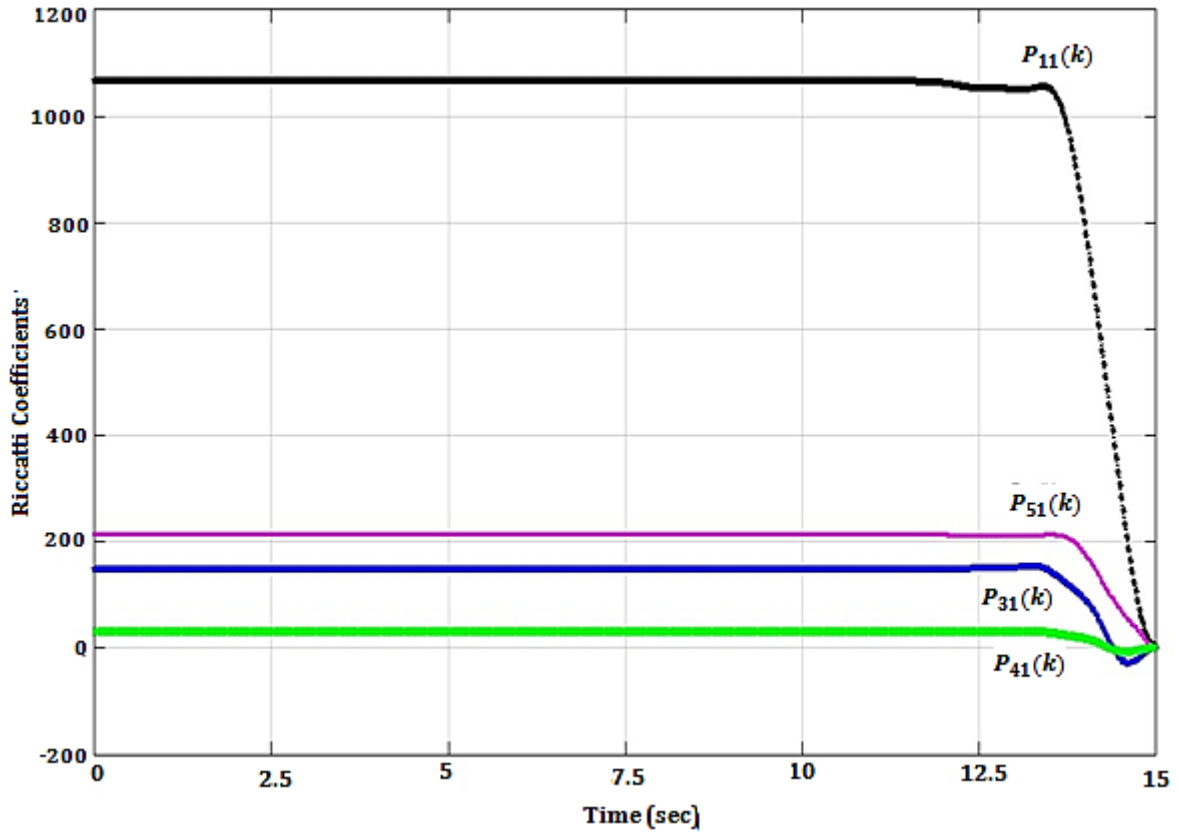


Figure 4.12. Riccati Coefficients behaviour for discrete-time linear State Regulator system.

The feedback control gain matrix L from Equation (4.16) is calculated, when $k = k_0$ as:

$$L = \begin{bmatrix} 0.0348 & 0.0582 & 0.1387 & 0.0434 & 0.0146 & 0.0424 & 0.0174 \\ 0.6516 & 0.1448 & 0.1440 & 0.0306 & -0.4294 & 0.2605 & 1.8553 \end{bmatrix}$$

and, the feedback control gain matrix L from Equation (4.16) is calculated respectively when $k = k_{f-1}$, as:

$$L = \begin{bmatrix} -0.0000 & 0.0000 & -0.0042 & 0.0000 & 0.0000 & 0.0000 & 0.0157 \\ -0.0000 & -0.0000 & -0.0184 & -0.0000 & -0.0000 & -0.0000 & 0.1567 \end{bmatrix}$$

Figures (4.13), (4.14) and (4.15) show the closed loop system responses for the Area-1's frequency deviation, Area -2's frequency deviation and the tie-line power deviation respectively by considering initial set-points of 0.0 Hz and 150 MW for the tie-line power in each area. It is also assumed that both areas are set to maximum power generation capacity of 2000 MW (0.2 pu) in the beginning of the simulation.

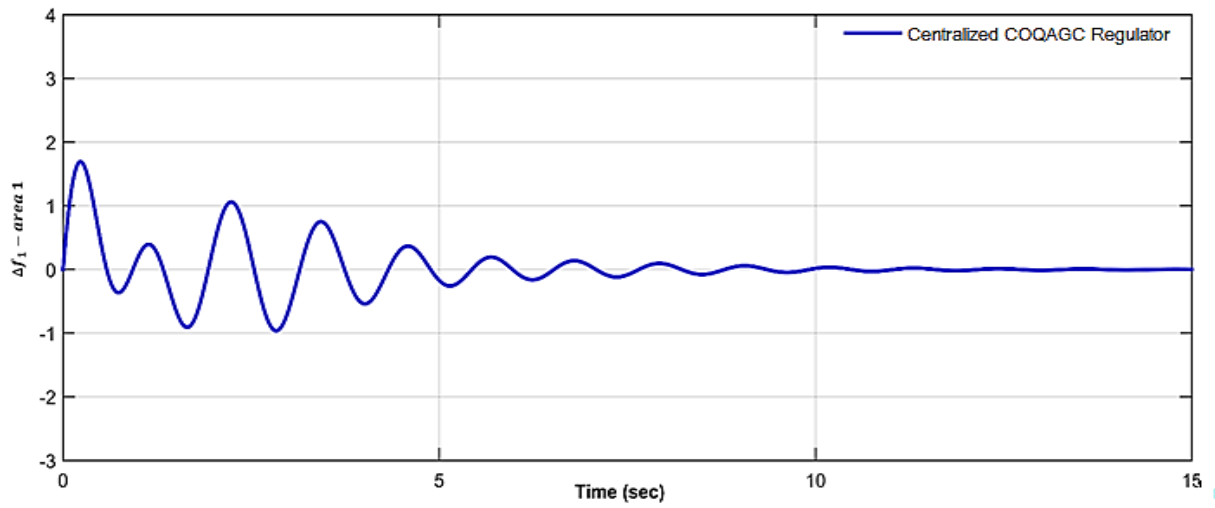


Figure 4- 13. Frequency deviation in area-1.

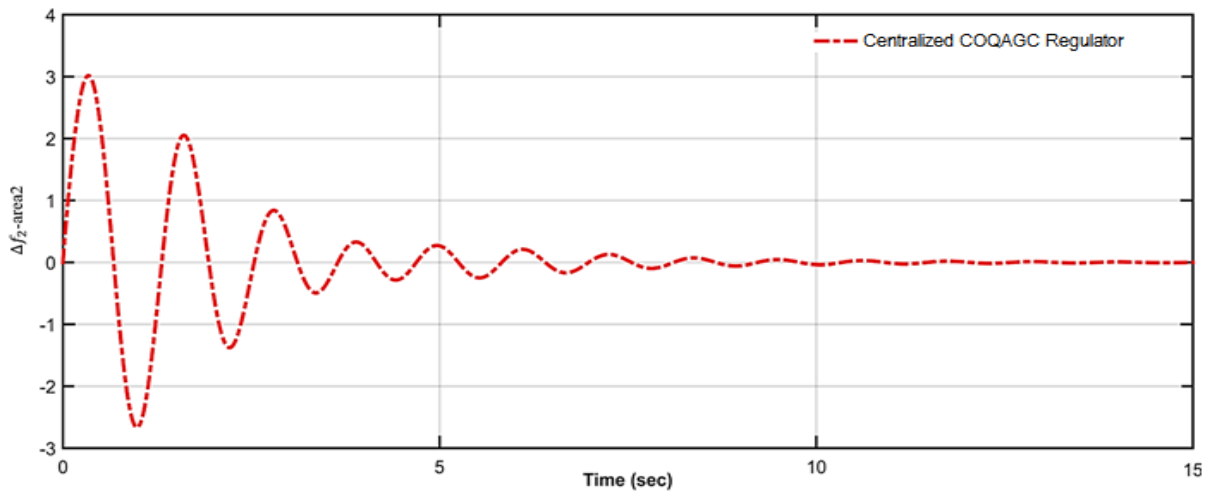


Figure 4.14. Frequency deviation in area-2.

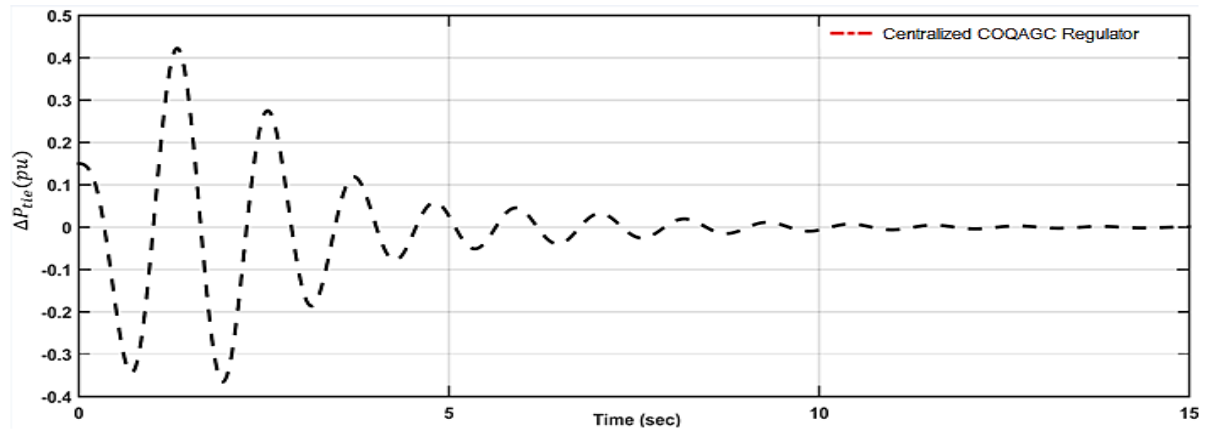


Figure 4.15. Tie-line power deviation between area 1 and 2.

To test the robustness of the centralized optimal quadratic COQAGC regulator against the disturbances such as step load perturbations $r^T = [\Delta P_{D1} \Delta P_{D2}]^T$ with large magnitude are added to each area. In this scenario, the initial tie-line power is set at 150 MW (0.15pu) and the initial values of the frequencies deviations in both areas are set at 0.0 Hz for each area. The simulation is run in MATLAB environment for 600 iterations with $k_0 = 0$ and $k_f = 600$ which are iteration numbers respectively. The magnitudes of the load

disturbances for the two areas are $\Delta P_{D1} = 500 \text{ MW}$ (0.5pu) and $\Delta P_{D2} = 500 \text{ MW}$ (0.5pu) respectively. The input load disturbances to the system are added at $t = 8$ seconds. The deviations of the frequencies (Δf_1 and Δf_2) and the tie-line deviation (ΔP_{tie}) are shown in Figures (4.16), (4.17) and (4.18) respectively. The centralized optimal quadratic regulator demonstrates faster disturbance rejection (robustness) but the subjected loads input increased the oscillation of the regulated system. This shows that the large load may make the system to be unstable. Load disturbance is added to the system at 8s, and is observed that sudden change in the frequency deviation as shown in Figures 4.16 and 4.17 respectively.

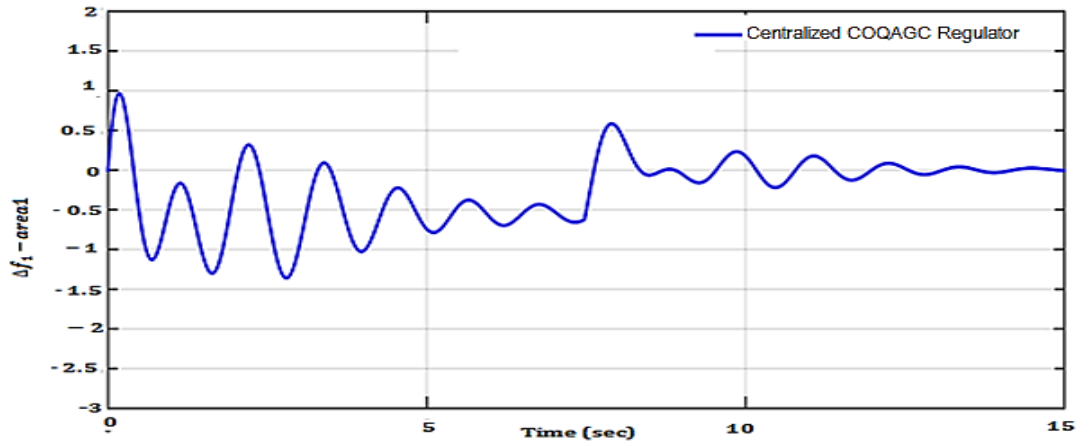


Figure 4.16. Frequency deviation-area 1 with SLP at $t = 8$ sec.

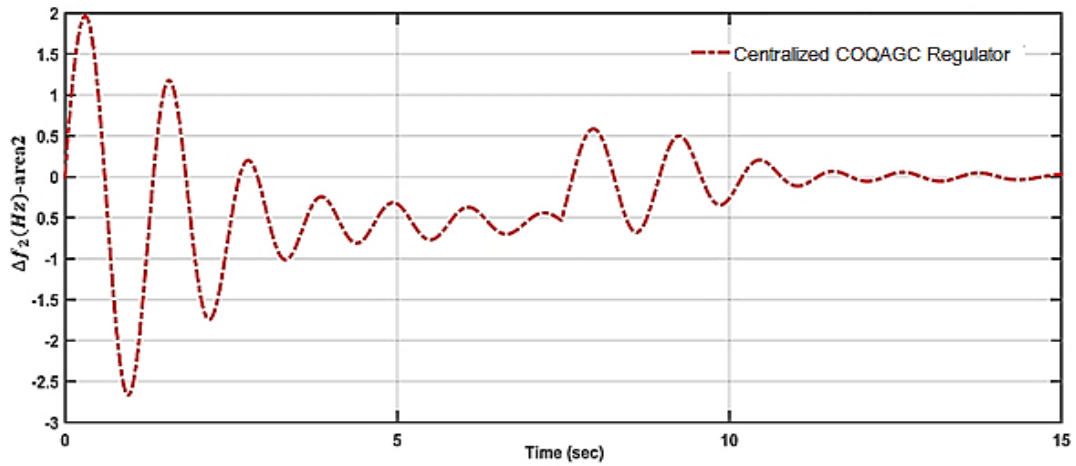


Figure 4.17. Frequency deviation-area 2 with disturbance input at $t = 8$ sec

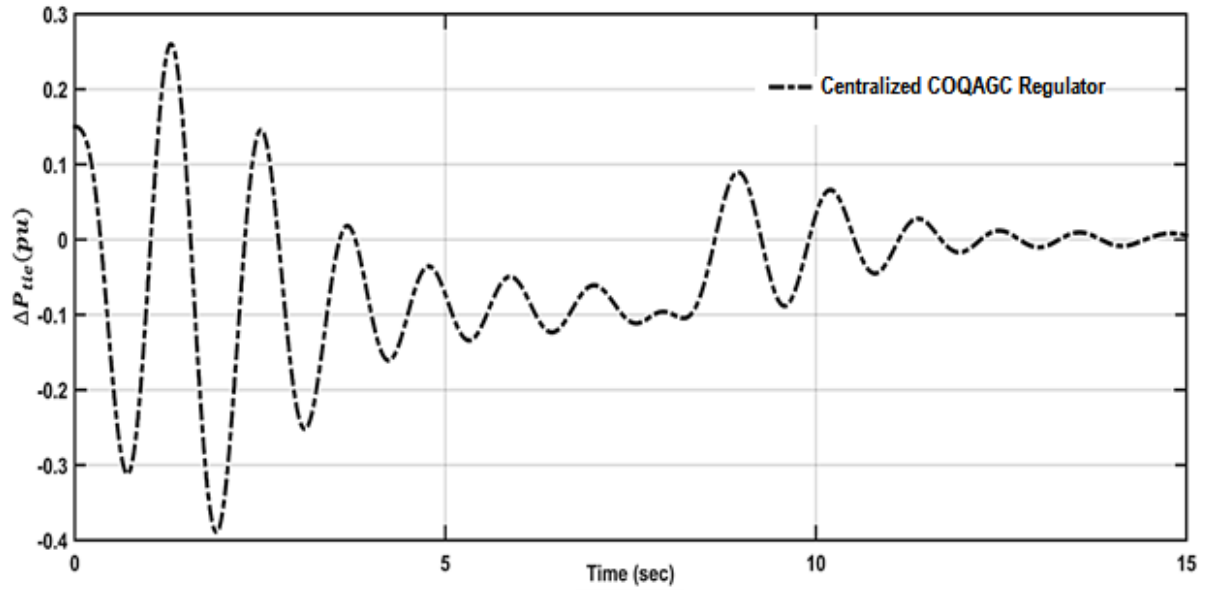


Figure 4.18. Tie-line power deviation with input disturbances at $t = 8$ sec.

4.5.2 Calculation of the centralised discrete COQAGC controller and performance investigation of the closed-loop system using MATLAB/SIMULINK Simulation

Similar simulation study is done for two area control power systems to test the closed-loop performance of a centralized discrete COQAGC controller with area control errors. Firstly, the augmented continuous and discrete models of the power system were obtained using one step Euler discretization procedure for converting continuous-time state-space models to discrete ones (Kögel & Findeisen, 2015). The cost function is minimized for the design of the centralized discrete COQAGC controller and state and control weighing matrices (Q_k and R_k) were estimated based on FMM. Also, the feedback gain matrix L is obtained using MATLAB environment. Then, the performance of a centralized discrete COQAGC controller is examined with and without disturbances as discussed in section 4.5.2.2.

4.5.2.1. Calculation of the continuous and discrete matrices of the augmented state space model of the two-area power system.

In same way as in the case of the centralized optimal quadratic regulator, the continuous augmented matrices, namely system state augmented matrix (A_a), the control input augmented matrix (B_a) and disturbance augmented matrix (Γ_1) when area control errors (ACEs) are taken in consideration are derived with respect to Equation (4.37) given in section 4.4.1, and Table 4.1 as follows:

$$\begin{aligned}
A_a = & \begin{bmatrix} 0 & 0.545 & 0 & 0 & -0.545 & 0 & 0 & 0 & 0 \\ -6.024 & -0.050 & 6.0024 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.333 & 3.333 & 0 & 0 & 0 & 0 & 0 \\ 0 & -5.208 & 0 & -12.500 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & -0.050 & 6.024 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.333 & 3.333 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5.208 & 0 & 12.500 & 0 & 0 \\ 1 & 0.425 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0.425 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
& B_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 12.500 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 12.500 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \Gamma_a = \begin{bmatrix} 0 & 0 \\ -6.024 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -6.024 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (4.74)
\end{aligned}$$

and the output matrix C_a is,

$$C_a = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.75)$$

Similarly, the discrete augmented matrices are derived and given as shown in Equation (4.76). The discrete matrices A_a and B_a were used in the next section in simulating the centralized discrete COQAGC controller. These matrices were also involved in the calculation of the discrete Riccati matrix and in the simulation results of the closed-loop system. In the next section, a calculation of the discrete Riccati matrix, the centralized discrete COQAGC controller, and simulation results of the closed-loop system is discussed

$$A_{ak} = \begin{bmatrix} 1.0000 & 0.0171 & 0 & 0 & -0.0171 & 0 & 0 & 0 & 0 \\ -0.192 & 0.9984 & 0.892 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8953 & 0.1047 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1635 & 0 & 0.6075 & 0 & 0 & 0 & 0 & 0 \\ 0.892 & 0 & 0 & 0 & 0.9984 & 0.1892 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8953 & 0.1047 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1635 & 0 & 0.6075 & 0 & 0 \\ 0.0314 & 0.0133 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -0.0314 & 0 & 0 & 0 & 0.033 & 0 & 0 & 0 & 1 \end{bmatrix} \quad B_{ak} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.3925 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.3925 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (4.76)$$

4.5.2.2 The closed loop calculation of the discrete Riccati matrix, the discrete AGC controller and simulation results of system

In this section the area control errors are introduced. Referring to Equations (4.53 – 4.54), if $\beta_1 = \beta_2 = 0.425$ where β_i is the frequency bias change and $a_{12} = -1$, the area control errors, the integral of area control errors for both areas, and the cost function are obtained as follows:

$$\begin{aligned} ACE_1 &= x_1 + 0.425x_2 \\ IACE_1 &= \sum_{k=k_0}^{\infty} (x_1 + 0.425x_2) \\ ACE_2 &= -x_1 + 0.425x_5 \\ IACE_2 &= \sum_{k=k_0}^{\infty} (-x_1 + 0.425x_5) \end{aligned} \quad (4.77)$$

and

$$\begin{aligned} J &= \frac{1}{2} \sum_{k=k_0}^{\infty} (x_1 + 0.425x_2)^2 + (-x_1 + 0.425x_5)^2 + \{(x_8)^2 + (x_9)^2\} \\ &\quad + \{u_1^2 + u_2^2\} \end{aligned} \quad (4.78)$$

where x_1 is the tie-line power deviation, x_2 and x_5 are frequency deviations of area-1 and area-2 respectively, x_8 and x_9 are the integrals of ACE_1 and ACE_2 respectively.

The discrete state and control weighing matrices Q_k and R_k are derived on the basis of the transformation of the AGC functional to the state space optimal quadratic functional, as described in Section 4.4.2., as follows:

$$Q_k = \begin{bmatrix} 2 & 0.425 & 0 & 0 & -0.425 & 0 & 0 & 0 & 0 \\ 0.425 & 0.180625 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.425 & 0 & 0 & 0 & 0.180625 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad R_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

”

The solution of the matrix difference Riccati equation P_k from Equation (4.15) at steady state is obtained with an iteration process starting from the initial condition $P(0) = 0$ as:

$$P_k = \begin{bmatrix} 9.4839 & 2.8974 & 1.0236 & 0.0980 & -2.8974 & -1.0236 & -0.0980 & 5.3830 & -5.3830 \\ 2.8974 & 1.6653 & 0.7310 & 0.0846 & -0.3033 & -0.1258 & -0.0134 & 3.2043 & -0.2770 \\ 1.0236 & 0.7310 & 0.4380 & 0.0601 & -0.1258 & -0.0541 & -0.0060 & 1.3279 & -0.0634 \\ 0.0980 & 0.0846 & 0.0601 & 0.0094 & -0.0134 & -0.0060 & -0.0007 & 0.1492 & -0.0023 \\ -2.8974 & -0.3033 & -0.1258 & -0.0134 & 1.6653 & 0.7310 & 0.0846 & -0.2770 & 3.2043 \\ -1.0236 & -0.1258 & -0.0541 & -0.0060 & 0.7310 & 0.4380 & 0.0601 & -0.0634 & 3.2043 \\ -0.0980 & -0.0134 & -0.0060 & -0.0007 & 0.0846 & 0.0601 & 0.0094 & -0.0023 & 0.1492 \\ 5.3830 & 3.2043 & 1.3279 & 0.1492 & -0.2770 & -0.0634 & -0.0023 & 38.6746 & -0.0043 \\ -5.3830 & -0.2770 & -0.0634 & -0.0023 & 3.2043 & 1.3279 & 0.1492 & -0.0043 & 38.6746 \end{bmatrix}$$

where for examples, $P_{11} = 9.4839$, $P_{12} = P_{21} = 2.8974$, $P_{32} = 0.7310$ and $P_{99} = 38.6746$.in the same way, the behavior of 4 Riccati coefficients out of 81 coefficients for the steady-state problem can be obtained.

After the computation of the above matrices, based on the Equation (4.21), the constant feedback gain matrix values for the closed-loop system at the steady-state is found to be:

$$L = \begin{bmatrix} 9.3508 & 9.8614 & 7.9663 & 1.3717 & -1.4092 & -0.6593 & -0.0174 & 17.0789 & 0.2613 \\ -9.3508 & -1.4092 & -0.6593 & -0.0771 & 9.8614 & 7.9663 & 1.3717 & 0.2613 & 17.0789 \end{bmatrix}$$

where for examples, $L_{11} = 9.3508$, $L_{12} = 9.8614, \dots, L_{19} = 0.2513$ and $L_{21} = -9.3508$, $L_{22} = -1.4092, \dots, L_{29} = 17.0789$.The corresponding closed-loop system's Eigenvalues found as given in Table 4.2. All the real parts of Eigenvalues of the closed-loop system are located in left half plane of the complex plane which guarantees the system stability associated with oscillation due to the imaginary parts of Eigen's values.

Table 4 2: The systems' states and their corresponding closed-loop Eigenvalues

States	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Eigenvalues	-13.3	-0.54 + 4.1i	-0.54 - 4.11i	-1.7159	-13.3	-0.370	-1.35 + 3.45i	-1.35 - 3.15i	-0.37

The simulations are conducted for two different cases which are discussed below:

Case1: No load disturbances $w^T = 0$ is imposed to the power system, the parameters of two area system are initial tie-line power is set at 150 MW (0.15pu) and the initial values of the frequency deviations in both areas are set to 0.0 Hz for each area. The Centralized discrete COQAGC controller Equation (4.21), acting on the two area state model with area control errors and the integrals of area control errors. The MATLAB simulation is set to 600 iterations with $k_0 = 0$ and $k_f = 600$.

Figures 4.19 and 4.20 show the frequency deviations of centralized discrete COQAGC controller with area control errors for area 1 and area 2, and tie-line power deviation. From the simulations, it can be observed that the frequencies deviations converge to zero in less than 3 seconds. It is noted that the frequency deviation overshoot associated with no oscillation before the signals return to zero.

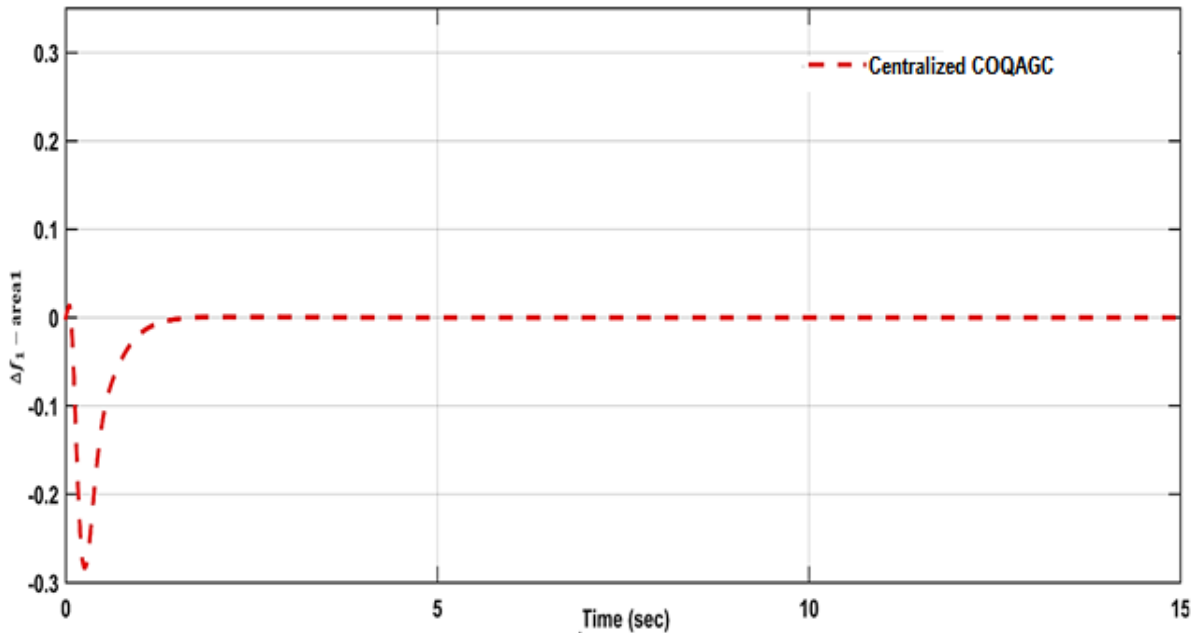


Figure 4.19. Frequency deviation in control area 1 without disturbances.

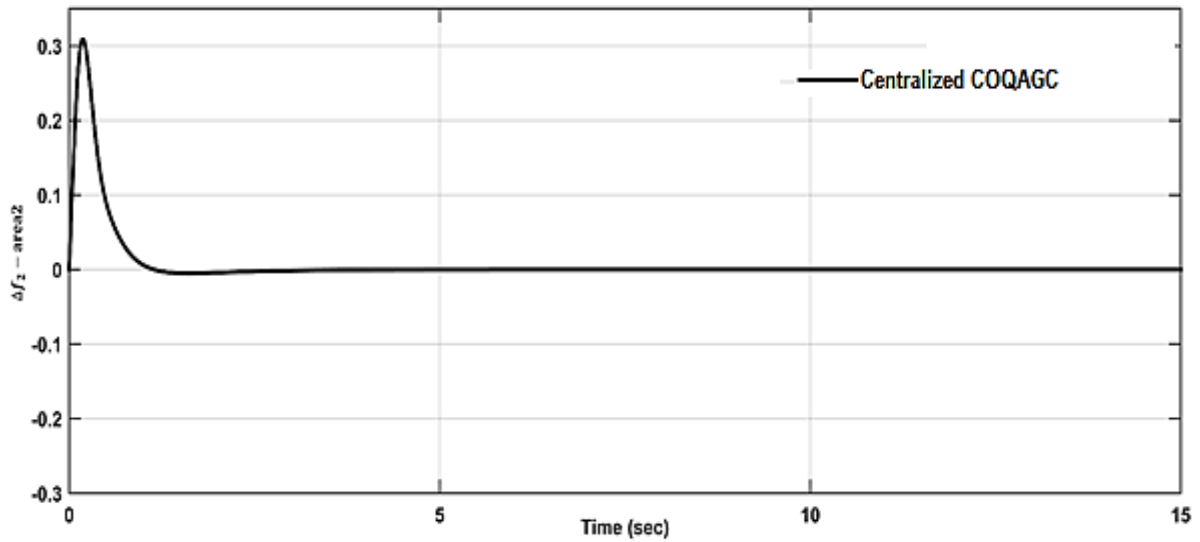


Figure 4.20. Frequency deviation in control area 2 without disturbances.

The centralized discrete COQAGC model without disturbance area provides that deviation of tie-line power between the two areas as shown in Figure 4.21. It is observed that the tie-line power deviation converges to zero in less than 2 seconds. Tie-line power deviation overshoots around the zero with oscillation before the signals return to zero.

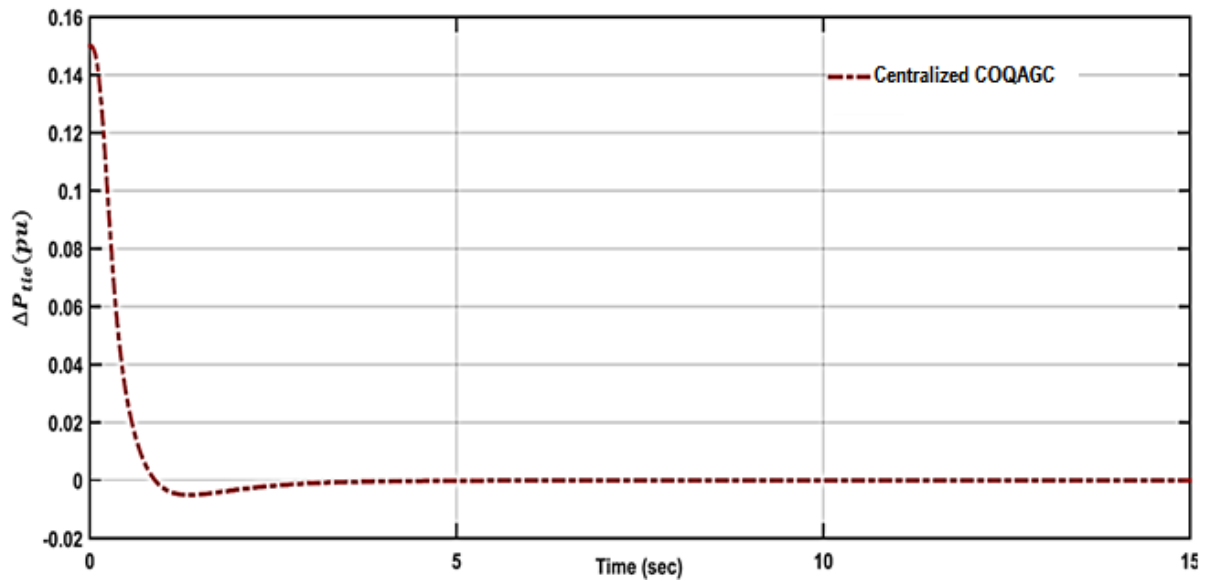


Figure 4.21. Tie-line power deviation-when no disturbance imposed.

Figure 4.22 shows area control errors for two area power system. It can be observed that the error control errors converge to zero in less than 5 seconds.

These simulation results revealed that the centralized discrete COQAGC controller with ACEs can bring the fluctuations associated with frequencies deviations, tie-line power deviation and area control errors to their nominal values in reasonable time as compared with the literature (Alrifai et al, 2011).

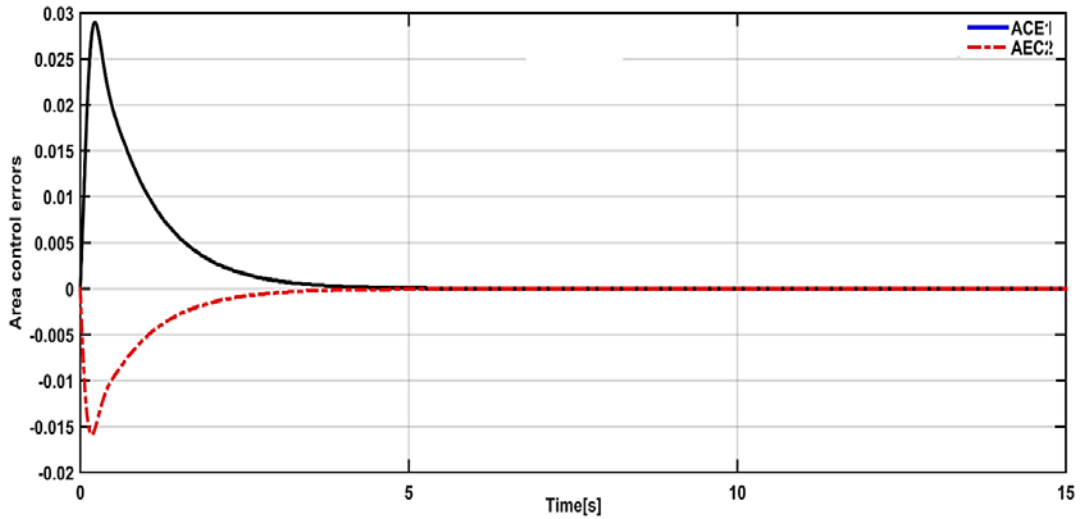


Figure 4. 22. Responses of the area control errors in both areas.

Case 2: Step load perturbations $w^T = [\Delta P_{D1} \ \Delta P_{D2}]^T$ with large magnitude are added simultaneously to each area. The initial tie-line power is set to 150 MW (0.15pu) and the initial frequency deviations are set to 0.0 Hz for each area. The centralized discrete COQAGC controller based on the functional minimization method is acted on the two area state model with area control errors and integrals of the area control errors. This section of the research work considered this mainly to test the feasibility of the developed controller. The MATLAB simulation is set to 600 iterations with $k_0 = 0$ and $k_f = 600$. The magnitude of the SLPs for the two areas are $\Delta P_{D1} = 100$ MW(0.1pu) and $\Delta P_{D2} = 100$ MW(0.1pu) respectively.

In this case, additional power system loads are added to the system at $t = 8$ second to show the day-day electricity usage case scenario (load demands can change at any second during the day). Figures 4.23 and 4.24 show the plots of the frequency deviations for area 1 and area 2 respectively with area control errors. It is observed that the frequency deviations converge to zero in less than 5 seconds. The centralized discrete COQAGC with ACEs demonstrates smaller undershoots and overshoots (oscillations) and faster disturbance rejection (robustness) in the frequency deviations Δf_1 and Δf_2 as compared to COQAGC regulator without area control errors.

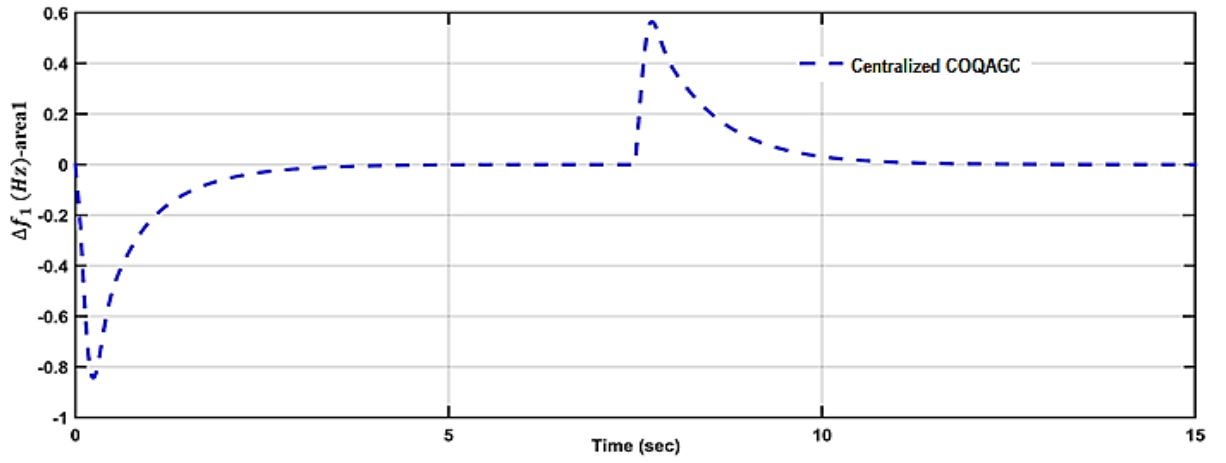


Figure 4.23. Frequency deviation area-1 with input disturbance.

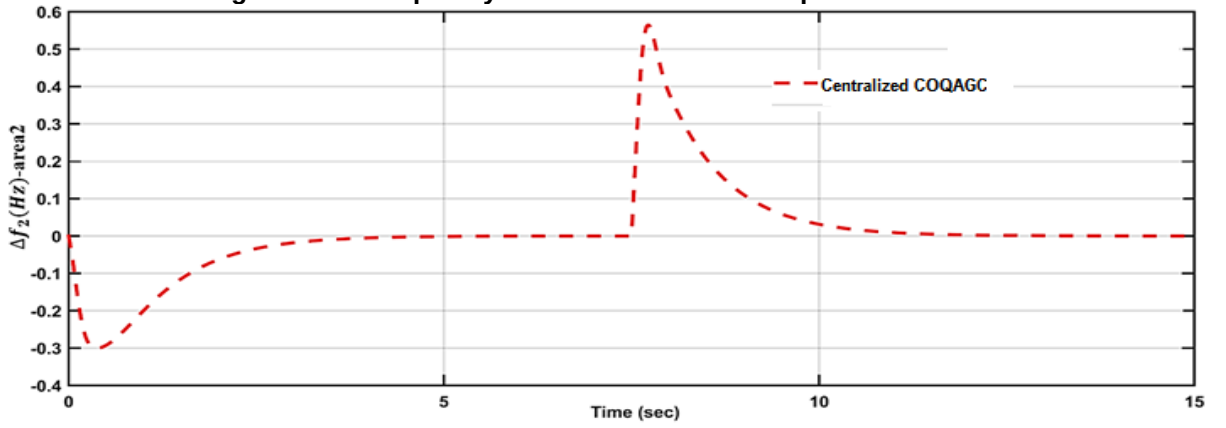


Figure 4.24. Frequency deviation area-2 with input disturbance.

The deviation in the tie-line power between the two areas is shown in Figure 4.25. The deviation converges to zero in less than 3 seconds with load disturbances added in the system at 8.0 seconds.

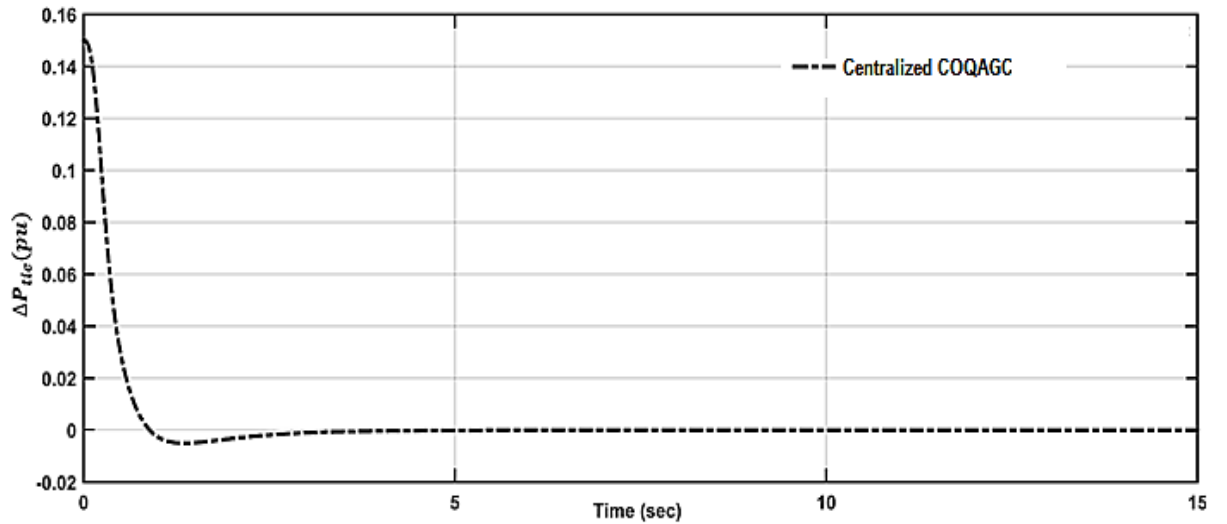


Figure 4.25. Tie-line power deviation-area-2.

Finally, Figure 4.26 shows the plot for area controls errors (ACE_1 and ACE_2) with load

disturbances added to the system at 8.0 seconds. It can be observed that area control errors converged to zero immediately after 10 seconds.

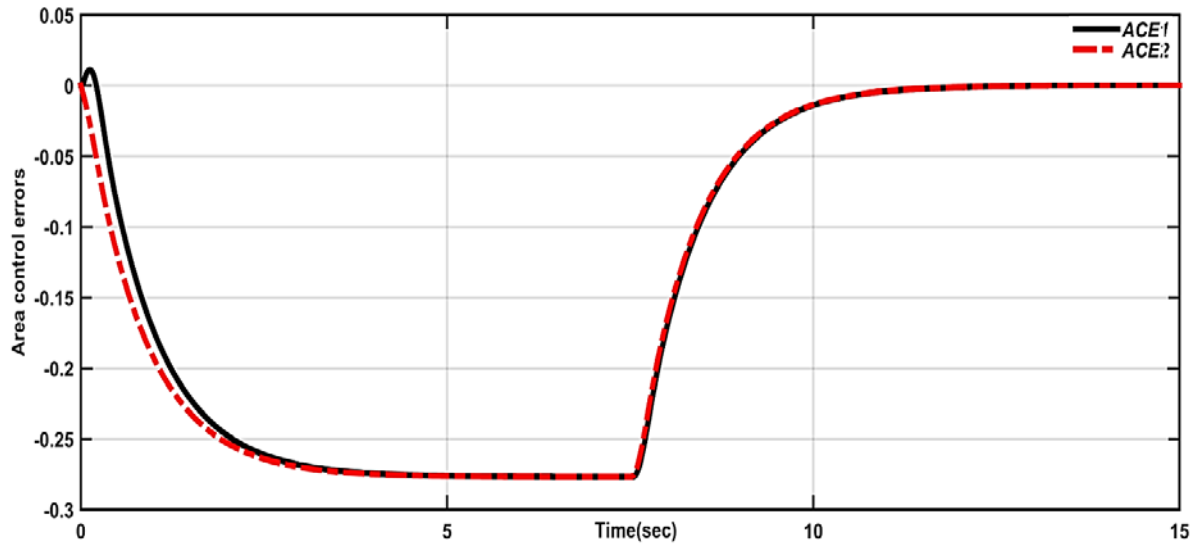


Figure 4.26. Responses of area control errors with simultaneous disturbances.

4.5.3 Comparative and analysis of the transition behaviour of the closed loop systems with and without area control errors using the developed optimal controllers

This section provides the comparative analysis of the closed system response with/without ACEs for the two area power systems. The parameter settings for the two cases are summarized in Table 4.3. From this table the initial parameters considered for both areas are as follows: Tie-line power deviation $\Delta P_{tie12} = 0.15$ pu and Turbine power $\Delta P_{ti} = 2$ pu. The frequency deviation $\Delta f = 50$ Hz is considered for the first case while the frequency deviation $\Delta f = 0.0$ Hz is considered for the second case. The input disturbance $\Delta P_{D1} = 0.1$ pu is considered for the first case, while $\Delta P_{D1} = 0.5$ pu is considered for the second case. Further, without area control errors in case 1 and with area control errors in case 2. A comparison is made between the responses of the centralized COQAGC regulator and Centralized discrete COQAGC controller with and without disturbances in terms of performance specifications in order to measure the robustness of the system against the disturbance rejection as shown in Figures 4.27-4.30. Table 4.4 shows the comparison of the frequency deviation for two cases with/without disturbances as well as without/with area control errors. From Figures 4.27-4.28, it can be observed that the frequency deviations performance with no disturbances and no ACEs outperform the one with disturbances when the comparison is made in terms of rising time, settling time, and peak time.

Table 4.3. Initial parameters settings for the two simulations cases.

Type of Controller	Settings Parameters						
	Case	Area	Δf	ΔP_{tie12}	ΔP_{ti}	AEC_i	ΔP_{Di}
COQAGC-Regulator with no disturbances	1.1	1	50HZ	0.15pu	2pu	0	0
		2	50HZ		2pu	0	0
COQAGC-Regulator- with disturbances	1.2	1	50HZ	0.15pu	2pu	0	$\Delta P_{D1} = 0.1pu$
		2	50HZ		2 pu	0	$\Delta P_{D2} = 0.1pu$
COQAGC- with no disturbances	2.1	1	0HZ	0.15pu	2pu	$\neq 0$	0
		2	0HZ		2pu	$\neq 0$	0
COQAGC- with disturbances	2.2	1	0HZ	0.15pu	2pu	$\neq 0$	$\Delta P_{D1} = 0.5pu$
		2	0Hz	0.15	2pu	$\neq 0$	$\Delta P_{D2} = 0.5pu$

Similar analysis is done for the Centralized discrete COQAGC controller response with/without the disturbance and with ACEs. It is seen from Figures 4.29 and 4.30, that the frequency deviations with no disturbances is much faster in terms of the settling time. It is therefore concluded that the ACEs added to the system enhanced the performance of the system but disturbances subjected to the system worsen the performance compared to the system with no disturbances and ACEs.

Table 4.4. performance of optimal AGC controllers without and with disturbances.

Frequency Deviations						
Type of Controller	Area	Rising time	Settling time	Overshoot	Peak time	Disturbance rejection response time
CQAGC-Regulator- with no disturbances	1	7 sec	10 sec	168.9%	0.22 sec	-
	2	10 sec	13 sec	300%	0.25 sec	-
COQAGC-Regulator- with disturbances	1	7.3 sec	12 sec	100%	0.25 sec	2.5sec
	2	10.3 sec	12.4 sec	300%	0.28 sec	3 sec
COQAGC with AECs-with no disturbances	1	0.9 sec	1 sec	28%	0.278 sec	-
	2	0.75 sec	1 sec	31%	0.2 sec	-
COQAGC with ACEs -- with disturbances	1	3 sec	3.5 sec	70%	0.225 sec	3.5 sec
	2	2 sec	3.5 sec	40%	0.3 sec	3.5 sec

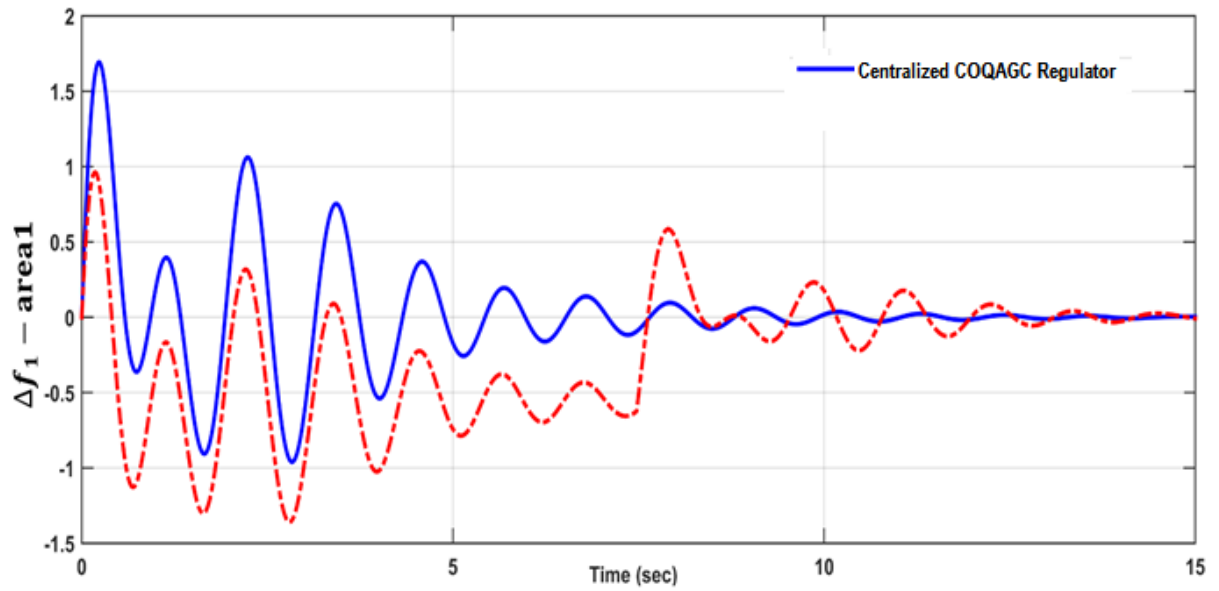


Figure 4.27. The closed-loop frequency deviation in area 1 for the case 1.

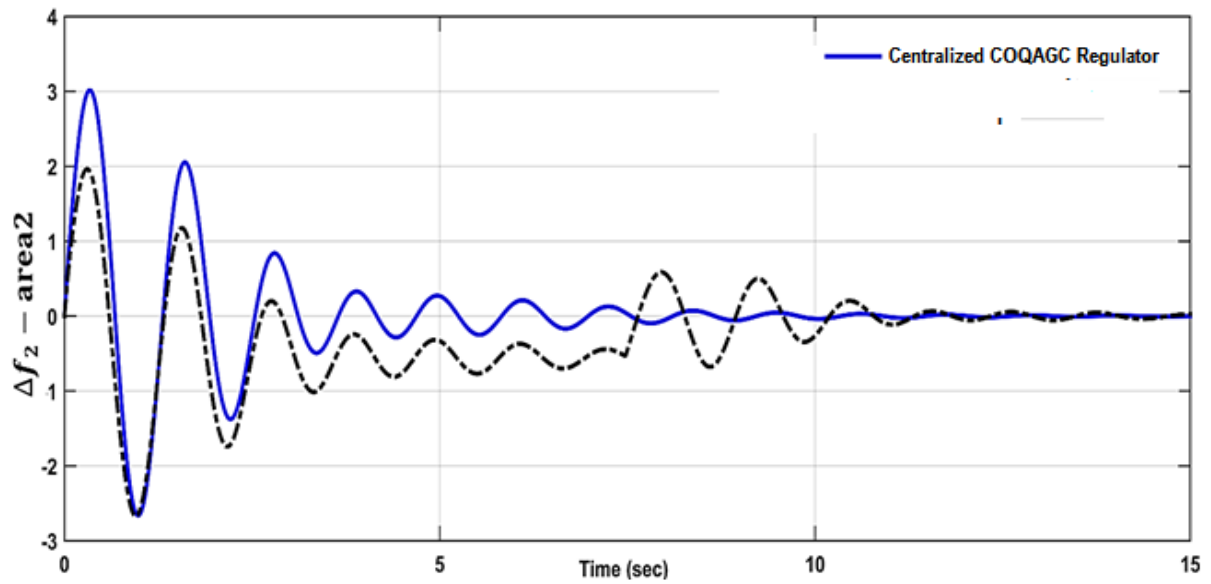


Figure 4-28. Figure 4.28: The closed loop response frequency deviation in area 2 for case 2

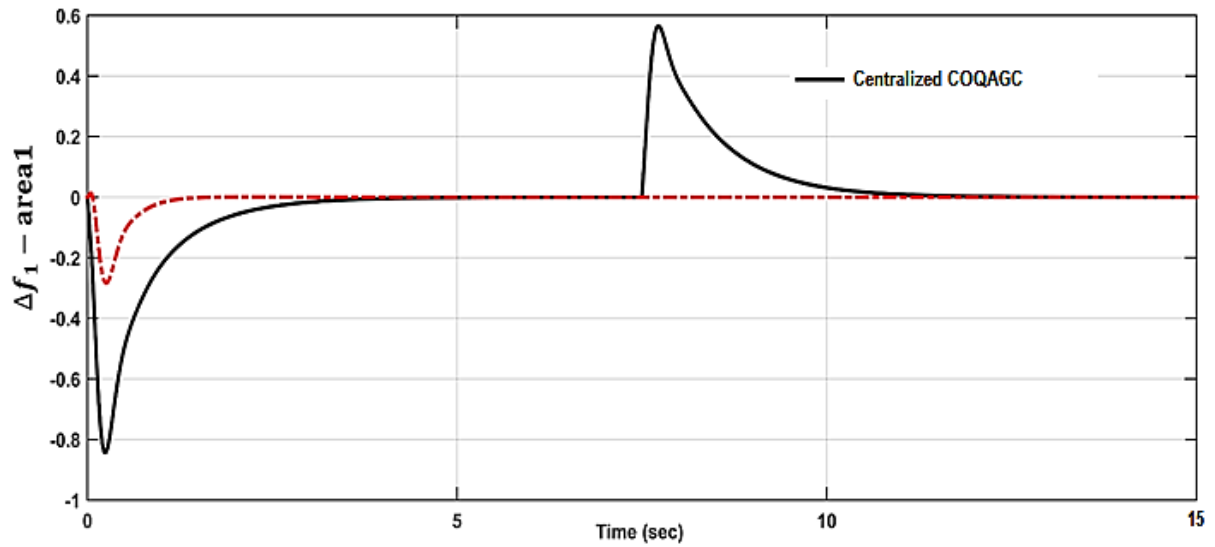


Figure 4- 29. The closed loop frequency deviation-area 1 with and without disturbances for the case 2

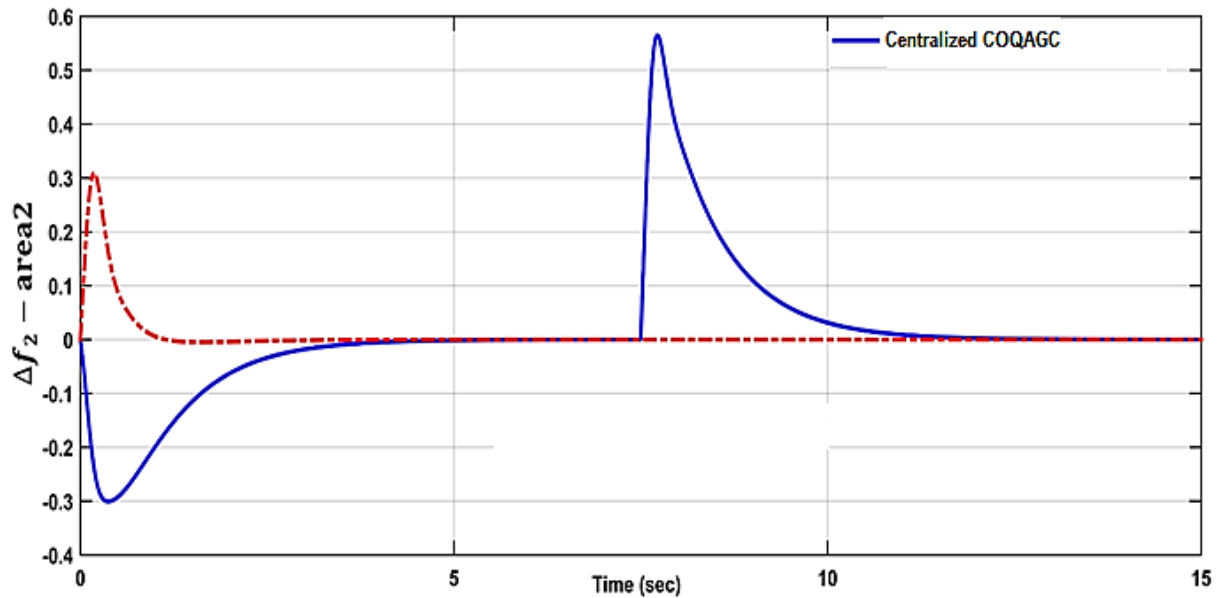


Figure 4.30. Figure 4.30. The closed-loop frequency deviation area 2 with and without disturbances for the case 2.

The results illustrate that Centralized discrete COQAGC controller with FMM provides good dynamics performance over the COQAGC regulator in terms of rising time, settling time, and overshoot. Both controllers show good disturbance rejection time response but centralized COQAGC-with ACEs have shorter disturbance rejection time response compared to COQAGC Regulator with no ACEs

Further, Table 4.5 shows the comparison of tie-line power deviation for the two cases without and with disturbances as well as without and with area control errors. The results thus obtained through the simulation depicts that COQAGC controller with FMM holds a good

performance in comparison with COQAGC -regulator performance characteristics which includes rising time, settling time overshoot and disturbance rejection time response respectively.

Table 4.5. Performance of Optimal AGC controllers without/with disturbances for Tie line power deviations.

Tie-line power deviations (pu)					
Type of Controller	Rising time	Settling time	Overshoot	Peak time	Disturbance rejection response time
COQAGC-Regulator with no disturbances	6 sec	7 sec	117%	1.25 sec	NA
COQAGC -Regulator- with disturbances	6 sec	7 sec	67%	1.25sec	2.5 sec
COQAGC- Controller using FMM with no disturbances	0.8 sec	1 sec	2 %	1.25 sec	NA
COQAGC- Controller using FMM with disturbances	0.8 sec	1 sec	2 %	1.25 sec	0 sec

The simulation results of the tie line with/without disturbances as well as without/with ACES can be seen in Figures 4.31 and 4,32 respectively. From Figure 4.31 it can be seen that the percentages overshoots, settling time and rising time, there are no changes in the tie-line responses without/with disturbances. On the other hand, from Figure 4.32, it can be observed that the results of the overshoot, rising time and settling time with/with disturbances and there is no ACEs are associated with underdamped oscillation with oscillation frequency 6.8 Hz which shows that one of the poles are located on the imaginary axis of the complex plane. The system closed-loop oscillation is increased while the overshoot is decreased when the power demand is subjected to system.

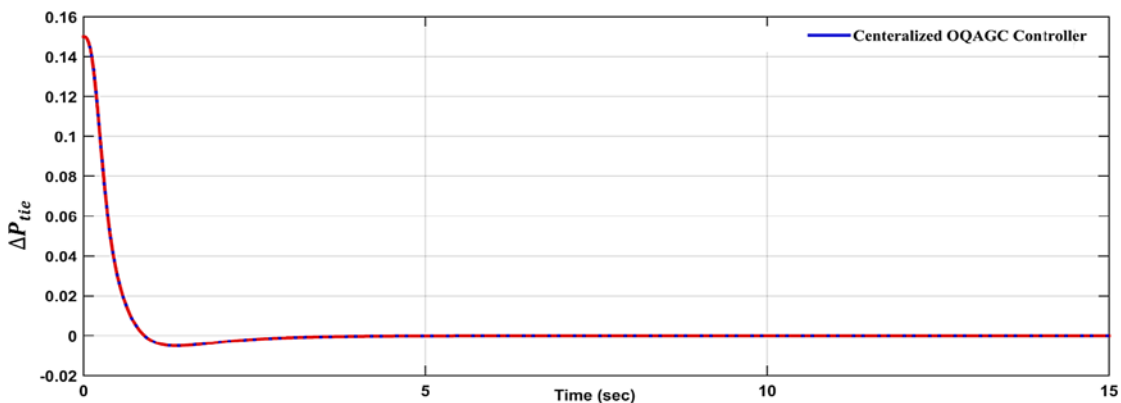


Figure 4.31. Tie line closed-loop responses a) black with disturbances, b) Blue with no disturbances for case 2.

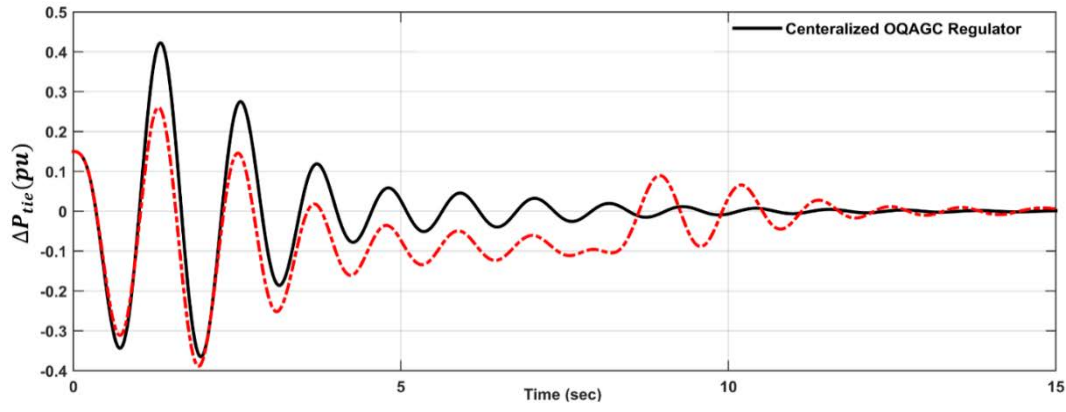


Figure 4- 32. Tie-line closed loop responses a) black with no disturbances, b) red with disturbances for case1

4.6 Case Study: Design of a Discrete COQAGC for Four Area Power Systems

4.6.1. Dynamics of four Area Interconnected Power System

In this section, a four area interconnected power system (FAIPS) is used as a case study for the analysis of discrete COQAGC (see Figure 5.33). FAIPS is made up of non-reheat, reheat, hydro, and gas turbines. Each control area is made up of one of these four generators. The four generator models are based on work given by (Paramar, Majhi & Kothari, 2012 and Kunder, 1994). Similarly, to assure zero steady error, the integral of area control errors is added to the block diagram. The integral of area control error is known as the integral controller in current linear control nomenclature. The optimal control rule will be established using either a full-state feedback method (a linear collection of all system states or an output feedback approach in recent optimization techniques. As a result, the later technique presupposes, as in this study, that all states of a complex interconnected power system are available.

Based on the principles of load flow equations, the linear model of a tie-line for four area power system can be developed as shown in Figure 4.34. This technique assumes that the tie-line resistance is negligible because of the high value of the inductance between the line, jX_{ij} . It is also assumed that the current I_{ij} flows from control area 1 to control area 2, control area 2 to control area 3, control area 3 to control area 4, control area 4 to control area 1 and that the resistance of the tie-line transmission is zero. The P_{r1} , P_{r2} , P_{r3} , and P_{r4} are the rated power capacities of the control areas 1, 2, 3 and 4 respectively.

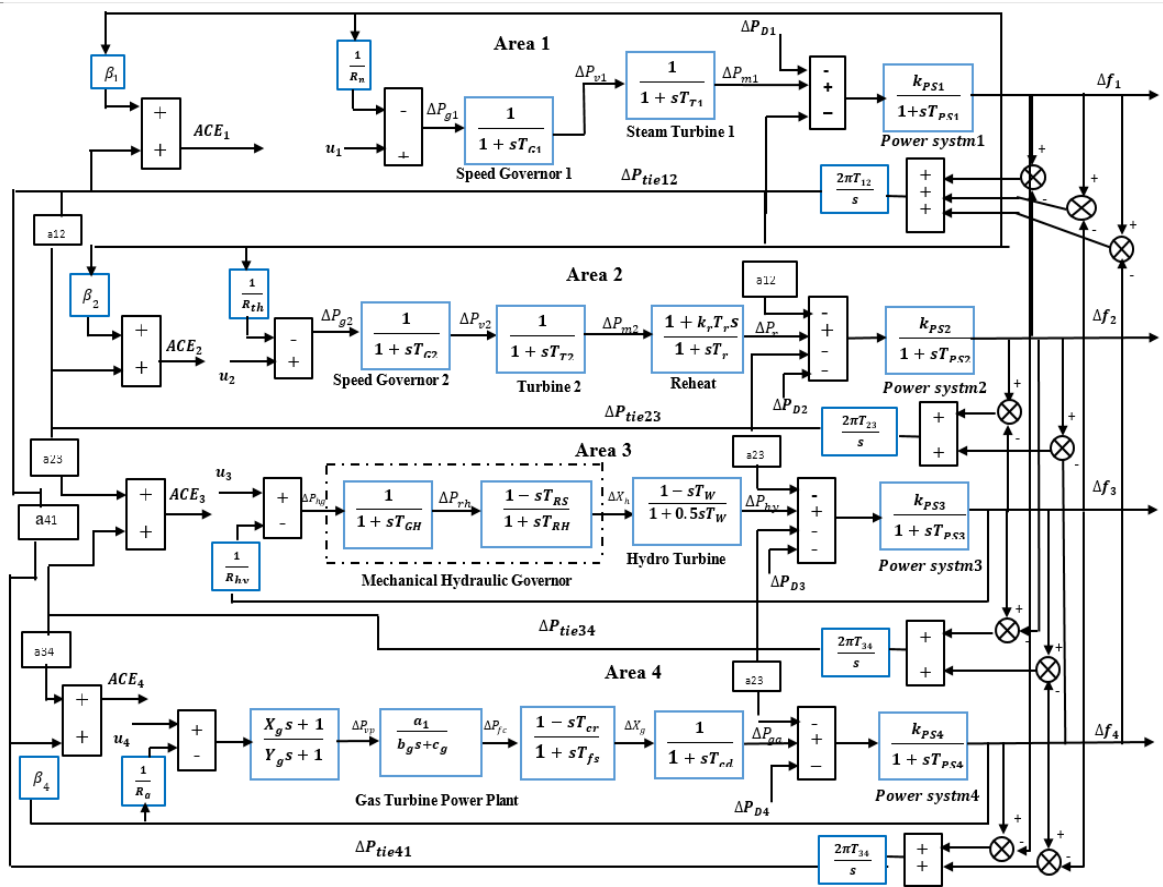


Figure 4.33. FAIPS with four integrated area control errors (Esmail & Krishnamurthy, 2023).

Then, the linear model of the Tie-line power deviations between areas (1 and 2), (2 and 3), (3 and 4) and (4 and 1) can be obtained as follows:

$$\begin{aligned}
 \Delta P_{tie12}(\text{pu}) &= \frac{2\pi T_{12}}{s} [3\Delta f_1 - \Delta f_2 - \Delta f_3 - \Delta f_4] \\
 \Delta P_{tie23}(\text{pu}) &= \frac{2\pi T_{23}}{s} [2\Delta f_2 - \Delta f_3 - \Delta f_4] \\
 \Delta P_{tie34}(\text{pu}) &= \frac{2\pi T_{34}}{s} [2\Delta f_3 - \Delta f_2 - \Delta f_4] \\
 \Delta P_{tie41}(\text{pu}) &= \frac{2\pi T_{41}}{s} [2\Delta f_4 - \Delta f_1 - \Delta f_3]
 \end{aligned} \tag{4.79}$$

Assuming the power system under study is a four area power systems i.e. $N = 4$ as illustrated in Figure 4.34. The state-space model of interconnected power system can be described as follows (Rakhshani, Remo & Mir, 2015, Deepak & Abraham, 2015):

$$\begin{aligned}
 \dot{x}(t) &= Ax(t) + Bu(t) + \Gamma w(t) \\
 y &= Cx(t) + Du(t)
 \end{aligned} \tag{4.80}$$

where $x(t) \in \mathbb{R}^{n \times 1}$ is the state vector, $u(t) \in \mathbb{R}^{2 \times 1}$ is the control input vector, and $w(t) \in \mathbb{R}^{2 \times 1}$ is the load disturbances input vector, whereas the matrices A , B , Γ , C and D have the appropriate dimensions. Equation (4.55) describes the variable vectors of a linear state-

space model for a four-area control power system as follows:

$$\begin{aligned}
 & \mathbf{x}^T \\
 & [\Delta f_1 \ \Delta P_{T1} \ \Delta P_{G1} IACE_1 \Delta P_{tie12} \ \Delta f_2 \ \Delta P_{T2} \ \Delta P_{TR} \ \Delta P_{G2} IACE_2 \Delta P_{tie23} \ \Delta f_3 \ \Delta P_{TW} \\
 & \mathbf{u}^T = [u_1 u_2 \ u_3 u_4]^T \\
 & \mathbf{w}^T = [\Delta P_{d1} \Delta P_{d2} \Delta P_{d3} \Delta P_{d4}]^T \\
 & \mathbf{y}^T = [\Delta f_1 \Delta f_2 \Delta f_3 \Delta f_4 \Delta P_{tie12} \Delta P_{tie23} \Delta P_{tie34} \Delta P_{tie41}]^T
 \end{aligned} \tag{4.81}$$

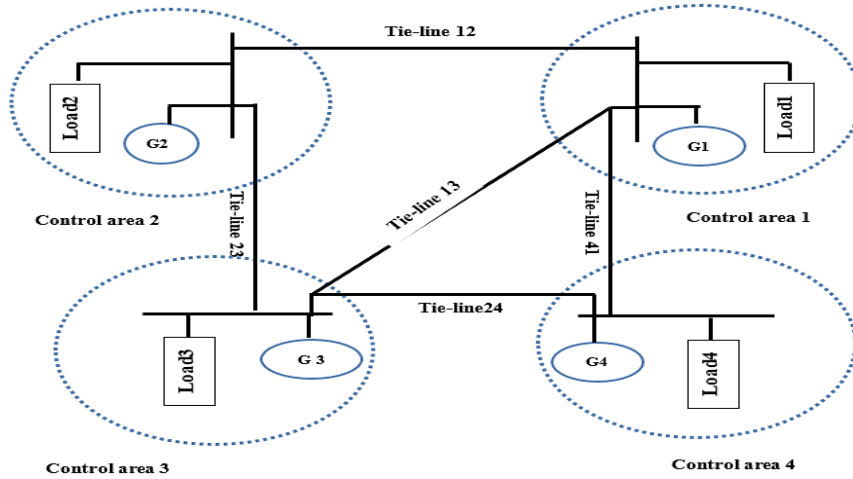


Figure 4 34. Proposed four area interconnected power system with tie-lines.

The matrices A, B, C, and D of the four interconnected power systems are shown below. These matrices were constructed by converting transfer functions into a set of first-order differential equations. The concept of modeling the interconnected power system as a state-space model can be found in the studies developed and represented by authors in the literature (Rakhshani, Remo, & Mir, 2015, Deepak & Abraham, 2015).

4.6.2. State Space Model of FAIPS

Multi area power system shown in Figure 4.33 with twenty-four state variables, defined as shown in Table 4.6.

Table 4.6. FAIPS variables and their corresponding states.

Variables	Corresponding states	Physical meaning
Δf_1	x_1	Frequency deviation for the 1 st area (Hz)
ΔP_{t1}	x_2	The power output deviation of the turbine for the first area (pu.MW).
ΔP_{g1}	x_3	The governor valve position deviation of the turbine for the first control area (pu.MW)
$IACE_1$	x_4	The first area's integral of area control error
ΔP_{tie12}	x_5	Tie-line power deviation between control areas 1 and 2
Δf_2	x_6	Frequency deviation for the 2 nd area (Hz).
ΔP_r	x_7	deviation in intermediate state of reheat turbine for second control area.
ΔP_{t2}	x_8	The deviation in intermediate state of reheater of reheat turbine
ΔP_{g2}	x_9	The governor valve position deviation of the turbine for the
$IACE_2$	x_{10}	The second area's integral of area control error
ΔP_{tie23}	x_{11}	Tie-line power deviation between control areas 2 and 3
Δf_3	x_{12}	Frequency deviation for the 3 rd area (Hz).
ΔP_{hv}	x_{13}	deviation in hydro unit output (pu).
ΔX_h	x_{14}	deviation in intermediate state of mechanical hydraulic governor
ΔP_{rh}	x_{15}	Deviation in intermediate state of hydro turbine governor for third area.
$IACE_3$	x_{16}	The third area's integral of area control error
ΔP_{tie34}	x_{17}	Tie-line power deviation between control areas 3 and 4
Δf_4	x_{18}	Frequency deviation for the fourth area (Hz)
ΔP_{ga}	x_{19}	Deviation in gas unit output –area four
ΔX_g	x_{20}	Deviation in intermediate state of gas turbine speed governor
ΔP_{gh}	x_{21}	Deviation in intermediate state of fuel system and combustor of
ΔP_{vp}	x_{22}	deviation in valve positioner of gas turbine
$IACE_4$	x_{23}	The fourth area's integral of area control error
ΔP_{tie41}	x_{24}	Tie-line power deviation between control areas 4 and 1

where, the disturbance vector is defined as $w_1 = \Delta P_{D1}$, $w_2 = \Delta P_{D2}$, $w_3 = \Delta P_{D3}$, and $w_4 = \Delta P_{D4}$. Following the description of all power system states, input control variables, and input disturbance variables as shown in Figure 4.33, the first ordinary differential equations (ODEs) for the four area power system are derived using the same method discussed in section 3.2

(Parmar, Majhi and Kothari, 2012) as follows:

$$\dot{x}_1 = -\frac{1}{T_{p1}}x_1 + \frac{k_{ps1}}{T_{p1}}x_2 + \frac{k_{ps1}}{T_{p1}}x_5 - \frac{k_{ps1}}{T_{p1}}\Delta P_{D1} \quad (4.82)$$

$$\dot{x}_2 = -\frac{1}{T_{t1}}x_2 + \frac{1}{T_{t1}}x_3 \quad (4.83)$$

$$\dot{x}_3 = -\frac{1}{T_{g1}R_n}x_1 - \frac{1}{T_{g1}}x_3 + \frac{\alpha_1}{T_{g1}}x_4 + \frac{\alpha_1}{T_{g1}}u_1 \quad (4.84)$$

$$\dot{x}_4 = \beta_1x_1 + x_5 - x_{24} \quad (4.85)$$

$$\dot{x}_5 = 2 * \pi T_{12}(x_1 - x_6 - x_{12} - x_{18}) \quad (4.86)$$

$$\dot{x}_6 = -\frac{k_{ps2}}{T_{p2}}x_5 - \frac{1}{T_{p2}}x_6 + \frac{k_{ps2}}{T_{p2}}x_7 + \frac{k_{ps2}}{T_{p2}}x_{11} - \frac{k_{ps2}}{T_{p2}}\Delta P_{D2} \quad (4.87)$$

$$\dot{x}_7 = -\frac{1}{T_{r1}}x_7 + \left(\frac{1}{T_{r1}} - \frac{k_{r1}}{T_{t2}}\right)x_8 + \frac{1}{T_{t2}}x_9 \quad (4.88)$$

$$\dot{x}_8 = -\frac{1}{T_{t2}}x_8 + \frac{1}{T_{t2}}x_9 \quad (4.89)$$

$$\dot{x}_9 = -\frac{1}{T_{g2}R_{th}}x_6 - \frac{1}{T_{g2}}x_9 + \frac{\alpha_2}{T_{g2}}u_2 \quad (4.90)$$

$$\dot{x}_{10} = \beta_2x_6 + x_{11} - x_5 \quad (4.91)$$

$$\dot{x}_{11} = 2 * \pi T_{12}(x_6 - x_{12} - x_{18}) \quad (4.92)$$

$$\dot{x}_{12} = -\frac{k_{ps3}}{T_{p3}}x_{11} - \frac{1}{T_{p3}}x_{12} + \frac{k_{ps3}}{T_{p3}}x_{13} + \frac{k_{ps3}}{T_{p3}}x_{17} - \frac{k_{ps3}}{T_{p3}}\Delta P_{D3} \quad (4.93)$$

$$\begin{aligned} \dot{x}_{13} = & \frac{2T_{rs}}{T_{gh}T_{rh}R_{hy}}x_{12} - \frac{2}{T_w}x_{13} + \left(\frac{2}{T_w} + \frac{2}{T_{rh}}\right)x_{14} - 2\left(\frac{1}{T_{rh}} - \frac{T_{rs}}{T_{gh}T_{rh}}\right)x_{15} + \\ & - \frac{2\alpha_3T_{rs}}{T_{rh}T_{gh}}x_{16} - \frac{2T_{rs}}{T_{rh}T_{gh}}u_3 \end{aligned} \quad (4.94)$$

$$\dot{x}_{14} = -\frac{T_{rs}}{T_{rh}T_{gh}R_{hy}}x_{12} - \frac{1}{T_{rh}}x_{14} + \left(\frac{1}{T_{rh}} - \frac{T_{rs}}{T_{rh}T_{gh}}\right)x_{15} + \frac{\alpha_3T_{rs}}{T_{rh}T_{gh}}x_{16} + \frac{T_{rs}}{T_{rh}T_{gh}}u_3 \quad (4.95)$$

$$\dot{x}_{15} = -\frac{1}{T_{gh}R_{hy}}x_{12} - \frac{1}{T_{gh}}x_{15} + \frac{\alpha_3}{T_{gh}}x_{16} + \frac{1}{T_{gh}}u_3 \quad (4.96)$$

$$\dot{x}_{16} = -x_{11} + x_{17} + \beta_3x_{12} \quad (4.97)$$

$$\dot{x}_{17} = 2 * \pi T_{23}(x_{12} - x_6 - x_{18}) \quad (4.98)$$

$$\dot{x}_{18} = -\frac{k_{ps4}}{T_{p4}}x_{17} - \frac{1}{T_{p4}}x_{18} + \frac{k_{ps4}}{T_{p4}}x_{19} + \frac{k_{ps4}}{T_{p4}}x_{24} - \frac{k_{ps4}}{T_{p4}}\Delta P_{D4} \quad (4.99)$$

$$\dot{x}_{19} = -\frac{1}{T_{cd}}x_{19} + \frac{1}{T_{cd}}x_{20} \quad (4.100)$$

$$\begin{aligned} \dot{x}_{20} = & -\frac{T_{CR}X_g}{T_f b_g R_g Y_g}x_{18} - \frac{1}{T_f}x_{20} + \left(\frac{1}{T_f} - \frac{T_{CR}}{T_f Y_g}\right)x_{21} + \frac{T_{CR}}{T_f}\left(\frac{1}{Y_g} - \frac{X_g c_g}{b_g Y_g}\right)x_{22} \\ & + \frac{\alpha_4 T_{CR} X_g}{b_g T_f Y_g}x_{23} + \frac{T_{CR} X_g}{b_g T_f Y_g}u_4 \end{aligned} \quad (4.101)$$

$$\dot{x}_{21} = -\frac{X_g}{Y_g b_g R_g}x_{18} - \frac{1}{Y_g}x_{21} + \left(\frac{1}{Y_g} - \frac{X_g c_g}{b_g Y_g}\right)x_{22} + \frac{\alpha_4 X_g}{b_g Y_g}x_{23} + \frac{X_g}{b_g Y_g}u_4 \quad (4.102)$$

$$\dot{x}_{22} = -\frac{1}{b_g R_g}x_{18} - \frac{c_g}{b_g}x_{22} + \frac{\alpha_4}{b_g}x_{23} + \frac{1}{b_g}u_4 \quad (4.103)$$

$$\dot{x}_{23} = -x_{17} + \beta_4 x_{18} + x_{24} \quad (4.104)$$

$$\dot{x}_{24} = 4\pi T_{23}x_{24} - 2\pi T_{23}x_{17} - 2\pi T_{23}x_3 \quad (4.105)$$

The first order differential equations (4.82-5.105) for FAIPS may be written as an interaction-oriented state space Equation, as explained in Chapter 3. (3.26). The corresponding vectors and matrices are as indicated in Figure 4.33:

$$\text{state vector, } x^T = [x_1 x_2 x_3 x_4 \cdots x_{24}]^T$$

$$\text{control vector, } u^T = [u_1 u_2 \quad u_3 u_4]^T \quad (4.106)$$

$$\text{disturbance vector, } w^T = [w_1 w_2 \quad w_3 w_4]^T = [\Delta P_{d1} \Delta P_{d2} \Delta P_{d3} \Delta P_{d4}]^T$$

$$\text{output vector, } y^T = [x_1 x_6 x_{12} \quad x_{18} x_5 x_{11} x_{17} x_{24}]^T$$

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$$B^T = \begin{bmatrix} 0 & 0 & \frac{\alpha_1}{T_{g1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\alpha_2}{T_{g2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\alpha_3}{T_{rh} T_{gh}} & \frac{\alpha_3}{T_{rh} T_{gh}} & \frac{\alpha_3}{T_{gh}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha_4 X_g}{b_g Y_g} & \frac{\alpha_4}{b_g} u_4 & 0 & 0 \end{bmatrix}$$

$$\Gamma^T = \begin{bmatrix} -\frac{k_{ps1}}{T_{p1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{k_{ps2}}{T_{p2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{ps3}}{T_{p3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{ps4}}{T_{p4}} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D = 0$$

4.6.3. Design of Centralized COQAGC for Four Area Interconnected Power System

The procedures that are described in sections 4.2 and 4.3 are used to design Centralized Discrete Optimal Quadratic AGC (COQAGC) controllers for the four control areas. Based on Equation (4.28), the cost function for four areas is

$$\begin{aligned}
 J = & \frac{1}{2} \sum_{k=k_0}^{\infty} \{ (AEC_1)^2 + (AEC_2)^2 + (AEC_3)^2 + (AEC_4)^2 \} \\
 & + \left\{ \left(\sum_{k=k_0}^{\infty} AEC_1 \right)^2 + \left(\sum_{k=k_0}^{\infty} AEC_2 \right)^2 + \left(\sum_{k=k_0}^{\infty} AEC_3 \right)^2 \right. \\
 & \left. + \left(\sum_{k=k_0}^{\infty} AEC_4 \right)^2 \right\} + \{ u_1^2 + u_2^2 + u_3^2 + u_4^2 \}
 \end{aligned} \tag{4.107}$$

According to Equations (4.29 and 4.30), the area control errors (ACE_1 , ACE_2 , ACE_3 and ACE_4) and the integral of area control errors ($IACE_1$, $IACE_2$, $IACE_3$ and $IACE_4$) of the four control areas are defined as follows:

$$\begin{aligned}
 ACE_1 &= \beta_1 \Delta f_1 + \Delta P_{tie1} \\
 ACE_2 &= \beta_2 \Delta f_2 + \Delta P_{tie2} \\
 ACE_3 &= \beta_3 \Delta f_3 + \Delta P_{tie3} \\
 ACE_4 &= \beta_4 \Delta f_4 + \Delta P_{tie4}
 \end{aligned} \tag{4.108}$$

and

$$\begin{aligned}
 IACE_1 &= \sum_{k=k_0}^{\infty} (\beta_1 \Delta f_1 + \Delta P_{tie1}) \\
 IACE_2 &= \sum_{k=k_0}^{\infty} (\beta_2 \Delta f_2 + \Delta P_{tie2}) \\
 IACE_3 &= \sum_{k=k_0}^{\infty} (\beta_3 \Delta f_3 + \Delta P_{tie3}) \\
 IACE_4 &= \sum_{k=k_0}^{\infty} (\beta_4 \Delta f_4 + \Delta P_{tie4})
 \end{aligned} \tag{4.109}$$

Where ΔP_{tie1} , ΔP_{tie2} , ΔP_{tie3} and ΔP_{tie4} are the sum of the tie-line deviations for the four areas respectively. From the block diagram shown in Figure 4.34, the sum of tie-line power deviations in area 1, area 2, area 3 and area 4 respectively are obtained as follows

$$\begin{aligned}\Delta P_{tie1} &= \Delta P_{tie12} + a_{34}\Delta P_{tie34} + a_{41}\Delta P_{tie41} \\ \Delta P_{tie2} &= a_{12}\Delta P_{tie12} + \Delta P_{tie34} \\ \Delta P_{tie3} &= a_{23}\Delta P_{tie23} + \Delta P_{tie34} \\ \Delta P_{tie4} &= a_{34}\Delta P_{tie34} + \Delta P_{tie41}\end{aligned}\tag{4.110}$$

Substituting ACE_1 , ACE_2 , ACE_3 , and ACE_4 and $IACE_1$, $IACE_2$, $IACE_3$, and $IACE_4$) from Equations (4.108 and 4.109) to Equation (4.107), the cost functional can be given by:

$$\begin{aligned}J = \frac{1}{2} \sum_{k=k_0}^{\infty} & ((\beta_1 \Delta f_1 + \Delta P_{tie1})^2 + (\beta_2 \Delta f_2 + \Delta P_{tie2})^2 + ((\beta_3 \Delta f_3 + \Delta P_{tie3})^2 \\ & + ((\beta_4 \Delta f_4 + \Delta P_{tie4})^2) \\ & + \{(IACE_1)^2 + (IACE_2)^2 + (IACE_3)^2 + (IACE_4)^2\} \\ & + \rho\{u_1^2 + u_2^2 + u_3^2 + u_4^2\})\end{aligned}\tag{4.111}$$

$\beta_1, \beta_2, \beta_3$ and β_4 are the frequency bias for the four area power system respectively, ΔP_{tie12} , ΔP_{tie23} , ΔP_{tie34} and ΔP_{tie41} are the tie-line deviations between areas 1 and 2, areas 2 and 3, areas 3 and 4 and areas 4 and 1 respectively and $a_{12} = a_{23} = a_{34} = a_{41} = -1$, is the constant coefficient that changes the sign of the tie-line power from one area to another. Hence, the vectors x for four-area interconnected power systems with one type of generation unit in each area according to equation (4.35) is defined as follows

$$x^T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} \\ x_{15} & x_{16} & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} & x_{22} & x_{23} & x_{24} \end{bmatrix}^T\tag{4.112}$$

Where x_4 , x_{10} , x_{16} and x_{23} are the integral of the Area control errors of areas 1, 2, 3 and 4 respectively. It can be observed that the functional of the cost function in Equation (4.107) can be presented in the form of Equation (4.109), where $f(\cdot)$ is a functional of area control errors, integral of area control errors and control efforts. By taking the partial derivatives of this function with respect to state variables as in Equations (4.111) and control input signals

in Equation (4.112), then, the matrices Q_k and R_k can be derived based on the minimization approach of the function as follows:

1. Rewrite and extend the functional $f(\cdot)$ as follows:

$$\begin{aligned} f(\cdot) = & \frac{1}{2} (\{[(\beta_1 \Delta f_1)^2 + 2\beta_1 \Delta f_1 \Delta P_{tie1} + \Delta P_{tie1}^2] \\ & + [(\beta_2 \Delta f_2)^2 + 2\beta_2 \Delta f_2 \Delta P_{tie2} + (\Delta P_{tie2})^2 + (\beta_3 \Delta f_3)^2 + 2\beta_3 \Delta f_3 \Delta P_{tie3} \\ & + (\Delta P_{tie3})^2 + (\beta_4 \Delta f_4)^2 + 2\beta_4 \Delta f_4 \Delta P_{tie4} + (\Delta P_{tie4})^2]\} \\ & + \{(IACE_1)^2 + (IACE_2)^2 + (IACE_3)^2 + (IACE_4)^2\} \\ & + \rho\{u_1^2 + u_2^2 + u_3^2 + u_4^2\}) \end{aligned} \quad (4.113)$$

where ρ is the participation factor. In this this paper we considered it 1 ($\rho = 1$) since we used only one type of generator in each area.

2. Define the state variables i.e. $x_1 = \Delta f_1$, $x_2 = \Delta P_{T1}$, $x_3 = \Delta P_{G1}$, $x_4 = IACE_1$, $x_5 = \Delta P_{tie12}$, $x_6 = \Delta f_2$, $x_7 = \Delta P_{r1}$, $x_8 = \Delta P_{T2}$, $x_9 = \Delta P_{G2}$, $x_{10} = IACE_2$, $x_{11} = \Delta P_{tie23}$, $x_{12} = \Delta f_3$, $x_{13} = \Delta P_{pen}$, $x_{14} = \Delta P_{HT}$, $x_{15} = \Delta P_{Gh}$, $x_{16} = IACE_3$, $x_{17} = \Delta P_{tie34}$, $x_{18} = \Delta f_4$, $x_{19} = \Delta P_{GT}$, $x_{20} = \Delta P_{GTF}$, $x_{21} = \Delta P_{GTG}$, $x_{22} = \Delta P_{VP}$, $x_{23} = IACE_4$, $x_{24} = \Delta P_{tie41}$.
3. Then substitute them into Equation (4.111) without touching the tie-line variables which becomes

$$\begin{aligned} f(\cdot) = & \frac{1}{2} (\{[(\beta_1 x_1)^2 + 2\beta_1 x_1 \Delta P_{tie1} + \Delta P_{tie1}^2] \\ & + [(\beta_2 x_6)^2 + 2\beta_2 x_6 \Delta P_{tie2} + (\Delta P_{tie2})^2 + (\beta_3 x_{12})^2 + 2\beta_3 x_{12} \Delta P_{tie3} \\ & + (\Delta P_{tie3})^2 + (\beta_4 x_{18})^2 + 2\beta_4 x_{18} \Delta P_{tie4} + (\Delta P_{tie4})^2]\} \\ & + \{(x_4)^2 + (x_{10})^2 + (x_{16})^2 + (x_{23})^2\} \\ & + \rho\{u_1^2 + u_2^2 + u_3^2 + u_4^2\}) \end{aligned} \quad (4.114)$$

4. Define the variables of the tie-lines power exchanges ΔP_{tie1} , ΔP_{tie2} , ΔP_{tie3} and ΔP_{tie4} from equation (4.106) and Figure 4.34 as $\Delta P_{tie1} = x_5 - x_{17} - x_{24}$, $\Delta P_{tie2} = -x_5 + x_{11}$, $\Delta P_{tie3} = -x_{11} + 2x_{17}$ and $\Delta P_{tie4} = -x_{17} + x_{24}$. Then substituting them into equation (4.109) which gives:

$$\begin{aligned}
f(.) = \frac{1}{2} \{ & [\beta_1^2 x_1^2 + 2\beta_1 x_1 (x_5 - x_{17} - x_{24}) + x_5^2 + x_{17}^2 + x_{24}^2 - 2x_5 x_{17} - 2x_5 x_{24} \\
& + 2x_{17} x_{24}] \\
& + [\beta_2^2 x_6^2 + 2\beta_2 x_6 (-x_5 + x_{11}) + x_5^2 - 2x_5 x_{11} + x_{11}^2] + [\beta_3^2 x_{12}^2 \\
& + 2\beta_3 x_{12} (-x_{11} + x_{17}) + x_{11}^2 - 4x_{11} x_{17} + 4x_{17}^2] + [\beta_4^2 x_{18}^2 \\
& + 2\beta_4 x_{18} (-x_{17} + x_{24}) + x_{17}^2 - 2x_{17} x_{24} + x_{24}^2] \} \\
& + \{x_4^2 + x_{10}^2 + x_{16}^2 + x_{23}^2\} + \rho\{u_1^2 + u_2^2 + u_3^2 + u_4^2\}
\end{aligned} \tag{4.115}$$

5. When the functional $f(.)$ in Equation (4.109) is partially differentiated with respect to the state variables $x_1 \ x_2 \ x_3 \ \cdots \ x_{24},$ the following first order differential equations are obtained:

$$\begin{aligned}
1/2 \, df(.)/dx_1 &= \beta_1^2 x_1 + \beta_1 x_5 - \beta_1 x_{17} - \beta_1 x_{24} \\
1/2 \, df(.)/dx_2 &= 0 \\
1/2 \, df(.)/dx_3 &= 0 \\
1/2 \, df(.)/dx_4 &= x_4 \\
1/2 \, df(.)/dx_5 &= \beta_1 x_1 + 2x_5 - \beta_2 x_6 - x_{11} - x_{17} - x_{24} \\
1/2 \, df(.)/dx_6 &= -\beta_2 x_5 + \beta_2^2 x_6 + \beta_2 x_{11} \\
1/2 \, df(.)/dx_7 &= 0 \\
1/2 \, df(.)/dx_8 &= 0 \\
1/2 \, df(.)/dx_9 &= 0 \\
1/2 \, df(.)/dx_{10} &= x_{10} \\
1/2 \, df(.)/dx_{11} &= -x_5 + \beta_2 x_6 + 2x_{11} - \beta_3 x_{12} - 2x_{17} \\
1/2 \, df(.)/dx_{12} &= -\beta_3 x_{11} + \beta_3^2 x_{12} + \beta_3 x_{17} \\
1/2 \, df(.)/dx_{13} &= 0 \\
1/2 \, df(.)/dx_{14} &= 0 \\
1/2 \, df(.)/dx_{15} &= 0
\end{aligned} \tag{4.116}$$

$$1/2 \, df(.) / dx_{16} = x_{16}$$

$$1/2 \, df(.) / dx_{17} = -\beta_1 x_1 - x_5 - 2x_{11} + \beta_3 x_{12} + 3x_{17} - \beta_4 x_{18}$$

$$1/2 \, df(.) / dx_{18} = \beta_4^2 x_{18} - \beta_4 x_{17} + \beta_4 x_{24}$$

$$1/2 \, df(.) / dx_{19} = 0$$

$$1/2 \, df(.) / dx_{20} = 0$$

$$1/2 \, df(.) / dx_{21} = 0$$

$$1/2 \, df(.) / dx_{22} = 0$$

$$1/2 \, df(.) / dx_{23} = x_{23}$$

$$1/2 \, df(.) / dx_{24} = -\beta_1 x_1 - x_5 + \beta_4 x_{18} + 2x_{24}$$

A similar concept can be used to control energy use, and the first-order partial derivatives with respect to control input signals can be found as follows:

$$df(.) / du_1 = u_1$$

$$df(.) / du_2 = u_2 \tag{4.117}$$

$$df(.) / du_3 = u_3$$

$$df(.) / du_4 = u_4$$

The state weighing matrix, Q_k , and the control weighing matrix, R_k can then be constructed using the above states partial (derivative Equation (4.116) and the control partial derivatives from Equation (4117):

$$Q_k = \begin{bmatrix} \beta_1^2 & 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 & 2 & -\beta_2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -\beta_2 & \beta_2^2 & 0 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & \beta_2 & 0 & 0 & 0 & 0 & 2 & -\beta_3 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_3 & \beta_3^2 & 0 & 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\beta_1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -2 & \beta_3 & 0 & 0 & 0 & 0 & 3 & -\beta_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & \beta_4^2 & 0 & 0 & 0 & 0 & 0 & \beta_4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\beta_1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

and

$$R_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrices of the power system and of the Q_k and R_k are used further for the calculation of the matrix of the Optimal Quadratic AGC controller by using the Equations (4.15-4.17). The flow chart of the COQAGC controller algorithm is shown in Figure 4.10. The first parts of the algorithm are made to define the cost functional of the AGC problem based on the design requirements, secondly make use of the minimization of the functional of the cost function for selecting the state and control weighing matrices, thirdly, write the cost function in Equation (4.103) in the form of Then, through an iterative process the solution of discrete Riccati Equation (4.15) can be found in the MATLAB environment (Mathworks, 2014). The minimal values of the state and control weighing matrices as well as the solution of discrete Riccati

Equation (4.15) made it possible to calculate the optimal feedback gain matrix L based on Equation (4.16) which in turn optimises the closed-loop of the interconnected power system.

4.6.4. Results and Discussions

In this section, the developed discrete optimal quadratic controller is tested using MATLAB/SIMULINK simulation, namely the discrete centralized optimal quadratic AGC controller with ACEs in a four-area power system. To demonstrate the feasibility of the optimal control technique, it is implemented in a discrete-time form with a sampling of 814 *ms*. The simulation study investigated the developed controller's performance with and without disturbances. The parameters of the four-area power system are given in Table 4.7

Table 4- 7 The parameters of the four-area power systems (Parmar,Majhi, & Kothar,2012, Kundur,1994, Rakhshani, Remo, & Mir,2016).

Parameter	Area1	Area 2	Area 3	Area 4	Unit
P_{Ri}	2000	2000	2000	2000	MW
T_{psi}	20	20	20	20	sec
T_{ti}	0.3	0.3	0	0.23	sec
T_{r1}	0	0	1	0	sec
K_{r1}	0	0	0.3	0	-
K_{psi}	120	120	120	120	Hz/pu , MW
T_i	0.0867	0.0867	0.0867	0.0867	MW/Hz
T_{Gi}	0.08	0.08	0.2	0.6	sec
T_{cd}	0	0	0	0.2	sec
T_{cr}	0	0	0	0.001	sec
bg	0	0	0	0.05	sec
C_g	0	0	0	1	-mm
Y_g	0	0	0	1	mm
T_{cb}	0	0	0	0.01	sec
T_{rh}	0	0	28.75	0	sec
T_w	0	0	1	0	sec
T_R	0	0	0.11		-
R_i	2.4	2.4	2.4	2.4	Hz/pu , MW
B_i	0.425	0.425	0.425	0.425	
D_i	0.0833	0.0833			N/m.sec
$\Delta\delta$	30°	30°	30°	30°	Degree

4.6.4.1 Calculation of the continuous and discrete matrices of the four-area power system

The state matrix A , the input matrix B , and disturbance matrix Γ from subsection 4.6.1 are obtained using the values of the parameters of non-reheat reheat, hydro and gas turbines represented in Table 4.7 and the values of the matrices are given below.

$A =$	-0.05	6	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	3.33	-3.33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	-5.21	0	-12.5	6.79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0.43	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	
	1.632	0	0	0	0	-0.545	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	6	-0.0	6	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	-3.33	3.33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	-1.563	0	-1	1	2.038	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	-5.21	0	0	0	6.7935	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	-1	0.425	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1.088	0	0	0	0	0	-0.5438	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	6	-0.05	6	0	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0.7246	-1.6	1.6696	1.6696	-0.5687	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	-0.3623	0	-0.0348	-0.8348	0.0023	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	-2.0833	0	0	-5	1.6349	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	-1	0.425	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	-1	0	0	0	0	0	0	1.0877	0	0	0	0	0	-0.5438	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	-0.0500	6	0	0	0	0	0	0	0	0	0	0	-6	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.2174	0	-4.347	4.3043	-0.4783	0.0728	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	-1	-11	1.6733	0	0	0	0				

$$\mathbf{B}^T = \begin{bmatrix} 0 & 0 & 12.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.75 & 12.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.7391 & 0.8696 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5217 & 12 & 20 & 0 & 0 \end{bmatrix}$$

$$\Gamma^T = \begin{bmatrix} -6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The continuous-time matrix A , B and E are converted to discrete-time matrices A_k, B_k and Γ_k using Euler discretization procedure discussed in section 4.5.1.1, as follows:

$$\begin{aligned}
IACE_2 &= \sum_{k=k_0}^{\infty} (0.425x_6 - x_5 + x_{11}) \\
IACE_3 &= \sum_{k=k_0}^{\infty} (0.425x_{12} - x_{11} + 2x_{17}) \\
IACE_4 &= \sum_{k=k_0}^{\infty} (0.425x_{18} - x_{17} + x_{24})
\end{aligned} \tag{4.119}$$

Substituting equations (4118) and (4119) into equation (4.111), the cost function becomes

$$\begin{aligned}
J = \frac{1}{2} \sum_{k=k_0}^{\infty} \{ & [0.180625x_1^2 + 0.85x_1(x_5 - x_{17} - x_{24}) + x_5^2 + x_{17}^2 + x_{24}^2 - 2x_5x_{17} \\
& - 2x_5x_{24} + 2x_{17}x_{24}] \\
& + [0.18062x_6^2 + 0.85x_6(-x_5 + x_{11}) + x_5^2 - 2x_5x_{11} + x_{11}^2] \\
& + [0.18062x_{12}^2 + 0.85x_{12}(-x_{11} + x_{17}) + x_{11}^2 - 4x_{11}x_{17} + 4x_{17}^2] \\
& + 0.18062x_{18}^2 + 0.85x_{18}(-x_{17} + x_{24}) + x_{17}^2 - 2x_{17}x_{24} + x_{24}^2] \} \\
& + \{x_4^2 + x_{10}^2 + x_{16}^2 + x_{23}^2\} + \{u_1^2 + u_2^2 + u_3^2 + u_4^2\}
\end{aligned} \tag{4.120}$$

where x_5 , x_{11} , x_{17} and x_{24} are the tie-line power deviations between areas 1 and 2, 2 and 3, 3 and 4 and 4 and 1 respectively, x_1 , x_6 , x_{12} and x_{18} are frequency deviations of area-1 to 4 respectively, x_4 , x_{10} , x_{16} and x_{23} are the integrals of ACE 1 to 4 respectively.

The state and control weighting matrices Q_k and R_k are obtained on the basis of functional minimization procedure discussed in section 4.2 and according to Equation (4.116) as follows:

[illegible]

	17.9	12.97	2.21	7.12	26.3	9.983	11.66	7.42	0.249	-11.61	39.48	-4.76	1.46	18.1	-0.99	-23.6	16.3	-2.1	0.05	1.29	0.76	-0.29	-0.63	-4.75
	12.9	13.22	2.70	9.15	4.48	0.566	1.948	2.66	0.156	-7.717	8.127	-2.04	-1.36	6.26	-0.95	-10.5	5.67	-3.2	-1.61	0.04	0.54	-0.28	-0.45	-5.29
	2.2	2.70	0.593	2.08	-0.560	-0.399	-0.184	0.23	0.025	-1.733	-0.120	-0.25	-0.39	0.62	-0.18	-1.4	0.46	-0.7	-0.45	0.09	0.08	-0.05	-0.08	-1.01
	6.5	8.95	2.06	65.	-36.2	-22.74	-16.47	-5.47	-0.140	26.54	-108.1	-12.5	-40.4	-78.7	-63.0	64.	-61.3	-3.1	-1.96	-1.02	-1.2	0.63	16.00	-45.88
	27	5.47	-0.400	-32.5	163.2	92.38	84.30	36.78	0.729	-52.17	33.4	-10.9	57.2	154.3	0.49	-164	115.9	6.9	5.69	2.09	-0.54	0.43	-10.22	53.68
	11.3	1.29	0.296	-21.6	94.1	70.28	60.04	23.33	0.496	-16.60	236.5	-9.42	39.4	94.6	2.25	-102	61.9	3.2	1.19	-0.62	-0.20	0.02	-5.81	36.7
	12.6	2.54	-0.090	-14.2	83.9	58.47	52.91	22.63	0.600	-12.81	191.1	-10.9	25.3	72.4	0.89	-78	52.6	2.2	1.17	-0.16	-0.08	0.01	-4.17	26.09
	7.6	2.6	0.262	-4.65	35.9	22.16	22.03	11.45	0.555	-4.33	71.73	-5.20	6.08	24.6	-0.27	-27	21.9	0.27	1.17	-0.09	-0.01	-0.01	-1.37	8.96
	0.3	0.156	0.025	-0.129	0.697	0.471	0.582	0.550	0.150	0.994	1.954	-0.151	-0.07	0.247	-0.03	-0.3	0.69	-0.01	-0.01	-0.01	-0.01	0.1	-0.01	0.26
	-122	-7.95	-1.36	26.1	-54.8	-18.50	-14.47	-4.99	0.984	75.57	-111.6	-10.6	-42.7	-98.3	1.07	92	-70.7	-1.50	-1.73	-1.31	-0.65	0.26	13.53	-37.49
	44.6	10.7	0.230	-98.7	346.3	244.4	201.0	76.74	2.043	-105.3	916.9	-3.04	195.8	414.0	8.9	-446	270.6	8.06	0.57	-3.80	-0.47	-1.7	-32.58	173.56
$P =$	-4.85	-2.06	-0.244	-12.4	-11.19	-9.348	-11.11	-5.336	-1.252	-10.60	-1.689	21.5	20.7	10.6	0.70	-16	16.6	-5.9	-4.90	-1.52	1.08	-0.69	-5.46	16.27
	1.92	-1.26	-0.391	-40.2	60.85	42.48	27.67	6.800	-0.067	-42.83	206.1	20.5	86.3	122	8.33	-112	47.3	2.05	-1.98	-2.73	0.66	-0.64	-15.17	57.13
	21.1	7.42	0.750	-76.9	170.4	106.3	82.66	28.50	0.297	-98.56	451.10	0.6	121.6	311	-4.37	-285	191.6	2.80	0.6	-1.02	0.08	-0.18	-34.40	109.72
	-1.27	-1.10	-0.206	-0.926	-0.421	1.676	0.312	-0.557	-0.040	0.970	7.196	0.662	8.46	-3.80	3.60	6.1	-14.2	0.26	0.8	-0.01	0.15	-0.11	0.6	-0.10
	-27.3	-12.2	-1.64	55.5	-178.3	-112.5	-88.28	-31.54	-0.416	89.93	-476.1	-15.1	-108.2	-273	5.89	314	-218.4	1.37	2.11	2.09	1.76	-0.82	20.76	-112.78
	19.4	4.19	0.666	-56.5	125.8	68.47	59.55	25.12	0.744	-67.99	285.7	15.6	42.4	-175.9	-13.7	-211	240.5	-2.67	8.23	11.70	3.39	-0.80	-17.56	67.41
	-2.34	-3.39	-0.743	-3.33	6.135	2.753	1.689	-0.031	-0.013	-1.505	6.796	-6.03	2.32	3.44	2.12									

$$L = \begin{bmatrix} 1.149 & 1.868 & 0.449 & 1.729 & -1.317 & -0.500 & -0.428 & -0.038 & 0.012 & -0.810 & -0.920 & -0.089 & -0.299 & 0.158 & -0.120 & -0.702 & -0.104 & -0.527 & -0.395 & -0.156 & 0.031 & -0.027 & -0.036 & -0.597 \\ 1.405 & 0.683 & 0.087 & -0.640 & 5.211 & 3.072 & 3.342 & 2.169 & 0.253 & 0.550 & 10.406 & -0.832 & 0.374 & 2.920 & -0.117 & -3.420 & 3.462 & -0.026 & -0.011 & -0.022 & -0.005 & 0.001 & -0.1600 & 1.285 \\ 0.586 & 0.240 & 0.027 & 0.030 & 2.159 & 1.309 & 1.370 & 0.600 & 0.011 & -0.300 & 2.639 & -1.606 & -0.930 & 1.909 & -0.175 & -0.712 & 1.218 & 0.516 & 0.591 & 0.349 & 0.019 & 0.016 & -0.028 & -0.782 \\ 0.299 & 0.150 & 0.016 & -0.193 & -0.053 & -0.237 & -0.146 & -0.053 & -0.001 & -0.203 & -0.844 & 0.132 & -0.326 & -0.179 & -0.047 & 0.350 & 2.096 & 0.497 & 1.084 & 1.243 & 1.367 & -0.669 & 2.162 & -1.683 \end{bmatrix}$$

$$u(k) = -Lx(k) \quad (4.121)$$

and, the corresponding closed-loop system's Eigenvalues are found as given in Table 2. All the real parts of Eigenvalues of the discrete closed-loop are less than 1 which guarantees the system stability associated with oscillation due to the imaginary parts of Eigenvalues.

Table 4.8. The systems' states and their corresponding closed-loop Eigenvalues

States	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
Eigenvalue	-0.62	-0.08	0.43	0.8 + 0.34i	0.8- 0.34i	0.51	0.9 + 0.29i	0.9 - 0.29i	0.9 + 0.22i	0.9 - 0.217i	0.8+ 0.158i	0.8 - 0.16i
Statezs	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}
Eigenvalue	0.97 + 0.07i	0.97 - 0.07i	0.94	0.93 - +0.03i	0.93 - 0.03i	0.9 +0.04i	0.9- 0.04i	0.9	0.7	0.8	0.73	0.73

The simulations were conducted for two different cases which are discussed below:

Case1: load disturbances $w^T = 0.01pu$ are imposed on the power system at $t = 0$. The parameters of four area control are utilized. The initial tie-line power is set at 0 MW (0pu) and the initial values of the frequency deviations in four areas are set at 0 (0 Hz) for each area. The Discrete Centralized Optimal Quadratic AGC controller Equation (4.94), is assumed to act on the four-area state model when the area control errors are considered. The simulation is run in the MATLAB environment for 1800 iterations with $k_0 = 0$ and $k_f = 1800$.

Case 2: Step load disturbances $w^T = [\Delta P_{D1} \Delta P_{D2} \Delta P_{D3} \Delta P_{D4}]^T$ with are added simultaneously to each area. The initial tie-line power is set at 0 MW (0pu) and the initial values of the frequency deviations are set at 0.0 Hz for each area. The Centralized Optimal Quadratic AGC controller is assumed to act on the four-area state model when the area control errors are considered. We considered this mainly to test the feasibility of the developed controller and its robustness against disturbances. The simulation is run in the MATLAB environment for 1800 iterations with $k_0 = 0$ and $k_f = 1800$. The magnitude of the load disturbances for the two areas is $\Delta P_{D1} = 10$ MW (0.01pu), $\Delta P_{D2} = 10$ MW (0.01pu) , $\Delta P_{D3} = 10$ MW (0.01pu) and $\Delta P_{D4} = 10$ MW (0.01pu) respectively.

Figure 4.35 shows the frequency deviations of Centralized discrete COQAGC when area control errors are considered in areas 1, 2, 3 and 4 respectively. From the simulations, it can be observed that the frequency deviations converge to zero in less than 8 seconds due to

area control errors. It is noted that the frequency deviations undershoot and overshoot with an oscillatory response before the signals return to zero in the steady state.

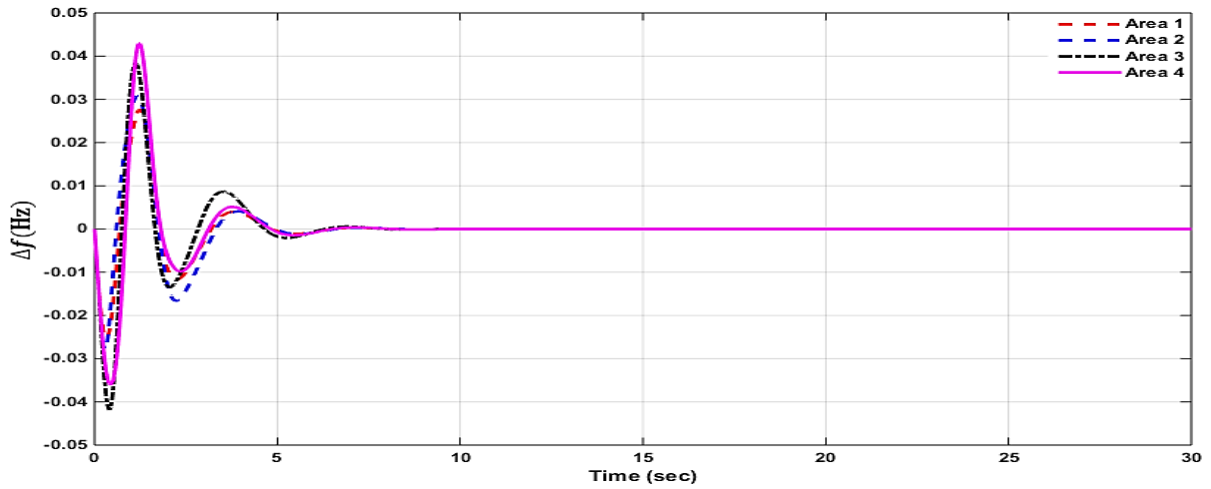


Figure 4. 35. Frequency deviations in control area 1 with 1% disturbance at $t = 0$ sec.

The centralized discrete COQAGC model with area control errors provides that deviation of tie-line power for the four areas shown in Figure 4.36. It is observed that the tie-line deviation converges to zero in less than 8 seconds. Tie-line power deviations for the four areas overshoot and undershoot around the zero with oscillation before the signals return to zero. Furthermore, we can see that the hydropower system was associated with the smallest overshoot, while the reheat turbine thermal power system was associated with the highest overshoot. For gas power system, the amplitude of the overshoot is constant compared to other power systems.

Figure 4.37 shows the integral of area control errors for four area power systems. It can be observed that the integral of area control errors converges to zero or constant values immediately after 5 seconds. The integral of area control error deviations for the hydro and gas power systems overshoot and undershoot around the zero with oscillation before the signals return to zero. But, the non-reheat and reheat thermal power systems produce the biggest amplitude with less oscillation around the zero.

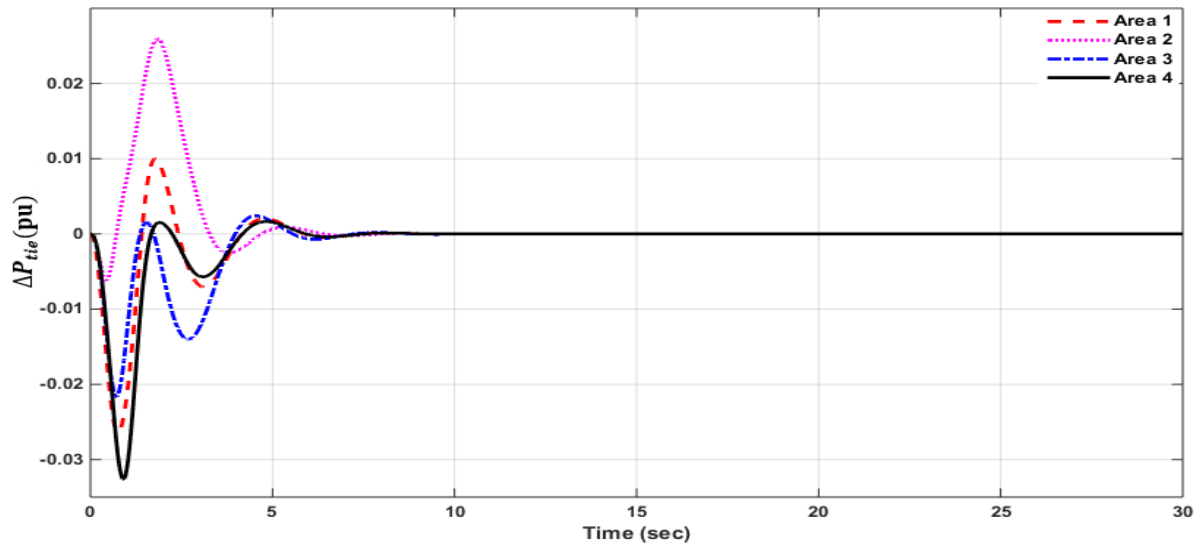


Figure 4- 36.Tie-line power deviations in four areas with 1% disturbances at t = 0 sec.

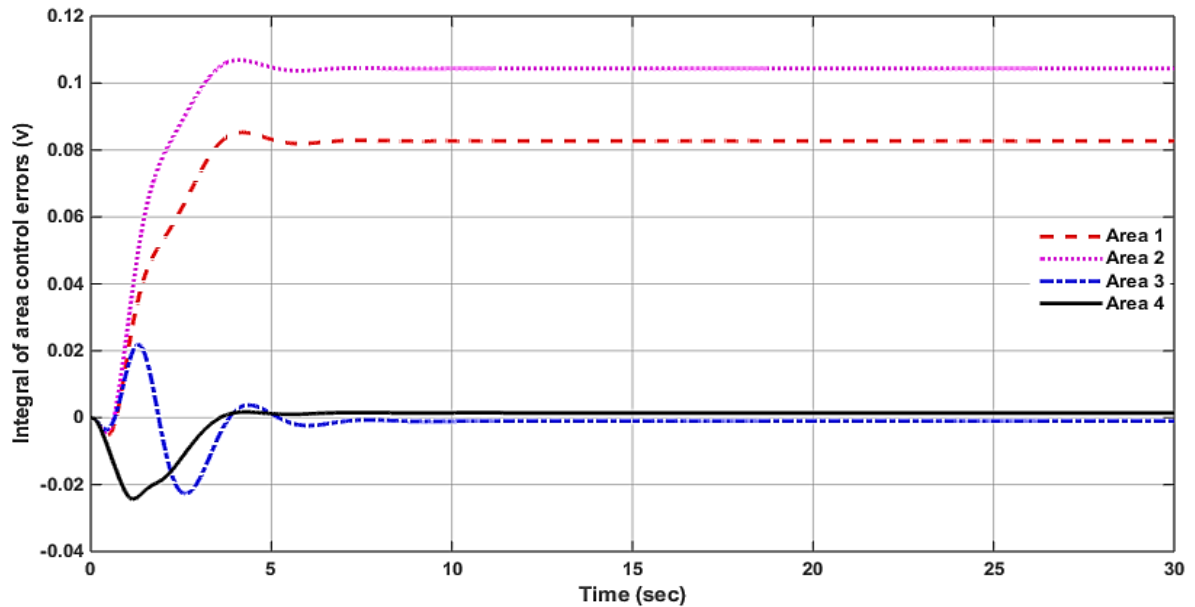


Figure 4.37. Integral of area control errors in four areas with no disturbances.

These results reveal that the centralized discrete COQAGC controller with ACEs can bring the fluctuations associated with frequency deviations, tie-line deviations, and the integral of area control errors to their nominal values within a reasonable time. In addition, the frequency deviations, tie-line deviations, and the integral of area control errors for the four areas are associated with overshoot and undershoot around the zero with oscillation during the transient period. Moreover, the non-reheat and reheat turbine thermal power systems were associated with the highest overshoot for the integral of the area control areas, while the hydro and gas power systems were associated with the smallest overshoot for the integral of the area control areas.

In case 2 simulations, 1% power loads are added simultaneously to the system random 5 second intervals since the load demands can change at any time during the day. Figure 4.38 shows the plots of the frequency deviations for areas 1 to 4 respectively, when area control errors are considered. It is observed that the COQAGC controller rejects the disturbances significantly at 4 seconds or more. The centralized discrete COQAGC has the greatest overshoot and undershoots in the frequency deviations Δf_3 and Δf_4 while all frequency deviations have the same the disturbance rejection (robustness) and same settling time.

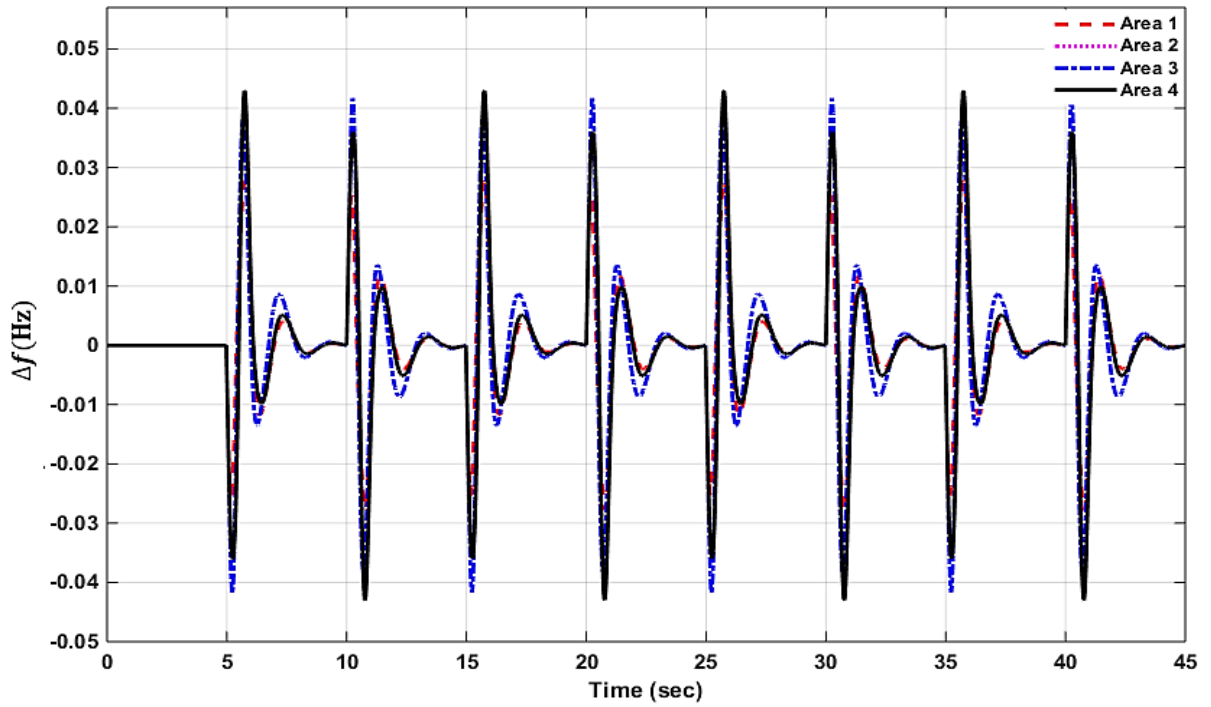


Figure 4.38: Frequency deviations of centralized discrete COQAGC for four areas with load disturbances.

The deviations in the tie-line power for the four areas are shown in Figure.4.39. The tie-line power deviation between areas 2 and 3 has the shortest undershoot among tie-line deviations while the tie-line power deviation between areas 4 and 1 has the fastest disturbance rejection (robustness) compared to other ones in the presence of large magnitude load disturbances.

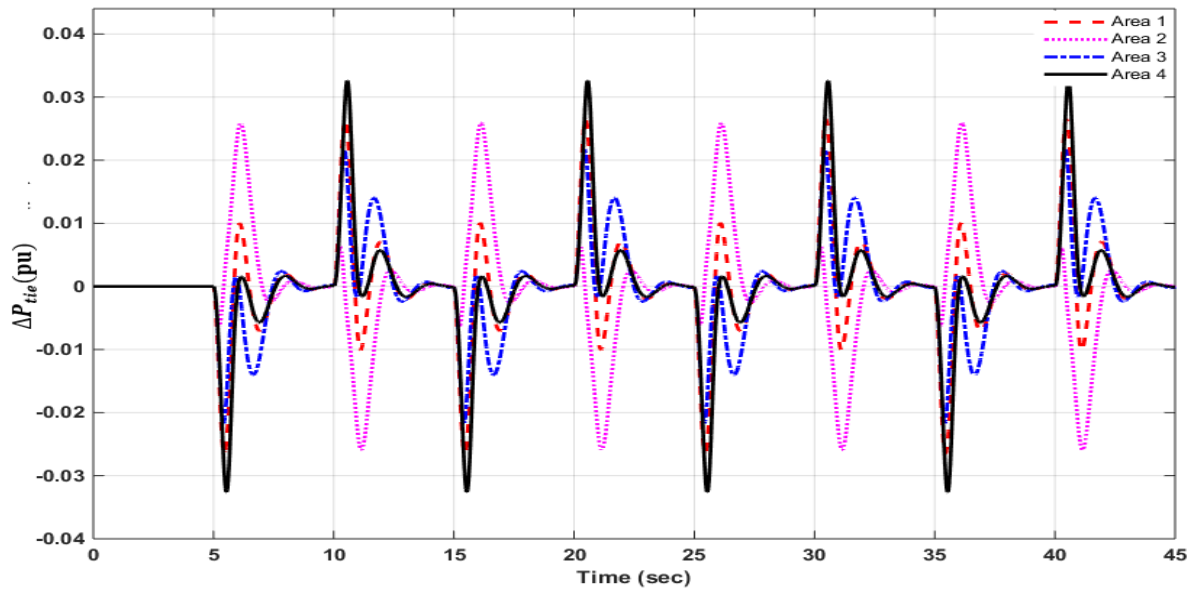


Figure 4.39. COQAGC Tie-line deviations for four area power systems with load disturbances.

Finally, Figure.4.40 shows the plots of the integral of area control errors (IACE₁ and IACE₂, IACE₃ and IACE₄) in the presence of load disturbances. It can be observed that IACE₄ has the fastest disturbance rejection (robustness), while ACE₁, IACE₂ and ACE₃ approximately have the same disturbance rejection when compared to IACE₄.

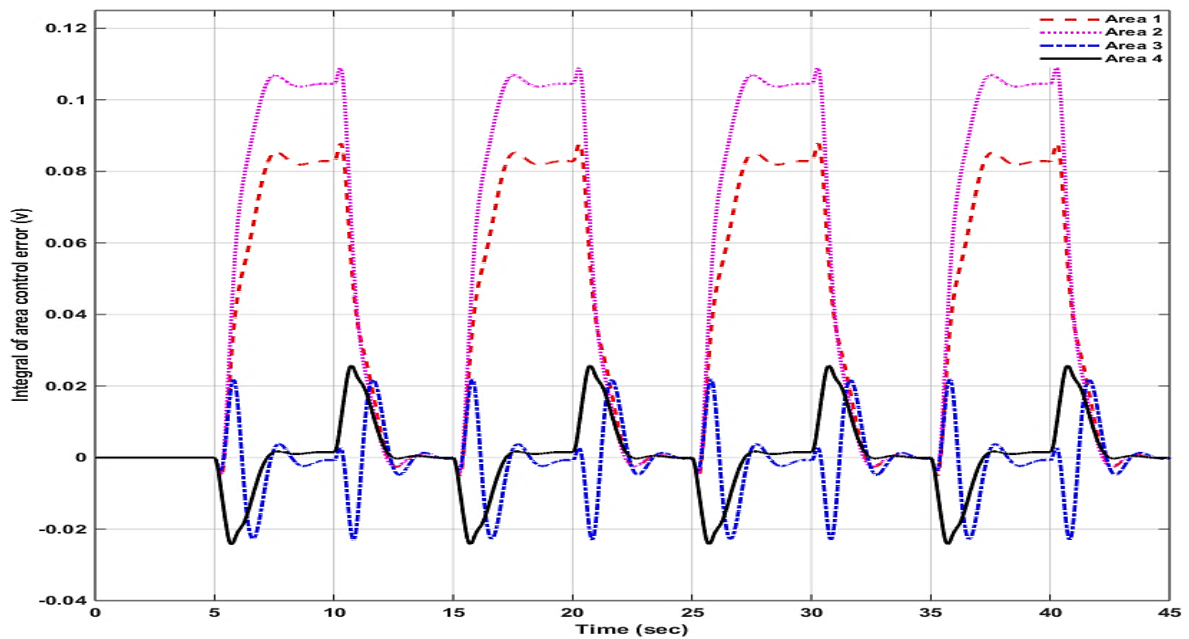


Figure 4.40. The integral of Area control errors for centralized discrete OQAGC with disturbances.

The parameter settings for the two cases are summarized in Table 4.9. Table 4.10 shows the comparison of the frequency deviation for two cases with disturbances at $t = 0$ sec,

simultaneous disturbances and with area control errors. Where the abbreviations Peak OS, Peak US, PT, ST, DRRT stand for Peak overshoot, Peak undershoot, Peak time, Settling time and Disturbance rejection response time respectively. The results illustrate that the centralized discrete COQAGC controller provides good dynamic performance in terms of rising time, settling time, and overshoot. The controller also shows good disturbance rejection time response.

Table 4.9. Parameters settings for the two simulations cases

Case	Δf_i	ΔP_{tieij}	AEC_i	ΔP_{Di}	$IAEC_i$	ΔP_T
1	0 Hz	0 Pu	0	0.01 pu	0	0
2	0 Hz	0Pu	0	0.01 pu	0	0

Table 4.10 Performance of centralized discrete COQAGC controller with two cases's disturbances.

Frequency Deviations, Tie-line power deviations and Integral of area control errors									
	Case 1					Case 2			
Area	State variables	Peak OS	Peak US	PT (sec)	ST (Sec)	Peak OS	Peak US	Tp (Sec)	DRRT (sec)
1	Δf_1	0.027	-0.024	0.425	8	0.028	-0.0253	0.75	4.75
2	Δf_2	0.031	-0.028	0.325	8	0.31	-0.028	0.7	4.3
3	Δf_3	0.04	-0.042	0.5	8	0.038	-0.042	0.7	4.35
4	Δf_4	0.043	-0.036	0.6	8	0.043	-0.036	0.72	4.36
1	ΔP_{tie12}	0.01	-0.0026	0.3	7.7	0.026	-0.0263	0.425	4.6
2	ΔP_{tie23}	0.026	-0.006	0.9	7.7	0.026	-0.006	1.15	4.35
3	ΔP_{tie34}	0.0024	-0.022	0.27	7.5	0.022	-0.04	0.43	4.4
4	ΔP_{tie41}	0.0015	-0.025	0.8	7.5	0.022	-0.033	0.58	4.4
1	$IAEC_1$	0.085	-0.005	0.62	7	0.0085	-0.005	2.55	8.32
2	$IAEC_2$	0.0107	-0.0046	0.72	7	1.07	-0.004	2.4	8.32
3	$IAEC_3$	0.022	-0.023	0.575	8.8	0.022	-0.0035	0.8	4.5
4	$IAEC_4$	0.0022	-0.024	0.02	6.8	0.022	-0.0243	2.75	4.1

To validate the performance of the proposed controller, a simulation results from the literature and COQAGC simulation results were compared. The performance of discrete COQAGC is compared with A hybrid neuro-fuzzy (ANFIS) controller presented by Prakash and Sinha, 2014 as shown in Tables 4.11 and 4.21. Both controllers are imposed with 1% Step load disturbance at $t = 0$ second (case1) for four area interconnected power system. The simulation results shown in Tables 4.11 and 4.12 show that discrete COQAGC is better in terms of peak undershoot and settling time and provides superior performance compared to ANFIS controller. The main disadvantage of the discrete COQAGC control structure is that the frequency deviations, tie-line deviations, and the integral of area control errors for the

four areas are associated significantly with overshoot and undershoot around the zero during the transient period. In addition, the system model grows and expands as the number of areas increases. This poses computational and design challenges.

Table 4.11: Comparative study of peak undershoot between COQAGC and ANFIS (Prakash and Sinha, 2014).

Controllers	Δf area 1	Δf area 2	Δf area 3	Δf area 4	ΔP_{tie} (thermal-thermal)	ΔP_{tie} (thermal-hydro)
ANFIS(Prakash and Sinha, 2014)	-0.061	-0.061	-.054	-0.054	-0.052	-0.054
COQAGC	-0.024	-0.028	-0.042	-0.036	-0.0026	-0.022

Table 4.12: Comparative study of settling time between COQAGC and ANFIS (Prakash and Sinha, 2014).

Controllers	Δf area 1	Δf area 2	Δf area 3	Δf area 4	ΔP_{tie} (thermal-thermal)	ΔP_{tie} (thermal-hydro)
ANFIS(Prakash and Sinha, 2014)	18	18	17	17	15	27
COQAGC	8	8	8	8	7.7	7.7

4.7 Sensitivity Analysis of the COQAGC Controller for FAIPS

The values of system parameters often change owing to environmental influences, equipment component replacement, changes in operating circumstances, and system component aging. The system's performance may suffer and it may become unstable (Arya & Kumar, 2016). As a result, sensitivity analysis is carried out to establish the resilience of the centralized discrete COQAGC controller at nominal levels across variations in operating load conditions and tie-line synchronization coefficients. The developed controller is compared with the Fuzzy gain scheduling controller (FGSC) from literature in order to study the sensitivity analysis using the identical operating conditions for both controllers (Arya & Kumar, 2016). The operational load conditions and tie-line synchronization coefficients are adjusted from their nominal values in Table 4.7 by $\pm 25\%$ and $\pm 50\%$ of the nominal 1 percent size of step load perturbation (SLP) in changing step size of 25%, as shown in Table 4.13.

Table 4.13. Variations in the four-area interconnected power system's specified parameters

Parameters Variations Rate	Variation of the tie-line synchronizing coefficient	Variations Load operating condition
Nominal	0.0866	1%
-25% of Nominal	0.06495	0.0075

-50% of Nominal	0.0433	0.005
+25% of Nominal	0.10825	0.0125
50% of Nominal	0.1299	0.015

Figures 4.41 and 4.42, show the simulated responses to frequency deviations and tie line power deviations for each modified condition produced by the proposed controller. The sensitivity analysis results are compared with the dynamic responses that obtained with the proposed controller at nominal value 1% size of SLP. The results clearly reveal that all dynamic responses of frequency deviations (Figure 4.41) and tie-line deviations (Figure 4.42) have a similar pattern of peak of undershoot (Peak US), peak of overshoot (Peak OS), and all have a settling period of 4 sec with small differences across areas. The settling times of non-reheat, hydro, and gas power plants, on the other hand, is almost the same, although the peak overshoot responses varied slightly. During the transient period, however, all output responses are associated with oscillations, with the exception of the reheat turbine plant's high peak overshoot response. Interaction factors between the subsystems produce variations in the peak overshoot response, which must be adjusted for.

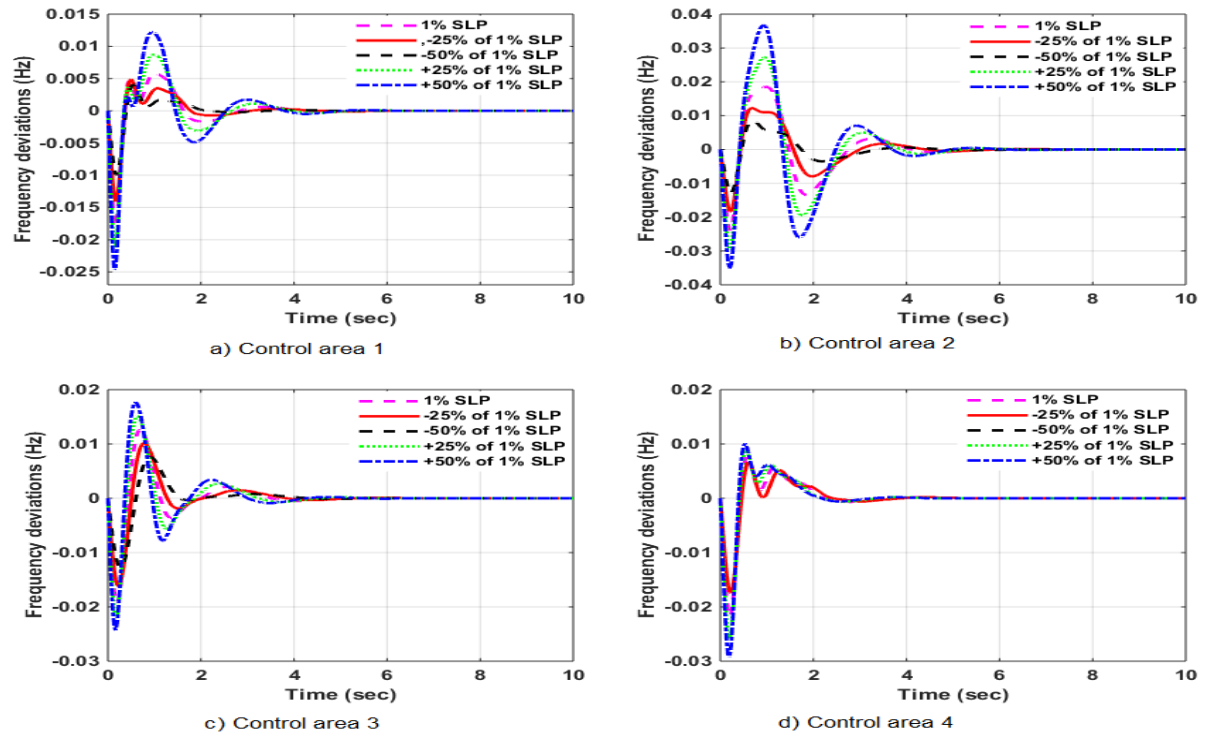


Figure 4.41. Dynamic response of Frequency deviations with a) Non-reheat turbine b) Reheat turbine c) Hydro turbine d) Gas turbine when changes in 1% SLP applied at $t = 0$ sec.

A close examination of all findings indicates that they are almost identical and have not varied much from the nominal values. As a result, the centralized discrete COQAGC controller is robust to fluctuations in load operating circumstances and tie-line synchronization coefficients. Sensitivity analysis simulation findings of frequency and tie line power deviations in terms of peak overshoot, peak undershoot and settling time for varied the system parameters and operational load conditions are summarized in Table 4.14.

The results of sensitivity analysis of centralized discrete COQAGC is compared with Fuzzy gain scheduling controller (FGSC) proposed by Arya & Kumar, 2016. The comparison is investigated for the highest undershoot and setting time for the frequency deviations and tie line deviation of thermal power systems with Non-reheat turbines (Figures 4.41(a) and 4.42(a)). The simulation results in Table 4.15 reveal that FGSC has superior dynamic responses of frequency deviations in terms of oscillations and peak of overshoot, compared to discrete centralized COQAGC controller while COQAGC has better dynamic response in term of peak undershoot.

Table 4.14 Sensitivity analysis of frequency and tie-line power deviations

Controller	Area	Variable	Peak OS (HZ)					Peak US (Hz)					ST (sec)				
		LSP%	1%	-25%	+25%	-50%	50%	1%	-25%	+25%	-50%	50%	1%	-25%	+25%	-50%	50%
COQAGC	1	Δf_1	0.005	0.01	0.021	0.004	0.012	-0.017	-0.014	0.009	-0.01	-0.03	4	4	4	4	4
	2	Δf_2	0.019	0.012	0.027	0.009	0.037	-0.024	-0.018	-0.03	-0.013	-0.035	5	5	5	5	5
	3	Δf_3	0.013	0.01	0.015	0.008	0.018	-0.019	-0.016	-0.022	-0.013	-0.024	4	4	4	4	4
	4	Δf_4	0.005	0.007	0.009	0.005	0.01	-0.013	-0.013	-0.026	-0.017	-0.029	4	4	4	4	4
	1 & 2	ΔP_{tie12}	0.001	0.0	0.003	0.0	0.006	-0.014	-0.009	-0.02	-0.004	-0.026	4	3	4	3	4
	2 & 3	ΔP_{tie23}	0.019	0.011	0.03	0.005	0.045	-0.012	-0.008	-0.018	-0.004	-0.024	4	4	4	4	5
	3 & 4	ΔP_{tie34}	0.0006	0.0003	0.0009	0.0001	0.0013	-0.011	-0.008	-0.0142	-0.005	-0.018	4	4	4	4	5
	4 & 1	ΔP_{tie41}	0.0005	0.0004	0.0006	0.0002	0.0005	-0.013	-0.009	-0.0195	-0.0045	-0.024	4	4	4	4	4

However, FGSC (Aya et al, 2016) and COQAGC controllers have almost same settling time for tie line deviations while Fuzzy gain scheduling controller achieves less peak undershoot for tie line deviations when compared to discrete centralized COQAGC controller. The variations in peak undershoot response and settling time is produced by interaction variables across control areas which must be adjusted for in the future studies.

Table 4.15: Comparative study of Peak OS and ST for sensitivity analysis between COQAGC and FGSC

Controller	Δf_1 (Non-reheat thermal system)		ΔP_{tie12} between thermal -thermal	
For 1% SLP, +25% of input signal is applied	TS	Peak US	ST	Peak US
FGSC(Arya and Kumar, 2016)	3 sec	-0.06	4 sec	-0.006
COQAGC (Developed)	4 sec	-0.021	4 sec	-0.02

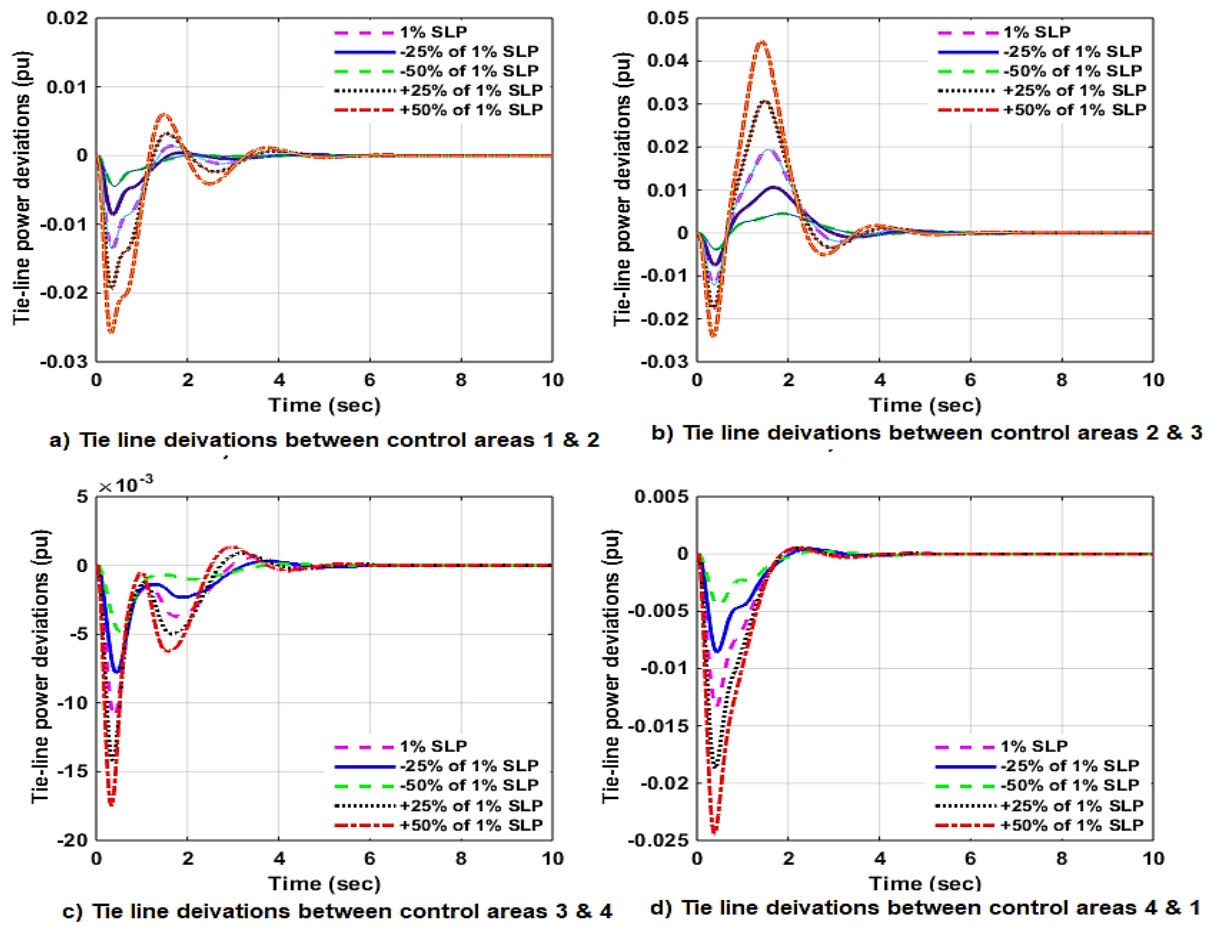


Figure 4.42. Dynamic responses of Tie-line power deviations with a) Non-reheat turbine b) Reheat turbine c) Hydro turbine d) Gas turbine when changes in 1% SLP applied at $t = 0$ sec.

4.8. Multi-area Power system model with renewable energy sources

To demonstrate the ability of the proposed centralized discrete COQAGC controller, the study is further extended to a multi-area multi-source interconnected power system with renewables as depicted in Figure 4.43. Area 1 comprises non-reheat thermal and hydro plants. Area 2 comprises a wind power plant. The linear model of a wind power plant includes the transfer function for the pitch actuator, the transfer function for the lag mechanism that matches the phase/gain characteristic of the model, and a blade characteristics block (Dasa, Aditya, & Kothari, 1999). It is crucial to include the important inherent requirements and the basic physical constraints in the model in order to get an accurate understanding of the AGC problem (Parmar, Majhi & Kothar, 2012, Mohanty, Panda & Hota, 2014, Arya Kumar & Ibraheem, 2016, Sahu, Gorripotu & Pand, 2016). The important constraints which affect the power system performance are boiler dynamics, Generation Rate Constraint (GRC), and Governor Dead Band (GDB). In this study, the GRC was integrated into the system model as illustrated in Figure 4.43 in order to test whether the proposed controller could be realistically implemented. It is assumed that the participation factors for thermal and hydro are 0.543470 and 0.3226034, respectively, and that the participation factor for wind turbines is 0.125 (Boukas & AL-Sunni, 2011).

4.8.1. State space equation of a multi-Source power system with renewables

The procedure discussed in section in section 4.1 is applied here to obtain the state space equation of a multi-source power system with renewable shown in Figure 4.43. The typical parameters of non-reheat thermals and Hydro plants are taken from Table 1, while wind turbine parameters are adopted from the work conducted by Sahu, Gorripotu & Panda, 2016), where $T_{p1} = 6$; $T_{p2} = 0.04$, $k_{p2} = 1.25$, $k_{p3} = 1.4$, $T_{g2} = 0.08$, $k_{bc} = 08$, $R_w = 2.4$ and $\beta_2 = 0.425$. The system in Figure 4.43 has 15 states variables where $x_1 = \Delta f_1$, $x_2 = \Delta P_{GN1}$, $x_3 = \Delta P_{V3}$, $x_4 = IACE_1$, $x_5 = \Delta P_{GH}$, $x_6 = \Delta X_H$, $x_7 = \Delta P_{RH}$, $x_8 = \Delta P_{tie12}$, $x_9 = \Delta f_2$, $x_{10} = \Delta D$, $x_{11} = \Delta H$, $x_{12} = \Delta H_1$, $x_{13} = IACE_2$, $x_{14} = \Delta P_{GN2}$, $x_{15} = \Delta P_{V4}$. As result the state, control input and disturbance vectors of a linear state-space model for a two area multi-source power systems can be described as follows:

$$\begin{aligned}
x^T &= [\Delta f_1 \ \Delta P_{GN1} \ \Delta P_{v3} IACE_1 \Delta P_{GH} \ \Delta X_H \ \Delta P_{RH} \ \Delta P_{tie12} IACE_2 \Delta P_{tie12} \ \Delta f_2 \ \Delta D \ \Delta H \ \Delta H_1 \ IACE_2 \Delta P_{GN2} \ \Delta P_{v4}]^T \\
u^T &= [U_{th1} U_{hy} \ U_w U_{th2}]^T \\
w^T &= [\Delta P_{D1} \Delta P_{D2}]^T \\
C^T &= [\Delta f_1 \Delta f_2 \Delta P_{tie12}]^T
\end{aligned} \tag{4.122}$$

The continuous the continuous state matrix A_m , the input matrix B_m , and disturbance matrix Γ_m are given below:

$$A_m = \begin{bmatrix}
-\frac{1}{T_{ps1}} & \frac{k_{ps1}}{T_{ps1}} & 0 & 0 & \frac{k_{ps1}}{T_{ps1}} & 0 & 0 & 0 & -\frac{k_{ps1}}{T_{ps1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{R_{n1} T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\beta_1}{T_{RH} T_{GH} R_{hy}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2T_{RS}}{T_{RH} T_{GH} R_{hy}} & 0 & 0 & 0 & -\frac{2}{T_W} & \frac{2}{T_W} & \frac{2}{T_{RH}} & \frac{2}{T_{RH}} & -\frac{2T_{RS}}{T_{RH} T_{GH}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{T_{RS}}{T_{RH} T_{GH} R_{hy}} & 0 & 0 & 0 & 0 & -\frac{1}{T_{RH}} & \frac{1}{T_{RH}} & -\frac{T_{RS}}{T_{RH} T_{GH}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{T_{GH} R_{hy}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{GH}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2\pi T_{12}}{9} & 0 & 0 & 0 & 0 & 0 & 0 & -2\pi T_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k_{ps2}}{T_{ps2}} & -\frac{1}{T_{ps2}} & \frac{k_{ps2} k_{hc}}{T_{ps2}} & 0 & 0 & 0 & 0 & \frac{k_{ps2}}{T_{ps2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & k_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{p2} T_{p1}}{R_w T_{p2}} & 0 & -1 & k_{p2} & -\frac{k_{p2} T_{p1}}{T_{p2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_w T_{p2}} & 0 & 0 & -\frac{1}{T_{p2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_{n2} T_{G2}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{G2}}
\end{bmatrix} \tag{4.123}$$

$$B_m^T = \begin{bmatrix}
0 & 0 & \frac{0.54347}{T_{G1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{0.3260842 T_{RS}}{T_{RH} T_{GH}} & \frac{0.3226084 * T_{RS}}{T_{RH} T_{GH}} & \frac{0.322684}{T_{GH}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{0.125 T_{p1} k_{p2}}{T_{p2}} & \frac{0.125}{T_{p2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{0.4344705}{T_{G2}} & 0
\end{bmatrix}$$

$$\Gamma_m^T = \begin{bmatrix}
-\frac{k_{ps1}}{T_{ps1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{ps1}}{T_{ps1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

4.8.2. Simulation Results of Multi-Source Interconnected Power System

the design procedures described in sections 4.2 and 4.3 are applied to the Multi-Area Power system model with renewable energy sources as shown in Figure 4.43. Based on the functional minimization method (FFM) in section 3, Equations (4.28) - (4.32), the state and control weighting matrices Q_m and R_m were formulated for a multi-area power system model with renewable energy sources. The excursions ΔACE_i , $IACE_i$ and control input u are substituted into Equation (4.28). As a result, the cost function J for the systems can be written as:

$$Q_m = \begin{bmatrix} 0.1806 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4250 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4250 & 0 & 0 & 0 & 0 & 0 & 0 & 2.0000 & -0.4250 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4250 & 0.1806 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, R_m = \begin{bmatrix} 0.5435 & 0 & 0 & 0 \\ 0 & 0.3226 & 0 & 0 \\ 0 & 0 & 0.1250 & 0 \\ 0 & 0 & 0 & 0.5444 \end{bmatrix} \quad (4.125)$$

According to equation (4.16), the numerical values of optimal feedback gains matrix is found as follows:

$$L_m = \begin{bmatrix} 0.3737 & 0.8423 & 0.2264 & 1.2059 & 1.3067 & 1.2566 & -0.5067 & -0.2141 & -0.0358 & -0.1855 & -0.0506 & 0.3073 & -0.1243 & -0.1521 & -0.0398 \\ 0.2297 & 0.3636 & 0.0877 & 0.4660 & 0.4872 & 0.8368 & -0.2258 & -0.0120 & -0.0804 & -0.1183 & -0.0172 & 0.0843 & -0.0753 & -0.1175 & -0.0252 \\ -0.1754 & -0.4655 & -0.1187 & 0.1967 & -0.5828 & -0.0179 & 0.0939 & 1.5877 & -0.3347 & 2.3510 & 0.5688 & -4.1123 & 1.6726 & 1.9994 & 0.5394 \\ -0.0461 & -0.1506 & -0.0389 & 0.1472 & -0.1878 & -0.0138 & 0.0315 & 0.5995 & 0.0192 & 0.8870 & 0.2752 & -1.7271 & 0.6529 & 0.7322 & 0.2111 \end{bmatrix} \quad (4.126)$$

The computer simulations were conducted with discrete COQAGC controllers' gains for two different cases. The simulation results are presented in Figures 4.44 to 4.47 and detailed explanations of those simulation results are presented below. Figures 4.44 to 4.45 show simulation results for frequency deviations and Tie-line power deviations. These are the results obtained when the step load perturbation (SLP) set at 0.01 pu in area1 at the initial time $t = 0$ while keeping the rest of the initial values to zeros.

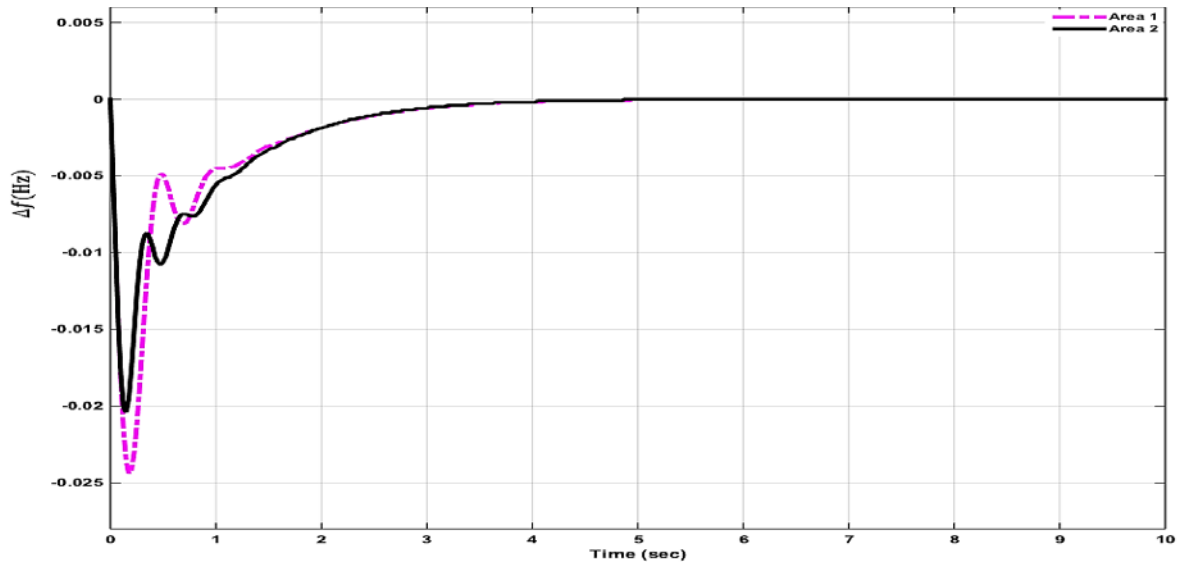


Figure 4.44. Frequency deviations for Areas 1 and 2

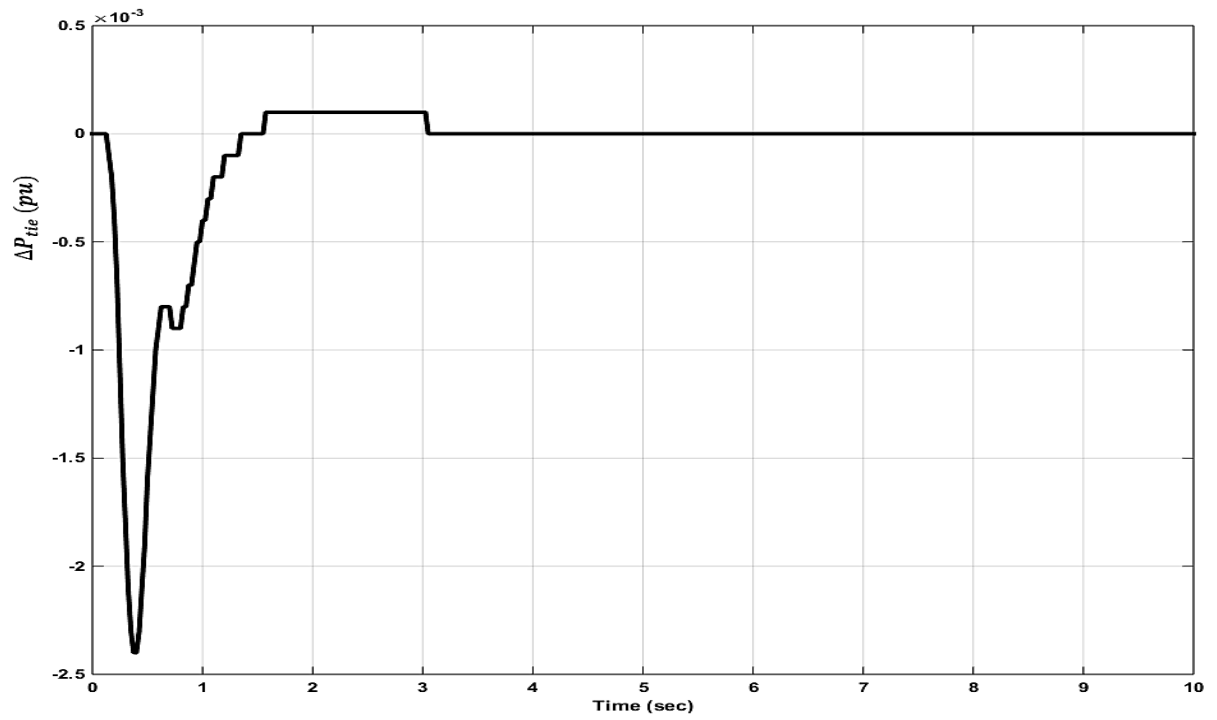


Figure 4.45. Tie-line power deviation between areas 1 and 2

simulation results for frequency deviations, and Tie-line power deviations are shown in Figures 4.46 and 4.47 respectively.

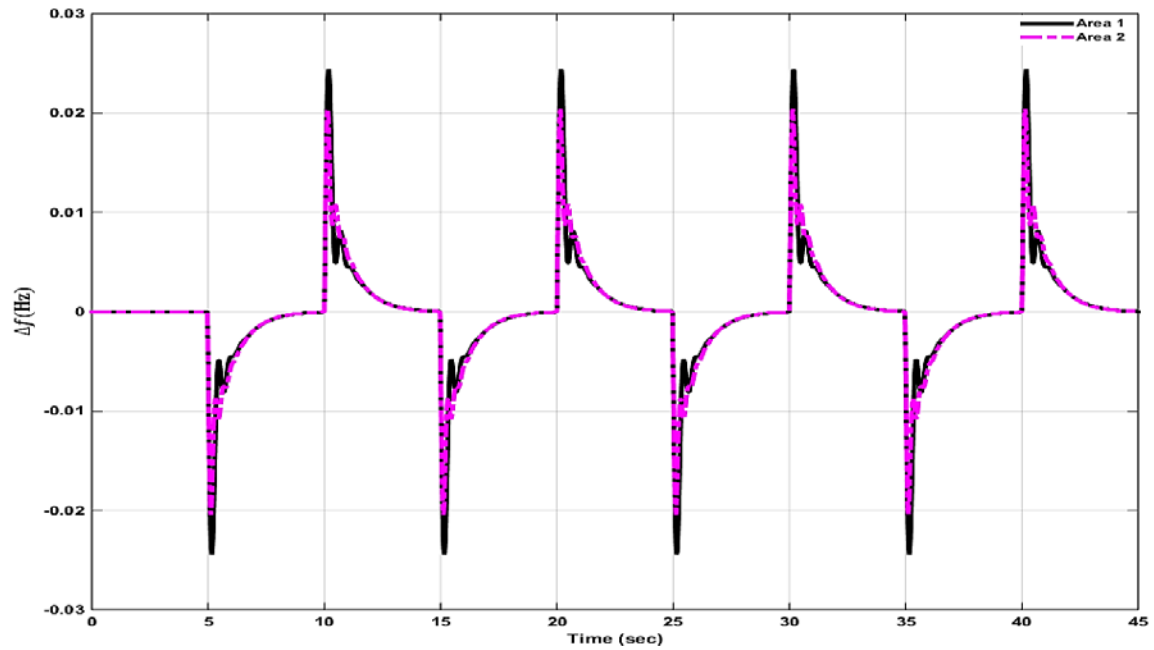


Figure 4.46. Frequency deviation results of two areas 1 and 2 with simultaneous SLPs

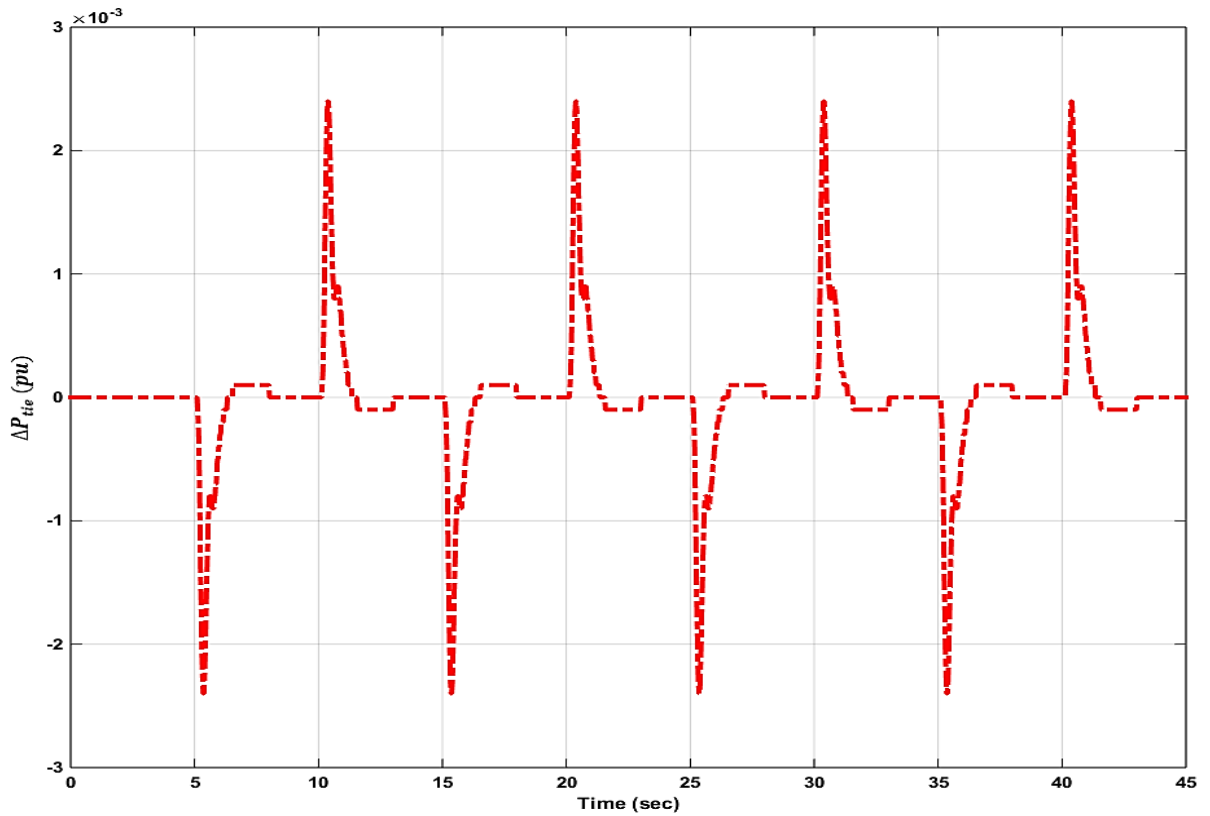


Figure 4.47. Tie-line power deviation result in both areas with simultaneous SLPs

The simulations result for frequency deviations and tie-line power deviations of both areas are summarized in Tables 4.16 and 4.17, respectively. It is observed that the discrete COQAGC controller achieves almost the same frequency deviation responses of peak overshoots when the Step Load Perturbations are applied respectively (SLP = 0.01 pu) at $t = 0$ sec and simultaneously at various time intervals. The area 2 has 18 % difference of peak overshoot compared to that of area 1 for both cases. The settling time of area 2 is smaller than the settling time of area 1 with zero steady state errors in both cases. The settling time and peak overshoot are same for the tie-line power deviations. The tie line deviation has lesser disturbance rejection time response when compared to disturbance rejection time response of the frequency deviations.

Table 4.16: Performance of COQAGC in terms of peak OS, TS, DRRT) and, Steady state error ($e_{ss}(\infty)$).

Frequency Deviations					
DOQAGC Controller	Area	ST	Peak OS	DRRT	$e_{ss}(\infty)$
SLP = 0.01 p.u at $t = 0$ sec	1	4.95 sec	-0.0244	-	0
	2	4.875 sec	-0.0201	-	0
Difference %	-	2%	18%	-	0%
Simultaneous SLP = 0.01 p.u at $t = 5$ sec interval	1	4.95 sec	-0.0244	4.85 sec	0
	2	4.875 sec	-0.0201	4.925 sec	0

Difference %	-	2%	18%	2%	0%
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Table 4.17: Performance of COQAGC in terms of Peak OS, ST and DRRT with simultaneous disturbances

Tie-line power deviations (pu)					
DOQAGC Controller	Areas	ST	Peak OS	DRRT	$e_{ss}(\infty)$
with SLP = 0.01p.u at t = 0	1 and 2	3.05 sec	-0.0024	-	0
Simultaneous SLP = 0.01p.u at 5 sec intervals	1 and 2	3.05 sec	-0.0024	3.05 sec	0

4.8.3. Generation Rate Constraint

An additional test of the effectiveness of the Centralized discrete COQAGC controller is conducted by considering the GRC of thermal and hydroelectric plants in the multi-source power system model discussed above. This study considers a GRC of 10%/min (0.0017pu/s) for a single non-reheat thermal plant and a GRC for the hydro plant of 270%/min (+0.045pu/s) for a raising generation, and 360%/min (−0.06pu/s) for a lowering generation for the hydro plant. Simulations are conducted for non-reheat and hydro-generating plants with and without the GRC with limits described above. Since the hydro-generating plant limits are wider when compared to 1% SLP, so 10% SLP is used to test the GRC simulation of the hydro plant. The effect of GRC on the frequency deviation responses of multi-source power system obtained with centralized discrete COQAGC controller at 1% and 10% step load perturbation are shown in Figures 4.48 and 4.49 respectively. These results show that the Centralized discrete COQAGC controller exhibits a larger peak overshoot and a longer settling time when using GRC. However, in these particular two cases, the dynamic of the multi-source power system with GRC meets the automatic generation control requirements as described in the literature (Parmar, Majhi & Kothari, 2012).

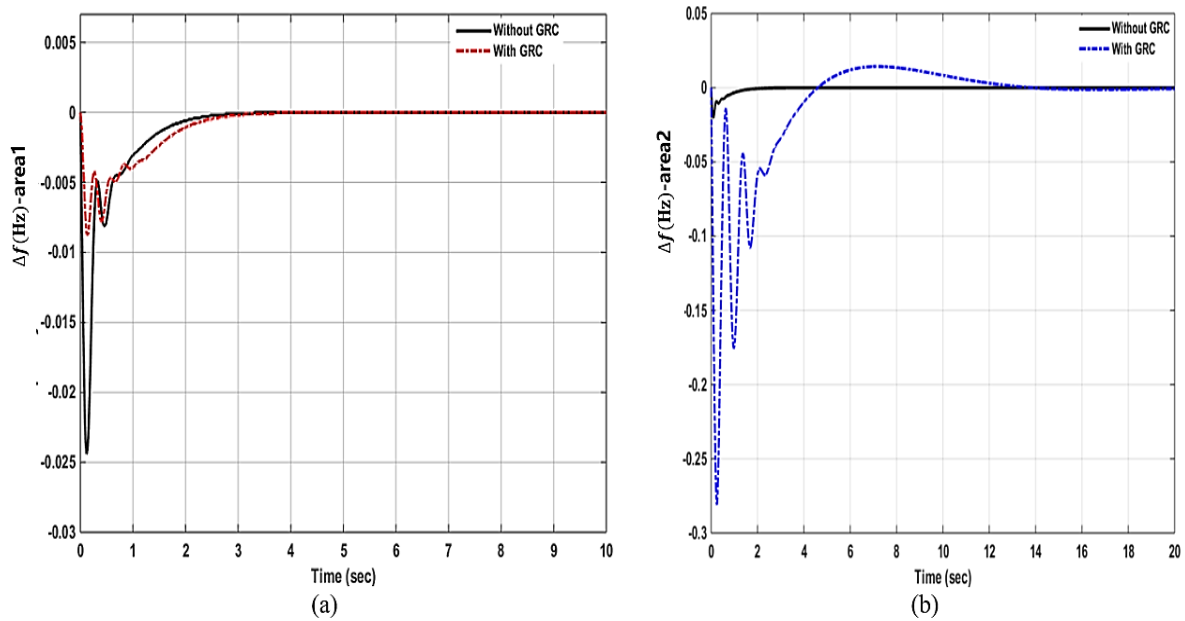


Figure 4- 48. Frequency deviation response to 1% SLP in multi-source power system with and without GRC for thermal plant (a) Area 1(b) Area 2.

It has been observed that the dynamic performance of the system deteriorates if both GRC limits are applied at the same time. It is therefore necessary to take GRC into account when studying the system realistically.

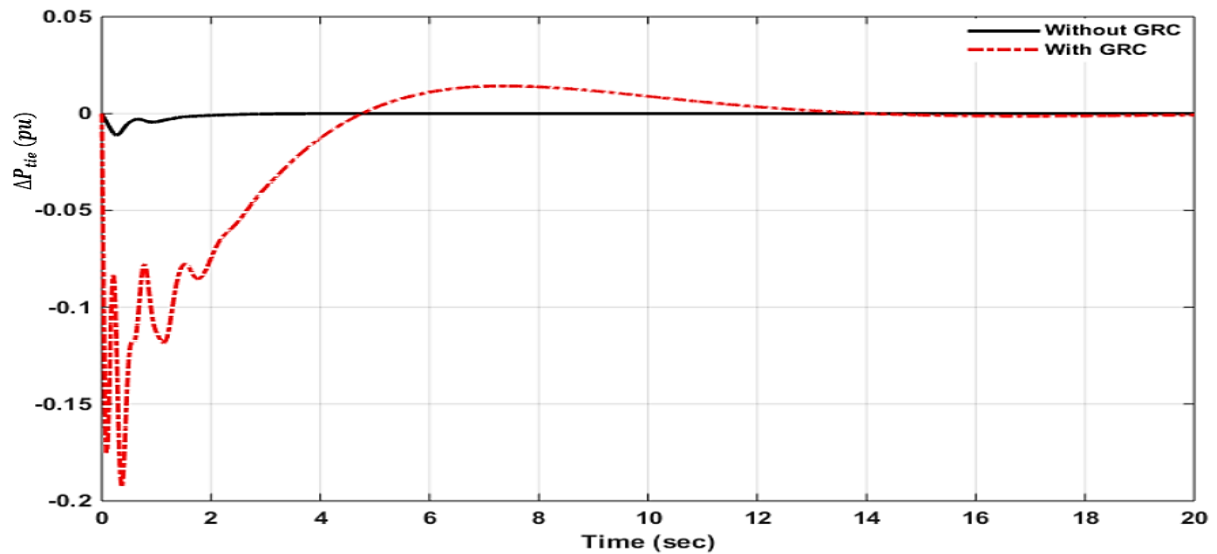


Figure 4-49. Tie-line deviations response to 1% SLP in multi-source power system with and without GRC

The results of GRC of discrete COQAGC are compared with optimal output control (Parmar, Majhi & Kothari, 2012). The comparison is investigated for the highest overshoot and setting time for the frequency deviation (Figures 4.48 (b)). The simulation results in Table 4.18 reveal that the discrete COQAGC controller has superior dynamic responses in terms of

peak of overshoot, and settling time compared to Full state feedback control (Parmar, Majhi & Kothari, 2012).

Table 4.18: Comparative study of peak OS with GRC for tie line power deviations between COQAGC and Full state feedback control

Controllers	Thermal power system	
	Settling time	Peak overshoot
Full state feedback control (Parmar, Majhi & Kothari, 2012).	25 sec	-0.091
COQAGC(Developed)	16 sec	-0.25

4.9. Summary

This chapter presents the design of centralized discrete-time optimal quadratic control systems using an optimal control theory and Lagrangian traditional multipliers method for the case of feedback control of a multi-area power system. Developed a centralized control schemes using 1) Discrete-time COQAGC regulators and 2) Centralized Discrete COQAGC controllers for multi-area interconnected power systems. The two-area interconnected power system (TAIPS) model with no area control errors has been considered in the case of the Centralized COQAGC regulator with and without the SLPs. TAIPS and four area interconnected power systems (FAIPS) with area control errors have been considered for Centralized discrete COQAGC controllers. The derivatives of the integral of area control errors have been added to the state vector of the TAIPS and FAIPS to achieve frequency and tie-line deviations with zero values at the steady state. The main contribution of this chapter is developed a general framework to design and assess the optimal quadratic control schemes for TAIPS and FAIPS. The chapter defines the cost function requirements with area control errors areas, integral area control errors and control efforts. The functional minimization method is used to select the best state and control weighing matrices.

It was concluded the COQAGC Centralized discrete controllers are robust against the SLPs in terms of steady-state design characteristics in comparison with centralized COQAGC regulators. The sensitivity study analysis proves that the developed centralized discrete COQAGC controller is robust in load operating circumstances and tie-line synchronization coefficients. The GRC analysis prove that the developed centralized controller meets the requirements of the automatic generation problem. The main disadvantage of the centralized control structure is that the system model grows and expands as the number of areas increases. This poses computational and design challenges.

However, in an industry perceptive there is a need for decentralized controller, which solves the complex multi-area interconnected systems (MAIPS). Therefore, next chapter focused on decentralized controller design for the MAIPS.

Chapter 5

Decentralized Discrete Optimal Quadratic AGC Control for Interconnected Power systems

5.1. Introduction

The discrete optimum decentralized quadratic automated generation control (DOQAGC) for an interconnected power system is formulated in this chapter. A DOQAGC control design begins by decentralizing large-scale systems and separating them into several subsystems through overlapping decomposition. Overlapping decomposition is accomplished by either extending the linked power system's state space equation or by utilizing tie line power exchanges that are a function of frequency variations (Alrfai et al, 2011 & Siljak, 2007). Both techniques strive to produce extended distinct subsystems. LQR control theory and weighting matrices are employed in the expanded space to create suboptimal discrete decentralized control rules, depends on the number of control areas in a multi-area interconnected power system. Furthermore, the weighting matrices of the cost sub-functions are calculated using the functional minimization technique (FMM) described in chapter 4, sections 4.3 and 4.4.2. In this chapter, the Discrete Centralized Optimal Quadratic Automatic Generation Control (COQAG) architecture and the resultant DOQAGC controllers are compared to identical two and four control areas in multi-area power systems.

This chapter's key contribution are as follows:

- The continuous and discrete state space equations of two identical subsystems in a multi-area power system with identical non-reheat turbines, as well as four subsystems in a multi-area power system with non-reheat, reheat, hydro, and gas turbines, have been formulated using overlapping decomposition technique.
- Based on the optimum control theory framework and FMM presented in Chapter 4, section 4.3 of centrally interconnected power systems, discrete DOQAGC algorithms are applied to subsystems with dynamic couplings.
- To minimize the sub-functional of the criteria, the design requirements for discrete sub-cost functions are based on area control errors, integrals of area control errors, and the rule of control action for subsystems.

- The dynamic response for 10% step load perturbations (SLPs) at $t=0$ and 10% simultaneous SLPs for two identical and four distinct control areas was explored using the DOQAGC and COQAGC.
- This study improves computational difficulties and results in a simple and methodical design approach applicable to complex interconnected power systems.

The remainder of the chapter is structured as follows: Section 5.2. discusses the decomposition of multi-area interconnected systems into subsystems. Section 5.3 develops the formulation of the optimisation problem and the functional minimization approach for subsystems. Section 5.4 presents the decomposed model of an identical two-area power system as well as DOQAGC simulation results for identical two subsystems. Section 5.5 determines the decomposition of four multi-area control systems using multi-source power generators (MSPGs) into four subsystems. Section 5.6 presents simulation data and a discussion of four subsystems, as well as a comparison of COQAGC and DOQAGC in terms of peak overshoot, settling time, disturbance rejection response time, and steady-state error. Finally, section 5.7 provides the chapter's conclusion.

5.2. Decomposing Interconnected Power Systems into Subsystems

Overlapping decomposition is a mathematical framework for transforming a given state-space matrix of a complex system into smaller dispersed matrices under particular circumstances, as explained in Chapter 3. It is mostly based on graph theory and may be used to address complicated challenges in systems engineering. Once the decentralization process is complete, we may design decentralized control laws and apply them on their respective subsystems using any linear or non-linear control design approach (Siljak, 1978) and (Siljak, 1991). When working with and evaluating simpler subsystem matrices rather than complicated ones, overlapping decomposition has two advantages: easy computational and simple qualitative analytic. This is mostly due to the difficulty and time required to compute the inverse or determinants of a high-order matrix (Kusekwa, 2010). As a result, it is a good idea to decompose a big complex system model into subsystem matrices using overlapping decomposition to simplify computations, save time, and develop local controllers for subsystems using conventional control methods.

Consider the discrete state space equation for the interconnected power system to demonstrate the use of overlapping decomposition:

$$X(k+1) = AX(k) + Bu(k) + \Gamma w(k)w(k) \quad (5.1)$$

where $X(k) \in \mathbf{R}^n$ is the state vector; $u(k) \in \mathbf{R}^r$ is the control input vector; $w(k)$ is the disturbance vector; A is the system state matrix; B is the control matrix; Γ is the disturbance

matrix, and C is the output matrix. Using an overlapping decomposition, Equation (5.1) can be decomposed into N subsystems, where N is the number of areas in the space of the interconnected power system. From equation (5.1) we can also decompose the state X^T of the interconnected system into N sub-vectors $\bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{X}_4, \dots$ and $\bar{X}_N$ with dimensions $n_1, n_2, n_3, \dots, n_N$. As a result, the full state vector and the full dimension of the interconnected power system can be written as follows:

$$X^T = [\bar{X}_1(k) \quad \bar{X}_2(k) \quad \bar{X}_3(k) \quad \bar{X}_4(k) \quad \dots \quad \bar{X}_N(k)]^T \text{ and} \\ n = n_1 + n_2 + n_3 + n_4 + \dots + n_N \quad (5.2)$$

Therefore, the state vector of equation (5.2) can be divided into N sub-vectors as follows:

$$\begin{aligned} \bar{X}_1(k) &= \begin{bmatrix} \Delta f_1 \\ \vdots \\ \vdots \\ IACE_1 \\ \Delta P_{tie1} \end{bmatrix} = \begin{bmatrix} x_{11} \\ \vdots \\ \vdots \\ x_{1n_1-1} \\ x_{1n_1} \end{bmatrix}, \bar{X}_2(k) = \begin{bmatrix} \Delta f_2 \\ \vdots \\ \vdots \\ \vdots \\ IACE_2 \\ \Delta P_{tie2} \end{bmatrix} = \begin{bmatrix} x_{21} \\ \vdots \\ \vdots \\ \vdots \\ x_{2n_2-1} \\ x_{2n_2} \end{bmatrix}, \bar{X}_3(k) = \begin{bmatrix} \Delta f_3 \\ \vdots \\ \vdots \\ \vdots \\ IACE_3 \\ P_{tie3} \end{bmatrix} \\ &= \begin{bmatrix} x_{31} \\ \vdots \\ \vdots \\ \vdots \\ x_{3n_3-1} \\ x_{3n_3} \end{bmatrix}, \dots, \bar{X}_N(k) = \begin{bmatrix} \Delta f_N \\ \vdots \\ \vdots \\ \vdots \\ IACE_{N-1} \\ \Delta P_{tieNn_N} \end{bmatrix} = \begin{bmatrix} x_{N1} \\ \vdots \\ \vdots \\ \vdots \\ x_{Nn_N-1} \\ x_{Nn_N} \end{bmatrix} \end{aligned}$$

where, $\bar{X}_1, \bar{X}_2, \bar{X}_3, \dots$ and $\bar{X}_N$ are the state vectors of area1, area 2, area 3, $\dots$ and area N respectively. In the same way, the input $u(k)$, the input disturbance ΔP_d and interaction $x_j(k)$ vectors are also decomposed into sub-vectors. The number of regions and generators in each area determines the dimensions of these sub-vectors. This notion is related to the diagonalization of a system's state-space equation. This may be accomplished by either altering the state space equation described by the eigenvectors of the state transition matrix or by using inclusion principles to expand and contract the state space equation. The extended and contracted interconnected power systems were linked using linear transformations from equation (3.36), as stated in section 3.3. As a result, discrete decentralized subsystems may be expressed and merged in discrete state-space equations as follows:

$$\begin{aligned}
\begin{bmatrix} \bar{X}_1(k+1) \\ \bar{X}_2(k+1) \\ \bar{X}_3(k+1) \\ \vdots \\ \bar{X}_N(k+1) \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{A}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{A}_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{A}_N \end{bmatrix} \begin{bmatrix} \bar{X}_1(k) \\ \bar{X}_2(k) \\ \bar{X}_3(k) \\ \vdots \\ \bar{X}_N(k) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{B}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{B}_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{B}_N \end{bmatrix} \begin{bmatrix} \mathbf{u}_1(k) \\ \mathbf{u}_2(k) \\ \mathbf{u}_3(k) \\ \vdots \\ \mathbf{u}_N(k) \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{F}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{F}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{F}_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{F}_N \end{bmatrix} \begin{bmatrix} \mathbf{z}_1(k) \\ \mathbf{z}_2(k) \\ \mathbf{z}_3(k) \\ \vdots \\ \mathbf{z}_N(k) \end{bmatrix} + \begin{bmatrix} \mathbf{\Gamma}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{\Gamma}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{\Gamma}_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{\Gamma}_N \end{bmatrix} \begin{bmatrix} \Delta \mathbf{P}_{d1}(k) \\ \Delta \mathbf{P}_{d2}(k) \\ \Delta \mathbf{P}_{d3}(k) \\ \vdots \\ \Delta \mathbf{P}_{dN}(k) \end{bmatrix}
\end{aligned} \tag{5.3}$$

where $\mathbf{u}_1(k), \mathbf{u}_2(k), \dots, \mathbf{u}_N(k)$ are sub-control input vectors, $\mathbf{z}_1(k), \mathbf{z}_2(k), \dots, \mathbf{z}_N(k)$ are sub-interaction variable vectors, $\Delta \mathbf{P}_{d1}(k), \Delta \mathbf{P}_{d2}(k), \dots, \Delta \mathbf{P}_{dN}(k)$ are sub-disturbance variable vectors, $\mathbf{u}_1(k), \mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_N$ are the subsystem state matrices, $\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_N$ are sub-control matrices, $\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_N$ are the sub-interaction matrices, and $\mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \dots, \mathbf{\Gamma}_N$ are sub-disturbance matrices.

By comparing equations (5.1) and (5.3), we can see that the overlapping decomposition of the interconnected power system leads in decoupled autonomous subsystems. This is the major rationale for the overlapping decomposition based on the inclusion principle. Furthermore, when disjoint decomposition fails, the decomposition notion can guarantee stability (Ikdea, Seljak, & While 1980). As a result, Figure 5.1 depicts the generic block diagram of the i^{th} subsystem of N subsystems. The integration area control error in Figure 5.1 represents the state-augmentation strategy based on the inclusion of integral action in the state-space model (IACE). When the power system is disrupted by a step load perturbation, state-augmentation is a technique for adding an integral action to a state space model in order to attain zero steady state-error.

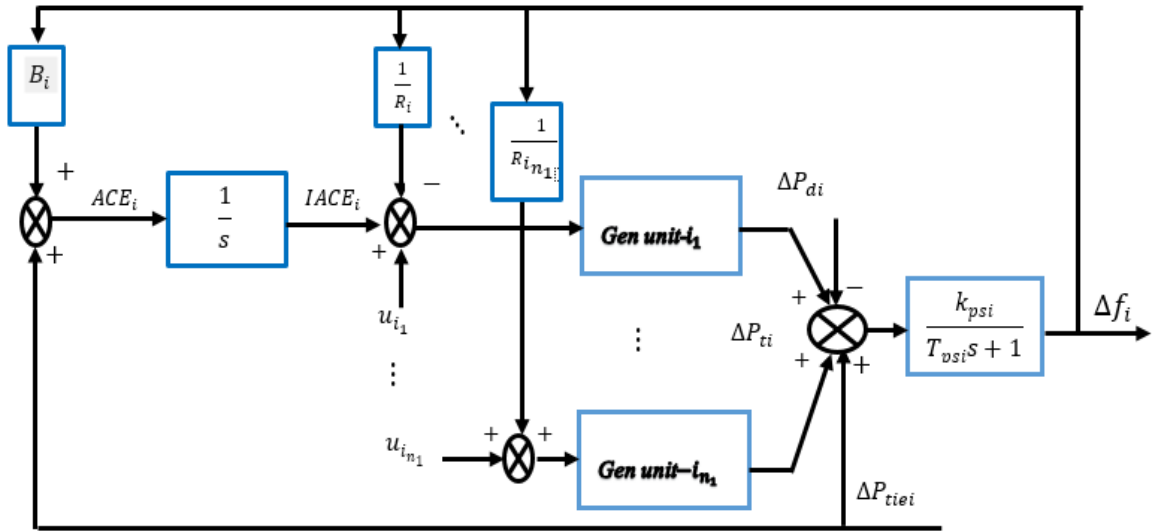


Figure 5.1. A decomposed and augmented model of an i^{th} subsystem(Bevrani, 2014)

Mathematically, the i^{th} integral of ACE_i can be written in discrete representation as:

$$IACE_i(k+1) = \sum_{k=k_0}^{\infty} AEC_i(k) \quad (5.4)$$

and, the discrete i^{th} tie-line power deviation between areas i and j can be recalled from equation (5.29) as

$$\Delta P_{tiei}(k+1) = 2\pi \frac{T_{ij}}{s} \left[\sum_{j=1}^2 \sum_{j \neq i} T_{ij} \Delta f_i(k) - \sum_{j=1}^2 \sum_{j \neq i} T_{ij} \Delta f_j(k) \right] \quad (5.5)$$

The number and type of generators in each control region determine the length of each subsystem.

5.2.1. Calculation procedure for Interaction Matrix L and interaction variables

In complex systems, inputs and outputs interact, which is one of the most difficult elements of regulating them. A coupling can cause signals to interact unexpectedly, disrupting other subsystems (MUGA, 2015). The relationships between subsystems in complex dynamic systems are defined using an interconnection matrix, which is a matrix with binary values and dimension, $(n+r+l)(n+r+l)$. It is derived from the structure of a system model based on graph theory by taking into account state matrix A, input matrix B, and output matrix C.

(Lunze, 1992 & Siljak, 2007). The interaction matrix L from equation (3.23) is decomposed mathematically into the matrix

$$L = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1(n+r+l)} \\ L_{21} & L_{22} & \cdots & L_{2(n+r+l)} \\ \vdots & \vdots & \ddots & \vdots \\ L_{(n+r+l)1} & L_{(n+r+l)2} & \cdots & L_{(n+r+l)(n+r+l)} \end{bmatrix} \quad (5.6)$$

where n represents the dimension of the state matrix A , r represents the dimension of the input matrix B , and l represents the size of the output matrix C . The coupling relationship between multiple subsystems is represented by the block, L_{ij} . When there is no connection between subsystems, the subscript, $L_{ij} = 0$ denotes zero binary values for all diagonal elements, but the subscript, L_{ij} , suggests one binary value for all diagonal elements. As a result, the, $L_{ij} \neq 0$ elements must meet the two qualities listed below.

- i. There is no coupling between subsystems

$$L_{11} = L_{22} = L_{33} = \cdots = L_{(n+r+l)(n+r+l)} = 0$$

- ii. There is coupling between subsystems

$$\begin{bmatrix} L_{12} & L_{13} & \cdots & L_{1((n+r+l)-1)} \\ L_{21} & L_{23} & \cdots & L_{2((n+r+l)-2)} \\ \vdots & \vdots & \ddots & \vdots \\ L_{(n+r+l)1} & L_{(n+r+l)3} & \cdots & L_{(n+r+l+1)((n+r+l)-1)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

From these two properties, it can be noted that the L_{ij} elements can be obtained as scalars or matrices depending on the number of the areas and number of the generators involved in each area. On the other hand, the interaction variables vector $\mathbf{z}^T = [z_1 \ z_2 \ z_3 \ \cdots \ z_N]$ are determined by using the L_{ij} elements and the frequency deviations $\Delta f_1, \Delta f_2, \Delta f_3, \cdots, \Delta f_N$ respectively.

To demonstrate how the interaction matrix may be used to determine interaction variables, and L_{ij} elements, The four areas of an interconnected power system with one generator in each area is considered for investigation. i.e. Non-reheat. Reheat, Hydro and Gas generators respectively as shown I Figure 5.2. Note, fir purpose of explanation, Figure 5.2 is recalled from chapter 4, Figure 4.

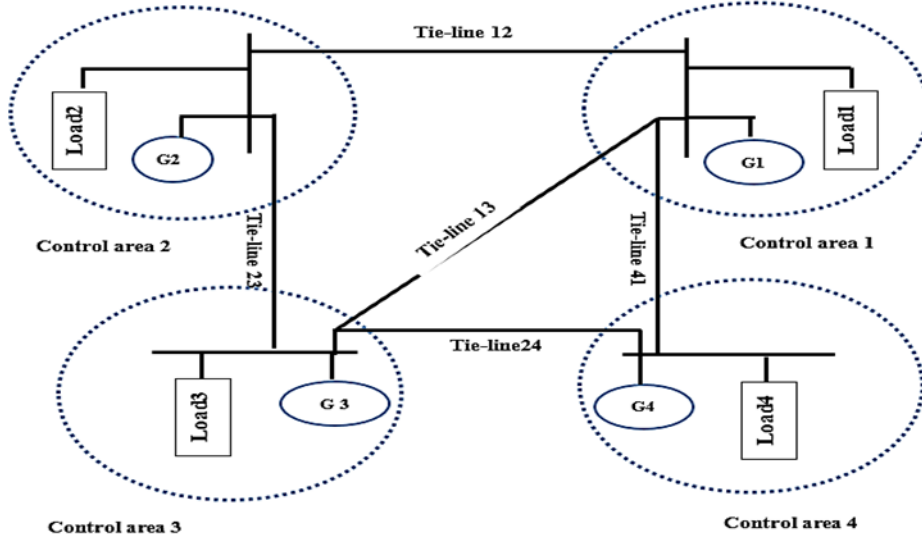


Figure 5. 2. four area interconnected power system with tie-line (Hasan & Hussein, 2014)

Hence, the interaction variables and L_{ij} elements for four areas are identified as:

- Area 1($i = 1$) with non-reheat generator

$$\mathbf{z}_1(t) = \sum_{\substack{j=1 \\ j \neq i}}^4 \mathbf{L}_{ij} \bar{\mathbf{x}}_j(t) = [\mathbf{L}_{12} \mathbf{L}_{13} \mathbf{L}_{14}] \bar{\mathbf{x}}_1(t), \therefore \mathbf{z}_1 = \begin{bmatrix} \mathbf{z}_{11} \\ \mathbf{z}_{12} \\ \mathbf{z}_{13} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{21} \\ \mathbf{x}_{31} \\ \mathbf{x}_{41} \end{bmatrix}, \quad (5.7)$$

and L_{ij} elements are determined as:

$$\mathbf{L}_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{L}_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{L}_{13} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{L}_{14} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Area 2($i = 2$) with reheat generator:

$$\mathbf{z}_2(t) = \sum_{\substack{j=1 \\ j \neq i}}^4 \mathbf{L}_{ij} \bar{\mathbf{x}}_j(t) = [\mathbf{L}_{21} \mathbf{L}_{23} \mathbf{L}_{24}] \bar{\mathbf{x}}_2(t), \therefore \mathbf{z}_2 = \begin{bmatrix} \mathbf{z}_{21} \\ \mathbf{z}_{22} \\ \mathbf{z}_{23} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{11} \\ \mathbf{x}_{31} \\ \mathbf{x}_{41} \end{bmatrix}, \quad (5.8)$$

and L_{ij} elements are determined as:

$$L_{22} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, L_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L_{23} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, L_{24} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Area 3($i = 3$) with hydro generator:

$$z_3(t) = \sum_{\substack{j=1 \\ j \neq i}}^4 L_{ij} \bar{x}_3(t) = [L_{31} L_{32} L_{34}] \bar{x}_3(t), \therefore z_3 = \begin{bmatrix} z_{31} \\ z_{32} \\ z_{33} \end{bmatrix} = \begin{bmatrix} x_{11} \\ x_{21} \\ x_{41} \end{bmatrix}, \quad (5.9)$$

and L_{ij} elements are determined as:

$$L_{33} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, L_{31} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L_{32} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, L_{34} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Area 4($i = 4$) with gas generator:

$$z_4(t) = \sum_{\substack{j=1 \\ j \neq i}}^4 L_{ij} \bar{x}_4(t) = [L_{41} L_{42} L_{43}] \bar{x}_4(t), \therefore z_4 = \begin{bmatrix} z_{41} \\ z_{42} \\ z_{43} \end{bmatrix} = \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix}, \quad (5.10)$$

and L_{ij} elements are determined as:

$$L_{44} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, L_{41} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L_{42} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, L_{43} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $L_{11} = L_{22} = L_{33} = L_{44}$, $L_{12} = L_{21} = L_{31} = L_{41}$, $L_{13} = L_{23} = L_{32} = L_{42}$, and $L_{14} = L_{24} = L_{34} = L_{43}$.

5.2.2. Discrete Interaction Sub-matrices F_i and interaction dynamics matrix A_{z_i} :

For N control regions, the discrete interaction sub-matrices are calculated using the mathematical equation for discrete tie-line power deviation, $\Delta P_{tie,i}(k+1)$ stated in Equation (5.11). There are two sorts of interaction variables between the several control domains in

this equation: coupling and non-coupling. The interaction sub-matrices are simply found using interaction equation (5.11), by examining the coupling interactions and disregarding the non-coupling interactions. Equation (5.36) may thus be reduced into sub-equations dependent on the number of areas.

$$\Delta P_{tie,i}(k+1) = 2\pi \left[\sum_{\substack{j=1 \\ j \neq i}}^N T_{ij} \Delta f_i(k) - \sum_{\substack{j=1 \\ j \neq i}}^N T_{ij} \Delta f_j(k) \right] \quad (5.11)$$

where $\Delta P_{tie,1}(k)$, $\Delta P_{tie,2}(k)$, $\Delta P_{tie,3}(k), \dots, \Delta P_{tie,N}(k)$ are the discrete tie-line power exchanges of the N control areas and T_{ij} is the torque coefficient between areas i and j , N is the number of control areas, $\Delta f_i(k)$ is the discrete frequency deviation in the i^{th} area, $\Delta f_j(k)$ is the frequency deviation in the j^{th} area. As result, the discrete interaction space matrix for the N control areas can be expressed as:

$$\begin{bmatrix} \Delta P_{tie,1}(k+1) \\ \Delta P_{tie,2}(k+1) \\ \Delta P_{tie,3}(k+1) \\ \vdots \\ \Delta P_{tie,N}(k+1) \end{bmatrix} = \begin{bmatrix} F_1 & 0 & 0 & 0 & 0 \\ 0 & F_2 & 0 & 0 & 0 \\ 0 & 0 & F_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & F_N \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \\ z_3(k) \\ \vdots \\ z_N(k) \end{bmatrix} \quad (5.12)$$

where $\Delta P_{tie,1}(k+1)$, $\Delta P_{tie,2}(k+1)$, $\Delta P_{tie,3}(k+1), \dots, \Delta P_{tie,N}(k+1)$ are the discrete tie-line power exchanges of the N control areas, $F_1, F_2, F_3, \dots, F_N$ are the interaction sub-matrices of the N control areas and $z_1, z_2, z_3, \dots, z_N$ are the interaction variables for the N control areas.

As a result of the preceding discussion, the interconnected power system's interaction sub-matrices comprise of four control areas, each having non-reheat, reheat, hydro, and gas generators, as illustrated in Figure 5.2, which may be calculated as follows:

Discrete tie-line power exchange $\Delta P_{tie,12}$ between areas 1 and 2 can be obtained as follows:

$$\Delta P_{tie,12}(k+1) = 2\pi T_{12}[\Delta f_1 - \Delta f_2] \quad (5.13)$$

Similarly, the discrete tie-line power exchange between areas 1 and 3 as

$$\Delta P_{tie,13}(k+1) = 2\pi T_{13}[\Delta f_1 - \Delta f_3] \quad (5.14)$$

and the discrete tie-line power exchange between areas 1 and 4 is given by

$$\Delta P_{tie,14}(k+1) = 2\pi T_{14}[\Delta f_1 - \Delta f_4] \quad (5.15)$$

Summing Equations (5.13), (5.14),(5.15) and assuming $T_{12} = T_{13} = T_{14}$, the total discrete tie-line power exchange between areas 1 and the other three areas is calculated as follows:

$$\Delta P_{tie,1}(k+1) = \Delta P_{tie,12}(k+1) + \Delta P_{tie,13}(k+1) + \Delta P_{tie,14}(k+1) = 2\pi T_{12}[3\Delta f_1 - \Delta f_2 - \Delta f_3 - \Delta f_4] \quad (5.16)$$

In the same way, discrete tie-line power exchange $\Delta P_{tie,21}$ between areas 2 and 1 is calculated as

$$\Delta P_{tie,21}(k+1) = 2\pi T_{21}[\Delta f_2 - \Delta f_1] \quad (5.17)$$

Similarly, the discrete tie-line power exchange between areas 2 and 3

$$\Delta P_{tie,23}(k+1) = 2\pi T_{23}[\Delta f_2 - \Delta f_3] \quad (5.18)$$

Summing Equations (5.17), (5.18),and assuming $T_{12} = T_{21} = T_{23}$, the total discrete tie-line power exchange between area 2 and the other two areas is calculated as

$$\Delta P_{tie,2} = \Delta P_{tie,21}(k+1) + \Delta P_{tie,23}(k+1) = 2\pi T_{12}[2\Delta f_2 - \Delta f_1 - \Delta f_3] \quad (5.19)$$

In the same way, the discrete tie-line power exchange $\Delta P_{tie,31}$ between areas 3 and 4 is obtained as follows:

$$\Delta P_{tie,12}(k+1) = 2\pi T_{31}[\Delta f_3 - \Delta f_1] \quad (5.20)$$

Similarly, the discrete tie-line power exchange between areas 3 and 2

$$\Delta P_{tie,3}(k+1) = 2\pi T_{32}[\Delta f_3 - \Delta f_2] \quad (5.21)$$

and the discrete tie-line power exchange between areas 3 and 4 is given by

$$\Delta P_{tie,34}(k+1) = 2\pi T_{34}[\Delta f_3 - \Delta f_4] \quad (5.22)$$

Summing Equations (5.20), (5.21),(5.22), and assuming $T_{12} = T_{31} = T_{32} = T_{34}$, the total discrete tie-line power exchange between area 1 and the other three areas is calculated as

$$\Delta P_{tie,3}(k+1) = \Delta P_{tie,31}(k+1) + \Delta P_{tie,32}(k+1) + \Delta P_{tie,34}(k+1) = 2\pi T_{12}[3\Delta f_3 - \Delta f_1 - \Delta f_2 - \Delta f_4] \quad (5.23)$$

Finally, the discrete tie-line power exchange $\Delta P_{tie,41}$ between areas 4 and 1 is obtained as follows:

$$\Delta P_{tie,41}(k+1) = 2\pi T_{41}[\Delta f_4 - \Delta f_1] \quad (5.24)$$

and, the discrete tie-line power exchange between areas 4 and 3

$$\Delta P_{tie,43}(k+1) = 2\pi T_{43}[\Delta f_4 - \Delta f_3] \quad (5.25)$$

Summing Equations (5.24, (5.25) and assuming $T_{12} = T_{41} = T_{43}$, the total discrete tie-line power exchange between area 4 and the other two areas is calculated as

$$\Delta P_{tie,4}(k+1) = \Delta P_{tie,41}(k+1) + \Delta P_{tie,42}(k+1) = 2\pi T_{12}[2\Delta f_4 - \Delta f_1 - \Delta f_3] \quad (5.26)$$

Based on equations (5.16), (5.19), (5.23), and (5.26), the tie-line power exchange equations can be written as:

$$\begin{aligned} \Delta P_{tie,1}(k+1) &= 2\pi T_{12}(3x_{11} - x_{21} - x_{31} - x_{41}) \\ &= 2\pi T_{12}(3z_{11} - z_{12} - z_{13} - z_{14}) \\ \Delta P_{tie,2}(k+1) &= 2\pi T_{12}(2x_{21} - x_{11} - x_{31} - x_{41}) = 2\pi T_{12}(2z_{22} - z_{21} - z_{23}) \\ \Delta P_{tie,3}(k+1) &= 2\pi T_{12}(x_{31} - x_{11} - x_{21} - x_{41}) \\ &= 2\pi T_{12}(3z_{33} - z_{31} - z_{32} - z_{34}) \\ \Delta P_{tie,4}(k+1) &= 2\pi T_{12}(x_{41} - x_{11} - x_{31}) = 2\pi T_{12}(2z_{44} - z_{41} - z_{43}) \end{aligned} \quad (5.27)$$

From equation (5.27) and ignoring the non-coupling interaction variables such $z_{11} = z_{22} = z_{33} = z_{44} = 0$ because $\dot{z}_{11} = \dot{z}_{22} = \dot{z}_{33} = \dot{z}_{44} = 0$, a four subsystems interaction matrices F_1 , F_2 , F_3 and F_4 can be obtained as follows:

$$\begin{aligned} F_1 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{12} & -2\pi T_{13} & -2\pi T_{14} \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{21} & -2\pi T_{23} & 0 \end{bmatrix}, \\ F_3 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{31} & 2\pi T_{32} & -2\pi T_{34} \end{bmatrix}, F_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{41} & 0 & -2\pi T_{43} \end{bmatrix} \end{aligned}$$

As can be seen, the interaction sub-matrices are calculated by adding discrete deviations of the tie-line power between regions when they are related with a given control area.

Additionally, coupling variables between control regions are employed to detect meaningful interaction signals. Because the tie-line power variation across areas varies with frequency, the interaction signals for each area are represented as frequency deviation variables. The position of these signals is thus determined by how the tie-line power deviation variables are specified in the power system matrix.

Next, the same principles are used to determine the dynamic model interconnection of each subsystem. Thus, the discrete equation of the formula $\mathbf{z}_i(k+1) = \mathbf{A}_{z_i} \mathbf{z}_i(k)$ for N control areas is

$$\begin{bmatrix} \mathbf{z}_1(k+1) \\ \mathbf{z}_2(k+1) \\ \mathbf{z}_3(k+1) \\ \vdots \\ \mathbf{z}_N(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{z1} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{A}_{z2} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{A}_{z3} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{A}_{zN} \end{bmatrix} \begin{bmatrix} \mathbf{z}_1(k) \\ \mathbf{z}_2(k) \\ \mathbf{z}_3(k) \\ \vdots \\ \mathbf{z}_N(k) \end{bmatrix} \quad (5.28)$$

On the basis of Equation (5.28) and Equations (5.7) - (5.10), the discrete dynamic sub-matrices of interconnections $\mathbf{z}_1$, $\mathbf{z}_2$, $\mathbf{z}_3$ and $\mathbf{z}_4$ of a power system with four control areas can be written in the following way:

$$\begin{aligned} \mathbf{A}_{z1} &= \begin{bmatrix} -\frac{K_{ps2}}{T_{p2}} & 0 & 0 \\ 0 & -\frac{K_{ps3}}{T_{p3}} & 0 \\ 0 & 0 & -\frac{K_{ps4}}{T_{p4}} \end{bmatrix}, \mathbf{A}_{z2} = \begin{bmatrix} -\frac{K_{ps1}}{T_{p1}} & 0 & 0 \\ 0 & -\frac{K_{ps3}}{T_{p3}} & 0 \\ 0 & 0 & -\frac{K_{ps4}}{T_{p4}} \end{bmatrix}, \\ \mathbf{A}_{z3} &= \begin{bmatrix} -\frac{K_{ps1}}{T_{p1}} & 0 & 0 \\ 0 & -\frac{K_{ps2}}{T_{p2}} & 0 \\ 0 & 0 & -\frac{K_{ps4}}{T_{p4}} \end{bmatrix}, \mathbf{A}_{z4} = \begin{bmatrix} -\frac{K_{ps1}}{T_{p1}} & 0 & 0 \\ 0 & -\frac{K_{ps2}}{T_{p2}} & 0 \\ 0 & 0 & -\frac{K_{ps3}}{T_{p3}} \end{bmatrix} \end{aligned} \quad (5.29)$$

where K_{ps1} , K_{ps2} , K_{ps3} and K_{ps4} are the gains of the power system with four control area respectively, $\mathbf{A}_{z1}$, $\mathbf{A}_{z2}$, $\mathbf{A}_{z3}$ and $\mathbf{A}_{z4}$ are dynamic sub-matrices for four control areas respectively, and T_{p1} , T_{p2} , T_{p3} and T_{p4} are the time constant for four control areas respectively.

According to equation (5.3) and equations (5.3-5.10) and equation (5.28), the state space equations for N interconnected subsystems can be written as follows:

$$\begin{aligned}
\dot{\bar{X}}_1 &= A_1 \bar{X}_1 + B_1 u_1 + F_1 \bar{X}_1 + \Gamma_1 \Delta P_{d1} \\
\dot{\bar{X}}_2 &= A_2 \bar{X}_2 + B_2 u_2 + F_2 \bar{X}_2 + \Gamma_2 \Delta P_{d2} \\
\dot{\bar{X}}_3 &= A_3 \bar{X}_3 + B_3 u_3 + F_3 \bar{X}_3 + \Gamma_3 \Delta P_{d3} \\
\dot{\bar{X}}_4 &= A_4 \bar{X}_4 + B_4 u_4 + F_4 \bar{X}_4 + \Gamma_4 \Delta P_{d4} \\
&\vdots \\
\dot{\bar{X}}_N &= A_N \bar{X}_N + B_N u_N + F_N \bar{X}_N + \Gamma_N \Delta P_{dN}
\end{aligned} \tag{5.30}$$

From equation (5.30), the state space equations for 4 interconnected subsystems can be obtained as follows:

$$A_1 = \begin{bmatrix} -\frac{1}{T_{p1}} & \frac{k_{ps1}}{T_{p1}} & 0 & 0 & \frac{k_{ps1}}{T_{p1}} \\ 0 & -\frac{1}{T_{r1}} & \frac{1}{T_{r1}} & 0 & 0 \\ -\frac{1}{R_n T_{g1}} & 0 & -\frac{1}{T_{g1}} & 0 & 0 \\ \beta_1 & 0 & 0 & 0 & 1 \\ 2\pi T_{12} & 0 & 0 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{G1}} \\ 0 \\ 0 \end{bmatrix}, F_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{12} & -2\pi T_{13} & -2\pi T_{14} \end{bmatrix}, \Gamma_1 = \begin{bmatrix} -\frac{K_{ps1}}{T_{p1}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.31}$$

$$A_2 = \begin{bmatrix} -\frac{1}{T_{p2}} & \frac{k_{ps2}}{T_{p2}} & 0 & 0 & 0 & \frac{k_{ps2}}{T_{p2}} \\ 0 & -\frac{1}{T_{r1}} & -\frac{1}{T_{r1}} - \frac{k_{r1}}{T_{t2}} & \frac{1}{T_{t2}} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{t2}} & \frac{1}{T_{t2}} & 0 & 0 \\ -\frac{1}{R_{th} T_{g2}} & 0 & 0 & -\frac{1}{T_{g2}} & 0 & 0 \\ \beta_2 & 0 & 0 & 0 & 0 & 1 \\ 2\pi T_{23} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{g2}} \\ 0 \\ 0 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{21} & -2\pi T_{23} & 0 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} -\frac{K_{ps2}}{T_{p2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.32}$$

$$A_3 = \begin{bmatrix} -\frac{1}{T_{p3}} & \frac{k_{ps3}}{T_{p3}} & 0 & 0 & 0 & \frac{k_{ps3}}{T_{p3}} \\ -\frac{2T_r}{T_{gh} R_{hy} T_{rh}} & -\frac{1}{T_w} & \frac{2}{T_{rh}} + \frac{2}{T_w} & \frac{2}{T_{rh}} - \frac{2T_r}{T_{gh}} & 0 & 0 \\ -\frac{T_{rs}}{T_{gh} R_{hy} T_{rh}} & 0 & -\frac{1}{T_{rh}} & \frac{1}{T_{rh}} - \frac{T_r}{T_{gh}} & 0 & 0 \\ -\frac{1}{R_{hy} T_{gh}} & 0 & 0 & -\frac{1}{T_{gh}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2\pi T_{34} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ \frac{2\alpha_3}{T_{rh} T_{gh}} \\ \frac{\alpha_3}{T_{rh} T_{gh}} \\ \frac{\alpha_3}{T_{gh}} \\ 0 \\ 0 \end{bmatrix}, F_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{31} & -2\pi T_{32} & -2\pi T_{34} \end{bmatrix}, \Gamma_3 = \begin{bmatrix} -\frac{K_{ps3}}{T_{p3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.33}$$

$$\begin{aligned}
A_4 = & \begin{bmatrix} -\frac{1}{T_{p4}} & \frac{k_{ps4}}{T_{p4}} & 0 & 0 & 0 & 0 & \frac{k_{ps4}}{T_{p4}} \\ 0 & -\frac{1}{T_{cd}} & \frac{1}{T_{cd}} & 0 & 0 & 0 & 0 \\ -\frac{x_g T_{cb}}{T_f b_g R_g} & 0 & -\frac{1}{T_f} & \frac{1}{T_f} & -\frac{T_{cb}}{T_f Y_g} & \frac{T_{cb}}{T_f} \left(\frac{1}{Y_g} - \frac{x_g c_g}{b_g Y_g} \right) & 0 \\ -\frac{x_g}{Y_g b_g R_g} & 0 & 0 & \frac{1}{Y_g} & \frac{1}{Y_g} & \frac{x_g c_g}{b_g Y_g} & 0 \\ -\frac{x_g}{b_g R_g} & 0 & 0 & 0 & -\frac{c_g}{b_g} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \beta_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\pi T_{23} \end{bmatrix}, B_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\alpha_4 x_g}{b_g Y_g} \\ \frac{\alpha_4}{b_g} \\ 0 \\ 0 \end{bmatrix}, F_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2\pi T_{41} & 0 & -2\pi T_{43} \end{bmatrix}, \Gamma_4 = \begin{bmatrix} -\frac{K_{ps4}}{T_{p4}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5.34)
\end{aligned}$$

The matrices corresponding to four decentralized subsystems are represented by equations (5.31) - (5.34). The four generated subsystem models will be coupled with the subsystem interaction matrix and the interconnection matrix in the next section to construct the subsystem optimization problem.

5.2.3. Optimization Problem Formulation

The obtained subsystem models in Equation (5.30) are coupled with their respective interaction matrices and interconnection matrices from Equations (5.12) and (5.28) to generate the models of the enhanced subsystems illustrated in Equation (5.35).

$$\begin{aligned}\bar{\mathbf{x}}_i(\mathbf{k} + 1) &= \mathbf{A}_i(\mathbf{k}) + \mathbf{B}_i\mathbf{u}_i(\mathbf{k}) + \mathbf{F}_i\mathbf{z}_i(\mathbf{k}) + \mathbf{\Gamma}_i\mathbf{w}_i(\mathbf{k}) \\ \mathbf{z}_i(\mathbf{k} + 1) &= \mathbf{A}_{zi}\mathbf{z}_i(\mathbf{k}), \quad i = 1, 2, 3, \dots, N\end{aligned}\quad (5.35)$$

where $\mathbf{Q}_i \in \mathbb{R}^{n \times n}$ and $\mathbf{R}_i \in \mathbb{R}^{1 \times 1}$ are symmetric positive semi-definite matrices, $i = 1, 2, 3, \dots, N$ $\bar{\mathbf{x}}_i(\mathbf{k})$ is i^{th} subsystem state vector, $\mathbf{u}_i(\mathbf{k})$ is i^{th} subsystem control law vector, $\mathbf{z}_i(\mathbf{k})$ and $\mathbf{z}_i(\mathbf{k} + 1)$ are the interaction variables, $\mathbf{w}_i(\mathbf{k})$ i^{th} subsystem input disturbance vector, $\bar{\mathbf{A}}_i$ is i^{th} subsystem state matrix and $\bar{\mathbf{B}}_i$ is i^{th} control input matrix, and $\mathbf{\Gamma}_i \in \mathbb{R}^{n_i \times r_i}$ is load disturbance matrix.

The vectors $\bar{\mathbf{x}}_i(\mathbf{k})$ and $\mathbf{z}_i(\mathbf{k})$ are combined to form augmented subsystem vector $\bar{\mathbf{y}}_i^T = [\bar{\mathbf{x}}_i(\mathbf{k}) \quad \mathbf{z}_i(\mathbf{k})]^T$ for the augmented subsystem. Therefore, based on the authors Siljak, 2007), Arifai et al, 2011 and Vlahakisa et al, 2019), the augmented decentralized state space model is written as follows,

$$\begin{bmatrix} \bar{\mathbf{x}}_i(\mathbf{k} + 1) \\ \mathbf{z}_i(\mathbf{k} + 1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_i & \mathbf{F}_i \\ 0 & \mathbf{A}_{zi} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{x}}_i(\mathbf{k}) \\ \mathbf{z}_i(\mathbf{k}) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_i \\ 0 \end{bmatrix} \mathbf{u}_i(\mathbf{k}) + \begin{bmatrix} \mathbf{F}_i \\ 0 \end{bmatrix} \mathbf{z}_i(\mathbf{k}) + \begin{bmatrix} \mathbf{\Gamma}_i \\ 0 \end{bmatrix} \mathbf{w}_i(\mathbf{k}) \quad (5.36)$$

with:

$$\bar{\mathbf{A}}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{F}_i \\ 0 & \mathbf{A}_{zi} \end{bmatrix}, \bar{\mathbf{B}}_i = \begin{bmatrix} \mathbf{B}_i \\ 0 \end{bmatrix}, \bar{\mathbf{F}}_i = \begin{bmatrix} \mathbf{F}_i \\ 0 \end{bmatrix} \mathbf{z}_i, \text{ and } \bar{\mathbf{\Gamma}}_i = \begin{bmatrix} \mathbf{\Gamma}_i \\ 0 \end{bmatrix}$$

where, $\bar{\mathbf{A}}_i$ is the augmented subsystem state matrix of $n_i + 1 \times n_i + 1$ dimension and $\bar{\mathbf{B}}_i$ is the augmented subsystem control input matrix, $\bar{\mathbf{F}}_i$ is the augmented interconnection state matrix, and $\bar{\mathbf{\Gamma}}_i$ is the augmented subsystem disturbance matrix, all of $n_i + 1 \times r_i$ dimension.

The optimization procedure used in section 4.2 is applied to find sub-optimal control law $\mathbf{u}_i(\mathbf{t}) \in \mathbf{R}^1$ that minimizes N subsystems cost functions. the cost sub-function J_i used for the optimization procedure is defined as follows:

$$\min \bar{J}_i = \frac{1}{2} \sum_{k=k_0}^{k_f=\infty} y_i^T(k) \bar{Q}_i y_i(k) + \bar{u}_i^T(k) R_i \bar{u}_i(k) \quad (5.37)$$

Subject to the constraint:

$$\begin{aligned} \dot{\bar{y}}_i &= \bar{A}_i \bar{y}_i + \bar{B}_i \bar{u}_i(k) + \bar{F}_i \bar{z}_i(k) + \bar{\Gamma}_i \bar{w}_i(k) \\ \bar{y}_i(0) &= \begin{bmatrix} \bar{x}_i(0) \\ \bar{z}_i(0) \end{bmatrix}, i = 1, 2, 3, \dots, N \\ \bar{z}_i(k) &= \sum_{\substack{j=1 \\ j \neq i}}^4 L_{ij} z_j(k), z_i(k) \in R_i \end{aligned} \quad (5.38)$$

where, $\bar{Q}_i$ is state weighting matrix and $\bar{R}_i$ is the control weighting matrix. With

$$\bar{Q}_i = \begin{bmatrix} Q_i & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \bar{R}_i = R_i$$

The original dimension of the vectors and zeros represent in the matrix of the individual subsystem or control area can be used to identify expanded order of the system.

To create a discrete decentralized optimum quadratic automatic generator control (DOQAGC) for the subsystem given by Equation (5.38), a feedback control gain Law, L_i must be found. which reduces the non-optimal cost function

$$\bar{u}_i(k) = -L_i \bar{y}_i(k) \quad (5.39)$$

The control feedback gain is determined using the steady-state ricatti equation solution, which is provided as:

$$P_i(k) = \bar{Q}_i + \bar{A}_i^T \left\{ P(k+1) - P(k+1) [\bar{B}_i^T P(k) \bar{B}_i + \bar{R}_i]^{-1} \bar{B}_i^T P(k+1) \right\} \bar{A}_i \quad (5.40)$$

The values of L_i which minimizes the subsystems cost functions, is given by:

$$L_i = [\bar{R}_i + \bar{B}_i^T P(k) \bar{B}_i]^{-1} \bar{B}_i^T P(k) \bar{A}_i \quad (5.41)$$

5.2.4. Method of Functional Minimization for Decentralized Subsystems

The functional minimization approach outlined in section 4.3 is employed in this part to build the state and control weighting matrices, Q_i and R_i for decentralized subsystems. The objective is to divide the cost function in Equation (4.50) into sub-cost functions depending on the state and control divisions, as shown in Equation (4.25). (5.29). As a result, the sub-cost function for the, i^{th} . region is defined as follows:

$$J_i = \frac{1}{2} \sum_{k=k_0}^{\infty} (AEC_i)^2 + \left(\sum_{k=k_0}^{\infty} AEC_i \right)^2 + u_i^2 \quad (5.42)$$

where AEC_i and $\sum AEC_i$ are the i^{th} area control error and integral of the i^{th} area control error respectively, u_i is the i^{th} control signal in the interconnected power system, **where** $i = 1, 2, 3, \dots, N$. The i^{th} area control error (AEC_i) is defined as follow:

$$AEC_i = \beta_i \Delta f_i + \Delta P_{tiei} \quad (5.43)$$

where, Δf_i is the frequency deviations for i^{th} area, the ΔP_{tiei} is the Tie-line power deviations from i^{th} area to other areas .

Similarly, the integral of the area control errors for the i^{th} control area can be defined by

$$IACE_i = \sum_{k=k_0}^{\infty} (\beta_i \Delta f_i + \Delta P_{tiei}) \quad (5.44)$$

where $IACE_i$ is the integrals of AEC_i for the i^{th} control area. Substituting Equations (5.43 - 5.44) to Equation (5.43), the sub-cost function becomes:

$$J_i = \frac{1}{2} \sum_{k=k_0}^{\infty} (\beta_i \Delta f_i + \Delta P_{tiei})^2 + (IACE_i)^2 + u_i^2 \quad (5.45)$$

with the sub-cost functional

$$f_i(.) = \frac{1}{2} ((\beta_i \Delta f_i + \Delta P_{tiei})^2 + (IACE_i)^2 + u_i^2) \quad (5.46)$$

This sub-functional, as stated in section 4.3, may be utilized systematically to create the weighting matrices, Q_i and R_i as follows:

1. Write the AEC_i and $IACE_i$ in terms of the state variables and substitute them into the sub-cost functional in Equation (5.46). This process makes the cost function to become a function of the state variable $\bar{x}_i$ and control signal u_i . It should be noted that the length of the state's vector is determined by the types and quantities of power generators in each control region, as well as the Tie-line configuration.

2. We take the first partial derivatives of the sub-functional $f_i(.)$ with respect to all states of i^{th} control area and the i^{th} area control signal.

In this regard, the partial derivatives with respect to frequency and the Tie-line states for i^{th} control area can be obtained in a form of first ordinary differential equations (ODEs) as follows:

$$\frac{df_i}{d\Delta f_i} = \beta_i^2 \Delta f_i + \Delta P_{tiei} \quad (5.47)$$

and:

$$\frac{df_i}{d\Delta P_{tiei}} = \beta_i \Delta f_i + \Delta P_{tiei} \quad (5.48)$$

In the same way, taking the partial derivatives with respect to the Integral of the i^{th} Area Control Error ($IACE_i$) states of i^{th} area control, the following first-order ODEs is obtained:

$$\frac{df_i}{dIACE_i} = IACE_i \quad (5.49)$$

Equations (5.47) - (7.48) can partition the state weighing matrix Q_k obtained in chapter 4 , section 4.3 into the following decentralized weighing matrices:

$$Q_k = \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \end{bmatrix} \quad (5.50)$$

Where the N is number of areas in the interconnected power systems. Similarly, the first-order partial derivatives with respect to control input signal for the i^{th} control area is obtained as follows:

$$\frac{df_i}{du_i} = u_i \quad (5.51)$$

Then, from the Equation (5.51) the control weighing matrix R_k is partitioned to decentralized control signals as

$$R_k = \begin{bmatrix} u_1 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & u_N \end{bmatrix} \quad (5.52)$$

5.2.5. Summary

The overlapping decomposition approach is widely explored and used to decompose FAIPS into four subsystems using different types of generators with non-reheat, reheat, hydro and gas turbines. For the decomposing subsystems of the multi-area interconnected power system, a generic form of decentralized state space equation is designed. The optimization problems are formulated using LQR and the augmenting subsystems. As a result, based on the number of regions in the interconnected power systems, the complex cost function is broken into simple sub-cost functions. The devised functional minimization techniques for choosing states and control weighting matrices ($\mathbf{Q}_k$ and $\mathbf{R}_k$). As a consequence, the overlapping decomposition approach and the functional minimization method have simplified and generated subsystems with smaller dimensions as compared to the complex central interconnected power system's dimensions.

5.3. Decomposed models of identical two are interconnected power system

This section describes two options for creating subsystems from an interconnected power system. The first scenario employs a two-area power system with identical non-reheat turbines, as detailed in Section 3.4. (3.35). The second scenario, on the other hand, features a more complicated interconnected power system comprised of four multi-area power systems with non-reheat, reheat, gas, and hydro turbines. The overlapping decomposition approach is used to decompose this power system into four subsystems. Interconnections between control areas, as well as the inclusion of integral of ACEs, will enhance the dimensions of the four subsystems. Section 5.5 develops and discusses the four formal subsystems in further detail. The standard parameters of non-reheat, reheat thermal, gas, and hydro generators are then numerically calculated to determine the applicable subsystem models (Hakimuddin et al, 2020 & Parmar, Majhi & Kothar, 2012). Table 5.8, in section 5.4, provides the numerical values of the non-reheat turbine, reheat turbine, gas, and hydro plants.

5.3.1. Decomposed Models of ITAPS considering non-reheat turbines

Section 3.4 decomposed the subsystem models for the ITAPS with non-reheat turbine. For decentralized discrete optimal quadratic AGC (DOQAGC) analysis, the block diagram of the i th subsystem depicted in Figure 5.3 is used. Each subsystem is made up of a thermal plant with a non-reheat turbine, a tie-line power deviation, and area control (Kunder,1994) (Arrafi et al, 2011). In accordance with current power system control nomenclature, the control law, u_i , shall be referred to as the state feedback control law and the integral controller, IACE-i, as the integral controller.

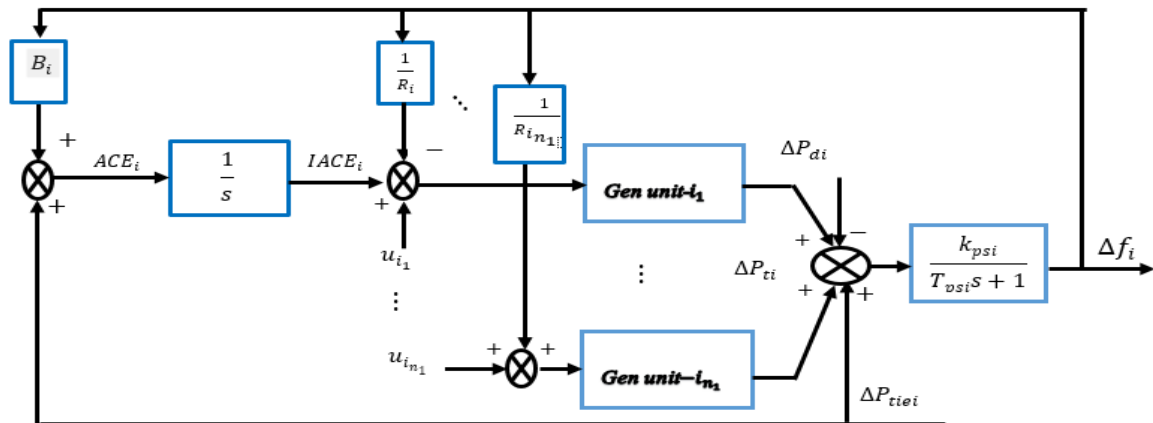


Figure 5.3. Single block diagram of ITAPS with non-reheat turbines, area control error (ACE_i), frequency deviation (Δf_i) and tie line power deviation (ΔP_{tiei}) (Bevrani, 2014).

The augmentation process indicated in Equation (5.11) is used for the subsystems specified in Equations (3.62) and (3.63), as well as the interconnections described in Equations (3.56). (3.57). Table 4.1 in Chapter 4 contains the characteristics and information for the thermal power system with non-reheat turbine. The numerical values for the subsystem matrices for area 1 and area 2 are as follows:

$$\bar{A}_1 = \begin{bmatrix} -0.05 & 6 & 0 & 0 & -6 & 0 \\ 0 & -3.3 & 3.3 & 0 & 0 & 0 \\ -5.21 & 0 & -12.5 & 12.5 & 0 & 0 \\ 0.425 & 0 & 0 & 0 & 1 & 0 \\ 0.5438 & 0 & 0 & 0 & 0 & -0.5438 \\ -6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 12.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_1 = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5.44 \end{bmatrix}, \quad (5.53)$$

$$\bar{A}_2 = \begin{bmatrix} -0.05 & 6 & 0 & 0 & 6 & 0 \\ 0 & -3.3 & 3.3 & 0 & 0 & 0 \\ -5.21 & 0 & -12.5 & 12.5 & 0 & 0 \\ 0.425 & 0 & 0 & 0 & -1 & 0 \\ 0.5438 & 0 & 0 & 0 & 0 & -0.5438 \\ -6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 12.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_2 = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5.44 \end{bmatrix} \quad (5.54)$$

For this task, these matrices are discretized using the step Euler approach and sample time outlined in Chapter 4.5.1 (Kgel & Findeisen, 2015) and (Boukas & AL-Sunni, 2011). As a consequence, the discrete decentralized matrices for the two subsystems are:

$$\bar{A}_{k1} = \begin{bmatrix} 0.9984 & 0.1884 & 0 & 0 & -0.1884 & 0 \\ 0 & 0.8953 & 0.1047 & 0 & 0 & 0 \\ -0.1635 & 0 & 0.6075 & 0.3925 & 0 & 0 \\ 0.0133 & 0 & 0 & 1 & 0.0314 & 0 \\ -0.0171 & 0 & 0 & 0 & 1 & 0.0171 \\ -0.1884 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{B}_{k1} = \begin{bmatrix} 0 \\ 0 \\ 0.2925 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_{k1} = \begin{bmatrix} -0.1884 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_{k1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.0171 \end{bmatrix}$$

$$\bar{A}_{k2} = \begin{bmatrix} 0.9984 & 0.1884 & 0 & 0 & 0.1884 & 0 \\ 0 & 0.8953 & 0.1047 & 0 & 0 & 0 \\ -0.1635 & 0 & 0.6075 & 0.3925 & 0 & 0 \\ 0.0133 & 0 & 0 & 1 & -0.0314 & 0 \\ -0.0171 & 0 & 0 & 0 & 1 & -0.0171 \\ -0.1884 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{B}_{k2} = \begin{bmatrix} 0 \\ 0 \\ 0.2925 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_{k2} = \begin{bmatrix} -0.1884 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_{k2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.0171 \end{bmatrix}$$

Each subsystem in Figure 5.3, has five sub-states variables where $\Delta f_1 = x_{11}$, $T_{t1} = x_{12}$, $T_{g1} = x_{13}$, $IACE_1 = x_{14}$, $\Delta P_{tie12} = x_{15}$, $\Delta f_2 = x_{21}$, $T_{t2} = x_{22}$, $T_{g2} = x_{23}$, $IACE_2 = x_{24}$, $\Delta P_{tie21} = x_{25}$. The interaction variables are defined as , $z_1 = x_{16}$ and , $z_2 = x_{26}$. The sub-cost functions J_i is given in Equation (5.45). Based on the FFM in Equations (5.46) - (5.52) and the design consideration discussed in section 4.3, the state and control weighting matrices $\bar{Q}_i$ and $\bar{R}_i$ were formulated for both subsystems. The sub-excursions ACE_i , $IACE_i$ and control input u_i are substituted into Equation (5.42). As a result, the sub-cost function J_i for two subsystems can be written as:

$$J_1 = \frac{1}{2} \sum_{k=k_0}^{\infty} [B_1^2 x_{11}^2 + 2\beta_1 x_{11} x_{15} + x_{15}^2] + (x_{14})^2 + u_1^2$$

$$J_2 = \frac{1}{2} \sum_{k=k_0}^{\infty} [B_2^2 x_{21}^2 - 2\beta_2 x_{21} x_{25} + x_{25}^2] + (x_{24})^2 + u_2^2 = \quad (5.55)$$

The partial differential equations with regard to the sub-state variables for the two area power systems are found using equation (5.46) as follows:

- Subsystem 1:

$$\begin{aligned} 1/2 \, df_1(.) / dx_{11} &= \beta_1^2 x_{11} + \beta_1 x_{15} \\ 1/2 \, df_1(.) / dx_{12} &= 0 \\ 1/2 \, df_1(.) / dx_{13} &= 0 \\ 1/2 \, df_1(.) / dx_{14} &= x_{14} \\ 1/2 \, df_1(.) / dx_{15} &= \beta_1 x_1 + x_{15} \end{aligned} \quad (5.56)$$

Furthermore, when the control input signal is considered along with the interconnection signal, the first-order partial derivatives are as follows:

$$\begin{aligned} df_1(.) / du_1 &= u_1 \\ df_1(.) / dz_1 &= 0 \end{aligned} \quad (5.57)$$

- Subsystem 2:

$$\begin{aligned}
1/2 \, df_2(\cdot)/dx_{11} &= \beta_2^2 x_{21} - \beta_2 x_{25} \\
1/2 \, df_2(\cdot)/dx_{12} &= 0 \\
1/2 \, df_2(\cdot)/dx_{13} &= 0 \\
1/2 \, df_2(\cdot)/dx_{14} &= x_{24} \\
1/2 \, df_2(\cdot)/dx_{15} &= -\beta_2 x_1 + x_{25}
\end{aligned} \tag{5.58}$$

and the first-order partial derivatives with respect to control input signal u_2 and interaction signal z_2 are obtained as follows:

$$\begin{aligned}
df_2(\cdot)/du_2 &= u_2 \\
df_2(\cdot)/dz_2 &= 0
\end{aligned} \tag{5.59}$$

The partial differential equations (5.56) -(5.59) can be organized in the sub-state weighting matrix $\bar{Q}_i$ and sub-control weighting matrix $\bar{R}_i$ as.

$$\bar{Q}_1 = \begin{bmatrix} \beta_1^2 & 0 & 0 & 0 & \beta_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{Q}_2 = \begin{bmatrix} \beta_2^2 & 0 & 0 & 0 & -\beta_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\beta_2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{R}_1 = \bar{R}_2 = 1$$

5.3.2. Simulation Results and Discussion

The design process described in section 5.2 is applied. Table 4.1 in Chapter 4 shows the subsystem parameters and data for the thermal power system with non-reheat turbine. L_i , the suboptimal feedback gains, are achieved. The simulation algorithms are created in MATLAB and run for 1800 iterations with, $k_0 = 0$ and, $k_f = 1800$.

For two separate scenarios, computer simulations were run with suboptimal controller gains. First, the simulated responses of the closed loop subsystems are derived by setting the starting values of the step load perturbations in both subsystems to 0.01 pu while maintaining the rest of the variables at zeros. Second, the simulation results of the subsystems are acquired for 1% SLP) which are introduced to each subsystem concurrently while the remainder of the variables are initially set to zero values. Here are the computational results for discrete sub-weighting matrices $\bar{Q}_i$ and $\bar{R}_i$:

$$\bar{Q}_{k1} = \begin{bmatrix} 1.0057 & 0 & 0 & 0 & 0.0133 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0314 & 0 & 0 \\ 0.0133 & 0 & 0 & 0 & 1.0314 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{Q}_{k2} = \begin{bmatrix} 1.0057 & 0 & 0 & 0 & -0.0133 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0314 & 0 & 0 \\ -0.0133 & 0 & 0 & 0 & 1.0314 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{R}_1 = \bar{R}_2 = 1$$

According to equation (5.66), the decentralized subsystems yield suboptimal feedback gains:

$$\bar{L}_1 = [7.438 \quad 7.1350 \quad 2.0331 \quad 3.0145 \quad -4.3292 \quad -2.4222]$$

and

$$\bar{L}_2 = [7.6062 \quad 7.0335 \quad 2.0209 \quad -1.0199 \quad 16.7879 \quad -4.9870]$$

Figures (5.4) to (5.6) show simulation results for Case1 for frequency deviations, tie-line power deviations, ACE integrals, and interconnection states. The frequency deviation of the subsystem 1 and 2 is shown in Figure 5.4 below. The Step Load perturbation (SLP) is applied at 0s, the both subsystems have maximum overshoot of -0.016 frequency deviations and the systems achieved steady state at about 9 seconds. It is noted that both subsystem 1 and 2 have similar frequency deviation response characteristics as shown in Figure 5.4.

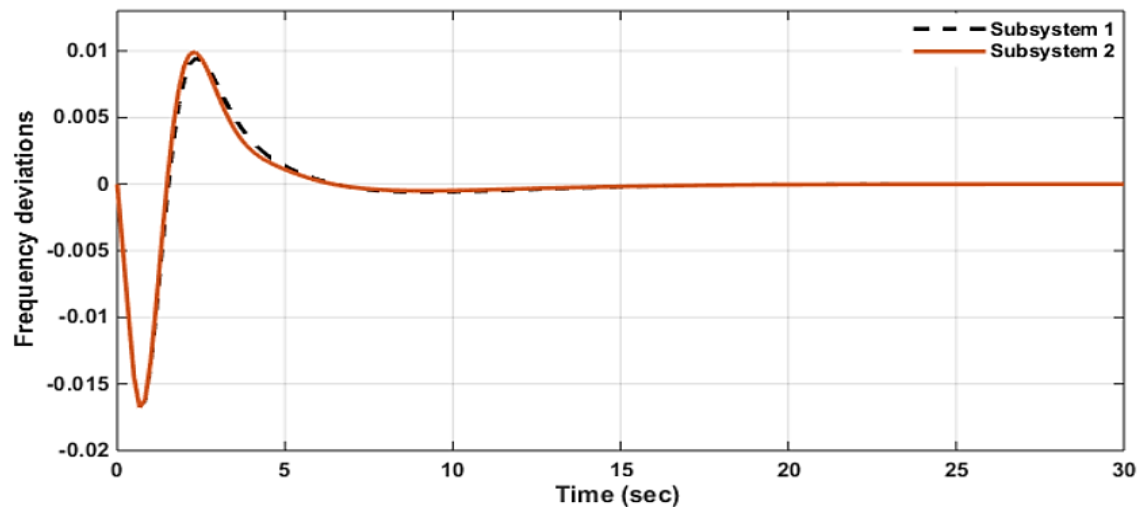


Figure 5.4. Frequency deviation for both subsystems.

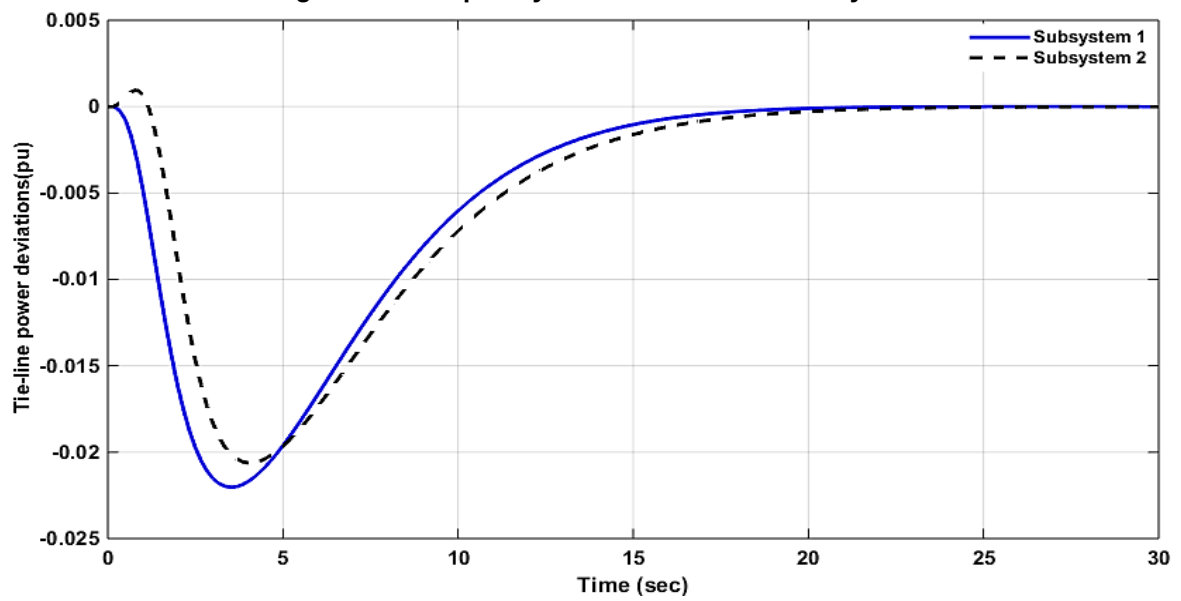


Figure 5.5. Tie-line power deviation for both systems.

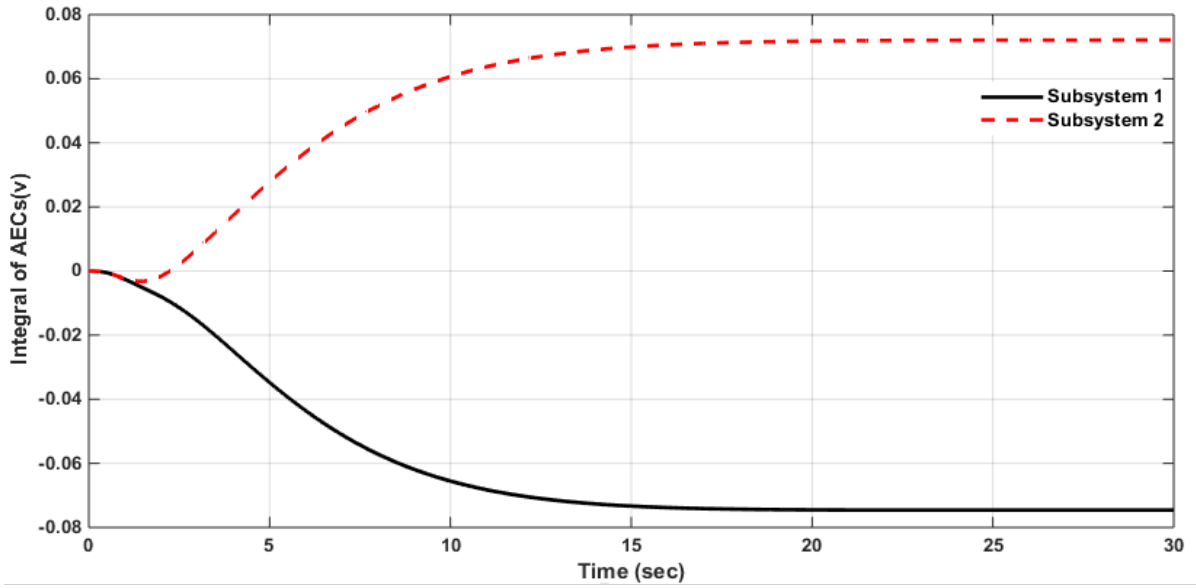


Figure 5.6. Integral of IACEs simulation result for both subsystems.

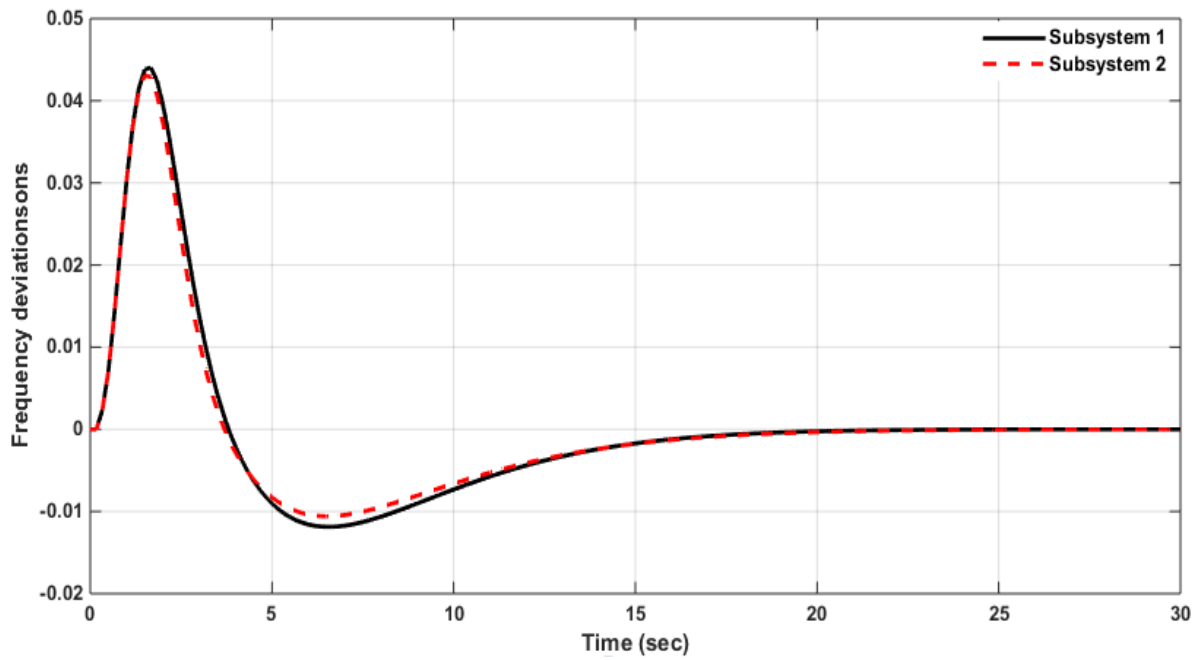


Figure 5.7. The interactions simulation results for both subsystems

Figures (5.8) to (5.11) show the simulation results for Case 2 for frequency deviations, tie-line power deviations, ACE integrals, and interconnection states, respectively.

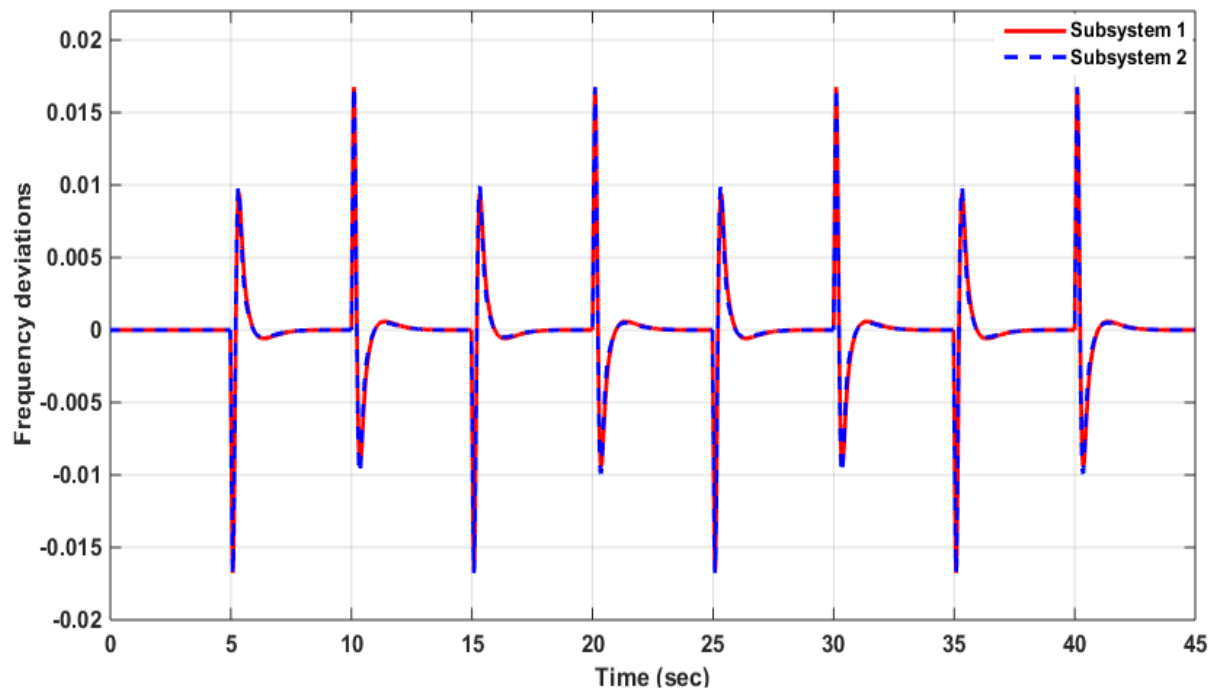


Figure 5.8. Frequency deviation results of both subsystems with simultaneous SLPs.

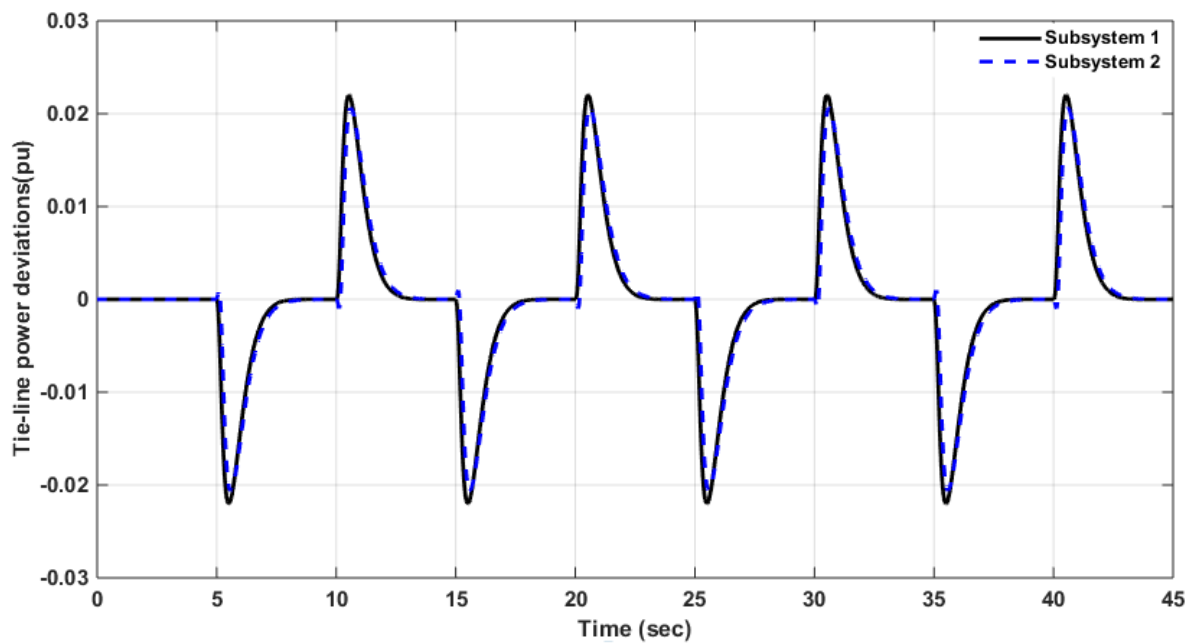


Figure 5.9. Tie-line power deviation result in both systems with simultaneous SLPs.

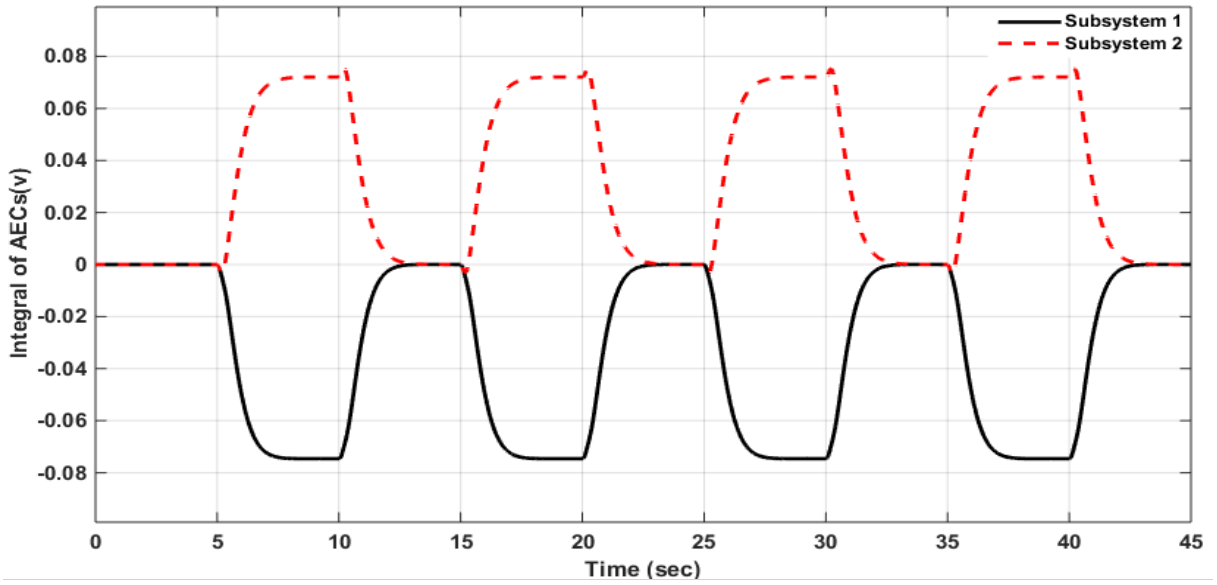


Figure 5.10. The simulation result of the integral of IACEs in both subsystems for simultaneous SLPs.

Tables 5.1 and 5.2 describe the simulation results for frequency deviations and tie-line power deviations for both subsystems. When step load perturbations are applied, the DOQAGC controller produces almost the same frequency deviation responses of settling time and overshoots for the two cases ($\Delta P_{Di}(0) = 0.01\text{pu}$) at $t = 0$ sec and simultaneous $\Delta P_{Di} = 0.01\text{Pu}$).

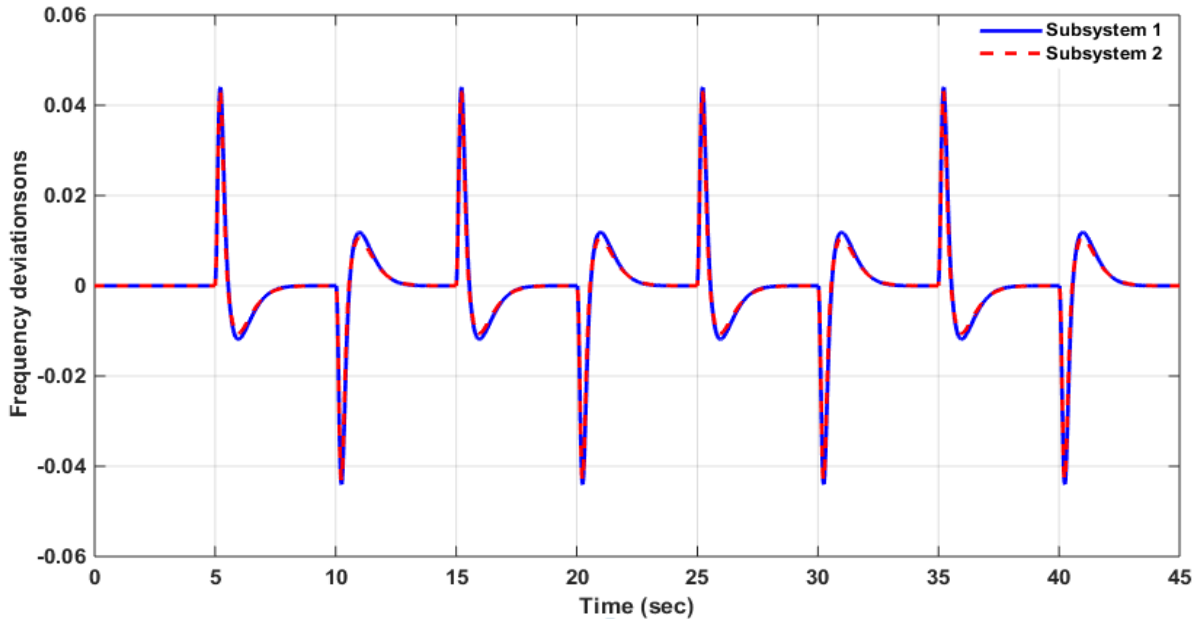


Figure 5.11. The interactions simulation results for both subsystems for simultaneous SLPs.

The settling time for the tie-line power deviations is the same, but the peak overshoot of subsystem 1 in both cases is 8.7 percent higher than that of subsystem 2. The peak OS of ΔP_{tie21} , is improved by 8.2 and 1.6 percent, respectively, for subsystem 2 compared to

subsystem 1. The disturbance rejection time responses for both subsystem cases are approximately same, 3 seconds for each subsystem.

Table 5.1. DOQAGC's performance in terms of Peak OS, settling time, and DRRT.

Frequency Deviations					
Criterion	Subsystem	ST (sec)	Peak OS	DRRT (sec)	Steady state error
$\Delta P_{Di} = 0.01 \text{ Pu}$ at $t = 0 \text{ sec}$	1	10	-0.016	-	0
	2	10	-0.016	-	0
Difference %	-	0%	0%	-	0%
Simultaneous $\Delta P_{Di} = 0.01 \text{ Pu}$	1	-	-0.016	1	0
	2	-	-0.016	1	0
Difference %	-	0%	0%	0%	0%

Table 5.2. DOQAGC performance in terms of Peak OS, ST, and DRRT with simultaneous disturbances

Tie-line power deviations (pu)					
Criterion	Subsystem	Settling time	Peak OS	DRRT	Steady state error
$\Delta P_{Di} = 0.01 \text{ Pu}$ at $t = 0 \text{ sec}$	1	15 sec	0.023	-	0
	2	15 sec	0.021	-	0
Difference %		0%	8.7%	-	0%
Simultaneous $\Delta P_{Di} = 0.01 \text{ Pu}$	1	-	0.023	3 sec	0
	2	-	0.021	3 sec	0
Difference%	-	-	8.7%	0%	0%

5.3.3 Comparative analysis of the ITAPS using COQAGC and DOQAGC control methods

This section compares the closed system responses for the centralized ITAPS and the decentralized subsystems. Table 4.3 summarizes the parameter settings for the two scenarios, with input disturbances set at 0.01pu and simultaneous disturbances set at 0.01pu MW. Figure 5.12 depicts the comparative simulation responses of scenario 1's centralized

ITAPS and decentralized subsystems. When compared to the DOQAGC, the COQAGC has a lower peak overshoot and a slightly longer settling time with 0% steady-state error. The quantitative comparison is summarized in Tables 5.4 and 5.5, where the peak OS of Δf_1 for DOQAGC is increased by percentage of 30% while the peak OS of Δf_2 and ΔP_{tie12} for COQAGC are increased higher by 12.5 % and 74 compared of that of DOQAGC. On the other hand, DOQAGC improved the settling time of Δf_1 and Δf_2 by 3%, and 2% respectively while COQAGC has better improvement percentage for settling time of ΔP_{tie12} by 37.5 of that of DOQAGC.

Figure 5.13 depicts the dynamic comparative responses of case 2's centralized ITAPS and decentralized subsystems. The dynamic responses of DOQAGC are shown to have a zero steady-state error, relatively more minor peak overshoot for Δf_1 and Δf_2 and a more significant peak overshoot for ΔP_{tie12} as compared to that of COQAGC. Additionally, DOQAGC exhibits zero steady-state error, shorter disturbance rejection time response for Δf_1 and Δf_2 and a longer disturbance rejection time response for ΔP_{tie12} as compared with COQAGC. As shown in Tables 5.6 and 5.7, DOQAGC showed an improvement of 53.3% and 53.3% over OQAGC in terms of Δf_1 , Δf_2 and while OQAGC showed an improvement of 71.4% in terms of ΔP_{tie12} over DOQAGC. In comparison to COQAGC, the DOQAGC improved the settling times of Δf_1 and Δf_2 by 36% and 36%, respectively. In contrast, COQAGC improved the settling time of ΔP_{tie12} by 30% compared to DOQAGC.

Table 5.3. Initial parameters settings for two simulations cases.

Type of Controller	Parameters Area			
	Case	Subsystem	Δf_i (Hz)	ΔP_{Di} (pu)
CQQAGC with $\Delta P_{Di} = 0.01$ pu, set at t = 0. sec	1.1	1	0.01	0.0
		2	0.01	0.0
CQQAGC with simultaneous disturbances set at 0.01puMW	1.2	1	0.0	0.01
		2	0.0	0.01
CQQAGC with $\Delta P_{Di} = 0.01$ pu, set at t = 0 sec	2.1.	1	0.01	0.0
		2	0.01	0.0
CQQAGC with simultaneous disturbances set at 0.01pu MW	2.2	1	0.0	0.01
		2	0.0	0.01

Table 5-4. Case 1 dynamic response comparison in terms of peak Overshoots (Peak OS)

Type of Controller	$\Delta f_1(Hz)$	$\Delta f_2(Hz)$	ΔP_{tie12}	Steady state error
COQAGC	-00.23	-0.014	-0.006	0
DOQAGC	-0.016	-0.016	-0.023	0
Reduction %	30%	-12.5%	-74%	%0

Table 5.5. Case 1 dynamic response comparison in terms of settling time(ST)

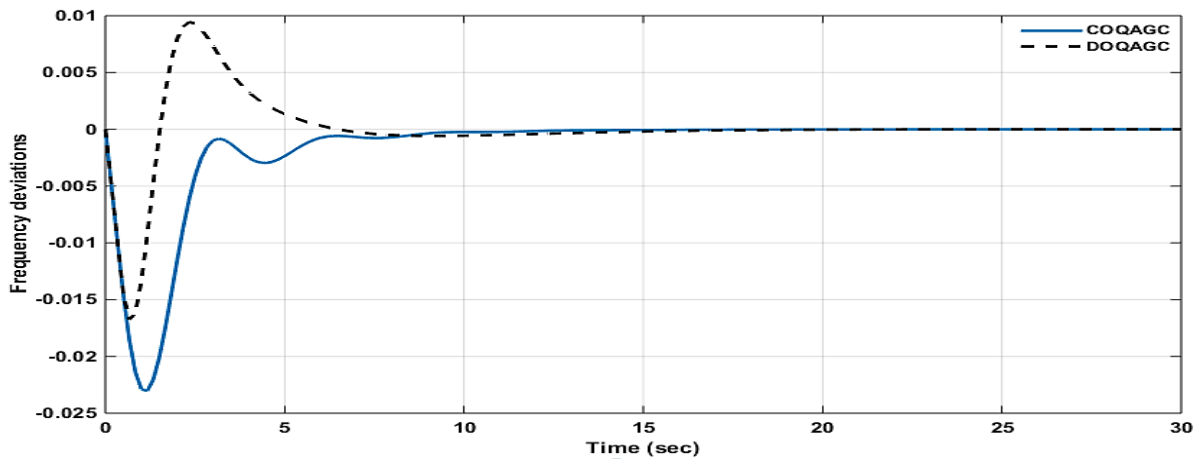
Type of Controller	$\Delta f_1(Hz)$	$\Delta f_2(Hz)$	$\Delta P_{tie}(u MW)$	Steady state error
COQAGC	9.6 sec	10 sec	10 sec	0
DOQAGC	9.3 sec	9.8 sec	16 sec	0
Reduction %	3%	2%	-37.5%	0%

Table 5-6. Case 2 dynamic response comparison in terms of peak overshoot(Peak OS)

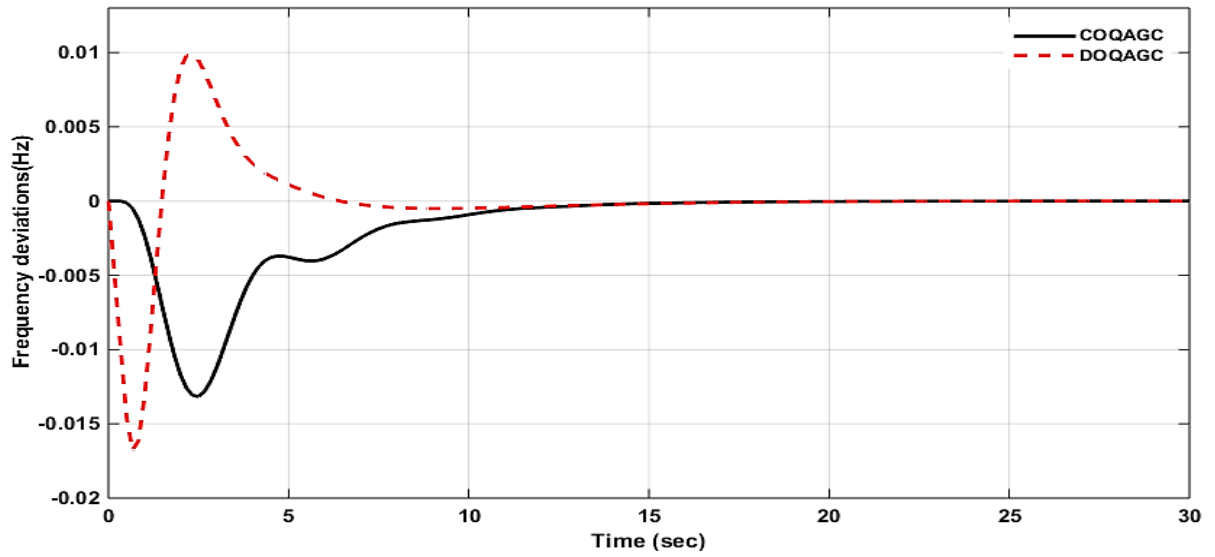
Type of Controller	$\Delta f_1(Hz)$	$\Delta f_2(Hz)$	$\Delta P_{tie12}(pu)$	Steady state error
COQAGC	-0.025	-0.025	-0.021	0
DOQAGC	--0.01166	-0.01166	-0.006	0
Reduction %	53.3%	53.3%	-71.4%	0%

Table 5.7: Case 2 dynamic response comparison in terms of disturbance rejection response time

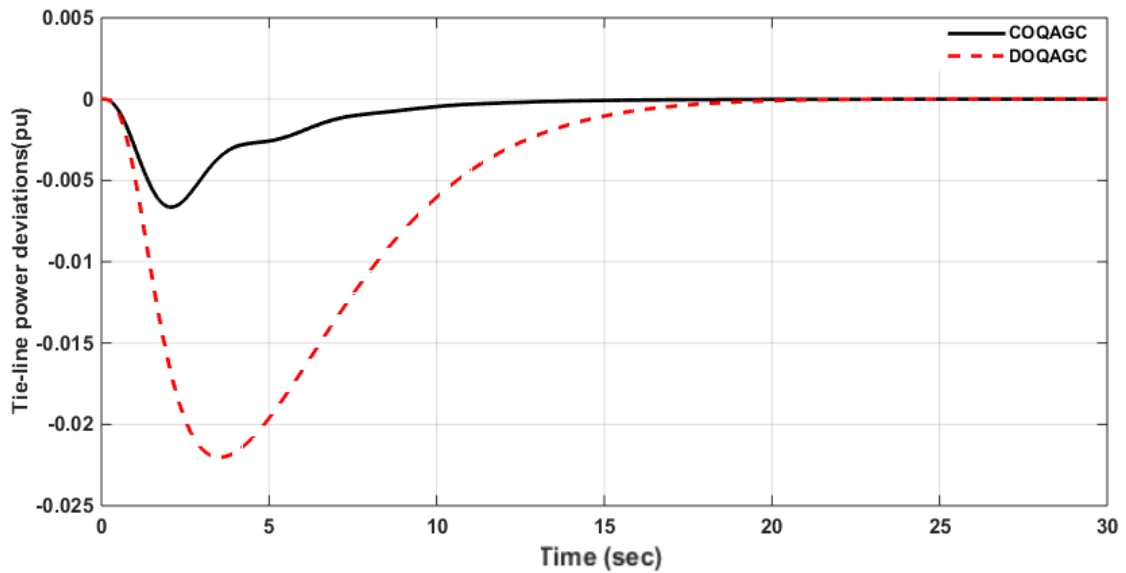
Type of Controller	$\Delta f_1(Hz)$	$\Delta f_2(Hz)$	$\Delta P_{tie}(pu MW)$	Steady state error
COQAGC	4.7 sec	4.7 sec	1.4 sec	0
DOQAGC	3.0 sec	3 sec	1.7 sec	0
Reduction %	36 %	36 %	-30%	0%



(a) Frequency deviation responses of subsystem1 and control area1.

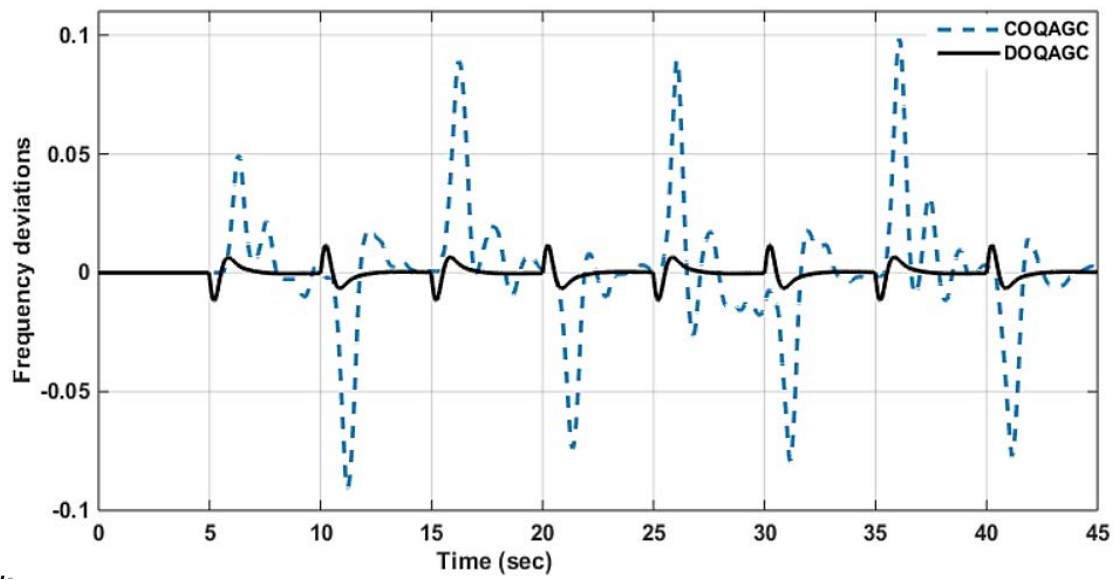


(b) Frequency deviation responses of subsystem 2 and control area2.

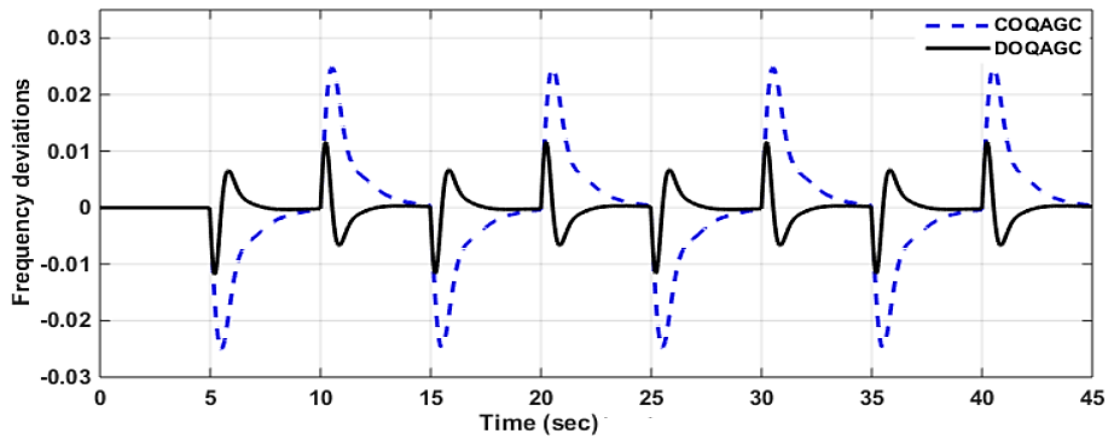


(c) Tie line power deviation responses of subsystem1 and ITAPS.

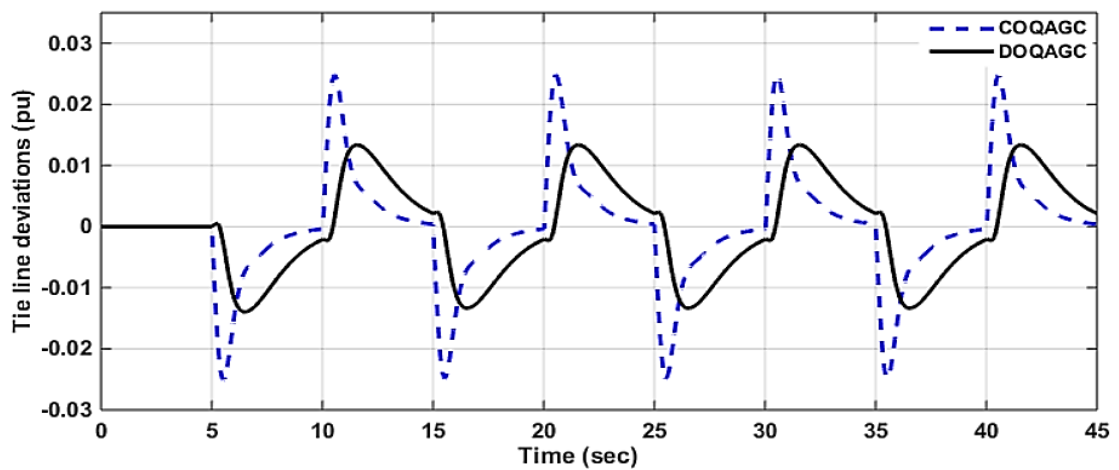
Figure 5. 12. Simulation results of ITAPS with non-reheat turbines.



(a) Frequency deviation responses of subsystem1 and control area1



(b) Frequency deviation responses of subsystem 2 and control area2.



(c) Tie line power deviation responses of subsystems and ITAPS

Figure 5.13 Simulation results of ITAPS with non-reheat turbines at simultaneous disturbances set at 0.01 pu.

5.4. Decomposing four multi-area control system with MSPGs into four subsystems

The state space equation for FAIPS derived in section 4.6.2 is split into four subsystems for DOQAGC controller construction and analysis in this part. The augmentation procedure given in Equation (5.36) is also used for the subsystems indicated in Equations (5.31) and (5.34), as well as the interconnection matrices described in Equation (5.36). The specifications and data of power systems comprising non-reheat, reheat, hydro, and gas turbines are shown in Table 5.8.

Table 5.8. standard parameters of Thermals, Gas and Hydro plants (Parmar, Majhi & Kothar, 2012 & Hakimuddin et al, 2020).

Parameters	Non-reheat thermal	Reheat thermal	Gas plant	Hydro plant
Rated Power	$P_R = 1200\text{ MW}$	$P_R = 800MW$	$P_R = 1000\text{ MW}$	$P_R400\text{ MW}$
Power system time constant	$T_{ps} = 20\text{ sec}$	$T_{ps} = 20\text{ sec}$		
Power system gain	$K_{ps} = 120\text{ Hz/pu}, MW$			
Turbine time constant	$T_{Ti} = 0.3\text{ sec}$	$T_{ri} = 10sec$	$T_{Wi} = 1.0ec$	$T_{CDi} = 0.2\text{ s}$
Tie-line constant	$T_{12} = 0.0866.$	$T_{12} = 0.0433$		
Governor time constant	$T_{Gi} = 0.08\text{ sec}$	$T_{Gi} = 0.08\text{ sec}$	$X_i = 0.6s$ (lead)	$T_{RHi} = 28.75\text{ s}$
Frequency dropping factor	$R_i = 2.4\text{ Hz/pu}, MW$			
Frequency Bias constant (B_i)				
Damping coefficient	$D_i = 08.33\frac{N}{m}.sec$	$D_i = 08.33\frac{N}{m}.sec$		
Coefficient of reheated steam turbine	-	$K_{ri} = 0.3$	-	-
Hydro turbine speed governor reset time	-	-	-	$T_{Ri} = 5\text{ s}$
Rated frequency	$F_r = 60\text{ Hz}$	$F_r = 60\text{ Hz}$	$F_r = 60\text{ Hz}$	$F_r = 60\text{ Hz}$
Power generation contribution	$K_{ti} = 0.4$	$K_{ti} = 0.4$	$K_{gi} = 0.2$	$K_{hi} = 0.5$

The matrices and vectors in subsystems are numerically computed as follows:

$$\bar{A}_1 = \begin{bmatrix} -0.05 & 6 & 0 & 0 & -6 & 0 & 0 & 0 \\ 0 & -3.3 & 3.3 & 0 & 0 & 0 & 0 & 0 \\ -5.21 & 0 & -12.5 & 12.5 & 0 & 0 & 0 & 0 \\ 0.425 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.5438 & 0 & 0 & 0 & 0 & -0.5438 & -0.5438 & -0.5438 \\ -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 12.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_1 = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5438 & -0.5438 & -0.5438 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} -0.05 & 6 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 1 & -2 & 3.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.3 & 3.33 & 0 & 0 & 0 & 0 & 0 \\ -5.21 & 0 & -12.5 & 12.5 & 0 & 1 & 0 & 0 & 0 \\ 0.425 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5438 & 0 & 0 & 0 & 0 & 0 & -0.5438 & -0.5438 & 0 \\ -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 12.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_2 = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5438 & -0.5438 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{A}_3 = \begin{bmatrix} -0.05 & 6 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0.7246 & -1.6 & 1.6696 & 1.6696 & 0 & 0 & 0 & 0 & 0 \\ -0.3623 & 0 & -0.0348 & -0.8348 & 0 & 0 & 0 & 0 & 0 \\ -2.0833 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 \\ 0.425 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.5438 & 0 & 0 & 0 & 0 & 0 & -0.5438 & -0.5438 & -0.5438 \\ -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{B}_3 = \begin{bmatrix} 0 \\ -1.74 \\ 0.87 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_3 = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5438 & -0.5438 & -0.5438 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{A}_4 = \begin{bmatrix} -0.05 & 6 & 0 & 0 & 6 & 0 & 6 & 0 & 0 & 0 \\ 0 & -5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.2174 & 0 & -4.3478 & 4.3043 & -0.4783 & 0 & 0 & 0 & 0 & 0 \\ -5 & 0 & 0 & -1 & -11 & 0 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 0 & -20 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0.425 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5438 & -0.5438 & 0 & -0.5438 \\ -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{B}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 12 \\ 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_4 = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -5.44 & 0 & 0 \\ -0.5438 & 0 & -0.5438 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

The inclusion of interconnection matrices increased the matrix dimensions of subsystems by 3 rows and 3 columns. As a consequence, subsystem dimensions have been increased based on the kind and number of generators in each area, i.e. where the new dimension equals the number of areas minus one. The Eigenvalues of the four subsystems are shown in Table 5.9. The subsystems, on the other hand, are not asymptotically stable since part of the eigenvalues are on the complex plane's right half plane. These eigenvalues were previously computed to supplement the interconnections matrices in the subsystem state space equations

Table 5.9. The eigenvalues of the four subsystems without interactions.

Subsystem1	Subsystem 2	Subsystem 3	Subsystem 4
0.5164	0	0	0
-1.1475	11.7735	-6.1578	-20.1484
3.4346	3.848	1.8903	2.0873+4.0448j
-0.0155	0.04304	-1.191+0.18j	2.0873-4.0448j
-0.0216	-1.6132	-1.191+0.18j	-7.212+4.5316j
-	-4.3221	-0.0365	-7.212-4.5316j
-	-	-	0.5438

The discrete matrices are constructed using a sample time $T = 0.0814$ seconds and four identity matrices I_1 , I_2 , I_3 and I_4 . based on Equation (4.46), chapter 4, section 4.5.1.1. The dimensions of identity matrices are determined by subsystem dimensions and connectivity matrices. As a consequence, the discrete decentralized matrices for the non-reheat, reheat, hydro, and gas generator subsystems may be determined as follows:

$$\bar{A}_{k1} = \begin{bmatrix} 0.9995 & 0.4884 & 0 & 0 & -0.4884 & 0 & 0 & 0 \\ 0 & 0.7287 & 0.1713 & 0 & 0 & 0 & 0 & 0 \\ -0.424 & 0 & -0.0175 & 1.0175 & 0 & 0 & 0 & 0 \\ 0.0346 & 0 & 0 & 1 & 0.0814 & 0 & 0 & 0 \\ 0.0443 & 0 & 0 & 0 & 1 & -0.0443 & -0.0443 & -0.0443 \\ -0.4884 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -0.4884 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -0.4884 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{B}_{k1} = \begin{bmatrix} 0 \\ 0 \\ 1.0175 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{F}_{k1} = \begin{bmatrix} -0.4884 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_{k1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0443 & -0.0443 & -0.0443 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{A}_{k2} = \begin{bmatrix} 0.9995 & 0.4884 & 0 & 0 & 0 & 0.4884 & 0 & 0 & 0 \\ 0 & 1 & -0.1628 & 0.2713 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7287 & 0.2713 & 0 & 0 & 0 & 0 & 0 \\ -0.424 & 0 & -0.0175 & 2.0175 & 0 & 0 & 0 & 0 & 0 \\ 0.0346 & 0 & 0 & 0 & 1 & 0.0814 & 0 & 0 & 0 \\ 0.0443 & 0 & 0 & 0 & 0 & 1 & -0.0443 & -0.0443 & 0 \\ -0.4884 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -0.4884 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -0.4884 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{B}_{k2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.0175 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_{k2} = \begin{bmatrix} -0.4884 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_{k2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0443 & -0.0443 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{A}_{k3} = \begin{bmatrix} 0.9995 & 0.4884 & 0 & 0 & 0 & 0.4884 & 0 & 0 & 0 \\ 0.059 & 0.8698 & 0.1359 & 0.1359 & 0 & 0 & 0 & 0 & 0 \\ -0.0295 & 0 & 0.9972 & -0.068 & 0 & 0 & 0 & 0 & 0 \\ -0.1696 & 0 & 0 & 0.583 & 0 & 0 & 0 & 0 & 0 \\ 0.0346 & 0 & 0 & 0 & 1 & 0.0814 & 0 & 0 & 0 \\ 0.0443 & 0 & 0 & 0 & 0 & 1 & -0.0443 & -0.0443 & -0.0443 \\ -0.4884 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -0.4884 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -0.4884 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{B}_{k3} = \begin{bmatrix} 0 \\ -0.1416 \\ 0.0708 \\ 0.0407 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_{k3} = \begin{bmatrix} -0.4884 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_{k3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0443 & -0.0443 & -0.0443 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{A}_{k4} = \begin{bmatrix} 0.9995 & 0.4884 & 0 & 0 & 0 & 0 & 0.4884 & 0 & 0 & 0 \\ 0 & 0.593 & 0.4070 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0177 & 0 & 0.6461 & 0.3504 & -0.0389 & 0 & 0 & 0 & 0 & 0 \\ -0.4070 & 0 & 0 & -0.9186 & -0.8954 & 0 & 0 & 0 & 0 & 0 \\ 0.9768 & 0 & 0 & 0 & -0.628 & 0 & 0 & 0 & 0 & 0 \\ 0.0814 & 0 & 0 & 0 & 0 & 1 & 0.0346 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0443 & -0.0443 & -0.0443 & -0.0443 \\ -0.4884 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -0.4884 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -0.4884 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{B}_{k4} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.9768 \\ 1.628 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Gamma}_{k4} = \begin{bmatrix} -0.4884 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \bar{F}_{k4} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0443 & 0 & -0.0443 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The numerical values of interconnection matrices are derived using Equation (5.29) and the parameters and data of the power systems supplied in Table 5.8:

$$A_{z1} = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}, A_{z2} = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}, \quad (5.60)$$

$$A_{z3} = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}, A_{z4} = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

Equations (5.2) and (5.3) were used to calculate the dimensions of the four subsystems (5.11). The overall number of states is 36, which is a mix of subsystem states. ($\bar{X}_1(k)$, $\bar{X}_2(k)$, $\bar{X}_3(k)$, and $\bar{X}_4(k)$), and the four subsystem interaction variables (z_1 , z_2 , z_3 and z_4). Thus,

the state vectors (X_1^T, X_2^T, X_3^T and X_4^T) are defined as given in equation (5.61) for the four subsystems respectively.

$$\begin{aligned}
X_1^T &= [\bar{X}_1(k) \quad z_1] = [x_{11} \ x_{12} \ x_{13} \ x_{14} \ x_{15} \ x_{16} \ x_{17} \ x_{18}]^T \\
X_2^T &= [\bar{X}_2(k) \quad z_2] = [x_{21} \ x_{22} \ x_{23} \ x_{24} \ x_{25} \ x_{26} \ x_{27} \ x_{28} \ x_{29}]^T \\
X_3^T &= [\bar{X}_3(k) \quad z_3] = [x_{31} \ x_{32} \ x_{33} \ x_{34} \ x_{35} \ x_{36} \ x_{37} \ x_{38} \ x_{39}]^T \\
X_4^T &= [\bar{X}_4(k) \quad z_4] = [x_{41} \ x_{42} \ x_{43} \ x_{44} \ x_{45} \ x_{46} \ x_{47} \ x_{48} \ x_{49} \ x_{410}]^T
\end{aligned} \tag{5.61}$$

Furhermore, the the dimensions of the subsystems (n_1, n_2, n_3 , and n_4) respectively for non-reheat, reheat, hydro and gas generators and their interactions were defined as follow:

$n_1 = 8, n_2 = 9, n_3 = 9$, and $n_4 = 10$, where $\Delta f_1 = x_{11}, \Delta P_{t1} = x_{12}, \Delta P_{g1} = x_{13}, IACE_1 = x_{14}, \Delta P_{tie12} = x_{15}, \Delta f_2 = x_{21}, \Delta P_r = x_{22}, \Delta P_{t2} = x_{23}, \Delta P_{g2} = x_{24}, IACE_2 = x_{25}, \Delta P_{tie23} = x_{26}, \Delta f_3 = x_{31}, \Delta P_{hy} = x_{32}, \Delta X_e = x_{33}, \Delta P_R = x_{34}, IACE_3 = x_{35}, \Delta P_{tie34} = x_{36}, \Delta f_4 = x_{41}, \Delta P_{ga} = x_{42}, \Delta P_f = x_{43}, \Delta P_{gh} = x_{44}, \Delta P_g = x_{45}, IACE_4 = x_{46}, \Delta P_{tie41} = x_{47}$. The interaction variables are defined as , $z_{11} = x_{16}, z_{12} = x_{17}, z_{13} = x_{18}, z_{21} = x_{27}, z_{22} = x_{28}, z_{23} = x_{29}, z_{31} = x_{37}, z_{32} = x_{38}, z_{33} = x_{39}$ and , $z_{41} = x_{48}, z_{42} = x_{49}, z_{43} = x_{410}$

The subsystems before augmentation process have rank of 5, 6, 6 and 7 respectively. Thus, augmented subsystems are controllable. The sub-cost functions J_i ($i = 1,2,3,4$) defined in Equation (5.45) and the design consideration discussed in the section 4.3 and Equations (5.46) – (5.47) are used to determine the state and control weighting matrices. Excursions $ACE_i, IACE_i$ and control input u_i are substituted into a J_i from Equation (5.45) so the sub-cost functions for the augmented four subsystems can be defined as:

$$\begin{aligned}
J_1 &= \frac{1}{2} \sum_{k=k_0}^{\infty} [B_1^2 x_{11}^2 - 2\beta_1 x_{11} x_{15} + x_{15}^2] + (x_{14})^2 + u_1^2 \\
J_2 &= \frac{1}{2} \sum_{k=k_0}^{\infty} [B_2^2 x_{21}^2 - 2\beta_2 x_{21} x_{26} + x_{26}^2] + (x_{25})^2 + u_2^2 = \\
J_3 &= \frac{1}{2} \sum_{k=k_0}^{\infty} [B_3^2 x_{31}^2 - 2\beta_3 x_{31} x_{36} + x_{36}^2] + (x_{35})^2 + u_3^2 = \\
J_4 &= \frac{1}{2} \sum_{k=k_0}^{\infty} [B_4^2 x_{41}^2 - 2\beta_4 x_{41} x_{47} + x_{47}^2] + (x_{46})^2 + u_4^2 =
\end{aligned} \tag{5.62}$$

After defining the sub-functionals $f_1(ACE_1, IACE_1, u_1)$, $f_2(ACE_2, IACE_2, u_2)$, $f_3(ACE_3, IACE_3, u_3)$, and $f_4(ACE_4, IACE_4, u_4)$ as in Equation (5.46), the partial differential equations with respect to the sub-state variables for the augmented four subsystems are obtained as follows:

- Subsystem1:

$$\begin{aligned}
1/2 \, df_1(.) / dx_{11} &= \beta_1^2 x_{11} - \beta_1 x_{15} \\
1/2 \, df_1(.) / dx_{12} &= 0 \\
1/2 \, df_1(.) / dx_{13} &= 0 \\
1/2 \, df_1(.) / dx_{14} &= x_{14} \\
1/2 \, df_1(.) / dx_{15} &= \beta_1 x_1 - x_{15} \\
1/2 \, df_1(.) / dx_{16} &= 0 \\
1/2 \, df_1(.) / dx_{17} &= 0 \\
1/2 \, df_1(.) / dx_{18} &= 0
\end{aligned} \tag{5.63}$$

and the first-order partial derivatives with respect to control input u_1 is obtained as follows:

$$df_1(.) / du_1 = u_1 \tag{5.64}$$

- Subsystem 2:

$$\begin{aligned}
1/2 \, df_2(.) / dx_{21} &= \beta_2^2 x_{21} + \beta_2 x_{25} \\
1/2 \, df_2(.) / dx_{22} &= 0 \\
1/2 \, df_2(.) / dx_{23} &= 0 \\
1/2 \, df_2(.) / dx_{24} &= \\
1/2 \, df_2(.) / dx_{25} &= x_{25} \\
1/2 \, df_2(.) / dx_{26} &= \beta_2 x_{21} + x_{26} \\
1/2 \, df_2(.) / dx_{27} &= 0 \\
1/2 \, df_2(.) / dx_{28} &= 0 \\
1/2 \, df_2(.) / dx_{29} &= 0
\end{aligned} \tag{5.65}$$

and the first-order partial derivatives with respect to control input signal u_2 is obtained as follows:

$$df_2(.) / du_2 = u_2 \tag{5.66}$$

- Subsystem3:

$$\begin{aligned}
1/2 \, df_3(.) / dx_{31} &= \beta_3^2 x_{31} - \beta_3 x_{36} \\
1/2 \, df_3(.) / dx_{32} &= 0 \\
1/2 \, df_3(.) / dx_{33} &= 0 \\
1/2 \, df_3(.) / dx_{34} &= 0 \\
1/2 \, df_3(.) / dx_{35} &= x_{35} \\
1/2 \, df_3(.) / dx_{36} &= \beta_3 x_{31} - x_{36} \\
1/2 \, df_3(.) / dx_{37} &= 0 \\
1/2 \, df_3(.) / dx_{38} &= 0 \\
1/2 \, df_3(.) / dx_{39} &= 0
\end{aligned} \tag{5.67}$$

and the first-order partial derivatives with respect to control input signal u_3 is obtained as follows:

$$df_3(.) / du_3 = u_3 \tag{5.68}$$

- Subsystem 4:

$$\begin{aligned}
1/2 \, df_4(.) / dx_{41} &= \beta_4^2 x_{41} + \beta_4 x_{47} \\
1/2 \, df_4(.) / dx_{42} &= 0 \\
1/2 \, df_4(.) / dx_{43} &= 0 \\
1/2 \, df_4(.) / dx_{44} &= 0 \\
1/2 \, df_4(.) / dx_{45} &= 0 \\
1/2 \, df_4(.) / dx_{46} &= x_{46} \\
1/2 \, df_4(.) / dx_{47} &= \beta_4 x_{41} + x_{47} \\
1/2 \, df_4(.) / dx_{48} &= 0 \\
1/2 \, df_4(.) / dx_{49} &= 0 \\
1/2 \, df_4(.) / dx_{410} &= 0
\end{aligned} \tag{5.69}$$

and the first-order partial derivatives with respect to control input signal u_4 is obtained as follows:

$$df_4(.) / du_4 = u_4 \tag{5.70}$$

The partial differential equations (5.63) -(5.70) can be organized in the sub-state weighting matrix $\bar{Q}_i$ ($i = 1,2,3,4$) and sub-control weighting matrix $\bar{u}_i$ ($i = 1,2,3,4$) as follows.

$$\begin{aligned}
\bar{Q}_1 &= \begin{bmatrix} \beta_1^2 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{Q}_2 = \begin{bmatrix} \beta_2^2 & 0 & 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{R}_1 = \bar{R}_2 = 1 \\
\bar{Q}_3 &= \begin{bmatrix} \beta_3^2 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{Q}_4 = \begin{bmatrix} \beta_4^2 & 0 & 0 & 0 & 0 & 0 & \beta_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \beta_4 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{R}_3 = \bar{R}_4 = 1
\end{aligned}$$

The following computational results are obtained for the discrete weighting matrices $\bar{Q}_i$ and $\bar{R}_i$:

$$\begin{aligned}
\bar{Q}_1 &= \begin{bmatrix} 1.0147 & 0 & 0 & 0 & -0.0346 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0814 & 0 & 0 & 0 & 0 \\ 0.0346 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{Q}_2 = \begin{bmatrix} 1.0147 & 0 & 0 & 0 & 0 & 0.0346 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0814 & 0 & 0 & 0 & 0 \\ 0.0346 & 0 & 0 & 0 & 0 & 1.0814 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{R}_1 = \bar{R}_2 = 1 \\
\bar{Q}_3 &= \begin{bmatrix} 1.0147 & 0 & 0 & 0 & 0 & -0.0346 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0814 & 0 & 0 & 0 & 0 \\ -0.0346 & 0 & 0 & 0 & 0 & 1.0814 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{Q}_4 = \begin{bmatrix} 1.0147 & 0 & 0 & 0 & 0 & 0 & 0.0346 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0814 & 0 & 0 & 0 & 0 \\ 0.0346 & 0 & 0 & 0 & 0 & 0 & 1.0814 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{R}_3 = \bar{R}_4 = 1
\end{aligned}$$

The sub-optimal feedback gains of the decentralized subsystems can be obtained as:

$$\begin{aligned}
\bar{L}_1 &= [1.0785 \quad 1.6556 \quad 0.92 \quad 1.0098 \quad 4.1627 \quad -1.1692 \quad -1.6789 \quad -1.6789] \\
\bar{L}_2 &= [2.4856 \quad 4.3134 \quad -0.349 \quad 2.6062 \quad 0.0411 \quad 5.4839 \quad -0.9966 \quad -0.6782 \quad -0.1194] \\
\bar{L}_3 &= [-16.4585 \quad -30.5384 \quad -44.8027 \quad -1.0013 \quad 0.0887 \quad -31.1132 \quad 5.5643 \quad 6.3552 \quad 6.3552] \\
\bar{L}_4 &= [4.7987 \quad 4.5247 \quad 4.3858 \quad 2.2303 \quad -1.0980 \quad 0.0421 \quad 13.6034 \quad -2.22 \quad 0.1636 \quad -1.8929]
\end{aligned}$$

5.5. Simulation Results and Discussions

Computer simulations are used to evaluate the effectiveness of decentralized control laws. The design method outlined in Section 5.2 and implemented in Section 5.3.2 is also used in this section. Table 5.8 shows the subsystem characteristics and statistics for thermal power systems with non-reheat, reheat, hydro, and gas turbines. The sub-optimal feedback gains, $L_i (i = 1, 2, 3, 4)$ are computed in the same manner as in section 5.2.3. Similarly, the simulation algorithms are constructed and run for 1800 iterations in the MATLAB environment with, $k_0 = 0$ and, $k_f = 1800$. The simulation findings for three separate instances with suboptimal controllers are given below.

5.5.1. Investigation of the closed loop performance of Subsystems when SLPs are applied at $t = 0$ sec.

The designed dynamic optimal control systems is investigated for step load perturbations (SLPs) imposed on the subsystems at $t = 0$ sec. Dynamic responses of the subsystems are obtained for 10% SLPs ($w_1 = \Delta P_{D1} = 0.01 \text{ pu}$, $w_2 = \Delta P_{D2} = 0.01 \text{ pu}$, $w_3 = \Delta P_{D3} = 0.01 \text{ pu}$ and $w_4 = \Delta P_{D4} = 0.01 \text{ pu}$) while all initial values of the other variables of the four subsystems were set at zero values. Figures 5.15 and 5.16 show the dynamic responses. It is clear that the subsystems' performances differ in terms of peak overshoot and settling time indices. However, the non-reheat turbine subsystem has the shortest settling time, whereas the hydro plant subsystem has the largest peak overshoot for both frequency and tie-line power deviations. During the transient phase, the frequency and tie-line power variations undershoot and overshoot with an oscillating response. Tables 5.10 and 5.11 show the summary performance for frequency deviations and tie-line power deviations.

Table 5.10. DCOQAGC's performance in terms of TS, Peak OS, and DRRT in Case 1.

Frequency Deviations					
Type of Controller	Subsystem	ST(Sec)	Peak OS of $\Delta f_i(\text{Hz})$	DRRT	Steady state error
DOQAGC	1	5 sec	-0.02	-	0
	2	10 sec	-0.02	-	0
	3	10 sec	0.033	-	0
	4	15 sec	0.024	-	0

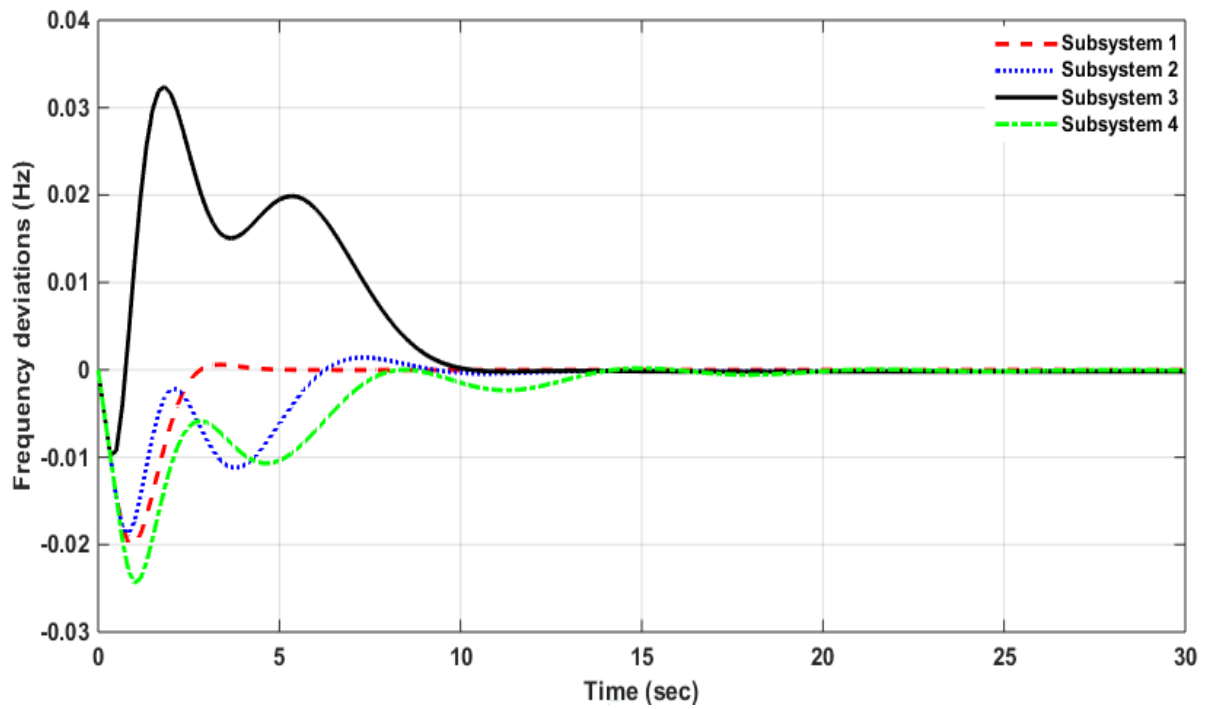


Figure 5.14. Frequency deviations in the four subsystems when disturbances are imposed at $t=0$.

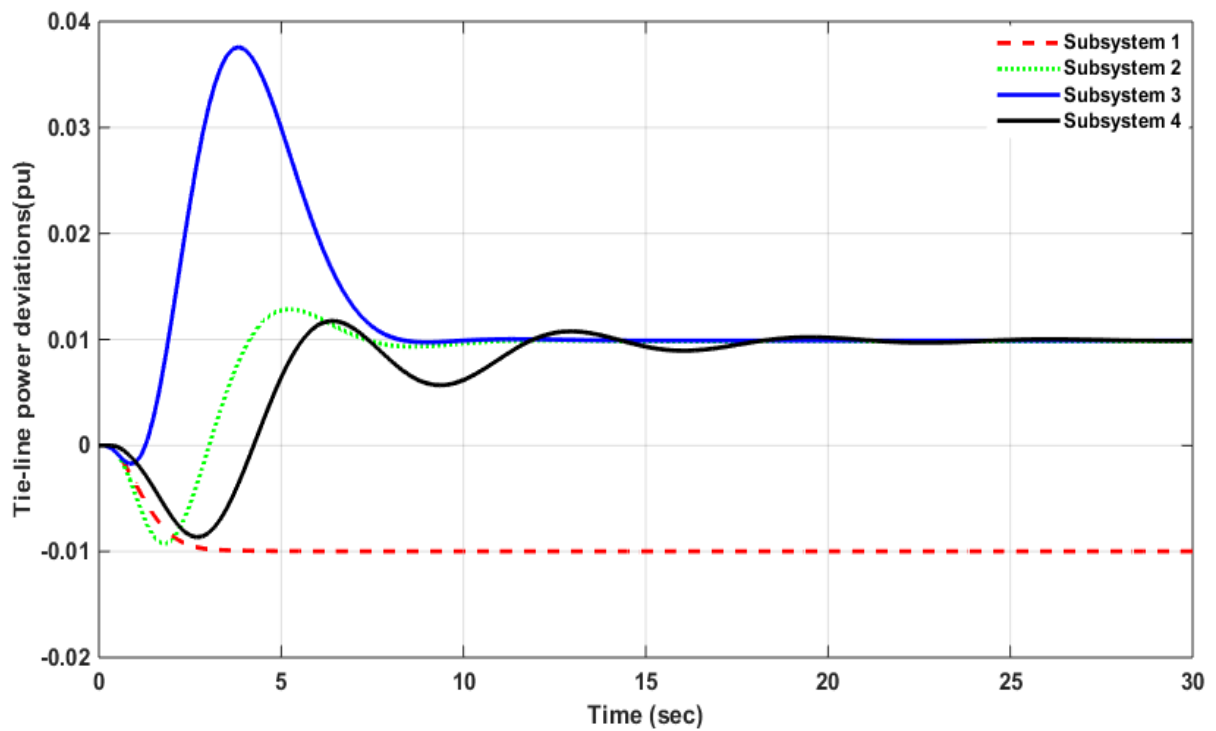


Figure 5.15. Tie-line power deviations in four subsystems when disturbances are imposed at $t=0$.

Table 5.11. DOQAGC's performance in terms of TS, Peak OS, and DRRT in Case 2.

Tie-line power deviations (pu)

Type of Controller	Subsystem	ST	Peak OS of $\Delta f_i(\text{Hz})$	DRRT	Steady state error
DOQAGC	1	5 sec	-0.0	-	0
	2	10 sec	0.013	-	0
	3	9 sec	0.038	-	0
	4	16 sec	-0.008	-	0

Similarly, the SLPs at interconnection states between various subsystems are considered. This is investigated by imposing the same disturbances at $t = 0$ sec, and taking interconnection states for a certain corresponding subsystems. Dynamic responses for the interaction states of the four subsystems ($\mathbf{z}_1^T = [z_{11} \ z_{12} \ z_{13}]^T$, $\mathbf{z}_2^T = [z_{21} \ z_{22} \ z_{23}]^T$, $\mathbf{z}_3^T = [z_{31} \ z_{32} \ z_{33}]^T$, $\mathbf{z}_4^T = [z_{41} \ z_{42} \ z_{43}]^T$) are depicted in Figures 5.16 to 5.19 respectively. Similarly, it can be shown that the interaction performances differ in terms of overshoot and settling time indices. However, the state interaction of the subsystem with the non-reheat turbine has the least peak overshoot and the shortest settling time, whereas the state interaction with the gas plant has the worst overshoot and the longest settling time. It is also worth noting that the frequency and tie-line power deviations oscillate with an oscillatory response before returning to zero in the steady-state. Table 5.12 provides an overview of the dynamic responsiveness of the interconnection states.

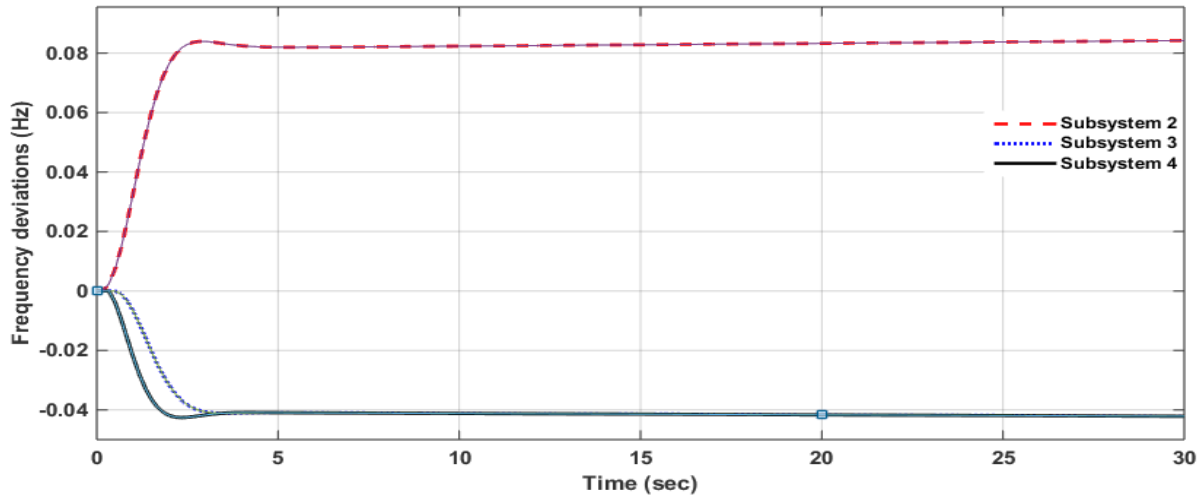


Figure 5.16. Responses of interaction states z_{11} , z_{12} and z_{13} operation against subsystem 1

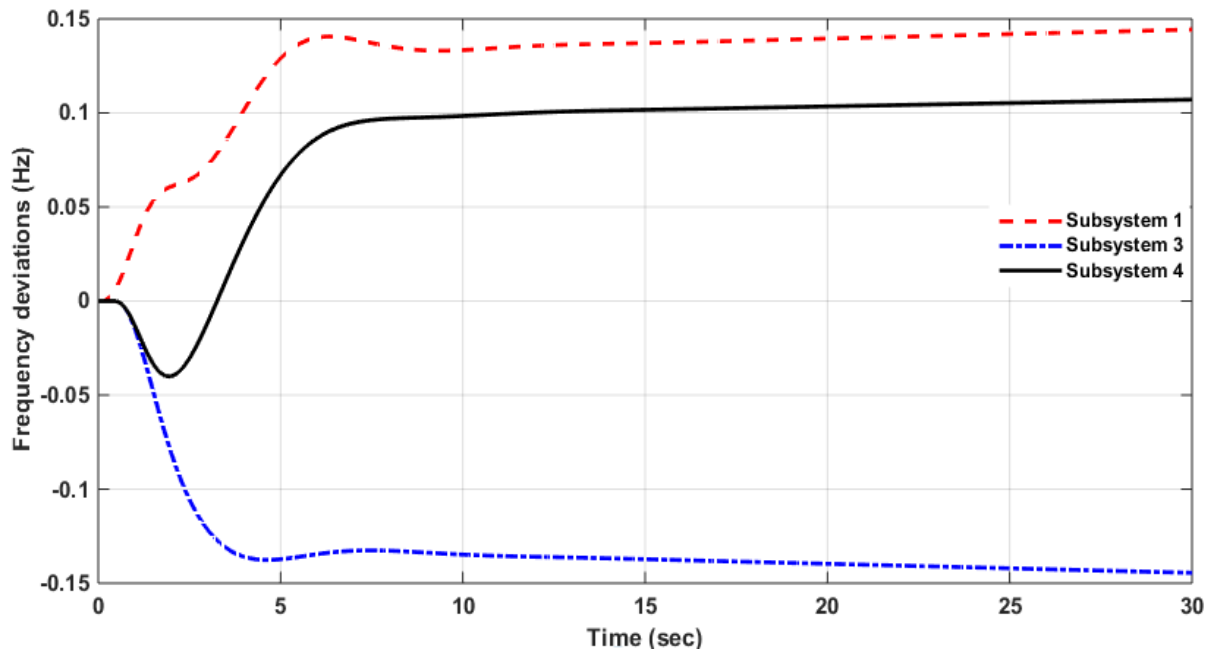


Figure 5.17. Responses of interaction states z_{21} , z_{22} and z_{23} operation against subsystem 2

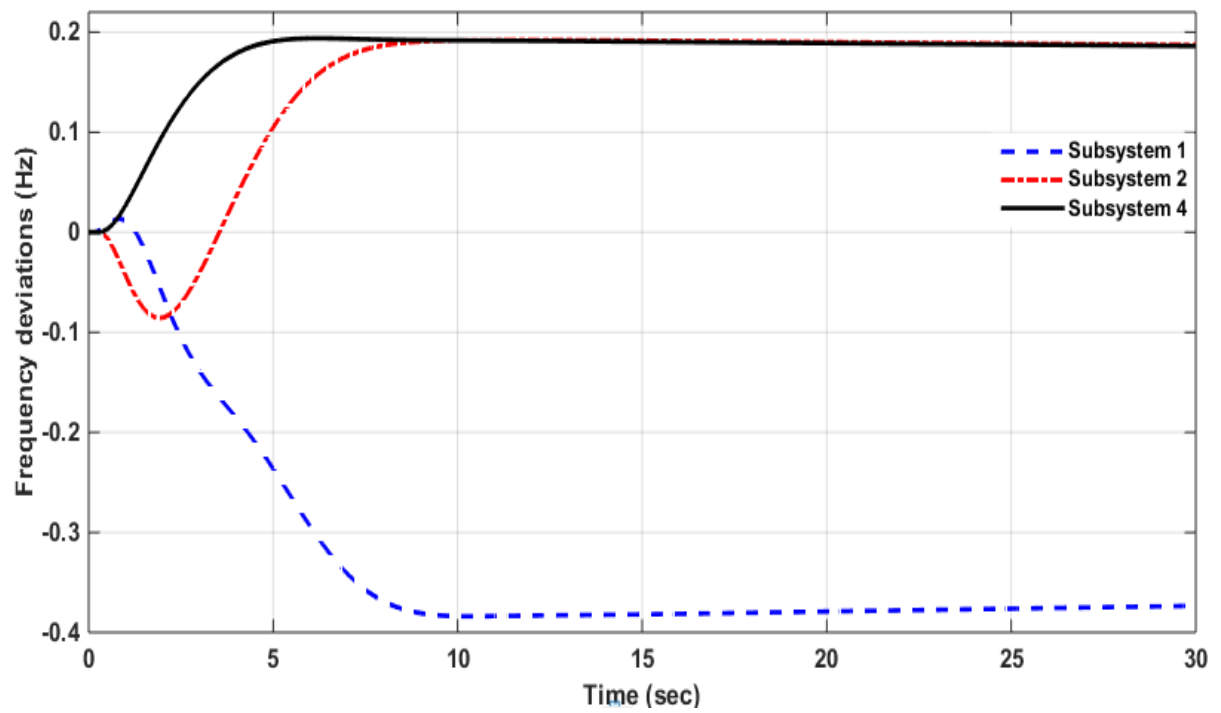


Figure 5.18. Responses of interaction states z_{31} , z_{32} and z_{33} operation against subsystem 3.

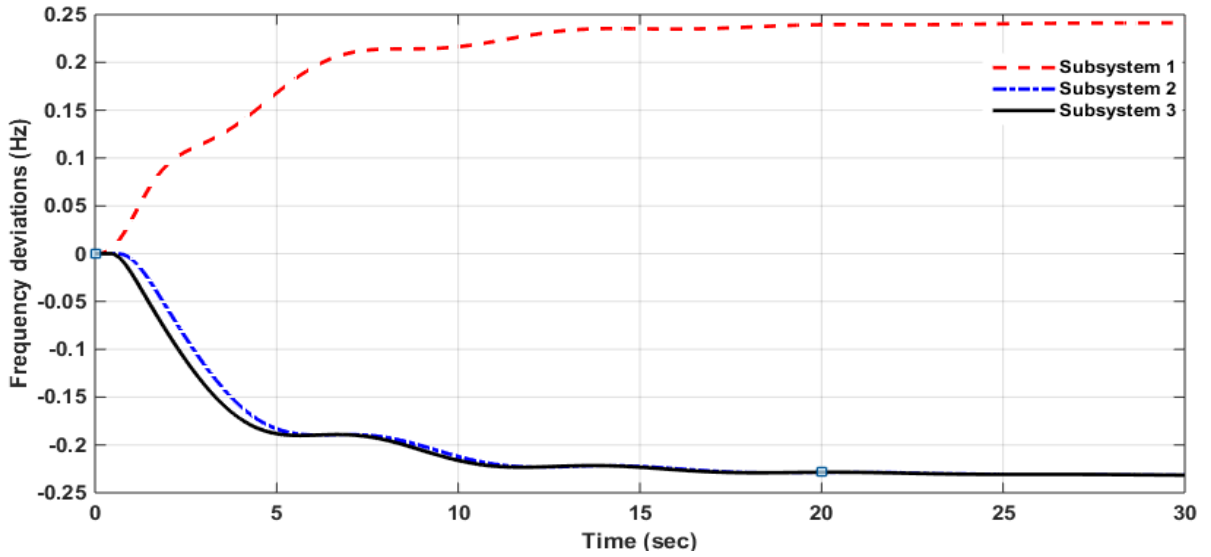


Figure 5.19. Responses of interaction states z_{41} , z_{42} and z_{43} operation against subsystem 4.

Table 5.12. Response of interconnection variables in terms of Peak OS and ST for case1

Interaction	Comparative	Subsystem 1	Subsystem 2	Subsystem 3	Subsystem 4
z_1	Peak OS	0	0.084	-0.041	-0.043
	ST	0	3.9 sec	3.9 sec	3.9 sec
z_2	Peak OS	0.1405	0	-0.14	-0.04
	ST	7 sec	0	8 sec	7 sec
z_3	Peak OS	-0.383	-0.086	0	0.194
	ST	10 sec	8 sec	0	8 sec
z_4	Peak OS	0.22	-0.22	-0.22	0
	ST	15 sec	17 sec	16 sec	0

5.5.2. Investigation of the closed loop performance of Subsystems when simultaneous perturbations are applied every 5 seconds intervals

In this section, investigations of the closed-loop performance of the four subsystems with interactions to step load disturbances ($w_1 = \Delta P_{D1}$, $w_2 = \Delta P_{D2}$, $w_3 = \Delta P_{D3}$ and $w_4 = \Delta P_{D4}$) for the separate designed discrete decentralized control laws are also investigated. A step perturbation of a magnitude of 1 % is added simultaneously every 5 seconds to each subsystem. All other variables in the four subsystems depicted in 4.33, section 4.6.1 have their initial values set to zero. Figures 5.20 and 5.21 exhibit dynamic responses to frequency and tie-line power variations. Similarly, to Case 1, dynamic responses for frequency variations are derived in terms of peak overshoot and settling time. In terms of settling time and peak overshoot, the responses of tie-line power variations deviate somewhat from Case

1. Good disturbance rejection controllers, however a subsystem with a non-reheat turbine has the shortest disturbance rejection time response, while a gas generator has the longest. However, due to the interconnections and nonlinear behavior of the generators of the four subsystem decentralized controllers, the dynamic responses of the four subsystem decentralized controllers differ.

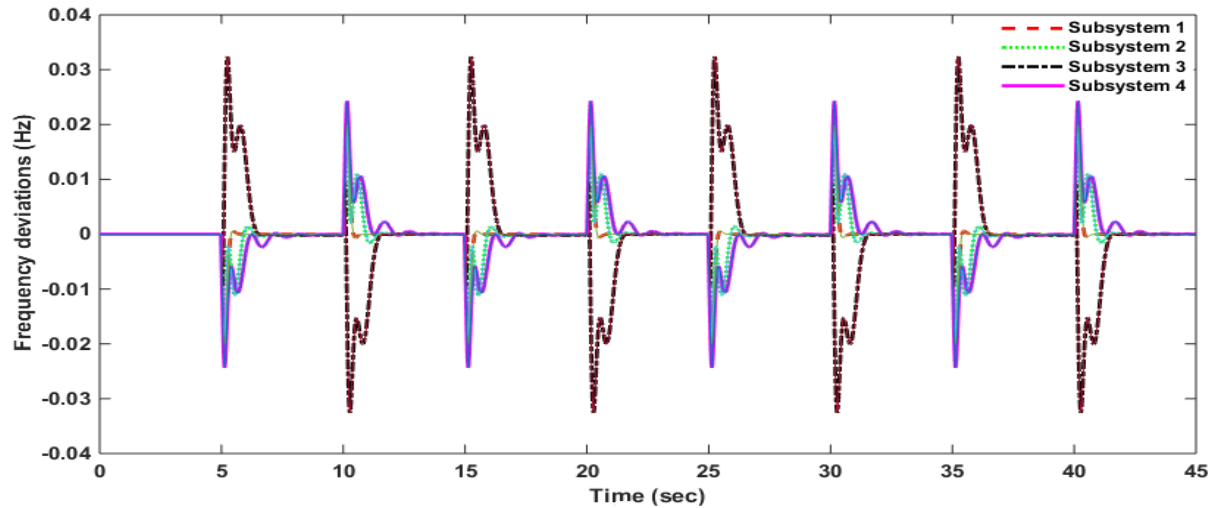


Figure 5.20. Frequency deviations of ODQAGC for four subsystems with load disturbances.

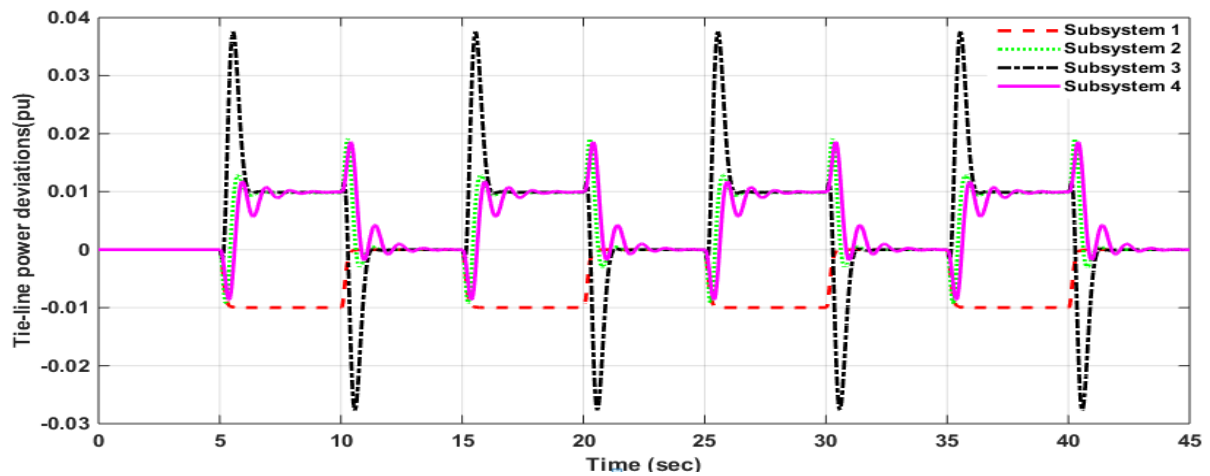


Figure 5.21. DOQAGC Tie-line deviations for four power subsystems with load disturbances

Table 5.13. DOQAGC performance in Case 2 for Frequency deviations.

Frequency Deviations					
Type of Controller	Subsystem	ST(sec)	Peak OS(Hz)	DRRT(sec)	Steady state error
DOQAGC	1	5	-0.02	1.2	0
	2	10	± 0.02	2.43	0
	3	10	0.033	1.85	0
	4	16	0.024	3.05	0

Table 5.14. DOQAGC's performance in Case 2- tie-line power deviations.

Tie-line power deviations (pu)					
Type of Controller	Subsystem	ST(sec)	Peak OS (pu)	DRRT (sec)	Steady state error
DOQAGC	1	5	-0.02	5	0
	2	3	0.013	1.80	0
	3	9	0.038	1.53	0
	4	16	-0.008	5.8	0

Figures 5.22–5.25 illustrate interaction state plots as a consequence of simultaneous SLPs on each subsystem. Similarly, these disturbances affect peak overshoot and settling time in maximum subsystems 2 and 3, as well as marginally in subsystem 4. The disturbance rejection time responses of the subsystems with non-reheat and reheat turbines are similar, while the disturbance rejection time responses of subsystems 3 and 4 are slightly longer. Subsystem 4 interaction states, on the other hand, have no peak overshoot, whereas subsystem 3 interaction states have the greatest peak overshoot. The dynamic response of interaction states in the four subsystems, however, is only partially resolved to zero at this point.

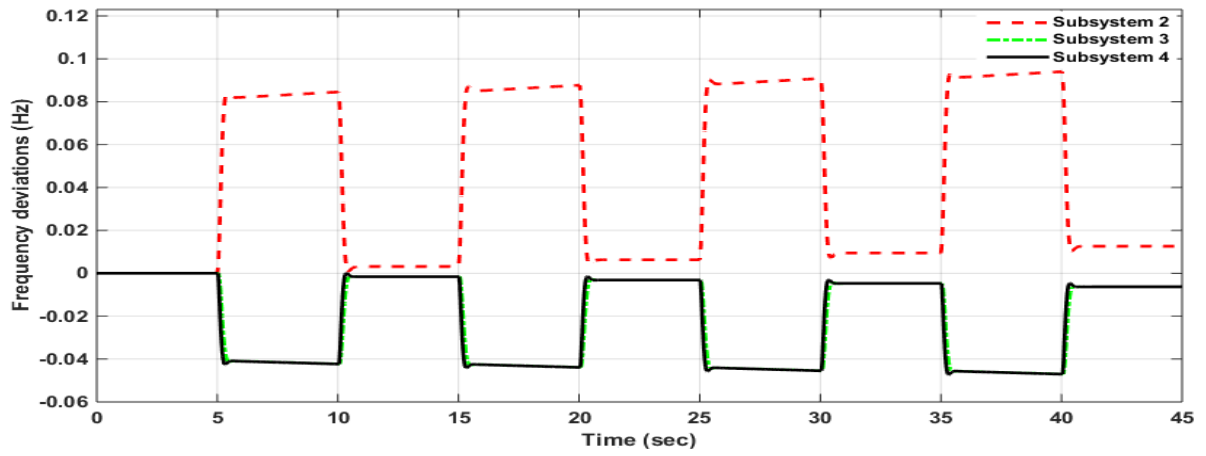


Figure 5.22. Responses of interaction states z_{11} , z_{12} and z_{13} working against subsystem 1.

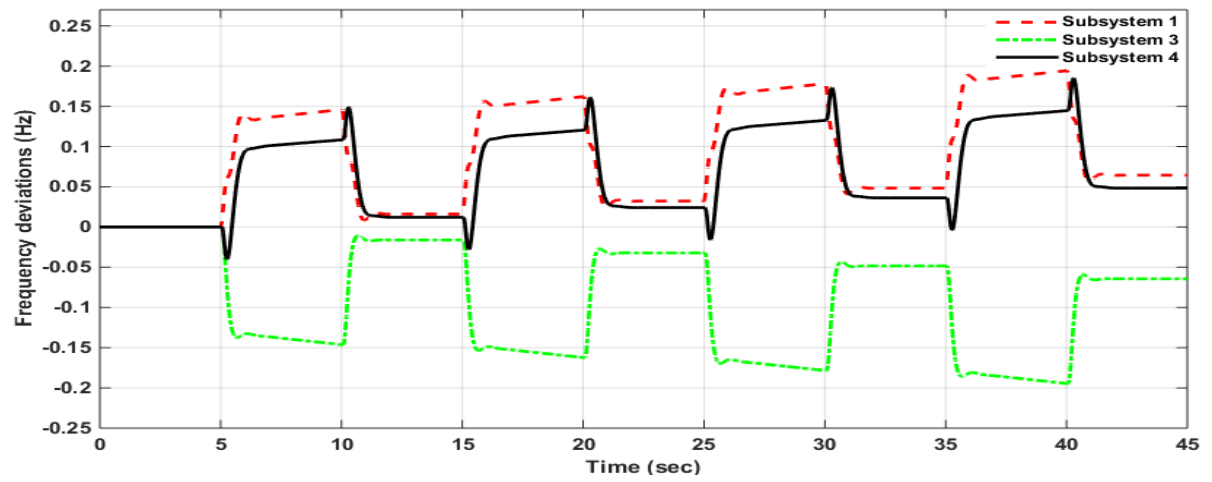


Figure 5.23. Responses of interaction states z_{21} , z_{22} and z_{23} operating against subsystem 2.

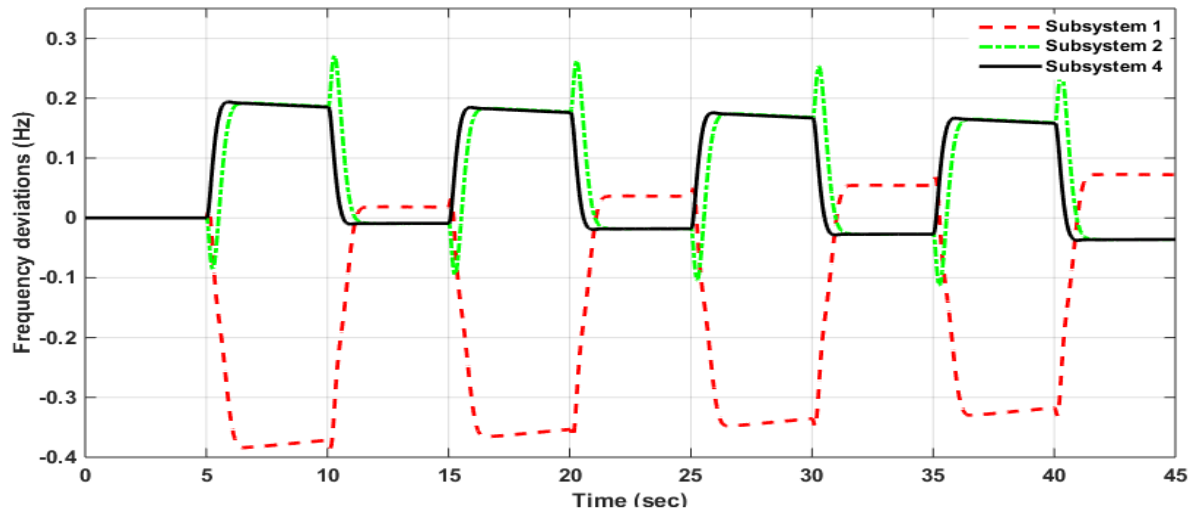


Figure 5.24. Responses of interaction states z_{31} , z_{32} and z_{33} working against subsystem 3

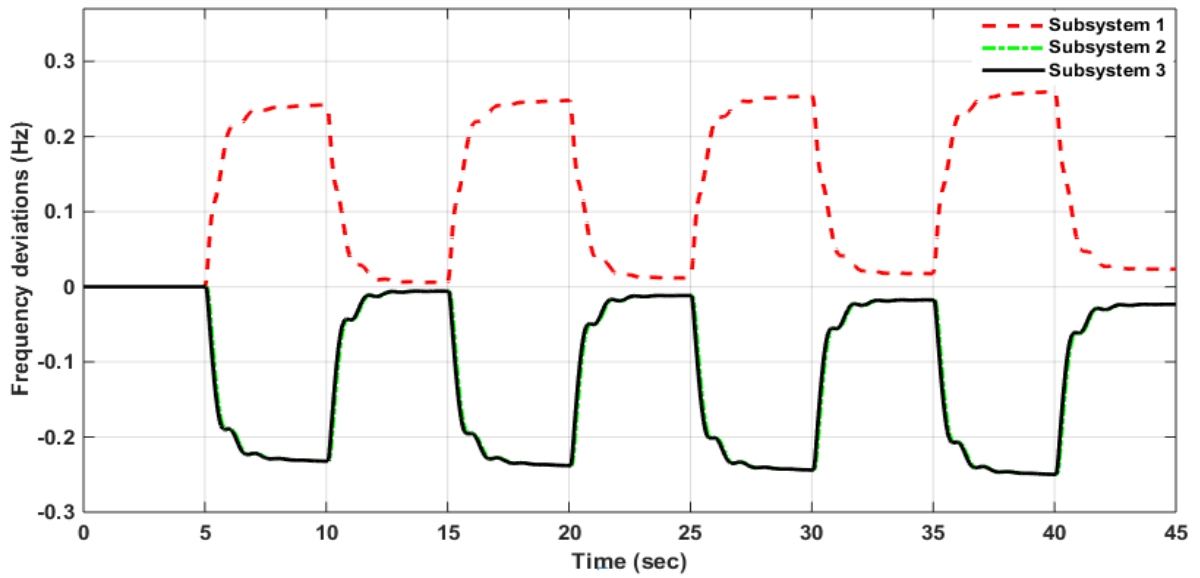


Figure 5.25. Responses of interaction states z_{41} , z_{42} and z_{43} operation against subsystem 4.

Table 5.15. Interconnection states responses in terms of peak of OS and settling time (ST) for case2

Interaction variables	Comparative Indices	Subsystem 1	Subsystem 2	Subsystem 3	Subsystem 4
z_1	Peak OS	-	0.084	-0.04	-0.04
	ST	-	5 sec	5 sec	5 sec
z_2	Peak OS	0.141	-	-0.14	0.14
	ST	5 sec	-	5 sec	5 sec
z_3	Peak OS	-0.38	0.27	-	0.19
	ST	6 sec	6 sec	-	6 sec
z_4	Peak OS	0	0	0	-
	ST	7 sec	7 sec	7 sec	-

5.5.3. Sensitivity Analysis

Sensitivity analysis is carried out to demonstrate the robustness of the decentralized optimal control laws for the closed loops of the four subsystems. In this scenario, the nominal parameters and data for the four generators listed in Table 5.7 have been modified by $\pm 25\%$ from their nominal values. It means initial parameters are varied $\pm 25\%$ for the considered non-reheat thermal, reheat thermal, gas and hydro plants in order to study the plant performances for varying initial system parameters and nonnominal parameters. At all parameter settings, an SLP of 1% is considered to test the performance of the four subsystems. Table 5.16 depicts the parameter value variations of the four power subsystems. Two scenarios are run to verify the decentralized optimal control laws' robustness. In the first scenario, variations in all subsystem parameters are taken into

account for 1% size of the SLPs while in the second scenario, only the operating load conditions and tie-line synchronization coefficients varied by $\pm 25\%$ from their nominal values (Arya & Kumar, 2016).

Table 5.16. Variations in the parameters of the four subsystems

Parameters And Variations	Non-reheat thermal		Reheat thermal		Gas plant		Hydro plant	
	+ 25%	- 25 %	+ 25	- 25 %	+ 25 %	- 25 %	+ 25 %	- 25 %
T_{ps}	25	15	25	15	25	15	25	15
K_{ps}	150	90	150	90	150	90	150	90
T_{Ti}	0.375	0.225	0.375	-0.225	-	-	-	-
T_{ri}	-	-	12.5	7.5	-	-	-	-
T_{Wi}	-	-	-	-	1.25	0.75	-	-
T_{CDi}	-	-	-	-	-	-	0.25	0.15
T_{ij}	0.10825	0.06495	0.10825	0.06495	0.10825	0.06495	0.10825	0.06495
T_{Gi}	0.1	0.06	0.1	0.06	-	-	-	-
X_i	-	-	-	-	0.75	0.45	-	-
Y_i	-	-	-	-	1.25	0.75	-	-
T_{RHi}	-	-	-	-	-	-	35.9375	21.5625
T_{GHi}	-	-	-	-	-	-	0.25	0.15
R_i	3	1.8	3	1.8	3	1.8	3	1.8
B_i	0.53125	0.31875		0.31875		0.31875		0.31875
D_i	10.4125	6.2475	10.4125	6.2475	10.4125	6.2475	10.4125	6.2475
K_{ri}	-	-	0.375	0.225	-	-	-	-
T_{Ri}	-	-	-	-	-	-	6.25	3.75
K_{ti}	0.5	0.3	0.5	0.3	-	-	-	-
K_{gi}	-	-	-	-	0.25	0.15	-	-
K_{hi}	-	-	-	-	-	-	0.625	0.375

Figures 5.26 and 5.27 demonstrate the simulated responses of the sensitivity analysis of frequency deviations and tie line power deviations with variations in all subsystems parameters. It can be observed that the Recatti equation and the control feedback gain are altered in this approach when the parameters are changed. As a consequence, the four subsystems decentralized control laws from Equation (5.14) may be produced for the variable and nominal parameters and their subsystem's feedback control gains are provided in Tables 5.17-5.19.

Table 5.17 the gains of the decentralized DOQAGCs for nominal parameters and data.

$\bar{L}_1 = [1.0785 \quad 1.6556 \quad 0.92 \quad 1.0098 \quad 4.1627 \quad -1.1692 \quad -1.6789 \quad -1.6789]$
$\bar{L}_2 = [2.4856 \quad 4.3134 \quad -0.349 \quad 2.6062 \quad 0.0411 \quad 5.4839 \quad -0.9966 \quad -0.6782 \quad -0.1194]$

$\bar{L}_3 = [-16.4585 \quad -30.5384 \quad -44.8027 \quad -1.0013 \quad 0.0887 \quad -31.1132 \quad 5.5643 \quad 6.3552 \quad 6.3552]$
$\bar{L}_4 = [4.7987 \quad 4.5247 \quad 4.3858 \quad 2.2303 \quad -1.0980 \quad 0.0421 \quad 13.6034 \quad -2.22 \quad 0.1636 \quad -1.8929]$

Table 5.18. the gains of the decentralized DOQAGCs for +25% of the nominal parameters and data.

$\bar{L}_1 = [1.7407 \quad 2.1612 \quad 1.2692 \quad 1.2459 \quad 4.1302 \quad -1.5129 \quad -2.0778 \quad -2.0778]$
$\bar{L}_2 = [3.6834 \quad 6.0762 \quad -0.2736 \quad 2.9142 \quad 0.0443 \quad 7.2157 \quad -1.4762 \quad -1.1087 \quad -0.1357]$
$\bar{L}_3 = [-15.38485 \quad -30.2777 \quad -41.4505 \quad -0.9960 \quad 0.1000 \quad -26.730 \quad 5.4834 \quad 6.2853 \quad 6.2853]$
$\bar{L}_4 = [6.9367 \quad 7.1990 \quad 6.7432 \quad 2.9339 \quad -1.4959 \quad 0.0558 \quad 20.2144 \quad -3.6115 \quad 0.2017 \quad -3.2066]$

Table 5.19. the gains of the decentralized DOQAGCs for -25% of the nominal parameters and data

$\bar{L}_1 = [0.3670 \quad 1.0854 \quad 0.5055 \quad 0.7647 \quad 3.9426 \quad -0.8083 \quad -1.2276 \quad -1.2276]$
$\bar{L}_2 = [1.4467 \quad 2.9416 \quad -0.3853 \quad 2.3072 \quad 0.0315 \quad 3.8449 \quad -0.6050 \quad -0.3422 \quad -0.0830]$
$\bar{L}_3 = [-21.3850 \quad -35.8613 \quad -58.3131 \quad -1.0068 \quad 0.0761 \quad -45.3401 \quad 6.7665 \quad 7.5296 \quad 7.5296]$
$\bar{L}_4 = [2.9982 \quad 2.5391 \quad 2.6584 \quad 1.6435 \quad -0.7539 \quad 0.0282 \quad 8.6132 \quad -1.2976 \quad 0.1184 \quad -1.0608]$

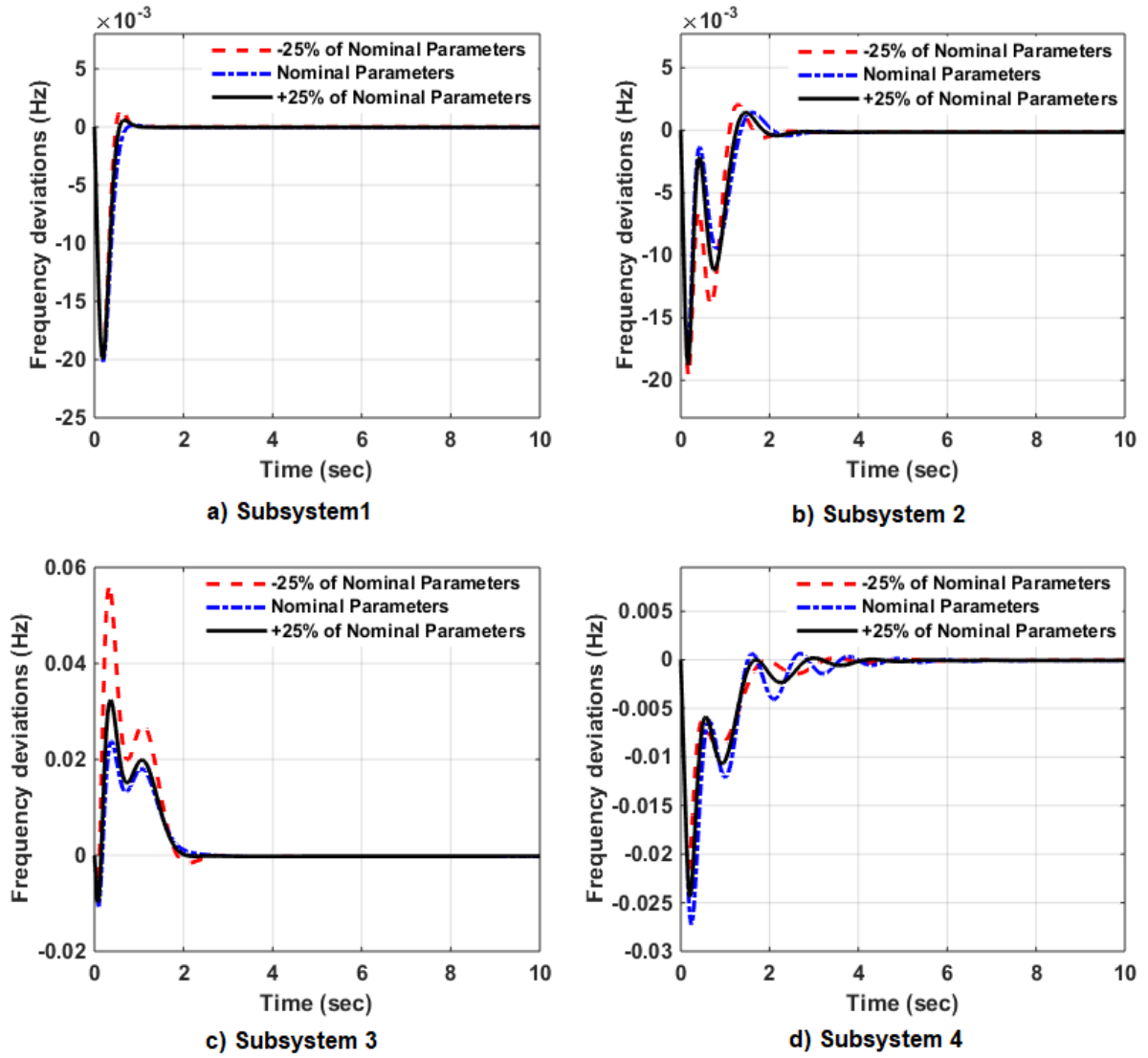


Figure 5.26. Frequency deviations response for 1% SLP in four subsystems with variation in all subsystems parameters.

A careful inspection of all resultss reveals that the proposed DOQAGC controller with parameters obtained at nominal values provides good performance for all variations in subsystems parameters on settling time for frequency and tie line power deivations results shown in Figures 5.26 and 5.27. However, the settling time of frequency and tie line power deivations for rehear thermal and hydro plants is about the same, 2 seconds each. However, the nonreheat power plant has the shortest settling time (1 sec) while the gas plant has the largest settling time (4 sec).

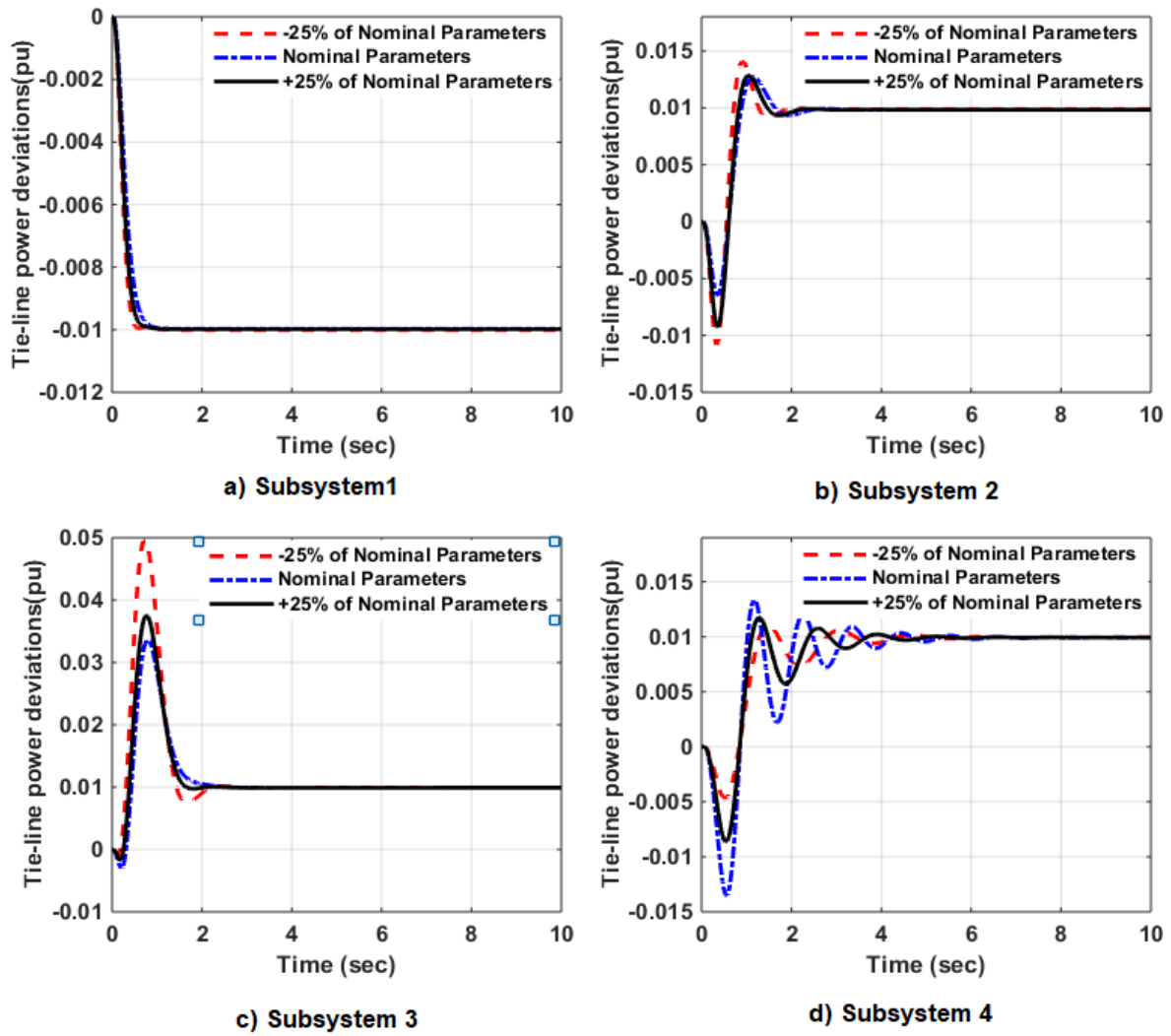


Figure 5.27. Tie line power deviations response to 1% SLP in four subsystems with variation in all subsystems parameters

The peak and under shoots responses of frequency and tie line power deviations are slightly differed with variations of the parameters. However, a reasonable undershoot and overshoots of frequency and tie line power deviations responses are still achieved in respect to variations. As a result, we can conclude that the proposed DOQAGC controllers developed with the subsystem parameters perform adequately when the system parameters vary. During the transient phase, however, all response times are related with oscillations with the exception of the temporal response of the non-reheat turbine subsystem. The changes in the peak overshoot response are caused by interaction variables between the subsystems, which must be compensated for. Table 5.20 summarizes the various performance indices for frequency and tie line deviations when all the parameters of the subsystem generators are varied.

Table 5.20. Analysis of sensitivity over the frequency and tie line deviations response performance indicators.

Control ler	State Variables and SLPS	Peak OS (Hz)			Peak US (Hz)			ST(sec)			$e_{ss}(\infty)$
		+ 25%	- 25 %	1%	+ 25 of	- 25 %	1%	+ 25 %	- 25 %	1%	
DOQAGC	SLP	1%	1%	1%	1%	1%	1%	1%	1%	1%	0
	Δf_1	0.011	0.0014	0	-0.02	-0.02	-0.02	1	1	1	0
	Δf_2	0.014	0.002	0.0013	-0.02	-0.01	-0.018	2	2	2	0
	Δf_3	0.032	0.06	0.024	-0.01	-0.02	-0.01	2	2	2	0
	Δf_4	0.001	0	0.001	-0.02	-0.022	-0.0027	4	4	4	0
	ΔP_{tie12}	0	0	0	0	0	0	1	1	1	0
	ΔP_{tie23}	0.003	0.004	0.013	-0.01	-0.001	-0.01	2	2	2	0
	ΔP_{tie34}	0.028	0.04	0.024	-0.0014	0	-0.003	2	2	2	0
	ΔP_{tie41}	0.002	0.001	0.0032	-0.01	-0.01	-0.014	4	4	4	0

Similarly, the operating load conditions and tie-line synchronization coefficients are varied by $\pm 25\%$ from their nominal values (Arya & Kumar, 2016). In this scenario, SLPs and tie-line synchronization coefficients are increased and decreased by $\pm 25\%$ % of their nominal values. The variations in load operating conditions and tie-line synchronization coefficients for each subsystem are shown in Table 5.21.

Table 5.21. Variations in load operating condition and tie-line synchronizing coefficients for each subsystem.

Parameters Variations	Variation of the tie-line synchronizing coefficient	Variations Load operating condition
Nominal	0.0866	1%
-25% of Nominal	0.06495	0.0075
+25% of Nominal	0.10825	0.0125

The dynamic responses of frequency and tie line deviations are examined for SLPs of diverse magnitude ranged from -0.0075 to 0.0125 in steps of 25% taking one at a time. Figures 5.28 to 5.29 show the frequency and tie line deviation responses. A reasonable settling time, undershoot and overshoots of frequency and tie line power deviations responses are still achieved in respect to variation in SLPs and tie line synchronization coefficients. However, as the magnitude of the SLPs is increased the responses of the

overshoot and undershoot increase too, while the settling time is constant for each subsystem hence this shows that the proposed DOQAGC control design is good irrespective of the SLPs and tie line synchronization coefficients variations.

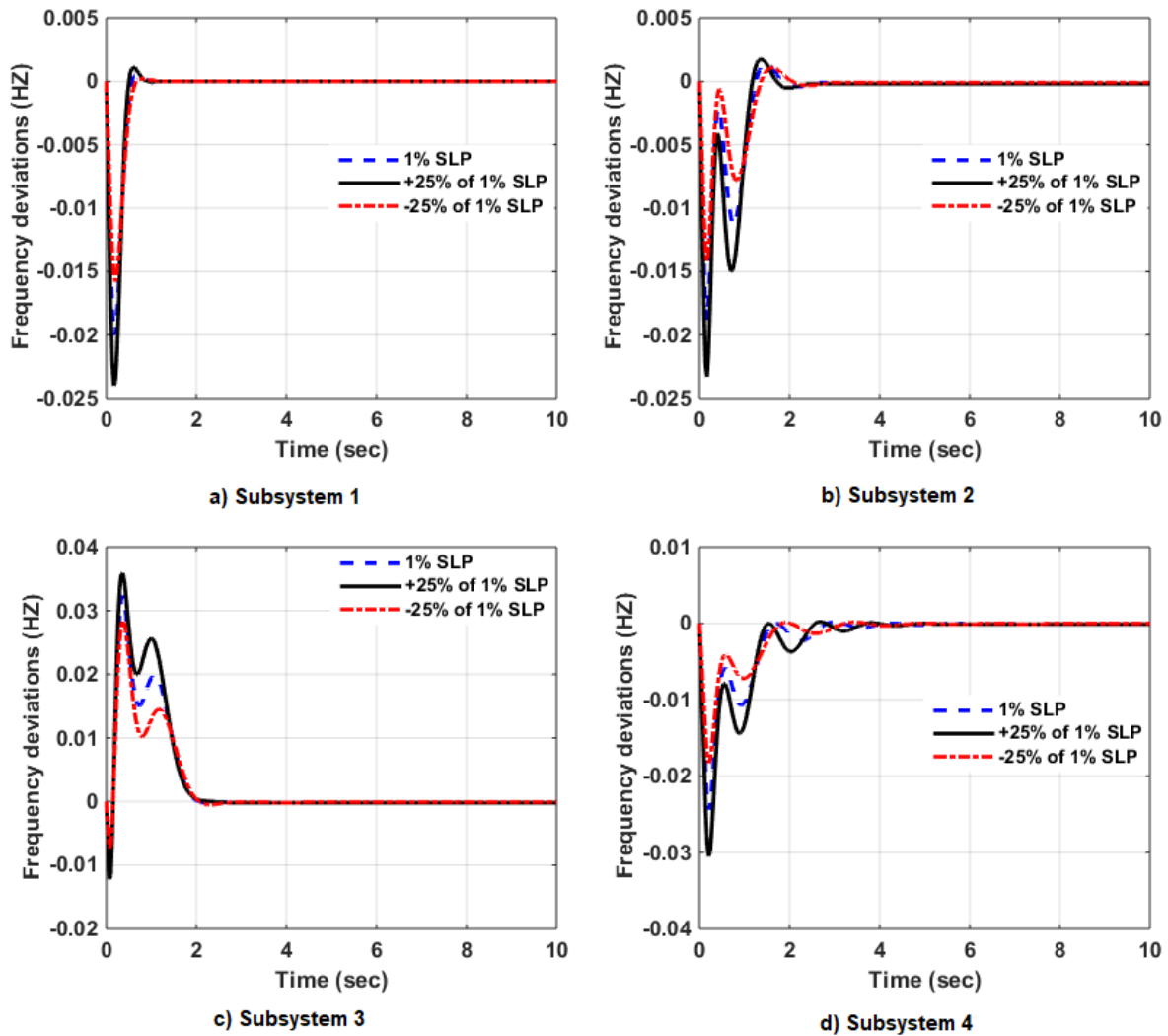


Figure 5.28. Frequency deviations responses with changes in SLP at $t = 0$ sec for four subsystems with variation in parameters.

Table 5.22 summarizes the various performance indices for frequency and tie line deviations when SLPs and tie line synchronization coefficients are varied. The frequency and tie line deviations results shown in Tables 5.20 and 5.22 appear to be identical, with just minor differences which is due to interaction variables. Therefore, the proposed DOQAGC controller performs well, delivering comparable outcomes and performances with 0% steady state error ($e_{ss}(\infty)$) values for both scenarios. In summary, the DOQAGC controller handles the variations of the parameters and operating load conditions while achieving subsystem stability and zero steady-state errors.

Table 5.22. Analysis of sensitivity over the frequency and tie line deviations response performance indicators when SLPs and tie line synchronization coefficients are varied.

Control ler	State Variables and SLPs	Peak OS (Hz)			Peak US (Hz)			ST(sec)			$e_{ss}(\%)$
		+ 25%	- 25 %	Nominal	+ 25 of	- 25 %	Nominal	+ 25 %	- 25 %	Nominal	
DOQAGC	SLP	1%	1%	1%	1%	1%	1%	1%	1%	1%	0
	Δf_1	0.001	0.001	0.001	-0.023	-0.016	-0.02	1	1	1	0
	Δf_2	0.002	0.002	0.002	-0.024	-0.016	-0.023	2	2	2	0
	Δf_3	0.033	0.002	0.036	-0.011	-0.007	-0.016	2	2	2	0
	Δf_4	0	0	0	-0.03	-0.018	-0.022	4	4	4	0
	ΔP_{tie12}	0	0	0	0	0	0	1	1	1	0
	ΔP_{tie23}	0.003	0.005	0.002	-0.017	-0.005	-0.08	2	2	2	0
	ΔP_{tie34}	0.034	0.021	0.028	0	0	0	2	2	2	0
	ΔP_{tie41}	0.022	0.01	0.002	-0.012	-0.05	-0.007	4	4	4	0

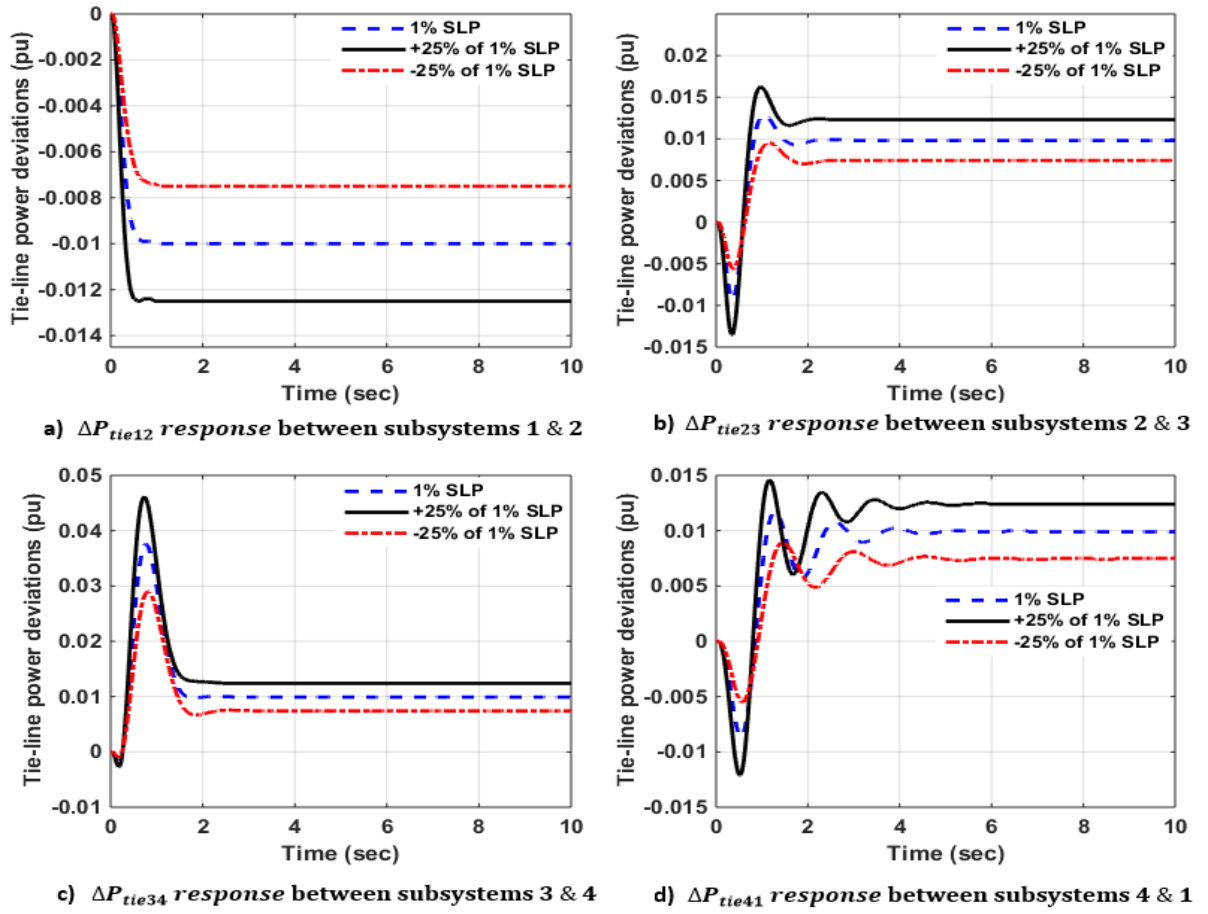


Figure 5-29. Tie line power deviations responses with changes in SLP at $t = 0$ sec for four subsystems.

5.5.4. COQAGC and DOQAGC Controllers Comparison

This section compares the closed system responses for the centralized FAIPS with the four decentralized subsystems. Table 5.23 summarizes the parameter settings for the two situations. Figures 5.30 and 5.31 depict the simulated responses of the centralized FAIPS and the decentralized four subsystems for Case 1. Following a thorough review of all replies, DOQAGC outperformed COQAGC in terms of peak overshoot and settling time with 0% steady-state error, ($e_{ss}(\infty)$) for frequency and tie line power variations. Except for the tie line power variation between areas 3 and 4, COQAGC has higher peak overshoot performance and a lower settling time between areas 2 and 3. Tables 5.24 and 5.25 summarize the quantitative comparison, where the peak OS and settling time of Δf_1 , Δf_2 , Δf_3 , Δf_4 , ΔP_{tie12} , ΔP_{tie23} and ΔP_{tie41} for DOQAGC are quite improved compared to that of COQAGC.

Table 5.23. Initial simulation parameter settings for the two cases.

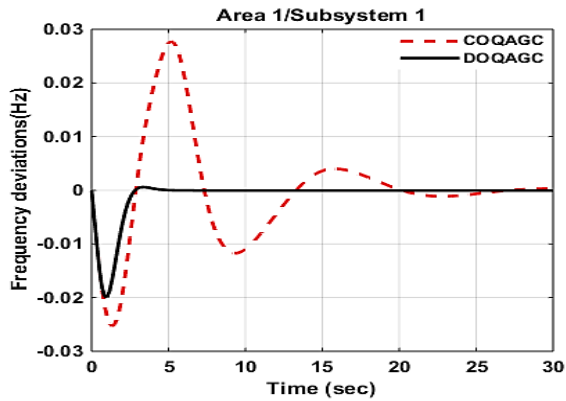
Type of Controller	Case	Area/Subsystem	Δf_i	ΔP_{Di}
CQQAGC with $\Delta P_{Di} = 0.01 \text{ puMW}$ when time at zero initial value	1.1	1	0.0 Hz	$\Delta P_{D1} = 0.01 \text{ pu}$
		2	0.0 Hz	$\Delta P_{D2} = 0.01 \text{ pu}$
		3	0.0 Hz	$\Delta P_{D3} = 0.01 \text{ pu}$
		4	0.0 Hz	$\Delta P_{D4} = 0.01 \text{ pu}$
DQQAGC with $\Delta P_{Di} = 0.01 \text{ puMW}$ when time at zero initial value	1.2	1	0. HZ	$\Delta P_{D1} = 0.01 \text{ pu}$
		2	0. HZ	$\Delta P_{D2} = 0.01 \text{ pu}$
		3	0. HZ	$\Delta P_{D3} = 0.01 \text{ pu}$
		4	0. HZ	$\Delta P_{D4} = 0.01 \text{ pu}$
CQQAGC with simultaneous disturbances set at 5 sec intervals	2.1	1	0.0 HZ	$\Delta P_{D1} = 0.01 \text{ pu}$
		2	0.0 HZ	$\Delta P_{D2} = 0.01 \text{ pu}$
		3	0.0 HZ	$\Delta P_{D3} = 0.01 \text{ pu}$
		4	0.0 HZ	$\Delta P_{D4} = 0.01 \text{ pu}$
DQQAGC with simultaneous disturbances set at 5 sec intervals	2.2	1	0 HZ	$\Delta P_{D1} = 0.01 \text{ pu}$
		2	0 Hz	$\Delta P_{D2} = 0.01 \text{ pu}$
		3	0 HZ	$\Delta P_{D3} = 0.01 \text{ pu}$
		4	0 Hz	$\Delta P_{D4} = 0.01 \text{ pu}$

Table 5.24. Case 1 dynamic response to frequency and tie-line power deviations in terms of peak OS.

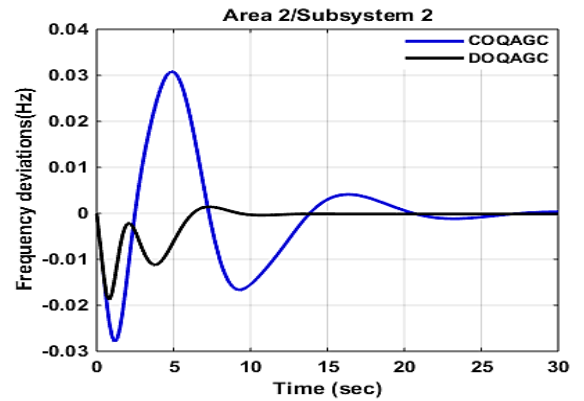
Type of Controller	Δf_1	Δf_2	Δf_3	Δf_4	ΔP_{tie12}	ΔP_{tie23}	ΔP_{tie34}	ΔP_{tie41}	$e_{ss}(\infty)$
COQAGC	0.028	0.03	0.038	0.041	-0.025	-0.02	-0.021	-0.032	0
DOQAGC	-0.02	-0.019	0.031	-0.024	0.01	-0.008	0.039	0.011	0
Reduction %	29%	37%	18%	41%	60%	60%	-46%	66%	0%

Table 5.25. Case 1 dynamic responses to frequency and tie-line power variances are compared in terms of settling time.

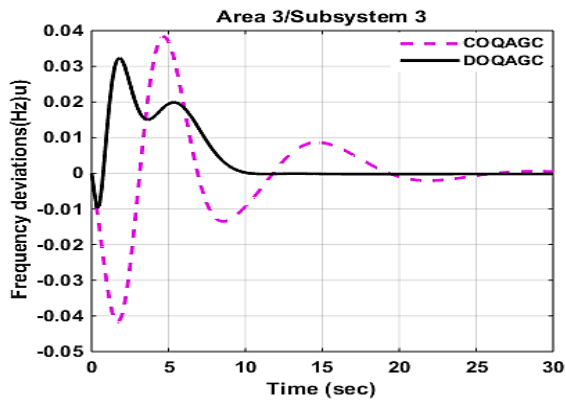
Type Controller of	Δf_1	Δf_2	Δf_3	Δf_4	ΔP_{tie12}	ΔP_{tie23}	ΔP_{tie34}	ΔP_{tie41}	$e_{ss}(\infty)$
COQAGC	13	14	12	13	18	3	16	16	0
DOQAGC	5	9	10	9	4	8	8	12	0
Reduction %	62%	36%	17%	31%	78%	-63%	50%	25%	0%



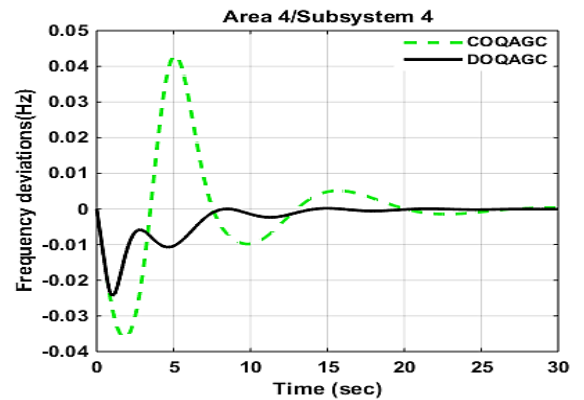
(a) Frequency deviations area 1 and subsystem1



(b) Frequency deviations area 2 and subsystem 2



(c) Frequency deviations area 3 and subsystem 3



(d) Frequency deviations area 4 and subsystem 4

Figure 5.30. Frequency deviation responses of four subsystems and four control areas.

Figures 5.32 and 5.33 depict the simulated responses of the centralized FAIPS and the decentralized four subsystems for Case 2. Following a thorough study of all responses, DOQAGC outperformed COQAGC in terms of peak overshoot, settling time, and disturbance

rejection response time for frequency and tie line power variations. Except for the tie line power variance between areas 3 and 4, the COQAGC offers higher peak overshoot performance.

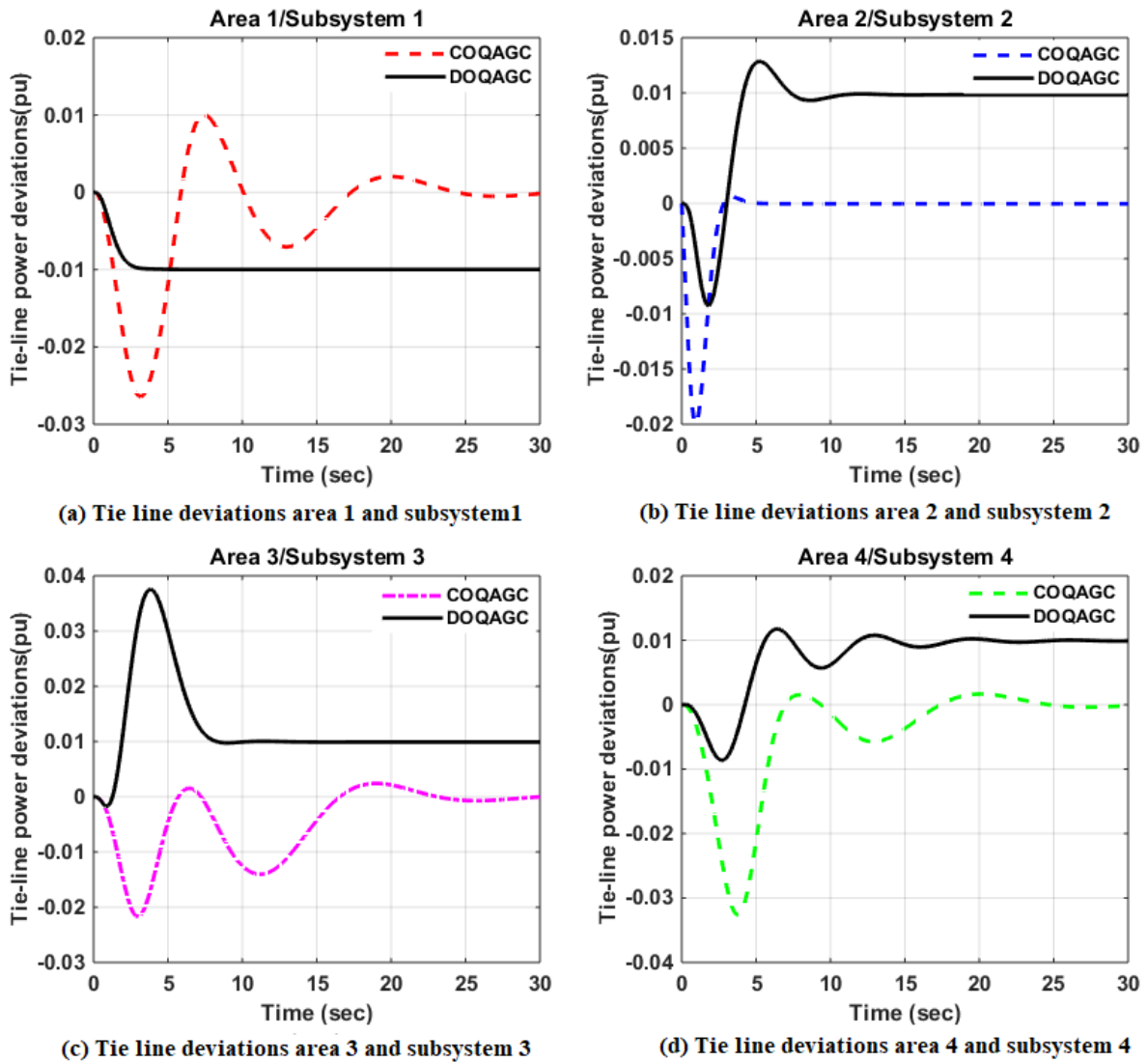


Figure 5.31. Tie line power deviation responses of four subsystems versus four control areas.

Tables 5.26, 5.27, and 5.28 highlight the quantitative comparison, where the OS peak, settling time, and disturbance rejection response time of Δf_1 , Δf_2 , Δf_3 , Δf_4 , ΔP_{tie12} , ΔP_{tie23} and ΔP_{tie41} DOQAGC results are noticeably better than COQAGC results.

Table 5.26. Case 2 dynamic responses to frequency and tie-line power deviations in terms of peak OS.

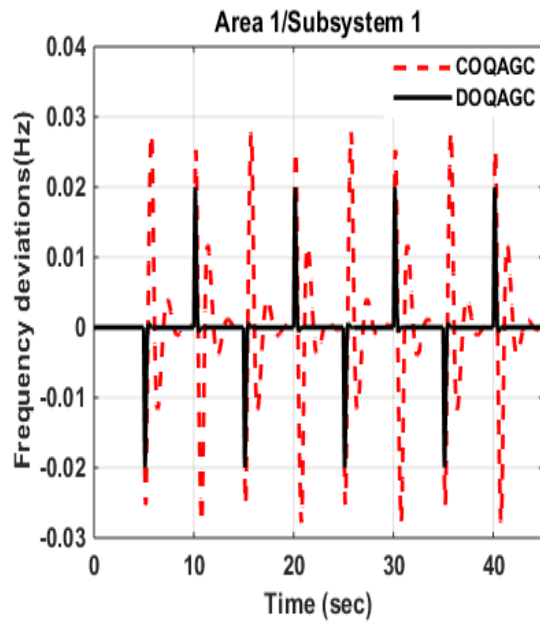
Type of Controller	Δf_1	Δf_2	Δf_3	Δf_4	ΔP_{tie12}	ΔP_{tie23}	ΔP_{tie34}	ΔP_{tie41}	$e_{ss}(\infty)$
COQAGC	0.028	0.03	0.042	0.043	-0.027	-0.026	-0.022	-0.033	0
DOQAGC	0.02	0.02	0.032	0.024	0.01	-0.019	0.038	0.019	0
Reduction %	29%	37%	24%	44%	63%	27%	-42%	42%	0%

Table 5.27. Case 2 dynamic responses to frequency and tie-line power variances are compared in terms of settling time.

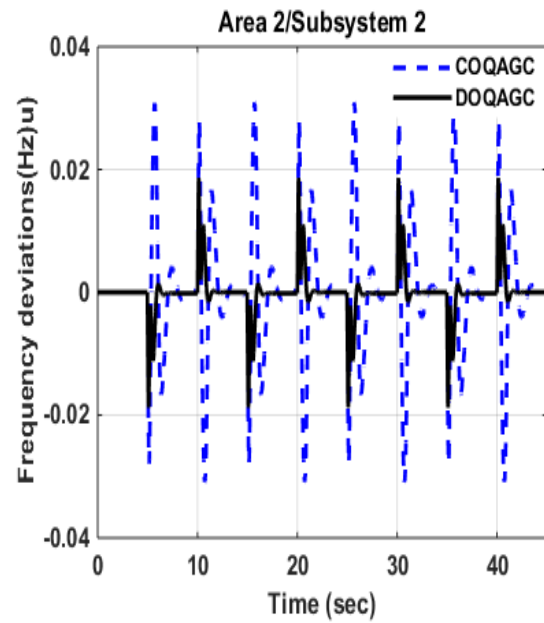
Type of Controller	Δf_1	Δf_2	Δf_3	Δf_4	ΔP_{tie12}	ΔP_{tie23}	ΔP_{tie34}	ΔP_{tie41}	$e_{ss}(\infty)$
COQAGC	4	4	4	4	4.4	3.5	4.3	3.7	0
DOQAGC	0.5	1.7	1.7	2.2	0.6	1.4	1.9	3.5	0
Reduction %	88%	58%	58%	45%	95%	60%	56%	5%	0%

Table 5.28. Dynamic responses to frequency and tie-line power deviations of disturbance rejection response time (DRRT) for Case 2 are compared.

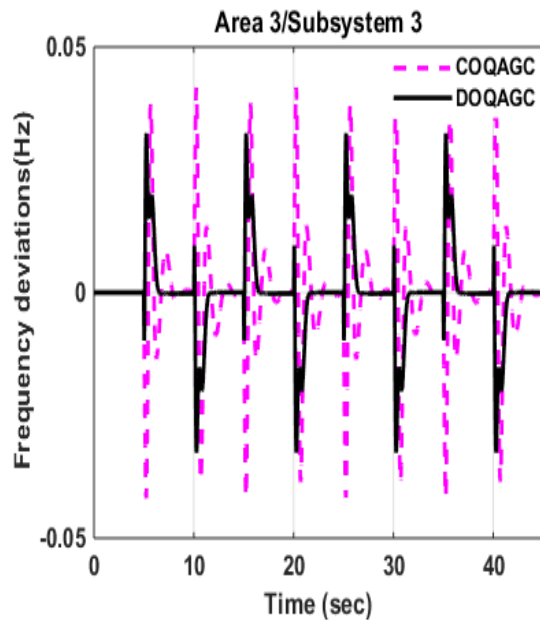
Type of Controller	Δf_1	Δf_2	Δf_3	Δf_4	ΔP_{tie12}	ΔP_{tie23}	ΔP_{tie34}	ΔP_{tie41}	$e_{ss}(\infty)$
COQAGC	4.2	4.2	4.8	4.8	4.9	4.2	4.5	4.5	0
DOQAGC	1	2	1.8	3	0.8	2	2	3.5	0
Reduction %	76%	52%	63%	38%	84%	52%	51%	22%	0%



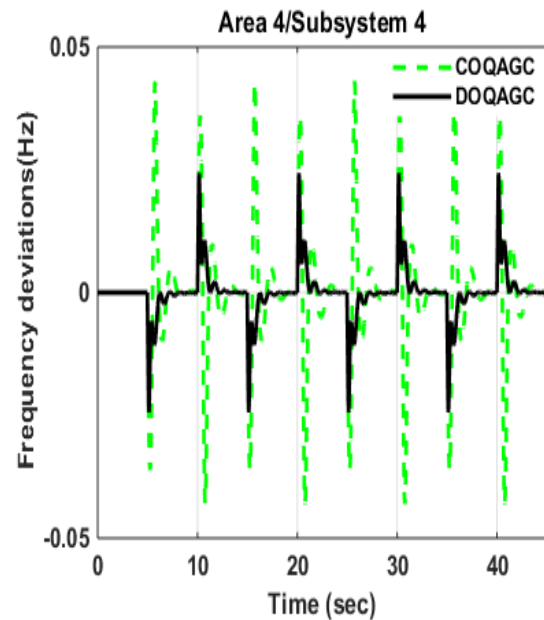
(a) Frequency deviations area 1 versus subsystem 1



(b) Frequency deviations area 2 versus subsystem 2



(c) Frequency deviations area 3 versus subsystem 3



(d) Frequency deviations area 4 versus subsystem 4

Figure 5.32. Responses to frequency deviations of four subsystems vs four control regions for simultaneous disturbances.

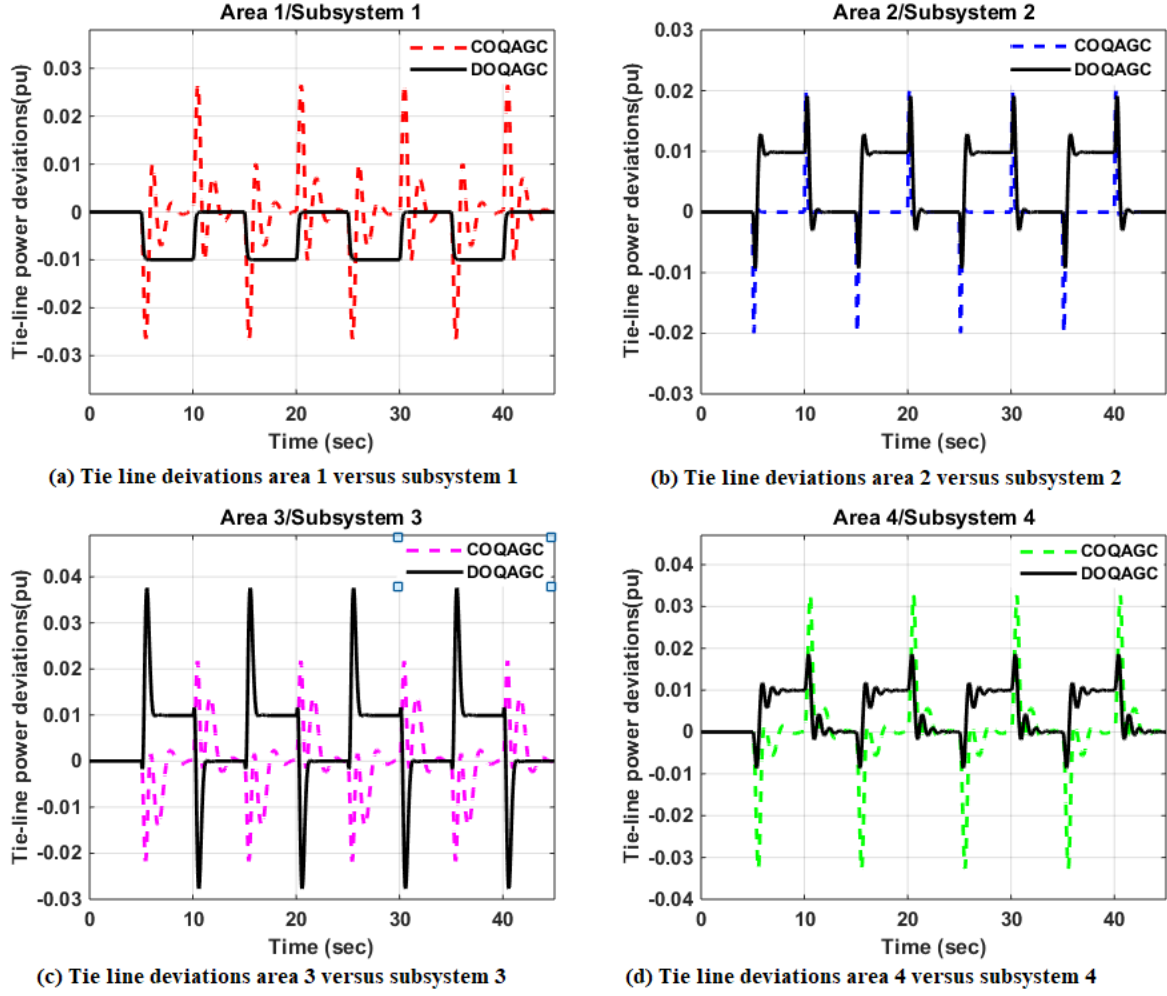


Figure 5.33. Tie line power deviation responses of four subsystems vs four control zones for simultaneous disturbances.

5.4.5. Proposed Controllers versus Literature Controllers

This section uses a comparative study to validate the performance of the developed controllers (COQAG and DOQAGC) vs. controllers from the literature. This comparison study includes various test cases performed in chapters 4 and 5, namely, a comparison of 1% SLPs applied at $t=0$ sec, simultaneous SLPs, and sensitivity analysis. The controllers' performance will be determined by dynamic response measures such as peak overshoot, settling time, steady state error ($e_{ss}(\infty)$), involved cost function approaches, and weighting matrix selection methods. Literature controllers being compared to the DOAGGC controller include COQAGC (Esmail & Krishnamurthy, 2023), and Full Stat Feedback Controller (FSFC) (Paramar, Majhi & Kothari, 2012). Decentralized optimized Integrated Sliding Mode Control load frequency Control (OISMCLFC) (Alhelou et al, 2023), Distributed Linear Quadratic Regulator Load Frequency Control (DLQR- LFC) (Vlahakis, Dritsas & Halikias, 2019), Robust Decentralized Load Frequency Control (RDLFC) (Dong, Zhang & Gao, 2012),

and Fuzzy Gain Scheduling Controller (FGSC) (Arya & Kumar, 2016). The sensitivity analysis was carried out for different system conditions and the simulation results are summarized in Tables 5.29 to 5.32.

Tables 5.29-5.32 summarize the comparison between control strategies presented by the authors from the literature to the simulation results of the proposed COQAGC and DOQAGC controllers. All of the findings in tables 5.29 – 5.32 show that the frequency and tie-line deviations were returned to zero at the steady state as predicted. The comparisons are based on non-reheat thermal generator findings for various interconnected power systems provided in this thesis are compared with the literature.

Table 5.29 summarizes the comparison between controllers from the literature and the results of the proposed COQAGC and DOQAGC when 1% SLPs are applied at $t = 0$ for sec frequency and tie line deviations. When compared to all other controllers, the DOQAGC controller has the best settling time (TS), 9 sec, while the FSFC controller achieves the best peak OS, 0.008 Hz.

Table 5.29. Comparison between the developed COQAGC and DOQAGC with FSFC and OISMCLFC controllers from the literature

Type of Controller: Frequency and tie line deviations when 1% SLPs are applied at $t = 0$ sec.	SLPs	Peak OS (Hz)		ST (sec)		$e_{ss}(\infty)$	Cost Function Method	Weighting matrices selection	Domain
		Δf_1	ΔP_{tie12}	Δf_1	ΔP_{tie12}				
COQAGC(Esmail & Krishnamurthy, 2023)	1%	0.03	0.027	14	0	0	LQR	FMM	Discrete
DOQAGC	1%	0.02	0.01	9	0	0	LQR	FMM	Discrete
FSFC(Paramar, Majhi & Kothari, 2012)	1%	0.006	-	15	-	0	LQR	Trial and error	Continuous
OISMCLFC (Alhelou et al, 2023)	1%	0.06	-	10	-	0	ISE	MPSO	Continuous

Table 5.30 summarizes the comparison between controllers from the literature and the results of the developed COQAGC and DOQAGC when 1% simultaneous SLPs are applied at specified time intervals for frequency and tie line deviations. When compared to all other controllers, the DOQAGC controller achieves the best disturbance rejection response time (DRRT), 1 sec and 2 secs for frequency and tie line deviations respectively. Furthermore, when compared to other controllers from the literature, DOQAGC obtains the best peak OS, 0.01 Hz, for tie line deviations, whereas RDLFC achieves the best peak OS, 0.002 Hz, for frequency deviations.

Table 5 30. Comparison between the developed COQAGC and DOQAGC with DLQRLFC and RDLFC controllers from the literature

Type of Controller: Frequency and tie line deviations when simultaneous SLPs are applied approximately at same time intervals.	SLPs	Peak OS (Hz)		DRRT (sec)		$e_{ss}(\infty)$	Cost Function Method	Weighting matrices selection	Domain
		Δf_1	ΔP_{tie12}	Δf_1	ΔP_{tie12}				
COQAGC(Esmail& Krishnamurthy, 2023)	1%	0.028	0.027	4.2	4.2	0	LQR	FMM	Discrete
DOQAGC	1%	0.02	0.01	1	2	0	LQR	FMM	Discrete
DLQRLFC(Vlahakis, Dritsas& Halikias, 2019	1%	0.075	0.085	2	5	0	LQR	Trial and error	Continuous
RDLFC(Dong,Zhang & Gao, 2012)	1%	0.002	0.017	3	3	0	-	-	Continuous

Tables 5.31 and 32 summarize the comparison between controllers from the literature and the results of the developed COQAGC and DOQAGC controllers when sensitivity analysis is considered. The findings reveal that the FGSC controller achieves the best peak OS, 0.003Hz, and 0 Hz for cases +25% of 1% and -25% of 1% SLPs, while DOQAGC and FGSC achieve the best peak OS, 0 Hz for frequency deviations when compared to all other controllers for the nominal case. Furthermore, DOQAGC and FGSC provide the best peak OS, 0.0 Hz for the tie line deviations for the instances, nominal, +25 % of 1 % SLP and -25% of 1% of SLP. In contrast, the findings in Table 5.32 show that DOQAGC achieves the best settling time, 1 sec for the cases nominal, +25 % of 1 % SLP, and -25 % of 1% SLP when compared to the other controllers for the frequency and tie-line deviations.

Table 5.31. Sensitivity analysis of the frequency and tie-line power deviations for non-reheat thermal power systems for peak OS.

Type of Controller	SLPs	Δf_1			ΔP_{tie12}			$e_{ss}(\infty)$	Cost Function Method	Weighting matrices selection	Domain
		%	+ 25%	- 25 %	Nominal	+ 25%	-25 %				
COQAGC(Esmail& Krishnamurthy, 2023)	\	0.021	0.01	0.005	0.003	0	0.001	0	LQR	FMM	discrete
DOQAGC	1%	0.011	0.014	0	0	0	0	0	LQR	FMM	discrete
FSFC(Paramar, Majhi& Kothari,2012)	1%	0.008	0.007	0.01	-	-	-	0	LQR	Trial and Error	continuous
FGSC controller (Arya & Kumar, 2016)	1%	0.0003	0	0	0	0	0	0	LQR	FMM	continuous

In General, the differences in the controller's performance findings are attributable to the methodologies used to formulate the cost function or the methods employed by the authors to establish the weighting matrices, as well as the kind of interconnected power systems considered in the study. For example, the LQR control design technique and FMM were used to develop the cost function and choose weighting matrices (Q and R) for the FGSC,

COQAGC, and DOQAGC controllers. In contrast, the LQR control design method and trial-and-error were used to develop the cost function and choose weighting matrices for the FSFC and DLQRLFC controllers. Furthermore, FSFC simulation results were obtained when a single area power system with non-reheat, reheat, hydro, and gas turbines was considered (Paramar, Majhi & Kothari, 2012). In contrast, COQAG and DOQAGC controller simulation results in this thesis were obtained for four control area interconnected power systems with non-reheat, reheat, hydro, and gas turbines. Similarly, for sensitivity analysis, the controller's findings vary due to the kind of interconnected power systems used in the simulation.

Table 5-32. Sensitivity analysis of the frequency and tie-line power deviations for non-reheat thermal power systems for settling time.

Type of Controller	SLPs	Δf_1			ΔP_{tie12}			$e_{ss}(\infty)$	Function Method	Weighting matrices selection	Domain
	%	+ 25%	- 25 %	Nominal	+ 25%	- 25 %	Nominal				
COQAGC(Esmail& Krishnamurthy, 2023)	1%	4	4	4	4	3	4	0	LQR	FMM	Discrete
DOQAGC	1%	1	1	1	1	1	1	0	LQR	FMM	Discrete
FSFC(Paramar, Majhi& Kothari,2012)	1%	15	15	15	-	-	-	0	LQR	Trail and Error	Continuous
FGSC controller (Arya & Kumar, 2016)	1%	4	4	4	4	4	4	0	LQR	FMM	Continuous

The FSGC is simulated using an identical two-area linked power system with a non-reheat turbine in each area, whereas the DOQAGC is simulated with four control interconnected power systems with non-reheat, reheat, hydro, and gas turbines. Tables 5.31 and 5.32 indicate that the DOQAGC results are competitive with FGSC in terms of settling time and peak OS when the FMM technique is applied. This implies that FMM can systematically simplify and ease the selection of weighting matrices. Furthermore, the decentralization of a four-area interconnected power system utilizing an overlapping decomposition approach decreases the computational difficulties and reduces the size of complex interconnected power systems. Consequently, DOQAGC enhances frequency and tie line deviation results and may be deployed in any complex interconnected power system.

5.6. Conclusion

The overlapping decomposition approach was utilized in this chapter to decompose a given state space equation of a power system into smaller subsystems, converting complicated engineering difficulties into simpler ones. The overlapping decomposition of identical two and four-area control power systems has been addressed. Consideration of interaction variables between subsystems and incorporation of ACE integrals into control areas increased the size of the subsystems.

The dynamic response of DOQAGC of simple and complicated power systems with identical two-area control power systems with non-reheat turbines and four-area control power systems with non-reheat, reheat, hydro, and gas turbines is studied. To demonstrate the overlapping decomposition method, the continuous and discrete state space equations of two area power systems with two identical non-reheat turbines control and four area control power system non-reheat, reheat, hydro, and gas turbines were formulated and later used for the COQAGC and DOQAGC comparative demonstration. The complicated optimisation issue has been reduced to local sub-optimization problems at the subsystem level due to the overlapping decomposition, decreasing the dimensions of the controllers and computing difficulties. The functional minimization approach was utilized to choose the weighting matrices Q and R of the sub-cost function while considering identical two and four-area control power systems. This strategy simplified and systematic selection process.

The simulation results obtained with DOQAGC were compared to those obtained with COQAGC and proposed controllers from the literature. The dynamic results are obtained for 1% SLPs at $t = 0$ and 1% simultaneous SLPs. All dynamic responses attained a steady state error of zero. The dynamic responses of DOQAGC and COQAGC to peak overshoot, settling time, and disturbance rejection reaction time were competitive when compared to the dynamic responses of TAIPS and two decentralized subsystems for 1% SLPs at $t = 0$ and 1% simultaneous SLPs. When the dynamic responses of four area control power systems and four decentralized subsystems were compared, DOQAGC outperformed COQAGC in all cases for 1% SLP at $t = 0$ and 1% simultaneous SLPs, with smaller peak overshoot, shorter settling time, and disturbance rejection response time, respectively, with zero steady-state error. The effect of robustness on subsystem parameter fluctuations was explored. The rigorous evaluation of all responses demonstrates that for parameters changing between the nominal parameter values and the ± 25 parameter values, the robustness delivers the same settling time and zero steady-state error, but the dynamic response is altered for peak overshoot. However, due to interaction variables across the subsystems, all responses fluctuate throughout the transitory period. This is one of the drawbacks of decentralized control concepts because it does not consider the effects of interconnection variables. The test of multi-source power system having a renewable resource with generation rate constraints (GRC) is not conducted for DOQAGC control due to time constraints. This test will do it in the future studies. The following chapter will discuss the conclusion and future recommendations.

Chapter 6

Conclusion and Future Recommendations

6.1. Introduction

The goal of this research was to develop software, control methods, and algorithms that can handle Automatic Generation Control (AGC) problems in Multi-Area Interconnected Power Systems (MAIPS) and reduce their computational complexity while achieving robustness against step load perturbations and modelling uncertainties.

This chapter concludes the thesis by validating the research hypotheses and summarizing the research questions' answers.

The literature review in Chapters 2, 3, 4, and 5 presents the various control strategies and modelling formulations related to AGC problems. It provides the context for the development of discrete centralized and discrete decentralized Optimal Quadratic Automatic Generation Control (COQAGC) and their performances in steady state time characteristics of SLPs, modelling uncertainties, and nonlinearities that are related to MAIPS and answers the research questions 1, 2, 3, and 4.

Chapter 3 includes a review of the literature on various types of decomposition methods, inclusion principles, the dynamics of Two-Area Interconnected Power Systems (TAIPS) with identical non-reheat turbines, and the use of overlapping decomposition to decompose TAIPS into two subsystems and answer research questions 1 and 6.

The literature review in Chapters 4 and 5 sets the stage for using optimal control theory and Lagrangian multipliers, as well as developing functional minimization methods (FMMs), to design centralized discrete COQAGC, centralized discrete COQAGC regulator, and discrete decentralized DOQAGC, and answer research questions 3, 4, and 5.

The simulation methodology and results in chapters 4 and 5 put the hypotheses to the test by applying the proposed discrete centralized optimal quadratic automatic generation control (COQAGC) and discrete decentralized optimal quadratic automatic generation control (DOQAGC) to various cases of interconnected power systems. The simulation results address the following research questions: 1, 2, 3, 4, (a & b), 5, and 7.

This chapter concludes with a summary of the research findings and recommendations for future research.

6.2. Overview of Findings

The research hypotheses are stated as follows:

1(a). Most studies describe interconnected power systems in the continuous time domain; however, the discrete controller is often implemented on embedded computers in communication channels, reducing closed-loop performance and making the power system unstable. The power system implementation can be enhanced by discretizing and designing the power system modelling and the controller using the Euler discretization method.

1(b). The design of centralized and decentralized discrete optimum quadratic AGC controllers using optimal control theory, Lagrange multipliers, multivariable control theory, and functional minimization method can successfully improve multi-area interconnected power systems.

2. The most widely utilized state and control weight matrix selection approaches, like as identity matrices, controllability and observability indices, and Hybrid Bacteria Foraging oriented particle swarm optimization (HBF-PSO)) methods, are insufficient for optimal AGC-based LQR in large-scale interconnected power systems. To accurately construct the state and control weighting matrices, the Functional Minimization Method, derived from the partial derivative of the function of the cost function, is applied.

3. Using an overlapping decomposition method and inclusion principles to divide a large-scale interconnected power system into independent subsystems simplifies design obstacles and minimizes computational time.

4. The development of discrete centralized and discrete decentralized control methods based on functional minimization, decomposition, and decentralization approaches coupled with the requirement of AGC problems can be highly beneficial in achieving robust performance against step load perturbations, modelling uncertainties, and nonlinearities such as generation rate constraints (GRC), which are the most important aspects of large-scale systems.

The research questions posed in Section 1.2 of the introduction are addressed below:

Research question 1: How are continuous state space equations built as a combination of first-order differential equations obtained from the transfer functions of various cases of multi-area interconnected power systems?

This research question was addressed in Chapters 3, 4, and 5 on multi-area interconnected power system modelling. Formulating the state space equations from the transfer functions allowed the modelling behaviour of the interconnected power system to be entirely captured

in the time domain. The derivation of first-order differential equations from the transfer functions of various subsystems in an interconnected power system is an effective method for developing a state space model for an interconnected power system with any number of control areas or generating units. Most researchers in the literature used this approach for either multiple control areas with one identical generator in each area or single control area with multi-source power generators such as thermal with a non-reheat turbine, hydro, thermal with reheat turbine, and gas plants for Linear Quadratic Regulator (LQR) based AGC problem (Parmar, Majhi & Kothar, 2012; Abdelhamid & El-Gaafary, 2018; Vlahakis, Dritsas, & Halikias, 2019; Panwar et al., 2022). However, in this study, state space equations for a two-area interconnected power system with identical non-reheat turbines in each area, a four-area interconnected power system with non-reheat, reheat, hydro, and gas turbines, and two-area multi-source generators are formulated in the continuous and discrete time domains. The continuous time state space equation for a two-area interconnected power system with identical non-reheat turbines is formulated in Chapters 3 and 4, while the continuous time state space equations for a four-area interconnected power system with non-reheat, reheat, hydro and gas turbines and two control areas with multi-source generators in each control area are formulated in chapter 4. It is worth noting that a four-area interconnected power system with non-reheat, reheat, hydro, and gas turbines proposed in this thesis is adopted from the work presented by Parmar, Majhi and Kothari (2012) in a single control area and Kundur, 1994. The dynamic of area control errors (ACEs) is augmented into the state space equations to achieve zero steady-state errors for frequency and tie line deviations. This addition expands the dimensions of the area control and adds more variables that depend on the number of control areas. The formulated continuous state space equations above are also decomposed into two subsystems, each with an identical non-reheat turbine, and four subsystems, each with a non-reheat turbine, reheat, hydro, and gas turbines, as demonstrated in Chapter 5. The overlapping decomposition method is an effective and feasible method for decomposing interconnected power systems into subsystems using inclusion principles (Gao & Yang, 2015).

Research question 2: Using Shannon's theorem and Euler's discretization criterion, how are discrete state space equations created for various cases of multi-area interconnected power systems?

This research question was addressed in Chapters 4 and 5 on sampling and discretizing the state space equations of a multi-area interconnected power system. Euler's discretization method helps discretize the model of the power system and AGC control laws since most developed state space equations in the literature are presented in the continuous time domain, specifically for the LQR-AGC problem; Abdelhamid & El-Gaafary, 2018, Vlahakis,

Dritsas, & Halikias, 2019), Panwar et al., 2022; Parmar, Majhi and Kothari, 2012). However, all the developed state space equations discussed in research question 1 are discretized in chapters 4 and 5 using Shannon's theorem and Euler's discretization criteria. The benefits and drawbacks of Euler's discretization method are discussed in Chapter 4, where it is described as an easy and direct discretization method. However, it provides an approximated discrete state space model.

Research question 3: a) What are the design requirements for selecting state and control weighting matrices with a minimal number of state variables using the Functional Minimization Method? b) Can the usage of FMM with few variables aid in the creation of the proposed controllers without the requirement for an observer to estimate the variables? c) What influence do the state and control weighting matrices based on the Functional minimization approach have on the proposed controllers COQAGC and DOQAGC?

This research question is addressed in chapters 4 and 5 when constructing the state and control weighting matrices for the proposed discrete centralized and discrete decentralized LQR AGC controllers. The design requirements for the functional minimization approach, such as area control errors (ACEs), integral area control errors (IACEs), and control effort (u), are transformed into the cost function to formulate the functional of the cost function. Most importantly, the functional minimization approach requires the minimization of ACEs, IACEs, and control efforts about power system variables at the steady state. For each type of MAIPS, three variables are used to formulate various functional minimization functions: squared area control errors, squared integral of area control areas, and squared control energy expenditure. The state and control weighting matrices (Q and R) are created by performing partial differentiations on the functional minimization functions. For discrete Centralized COQAGC, state and weight matrices for TAIPS, FAIPS, and N multi-area Interconnected Power systems (NAIPS) are constructed using the functional minimization method. This approach is used to construct state and control weighting matrices for the two decentralized subsystems, four decentralized subsystems, and N decentralized subsystems for different discrete decentralized DOQAGC controllers. Most LQR design methods for interconnected power systems assumed that all state variables were available to the feedback control law (Pillai et al., 2022). This is impractical because it is impossible to measure all state variables of large-scale interconnected power systems directly. To avoid this difficulty, state variables are typically estimated with an observer using feedback control law and state estimation via the Kalman filter. The construction of state and control weighting matrices from a few state variables, such as ACEs and IACEs, on the other hand, reduces the number of measurable state variables, making FMM more practical in implementing AGC control laws. Furthermore, most available FMM methods in the literature are constructed for

identical two-area power systems with continuous feedback control laws. Furthermore, the constructed state and control weighting matrices for NAIPS and N decentralized subsystems demonstrate and validate the capability of the functional minimization method for complex interconnected power systems. It facilitates and systematizes the selection of state and control weighting matrices, resulting in effective optimal state and control weighting matrices for complex interconnected power systems.

Research question 4: How are discrete centralized and discrete decentralized optimal quadratic AGC controller problems formulated utilizing optimal and suboptimal quadratic cost functions with Lagrange multipliers, subject to discrete state space equations and discrete sub-state space equations?

This research question was addressed in Chapters 4 and 5 on the development of optimization methods for MAIPS AGC problems. Modern optimal control theory, quadratic cost functions, Lagrange multipliers, Algebraic Riccati Equation (ARC), and FMMs are used to design COQAGC and DOQAGC controllers for various AGC scenarios. The discrete optimal AGC control strategy forms an optimal method for computing the feedback gains of the centralized and decentralized optimal AGC Controllers by minimizing the cost functions. The Evolutionary programming technique is used in the literature to find the best gains for decentralized LQR control in interconnected power systems (Bensenouci & Abdel Ghany, 2005). Furthermore, the majority of optimal LQR AGC problems for interconnected power systems are formulated in the continuous time domain using optimal control theory, quadratic cost functions, pole placement method, ARC, and the computed state and control weighting matrices by using various techniques such as estimation techniques as well as trial and error approach. Pole placement is a linear control technique that achieves the desired performance using the design specification's characteristic equations. It is helpful for Multi-input Multi-output (MIMO) systems but unsuitable for sizeable complex power systems due to the complexity of the large-scale power system characteristic equation. However, the COQAGC and DOQAGC control problems are formulated respectively using i) optimal quadratic cost function, Algebraic Riccati Equation (ARC), and functional minimization methods, ii) sub-optimal quadratic cost functions, overlapping decomposition method, Sub Algebraic Riccati Equations (SARC) and sub-functional minimization methods in this thesis. The Algebraic Riccati Equation is solved iteratively to obtain the optimal gain of the proposed COQAGC and DOQAGC control laws. In Chapter 4, discrete optimal LQR AGC problems with N control areas, TAIPS with two identical non-reheat turbines, and FAIPS with non-reheat, reheat, hydro, and gas turbines are formulated. Similarly, in Chapter 5, discrete optimal LQR AGC problems for N decentralized subsystems are formulated: two

decentralized subsystems with non-reheat turbines in each area and four decentralized subsystems with non-reheat, reheat, hydro, and gas turbines, respectively.

Research question 5: What improvements do discrete centralized and discrete decentralized optimal quadratic automatic generation (AGC) controllers provide to multi-area power systems in terms of robust performance against step load perturbations, modelling uncertainties, and nonlinearities such as generation constraints rate.

The design, implementation, and simulation of the centralized discrete optimal Quadratic AGC controller (COQAGC) for MAIPS are carried out in Chapter 4. First, the Function minimization method is used to construct the state and control weighting matrices for quadratic cost functions. Second, the two centralized COQAGCs are designed using optimal control theory, Lagrangian traditional multipliers, and state space equations, namely 1) centralized discrete- COQAGC regulators and 2) Centralized discrete COQAGC controllers. Following this, the centralized discrete COQAGC regulator and Centralized discrete COQAGC are tested on TAIPS with step load perturbations, sensitivity analysis, and generation rate constraints (GRC).

For TAIPS, simulation results show that the Centralized discrete COQAGC controller outperforms the LQR regulator regarding step load perturbation rejection and steady-state performance characteristics such as settling time, overshoot, and rising time. The limitation of the LQR regulator is that the frequency and tie-line deviations associated with oscillations and their outputs do not return to zero because the output responses are kept. They are continuously oscillating around zero. This is due to the imaginary parts of the closed-loop eigenvalues. This shortcoming is addressed by centralized discrete COQAGC, which adds an integral of area control errors to all control areas to return the frequency and tie-line deviations to zero. This process increases the dimensions of the interconnected power systems by one state variable in each area.

Similarly, the integral of area control errors is added to the FAIPS model to ensure zero steady errors for frequency and tie line deviations. This increases the dimension of the system model to twenty-four state variables for four generators. It is worth noting that the dimensions increase as the number of areas increases, resulting in a complex system. As a result, the system faces computational difficulties, and the number of states increases, making control design difficult.

Simulation results demonstrate the proposed controllers' performance for FAIPS and Multi-area-Multi Source interconnected power systems based on discrete COQAGC and discrete DOQAGC control laws. First, the closed loop of a discrete COQAGC controller's set point disturbance rejection capabilities (1 % SLPs and 1% Simultaneous SLPs at 5-sec intervals),

sensitivity analysis, and Generation Rate Constraint (GRC) are tested in Chapter 4. The analysis of the settling time, peak overshoot, and peak undershoot response indices for frequency and tie-line deviations show that the simulated response of step load disturbance rejection for the discrete COQAGC is good. However, the response of the frequency deviation and tie-line deviations is associated with oscillation during the transient period. The performance of the proposed discrete COQAGC control system is compared to that of a hybrid -neuro-fuzzy (ANFIS) for step load disturbance rejection perturbed to interconnected power systems at $t = 0$ sec. The simulation response with 1% SLPs indicates that discrete COQAGC performs better in undershoot and settling time when compared to ANFIS. The response of frequency deviations, tie line deviations, and Integral of area control errors (IACEs) were significantly associated with oscillations during the transient period due to the influence of the tie line deviations between control areas when compared to controllers from the literature (Esmail & Krishnamurthy, 2023).

The simulation of sensitivity analysis is also investigated when the operating load conditions and tie line synchronization coefficients are varied by $\pm 25\%$ and $\pm 50\%$ from their nominal values. The results indicate that the effect of parameter variation is minimal in settling time. All responses have similar patterns of peak overshoot and undershoot, though all simulation responses are associated with oscillation during the transient period due to interconnection signals between control areas. COQAGC and FGSC controllers are compared in terms of sensitivity analysis. The discrete COQAGC controller effectively reduces peak undershoot, whereas the FGSC controller effectively reduces oscillations and peak overshoot. Regarding settling time, both controllers are shown to be equally effective. The competitive performance of these two controllers is because both are designed using the LQR control method, and weighting matrices are chosen using the FMM approach. Their results, however, differ due to the use of different types of interconnected power systems, i.e., TAIPS and FAIPS.

Similarly, the discrete COQAGC's performance is demonstrated on a multi-area, multi-source interconnected power system with a renewable energy source, and its dynamic response is compared to that of the ANFIS controller. The discrete COQAGC controller outperforms the ANFIS controller regarding dynamic response, with lower peak undershoot, shorter settling time, and zero steady-state error for frequency deviation and tie-line deviations.

Generation rate constraints (GRCs) are demonstrated and compared for multi-area - multi-source interconnected power systems. The GRC demonstrated the effects of limiting the output of a generating plant. As a result of GRC implementation influencing the performance of the discrete COQAGC controller, the peak overshoot and settling time increase. The discrete COQAGC controller outperforms the FSFC regarding peak overshoot and settling

time. As a result, GRC must be considered for the implementation of real interconnected power systems.

Second, the performance of the DOQAGC controller is demonstrated in FAIPS for set point disturbance rejection capabilities (1% SLPs and 1% Simultaneous SLPs at 5-sec intervals), and Sensitivity analysis in Chapter 5 and its simulation results are compared with controllers from literature. All comparison is based on thermal generators with non-reheat turbines.

The impact of step load disturbance rejection applied $t = 0$ sec is demonstrated on FAIPS, and its simulation results are compared with controllers from literature for thermal generators with non-reheat turbines. The DOQAGC controller outperformed COQAGC, FSFC, and OISMC-LFC controllers by achieving a better and lesser dynamic response regarding settling time for frequency deviations. In contrast, FSFC controllers outperformed COQAGC, DOQAGC, and OISMC-LFC controllers by achieving better and smaller dynamic responses in peak overshoot.

Similarly, the impact of simultaneous step load disturbance rejection applied at 5-sec intervals is demonstrated on FAIPS. Its simulation results are compared with controllers from literature for thermal generators with non-reheat turbines. The DOQAGC controller outperformed COQAGC and DLQRLFC controllers by achieving a better and lesser dynamic response regarding disturbance rejection response time (DRRT) for frequency and tie line deviations. Furthermore, the DOQAGC controller outperformed RDLFC by achieving the most minor peak OS for tie-line deviations. In contrast, RDLFC outperformed COQAGC, DCOQAGC, and DLQRLFC controllers by achieving a smaller dynamic response in terms of peak overshoot for the frequency deviations.

Finally, the sensitivity analysis for the DOQAGC controller is performed by varying operational load conditions and tie line synchronization coefficients to wide parameter variation of $\pm 25\%$ from their nominal values. All dynamic responses of the sensitivity analysis appear to be satisfactory, and the proposed controllers worked effectively under these variations. Regarding frequency deviations, the FGSC controller has the lowest peak OS to parameter variation ($\pm 25\%$) compared to the COQAGC, DOQAGC, and FSFC controllers. Compared to COQAGC and FSFC controllers, DOQAGC and FSFC controllers have the smallest peak OS for tie line deviations for nominal and $\pm 25\%$ parameter variation. Furthermore, the DOQAGC controller achieves the best settling time for frequency and tie line deviations compared to COQAGC, FGSC, and FSFC controllers. DOQAGC and FGSC controller findings for sensitivity analysis differ, which could be due to the type of interconnected power systems considered in the simulations (Arya & Kumar, 2016; Esmail & Krishnamurthy, 2023). Most simulation results for set point perturbation rejection capabilities

(1% SLPs and 1% Simultaneous SLPs at 5-sec intervals), as well as Sensitivity analysis, showed that the DOQAGC controller has better performance when compared to controllers from the literature and the proposed COQAGC in terms of the setting time overshoot responses for frequency and tie line deviations. As a result, FMM can systematically simplify and facilitate the selection of weighting matrices. Furthermore, using an overlapping decomposition approach to decentralize a four-area interconnected power system reduces computational difficulties and the size of complex interconnected power systems. DOQAGC thus improves frequency and tie line deviation results and is suitable for use in any complex interconnected power system.

Research question 6: What impact does the overlapping decomposition approach have on the design of discrete decentralized optimal quadratic automatic generation (DOQAGC) and multi-area interconnected power systems in terms of AGC design issues and computation difficulties?

This question was answered in Chapters 3 and 5 in decomposing the multi-area interconnected power systems into separate subsystems. In this technique, the areas represent the subsystems, while the deviations in tie line power are the overlapping parts. TAIPS with identical two non-reheat turbines, FAIPS with non-reheat, reheat, hydro, and gas turbines, and N multi-area interconnected power systems are decomposed into two subsystems, four subsystems, and N subsystems using overlapping decomposition methods for decentralized AGC control problem. Therefore, overlapping decomposition is a proper technique for producing subsystems with lesser dimensions compared to the original systems. Furthermore, it simplifies the computation, saves time, and enables the development of local subsystem controllers using linear and nonlinear control methods.

Research question 7: What software environment is utilized to build and test control algorithms for a multi-area interconnected power system?

This research question is answered, implemented in chapters 4 and 5. The developed controller's algorithms are programmed and implemented in MATLAB/SIMULINK environment. The benefit of MATLAB/ SIMULINK packages is that they allow complex feedback control system calculation and various hardware simulations (Martyanov, Solomin & Korobotov, 2015). Thorough data analysis and the numerous simulated case studies are utilized to test the developed COQGAC and DOQAGC algorithms as summarized in Table A.1. These case studies are tested on the developed software programs and simulated the closed-loop systems for both proposed centralized and decentralized AGC controllers in MATLAB /SIMULINK environment for set-point disturbance rejections, robustness against modelling uncertainties and generation rate constraints (GRC). A comparative analysis is

performed between the discrete decentralized optimal quadratic automatic generation control (DOQAGC), the discrete centralized optimal quadratic automatic generation control (DOQAGC), and a controller from the literature. Therefore, Software programs developed and implemented with Discrete LQR optimal control algorithms and weighting matrices are essential in assessing the dynamical simulation performances of identical two areas, four areas, and any interconnected power systems problems.

6.3. Summary of conclusions

This thesis has included a literature review on an advanced control method for solving AGC problems in multi-area interconnected power systems. The literature review publications were grouped according to power system modelling: single and multi-area interconnected power systems, control techniques, and nonlinearities associated with AGC problems in multi-area power systems. Different types of control methods are used in AGC problems, such as decentralized and centralized control techniques, soft computing and intelligent control schemes, robust control techniques, and optimal control methods based on linear quadratic regulators, as well as other types of control methods, such VSC/SMC, DMPC and DADRC are reviewed. The literature review finds that it is beneficial to consider physical constraints associated with the multi-area interconnected power systems such as governor dead band (GDB), generation rate constraints (GR), Boilers dynamics (BD), and time delays due to communications channels as well as the sensitivity analysis to test the practicality of the proposed controllers. The literature review in Chapters 3, 4, and 5 represents the mathematical modelling of interconnected power systems, i.e., identical TAIPS FAIPS. It reviews the importance of decomposition methods and decentralization, such as overlapping decomposition methods and inclusion principles with the ability to decompose the interconnected power systems into subsystems. The literature review in Chapter 4 provides the context of using the optimal control theory and Lagrange multipliers and formulating the Functional Minimization Method (FMM) for designing centralized discrete COQAGC and centralized discrete LQR regulators. The literature review in Chapter 5 presents the formulation of the suboptimal AGC control problem and quadratic sub-cost functions as well as the construction of sub-state and sub-control weighting matrices using methods such as suboptimal optimal control theory, Lagrangian traditional multipliers, discrete state space equations and sub-functional minimization approach for each subsystem separately. The advantages of decomposing and decentralizing the interconnected power systems into subsystems using a method such as overlapping decomposition reduce the complicated optimization problems down to local sub-optimization problems at the subsystems level, resulting in decreased dimensions of the controllers and the subsystems as well as improved the computational time. Furthermore, the functional minimization approach simplified and

systematized the weighting matrices' selection process, resulting in considering the LQR AGC control-based FMM for the real-time implementation of the large, interconnected power systems.

The design, implementation, and simulation tests in chapters 4 and 5 focus on improved control methods and algorithms for solving AGC problems in multi-area interconnected power systems that will reduce computational difficulties and have flexibility, reliability, and robustness against load perturbations. In this research, a new design of discrete COQAGC and discrete DOQAGC control design strategies and associated algorithms are developed in MAIPS to address the challenges associated with AGC problems. The results show that Discrete DOQAGC is more reliable against modelling uncertainties and SLPs than discrete COQAGC and most controllers from the literature and has lower computational requirements.

Further studies and development of a suitable multilevel control approach are required to apply Discrete LQR AGC control techniques and optimization algorithms based on Functional minimisation methods in interconnected power systems to reduce the impacts of the interaction signals.

6.4. Publications

1. Esmail, M, Tzoneva, R & Krishnamurthy, S. 2017, Review of Automatic Generation Control in Deregulated Environment, IFAC-Papers Online, Science Direct, Vol.50, no.2, pp.88-93.
2. Esmail, M., Krishnamurthy, S, 2023. Discrete optimal quadratic AGC-based cost functional minimisation for interconnected power systems. Nature: Scientific Reports no. 13, vol. 2752, pp.1-37.
3. M. Esmail and S. Krishnamurthy, "Discrete Centralized AGC Using LQR-Based Cost Functional Minimization for Multi-Area Interconnected Power Systems," in IEEE Access, vol. 12, pp. 20426-20440, 2024, doi: 10.1109/ACCESS.2024.3361280.
4. Esmail, M & Krishnamurthy, S. 2023. Comparative Analysis of Physical Constraints between Centralized/ Decentralized Discrete Optimal Quadratic AGC based Cost Functional Minimization for interconnected power, International Journal of Electric Power Systems Research, Science Direct. (Planning to submit Feb 2024)

6.5. Future research

Future research can extend the current research into investigating the performance of the developed DOQAGC method and algorithm considering physical constraints such as generation rate constraints for multi-source generators in each area, i.e., mixing traditional generators with distributed energy resources, such as solar and wind turbines and compared with COQAGC method. The developed DOQAGC control methods and algorithms can be tested in a closed-loop AGC using a multilevel control approach in a Real-Time Digital Simulator (RTDS). Implementation and testing of AGC closed-loop control methods and algorithms using IEC 61499 and IEC 61850 standards can be investigated to provide interoperability, configuration, and flexibility between the subsystems of multi-area interconnected power systems. Further, Microgrids problems can also be solved using the developed methods and algorithms. The analysis of communication delays, Boiler Dynamics (BD), and Governor Dead Band (GDB) effects can be extended to test the performance of the developed control methods under these conditions. Further work can explore the distributed LQR control techniques and optimisation algorithms based on Functional minimization methods that can be used in the operation and performance of interconnected power systems with multi-source generators in each control area.

6.6. Chapter Conclusion

This chapter provides an overview of the thesis's findings, developed techniques and tools, domains it applies to, summary of conclusions, potential follow-up research to strengthen the thesis, and potential publications.

BIBLIOGRAPHY

- Abdelhamid, A. A.M & El-gaafary. A. A. M. 2018. Optimal decentralized LQR control to enhance multi-area LFC system stability. *Hindawi mathematical problems in engineering*, 2018,1-11.
- Abdelhamid, A. M. & El-Gaafary, A. A. M. 2018. Optimal decentralized LQR control to enhance multi-area lfc system stability. *Hindawi-Mathematical Problems in Engineering*, 2018:1-12.
- Abdel-Magid, Y. I., & Abido, A. 2003. AGC tuning of interconnected reheat thermal systems with particle swarm Optimization. Paper presented at 10th IEEE International Conference on Electronics , Circuits and Systems, 2003. ICECS-2003. Proceedings of the 2003, Sharjah, United Arab Emirates, 2003,1 :376 – 379.
- Abdel-Majid, Y.L. & Dawoud, M.M. 1997. Optimal AGC tuning with genetic Algorithms. *Electric Power System Research*, 38 (1996): 231-238. Available from: ScienceDirect Online. [11 March2018].
- Academic Press, Inc.
- Ahamed, T.P., Nagendra Rao, P. S., & Sastry. P.S. 2002. A reinforcement learning approach to automatic generation control. *Electric Power Systems Research*, 63: 9-26..
- Ahmadi, A., Aldeen, M. & Abdolmaleki, M. 2014. Decentralised stabilisability condition for expanded and contracted LTI systems and quotient fixed modes. 2014 European Control Conference (ECC), June 24-27, 2014. Strasbourg, France,406-411.
- Alhelou, H. H., Nagpal, N., Kassarwan, N. & SIANO, P. 2023. Decentralized optimized integral sliding mode-based load frequency control for interconnected multi-area power sys-tems, *IEEE Access*,11, 32296-32307.
- Alhelou, H. Haes., Hamedani-Golshan, M., Zamani ,E. E., Heydarian-Forushani , R., & Siano , P. 2018. Challenges and opportunities of load frequency control in conventional, Modern and future smart power systems: A comprehensive review. *Energies*, 11(10):1-35.
- Ali, E.S., & Abd-Elazi. S.M. 2013. BFOA based design of PID controller for two area Load Frequency Control with nonlinearities. *Electrical Power and Energy Systems*,51:224-231.
- Alrifai, M.T., Hassan, M. F., & Zribi, M .2011.Decentralized load frequency controller for a multi-area interconnected power system. *International Journal of Electrical Power & Energy Systems*,33(2):198-209.
- Ansari, j., Abbasi, A.R., & Firouzi, B. B. 2022. Decentralized LMI-based event-triggered inte-gral sliding mode LFC of power systems with disturbance observer. *International Journal of Electrical Power & Energy Systems*,138:1-13.
- Ansarian, M., Shakouri, G. H., Nazarzadeh, J., & Sadeghzadeh, S. M .2006. A Novel Neuro-Optimal Approach for Decentralized LFC Design in Multi-Area Power System. Paper pre-sented at 2006 IEEE International Power and Energy Conference, Putra Jaya, 2006: 167-172.
- Ansarian, M., Shakouri, G.H., Nazarzadeh, J. & Sadeghzadeh, S. M. 2006. A Novel Neuro-Optimal Approach for Decentralized LFC Design in Multi-Area Power System. Paper pre-sented at 2006 IEEE international power and energy conference, Putra Jaya, Malaysia, 2006,167-172.
- Arya, Y. & Kumar, N. 2016. AGC of a multi-area multi-source hydrothermal power system interconnected via AC/DC parallel links under deregulated environment. *International journal of electrical power energy systems*, 75, 127–138.
- Arya, Y. 2019. A new optimized fuzzy FOPI-FOPD controller for automatic generation con-trol of electric power systems. *Journal of the Franklin Institute*,356(11): 5611-5629.

- Arya, Y. 2019. A new optimized fuzzy FOPI-FOPD controller for automatic generation control of electric power systems. *Journal of the Franklin Institute*, 356, 5611–5629.
- Arya, Y., & Kumar, N. 2016. Fuzzy Gain Scheduling controllers for automatic generation control of two-area interconnected Electrical Power Systems. *Electrical Power components systems*, 44 (7):737-751.
- Arya, Y., Kumar, N. & Gupta, S. K. 2017. Optimal automatic generation control of two-area power systems with energy storage units under deregulated environment. *Journal of renewable sustainable energy*, 9 (6),064105(1-20).
- Arya, Y., Kumar, N. & Gupta, S.K. 2012. Load frequency control of a four-area power system using linear quadratic regulator. *International journal of energy sciences*, 2(2): 69-76.
- Azimi,M.M., Afzalian, A.A., & Ghaderi, R. 2021, Decentralized output feedback stabilization based on modified event-trigger scheme for load frequency control of an interconnected microgrid. *Iranian Journal of Science and Technology, Transactions of Electrical Engineering*, 45:1411–1426.
- Azzam, M. 1999. Robust automatic generation control. *Energy Conversion Management*, 40:1413–1421.
- Azzam, M. 1999. Robust automatic generation control. *Energy conversion and management*, 40,413-1421.
- Bakken. B.H., & Grand. O.S. 1998. Automatic generation control in a deregulated power system. *IEEE Transactions on Power Systems*,13(4):1401–1406.
- Bakule, L. 2008. Decentralized control: An overview. *Annual Reviews in Control*, 32(1):87-98.
- Barakat, M., Donkol, A., Hamed, H.F.A. 2022. Controller parameters tuning of water cycle algorithm and its application to load frequency control of multi-area power systems using TD-TI cascade control. *Evolving Systems*,13:117–132.
- Bekhouche, N. (2002). Automatic generation control before and after deregulation. *Proceedings of the Thirty-Fourth Southeastern Symposium on System Theory (Cat. No.02EX540)*, Huntsville, Al, USA, 2002: 321-323.
- Bengiamen, N.N., & Chan, W. 1978. Multilevel load-frequency control of interconnected power systems. *IET Digital Library*,125 (6): 521-526.
- Bensenouci, A. & Abdel Ghany, A. M. 2005. Optimal discrete-time output feedback control for multi-area load frequency control using evolutionary programming, *The IEEE International Symposium on Industrial Electronics (ISIE2005)*, Dubrovnik, Croatia, June 20-23, 2005, 234-240.
- Bevrani, H, Mitani.Y., & Tsuji, K. 2003. A scenario on load-frequency controller design in a deregulated power system. *SICE 2003 Annual Conference (IEEE Cat. No.03TH8734)*, Fukui, Japan,2003, 3: 3148-3153.
- Bevrani, H, 2004, *Decentralized Robust Load-Frequency Control Synthesis in Restructured Power Systems*. PhD dissertation, Osaka University, Japan.
- Bevrani, H. 2014. *Robust power system frequency control*. 2nd ed. Switzerland: Springer International Publishing.
- Bevrani, H., & Hiyama, T. 2007. Robust load frequency regulation: A real-time laboratory experiment. *Optimal. Control Applications and. Methods*, 28: 419-433.
- Bevrani, H., & Hiyama, T. 2011. *Automatic generation control (AGC): Fundamentals and concepts*. Intelligent automatic generation control. New York, USA: Taylor and Francis Group;11-36.

- Bevrani, H., Mitani, Y., & Tsuji, K. 2004. Robust decentralised load-frequency control using an iterative linear matrix inequalities algorithm. *IEE Proceeding -Generation, Transmission and Distribution*, May, 2004,.151(3), 347–354.
- Bevrani. H., Mitani.Y., Tsuji. K., & Bevrani. H. 2005. Bilateral based robust load frequency control. *Energy Conversion and Management*,46(7-8):1129-1146.
- Bhatt. P., Roy. R., & Ghoshal. S. P. 2010. Optimized multi area AGC simulation in restructured power systems. *Internmtional Journal of Electrical Power Energy Systems*, 32: 311–322.
- Bimarta, R.V, Tran, T., & Kim, K. H. 2018), Frequency-adaptive current controller design based on LQR state feedback control for a grid-connected inverter under distorted grid. *Energies*,11(2674):1-29.
- Calovic, M. S. (1972). Linear regulator design for a load and frequency control. *IEEE Transactions*, 91: 2271-2285.
- Çelik, E. 2021. Design of new fractional order PI–fractional order PD cascade controller through dragonfly search algorithm for advanced load frequency control of power systems. *Journal of Soft Computing*, 25:1193–1217.
- Chan, W., & Hsu, Y.Y. 1981. Automatic generation control of interconnected power systems using variable-structure controllers. *IET Digital Library*, 128 (5): 269 -279.
- Chan, W., Mem, S. & Hsu, Y. 1981. Automatic generation control of interconnected power systems using variable-structure controllers. *IEE proceedings d-control theory and applications*,138(6)579-585.
- Chaturvedia, D. K., Satsangia, P. S., & Kalrab. P.K. 1999. Load frequency control: A generalised neural network approach. *Electrical Power and Energy Systems*, 21:405-415.
- Chen, X. B., & Stanković, S. S. 2005. Decomposition and decentralized control of systems with multi-overlapping structure. *Automatica*, 41(10):1765-1772.
- Chitu, C. 2013. Controller design for an electric power steering system based on LQR techniques. *International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, 32(3): 763-775.
- Christie, R. D., & Bose. A. 1996. Load frequency control issues in power system operations after deregulation. *IEEE Transactions on Power Systems*,11(3):1191-1200.
- Chu, D., Ohta, Y. & Siljak, D. D. 2009. Inclusion principle for descriptor systems. *IEEE transactions on automatic control*, 54(1),3-18.
- Cohn, N. 1956. Part III: Power apparatus and systems. *Transactions of the American Institute of Electrical Engineers*, 75 (3).
- Das, D, Kothari, M L, Kothari, D P & Nanda, J (1991) Variable structure control strategy to automatic generation control of interconnected reheate thermal system. *IET Digital Library*, 138(6): 579-585.
- Das, D., Aditya, S.K., & Kothari, D.P. 1999. Dynamics of diesel and wind turbine generators on an isolated power system. *Electrical power and energy systems*, 21(3):183-189.
- Das, S., Pan, I., Halder, K., Das, S., & Gupta, A. 2013. LQR based improved discrete PID controller design via optimum selection of weighting matrices using fractional order integral performance index. *Applied Mathematical Modelling*, 37(6): 4253-4268.
- Das. S., Mukhopadhyay. A., Roy. A., Abraham.A., & Panigrahi, B.K. 2011. Exploratory power of the harmony search algorithm: analysis and improvements for global numerical optimization. *IEEE Transaction Power Systems, Man and Cybernetics*,41(1): 89-106.

- Debbarma, S., Saikia .L. C., & Sinha. N. 2013. AGC of a multi-area thermal system under deregulated environment using a non-integer controller. *Electrical Power Systems Research*: 175–183
- Deepak, M., & Abraham, R.J .2015. Load following in a deregulated power system with Thyristor Controlled Series Compensator," *International Journal of Electrical Power & Energy Systems*,65:136-145
- Delassi, A., Arif, S., & Mokrani, L. 2015. Load frequency control problem in interconnected power systems using robust fractional $PI^{\lambda}D$ Controller. *Ain Shams Engineering Journal*,9(1):77-88.
- Demiroren. A., & Zeynelgil, H. L. 2007. GA application to optimization of AGC in three area power system after deregulation. *International Journal of Electrical Power & Energy Systems*, 29(3):230-240.
- Dey, P.,Saha, A.,Srimannarayana, P., Bhattacharya, A., & Marungsri, B.2022. A Realistic approach towards solution of load frequency control problem in interconnected power systems, *Journal of Electrical Engineering & Technology*,17:759-788.
- Donde, V., Pai. M. A., & Hiskens, I. A. (2001). Simulation and optimization in an AGC system after deregulation. *IEEE Transction in Power Systems*,16 (3):481-489.
- Dong, L., Zhang, Y., & Gao, Z. 2012. A robust decentralized load frequency controller for interconnected power systems. *ISA Transactions*, 51(3): 410-419.
- Drouin, M., Abou-Kandil, H., & Mariton, M. 1991. *Control of Complex Systems Methods and technology*. New York, USA: Plenum Press, Springer.
- Edwards, C., & spurgeon. S. 1998. *Sliding mode theory and applications*. UK: CRC Press.
- Elgerd, O. 1981. Control of electric power systems. *IEEE Control Systems Magazine*,1(2):4-16.
- Elgerd, O. I., & Fosha, C. 1970. Optimum megawatt frequency control of multi-area electric energy systems. *IEEE Transctions of Power Apparatus Systems*,89,(4):556-563.
- El-Kébir, B., & Fouad M. A. 2011. *Mechatronic systems analysis, design and implementation*. Verlag Berlin Heidelberg, Germany: Springer.
- Esmail, M., & Krishnamurthy, S. 2023. Discrete optimal quadratic AGC-based cost functional minimization for interconnected power systems. *Nature: Scientific Reports*,13(2752):1-37.
- Esmail, M., Tzoneva,R., & Krishnamurthy, M. 2017. Review of Automatic Generation Control in Deregulated Environment. *IFAC-PapersOnLine*, 50(2): 88-93.
- Feliachi, A (1996). Load frequency control in a deregulated environment. *Proceedings of the 1996 IEEE International Conference on Control Applications (IEEE International Conference on Control Applications held together with IEEE International Symposium on Intelligent Control)*, Dearborn, MI, USA, 1996: 437-441.
- Feliachi, A. 1987. Optimal decentralized load frequency control. in *IEEE Transactions on Power Systems*, 2(2):379-385.
- Filha, B. G., de Castro, A. C., Moreira, F. A., & Araujo, J.M. 2016. Design of a robust h_1 controller for a load-frequency power system. *International Journal of Innovative Computing, Information and Control*,12(1):321-331.
- Filha, B. G., de Castro, A.C., Moreira, F.A. & Araujo, J. M. 2016. Design of a robust H_{∞} controller for a load-frequency power system. *International journal of innovative computing, information and control*,12(1), 321-331.
- Fogel, D. B. 1995. *Evolutionary computation: Toward a new philosophy of machine intelligence*. New Jersey, USA: IEEE Press.

- Fosha, C. E., & Elgerd, O. I. 1970. The megawatt frequency control problem: A new approach via optimal control theory. *IEEE Transactions. Power Apparatus Systems*, 89(4): 563–577.
- Geem, Z. W., Kim J.H., & Loganathan, G.V. 2001. A new heuristic optimization algorithm: harmony search. *Simulations Journal*, 76 (2): 60–80.
- Golpîra, H., Bevrani, H., & Golpîra, H. 2011. Application of GA optimization for automatic generation control design in an interconnected power system. *Energy Conversion and Management*, 52(5): 2247–2255. Available: ScienceDirect [11 March 2018].
- Guo, J. 2022. The Load frequency control by adaptive high order sliding mode control strategy. *IEEE Access*, 10: 25392-25399.
- Hakimuddin, N., Khosla, A., & Garg, J. K. 2020, Centralized and decentralized AGC schemes in 2-area interconnected power system considering multi source power plants in each area, *Journal of King Saud University - Engineering Sciences*, 32(2):123-132.
- Hao, Y., Tan, W., & Ali, D. 2014. Decentralized active disturbance rejection control for LFC in deregulated environments. *Proceedings of the 33rd Chinese Control Conference*, July 28–30, 2014, Nanjing, China: 3719–3724.
- He, J. B., Wang, Q. G., & Lee, T. H. 2000, PI/PID controller tuning via LQR approach. *Chemical Engineering Science*, 55, (13): 2429–2439.
- Hosseini, H., & Etemadi, A. H. 2008. Adaptive neuro-fuzzy inference system based automatic generation control. *Electric Power Systems Research*, volume. 78(7):1230–1239.
- Hota, P. K., & Mohanty, B. 2016. Automatic generation control of multi-source power generation under deregulated environment. *International Journal of Electrical Power & Energy Systems*, 75: 205–214.
- Ibraheem, H.N., Hussein, A. A. 2014. Decentralized automatic generation control of interconnected power systems incorporating asynchronous tie-lines, *Springerplus*, 3 (744):1–8.
- Ibraheem, N.H. & Kumar, P. 2013. Optimal automatic generation control of interconnected power system considering new structures of matrix Q. *Electrical Power Components Systems*, 41(2):136–156.
- Ibraheem., Kumar, P., Hasan, N. & Singh, Y. 2012. Optimal automatic generation control of interconnected power system with asynchronous tie-lines under deregulated environment. *Electrical power component systems*, 40, (10), 1208–1228.
- Ikeda, M., & Šiljak, D. D. 1986). Overlapping decentralized control with input, state, and output inclusion. *Control Theory and Advanced Technology*, 2:155–172.
- Ikeda, M., Šiljak, D.D., & White, D. E. 1981. Decentralized control with overlapping information sets. *Journal of Optimization Theory and Applications*, 34: 279–310.
- Indulkar, C. S., & Raj, B. 1995. Application of Fuzzy Controller to automatic generation Control. *Electric Machines & Power Systems*, 23, (2): 209–220.
- Jain, S. K., Chakrabart, S., & Singh, S. N. 2013. Review of load frequency control methods, Part-II: Post-deregulation scenario and case studies. Paper presented at International Conference on Control, Automation, Robotics and Embedded Systems (CARE), Jabalpur, India, 2013:1–7.
- Jaleeli, N., VanSlyck, L. S., Ewart, D. N., Fink, L. H., & Hoffmann, A. G. 1992. Understanding automatic generation control. *IEEE Transactions on Power Systems*, 7(3):1106–1122.
- Kathori, D. P., & Nagrath, I.J. 2011. *Power System Analysis*, 4th ed. McGraw Hill, 2011.

- Khadanga, R. K., Kumar, A., & Panda, S. 2021. A novel sine augmented scaled sine cosine algorithm for frequency control issues of a hybrid distributed two-area power system. *Journal of Neural Computing and Applications*,33: 12791–12804.
- Khezri, R., Oshnoei, A., Oshnoei, S., Bevrani, H., & Muyeen, S. M. 2019. An intelligent coordinator design for GCSC and AGC in a two-area hybrid power system. *Applied Soft Computing*,76: 491-504
- Khuntia, S. R., & Panda, S. 2012. Simulation study for automatic generation control of a multi-area power system by ANFIS approach. *Applied Soft Computing*,12: 333-341.
- Khuntia, S.R. & Panda, S.2012. Simulation study for automatic generation control of a multi-area power system by ANFIS approach. *Journal of applied soft computing*,12, 333-341.
- Ko, H. S., Leeb, K.Y., & Kim, H. C. 2004. Intelligent based LQR controller design to power system stabilization. *Electrical Power Systems Research*, 71:1-9
- Kothari, D. P., & Nagrath, I. J. 2003. *Modern power system analysis*. 3rd ed. Singapore: McGraw-Hill.
- Krishnamurthy, S. 2013. Development of Decomposition Methods for Solutions of A multi-area power Dispatch Optimisation Problem. Doctor of Technology Thesis, Cape Peninsula University of Technology, Cape Town, South Africa.
- Kumar, D .M. V. 1998. Intelligent controllers for automatic generation control. *Proceedings of IEEE TENCON 98*. Paper presented at IEEE Region 10 International Conference on Global Connectivity in Energy, Computer, Communication and Control, NewDelhi, India, 1998, 2 :557-574.
- Kumar, I. P., & Kothari, D. P .2005. Recent philosophies of automatic generation control strategies in power systems. *IEEE Transactions on Power Systems*, 20(1): 346-357.
- Kumar, I.P. & Kothari, P. 2005. Recent philosophies of automatic generation control strategies in power systems,' *International IEEE transactions on power systems*, 20(1): 346-357.
- Kumar, J., Ng, K-H., & Sheble. G. 1997. AGC simulator for price-based operation. I. A model. *IEEE Transactions on Power Systems*12 (2): 527-532.
- Kundur, P. 1994. *Control of Active Power and Reactive Power*. Power system stability and control. USA, California, McGraw-Hill Education: 581-595.
- Kögel, M., & Findeisen, R. 2015. Discrete-time robust model predictive control for continuous-time nonlinear systems. Paper presented at 2015 American Control Conference (ACC), Chicago, IL, USA, 1-3 July, 2015: 924-930.
- Li, H., & Shi, Y. 2013. Distributed model predictive control of constrained nonlinear systems with communication delays. *Systems & Control Letters*, 62(10): 819–826.
- Lin, F. 2007. *Robust control design: an optimal control approach*. Hoboken, New Jersey: Wiley-Interscience.
- Lunze, J. 1992. *Feedback Control of Large-Scale System*. UK: Prentice Hall International Ltd.
- Ma, M., Chen, H., Liu, X., & Allgower, F. 2014. Distributed model predictive load frequency control of multi-area interconnected power system. *International Journal of Electrical Power & Energy Systems*, 62: 289-298.
- Ma, M., Zhang, C., Liu .X & Chen, H. 2017. Distributed model predictive load frequency control of the multi-area power system after deregulation. *IEEE Transactions on Industrial Electronics*,64(6): 5129-5139.
- Martyanov, A.S., Solomin, E.V. & Korobotov, D.V. 2015. Development of control algorithms in Matlab/Simulink. *Procedia engineering journal*,129(2015): 922-926.

- Meliopoulos, A. P. S., Cokkinides, G. J., & Bakirtzis, A. G. 1999. Load-frequency control service in a deregulated environment. *Decision Support Systems*, 24 (3–4): 243-250.
- Mohanty, B., Panda, S., & Hota, P. K. 2014. Differential evolution algorithm based automatic generation control for interconnected power systems with non-linearity. *Alexandria Engineering Journal*, 53: 537–55.
- Muga, J.N. 2015. Design and implementation of IEC 61499 standard-based Nonlinear controllers using functional block programming in Distributed control platform. DTech Thesis, Cape Peninsula University of Technology, Cape Town, South Africa.
- multi-area power systems. *Energy Convers. Managements*. 52 (5), 2015-2023.
- Naidu, k., Mokhlis, H, Bakar, A.H.A., Terzija, v., & Illias, H.A. 2014. Application of firefly algorithm with online wavelet filter in automatic generation control of an interconnected reheat thermal power system, *Electrical Power and Energy Systems*, 63 :401-413.
- Namara, P Mc., Meere, R., O'Donnell, T., & McLoone, S. 2016. Control strategies for automatic generation control over MTDC grids. *Journal of Control Engineering Practice*, 54: 129-139.
- OĞUZ. Y. 2011. Fuzzy PI control with parallel fuzzy pd control for automatic generation control of a two-area power systems. *Gazi University Journal of Science*, 24(4): 805-816.
- Pandey, S K., Mohanty, S R & Kishor, N 2013 'A literature survey on load–frequency control for conventional and distribution generation power systems,' *Journal of Renewable and Sustainable Energy Reviews*, vol.25. pp.318 -334.
- Pandey, S. K., Mohanty, S.R. & Kishor, N. 2013. A literature survey on load–frequency control for conventional and distribution generation power systems. *Journal of renewable and sustainable energy reviews*, 25, 318-334.
- Panwar. A., Agarwal, V., Sharma, G. & Sharma, S. 2022, Design of a novel agc action for a linked hydro governing system, *Electric power components and systems*, 49(15), 1201-1211.
- Paramar, K. P. S. 2013. State Space based load frequency control of multi-area power systems, PhD Thesis, Indian Institute of Technology Guwahati, India.
- Parmar, K. P., Majhi, S., & Kothar, D. P. 2012. Load frequency control of a realistic power system with multi-source power generation. *International Journal of Electrical Power and Energy Systems*, 42(1):426-433.
- Pathak, N., Nasiruddin, I., & Bhatti, T. S. 2015. A More realistic model of centralized automatic generation control in real-time environment. *Electric Power Components and Systems*, 43(19): 2205-2213.
- Penda, J. S., Mohanty, B., & Hota, P. K. 2013. Hybrid BFOA-PSO algorithm for automatic generation control of linear and nonlinear interconnected power systems. *Applied Soft Computing*, 13: 4718-4730.
- Prakash, A., Singh, P., Kumar, K., Panda. 2022. Design of a Reduced Order WADC for Wind Turbine System-Integrated Power System. *IEEE Transactions on Industry Applications*, 58(3):3250-3260.
- Prakasha, S. & Sinhab. S. K. 2014. Simulation based neuro-fuzzy hybrid intelligent PI control approach in four-area load frequency control of interconnected power system. *Journal of applied soft computing*, 23,152–164.
- Prakasha, S., & Sinhab, S. K. 2014. Simulation based neuro-fuzzy hybrid intelligent PI control approach in four-area load frequency control of interconnected power system. *Applied Soft Computing*, 23: 152–164.

- Rahman, A., Saikia, L. C., & Sinha, N. 2015. Load frequency control of a hydro-thermal system under deregulated environment using biogeography-based optimized three degree-of-freedom integral-derivative controller. *IET Generation, Transmission & Distribution*, 9(15): 2284-2293.
- Rajesh, K.S., Dash, S.S., & Rajagopal, R. 2019. Hybrid improved firefly-pattern search optimized fuzzy aided PID controller for automatic generation control of power systems with multi-type generations. *Swarm and Evolutionary Computation*, 44: 200-211.
- Rakhshani, E., Remon, D., Cantarellas, A. M., & Rodriguez, P. 2016. Analysis of derivative control based virtual inertia in multi-area high-voltage direct current interconnected power systems. *IET Generation, Transmission & Distribution*, 10(6): 1458-1469.
- Ram, P., & Jha, A. N. 2010. Automatic generation control of interconnected hydro-thermal system in deregulated environment considering generation rate constraints. Paper presented at 2010 International Conference on Industrial Electronics, Control and Robotics, Rourkela, India, 2010:148-159.
- Rangi, S., Sheilza J. & Arya, Y. 2022. Utilization of energy storage devices with optimal controller for multi-area hydro-hydro power system under deregulated environment. *Sustainable energy technologies and assessments*, 52, 1-15.
- Rerkpreedapong, D., & Feliache, A. 2002. Decentralized load frequency control for load following services. Paper Presented at 2002 IEEE Power Engineering Society Winter Meeting Conference proceedings, New York, NY, USA, 2002, 2: 1252-1257.
- Rerkpreedapong, D., Hasanovic, A., & Feliachi, A. 2003. Robust load frequency control using genetic algorithms and linear matrix inequalities. *IEEE Transaction Power Systems*, 18 (2): 855–861.
- Sabri, Y., EL Kamoun, N. & Lakrami, F. 2019. A Survey: centralized, decentralized, and distributed control scheme in smart grid systems. *IEEE 2019 7th Mediterranean Congress of Telecommunications (CMT)*, Fez, Morocco, 2019, 1-11.
- Sahu, k. S., Panda, S., & Rout, U.K. 2013. DE optimized parallel 2-DOF PID controller for load frequency control of power system with governor dead-band nonlinearity. *Electrical Power and Energy Systems*, 49:19-33.
- Sahu, P. C., Prusty, R. C., & Panda, P. S. 2020. Approaching hybridized GWO-SCA based type-II fuzzy controller in AGC of diverse energy source multi area power system. *Journal of King Saud University - Engineering Sciences*, 2(3): 186-197.
- Sekhar, K. C., & Vaisakh, K. 2013. Application of VSC/SMC controllers for automatic generation control in deregulated environment. Paper Presented at 2013 7th International Conference on Intelligent Systems and Control (ISCO), Coimbatore, Tamil Nadu, India, 2013: 210-218.
- Shang-Guan, X., Chance, Z., lin, J., William, S.J. & Min, W. 2020. Sampled-data based discrete and fast load frequency control for power systems with wind power. *Applied energy*, 259(114202)1-11.
- Shankar, R., Chatterjee, K. & Bhushan, R. 2016. Impact of energy storage system on load frequency control for diverse sources of interconnected power system in deregulated power environment. *International Journal of Electrical power energy systems*, 79, 11-26.
- Sharma, G., 2019. Optimal AGC design for diverse sources of power generations in each area using output vector feedback control technique. *JERA*, 45, 99-114.
- Sharma, G., Nasiruddin, I., Niazi, K.R. & Bansal, R. C. 2016. Robust automatic generation control regulators for a two area power system interconnected via AC/DC tie-lines considering new structures of matrix Q. *IET generation. transmission & distribution*, 10(14), 3570–3579

- Shayeghi, H., Shayanfar, H. A., & Jalili, A. 2009. Load frequency control strategies: A state-of-the-art survey for the researcher. *Energy Conversion and Management*, 50(2): 344-353.
- Shiva, C. K., & Mukherje, V. 2016. Design and analysis of multi-source multi-area deregulated power system for automatic generation control using quasi-oppositional harmony search algorithm. *Electrical Power and Energy Systems*, 80: 382-395.
- Shiva, C. K., & Mukherje, V. 2015. Automatic generation control of interconnected power system for robust decentralized random load disturbances using a novel quasi-oppositional harmony search algorithm. *Electrical Power and Energy Systems*, 73: 991-1001.
- Shree, S. B., & Kamaraj, N. 2016. Hybrid Neuro Fuzzy approach for automatic generation control in restructured power system. *International Journal of Electrical Power & Energy Systems*, 74: 274-285.
- Siljak, D. D. 1978. *Large-Scale Dynamic Systems: Stability and Structure*: North-Holland publisher.
- Siljak, D. D. 1991. *Decentralized control of complex systems*, 184, New York, USA:
- Tan, W & XU, Z 2009 'Robust analysis and design of load frequency controller for power systems,' *Electric Power systems Research*. Vol. 79, PP: 846-853.
- Tan, W. 2011. Decentralized load frequency controller analysis and tuning for
- Tan, W. 2011. Load frequency control: Problems and solutions, *Proceedings of the 30th Chinese Control Conference*, Yantai, China, 2011: 6281-6286.
- Tlili, A. S., & Braiek, N. B. 2009. Decentralized Observer based Guaranteed Cost Control for Nonlinear Interconnected Systems. *International Journal of Control and Automation*, 2(2): 29-46.
- Tomsovic, K., Bakken, D. E., Venkatasubramanian, V., & Bose, A. 2005. Designing the next generation of real-time control, communication, and computations for large power systems. *Proceedings of the IEEE*, May, 2005, 93 (5) 965-979.
- Toulabi, M., Shiroei, M., & Ranjbar, A. 2014. Robust analysis and design of power system load frequency control using the Kharitonov's theorem. *Electrical Power and Energy Systems*, 55:51-58.
- Trave, L., Titli, A., & Tarras, A. 1989. *Large scale systems: decentralization structure constraints and fixed modes* (Lecture notes in control and information sciences, 120), Berlin Heidelberg, Germany: Springer.
- Tungadio, D. H, Ramesh, B & Mukwanga, S, (2018). "Optimal Control of Active Power of Two Micro-Grids Interconnected with Two AC Tie-Lines", *Electrical Power Components and Systems*. Vol.45. no.19, pp.2188-2199 .
- Tyagi, M. B., & Srivastava, S. C. 2006. A decentralized automatic generation control scheme for competitive electricity markets. *IEEE Transactions on Power Systems*, 21(1): 312-320.
- Ullah, K., Basit, A., Ullah, Z., Aslam, S., & Herodotou, H. 2021. Automatic generation control strategies in conventional and modern power systems: A Comprehensive Overview. *Energies*, 14 (2376):1-43.
- Vlahakis, E., Dritsas, L., & Halikias, G. 2019. Distributed LQR design for a class of large-scale multi-area power systems. *Energies*, 12(14): 6-64.
- Vrdoljak, k., Periç, N. & Petrovi, Ć. I. 2010. Sliding mode based load-frequency control in power systems. *Electric power systems research*, 80, 514-527
- Vrdoljak, k., Periç, N., & Petrovi, Ć. I. 2010. Sliding mode based load-frequency control in power systems. *Electric Power Systems Research*, 80: 514-527.

- Vu, V-P. 2022. A polynomial Decentralized Controller Design for a Large- Scale Nonlinear System: SOS Approach. *IEEE Access*,10: 44008-44022.
- Wang, Y., Zhou, R., & Wen, C. 1993. Robust load-frequency controller design for power systems. *IEE Proceedings C (Generation, Transmission and Distribution)*, 140(1):11–16.
- Wang, Z.Q. & Sznaiar, M.1994. Robust control design for load frequency control using μ synthesis. Paper presented at conference record southcon March,1994, Orlando, FL, USA,186-190.
- Wood, A. J., Wallenberg, B. F., & Sheblé, G. B. 2014. Control of generation. 3 ed. Power generation, operation, and control. Hoboken, New Jersey.USA, John Wiley & Sons, Inc: 468-500.
- Xiaohua, L., Jiawang, G. & Yang, L. Multi-overlapping decentralized control design based on dynamic output feedback for large scale interconnected systems with radial structure. The 27th Chinese Control and Decision Conference (2015 CCDC), Qingdao, China, 5918-5923.
- Yan, R., Saha, T.K, Modi, N., Al Masood, N. A. & Mosadeghy, M. 2015. The combined effects of high penetration of wind and PV on power system frequency response. *Journal of applied energy*,145:320–330.
- Yang, L., Liu, T. & Hill, D. J. 2021. Decentralized event-triggered frequency regulation for multi-area power systems. *Automatica*, 26 (109479), 1-11.
- Yang, T. C., Ding, Z.T. & Yu, H. 2002. Decentralised power system load frequency control beyond the limit of diagonal dominance. *Electrical power and energy systems*,24,173-184.
- Yang, T., Zhang, Y., Li, W., & Zomaya, A, Y. 2020. Decentralized networked load frequency control in interconnected power systems based on stochastic jump system theory. *IEEE Transactions on Smart Grid*, 11, (5): 4427-4439.
- Yin, L., & Wu, Y. 2022. Mode-decomposition memory reinforcement network strategy for smart generation control in multi-area power systems containing renewable energy. *Applied Energy*,307(8266):1-19.
- Zecevic, Al., & Šiljak, D. D. 1994. Balanced decompositions of sparse systems for multilevel parallel processing. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 41(3): 220–233.
- Zeynelgil, H. L., Demiroren, A., & Sengor, N.S. 2002. The application of ANN technique to automatic generation control for multi-area power system. *Electrical Power and Energy Systems*,24(5): 345-354.
- Zhu, F., Zhou, X., Zhang, Y., Xu, D., & Fu, J. 2021. A load frequency control strategy based on disturbance reconstruction for multi-area interconnected power system with hybrid energy storage system, *Energy Reports*, 7: 8849-8857.
- Zou. Y., Lam J., Niu, Y., & Li, D. 2015. Constrained predictive control synthesis for quantized systems with markovian data loss. *Automatica*, 55: 217–225.