REAL-TIME STATE ESTIMATION OF A DISTRIBUTED ELECTRICAL POWER SYSTEM UNDER CONDITIONS OF DEREGULATION

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by

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DECLARATION

I, **Mashauri Adam Deusdedith Kusekwa**, declare that the contents of this thesis represent my own unaided work, and that the thesis has not previously been submitted for academic examination towards any qualification. Furthermore, it represents my own opinions and not necessarily those of the Cape Peninsula University of Technology.

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Signed **Date**

ABSTRACT

Static state estimation is a mathematical procedure in which physical measurements from sub-stations and physical model are combined in an optimal way. That is, the measurements obtained from the sub-stations are used with the physical model and the states of the power system are selected or calculated such that the states match the measurements in some best way. The states of a power system are the bus voltage magnitude and voltage angle of each bus of the system.

Static state estimation is an increasingly common part of electrical power utility energy management systems (EMS). It plays a critical part in a day-to-day operation of a power system utility. The system measurements obtained from static estimation are used for real-time operations like optimal power flow calculations and contingency analysis. Proper system operations with regards to avoidance of insecure conditions includes situational awareness, therefore, the static estimator plays an important role in power system security. A further motivation: in increasingly electrical power deregulation, more economic operations mean savings for customers and electrical power provider alike. Economic benefit might be realized if system operators have a more accurate situational awareness of the system through improved power system state estimator.

The objective of the study was to develop method, algorithm and MATLAB program for solution of power system state estimation using parallel processing techniques. In achieving the objective, the study has concentrated on development of an approximate Tanzanian power system network model comprising of 30 buses and used as a case study; decomposing the bus admittance matrix of the model into 3 interconnected sub-systems; development of mathematical model for real and reactive power injections, real and reactive power flows in the transmission lines and tie-lines connecting the sub-systems; development of measurement data model for voltage magnitude, real and reactive power injections, real and reactive power flows; formulating of a constrained weighted least absolute value state estimation problem; development of decomposition-coordination method and algorithm; and formulating algorithm and MATLAB program for solving the constrained state estimation problem using parallel processing technique.

Application of decomposition-coordination method and algorithm in a solution of an optimization problem such as power system state estimation and using of parallel processing techniques is a feasible means to operate, monitor and control of a large-scale and interconnected system efficiently within a limited time. The method improves operation stability of the systems; it increases compatibility, transmission and simplifies data and information processing.

Developed models and obtained results can be used by Institutions and universities for system monitoring and control, improvement, teaching, research and development purposes.

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GLOSARY

Terms/Acronyms/ Abbreviations	Definition/Explanation
Optimization	Is the process of finding the best way of using resources at the same time not violating any of the constraints that are imposed
Linear Programming	Is a technique for optimization of a linear objective function subject to linear equality and linear inequality
Non-Linear Programming	Is a technique of solving a system of equalities and inequalities, collectively termed as constraints
Interior Point Methods	Are certain class of algorithms to solve linear and non linear convex optimization problems
Algorithm	An iterative method that generates a sequence of the form $x^{k+1} = X\_k(A\_k(x^k))$ where x^0 is given as the initial point; *A_k* is an algorithm map that yields a set of policies given the current point x^k; and *A_k* is a selection function.
Augmented Lagrangian	The Lagrangian augmented by a term retaining the stationary properties of a solution to the original mathematical program but alters the Hessian in the subspace defined by the active set of constraints. The added term is called a penalty term , this term decreases the value of the augmented Lagrangian for *x* off the surface defined by the active constraints
Linearization	The strategy of linearization is to formulate a non-linear programming as a linear programming through introduction of auxiliary variables and constraints that are used to circumvent the non-linearities.
LU Decomposition	LU decomposition of matrix **A** is a process of finding a lower triangular matrix (**L**) and an upper triangular matrix (**U**) for which **A = LU**
Decomposition Method	The idea of decomposing a mathematical program into two or more sets of variables and associated constraints. The purpose is to separate some portion with special structure from the rest of the mathematical program
Dual	Is a form of mathematical programming with the property that its objective is always a bound on the original mathematical programming, called the primal
Aggregation	This is combining units to reduce problem dimension. One form of aggregation is to combine basic entities at the data level such a regions, time periods, and materials. The dimensions of the mathematical programming, which include numbers of variables and constraints, are generally reduced by aggregation at the entity level.
Variable(s)	Variable(s) usually represent things that are known at the start of solving an optimization problem but can be adjusted or controlled in order to find values of the variables that provide the best value of the objective function
Objective function	This is a mathematical expression that combines the variables to express the goal of solving the optimization problem.
Constraints	These are mathematical expressions that combines the variables to express limits on the possible solutions

Variable Bounds	A possibility in optimization problem which permits the variables to take on any value from minus infinity to plus infinity ($-\infty$ to $+\infty$)
Power System State Estimation	Is a procedure of processing redundant measurements available together with the knowledge of the network topology and line model parameters to provide a reliable, accurate, and complete set of data for the real-time monitoring and control of power systems
Task	A logically discrete section of computational work. A task is typically a program or program-like set of instructions that is executed by a processor
Parallel Task	A task that can be executed by multiple processors safely (yields correct results)
Serial Execution	Execution of a program sequentially, one statement at a time. In the simplest sense, this is what happens on a one processor machine. However, virtually all parallel tasks will have sections of a parallel program that must be executed serially.
Parallel Execution	Execution of a program by more than one task, with each task able to execute the same or different statement at the same moment in time
Shared Memory	From a strictly hardware point of view, this describes a computer architecture where all processors have direct (usually bus based) access to common physically memory. In a programming sense, it describes a model where parallel tasks all have the same "picture" of memory and can directly address and access the same logical memory locations regardless of where the physical memory actually exists
Distributed Memory	In hardware, refers to network based memory access for physical memory that is not common. As a programming model, tasks can only logically "see" local machine memory and must use communications to access memory on other machines where other tasks are executing
Communication	Parallel tasks typically need to exchange data. There are several ways this can be achieved, such as through a shared memory bus or over a network, however the actual event of data exchange is commonly referred to as communications regardless of the method employed
Synchronization	The coordination of parallel tasks in real-time, very often associated with communications. Often implemented by establishing a synchronization point within an application here a task may not proceed further until another task(s) reaches the same or logically equivalent pint. Synchronization normally involves waiting by at least one task, and can therefore cause a parallel application's wall clock execution time to increase
Granularity	In parallel computing, granularity is a qualitative measure of the ratio of computation to communication.
Coarse	Relatively large amounts of computational work are done between communication events.
Fine	Relatively small amount of computational work are done between communication events.
Analysis	Set of actions, carried out "a posteriori", to examine the power system behaviour. It is part of the control activities.
Control	Set of actions, excluding maintenance, oriented towards the

	implementation of the highest economy in electric power supply, compatible with the security criteria. System control function includes the implementation of the manoeuvres (switching and loadind) in normal and emergency conditions, and the power system monitoring.
Monitoring	Action by which the present working state of a power system is detected.
Operation	Whole process of power system control and maintenance
Operating Limits	Current or voltage limits should not be violated to avoid exist from "normal secure working state"
Operating Reserve	Generation capacity, above system load demand, required to provide for regulation, load forecasting error, equipment failures. It consists of the spinning and part of the cold reserve.
Cold Reserve	Part of operating reserve not connected to the system but capable of serving load demand within specified time, or interruptible load that can be removed from the system in a specified time.
Spinning Reserve	Part of operating reserve provided by partially loaded generators and ready to serve additional demand
Real- time	Time span from present time to a few minutes ahead
Tie-line	High voltage line interconnecting two power systems (or control areas)
Unbundling	Accounting separation of the activities inside a vertically integrated company (i.e. generation division, transmission division, distribution division, etc.).
Ancillary Services	Regulation functions, associated with the generators or the high voltage transmission network, to operate a generation and transmission power system. In some cases also the distribution network can supply some services (e.g. VAr contribution through the capacitor banks installed in the medium voltage netwok)
DCT	Distributed Computing Toolbox
PCT	Parallel Computing Toolbox
PDLB	Primal-Dual Logarithmic Barrier
IPD	Inexact Penalty Decomposition
CPUT	Cape peninsula University of Technology
DIT	Dar es Salaam Institute of Technology
SE	State Estimation
DSE	Dynamic State Estimation
WLAV	Weighted Least Absolute Value
WLS	Weighted Least Square
LP	Linear Programming
NLP	Non-Linear Programming
IPM	Interior Point Method
IPDPF	Infeasible Primal Dual Path Following
N-R	Newton-Raphson Method
MATLAB	Matrix Laboratory

SVC	Static Var Compensator
SCADA	Supervisory Control And Data Acquisition
EMS	Energy Management System
ESI	Electricity Supply Industry
IEEE	Institution of Electrical and Electronics Engineers
kV	Kilovolt
HV	High Voltage
EHV	Extra High Voltage
PPs	Power Producers
RTOs	Regional Transmission Operators
LAN	Local Area Network
WAN	Wide Area Network
FACTS	Flexible Alternating Current Transmission Systems
CDC	Central Dispatch centre
OPF	Optimal Power Flow
KKT	Karush-Kuhn-Tucker
PSSE	Power System State estimation
CD	Compact Disk
CPU	Central Processing Unit
ROM	Read Only Memory
RAM	Random Access Memory
SAUPEC	Southern African Universities Power Engineering Conference
AFRICON 2007	African Conference 2007
NE	Normal Equation
OT	Orthogonal Transformation
HAM	Hactel's Augmented Matrix
EKF	Extended Kalman Filter
KF	Kalman Filter
LAV	Least Absolute Value
SLP	Sequential Linear Programming
VAr	VoltAmpere reactive
LS	Least Squares
IPA	Interior Point
IRLS	Iteratively Re-weighted Least Square
IPMNL	Interior Point Method for Non-Linear Norm
SQP	Sequential Quadratic Programming
USA	United States of America
IPPs	Independent Power Producers
DITSCO	Distribution Transmission Companies

TANESCO	Tanzania Electric Supply Company Limited
IMF	International Monetary Fund
ECNZ	Electricity Cooperation of New Zealand
NETA	New Electricity Trading Arrangement
EU	European Union
FERC	Federal Energy Regulatory Commission
UK	United Kingdom
VIU	Vertically Integrated Utility
CM	Congestion Management
SCOPF	Security Constrained Optimal Power Flow
IGO	Independent Grid Operator
TSO	Transmission System Operator
ADR	Alternative Dispute Resolution
C	Customer (Electricity)
LF	Load Flow/Forecasting
ED	Economic Dispatch
SA	Stability Analysis/Assessment
ASA	Ancillary Service Auction
MEM	Ministry of Energy and Minerals: Tanzania
EWURA	Electricity, Water, Utilities Regulatory Authority
IPTL	Independent Power Tanzania Limited
ZSFPC	Zanzibar State Fuels and Power Corporation
MEP	Mtwara Energy Project: Tanzania
MW	Megawatt
AC	Alternating Current
SAPP	Southern Africa Power Pool
EAPMP	East Africa Power Master Plan
SSEA	Strategic/Sectoral Social Environment Assessment
ZTK	Zambia-Tanzania-Kenya
HFO	Heavy Fuel Oil
SONGAS	Songo Songo Gas Supply
HVAC	High Voltage Alternating Current
SVP	Shared Vision Program
NBRPTP	Nile Basin Regional Power Trade Project
NELSAP	Nile Equatorial Lakes Subsidiary Action Program
DRC	Democratic Republic of Congo
EARPP	East African Regional Power Plan
EAC	East African Community

EAPP	East Africa Power Pool
kVA	Kilovoltamperes
MVA	Megavoltamperes
ANSI	American National Standard Institute
TCUL	Tap-Changing Under Load
KVL	Kirchhoff's Voltage Law
KCL	Kirchhoff's Current Law
MFP	Ministry of Finance and Planning
MVAr	Megavolt-Ampere reactive
SQD	Successive Quadratic Programming
PDIPM	Primal-Dual Interior Point Method
ADCC	Area Dispatch Control Centre
BDM	Bender's Decomposition Method
MP	Master Problem
MIP	Mixture Integer Programming
GDB	Generalized Bender's Decomposition
DWDM	Dantzig Wolfe Decomposition Method
OPGV	Optimization Problem with Global Variable
SLICQ	Strong Linear Independence Constraints Qualification
TCSC	Thyristor Controlled Series Capacitor
LR	Lagrangian Relaxation
ALR	Augmented Lagrangian Relaxation
TCPAR	Thyristor Controlled Phase Angle Regulator
SPM	Shared Memory Processing
MPI	Message Passing Interface
SPMD	Single Program Multiple Data
SPSD	Single Program Single Data
MPMD	Multiple Program Multiple Data
CCS	Computer Cluster Server
MDCE	MATLAB Distributed Processing Engine
z	Measurement Vector
x	State Vector
ε	Measurement Error Vector
r	Measurement residual
$h(\bullet)$	Vector of non-linear functions related to state vector for the error free measurements
$x^{\bullet}$	Estimated value of the State Vector
$J(x)$	The objective/Cost function

W	Weighting matrix
$G(x)$	The gain matrix
$\Delta z = z - h(x)$	Correction on measurement
$\Delta x = x - x^0$	Correction on State Vector
Q_T	Orthogonal matrix
Q^T	Transpose of orthogonal matrix
R_u	Upper triangular matrix
Δy	Correction on y
$f:$	n-dimensional non-linear state transition function (DSE)
$w(t)$	System noise vector
$v(t)$	Normal error affecting the measurements (DSE)
F	Jacobian matrix of v(t)
B	Jacobian matrix of w(t)
K	The Kalman gain
k	Iteration counter
Σ	Covariance of the estimated state vector (DSE)
m	Number of unconstrained measurements
n	State vector number
N	Number of buses
A	A matrix of dimension mxn
b_c	A constant number
c	A constant number
y_{mxn}	A dual variable
$\mathbf{A}^\mathrm{T}$	Transpose of matrix **A**
$u_1, \cdots, u_m$	Set of independent variables (input, demand and control)
$x_1, \cdots, x_n$	Set of dependent (state) variables
$g(x,u) = 0$	Equality constraints
$f(x,u) \leq 0$	Inequality constraints
$\mathbf{c}^\mathrm{T}$	Transpose of matrix **c**
ρ	Lagrange multiplier (equality constraints)
λ	Lagrange multiplier (inequality constraints)
β	Lagrange multiplier (Lower slack variables)
π	Lagrange multiplier (upper slack variables)
u	Upper slack variables

l	Lower slack variables
n_R	Real number
z_b	Bounded measurement
$y^{\bullet}$	Optimal value of y
y	A vector of slack variables
s	A vector of slack variables
$s^{\bullet}$	Optimal value of s
$e = [1,1,\cdots,1]^T$	A matrix of all ones
m_E	Dimension of equality constraints (number)
m_L	Dimension of inequality constraints (number)
$\lambda^{\bullet}$	Optimal value of λ
$\beta^{\bullet}$	Optimal value of β
J	The Jacobian matrix of *h(x)*
τ	Penalty coefficient
ν	Positive constant (weighting parameter)
p^T	The transpose of Lagrangian Multiplier associated with decomposition-coordination
y_d	Dual variable
μ	Barrier parameter
α	Step Length
V_B	Base Voltage
I_B	Base Current
Z_B	Base Impedance
Z	Phasor impedance
I	Phasor current
R	Phasor resistance
C	Phasor capacitance
L	Phasor inductance
P_B	Base real power
P	Real power
Q	Reactive power
S	Complex power (apparent power)
δ	Angle between voltage and current
X	Phasor reactance

$i(t)$	Instantaneous current
r_w	Winding resistance
i	Representation of node i
j	Representation of node j
ψ	Total flux linkage
$v(t)$	Instantaneous voltage
v_a	Phase 'a' instantaneous voltage
v_b	Phase 'b' instantaneous voltage
v_c	Phase 'c' instantaneous voltage
v_f	Field instantaneous voltage
i_a	Phase 'a' instantaneous current
i_b	Phase 'b' instantaneous current
i_c	Phase 'c' instantaneous current
ψ_a	Phase 'a' flux linkage
ψ_b	Phase 'b' flux linkage
ψ_c	Phase 'c' flux linkage
ψ_f	Field flux linkage
r_f	Field winding resistance
L_a	Phase 'a' inductance
L_b	Phase 'b' inductance
L_c	Phase 'c' inductance
ψ_s	Total stator flux linkage
I_s	Total stator current
L_s	Total stator inductance
L_{sf}	Mutual stator-field inductance
L_{ff}	Self inductance
a	Phase 'a'
b	Phase 'b'
c	Phase 'c'
d	Direct axis
q	Quadrature axis

o	O axis
B_D	Direct Blondel's Transformation
i_d	Current flowing in the d-axis
i_q	Current flowing in the q axis
i_0	Zero sequence current
$\theta = \omega t$	Electrical degree
V_{B_D}	Direct Blondel's transformation stator voltage
B_D^{-1}	Inverse Direct- Blondel's Transformation
ψ_{B_D}	Direct -Blondel's transformation stator flux
V_s	Stator voltage
L_d	Direct axis synchronous reactance
L_q	Quadrature axis synchronous reactance
L_0	Zero sequence inductance
$\omega = \frac{d\theta}{dt}$	Stator angular frequency in electrical degree per second
r_s	Stator winding
S_G	Complex power generated
P_G	Real power generated
Q_G	Reactive power generated
P_d	Real power demand
Q_d	Reactive power demand
V_t	Generator's terminal voltage
I_t	Generator's terminal current
E	Electromotive force in the generator
V	Phasor voltage
V_i	Voltage at node *i*
V_j	Voltage at node *j*
I_i	Current flowing out of node *i*
I_j	Current flowing out of node *j*
t	Transformer turns ratio
Y_i	Shunt admittance at node *i*

Y_j	Shunt admittance at node *j*
Y_{ij}	Line admittance between nodes *i* and *j*
V_1	Voltage at terminal 1
V_2	Voltage at terminal 2
I_1	Current at terminal 1
I_2	Current at terminal 2
Y_{bus}	Ybus admittance matrix
$ABCD$	Transmission Line parameters
π	Nominal π circuit
$R_e[]$	Real part
$\mathrm{Im}\,ag[]$	Imaginary part
δ_{Sl}	Angle at slack node
V_{Sl}	Voltage magnitude at slack node
dP	Real power correction
dQ	Reactive power correction
Y	Shunt admittance
U_c	Control vector
T_n	Tap changing transformer number
S_n	Phase-shifting transformer number
D	Disturbance variable
N_g	Generator number
L_n	Total number of transmission line in the power system
S_i	Injected complex power
P_i	Injected real power
Q_i	Injected reactive power
$\delta_{ij} = \delta_i - \delta_j$	Angle difference between nodes *i* and *j*
R_c	Covariance matrix
σ	Standard deviation
$G_{ij} + jB_{ij}$	The ijth elements of the complex Ybus admittance matrix
$g_{ij} + jb_{ij}$	Series admittance of the branch connecting nodes *i* and *j*
$g_{ij}^{sh} + jb_{ij}^{sh}$	Shunt admittance of the branch connected to node *i*

G	Lagrangian matrix of g(x)
H	Lagrangian matrix of h(x)
G^T	Transpose of *G*
H^T	Transpose of *H*
F	Lagrangian matrix of *f(x)*
F^T	Transpose of *F*
$\hbar$	A constant number
λ	Duality gap
χ	Predefined tolerance
y	Local variable
$\aleph$	A polyhedral set representing constraints of a special structure
Γ_B, Γ_N	Feasible solution
α_c	Equality constraint according to Dantzig-Wolfe Decomposition
z_t	Target variable
N_S	Number of sub-systems
n_{set}	Number of subset
Δu	Correction on *u*
Δl	Correction on *l*
$F_k^{\bullet}(x)$	Optimal value function associated with Tammer's decomposition method
ϖ	Penalty parameter associated with Inexact Penalty Decomposition method
$p_k^{\bullet}(z_p)$	Optimal value-function associated with collaborative Optimization
N_{tl}	Number of tie-lines
z_{tl}	Vector of measurement for tie-line of the *ith* sub-system
h_{tl}	Non-linear function relating vector of measurements for tie-line and state vector
$y_{i,inj}$	Interconnectionmeasurements in tie-lines
$\rho_{i,inj}$	Vector of Lagrange multipliers
$\lambda_{i,tl}$	Vector of Lagrange multipliers
$y_{i,inj}, y_{i,tl}$	Prediction variables (interconnection and tie-line)
$z_{i,V}$	Vector of voltage measurement of *ith* sub-system
$z_{i,inj}$	Vector of real and reactive power injection measurements of *ith* sub-system
$z_{i,flow}$	Vector of real and reactive power flows of the *ith* sub-system

$\hat{V}_i$	Estimated voltage magnitude (calculated)
$\varepsilon_{i,V}$	Measurement error of voltage magnitude of *ith* sub-system

CHAPTER ONE: INTRODUCTION

1.1 Background

An electrical power system consists of three principal divisions: The generating system, the transmission system and the distribution system; Transmission lines are the connecting links between the generating plants and the distribution system and lead to other power systems over interconnections. A distribution system connects all individual loads to the transmission lines at substations that perform voltage transformation and switching functions.

Energy is converted at the generating plants from fossil fuels, hydro and nuclear into electrical energy. This electrical energy is then sent through a transmission system to loads at various places where it is usually converted into other useful forms of energy.

The role of electrical energy has grown steadily in the past few years in both scope and importance with time, and electrical energy is increasingly recognized as a key component to societal and economic progress. Therefore, reliable supply of electrical energy and reliable power system in general constitute the foundation of all prospering societies. As societies and economies develop further and faster, it is believed that electrical energy shortages and transmission bottleneck phenomena will persist. Under these circumstances, rotating blackouts will usually practice with certain regularity to avoid catastrophic failures of the entire power system. However, interconnections are serving to avoid blackout.

The primary requirement for interconnections of power systems is the sharing of responsibilities among the utility companies in case of emergency or interruption in power supply. For example, in an emergency, a system that is experiencing problem could draw upon the reserve generation from neighbouring systems. However, with all benefits brought by interconnections, more challenges are surfacing now, especially in the area of state monitoring and control of these interconnected systems.

In this situation, a proper question is? What effective approaches can be employed to monitor and control a complex power system composed of dynamic, interactive and non-linear entities with unscheduled discontinuities?[Shahidehpour & Wang, 2003]. An effective approach is to have or design means to intervene in the faulted system locally at place where the disturbances have originated in order to stop the problem from propagating through the system. This approach advocate distributed processing.

Distributed processing not only allows solving problem locally but enhances the reliability and improves flexibility and efficiency of power system monitoring and control.

The operation of power systems could be further optimized with innovative monitoring and control concepts. To realize this, effective tools should be designed, developed and included in the assessment of the system vulnerability mechanisms for the prevention of catastrophic failures. Power system state estimation is a very useful tool for monitoring and control of real-time state of the power systems.

Interconnection of power systems is transforming energy management system (EMS) components such as state estimation (SE) from an important application into critical one. SE is an essential component [Chawasak et al, 2005] of every energy management system (EMS) for security, [Rakpenthai et al, 2005; Zhengchun et al, 2005] monitoring and control of power systems. It is used to provide a best estimate of the system state based on the real-time system measurements and a pre-determined system model.

1.2 Static State Estimator

Static state estimation is a mathematical process for converting redundant noisy, uncertain measurements into reliable estimate of the state of a power system; the state of a power system is defined as the voltage magnitude and phase angle of each bus of a power system. In order to identify the current operating state of the system, state estimators facilitate accurate and efficient monitoring of operational constraints on quantities such as bus voltage magnitudes or transmission line loading. State estimators provide a reliable real-time database of the power system [Abur, A. & Exposito, A. G., 2004] including the existing state on which contingency analysis and security assessment functions can be taken to determine any required corrective actions.

Static State Estimators include the following functions:

- Topology processor
- Observability analysis
- State estimation solution
- Bad data processing
- Parameter and structural processing

State estimator constitutes the core of the on-line security analysis function. It is a critical element of energy management systems. It acts like a filter between raw measurements received from the substations and all the advanced application functions that require the most reliable database for the current state of the power system.

Deregulation of the electrical power industry has transformed state estimation from an important application into a critical one. Many critical commercial issues in the power market today, such as congestion management (CM), need to be founded and justified on a precise model of power system which is derived from state estimation procedure. Therefore, improvement of the state estimation to achieve a more accurate and more reliable state i.e. voltage magnitudes and phase angles is an important task

1.3 Objective of the Thesis

The objective of the thesis is: *to develop method, algorithm and MATLAB program for solution of power system state estimation using parallel processing techniques*. The objective is extended to include the study to determine *the effectiveness of parallel processing on the solution of the state estimation problem.*

The research has concentrated on formulating an improved robust state estimation optimization problem and developing its solution method that can work in large-scale, interconnected, and deregulated power systems. First, the power system is decomposed into overlapping subsystems, using bus admittance matrix. The approach allows seeing clearly the tie-lines connecting the overlapping subsystems. This method is contrary to the previous [Aguado, et al, 2001] where decomposition was based on physical arrangement or trial and error method. Next, the border variables are duplicated and included in the two neighbouring subsystems. A two level structure is proposed which includes the first level (sub-system) and the second level (coordination); a decomposition-coordination algorithm is developed to implement the structure. Each sub-system processes its sub-problem using sub-system measurements but exchanges few and dedicated border information with the coordinator. Processing of the overall state estimation problem is achieved in a parallel processing mode.

Application of decomposition-coordination method in a solution of an optimization problem such as state estimation and using parallel processing is a feasible means to operate large-scale and interconnected system efficiently within a limited time. The method improves operation stability of the systems. It increases compatibility, transmission, and simplifies data and information processing. It is reliable, with shorter computing time; easier gross error detection and identification, less time of data and information transmission and exchange.The method is applied to decompose the 30- bus Tanzanian power system network model.

1.4 Motivation

The challenge set by growing of power system interconnections is how to improve state estimators. This challenge and other challenges brought by changes in system structure motivates to conduct this investigation on existing state estimation methods and algorithms and develop a robust state estimator, which will be used in solution of large-scale, interconnected and deregulated power systems. In addition, the potential application of interior point methods in the successive linear solution of the state estimation problem is also a challenge. *To be more specific, the main objective of this research is to develop and systematically evaluate the primal-dual logarithmic barrier algorithm for the efficient solution of state estimation problem.* Although interior point methods (IPM) have received intensive study and achieved significant developments, there are still several challenging issues that deserve more investigation to further improve the performance of the methods.

The research question is how to apply the interior point method (IPM) for solution of large-scale interconnected state estimation problem. The research investigation is motivated by the developed and shown in the literature characteristics of this method. The existing challenges in the application of the method as how to adjust the barrier parameters and a Newton step length, how effectively to choose an initial point, how to reduce the number of iterations have to be considered during development of the decomposition-coordination method for solution of state estimation problem.

In order to exploit the full computational potential of the interior point methods for state estimation problem, it is essential to investigate all these challenges that have an influence on the algorithm for the case of interconnected state estimation problem. The following provides the motivation for the present thesis:

- The current successful applications and experience on using linear programming (LP) to solve various non-linear power system problems [Abur & Celik, 1991; Abur & Celik, 1993; Owen et al., 1978;Dantzig, 1963; Vanderbei,2001]
- The attractive property of LP methods in terms of the solution efficiency and reliability [Harris, 1970; Ravi & Wendel, 1984; Chieh, 1990]
- The critical need for a fast, robust and reliable solution of large-scale optimization problems in power system real-time monitoring and control [Singh & Alvarado, 1994; Monticelli, 2000; Abur & Exposito, 2004]

- The large-scale problem solving capability of interior point methods due to their polynomial complexity and computational efficiency as evidenced by the encouraging results from many applications [Mcshane et al., 1989; Meggido, 1988; Momoh, 1992; Momoh & Austin, 1993; Zountendijk, 1970; Luenberger, 1973; Ponnambalam et al., 1992]]
- The performance of the algorithms is closely related to several factors such as barrier parameter, initial point, step length and others. Therefore, customizing these factors to state estimation application can possibly speed up convergence [Clements et al, 1995; Wei et al, 1998; Singh & Alvarado, 1994; Wei et al, 1996; Bakry & Tapia, 1996]
- The attractive property of decomposition-coordination method, parallel processing in terms of solution efficiency and reliability [Lin, 2003;Bertsekas & Tsitsiklis, 1989; Lin & Lin,C., 1994; Conejo et al, 2007; Nogales et al, 2003; Conejo & Aguado; 2000; Abur, 2005;Carvalho & Barbosa, 1998]
- To the author's knowledge, the application of interior point methods for the solution of power system state estimation of the Tanzanian Power System has not been thoroughly investigated.

1.5 Contribution of the Thesis

The main results of this thesis contain contribution to several fields of electrical power engineering, namely, the load-flow and state estimation of power systems. The State estimator developed using decomposition-coordination method and parallel processing method provides a feasible approach to system operator (s) to monitor and control ever expanding, interconnected and deregulated power system of the future. More specifically key contributions can be briefly summarized as follows:

- Development of an approximate Tanzanian Power System Network model comprising of 30 buses. Bus admittance matrix of the model and its values was prepared and calculated and used in load flow analysis.
- Decomposition of the bus admittance matrix of the Tanzanian Power System model into 3 interconnected sub-systems
- Development of mathematical models for real and reactive power injections, real and reactive power flows in transmission lines and tie-lines of sub-systems resulting from bus admittance matrix decomposition
- Development of measurement model which has 3 parts determined by the type of measurement available in the system. The models are for voltage magnitude measurement, real and reactive injection data, real and reactive power flows in transmission lines and tie-lines. These models are applied in solving the power system state estimation problem under decomposition environment
- Development of decomposition-coordination method and algorithm in order to solve the power system state estimation problem. The method is implemented using two level structures. The first level for solving PSSE of the sub-system and the second level for coordination purpose of the computation.

- Formulation of constrained weighted least absolute value (WLAV) state estimation problem using primal-dual logarithmic barrier interior point method (PDLB).
- Development of algorithm and MATLAB program for solving the constrained WLAV state estimation problem

1.6 Thesis Outline

The thesis is organized into 11 chapters and 4 appendixes. Each chapter begins with an introduction describing the topic the chapter will present.

Chapter 1 briefly presents the background information of an electrical power system structure and evolvement of system interconnection, definition of static state estimator, its function and role it plays in a power system; objective of the thesis; motivation of conducting the research and contribution of the thesis. The chapter concludes by giving the thesis outline.

Chapter 2 reviews static and dynamic state estimation methods. Their mathematical formulations, weaknesses and strengths are explored. Brief review on interior point methods (IPMs) and their applications in solving linear and non-linear programming problems is discussed. The chapter concludes by making comparison of hierarchical and distributed processing of state estimation of large-scale power systems.

Chapter 3 addresses the concept of deregulation of power systems; motivation and reasons for deregulation are explored. Power sector deregulation process around the world is presented. Deregulated power systems structures and the role of state estimation in these structures are pointed out. The chapter concludes by introducing Tanzanian power sector status, structure, deregulation process and its impact; generation capacities and option to increase generation responding to customer requirements; high voltage transmission status and extension and corporation with regional and international power organizations.

Chapter 4 presents modelling process (formulating of mathematical models) of electrical system components. The chapter begins with discussion of the role of per unit system, its highlights how useful per unit system is for simplification of computation of electrical quantities. Next, formulating of mathematical models of AC synchronous generator, tap-changing and regulating transformers, transmission line (short, medium and long-length), FACT devices and composite loads is presented. The models developed in this chapter are very useful for load flow analysis, power system state estimation studies, optimal power flow studies, fault calculations etc.

Chapter 5 is one of the most important chapters of the thesis. It thoroughly covers data acquisition and classification methods and how to prepare database using MYSQL software; system network modelling of the Tanzanian power system using models developed in chapter 4. The chapter concludes by presenting procedures of decomposing the system network (Ybus matrix) into 3 overlapping sub-systems in form of block matrices. The sub-systems resulting from this chapter form a basis of developing decomposition-coordination algorithms

Chapter 6 is dedicated to comprehensive presentation of load flow solution of an integrated power system during normal operation. First, several methods used in the solution of load flow problem are described in brief. Next, the Newton-Raphson method for the solution of non-linear algebraic equations is discussed. An algorithm and MATLAB program developed using Newton-Raphson method is applied in solving the load flow of a practical power system i.e. the Tanzanian system network model built in chapter 5

Chapter 7 is the most important chapter in the thesis. The chapter is dedicated to state estimation problem formulation. The weighted least absolute value (WLAV) criterion is used in formulating the problem. Primal-dual logarithmic barrier a class of interior point method is applied in developing an algorithm, which is employed in preparing a MATLAB program. The program developed in this chapter is applied in solving the WLAV state estimation.

Chapter 8 is also one of the most important chapters of the thesis. The chapter is dedicated to formulating a decomposition method and algorithm. First, a brief review of decomposition methods in solving large-scale linear and non-linear programming problems is presented. Next, procedures of formulating decomposition-coordination method and algorithms are explored. Two level structures of solving a static optimization problem i.e. state estimation are described and presented. The chapter concludes by illustrating the first level sub-system and second level coordination algorithms.

Chapter 9 addresses parallel and distributed computing concepts. Overview of parallel and distributed systems; design of parallel algorithms; parallel computer architecture is presented. Brief description of distributed computing toolbox (DCT), parallel computing toolbox (PCT), their configuration and interaction with MATLAB distributed computing engine (MDCE); MATLAB distributed computing server (MDCS) and comparison analysis is given. The chapter concludes with programming parallel applications in MATLAB and application of parallel computing in solving of multi-area state estimation problem. Part of the MATLAB programs developed in this chapter is applied in the simulation process of chapter 10.

Chapter 10 is dedicated to simulation process and computational results. First, load flow, WLS state estimation and WLAV state estimation results obtained using integrated system network model are presented, analysed and discussed. Next, results from parallel processing of state estimation using a decomposed model are presented, analysed and discussed. The chapter concludes by giving general conclusions of the obtained simulation results.

Chapter 11 recapitulates conclusion from the thesis. It presents the deliverables in design and development; program (software) improvement and development and validation of system network model. Application of the thesis results, future research directions and publications in connection with the thesis is pointed out.

Appendixes

Appendix A lists names of power system networks applied in this thesis.

Appendix B gives input data for computation of load flow and state estimation analyses

Appendix C lists MATLAB files and programs applied in solving the load flow and state estimation problems.

Appendix D gives bus admittance matrix of the Tanzanian power system network

1.7 Conclusion

The basic objective of this chapter was to give a background information of power systems structures, definition of static state estimator, its functions and role in power systems; objective of the thesis, motivation of conducting the research, contributions from the thesis and provide general thesis outline.

Chapter 2 covers review of static and dynamic state estimation methods. Their mathematical formulations, weaknesses and strengths are explored; and propose a suitable method for solving the state estimation formulated in this thesis.

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

This chapter introduces power system state estimation (PSSE) methods applied in solving state estimation problem in an electrical power industry.The chapter provides a review of the state of art of the methods. The review covers weighted least squares (WLS), Dynamic state estimation (DSE), weighted least absolute value (WLAV) and robust state estimation methods. A review of previously proposed methods for hierarchical and distributed state estimation is also provided. The chapter concludes by giving a comparative analysis of PSSE methods.

2.2 Static State Estimation (SE) Methods

The subject of state estimation procedure is vast in literature; it is selected to review only those topics that are relevant to the objective of the thesis. It is difficult to cover the almost four decades of publications for the active research in theory and practice of state estimation, also the list of researchers, scholars and contributors in this aspect is quite long. There are many aspects of the overall state estimation procedure of the problem solution, but since the focus of the thesis is on numerical (mathematical) methods for solution of state estimation, therefore, an overview of specific methods as they are needed in the thesis is presented and discussed.

2.2.1 Mathematical Formulation of the Problem for state estimation: An overview

Application of estimation theory in monitoring and control of electrical power system was first proposed by Schweppe and Wildes in 1970s [Schweppe, et al, 1970] .Schweppe after establishing the motivation for SE as an important tool in operation, monitoring and control of power system, formulated a SE based on the weighted least square (WLS) estimation criterion.

The mathematical model of the state estimation is based on the mathematical relations between the measurements *z* and the state variable *x*. The equation relating the measurements to the state variables and the measurement error ε is given by:

$$z = h(x) + \varepsilon \qquad (2.1)$$

where

$z \in \Re^{mx1}$ is the measurement vector

$x \in \Re^{nx1}$ is the state vector

$h(\bullet) \in \Re^{mx1}$ a vector of non-linear functions related to state vector for the error free measurements

$\varepsilon \in \Re^{mx1}$ is the measurement error vector

Uncertainties arising from telemetered measurements, errors in mathematical models, unexpected system change and communication errors were also considered in the model. The proposed WLS state estimation was referred to as normal equation (NE) state estimation.

The estimated value of the state $x^{\bullet}$ is obtained by minimizing the WLS function i.e.

$$\min_{x} J(x) = [z - h(x)]^T W[z - h(x)] \quad (2.2)$$

where

$W \in \Re^{mxm}$ is a diagonal matrix whose elements are the reciprocal of the error variances. W is the matrix of weighting factors on the measurements. The solution of WLS problem gives the estimated state $x^{\bullet}$, which must satisfy the following optimality conditions:

$$\frac{\partial J(x)}{\partial x} = 0 \Rightarrow -H^T(x)W[z - h(x)] = 0 \quad (2.3)$$

The estimate $x^{\bullet}$ is obtained by an iterative scheme which calculates the corrections Δx at each iteration by solving

$$G(x)\Delta x = H^T(x)W\Delta z \quad (2.4)$$

where

$H(x) = \frac{\partial h(x)}{\partial x} =$ Jacobian matrix

$\Delta z = z - h(x)$

$\Delta x = x - x^0$

$G(x)\Delta x = H^T(x)W\Delta z =$ Gain matrix

$\mathbf{G(x)} \in \Re^{nxn}$

The gain matrix is a sparse and symmetric matrix; its dimension is that of the state vectors of the system. It is used in solving the system state by performing sparse matrix triangular factorization of **G(x)**.

The development made by Schweppe was explored by Larson and Tinney [Larson et al, 1970(a), Larson et al, 1970(b)]. They suggested the possibility of using the WLS estimator to real power system. Great attention was focused on linear and non-linear estimation theory, measurement scheme, location, type and number of measurements to be made as they are important to the implementation of the state estimator. Computational aspect of WLS estimation was treated in detail, computational results for on-line state estimation algorithm was applied to a 400 buses network. To improve the estimator, Dopazo and his colleagues' [Dopazo et al, 1970 (a); Dopazo et al, 1970(b)] demonstrated that, a measurement set consisting solely of real and line power flows has a computational advantage and substantial enhancement of estimator robustness. In their approach, fictious line elements voltages are calculated directly from measured power flows. Despite of its advantages over the previous estimator, the main drawback of the proposed approach is its prohibitive number of line measurements that are required in its implementation.

Enhancement of WLS estimator continued to be a major focus; Merrill and Schweppe [Merrill & Schweppe, 1971] formulated an estimator that was able to suppress bad measurement. One of the characteristic of the estimator is; the objective function is non-quadratic and is reduced to quadratic objective function in the absence of gross errors. Successful re-weighting of large residual terms makes the objective function to become less sensitive to large errors, therefore allowing the estimator to provide a modest degree of bad measurement rejection.

It was later established that the normal equation (NE) state estimation method was prone to ill-conditioning problem. The causes for ill-conditioning problem were established to include:

- Disparity in weighting factors [Aschmoniet et al, 1977]
- Large number of injection measurements [Gu et al, 1983]
- Connection of short (low impedance) and long transmission lines (high impedance) [Monticelli et al,1985]

The causes affect the estimator and result in degradation of estimator accuracy and make the gain matrix to be singular, which introduces numerical difficulties into estimation processes. A procedure to eliminate the ill-conditioning problem of the estimation via successive applications of householder orthogonal transformations known as the Golub's method was proposed by Simoes-Costa [Simoes-Costa, 1981]. It incorporates only perfect conditioned matrices while avoiding the explicit computation of the gain matrix.

Orthogonal transformation

The objective function of the linear WLS problem at each iteration is:

$$
\begin{aligned}
J(\Delta x) &= [\Delta z - H\Delta x]^T W[\Delta z - H\Delta x] \\
&= [\Delta\tilde{z} - \tilde{H}\Delta x]^T [\Delta\tilde{z} - \tilde{H}\Delta x] \\
&= \left\| \Delta\tilde{z} - \tilde{H}\Delta x \right\|^2
\end{aligned} \tag{2.5}
$$

Let Q be an orthogonal matrix, such that $Q^T Q = I$

$$
Q\tilde{H} = \begin{bmatrix} R \\ Q \end{bmatrix} \tag{2.6}
$$

where

R is an upper triangular matrix. Then,

$$
\begin{aligned}
J(\Delta x) &= [\Delta\tilde{z} - \tilde{H}\Delta x]^T Q^T Q[\Delta\tilde{z} - \tilde{H}\Delta x] \\
&= \left\| Q\Delta\tilde{z} - Q\tilde{H}\Delta x \right\|^2 \\
&= \left\| \Delta y_1 - R\Delta x \right\|^2 + \left\| \Delta y_2 \right\|^2
\end{aligned} \tag{2.7}
$$

where

$$
Q\Delta\tilde{z} = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} \tag{2.8}
$$

The minimum of $Q\Delta z^{\sim}$ occurs at $R\Delta x = \Delta y$

To overcome the ill-conditioning of the problem, the following techniques were proposed:

- Inclusion of constraints in the NE [Aschmoniet et al, 1977]
- Use of orthogonal transformation (OT) method [Simoes-Cost, 1981; Monticelli et al, 1985; Vempati et al, 1991]
- Use of Hybrid method [Gu et al, 1983; Monticelli et al, 1985]
- Use of Hactel's Augmented Matrix (HAM) [Gjelsvick, 1985]

The idea of separating the zero injection measurements (virtual measurements) from the telemetered measurements and treat them as equality constraints was accepted and seemed to be a break through in solving the normal equation WLS state estimation problem. The measurement z includes only the telemetered and the pseudo measurements in case they are available. The aim of inclusion of equality constraints is to restore observability to those parts of the power system network that are permanently or temporarily unobservable. The formulation includes that the equality constraints result into NE/C, and then WLS state estimation problem is solved by application of Lagrange multipliers. Inclusion of equality constraints tackled the ill-conditioning problem only on the disparity in weighting factors, but it does not eliminate the problem, but it reduced the difference in measurements.

However, more efforts were involved to eliminate the other causes. Simoes-Costa and GU [Simoes-Costa, 1981(a); Simoes-Costa, 1981 (b); GU, et al, 1983] proposed orthogonal transformation technique, aiming to eliminate the gain matrix in the solution of the state estimation (eqn 2.4).

The orthogonal transformation method shows highest numerical stability compared to the NE estimator. Its drawback is that it requires much computation.

Gu and Monticelli [Gu et al, 1983; Monticelli et al, 1985] improved the orthogonal transformation technique by proposing the Hybrid technique by using forward and backward substitution approach to obtain the solution. The proposed technique has reasonable numerical stability and computationally efficient and acceptable implementation complexity. Further improvement was suggested by Gjelsvick [Gjelsvick, 1985] who introduced the Hactel's augmented matrix technique. Virtual measurements (zero injections) and incremental residuals are taken as equality constraints and augmented in the coefficient of iteration of the normal equation. The technique avoids formation of the gain matrix. However, the dimension of coefficient matrix is large, therefore it requires more computation. The technique has reasonable compromise between numerical stability, computational efficiency, and acceptable complexity implementation. But the technique is a bit more complex in real application.

Improvement of WLS state estimation haunted researchers, their focus was to avoid or bypass the formation of a gain matrix ***(G(x))***. Robert Amerongen [Amerongen, 1991] suggested use of Givens orthogonal transformation technique with two features. The proposal addressed the inclusion of virtual measurements and treats them as real telemetered measurements from the sub stations; hence, the virtual measurements are no longer error free as before. The estimator developed from the proposed technique does not show any degradation in numerical stability. However, the drawback of the technique is on complexity in computation when virtual measurements are included rather than when there are only real telemetered measurements. Pandian et al [Pandian et al, 1998] suggested an estimator based on variable pivot column oriented Givens rotation technique. In his proposal, rows and columns ordering technique is applied in order to reduce the computational effort required in factorization of the matrix. The technique shifts from the common use of oriented QR decomposition of the matrix in column. It shows superiority when compared to the conventional row oriented QR decomposition technique. However, the approach was silent on how it can detect and identify outliers in the measurements.

Simoes-Costa and Gouvea [Simoes-Costa & Gouvea, 2000] included equality and inequality constraints in the Givens rotation technique. The aim is to apply constrained orthogonal state estimation to external system modelling. In the approach, equality constraints are used to model the zero injections in the system. Bounds based on physical as well as operation limits are included in the model. Quantities such as VAr limits on generators, virtual measurements, and transformer tap position setting limits are included as inequality constraints. Inclusion of these quantities, large variation of the external estimates caused by poor monitoring of the external estimates is avoided. The technique is effective in reducing power injection deviations in the system. The estimator developed using the technique shows good performance when tested in three different testing systems (IEEE30, IEEE118, and IEEE 340 nodes).

Zhengchun [Zhengchun et al, 2005] proposed incorporating virtual measurements as equality constraints, and solved the WLS estimator by performing QR decomposition on two portioned matrices, then solved a linear equation set involving a sparse triangular matrix. The technique circumvents the cross product of the Jacobian matrix, which can cause the loss of information in the process of computation. The proposed technique is stable, robust, and theoretically good. The only issue not discussed in the proposal is how bad data measurement is detected and identified from the telemetered measurements.

Idea of using soft computing on solving WLS state estimation was proposed by Faridon Shabani [Shabani et al, 1996]. His idea is based on using fuzzy-logic. In the proposed technique, the initial estimate is obtained by WLS criterion, and then all current estimates are updated using fuzzy technique reasoning in order to obtain the optimal estimate. The approach adaptively adjusts the weighting factor in the estimator and improves the computational performance. The technique can be implemented in large-scale power system. The difficulty arising from applying the technique is how to conduct the training.

As conclusion: Normal equation (NE) method is the easiest to implement, but its drawback is the ill-conditioning problem. The drawback is contributed by formation of a gain matrix. Several techniques have been proposed to deal with the problem. They include: orthogonal transformation (OT), Hybrid, and Hactel's augmented matrix. The techniques avoid or bypass the formation of a gain matrix. The techniques have the highest numerical stability, computationally efficiency and acceptable implementation complexity. Both Hybrid and Hactel's technique (Table 2.1) have shown compromise between numerical stability and complexity of implementation. Orthogonal transformations have shown robustness and at the same time retain the sparsity characteristics of the normal equation (NE). An effective compromise to improve the existing techniques is by combining techniques that have shown good numerical stability and computation efficiency. Combining Hybrid technique incorporating equality and inequality constraints can improve numerical stability and computation efficiency and therefore be used in large-scale power system

Table 2.1: Comparison of estimation methods based on WLS criterion

Serial Number	Technique	Advantage	Disadvantage
1	Normal Equation (NE)	High computational efficiency, with the fast P-Q decoupling techniques	Give rise to ill-conditioning problem
2	Normal equation with constraints NE/C	Better numerical stability compared to NE without equality constraints	More sophisticated approach is required because of triangular factorization
3	Orthogonal Transformation (OT)	Has higher numerical stability	Requires more computation and it has to use the P-Q decoupling techniques
4	Hybrid Technique	Reasonable compromise between numerical stability and computational efficiency, and acceptable implementation complexity	Not stable and efficient in particular for large-scale power system
5	Hactel's Augmented Matrix (HAM)	Stable and efficient, particularly for large-scale power system	More complicated in implementation

2.2.2 Dynamic State Estimation (DSE)

Dynamic state estimation (DSE) is one of the effective processes that can provide information on the state of the power system (Voltage magnitude and voltage phase angles) at present time instantly and in future time which is necessary for security, and supervision of the power system. Dynamic state estimation has two steps in its implementation. The first step is for prediction purpose and the second is for filtering of errors. The estimation is normally carried out at a time interval of few minutes (time samples). At each time sample *(t)*, the state *x (t)* is estimated from the observation *z (t),* which in most cases are generally measurements corrupted by noise. Using the power system dynamic model, two equations can be formulated: First, the measurement equation model, formulated as:

$$z(t) = h(x(t)) + v(t) \tag{2.9}$$

where

$z(t)$: is the m-dimensional measurement vector

$h(\bullet)$: is an m-dimensional non-linear function relating measurement to the state

$v(t)$: is the normal error affecting the measurements

The second part: state space transition, formulated as:

$$x(t+1) = f(x(t), w(t), t) \tag{2.10}$$

where

f : is an n-dimensional non-linear state transition function

w : is the system noise vector

Vectors $v(\bullet)$ and $w(\bullet)$ are white noise, uncorrelated between them, and with covariance matrices

$$E\left[v(t)v(l)^T\right] = R(t)\delta_{xl}\forall t,l \tag{2.11}$$

$$E\left[w(t)w(l)^T\right] = Q_{DSE}(t)\delta_{xl}\forall t,l \tag{2.12}$$

The solution of dynamic state estimation problem is normally obtained by using the Extended Kalman Filter (EKF) method which requires linearization of equations (2.10) and (2.11). The two EKF equations are divided into two steps: Prediction and Filtering steps.

Prediction step

$$X(t+\frac{1}{t}) = f(X.{}^{t}\!/_{t}) \tag{2.13}$$

$$\Sigma(t+\frac{1}{t}) = F(t)\Sigma({}^{t}\!/_{t})F(t)^T + B(t)Q_{DSE}(t)B^T(t) \tag{2.14}$$

F, B are Jacobian matrices of v and w

Filtering Step

$$X(t+\frac{1}{t+1}) = X(t+\frac{1}{t}) + K(t+1)\left[z(t+1) - h(X(t+\frac{1}{t}))\right] \tag{2.15}$$

$$K(t+1) = \Sigma(t+\frac{1}{t})H^T(t+1)\left[H(t+1)X\Sigma(t+\frac{1}{t})H^T(t+1) + R(t+1)\right]^{-1} \tag{2.16}$$

$$\Sigma(t+\frac{1}{t}) = \left[I - K(t+1)H(t+1)\right]X\,\Sigma(t+\frac{1}{t}) \tag{2.17}$$

where

Σ: Covariance of the estimated state vector

H_{DSE} :is the Jacobian of $h\ (x)$

K_G: the Kalman gain.

Equations (2.10-2.17) are applied to solve the dynamic state estimation. Debs and Larson [Debs & Larson, 1970] proposed the dynamic state estimation in the same year when Schweppe proposed the static estimation, since their first publication of DSE has not gained popular application in electrical power systems compared to the static WLS estimation. Leite da Silva et al [Leite da Silva et al, 1987] suggested dynamic models after making comparison of performance of dynamic tracking estimators operating in the quasi-static conditions. He established the advantages of the previous estimators proposed by Debs and Larson in terms of prediction and filtering processes and comes out with a new dynamic state estimator with reliable scheme on detection and identification of bad data or sudden changes in operating points of the power system. The proposed estimator can be implemented in any existing real-time state estimation algorithm in a centralized operation.

Implementation of dynamic state estimation in decentralized environment posed a challenge to researchers. Mandal and Shinha [Mandal & Shinha, 1997] suggested hierarchical dynamic state estimation in which in the non-llinear measurement function are incorporated. In their proposal, the estimator tracks the system state very well under various operating conditions of the system. They suggested decomposition of the system topologically into a number of small subsystems. In the approach, the local estimator estimates the states for a subsystem using only local measurements. The second level estimates the overall system by coordinating all local sub systems states. The algorithm proposed was suitable for distributed computing with the first level estimation carried out at local centres and the coordination level estimates to be carried out using interconnection measurements i.e. of tie lines. The approach reduced the computation time and also the number of telemetered measurement to the central centre. However, the approach is silent, if there is communication between the subsystems, and the role of virtual measurements in the problem formulation.

Durga Prasad and Thakur [Durga Prasad & Thakur, 1998] improved the EKF method and developed the Kalman Filter (KF) dynamic state estimation. The approach is to use the complex line flow measurements to obtain the complex element voltages at the filtering stage; Holt's smoothing technique is adopted at the prediction stage. The objective is to eliminate the heavy computational burden at the filtering stage. The advantage of the technique lies on the fact that the covariance matrices are reduced, which results in computational saving. How the bad data are treated, the technique is silent. Need for robust filtering motivated Thakur and Shinha [Thakur & Shinha, 2000] to develop a robust dynamic state estimation model that included robust filtering scheme in which statistical approach of M-estimation or Huber estimation [Huber,P.J., 1964] is used. The model includes the prediction stage, filtering stage, and robust filtering scheme to treat the bad data measurements in the telemetered measurements. The developed estimator performs better than the extended Kalman Filter (EKF) at both filtering and prediction stages and is simple in real-time implementation.

As conclusion: Dynamic state estimator is more efficient compared to static WLS estimator. It has features such as prediction stage, detection and identification of measurement errors, topology observability, and good filtering processes, but its implementation in power industry is still low. This is attributed to lack of realistic model, complex nature of the dynamic problem, time-consuming in designing the model, computational burden of filtering and lack of sufficient literature.

2.2.3 WLAV State Estimation

An alternative approach of solving power system state estimation problem is using the weighted least absolute value (WLAV) criterion. The criterion is more robust compared to the WLS one. The estimate is obtained by solving a sequence of linear programming (LP) problems. The drawback associated with WLAV criterion is its poor computation efficiency for large problem [Abur & Celik, 1991].

The weighted least absolute value (WLAV) state estimation problem is usually formulated as:

$$\begin{aligned} &\min J(x) = \sum_{j=1}^{m} W_j |\varepsilon_j| \\ &s.t. \quad z = h(x) + \varepsilon \end{aligned} \tag{2.18}$$

where

$z \in \Re^{mx1}$: is the measurement vector

$x \in \Re^{nx1}$: is the state vector

$\varepsilon \in \Re^{mx1}$: is the measurement error vector

$W \in \Re^{mx1}$: is the weighting factor

$h(.) \in \Re^{mx1}$: is the non-linear function relating state vector with the measurement.

Equation (2.18) is applied in solving the WLAV state estimation problem.

Irving [Irving et al, 1978] introduced the first practical weighted least absolute value (WLAV) criterion in solving the state estimation problem. He presented the problem as a linear programming (LP), aiming in minimizing the sum of residual modules. The LP estimator shows exceptional property of outright bad data rejection, i.e. reducing the expensive method of data analysis. However, the LP estimator satisfies a subset of measurement equation exactly $(n = m)$, assigning zero residual to these elements and rejecting the other measurements $(m - n)$. Hence, LP estimator is insensitive to bad data measurement; its filtering capacity is inferior to that of WLS estimator. Kotiuga [Kotiuga, 1982] suggested a more generalized WLAV estimator functionally equivalent to LP estimator. The proposed estimator seemed to be more efficient owing to the exploitation of the special structure of the power system measurement equations. Its outstanding robustness is attributed to its interpolative property by which the most erroneous measurements are completely discarded in arriving at the estimate of the system state.

Reduction of computation burden on LP estimator was suggested by Lo and Mohmoud [Lo &Mahmoud, 1986]. He developed a decoupled LP state estimator, which resembled that of the decoupled power flow. The estimation problem is decomposed into two smaller P-Q and Q-V problems. The two problems are solved sequentially to obtain the final solution. The elements of Jacobian matrix are evaluated at first iteration and held constant for all subsequent iterations. The technique avoids the considerable computation that requires updating the element at each start of iteration. The technique shows good computation characteristics. Abur and Celik [Abur & Celik, 1991] worked on improvement of computation burden in order to improve efficiency of the estimator. They introduced the use of Barrodale and Roberts's algorithm in the LP estimator, re-formulated the problem, and used simplex method in its solution. The sparsity of the problem and structure of measurement equations are fully utilized. The suggested technique reduces the dimension of basic matrix to be factorized and updated at the simplex method iterations. The technique is efficient and can be used in large power systems.

El-Keib and Singh [El-Keib & Singh, 1992] introduced the primal and dual formulation to the original decoupled LP estimator. Introduction of dual formulation aimed at improving the solution time of the existing LP estimators. The proposed estimator for the first time accommodates voltage measurement, which in the previous estimators was not even considered and assumed to be impossible. A set of tests using the IEEE 30, 57 and 118 test node systems were conducted to compare the performance of primal and dual formulation. Numerical result shows superiority of the dual formulation estimators over the primal formulation. In addition, the dual decoupled estimator shows perspectives in improving time computation; hence more investigation is required to improve it.

Constrained state estimator formulated as least absolute value (LAV) and using simplex method to find the solution was first proposed by Abur and Celik in 1992 [Abur & Celik, 1992]. The LAV estimator handles both equality and inequality constraints available in the electrical power system. The constraints include the zero injections at switching substations, bound imposed on the reactive power injection at the generator buses, the lower and upper limits on the tie-lines power flows at the area or boundaries. The approach complements with that suggested by Simoes-Costa and Gouvea in 2000, the difference is in the criterion used, while Simoes-Costa used WLS, Abu applied the LAV criterion. Abur thought that inclusion of equality and inequality constraints in the LAV estimator could provide more reliability by reducing the number of simplex iterations and CPU time. The estimator is tested on IEEE 14, 57 and 118 test node system as well as 498 and 986 Texas nodes. It was established that the estimator converges faster when compared to unconstrained estimator. This suggests that equality and inequality constraints have a role to play in improving convergence of an estimator.

Singh and Alvarado [Singh & Alvarado, 1994] suggested application of interior point methods (IPMs) in solving the weighted least absolute value (WLAV) power system state estimation (PSSE) problem. First, the problem is formulated as primal and dual, then the IPMs is used in solving the two problems

Primal problem

$$\begin{aligned} &\min c^T x \\ &s.t. \quad Ax = b \\ &\qquad\quad x \geq 0 \end{aligned} \tag{2.19}$$

where

$c \in R^{nx1}; x \in \Re^{nx1} : b \in \Re^{mx1}; A \in \Re^{mxn}$

Dual problem

$$\begin{aligned} &\max b^T y \\ &s.t \quad A^T y + s = c \\ &\qquad\quad s \geq 0 \end{aligned} \tag{2.20}$$

where

$y \in \Re^{mx1}$: are the dual variables

$s \in \Re^{nx1}$: is the vector of dual slack variables

Barrier function is used in solving the primal problem and the affine-scaling method is applied in solving the dual problem. The global problem i.e. primal and dual together, is solved by the sequential linear programming method (SLP). In addition, an augmented formulation is applied to solve the least square problems emerged from the solution process. The augmented approach is employed because it can permit easy treatment of free variables which is always a concern in the primal problem formulation. The two problems were solved and their results compared. Results show that dual formulation has an added advantage over the primal especially in the number of iterations and work per iteration. Clements et al [Clement et al, 1994] suggested application of direct interior point to the estimation problem. In his suggestion, he included equality as well as inequality constraints such as reactive power (VAr) limits on generators. Transformer tap setting positions and unmeasured loads are included and modelled as unknown quantities and bounded. Clements work extended the previous work proposed by Abu and Celik, Simoes-Costa and Gouvea by including of equality and inequality constraints as well as unmeasured measurements to improve the reliability of an estimator.

Logarithmic barrier function is appended to the objective function, slack variables are introduced and employed to treat the inequality constraints. Three different systems ranging from 6 to 118 bus test systems, in which VAr limit is modelled as inequality constraints were tested. Computational results from the test systems were compared with the conventional least square (LS) estimator. The overall result shows to be comparable. The method is implementable even by using the existing least square estimation software. Chiite and Swarup [Chiite & Swarup, 2003] extended the method proposed by Clements of using logarithmic barrier function by including a new variable, the measurement residual error along with the constraints. Newton method is employed in linearizing the non-linear equation developed from the Karush-Kuhn-Tucker (KKT). The approach is efficient and good under practical situations when there are missing measurements. It is apparent that inclusion of equality and inequality constraints is important and improves reliability and computation efficiency of the WLAV estimator, hence, more investigation is required to enhance the estimator and make it applicable to large-scale, interconnected and deregulated power systems.

Wei et al [Wei et al, 1998] proposed an interior point algorithm to solve the weighted non-linear norm (IPMNL) state estimation problem basing on the perturbed Karush-Kuhn-Tucker (KKT) condition on the primal problem. The approach is that two correction equation schemes are suggested in order to enhance the computational efficiency. First, a new interior point algorithm (IPA) using the perturbed KKT conditions for the weighted non-linear norm state estimation is developed, second; two novel schemes are proposed similar to that of nodal admittance matrix of the power flow problem, which can hold a four by four (4x4) structure block. Using this approach, the fill-ins are dramatically reduced compared to that of the primal schedule when correction equation is factorized. Numerical simulation on the proposed method is conducted using test systems ranging from 5 to 1047 buses

Japanese and Chinese system applications demonstrated that the number of iterations in all simulations is less than ten (10) and always constant even if there is change in measurement redundancy. The proposed method and its algorithm are promising for use in large-scale power systems.

Ramirez and Barocio [Ramirez &Barocio, 2000] suggested use of LP in solving the SE employing primal-dual predictor corrector algorithm. The algorithm could be applied as an alternative to the simplex method. The primal-dual formulation is employed due to its efficiency and reliability as suggested by Singh and Alvarado. Ramirez's proposal suggests a successive linearization in the constraints contrary to applying a direct interior point in solving the non-linear problem as suggested by Clements et al. An algorithm basing on Mehrotra's [Mehrotra, 1992] predictor corrector is developed and applied it makes the direction search towards the optimal point easily, contrary to simplex method that has a strategy of searching of an optimal point on the vertex of the feasible region.

Simulations conducted with the algorithm gave more precise solution than those obtained from the conventional WLS estimator, in the case of presence of gross errors in the power flow lines and injection measurements. In addition, the method exhibits a similar sensitivity to the leverage points as that of WLS estimator and can perform better under critical error conditions. Jabr [Jabr, 2005] proposed the Huber-M method that includes equality and inequality constraints; however, his proposed technique is different to the previous Huber-M method, which was formulated as unconstrained non-linear programming (NLP) problem. The solution is obtained using an iterative-reweighed method and the equality constraint is modelled by assigning larger weight. However, assigning large weight causes large disparity in measurements that results in ill-conditioning problem and degrades the convergence of the estimator.

In the proposed method the problem is formulated as a non-linear programming one (NLP) in which the functions in the problem are twice continuously differentiable. The solution of the problem, which is a complete optimization problem, is obtained using primal-dual interior point method. The constraints are handled in a straightforward mode not only for equality equations but also for inequality ones. The IPM algorithm described by the suggested method is based on the symmetric form of optimization problem. Results from the simulation of the proposed method show that even in the presence of outliers, the equality constraints are effectively modelled as zero bus injections. The result of numerical computation reveals that by imposing inequality constraints, the method is useful even in the absence of complete system observability.

Jabr [Jabr, 2006] proposed the sequential linear programming (SLP) algorithm. The method is based on solving the estimation problem through a sequence of linearized LAV problems. Instead of converting the linearized LAV problems into linear programming problems, a solution is obtained directly through iteratively re-weighted least squares (IRLS) method i.e. through adjusting the weight dynamically during the iterations. At the heart of IRLS algorithm is a sparse least square solver similar to that employed in the conventional LS estimator. The method is more efficient than other re-weighted least squares. However, its drawback is that requires an already installed LS estimator

Electrical power system is expanding rapidly and becoming very large in scale, more interconnected, and complex. Centralized monitoring and control of these power systems is becoming more difficult and expensive. The feasible approach is decomposition of the systems into smaller areas, which could be assumed to be in line with deregulation. Implementation of parallel and distributed processing is the best way of processing of the power system state estimation problem particularly for large-scale power systems.

As Conclusion: Weighted least absolute value (WLAV) criterion and interior point methods (IPMs) are promising methods in solving the power system state estimation (PSSE) problem. Inclusion of equality and inequality constraints in the PSSE problem formulation and in its algorithm enhances the reliability and computation efficiency. Finding from literature review (Table 2.2), indicates; WLAV criterion and interior point methods have been used in solving the estimation problem in a centralized and structured (regulated) power system. A challenge is to employ the methods in a deregulated system using decomposition-coordination, parallel, and distributed processing approaches, which is the direction of the present research project.

2.3 Robust State Estimation Methods

Glavic and Wehenkel [Glavic & Wehenkel, 2004] published a survey on interior point methods (IPMs) their application to power system, and research opportunities brought by IPMs. The survey recommends IPMs in solving engineering optimization problems. The survey outlines the advantage brought by IPMs over simplex methods in solving linear programming (LP) problems and its extension to solving non-linear programming (NLP) problems in electrical power system. Interior point method has been used by several researchers in solving the power system state estimation problem, for example Singh and Alvarado, Clements et al, Wei et al, Ramirez, Chiite and Swarup, employed the method in solving the SE problem. Hence, SE problem can be formulated as an optimization problem and effectively solved by IPMs.

Under normal steady-state operating conditions, all customer demands are met and all equipment is operating below its rated capacity. Theoretically speaking, the requirement of meeting customer demands is expressed mathematically by means of a set of equations (equality constraints) of the type:

Table 2.2: Summary of selected state estimation method

Name	Year	Criterion	Method	Problem	System	IEEE Type	Status	condition
Amerongen	1991	WLS	OT	Steady	1	30	3	4
Lin,S.Y.	1992	WLS	NE	Steady	2	30	2	4
Lin & Lin,C	1994	WLS	NE	Steady	2	30 & 118	2	4
Shabani et al	1996	WLS	HM	Steady	1	30 & 118	3	4
Pandian et al	1998	WLS	OT	Steady	1	5,14 & 1044	3	4
Simoes-Costa	2000	WLS	OT	Steady	1	14,30 57&118	3	5
Reza	2000	WLS	NE	Steady	2	8000	2	4
Korres,G.	2002	WLS	NE	Steady	1	14	1	5
Madtharad	2003	WLS	SVD	Steady	1	14	1	4
Rakpenthai	2005	WLS	OT	Steady	1	14 & 30	3	4
Al-Othman	2006	WLS	SQP	Steady	1	14 & 30	3	4
Roués. et al	1988	DSE	EKF	Dynamic	1	118	3	4
Mandal et al	1995	DSE	EKF	Dynamic	1	14 & 30	3	4
Mandal & Sinha	1997	DSE	EKF	Dynamic	1	118	3	4
Durga & Thakur	1998	DSE	KF	Dynamic	1	4,14 & 30	3	4
Shinha & Mandal	1999	DSE	HDSE	Dynamic	2	118	2	4
Thakur & Shinha	2000	DSE	EKF	Dynamic	1	30 & 118	3	4
El-Keib & Singh	1992	WLAV	LP	Steady	1	30,57 & 118	3	4
Abur & Celik	1992	WLAV	LP	Steady	1	14 &118	3	5
Singh & Alvarado	1994	WLAV	IPM	Steady	1	5,30 &118	3	4
Clement et al	1995	WLAV	IPM	Steady	1	6,30 & 118	3	5
Wei at al	1998	WLAV	IPM	Steady	1	5,14,57 118 &1024	3	5
Ramirez	2000	WLAV	IPM	Steady	1	14	3	5
Chiite & Swarup	2003	WLAV	IPM	Steady	1	30 & 118	3	5
Jabr, R.A.	2005	WLAV	Huber-M	Steady	1	14,57 & 118	3	5
Jabr ,R.A.	2006	LAV	SLP	Steady	1	14,30 57& 118	3	5

1: Centralized **2**: Distributed **3**: Regulated **4**: Unconstrained **5**: Constrained

$$\begin{aligned}
&g_1(x_1, x_2,..., x_n; u_1,..., u_m) = 0 \\
&g_2(x_1, x_2,..., x_n : u_1,..., u_m) = 0 \\
&\quad\vdots \\
&g_n(x_1, x_2,..., x_n; u_1, ..., u_m) = 0
\end{aligned} \tag{2.21}$$

where

$x_1,\cdots,x_n$ are set of dependent (state) variables and $u_1,\cdots,u_m$ are a set of independent (input, demand, or control) variables. Typically these equality constraints correspond to the so called power flow equations. The constraints relative to the equipment can be written, in general, in the following form:

$$\begin{aligned} c_1(x_1,x_2,...,x_n;u_1,...,u_m) &\le 0 \\ c_2(x_1,x_2,...,x_n;u_1,...,u_m) &\le 0 \\ &\vdots \\ c_n(x_1,x_2,...,x_n;u_1,...,u_m) &\le 0 \end{aligned} \tag{2.22}$$

They correspond to items such as upper and lower limits on power generation by a given unit (real and reactive powers), current limits on transmission lines and transformers, and so on. Mathematically, the formal operating state is defined whenever the utility system considered satisfies relations (2.21) and (2.22). Following certain disturbance events (short-circuits due to faults, loss of generation, loss of load, and others) some of the inequality constraints may be violated. For example, a line may become overloaded, or a system frequency may drop below a certain limit. In these cases the system is in the emergency operating state.

Finally, the system may exist in a situation where only a fraction of the customers are satisfied without overloading any equipment. In this case only a portion of the system is in the normal state. As a result, not all the equality constraints are satisfied, but the inequality constraints are. Such a state is called the restorative operating state. Symbolically, equations (2.21) and (2.22) can be written in the following form:

$$\begin{aligned} g(x,u) &= 0 \\ c(x,u) &\le 0 \end{aligned} \tag{2.23}$$

With this notation, the definition of the three operating states can be summarized as follows.

Normal state

$$\begin{aligned} g(x,u) &= 0 \\ c(x,u) &\le 0 \end{aligned} \tag{2.24}$$

Emergency state

$$\begin{aligned} g(x,u) &= 0 \\ c(x,u) &= 0 \end{aligned} \tag{2.25}$$

Restorative state

$$\begin{aligned} g(x,u) &\ne 0 \\ c(x,u) &\le 0 \end{aligned} \tag{2.26}$$

Therefore, the security of any power system is defined relative to its ability to withstand a number of postulated disturbances. A normal state is said to be secure, following any one of the postulated disturbances, the system remains in the normal state. Otherwise, it is insecure.

Optimization problems arise naturally in many engineering applications. Control problems can be formulated as optimization problems in which the variables are inputs and states, and the constraints include the model equations for the plant. At successively higher levels, optimization can be used to determine set of points for optimal operations (power system state estimation), to design processes and plants, and to plan for future capacity. Hence, optimization problems contain the following key ingredients:

- Variables that can take on a range of values as well as variables that are real numbers, integers, or binary;
- Constraints that define allowable values or scopes for the variables, or that specify relationship between the variables;
- An objective function that measures the desirability of a given set of variables

The optimization problem is to choose from among all the variables that satisfy the constraints the set of values that either minimizes or maximizes the objective function. Therefore, it is apparent that power system state estimation consists of all the above mentioned key ingredients and can be formulated as an optimization problem.

Several mathematical optimization methods have been proposed over the years to solve the PSSE problem as has been described in the previous sub-sections in this chapter. Listed below are some of the mathematical methods used: Linear programming (LP), Simplex method, Non-linear programming (NLP), and interior point methods. These methods are briefly reviewed in the following subsections.

2.3.1 Linear Programming (LP) Method

Linear optimization in which the constraints and the objective function are linear functions of the variables is usually known as "linear programming". In LP problems, a linear function of real variables is minimized over a region defined by linear constraints. The problem can be expressed in a standard form as given by [Min et al, 2005; Alsac et al, 1990)

$$\begin{aligned} &\min c^T x \\ &s.t. \quad Ax = b \\ &\qquad\quad x \geq 0 \end{aligned} \tag{2.27}$$

where

x is a vector of "n_r" real numbers, while $\boldsymbol{A}x=b$ is a set of linear equality constraints, and $x \geq 0$ indicates that all components of x are required to be non-negative. The dual of problem (2.23) is

$$\begin{aligned} &\max b^T y \\ &s.t. \quad A^T y + s = c \\ &\qquad\; s \geq 0 \end{aligned} \tag{2.28}$$

where

$c \in \Re^n$ is a vector , which defines the objective function $b \in \Re^m$ defines the equality constraints, $\lambda \in \Re^m$ is a vector of Lagrange multipliers $s \in \Re^n$ is a vector of dual slack variable, and finally $\boldsymbol{A} \in \Re^{mxn}$. These two problems are intimately related, and algorithms typically solve both of them simultaneously.

An important question when LP problem is solved is: How optimality of the computed solution is verified? One way of proving optimality is to generate a lower bound on the objective value. By definition, a valid lower bound z_b must satisfy a condition:

$$c^T x \geq z, \forall x \in X \tag{2.29}$$

where

X is a set determined by the constraints in eqn (2.27) feasible region

If solution $x^* \in X$ has been generated such that

$$c^T x^* = z \tag{2.30}$$

x^* is optimal solution due to the fact that "z_b" is a measured; here it is a lower bound on the optimal objective value. A method to generate valid lower bounds is through the definition of the dual problem. To each LP problem, called primal problem, there is corresponding dual problem in the form [Capitanescu et al, 2006],

The pair (y, s) is a dual feasible solution, if it satisfies the constraints. Also, (x, y, s) is said to be primal and dual feasible if x and (y, s) is primal and dual feasible, is called duality gap, and $x^T s$ is called complementary gap. Some useful definitions in regard to primal-dual approach are given by Yan in [Yan et al, 1999]

Primal feasibility: $Ax^* = b, x^* \geq 0$ (2.31)

Dual feasibility: $A^T y^* + s^* = c, s^* \geq 0$ (2.32)

Optimality: $c^T x^* - b^T y^* = 0$ (2.33)

Thus, in practice the optimality of a computed solution can always be verified by checking primal and dual feasibility and that the dual gap is zero. In both, primal and dual cases, a set of first order optimality conditions to the barrier problem is obtained. Combining these two sets of optimality conditions gives (Yan et al, 1999],

$$\begin{aligned} &Ax = b, x > 0, \\ &A^T y + s = c, s > 0 \\ &xs = \rho e \end{aligned} \tag{2.34}$$

where

$e = [1, 1, 1 \ldots, 1]^T$

These conditions are called the perturbed Karush-Kuhn Tucker (KKT) conditions, because they are identical to the KKT conditions of the original LP problem, except the complementary conditions have been perturbed by ρ. Hence, a solution to (2.34) for a sufficiently small ρ is a good approximation to the optimal solution [Capitanescu et al, 2006]. When the vectors x^*, y^*, s^* satisfy the following optimality conditions as suggested by Min [Min et al, 2005]:

$$\begin{aligned} &A^T y^* + s^* = c \\ &Ax^* = b \\ &x^* \geq 0, s^* \geq 0 \\ &(x^*)^T s^* = 0 \end{aligned} \tag{2.35}$$

Then, x^* solves the primal problem and (y^*, s^*) solves the dual problem.

The popularity of LP as an optimization standard stems from its direct applicability to many interesting problems, the availability of good general purpose algorithms, and the fact that in many real-world situations, the inexactness in the model or data means that the use of a more sophisticated non-linear model is not warranted. In addition LP does not have multiple minima, as may be the case with non-convex optimization problems. i.e. any local solution of a linear program-one whose function value is no larger than any feasible point in its immediate vicinity-also achieves the global minimum of the objective function over the whole feasible region. LP based methods are fast and efficient in determining binding constraints, but have difficulty with marginal losses.

2.3.2 Simplex Methods

Prior to 1987, all commercial programs for solving general LP made use of a simplex algorithm. Simplex algorithm, invented in 1947 by Dantzig, had fascinated optimization researchers for many years because its performance on practical problems is usually far better than the theoretical worst case. The simplex algorithm generates a sequence of feasible iterates x^k fro primal problem, where each iterate typically has the same number of non-zero (strictly positive) components as there are rows in ***A*** matrix. This iterates are used to generate dual variables y and s such that two other optimality conditions are satisfied, namely

$$\begin{aligned} A^T y^K + s^K &= c \\ \left(x^K\right)^T s^K &= 0 \end{aligned} \tag{2.36}$$

If the remaining condition $s^k \geq 0$ is also satisfied, then the solution has been found and the algorithm terminates. Otherwise, one of the negative components of s^k is chosen to allow the corresponding components of x to increase from zero. To maintain feasibility of the equality constraint $\boldsymbol{A}x=b$, the components that were strictly positive in x^k will change. One of them will become zero when the new component is increased to a sufficiently large value. When this happens, it has to stop and denote the new iterate by x^{k+1}.

Each iteration of the simplex method is relatively inexpensive. It maintains a factorization of the sub matrix of ***A*** that corresponds to the strictly positive components of *x;* typically, simplex methods converge in a number of iterations that is about two to three times the number of column. Simplex method is widely used in commercial software to solve a great variety of problems (production planning, transportation, scheduling, etc) when the function to optimize and its associated set of constraints are linear. Abur and Celik employed the method in solving the SE problem and it was successful.

2.3.3 Non-Linear Programming (NLP)

Non-linear programming (NLP) problems are constrained optimization problems with non-linear objective function or constraint functions. However, it is still assumed that all functions in question are smooth (i.e. at least twice differentiable), and that the variables are all real numbers. If any of the variables are required to take on integer values, the problem is a mixed-integer non-linear programming problem. Mixed-integer non-linear programming is not considered in this thesis. For the purposes of description, the following problem formulation is used [Alsac et al, 1990; Mamoh et al, 1999; Torres et al, 1999; Wei et al, 1998]

$$\min f(x)$$
$$s.t. \quad g(x) = 0 \tag{2.37}$$
$$c(x) \leq 0$$

where

x is a vector of "n_R" real variables, f is a smooth real-valued function, and g and c are smooth functions with dimension m_E and m_L, respectively [Mamoh et al, 1999; Wei et al, 1998]. Algorithms for NLP problem are more varied than those for LP. The major approaches represented in commercial software packages are sequential quadratic programming (SQP), reduced gradient, sequential linearly constrained, and augmented Lagrangian methods [Alsac et al, 1990]. The later is known as method of multipliers. Extension and successful interior point (IP) approaches for LP to NLP problems is now the subject of intense ongoing research among optimization Gurus, but little production software of these approaches have been in the market.

Disadvantages of NLP are that most of the algorithms cannot guarantee convergence to the global minimum i.e. the value x^* that minimizes f over the entire feasible region. At best, they find a point that yields the smallest value of f over all points in some feasible neighbourhood of itself. An exception occurs in convex programming. The second disadvantage of NLP over LP is that general –purpose software is somewhat less effective because the non-linear paradigm encompasses such a wide range of problems with a great number of potential pathologies and eccentricities. Even to a minimum value x^*, algorithms may encounter difficulties because the solution may degenerate, in the sense that certain of the active constraints become dependent, or are only weakly active.

Algorithms for special cases of the NLP problem, such as problems in which all constraints are linear or the only constraints are bound on the components of x, tend to be more effective than algorithms for the general problem because they are more able to exploit the special properties [Wei et al, 1998]. Local solutions of the NLP can be characterized by a set of optimality conditions analogous to those explained for LP problem. After introduction of Lagrange multipliers λ and π for the constraints $c(x) \leq 0$ and $g(x) = 0$, respectively, the Lagrange function for the problem is written as:

$$L(x, \lambda, \pi) = f(x) + \lambda^T c(x) + \pi^T g(x) \tag{2.38}$$

The first –order optimality conditions (commonly known as the KKT conditions) are satisfied at a point x^* if there are multiplier vector λ^* and π^* such that:

$$\nabla_x L(x^*, \lambda^*, \pi^*) = 0$$
$$g(x^*) = 0$$
$$c(x^*) \le 0, \lambda^* \ge 0 \qquad (2.39)$$
$$(\lambda^*)^T c(x^*) = 0$$

where

∇ is the Jacobian matrix of the Lagrangian

The active constraints are those for which equality holds x^*. All the components of $g(x)$ are active by definition, while the active components of $c(x)$ are those for which $c_i(x^*) = 0$. When the constraint gradient satisfies certain regularity conditions at x^*, the KKT conditions are necessary for x^* to be a local minimizer of the NLP, but not sufficient. A second-order sufficient condition is that the Hessian of the Lagrangian, the matrix $\nabla_{xx}L(x^*, \lambda^*, \pi^*)$, has positive curvature along all directions that lie in the null space of the active constraint gradients, for some choice of multipliers λ^* and π^* that satisfy the KKT conditions. i.e., it is required

$$w^T \nabla_{xx} L(x^*, \lambda^*, \pi^*) w > 0 \qquad (2.40)$$

for all vectors w such that

$$\nabla g(x^*)^T w = 0 \qquad (2.41)$$

and

$$\nabla c_i(x^*)^T w = 0 \text{ for all indices } i \qquad (2.42)$$

NLP can be used in finding a solution of state estimation (SE) problems.

2.4 Interior Point Methods

2.4.1 Development of IPMS

For decades, the simplex method discussed in section 2.3.2 proposed by Dantzig in 1947 has been the most widely used algorithm for solving linear programming (LP) problems. However, due to its vertex following property; the solution time to Simplex method may grow exponentially for some specifically constructed problems. This behaviour motivated researchers to start developing a linear programming method with lower combinatorial complexity. In 1978, Khachiyan [Khachiyan, 1979] a Russian researcher first developed a polynomial algorithm by applying the ellipsoid method to linear programming problem. Although his method cannot compete with the simplex method practically, it indeed has significant theoretical implications for combinatorial optimization.

In 1984, Karmarkar [Karmarkar 1984] introduced his projective algorithm, which not only had a polynomial time property, but also was much faster than the Simplex method. His method is called Interior point, because it searches an optimal point through the interior of the feasible region. Since then, a substantial number of contributions have been made towards the theoretical analysis and practical implementations of interior point method and its many variants. These variants can be classified into four categories: projective methods, affine scaling methods, potential reduction methods and path following methods. The four variants are briefly discussed in the following sub-sections:

Projective Method

Projective method grows out of Karmarkar's projective method. The method is based on two fundamental ideas:

- If the current solution is near the centre of the feasible region, then it would automatically goes closer to the optimal solution by moving in the steepest descent direction of the objective function;
- The solution space can be transformed so as to place the current solution point near the centre of the transformed feasible region. Without changing the problem in any essential way, such transformation can be done by using an appropriate type of projective transformations.

By formulating a linear programming (LP) problem as a special standard form, Karmarkar assumed that its optimal value is well known. Karmarkar later relaxed his assumption by using a lower bound to estimate the optimal value, and updating the lower bound at each iteration. Todd and Burrell [Todd & Burrell, 1986] propose a method to obtain the lower bound from the dual problem. Karmarkar's algorithm required $O(nL)$ iterations, where n is the number of variables and L is the number of bits required to record the problem. Each iteration involves the calculation of projection step which, in turn needs $O(n^3)$ arithmetic operations. He proposed the idea of inexact projection that leads to an average reduction of $O(\sqrt{n})$ in the worst case bound, resulting in $O(n^{2.5})$ arithmetic operations per iteration. Gay [Gay, 1991] applied the same idea to the dual problem. Anstreicher [Anstreicher, 1989] developed a combined phase I and phase II projective algorithm in order to relax the initially feasibility assumption. All of the developed algorithms need to Karmarkar's standard form. Moreover, these algorithms require to estimate the lower bound of the optimal value and to do the non-linear projective transformation at every iteration.

Affine- Scaling Method

Affine Scaling methods were originally suggested by Dikin in 1967 [Dikin 1967], and later studied by several linear programming IPM developer. Barnes [Barnes, 1986] and Vanderbei et al [Vanderbei, 1986] proposed what is now known as primal affine-scaling method as a variant of Karmarkar's projective method. Adler et al [Adler et al., 1989] suggested applying the method to the dual problem, resulting in a dual affine scaling method. Both methods require Karmarkar's standard form and can work on the general linear programming problem. The methods use a linear transformation rather than the costly non-linear projective transformation. The global convergence of the affine- scaling methods has been proved by several researchers. Although there has been no evidence of polynomial complexity for the developed algorithms, the main disadvantage of the affine- scaling methods is that, they do not have the centering direction to keep variables far away from the boundary. A small step size must be imposed to avoid numerically instability. This phenomenon causes the algorithms to take more iterations as suggested by McShane [McShane, 1989]

Potential Reduction Method

The Potential Reduction method was first proposed by Todd and Ye [Todd & Ye 1992; 1990]. Their method adopts Karmarkar's idea of using an appropriate potential function to measure the progress of an algorithm but avoids applying expensive projective transformation at each iteration.

The method applies the idea of affine- scaling to reduce the potential function by searching along the projected gradient of the potential function. Thus, the potential reduction function method has the features of both projective methods and affine – scaling methods. In order to determine the optimal step size when working with the method, a linear search has to be carried out at every iteration, which can be costly in computation. The potential reduction methods was later studies by several prominent researchers including Ye [Ye, 1991], Freund [Freund, 1991], Gonzaga [Gonzaga, 1994], Anstreicher [Anstreicher, 1991] etc. For Reduction function method, the best complexity achieved so far is $O(\sqrt{n}\, L)$ iterations as proposed by Todd and Ye [Todd &Ye, 1990]. However, their computational performance relies on proper potential function as well as an efficient linear search algorithm

Logarithmic Barrier Method

Logarithmic barrier method is based on applying Newton's method to follow the central path of the feasible region. The central path is formed by the optimal solutions of a family of problems defined by a logarithmic barrier function. The logarithmic barrier function is attributed to Frisch [Frisch, 1955] and is studied in detail by Fiasco and McCormick [Fiasco& McCormick, 1968] for non-linear optimization. The idea of central trajectories was suggested by Karmarkar [Karmarkar, 1984] and has been extensively examined by Bayer and Lagarais [Bayer &Lagarais, 1989] and Megiddo and Shub [Megiddo & Shub, 1986]. Megiddo suggested applying the logarithmic barrier method to the primal and dual problems simultaneously. The idea was developed by Kojima et al [Kojima et al., 1989] into primal-dual path following algorithm which requires $O(nL)$ iterations. Later, Monteiro and Adler [Monteiro & Adler, 1989] improved Kojima et al' results by employing ideas proposed by Gonzaga [Gonzaga, 1987] and Karmarkar [Karmarkar, 1984] to get a primal-dual algorithm, which requires $O(\sqrt{n}\, L)$ iterations, the best worst-case complexity to date. Since then, several variants of the primal-dual path following method have been suggested and extensively studied by outstanding researchers including the primal-dual algorithm of McShane et al [McShane et al., 1989] and Lustig et al [Lustig et al., 1991], the Predictor –Corrector algorithm suggested by Mizuno et al [Mizuno et al., 1990] and Mehrotra [Mehrotra,1992] and the infeasible algorithms of Zhang [Zhang,1992], Mizuno [Mizuno,1992] and Vanderbei [Vanderbei,1993]

The typical features of the Logarithmic barrier function or Path following method comes from several aspects such as:

- Following the central path allows the algorithm to take a large step towards the optimal point (solution);
- Use of Newton direction produces quadratic convergence speed;
- Using different step lengths in primal and dual space results in fast convergence.

As conclusion: This class of method performs extremely well in practice, comparing favourably to other interior point methods discussed in previous sub-sections. The method not only has the best polynomial complexity but is computationally efficient.

2.4.2 IPDPF Method

The infeasible primal-dual interior point path following (IPDPF) method is based on one-phase primal-dual path following [Vanderbei, R.J., 1993]. The original method operates on LP problems that have only upper bounds. The method can be extended to handle a general LP problem with lower (l) and upper (u) bounds.

The basic idea of the method is to solve a constrained optimization problem as a sequence of unconstrained problems. Its theoretical foundation is based on 3 methods [Wu et al, 1994]: Logarithmic barrier method for optimization with inequalities, Lagrange method for optimization with equalities and Newton-Raphson method for solving the non-linear equations of the Karush-Kuhn-Tucker (KKT). The method can be derived by introducing slack variables to convert the bound constraints into equality constraints.

The logarithmic barrier method is applied to eliminate the inequality constraints by incorporating them into a logarithmic barrier term that is appended to the objective function, in this berth the original problem is converted to a sequence of problems parameterized by the barrier parameter (μ) and the parameter is reduced towards zero as the iteration progress.

IPDPF does not solve the non-linear KKT equations exactly. Rather, it applies the one-step Newton-Raphson method to find the search direction. To summarize IPDPF can work in the following succession:

- Start with any initial point satisfying the slack variables which must be greater than zero and barrier parameter $\mu > 0$
- Apply one Newton-Raphson method to find a point closer to the central path and accept this value
- Reduce μ appropriately and start over again until the primal and dual feasibility is attained and the duality gap is smaller than a predefined tolerance.

Advantages shown by the method include:

- Does not require feasible point during the optimization process
- It allows separate step-lengths in the primal and dual places this increases the efficiency of the method and significantly reduce the number of iterations to convergence [McShane et al, 1989]
- It uses the complementary option [Lustig et al, 1994; Vanderbei, R.J., 1993] in determining the barrier parameter μ. This option is a abetter choice because it is related to the barrier parameter μ
- No initial feasible point is required. However, the primal and dual slack variables must be strictly positive.
- The algorithm developed using the method terminates when the feasibility and optimality conditions are satisfied [Lustig et al, 1991]

From its good computation features, the method has been chosen for the solution of power system state estimation problem in this thesis.

In the past 15 years, several efficient interior point methods have been suggested and applied. Lustig and Shanno [Luistig & Shanno, 1994], Zhang [Zhang, 1992] have made significant contribution in this area. Although simplex method has improved a lot in recent years [Bixby, 1995], [Forrest & Godfarb, 1992] and [Suhl, 1990] extensive tests have demonstrated conclusively that robust IPM can solve large-scale LP problems substantially faster the simplex method.

The important note when using interior point method is how to terminate an IPM algorithm. Contrary to the simplex algorithm, IPM algorithm never generates the exact optimal solution; instead it generates an infinite sequence converging towards an optimal solution. Hence, it is necessary to be able to terminate an IPM after a finite number of iterations and then report the exact optimal solution. However, this problem is sufficiently and succinctly described by Ye's finite termination scheme [Ye, 1992]. A closely related problem is to generate an optimal basic solution from an optimal interior point solution.

Interior point methods at present are very reliable optimization tool. They are, and will continue to be used in solving optimization problems such as optimal power flow, and power system state estimation just to mention few, problems in power system. Indeed a complete theory of interior point methods has been developed. Moreover, basing on this theory many efficient implementations have and will be developed. In fact, this trend of algorithm and improvement in computer hardware make many linear programming problems, to be solved routinely today and in the future than a decade or two decades ago. Even though interior point methods are not going to improve dramatically over the next decade, but it could be predicted that significant improvements in the current implementation particularly in solving power system state estimation problem of large-scale and interconnected system will be achieved.

2.5 Hierarchical State Estimation

Use of hierarchical and distributed state estimation structure in processing state estimation is not new. Dy-Liacco [Dy-Liacco, 1994] demonstrated a control centre in which hierarchical structure is employed. In the demonstrated structure, each area has its own dispatch centre and a central dispatch centre coordinated the integrated processing. Hierarchical concept has been used in processing large-scale optimal power flow (OPF) studies, [Cohen and Zhu, 1984]. It has also been applied to WLS state estimation problem [Kobayashi et al, 1974; Van Cutsem et al, 1981; Mukai et al, 1982]. The objective of all these work was mainly to reduce computing time, memory requirements and data exchange between areas. Basing on different decomposition schemes or special requirements on accuracy, information exchange and so on, there are many different hierarchical processing schemes for PSSE problem. Some of these schemes are briefly reviewed

Method by Kobayashi et al [Kobayashi et al, 1974], the method proposes decomposition of system network into several non-overlapping areas connected by tie-lines. Each tie-line only belongs to one or two neighbouring areas. Two level state estimation processing is created. The first level state estimation is conventional. The second level estimation has two steps in its implementation. First step estimates the boundary bus status; using the results from the first level estimation and first step, and then the second step estimates the global state. However, the injection measurements on the boundary buses are not considered in the first level, but they can be included at the first step of second level. There is no communication between the established levels. The proposed method is complex and difficult to implement.

Method by Van Cutsem et al [Van Cutsem et al, 1981], it proposes system network to be decomposed into several non-overlapping areas connected by tie-lines. The tie-lines do not belong to any area, thus the flow measurements on tie-lines are not included in the first level state estimation. The first level estimation is conventional one. The boundary bus values are sent to the second level state estimation. These boundary states are used as pseudomeasurements together with the tie-line flow measurements. Because of the nature of decomposition scheme, the injection measurements on the boundary buses are not used. The schemes do not allow communication between levels.

Method by Mukai et al [Mukai et al, 1982], it suggested decomposition of large-scale optimization problem, i.e. SE problem into subproblems, solve the problems in parallel computers and coordinate the result with the central computer (coordinator). A communication exists between the areas and the central computer on convergence status. The suggested method gives global optimal solution, and has fast computing speed. But, observability analysis and bad data detection and identification are difficult because the decomposition is based on the structure of the gain matrix.

Interest on distributed processing of power system state estimation attracted engineers and researchers. Kores [Kores & Contaxis, 2000] proposed a distributed state estimator that is based on two level state estimation; distributed observability analysis, and distributed bad data processing. The second-level estimation is accomplished using a reduced model. Iwamoto et al [Iwamoto et al, 1989] developed hierarchical state estimation method using a fast rectangular-coordinated approach. Two levels are suggested. The upper level, where the optimal tie-line bus voltages are evaluated, and the lower level, the optimal state of each subsystem is determined. The proposed method makes use of an extension of a fast second-order load flow technique that makes possible to employ a fixed Jacobian matrix in the hierarchical algorithm.

Zhao [Zhao & Abur, 2003] examined the hierarchical state estimation on large-scale power networks using existing local estimators along with synchronized phasor measurements. Lakshaminarasimhan [Lakshaminarasimhan and Girgis, 2006] proposed the use of hierarchical state estimation to coordinate the state outputs of wide area network. He addressed issues of incoherency and delay/absence of state output from one or more areas. Patel [Patel and Girgis, 2006] implemented hierarchical state estimation on an 1896 bus power system.

Lin [Lin, S.Y., 1992] developed a robust and computationally efficient distributed state estimator to solve the weighted least square (WLS) problem using distributed computation. The developed algorithm offers four advantages over other estimators: reduction of the time skew problem, freedom from power network topological errors, easy identification of unobservable states, and detection and identification of bad data in the measurements. El-Keib et al [el-Keib et al, 1992] developed a linear programming (LP) state estimation algorithm for large-scale interconnected power system. The algorithm uses the Dantzig-Wolfe decomposition. By using a multiple-column strategy and several modifications, to the original algorithm, the performance of the new algorithm is better compared to the original.

Ebrahimian [Ebrahimian & Baldick, 2000] presented an application of distributed algorithm to solve the power system state estimation, employing the auxiliary problem principle (APP) concept. The algorithm requires minimum modification to existing WLS state estimator. Aguado et al [Aguado et al, 2001] employed decomposition method in solving the SE problem in a distributed way. He presented and compared two decomposition methods based on Lagrangian Relaxation and Augmented Lagrangian algorithms. In addition, he showed that using decomposition methods reduces discrepancies among variables estimated at boundary nodes by different control areas.

The main incentives of moving from centralized state estimation towards distributed and hierarchical approaches are: First, local estimation in areas eliminates the necessity for communication between areas and the central dispatch centre (CDC). Hence, communication bandwidth is reduced. Second, local estimators, in hierarchical and distributed estimation deal with much less dimensional data compared to the centralized one. Consequently, computational power requirement decreases with the use of hierarchical and distributed computation approaches.

2.6 Comparative Analysis of SE Methods

In light of the review presented in this chapter, it can be noted that despite remarkable advances in mathematical optimization methods, numerical methods have yet to achieve fast and reliable real-time application in state estimation optimization. Considerable efforts are required to avoid mathematical traps such as ill-conditioning and convergence difficulties. Mathematical methods often use approximation to limit complexity leading them to lose desirable performance of the problem. Comparative analysis of the state estimation methods are provided in Table 2.3.

It can be pointed out that:

- The complexity and the size of the problem may affect the solution time dramatically when applying optimization method. Therefore, the use of mathematical optimization techniques is constrained when the problem is both large and complex;
- Mathematical optimization methods often require relatively long times to respond. Therefore, a rather high degree of predictability is required for optimization method to work efficiently if a short response time is needed.
- In mathematical optimization, it is assumed that computations and communication will occur as planned and strategies for handling lack of communication are not well known, therefore, mathematical optimization is not robust with respect to failures in computation and communication.

In order to address these difficulties attention is directed towards:

- Robust, adaptive, high-bandwidth control algorithm large-scale structurally decentralized system;
- Faster computation capability to approach real-time optimization, coordination and control;
- Proven robust hierarchical control algorithms in order to overcome the limitations in achieving wide-area integrated centralized controller performance; and
- Parallel and distributed algorithms for faster computing.

Table 2.3: Comparative analysis of state estimation methods

Method/Algorithm	Advantages	Disadvantages	Challenges
Normal Equation (NE-WLS)	High computational efficiency with the fast P-Q decoupling	Give rise to ill-conditioning problems	To deal will ill-conditioning problem
Normal Equation with constraints (NE/C-WLS)	Better numerical stability compared to NE without equality constraints	More sophisticate in application due to triangular factorization	Improve triangular factorization
Orthogonal Transformation(OT-WLS)	Has higher numerical stability	Needs more computation Needs the P-Q decoupling technique	Improve the P-Q decoupling technique
Hybrid Method (HM-WLS)	Compromise between numerical stability and computation efficiency	Not stable and efficient for large-scale power system	Improve stability and computation efficiency.
Hactel's Augmented Matrix (HAM-WLS)	More stable and efficient for large-scale power systems	Complicated in implementation	Improve implementation in large-scale power systems
Least Absolute value (LAV-WLAV)	Does not follow statistical distribution arrangement(Gaussian distribution assumption)	Considers only part "*n*" of measurements, does not work on the rest "*m-n*" measurements	To deal with "outliers" bad data
Simplex Method (LP)	Computationally more efficient especially for large and sparse problems	Has difficulties on: ▪ Initialization ▪ Iteration ▪ Terminating of algorithm	Improve on ▪ Initialization of algorithm ▪ Iteration ▪ Terminating of algorithm
Interior Point Methods (IPMs-LP/NLP)	Computationally efficient	Difficulties on: ▪ Problem formulation ▪ Algorithm ▪ Search direction	Improve on ▪ Problem formulation ▪ Algorithm ▪ Search direction

2.7 Conclusion

In this chapter, state estimation (SE) methods have briefly reviewed. Interior point method (IPM) is discussed in detail. The hierarchical and distributed estimation processing is briefly reviewed, as well as their advantages and disadvantages are mentioned and compared.

In deregulated power system, which may comprises of several areas, when scheduling power transactions, a system-wide SE solution is needed. Such a solution may be obtained by central state estimator, which will collect the measurements of the entire system and solve the very large state estimation problem. The computation will be heavy and challenging. Another challenge is that the area dispatch centres existing in the system are reluctant to share network application data among them because of business and competition confidentiality purposes. Thus, a good compromise would be for individual area dispatch centre (ADC) to run for its own state estimation and to have a central coordinator to coordinate their results to determine the state of the entire system.

In such a case, it is necessary to have a new scheme to solve the multi-area SE problem. This thesis works for designing of a new scheme of multi-area distributed processing of SE problem.

Chapter 3 presents review on deregulation of power systems, motivation and process of deregulation of power system around the world; its advantages and disadvantages are presented. The chapter also highlights the deregulation process going on the Tanzania Power system.

CHAPTER THREE: DEREGULATION OF POWER SYSTEMS

3.1 Introduction

Since 1990s, electrical power industries around the world have been experiencing a period of deregulation in which new structures in the industry are being formed. The deregulation programmes have included privatization of state owned enterprises, separation of potentially competitive segments i.e. generation and distribution from natural monopoly segments of distribution, and transmission, creation of competitive wholesale and retail markets, and application of performance based regulatory mechanisms to the remaining regulated segments.

Deregulation in the USA, for example, has led to establishment of independent power producers (IPPs), which are responsible for power supply in a deregulated power system, regional transmission providers (RTOs), responsible for transmitting and wheeling bulk electrical energy across the grid, and bulk electrical energy consumers (DISTCOs), which are responsible for buying bulk electrical energy from a power market place and distribute to end-users. The deregulation process differs from one place or from one country to another. In the developed countries, the three structures are in place and operating, while in developing countries initial stages of deregulation i.e. permission of power producers to operate is taking place.

Electrical power sector around the world evolved with primary regulated geographic monopolies that were either state owned or private owned. The primary components of electricity supply i.e. generation, transmission, distribution, and retail supply were integrated within individual electrical utilities. These firms in turn had de-facto exclusive franchises to supply electricity to industrial, commercial, and domestic retail end-users within a defined geographical area or a country. The performance of these regulated monopolies varied widely across countries.

In many developing countries, electrical power sector was and is still in great part characterized by poor service quality, low labour productivity, high power system losses, inadequate investment in power supply facilities, unavailability of service to large portion of the population, and low tariffs to cover costs and support new investment [World Bank, 1994]. Industrial customers such as mining companies had to respond to frequent system outages by building their own isolated generating plants, increasing their cost of doing business. However, the sector performance in developed countries before the 1990s was generally much better compared to developing countries, but high operating costs, construction cost overruns on new facilities, and high retail prices required to cover these costs stimulated pressures for changes that would reduce costs in retail [Joskow, 1998a; Joskow, 2000].

The deregulation goal has been to create new governance arrangements for the electrical power sector that provide long-term benefits to end-users. The benefits are to be realised by relying on competitive wholesale markets for electrical power to provide better incentives for controlling construction and operating costs of new and existing generating capacity, to encourage innovation in power supply technologies, monitoring, control, and to shift the risk of operating mistakes to suppliers and away from end-users. Retail competition or end-user choice is supposed to allow end-user to choose the retail power supplier offering the price or service quality combination that best meet their needs.

In this chapter, deregulation of power systems, its motivation, and review of electrical power sector deregulation processes, deregulated power system structures, deregulation of Tanzania power sector, and the role of power system state estimation in deregulated power sector are considered.

3.2 Motivation for Deregulation

Traditionally, the electricity industry was and is in great part still a monopoly industry with a vertical organization structure. In a vertically integrated environment, utility companies were and are still responsible for generation, transmission and distribution of electricity in a given location, country or geographical area. Such companies are state owned. In Tanzania, the utility company-Tanzania Electric Supply Company Limited (TANESCO) is state owned. But in the last two decades, and especially during the 1990s, the power sector has been undergoing a drastic restructuring all over the World. The old and monopolistic electricity markets are being replaced with deregulated electricity market open to competition. Different forces have driven the electricity market towards deregulation. Not all forces that are behind the restructuring are the same in all countries all over the World. Furthermore, each country has its own reason for restructuring. The reasons are not the same so, deregulation differ from one country to another. However, there are two common reasons which are the same to all countries. These reasons are categorised as technological and economic. Technological development of High voltage (HV) networks during the 1960s and 1970s contributed to possible transmission of bulk electricity over long distances. This is one of the necessary conditions in order the electricity market to be opened to producers that are located far from the main customers [Stoft, 3003] but power is transmitted easily to customers to satisfy their needs.

Improvement of power generation technologies; it is now possible for generation expansion decision to be done by small companies [Alba Rios, 3000, pp.6-18]. The expansion of gas network is an additional reason that makes investments in gas (natural gas) power plant easily realisable. Another mixed technical-ecological cause is the inclination of modern society for an increase in power produced by renewable sources. The emerging of IPPs who operate, mostly, wind power units (in Western Europe and the USA) gives a further competitive character to electricity industry despite the fact that the IPPs survive due to subsidies from the state. Improvement of transmission technologies has resulted in an efficient grid operation by transmission companies. (Refer Regional Transmission Organization (RTO) in the USA). Devices such as flexible alternating current transmission (FACT) now in operation in transmission lines enable a better control over electrical feature of many of the grid in developed and developing countries. Thus, the separation of generation and transmission decision is easier.

Beyond the technical improvement that has been taken place from the 1960s to 1990s, a set of economical reasons may be considered as the main force behind the electric industry restructuring. The main economic idea behind restructuring is that a well operated competitive market can guarantee both cost minimization and average electricity price hold at minimum i.e. affordable tariff. Economists believe that an open market provides stronger incentive to supplier of electricity in order to apply cost minimizing procedures than normally regulated market. In addition, the positive characteristic of a competitive market is its ability to drive the price, i.e. tariffs towards the marginal cost. Of course, in order for this advantage to be relialised, the market has to be well designed and organized. Another economic reason is the inability of the countries with high national debt, especially the developing countries to meet the necessary investment in state owned utility companies [Wolak & Patrick, 1999]. The only solution for these countries is to restructure their electric industry. Through the restructuring process, these countries achieve two objectives:

- Free up public funds and make them available to serve the national debt or other demands;
- The state collects an essential amount from the state owned utilities.

Furthermore, besides technical and economic reasons, pressure of some multilateral organisations such as the World Bank and international monetary fund (IMF) sets as a requirement opening of markets including the electricity industry in order to support a country financially. Consequently, the electricity industry of many developing countries financed by the World Bank opened up their electricity industries for competition.

3.3 Power Sector Deregulation: An overview

The first deregulated electricity market world-wide was that of Chile. Chile introduced electricity competition in 1983 by giving the right to large end users to choose their supplier and negotiate the price. Chile realised later explicit market mechanisms in order to determine the generator's dispatch and the wholesale electricity price [Osana, 2003]. Thus, competition among the producers started. The experiment with Chile's deregulation was successful and so, Argentina in 1993 opened its electricity market into competition followed by Peru in 1993, Bolivia and Colombia in 1994 and other countries of Central America in 1997. Deregulation of electricity in Latin America has led to an essential improvement of electricity industry and power sector as a whole. The current trend in this area is now the development of electricity market covering large part of Latin America beyond the first countries to implement deregulation process. [Rudnick & Zolezzi, 2001]

Electricity deregulation in Oceania has also a long history since 1987, when the government of New Zealand began the process of deregulation by setting up the electricity cooperation of New Zealand (ECNZ), later on, in 1988, the system operator, Transpower, was set up by ECNZ. Performance of the set up Institutions has brought the New Zealand electricity market among the most successful paradigms of electricity deregulation. The most present issue on deregulation of electricity industry in New Zealand is the introduction of the financial transmission rights as a tool for hedging transmission congestion cost giving incentives for grid expansion investments [Girdwood, 2003]

In Asia, the deregulation of electricity industry has undergone several changes. The most economic powerful country in the region, Japan, started electricity deregulation in 1995. The introduction of competition in power sector was achieved by promoting the entry of Independent Power Producers (IPPs) into the wholesale electricity market [Asano & Okada, 1998]. These IPPs were eligible to bid only in service areas outside from the area where they are located. In February 2003, the Electric Industry Committee (EIC) issued a directive demanding the establishment of electricity or Power Exchange (PE). In this case, consumers with demand more than 50KW were able to choose their suppliers since April 2005 [Power line, 2003]

In China, electric industry deregulation started in mid 1980s. Private generation i.e. IPPs has been allowed since then. In 2003, all state owned utility companies were transformed into commercial companies. From 2001 to 3005, the World Bank supported financially the government of China a five year plan to deregulate the electricity industry [East Asia and Pacific Regional office, February 2003, World]

In India, some states have launched electricity industry deregulation in mid 1980s. The objectives of deregulation in India are different from the rest of the World. In India, the first objective of deregulation targeted to make electricity accessible for each household [Monari, October 2003], the second objective is reduction of electricity supply cost and increase electricity service quality. The increase of electricity service quality could be achieved through improvement of operational efficiency and good governance of electricity industry.

In Europe, England started the procedure of electricity deregulation in 1989. The same year, the parliament adopted the electricity act inaugurating a sweeping deregulation and privatisation of power sector. In 1990, the new electricity industry act comes to being. Its operation mode was mandatory pool market. In 1994, consumers with demand more than 100MW become eligible to participate to the electricity market. In 1998, the deregulation degree in England reached 100 per cent by including all segment of electricity market. In 3001, New Electricity Trading Arrangement (NETA) was adopted. The new market emerged after adoption of NETA, tries to trade electricity like any other commercial commodity [Hesmondhalgh, 2003]. The second country in Europe to embark on electricity market deregulation after England was Norway. The beginning of electricity deregulation in Norway was in 1990 by adopting the Energy Act. In 1995, Swedish electricity industry was also reformed and together with Norwegian electricity market established the Nord Pool which was launched in early 1996. [Flatabo Doorman, Grande, Randen & Wangensteen May 2003]. Finland becomes a member of Nord Pool in 1998, followed by West Denmark in 1999 and finally the East Denmark in 2000. The overall performance of Nord Pool brings it to among the most successful paradigms of electricity industry deregulation.

In European Union (EU), with the exception of the United Kingdom (UK), deregulation has been launched since 1996 by the adoption of electricity directive 96/93/EC [Council and European Parliament Directive 96/93/eC, 1997]. The directive resulted from many years of negotiation between the member countries. However, the directive does not define a common guideline for the deregulation. Therefore, deregulation process in EU has followed different paths between the member countries.

In the USA, the first step towards the opening of electricity market was the adoption of Public Utility Regulatory Policies Act by the USA government in 1978. The Act ordered the utilities to purchase power from particular IPPs. In 1993, the Energy Policy Act provided the power to Federal Energy Regulatory Commission (FERC) so as to require from the utilities transmission service for the whole sale customers. The real phase of electricity deregulation in USA started in 1996. In this year, FERC issued two directives of "Promoting whosale competition through open Access Non-discriminating. Transmission Service by Public Utilities" the directive demanded all public utilities to provide non-discriminatory open access transmission service [FERC 1996]. The second directive instructed the utility companies to develop an internet-based system, which could enable the exchange of information about the transfer capacity of transmission lines. The electricity deregulation in USA has followed different paths in the numerous USA states, which have their own separate electricity market.

In Africa, some countries have begun electricity deregulation process in 1990s. The countries of Northern Africa, participated together with member of EU countries to build up the Mediterranean electric ring, an electrical network around all countries of Mediterranean region. In 1999, the government of Nigeria adopted a comprehensive privatisation programme that was completed in 3004. Deregulation of state owned electricity industry was among the issues for implementation in the third phase of privatisation programme. [Africa Region of the Word Bank May 2001]

In South Africa, the regulated electricity supply is performing well. However, the government has planned deregulation of power sector in the future.

3.4 Deregulated Power System Structures

In regulated power system, generation, transmission and distribution were typically, owned, controlled and operated as single entity, the vertically integrated utility abbreviated as (VIU). The central operator will dispatch the system having the full knowledge of operational costs and constraints of the system. Thus, in this structure, the use of transmission line by other entities was relatively limited. Even now, in many developing countries, transmission line is in great part owned by the government through utility companies, so congestion management was not a term that was used. Congestion management (CM) is the responsibility of the entity that operates and controls the interconnected system [Bompard et al, 3003].

Congestion management in the regulated system was taken as a problem and formulated as the optimization of some objective function subject to satisfying the various constraints considered. CM was included in the optimal power flow (OPF) problem for its solution. The security constrained optimal power flow (SCOPF) was used to explicitly consider contingency conditions. Hence, in deregulated power system CM is a key component and has to be considered and taken carefully, therefore a body has to be established to deal with it because it is important in market set up.

The deregulation of electricity industry in developed countries has spawned the introduction of new independent power producers (IPPs) responsible for the power supply; independent grid operators (IGO); typically called transmission system operators or organizer (TSO); independent system operators (ISO); or regional transmission organization (RTOs), which are responsible for transmitting and wheeling the bulk electrical power across power grids of deregulated system; and distribution companies (DISTCO) which buy bulk electrical power from a power marketplace and distribute it to power customers. An important task of TSO is congestion management and pricing of electrical power. These organizations are described in brief as follows:

IPPs: An IPP may have several power plants that are geographically distributed in power system. Permission to commission a generating plant is granted by a regulatory authority after satisfying the set requirement

RTO: The objective of establishing RTOs is to overcome pancaked transmission tariffs and to provide transmission customers with better and non-discriminatory service. RTO creation signifies unification of independent system operator (ISOs). The general objectives of RTOs are:

- To provide open access to transmission system and all services under its control at non-pancaked rates pursuant to a single, unbundled grid-wide tariffs that applies to all eligible users in a non-discriminatory manner.
- To have the primary responsibility in ensuring short-term reliability of grid operations.
- To have control over the operation of interconnected facilities.
- To identify constraints on the transmission system and be able to take operational to relieve those constraints within the trading rules established by the regulatory body.
- To establish appropriate incentive for efficient management and administration.
- To have ancillary service pricing policies that should promote the efficient use and investment in generation, transmission and distribution.
- To make transmission information publicly available on a timely basis via an electronic information network consistent with regulatory body requirement.

- To develop mechanisms to coordinate with neighbouring RTOs
- To establish an alternative dispute resolution (ADR) process to resolve disputes [Hogan, 1998]

DISTCO: Unlike distribution utilities in regulated power systems, which were responsible for the operation, reliability of distribution network; DISTCO in deregulated power systems may not be responsible for monitoring and control of distribution system. In some countries, municipal and city councils act as distribution companies and they buy bulk electrical power and distribute to their customers.

A common need in deregulated power systems, is that of operating transmission system independently of various market players [FCR, 1999; Singh & Papalexopoloulos, 1998]

Deregulated power system structure is shown in Figure 3.1

3.5 State Estimation in Deregulated Power Systems

The aim of state estimation (SE) as has been described in chapter two is to provide a reliable, accurate and complete set of data for real-time monitoring and control of power systems. By computing available measurements at the central dispatch centre (CDC) collected by SCADA, together with the knowledge of the network topology and transmission line model parameters, SE can obtain an accurate estimate of the state variables, which include complex bus voltages. Given an accurate estimate of complex bus voltages, exact knowledge on network topology, and precise values of the transmission line parameters, accurate values for all system quantities such as real and reactive power injections, real and reactive power flows can be computed via power flow equations and Kirchhoff's laws.

Although SE resembles a generalized load flow process, the implementation of SE is much more complex than that of load flow. Before SE can be computed, some preliminary functions such as measurement filtering, network topology processing, observability analysis, and bad data detection and identification have to be performed in order to implement SE.

State estimation is a key and powerful tool for power system operators to monitoring and controlling power systems. It is a core function of energy management systems (EMS) at the central dispatch centres (CDCs). The performance and optimization of modern power system depends heavily on the performance of the estimator that the EMS utilizes. SE is mostly computed in a serial manner at the CDC and therefore it may be sometimes difficult in meeting the time requirements for on line power system monitoring and control when a large amount of measurements are required to be processed within a limited time. Hence, a chance of improving or designing effective, robust and fast method is inevitable.

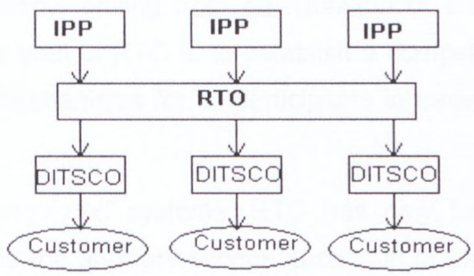


Figure 3.1: Deregulated Power System Structure

In regulated power system, CDC perform monitoring and control of the system through SCADA/EMS .SCADA is responsible for remote measurements and control, while EMS represents a set of senior online applications software for system optimization and control. SE is a key component of EMS. It provides accurate database for the following applications [Shahidehpour & Wang, 2003]

- Optimal power flow (OPF)
- Load forecasting (LF)
- Economic dispatch (ED)
- Stability analysis (SA)
- Security assessment
- Restoration strategy

Figure 3.2 shows specific functions of SCADA/EMS system and the role of SE in regulated power system.

Capital letters in Figure 3.2 denote:

A: On-line Load-Flow

B: Optimal Load-Flow

C: Load Forecasting

D: Economic dispatch

E: Stability analysis

F: Security Assessment

G: Restoration of Power Strategies

There are independent power producers (IPPs), regional transmission organization (RTO) and distribution companies (DISTCO) in deregulated power systems. In this new structure, the centralized optimization and control of CDC cannot in any way meet the new functions and requirements arising from deregulation of the systems. CDCs are replaced by RTO/ISOs. The goal of RTO is to establish a competitive environment and non-discriminatory access mechanisms for all participants in trading of electrical energy as explained in section 3.4

Compared with CDCs of regulated systems, RTO has new functions, including the congestion management and the ancillary service auction in order to meet deregulation requirements. The typical EMS monitoring and control functions of RTO in deregulated system are shown in Figure 3.3 [Shahidehpour & Wang, 2003]. SE is a key component of EMS. It provides accurate database for the following applications:

- Congestion management (CM)
- Load forecasting (LF)
- Ancillary service auction (ASA)
- Security assessment (SA)
- Stability analysis
- Restoration strategy

Optimal power flow process is included in the congestion management under security constrained optimal power flow of the transmission lines. Hence, PSSE is an important component in both EMS of regulated and deregulated power systems. Its function is even more important in deregulated systems; it determines the sustainability and competitiveness of deregulated power system's market.

Capital letters in Figure 3.3 denote:

A: On line Load Flow

B: Congestion Management

C: Load Forecasting

D: Auxiliary Service Auction

E: Stability Analysis

F: Security Assessment

G: Restoration of Power Strategies

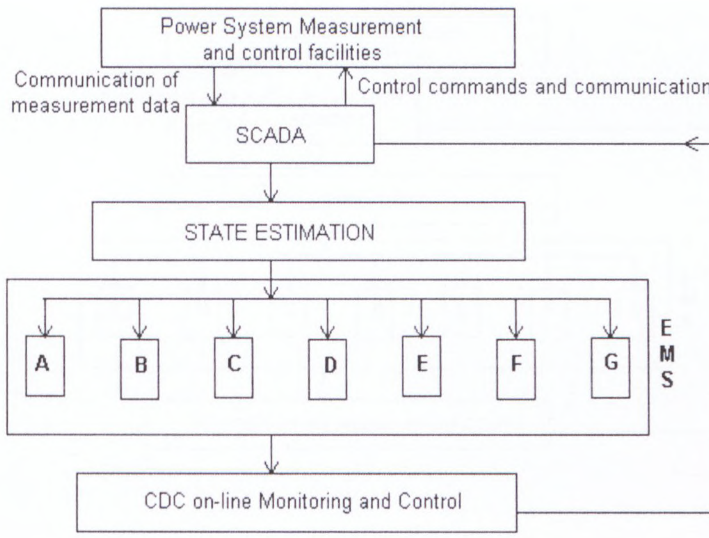


Figure 3.2: Specific functions of SCADA/EMS under regulated systems

3.6 Deregulation of Tanzania Power Sector

3.6.1 Tanzania Power Sector Status

Overall governance of the electricity industry is the responsibility of the Ministry of Energy and Minerals (MEM). MEM's role in the energy sector is restricted to the development of policy, institutional setting, legislative and regulatory framework. Energy, Water Utilities Regulatory Authority (EWURA) is a regulatory council responsible for licensing, tariff regulation and quality of service regulation and monitoring of electricity, water, petroleum and natural gas sector. EWURA is instrumental in protecting interests of the users with regards to the supply of regulated goods and service

.The electricity industry is dominated by Tanzania Electric Supply Company Limited (TANESCO), a government –owned vertically integrated electricity company operating in the mainland Tanzania is also responsible for most small isolated networks that are not interconnected to the national transmission system. Other players on the mainland include Independent Power Tanzania Limited (IPTL), Songas Limited, and ALSTOM in Mwanza as IPPs with off-take contracts with TANESCO. Electricity supply in Zanzibar is the sole responsibility of the Zanzibar State Fuels and Power Corporation (ZSFPC), a vertically integrated monopoly company. The government of Zanzibar has jurisdiction over energy policy in Zanzibar, and is independent of policy developments on the mainland.

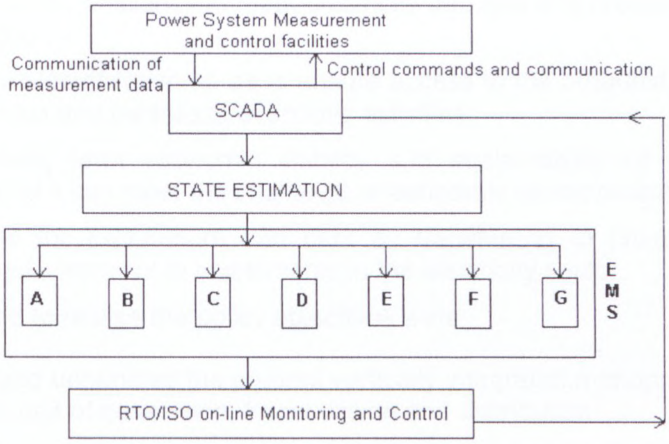


Figure 3.3: Specific EMS functions under deregulated systems

There is a considerable degree of installed capacity in off-grid power supply and auto-generation (mainly for mining companies), primarily due to TANESCO not having the necessary financial resources to extend the national transmission grid. The electricity supply industry (ESI) structure of Tanzania is shown in Figure 3.4

The initiation of electrical power sector reform in Tanzania was catalysed by a combination of macro-reform [www.gsb.uc.ac.za/mir. 1996] priorities, national energy policy, electric power sector conditions and international donor priorities. In 1993, the government of Tanzania expanded macro-economic reforms started under structural adjustment programmes in the mid-1980s to include sector focused objectives such as electric power sector [Wangwe et al, 1998]. In 1993, the first national energy policy was published, which included intensions to involve the private sector in development of energy sector [MEM, 1993].

In the same year, facing a drought induced electricity crisis and extensive load shedding, the Tanzania government lifted the state utility's monopoly on generation to attract private generation and alleviate shortages, which paved away to the country's two Independent Power Producer (IPP) projects. In addition, the reform imperative was reinforced by changes in World bank lending policy, as the World Bank made electric power sector reforms a condition for electric sector lending in 1993 [World Bank, 1993]

In October 1999, the government approved a new electricity industry policy and restructuring framework. [htt://www.prsctz.com, 1999]. The main objectives of the policy are:

- To increase sector efficiency to meet electricity demand and provide for sufficient reserve margin;
- To accelerate electrification so as to ensure access to the broadest cross-section of the population and centres of economic activities;
- To ensure long term economic viability and sustainability of the electricity industry, so that it can meet the challenge of economic development; and
- To reduce sector expenditure and debt by transferring to private capital the commercial risks inherent in investments in the electricity sector.

The strategies adopted to realise the policy objectives were:

- Restructure and unbundled the present vertically integrated monopoly utility along the functional unit of generation, transmission and distribution
- Introduce competition in the sector where applicable while seeking to safeguard stakeholders and consumers interest through regulation, and
- Review the electricity ordinance and enact new industry legislation to capture structural changes in the electricity sector and provide for private sector public electricity supplies.

The driving model of Tanzania's electric power sector reform was originally aimed at restructuring and unbundling the electric sector for eventual privatization. At the time of initial reforms, TANESCO, the national utility company, was already corporatized; the firm was operating under Tanzania's Company Ordinance act since 1931. During the 1970's to mid-1980' period, the company performed adequately, yet towards the end of 1980's the company's performance gradually declined [Katyega, 2004]. Despite its corporatized status, from the early 1990's the firm recorded poor technical and financial performance, making the status quo operation increasingly untenable.

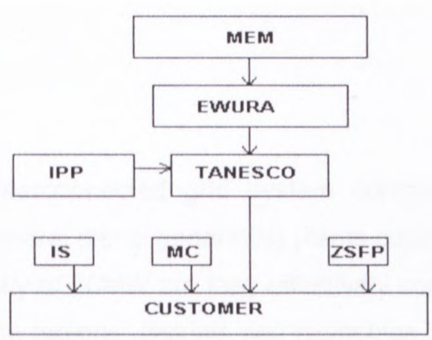


Figure 3.4: Electricity Supply Industry (ESI) Structure: Tanzania

TANESCO was unable to cover its operation and maintenance cost and debit service repayments from its revenue collection which fell during the 1990's. With diminished revenue for maintenance, outages and distribution losses increased during the same period [Katyega, 2004]. Efforts were made to commercialise and improve TANESCO's operation in the 1990's via support of the World Bank. However, despite these efforts including introduction of prepayment electricity meters, loss reduction measures and contracting services, TANESCO remained in a weak financial position by the late 1990's and the company performance deteriorated to unprecedented levels.

After a decade of reform efforts and introduction of Independent Power Producers (IPP) projects, TANESCO remains a vertically integrated utility but no longer holds a monopoly in generation. The two main IPPs, IPTL and SONGAS now contribute to generation, in addition to TANESCO's state owned hydro and small diesel facilities. Small amount of imports and self-generators also contribute to the Electricity Supply Industry (ESI). IPTL, SONGAS and Mtwara Energy Project (MEP) have made the Tanzania power system to be one of deregulated system in East Africa. Therefore, the power system state estimation proposed in the thesis, will work under the conditions of deregulated power system.

Objectives of deregulation of power industry include: technical, economical and pressure from some of multilateral organizations such as the World Bank and International Monetary Fund (IMF). However, the anticipated results from deregulation process have been observed as:

- An increase of power sector efficiency
- Ensuring economic viability and sustainability of the power industry

Reducing sector expenditure and debt by transferring to private sector the commercial risk inherent

3.6.2 Generation System

TANESCO operates an interconnected grid system comprising of hydropower and thermal plants. There are several diesel generating plants connected to the National grid. These have installed capacity of 80MW but they effectively contribute about 50 MW due to operation problems. Some regions, districts and townships are dependent on isolated diesel generators which are not connected to the national grid. These have installed capacity of 36.33MW but they are effectively contributing about 15MW due to aged machinery and lack of spare parts.

Two private independent power projects (IPPs) which are connected to the National grid are Independent Power Tanzania Limited (IPTL) with 103 MW installed capacity and

SONGAS (Songo- Songo gas-to electricity) project with installed capacity of 203 MW. The government through TANESCO owns a thermal plant with a capacity of 102 MW commissioned in November 2008, it also imports 4 MW from Uganda to Kagera region at 132 kV and about 1 MW from Zambia to the Mbozi district at 33 kV.

3.6.3 Transmission System

TANESCO owns transmission (HV) and distribution lines of different voltage levels scattered all over the country. In this thesis distribution lines are not considered. The high voltage transmission lines are estimated to comprise of 3,634.40 km of system voltage 220kV; 1,441.50 km of 132kV and 486 km of 66 kV, totalling to 5,553 km by the end of December, 2006 [www.tanesco.com, 2009]. High voltage transmission lines use pylon made of steel. Almost all transmission lines are radial single-circuit lines. The system is all alternating current (AC) operating at 50 Hz.

National Grid

The interconnected national grid had a peak-suppressed demand of 653 MW in 2007(SAPP statistics), which covered the following demand centres:

- Arusha including Manyara
- Dar es Salaam including Coast
- Dodoma
- Iringa
- Kilimanjaro
- Musoma
- Mbeya
- Morogoro
- Mwanza
- Shinyanga
- Singida
- Tabora
- Tanga
- Zanzibar

Isolated demand centres

The total non-coincident peak demand of the isolated (non-grid) centre in 2006 was 38.7 MW. These include the following centres:

- Kagera
- Kigoma
- Lindi
- Mafia Island
- Mtwara
- Pemba Island

- Rukwa
- Ruvuma

It should be noted that the southern part of the country, Mtwara, Lindi and Ruvuma regions are not connected to the national grid, however, through Mtwara power project they will be connected in the near future.

3.6.4 Power Generation Options

The TANESCO 2003 Power Master Plan identified new power generation options to be implemented in future because peak power demand and energy forecast up to 2025 is increasing (Figure 3.5 and 3.6)[SAPP Statistics, 2008]. The options are the same as those evaluated in the East Africa Power Master Plan (EAPMP). The World Bank in 2005 financed strategic/sectoral, social and environment assessment (SSEA) studies of power development options. Subsequent to the 2003 plan update, further development has been made on the Zambia-Tanzania-Kenya (ZTK) inter-connector, the Kiwira coal thermal plant and the Mnazi bay natural gas resource.

Hydropower Generation Options

The following hydro sites have been identified and evaluated and are included in the current TANESCO re-assessment options and are shown in Table 3.1

Table 3.1: Hydropower generation options

Serial No.	Project Name	Estimated Power [MW]	Fuel Source
1	Igamba Falls	20	Malagarasi River
2	Kakono	53	Rufiji River
3	Lower Kihansi	120	Rufiji River
4	Mandera	21	Pangani Falls
5	Mpanga	144	Rufiji River
6	Masigira	118	Ruhuhu River
7	Ruhudji	358	Ruhudji
8	Rumakali	222	Rumakali
9	Rusumo	62	Rusumo Falls
10	Songwe	340	Songwe Falls
11	Stieglers Gorge	1,200	Rufiji River

Source: TANESCO, 2007

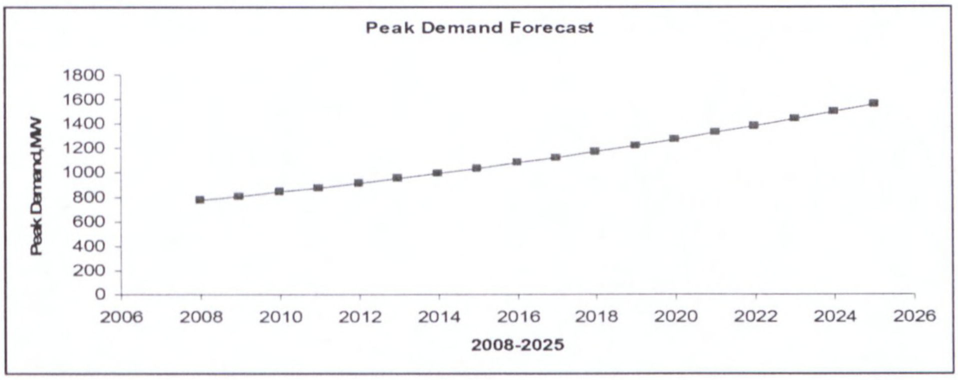
Thermal Generation Options

The new thermal generation options include some specific projects that are in development pipeline, and other generic options. These are listed in Table 3.2

Table 3.2: Thermal generation options

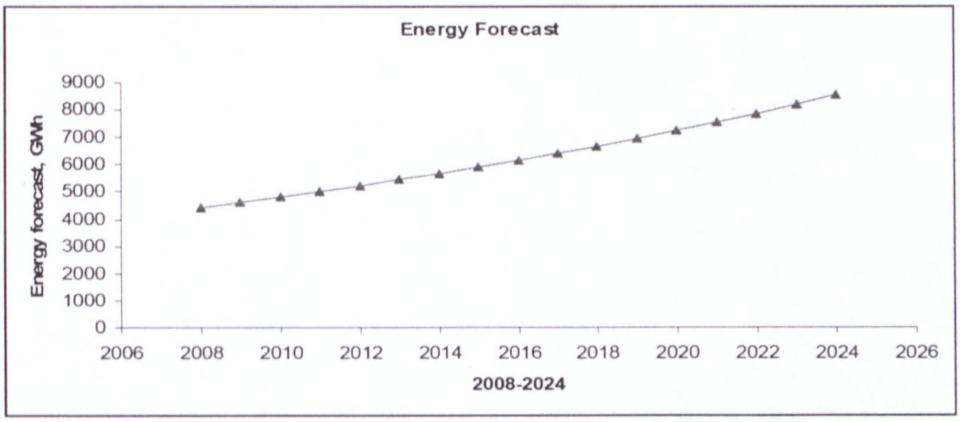
Serial No.	Project Name	Estimated Power [MW]	Fuel Source
1	Kiwira	200	Coal
2	Mchuchuma I	200	Coal
3	Mchuchuma II	200	Coal
4	Kinyerezi	180	Natural gas
5	Tegeta	45	Natural gas
6	Alstom	60	HFO

Source: TANESCO, 2007



Source: SAPP Statistic-2007

Figure 3.5: Peak power demand forecast: 2008-2025



Source: SAPP Statistic-2007

Figure 3.6: Electrical energy forecast: 2008-2024

3.6.5 Transmission Line Extensions

Development of hydro and thermal generating plants in the option list will force extension and construction of new transmission lines. Additional of thermal generation for a total of 180 MW at Kinyerezi in the vicinity of the city of Dar es Salaam will require a transmission line of 132- kV connection at Kinyerezi. The 330-kV interconnection from Zambia to northern Tanzania, purchases of electricity from Uganda and other neighbouring countries to the north of the country will force reinforcement on the 220-kV transmission system.

The introduction of the interconnection with Zambia will change the dynamic characteristics of the power system, and improvement should be pursued in tuning power system stabilizers and specifying such equipment for new generators. Addition of Ruhudji generating plant will require two new transmission lines: one from the new plant via Mufindi to Kihansi and the other directly to Kihansi. Addition of generating plant at Rumakali will require 220-kV lines to Mbeya and Mufindi.

From year 2004, the single-line to the north of the country has sufficient capability, however, with more mining companies operating in those areas and others to start operation in the near future, a second line will be needed from Iringa, Singida and Shinyanga. By the year 2010, a second line from Shinyanga to Mwanza will be needed. Difficult in supplying the forecasted load demand growth (Tables 3.5 and 3.6) using the 220-kV voltage standard and future inter-regional power transfer requirements, a 330-kV line from Singida to Arusha should be introduced.

In longer term, the 440-kV standard may be justifiable to supply the northern area of the country and to cater for international power-transfers; however, 220-kV transmission will continue to be the best choice for power transfer to the coast areas.Transmission line extensions are summarized in Table 3.3

Table 3.3: Transmission Line Extensions

Serial No.	From	To	Distance [km]	Voltage Level [kV]	Year of Completion
1	Shinyanga	Mwanza II	139	220	2010
2	Mbeya	Singida	487	330	2010
3	Singida	Arusha	316	330	2012
4	Iringa	Mtera	120	220	2017
5	Ruhudji-Mufindi	Kihansi	200	220	2016
6	Ruhudji	Kihansi	150	220	2016
7	Kidatu-Morogoro	Ubungo	310	220	2010
8	Mchuchuma	Mufindi I	283	220	2022
9	Mchuchuma	Mufindi II	283	220	2024
10	Rumakali	Mbeya	85	220	2027
11	Rumakali	Mufindi	268	220	2027

Source: TANESCO, 2007

Implementation of the generation options and extension of transmission lines can not be undertaken by the government through TANESCO alone, private sector should be involved. Already the National Parliament has enacted a law (Electricity act 2008) that allows competition in the power sector (electricity, oil and gas). Thus, more independent power producers (IPPs) to be installed and establishment of regional transmission organizations (RTOs) operating in the Tanzania power sector in the near future are expected. To date, IPTL, SONGAS are the two IPPs operating in the city of Dar es Salaam. Artumas, another IPP generates and supplies electricity in Mtwara region. Its service will be extended to Lindi and Ruvuma regions in future.

When all these generation options become fully operational, they will add up to more than 1,500MW into the National Grid. This will suffice the consumer demand, which at present is increasing at a rate of more than 15 percent annually. At the same time, Tanzania power sector will be one of the deregulated systems in the East African region.

3.7 Interconnections

TANESCO actively participates and corporates with government and other utility companies in order to enhance electrical power development in the country. TANESCO is active member in the following areas and bodies

3.7.1 Southern African Power Pool (SAPP)

The southern African power pool (SAPP) was established in 1995 by electricity utilities in 12 countries of the SADC region as an effort to pool their electricity –supply resource for their mutual benefit. The main goals of SAPP are: to provide reliable and economical electricity supply to the consumers of each of the SAPP members, consistent with reasonable utilization of natural resources and the effect on the environment, coordination and cooperation in the planning and operation of various systems to minimize costs while maintaining reliability and equitable sharing of the resulting benefits. The SAPP co-ordination centre is based in Harare, Zimbabwe. A 330 kV line has been proposed to connect Mbeya in Tanzania to Pensulo substation near Serenje in Zambia, to form the first part of the 1,600 km 400 HVAC Zambia-Kenya (ZTK) Interconnector.

3.7.2 Nile Basin Regional Power Trade Project (NBRPTP)

This proposal for a Nile Basin Regional Power Trade Project (NBRPTP) has been developed under the Shared Vision Program (SVP) of the Nile basin initiative (NBI). The project aims to establish the institutional means to coordinate the development of regional power markets among the Nile countries and build analytical capacity and provide technical infrastructure to manage Nile basin resources in development through equitable utilization of and benefit from the common Nile basin water resources. The Dar es Salaam declaration on regional electric power trade was signed on 20thMay 2003.

3.7.3 The Nile Equatorial Lakes-Subsidiary Action Program (NELSAP)

The NELSAP power development and trade aims at development of infrastructure consisting of small-scale hydropower development in critical areas, and strengthening transmission interconnections between several countries in the NELSAP region which includes: Burundi, DRC, Kenya, Rwanda, Tanzania and Uganda.

3.7.4 East African Regional Power Plan (EARPP)

East Africa, as a region, possesses adequate energy resources for her development. Under the auspices of the East African Community (EAC), the East African power master plan study is being carried out to define the least cost expansion programme for the development of combined power generation systems of Kenya, Uganda and Tanzania. TANESCO also participates fully in the East Africa Community committees whose major objective is to prepare the East Africa Power Master Plan and the energy committee for the proposed Zambia-Tanzania-Kenya (ZTK) interconnection. There are also plans to set an East Africa Power Pool (EAPP) whose objective is set a framework for power exchanges between utilities of the member states.

3.8 Conclusion

This chapter has presented an overview of deregulation of power systems; its motivation; objectives; advantages and disadvantages; deregulated power system structures and the role of power system state estimation in deregulated systems.

In deregulated systems, IPPs, RTOs and DISTCOs structures are formed. The central dispatch centres (CDCs) of the regulated systems can not meet the new functions and requirements arising from deregulated systems. The CDCs are replaced by RTO/ISOs. In these systems, SE is a key component of the energy management system (EMS) functions, because it provides accurate database for congestion management and ancillary service auction. Its role is more important, it determines the sustainability and competitiveness of deregulated power system's market.

Deregulation of Tanzania power sector is briefly presented; sighting Tanzania government electricity industry policy and restructuring framework; purposes of introducing the policy, its main objectives, and strategies adopted to realize the policy. It also describes the Tanzania power system status; generation options; transmission line extension; future interconnections and trends of the sector. Thus, the state estimation proposed in this thesis will be implemented in a deregulated Tanzania power system and regional interconnected systems either in the East African Power Pool (EAPP) or Southern African Power Pool (SAPP).

Chapter 4 presents system component modelling. AC synchronous generator, transformers, transmission lines (short, medium, and long-length lines), FACTS devices and composite loads models are developed; these models are used in building of a network model of the Tanzanian Power System Network of chapter 5.

CHAPTER FOUR: SYSTEM COMPONENT MODELLING

4.1 Introduction

In the broadest sense, a model is a systematic representation of an object or event in idealized and abstract form [Mortensen, D.C., 1972]. It clarifies the structure of complex events, reducing complexity to simpler and more familiar terms. It provides new ways to conceive a hypothetical ideas and relationships and offers insights.

In the following sections mathematical models for AC synchronous generator, tap-changing and regulating transformers, short, medium and long-length transmission lines, FACTS devices and composite loads are derived. These models are very useful in load flow studies, power system state estimation (PSSE) studies, optimal power flow (OPF) and simulation of faults.

4.2 Per-Unit System

In electrical power engineering a per-unit system is the expression of the system quantities as fraction of a defined unit quantity. All calculations are simplified because quantities are expressed as per-unit regardless of the voltage level. Similar types of apparatus will have impedances, voltage drops and losses that are the same when expressed as a per-unit even if the unit size varies widely. A per-unit system provides unit for power, voltage, current, impedance, and admittance. Only two of the mentioned are independent, usually power and voltage. All other quantities are specified as multiples of selected base values.

Relationship between units

The relationship between units in a per-unit system depends on whether the system is single-phase or three-phase. Per-unit system is based on the following formulae:

$$per\ unit = \frac{actual\ quantity}{base\ \ quantity} \tag{4.1}$$

By using (4.1), all quantities are expressed as ratios of some base value or values. When making analysis of a real system, all parameters such as voltage, current, power, and impedance are converted to per-unit. After convention, the entire systems then analyzed and the local results calculated. In this thesis the subscript *pu* indicates a per unit value throughout, the subscript *B* indicates a base value, and no indication of subscript indicate an actual value such as Amperes, Ohms or Volts.

Fundamental Process for changing to per-unit System

The per-unit description is obtained through 5 basic steps

- Selection of the base(s) for the system.
- Conversion of each known quantity (Volts, Amps, Ohm, etc to per-unit values.
- Drawing the system impedance diagram using the per-unit values.
- Performing the circuit analysis to determine the current magnitudes in per-unit.
- Converting per-unit currents and voltages from step 4 above into Volts and Ohms.

Selection of the base quantities

Selected base values must satisfy the fundamental laws of electricity, including Ohm's law

$$V_B = I_B Z_B \tag{4.2}$$

V_B is the base voltage

I_B is the base current

Z_B is the base impedance

If (4.2) is true, then (4.3) is also true

$$\frac{V}{V_B} = \frac{IZ}{I_B Z_B} \Rightarrow V_{pu} = I_{pu} Z_{pu} \tag{4.3}$$

V is the actual voltage

I is the actual current

Z is the actual impedance

To satisfy the above logic, the base quantities are selected using the rules given in Table 4.1. After the base values have been selected or computed, the per-unit values in the system are calculated using (4.4), (4.5) and 4.6)

$$Z_{pu} = \frac{Z(\Omega)}{Z_B} \tag{4.4}$$

$$I_{pu} = \frac{I(amperes)}{I_B} \tag{4.5}$$

$$V_{pu} = \frac{V(Volts)}{V_B} \tag{4.6}$$

Changing Bases

At a time the need arises to convert a per-unit value to another base. For example transformer nameplate percent impedance is normally given using the transformer full load **kVA** as the base. Thus, the transformer impedance must be converted to the **kVA$_B$** being used in the study. Equation (4.7) is used in the conversion procedure:

$$Z = Z_{pu}^{Old} Z_{B}^{Old} = Z_{pu}^{New} Z_{B}^{New} \tag{4.7}$$

Eqn (4.7) is re-arranged into

$$Z_{pu}^{New} = Z_{pu}^{Old} \cdot \frac{Z_{B}^{Old}}{Z_{B}^{New}} \tag{4.8}$$

Substituting

$$Z_{pu}^{New} = Z_{pu}^{Old} \cdot \frac{\dfrac{(kV_B^{Old})^2 x1,000}{kVA_B^{Old}}}{\dfrac{(KV_B^{New})^2 x1,000}{kVA_B^{New}}} \tag{4.9}$$

Equation (4.9) is reduced to:

$$Z_{pu}^{New} = Z_{pu}^{Old} \left(\frac{kV_B^{Old}}{kV_B^{New}} \right)^2 \cdot \frac{kVA_B^{New}}{kVA_B^{old}} \tag{4.10}$$

Table 4.1: Base selection criteria

Base Quantity	Symbol Applied	How Selected
kVA	kVA$_B$	Select at random. 10MVA or 100MVA are commonly used: However, some equipment such as transformers and generators has their impedance expressed using their nominal full-load as the base
Volts	kV$_B$	Select the voltage at the location where equipment is located. For example, a cable located in the 44.5kV system would use 44.5kV as its voltage base. Note that the line-line kV is used.
Current	I$_B$	$I_B = \frac{kVA_B}{kV_{LL}\sqrt{3}}$
Impedance	Z$_B$	$Z_B = \frac{kV_B x1,000}{I_B\sqrt{3}} \Rightarrow Z_B = \frac{(kV_B)^2 x1,000}{kVA_B}$

The voltage terms in (4.10) are used only when a device is being employed in a voltage different from the base upon it was calculated before.

Single-Phase System

Assuming that the independent base values are power and voltage, it can be written:

$$P_B = 1pu \tag{4.11}$$

$$V_B = 1pu \tag{4.12}$$

Alternatively, the base value for power may be given in terms of reactive apparent power, in which case

$$Q_B = 1pu \tag{4.13}$$

$$S_B = 1pu \tag{4.14}$$

The rest of the units can be derived from power and voltage using the equation *S=VI, P=Scosδ, Q=Ssinδ and V=IZ, Z* being represented by;

$$Z = R + jX = Z\cos\delta + jZ\sin\delta \tag{4.15}$$

where

$$I_B = \frac{S_B}{V_B} = 1pu \tag{4.16}$$

$$Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{I_B V_B} = \frac{V_B^2}{S_B} = 1pu \tag{4.17}$$

$$Y_B = \frac{1}{Z_B} = 1pu \tag{4.18}$$

Three-Phase System

Power and voltage are specified in the same way as in single-phase systems. However, due to differences, these terms are usually represented in three-phase systems by other relations; the relationships for the derived units are different. Specifically, power is given as total (not per-phase) power, and voltage is line-to-line voltage. In three-phase systems the equations *P=Scosδ and Q=Ssinδ* also hold. The apparent power *S* now equals to

$$S_B = \sqrt{3}V_B I_B \tag{4.19}$$

$$Z_B = \frac{V_B}{\sqrt{3}I_B} = \frac{V_B^2}{S_B} = 1pu \tag{4.20}$$

$$Y_B = \frac{1}{Z_B} = 1pu \qquad (4.21)$$

where

R is the resistance

X is the reactance

S is the apparent power

δ is the phase angle

P is the real power

Q is the reactive power

S_B is the base apparent power

4.3 Modelling of AC synchronous generator

Synchronous generators generate the overwhelming majority of electricity in the modern power systems. Most of the generators are three-phase and are designed to operate at constant frequency. A synchronous generator has two basic parts: the stator and the rotor. The stator is a hollow laminated ferro-magnetic structure with longitudinal slots on the inside surface. The coils are placed in the slots and are interconnected to form three separate windings. There are two basic rotor designs: Salient and non-salient or cylindrical. The salient construction has poles that project from the rotor. These poles exhibit a narrow air-gap under the pole structure and wider air-gap between poles. Cylindrical pole rotor consists of a cylindrical rotor, often made from a single steel forging, in which the field winding is embedded in longitudinal slots machined in its surface.

Synchronous generators are of two major types depending on the speed at which they are driven, that is high speeds such as 4,000 and 1,500 revolutions per minute with two or four poles, and lower speeds over a wide range from 150 to 600 revolutions per minute. High peripheral speed requires non-salient (cylindrical) pole rotor and low peripheral speed requires silent pole rotor. Detailed information of synchronous generator design, construction, operating characteristics and cooling can be traced in [Adkins, 1957; Fitzgerald & Kingsley, 1966; Vadhera, 1981; Bergen & Vittal, 2000].

Electromechanical torque results when the stator and rotor fields react with each other. In synchronous generator, electromechanical torque opposes rotation of the rotor so that the mechanical torque is applied by the prime mover to sustain the rotation. Changing the mechanical torque input by the prime mover only changes the electrical torque output of the generator. Power System is made up of several synchronous machines connected in parallel. The machines are connected in parallel so as to maintain Continuous supply of power after machine falls out of service. In order to connect the machines in parallel, certain conditions must be met as according to ANSI/IEEE standard, 115-1999. These standards stipulate:

- The machines should rotate at the same synchronous speed
- The voltage magnitude of the machines must be the same
- The current magnitude of the machines must be the same
- The frequency must be the same

Once the mentioned conditions are fulfilled, then the machines can therefore be connected in parallel so that the system can have continuous, adequate and stable power supply. Should any uncertainties occur in the system, the above conditions will be violated and the machines will fall out of synchronism, which will leads to instability of the system. The mechanism by which interconnected synchronous machines maintain synchronism with each other is through restoring forces, which act whenever there are forces tending to accelerate or decelerate one or more machines with respect to each other.

4.3.1 AC synchronous generator model

With full knowledge of the time varying inductances of synchronous generator as discussed and presented in [Fitzgerald &Kingsley, 1966; Vadhera, 1981] the general machine equations can be developed (by machine equations means synchronous generator equations), these equations are critical for developing machine model.

Considering motoring mode of a machine [Vadhera, 1981], that is current flowing into the winding as shown in Figure 4.1

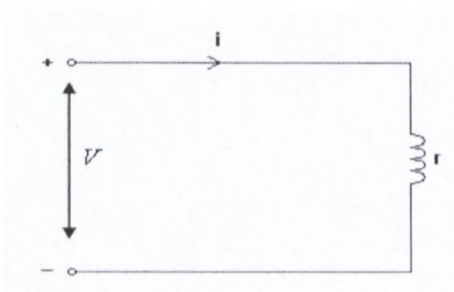


Figure 4.1: AC synchronous generator phase winding

For any of the three windings, the relationship in terms of the applied voltage and the input current is:

$$v = ri + \frac{d\Psi}{dt} \tag{4.22}$$

where

ψ: flux linkage

i_{ins}: Instantaneous current

r_w: winding resistance

With the subscript ***a***, ***b***, ***c*** and *f* attached to eqn (4.22) to denote the three-phases and the field winding, eqn (4.22) is written for each of the four windings as:

$$\begin{aligned}
v_a &= r_a i_a + \frac{d\Psi_a}{dt} \\
v_b &= r_b i_b + \frac{d\Psi_b}{dt} \\
v_c &= r_c i_c + \frac{d\Psi_c}{dt} \\
v_f &= r_f i_f + \frac{d\Psi_f}{dt}
\end{aligned} \tag{4.23}$$

In terms of self and mutual inductances ***L***, the flux linkages are given by:

$$\begin{aligned}
\Psi_a &= L_{aa} i_a + L_{ab} i_b + L_{ac} i_c + L_{af} i_f \\
\Psi_b &= L_{ba} i_a + L_{bb} i_b + L_{bc} i_c + L_{bf} i_f \\
\Psi_c &= L_{ca} i_a + L_{cb} i_b + L_{cc} i_c + L_{cf} i_f \\
\Psi_f &= L_{fa} i_a + L_{fb} i_b + L_{fc} i_c + L_{ff} i_f
\end{aligned} \tag{4.24}$$

Re-arranging and substituting (4.24) into (4.23), the following eqns are obtained:

$$\begin{aligned}
v_a &= r i_a + \frac{d}{dt}\left(L_{aa} i_a + L_{ab} i_b + L_{ac} i_c + L_{af} i_f\right) \\
v_b &= r i_b + \frac{d}{dt}\left(L_{ba} i_a + L_{bb} i_b + L_{bc} i_i + L_{bf} i_f\right) \\
v_c &= r i_c + \frac{d}{dt}\left(L_{ca} i_a + L_{cb} i_b + L_{cc} i_c + L_{cf} i_f\right) \\
v_f &= r_f i_f + \frac{d}{dt}\left(L_{fa} i_a + L_{fb} i_b + L_{fc} i_c + L_{ff} i_f\right)
\end{aligned} \tag{4.25}$$

In practice, $r_a = r_b = r_c = r_w$

Eqn (4.25) can be written in a compact matrix notation:

$$V = Ri + \frac{d}{dt}(Li) \tag{4.26}$$

Where the following vectors and matrices are defined:

$$V \underline{\underline{\Delta}} \begin{bmatrix} v_a \\ v_b \\ v_c \\ \cdots \\ v_f \end{bmatrix}, \qquad I \underline{\underline{\Delta}} \begin{bmatrix} i_a \\ i_b \\ i_c \\ \cdots \\ i_f \end{bmatrix} \tag{4.27}$$

$$R \underline{\underline{\Delta}} \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r_f \end{bmatrix} \tag{4.28}$$

$$L \underline{\underline{\Delta}} \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & \vdots & L_{af} \\ L_{ba} & L_{bb} & L_{bc} & \vdots & L_{bf} \\ L_{ca} & L_{cb} & L_{cc} & \vdots & L_{cf} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ L_{fa} & L_{fb} & L_{fc} & \vdots & L_{ff} \end{bmatrix} \tag{4.29}$$

Substituting eqns (4.27-4.29) into (4.24) complex equations which are difficult to be solved are obtained. Their solutions are simplified by Blondel's transformation or Park's transformation discussed in [Bergen & Vittal, 2000].

Basic machine equations in d, q, o variables

The flux linkage relation of (4.24) becomes simpler when is expressed in new variables *d, q* and *o.* The variables are: direct axis synchronous reactance, the quadrature axis synchronous reactance and the zero sequence inductance respectively. Thus, (4.24) can be written in a compact matrix notation:

$$\Psi = LI \tag{4.30}$$

In expanded form eqn (4.30) is given by:

$$\Psi \underline{\underline{\Delta}} \begin{bmatrix} \Psi_S \\ \cdots \\ \Psi_f \end{bmatrix} = \begin{bmatrix} L_S & \vdots & L_{sf} \\ \cdots & \vdots & \\ L_{sf}^T & \vdots & L_{ff} \end{bmatrix} \begin{bmatrix} I_S \\ \cdots \\ i_f \end{bmatrix} \tag{4.31}$$

where

Ψ is total the flux linkage

Ψ_S is the stator flux linkage

Ψ_f is the field flux linkage

The dotted line in (4.31) is provided to make a distinction between the field winding (rotor) and armature winding (stator). Eqn (4.31) may be further expanded as follows:

$$\Psi_S \underline{\Delta} \begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}, \qquad I_S \underline{\Delta} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{4.32}$$

$$L_S \underline{\Delta} \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \qquad L_{sf} \underline{\Delta} \begin{bmatrix} L_{af} \\ L_{bf} \\ L_{cf} \end{bmatrix} \tag{4.33}$$

Eqn (4.31) is transformed from phase axis ***a*, *b*,** and ***c*** into *d*, *q* and *o* variables by applying direct Blondel's transformation and inverse Blondel's transformation. The objective of using the transformation is to write armature (stator) winding and field winding (rotor) equations separately, such that:

$$\Psi_S = L_S I_S + L_{sf} i_f \tag{4.34}$$

$$\Psi_f = L_{sf}^T I_S + L_{ff} i_f \tag{4.35}$$

The Direct Blondel's transformation is expressed by:

$$I_B \underline{\Delta} B I_S \tag{4.36}$$

$$I_S \underline{\Delta} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \qquad I_B \underline{\Delta} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \tag{4.37}$$

I_S is the stator current

$$i_d = \frac{2}{3}\left[i_a \cos\theta + i_b \cos\left(\theta - 120^0\right) + i_c \cos\left(\theta + 120^0\right)\right] \tag{4.38}$$

$$i_q = \frac{2}{3}\left[-i_a \sin\theta - i_b \sin\left(\theta - 120^0\right) - i_c \sin\left(\theta + 120^0\right)\right)] \tag{4.39}$$

$$i_0 = \frac{1}{3}\left(i_a + i_b + i_c\right) \tag{4.40}$$

The currents $\boldsymbol{i_d}$, i_q and i_0 are known as direct axis, quadrature-axis and zero-sequence currents.

$$B \underline{\underline{\Delta}} \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-120^0) & \cos(\theta+120^0) \\ -\sin\theta & -\sin(\theta-120^0) & -\sin(\theta+120^0) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{4.41}$$

$$B^{-1} \underline{\underline{\Delta}} \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-120^0) & -\sin(\theta-120^0) & 1 \\ \cos(\theta-120^0) & -\sin(\theta+120^0) & 1 \end{bmatrix} \tag{4.42}$$

where

i_d is the current in the d axis

i_q is the current in the q axis

B_D is the Blondel's transformation

$\theta = \omega t$ is the electrical degree

Using eqns (4.38-4.42), the Blondel's transformation for stator flux linkages and its associate inverse transformation can be written as:

$$\Psi_B = B_D \Psi_S \tag{4.43}$$

$$\Psi_S = B_D^{-1} \Psi_B \tag{4.44}$$

Similarly, Blondel's transformation for stator voltages and its associate inverse one are:

$$V_B = B_D V_S \tag{4.45}$$

$$V_S = B_D{}^{-1} V_B \tag{4.46}$$

$$I_S = B_D^{-1} I_B \tag{4.47}$$

Applying (4.45) and (4.47) to (4.35) yields

$$B_D^{-1}\Psi_B = L_S B_D^{-1} I_{BD} + L_{sf} i_f \tag{4.48}$$

Pre-multiplying (4.48) by **B**, flux is obtained as:

$$\Psi_B = B L_S B^{-1} I_B + B L_{sf} i_f \tag{4.49}$$

Similarly equation (4.35) is written as:

$$\Psi_f = L_{sf}^T B_D^{-1} I_{BD} + L_{ff} i_f \tag{4.50}$$

Simplifying matrices of (4.49) and (4.50) the following useful equations are derived:

$$\Psi_d = L_d i_d + M_f i_f \qquad (4.51)$$

$$\Psi_q = L_q i_0 \qquad (4.52)$$

$$\Psi_0 = L_0 i_0 \qquad (4.53)$$

$$\Psi_f = L_{ff} i_f + 1.5 M_f i_d \qquad (4.54)$$

where

$$L_d = L_S + M_S + 1.5 L_m \qquad (4.55)$$

$$L_q = L_S + M_S - 1.5 L_m \qquad (4.56)$$

$$L_0 = L_S - 2 M_S \qquad (4.57$$

In a similar manner in *d, q* and *o* variables, the terminal voltage relationship is given by:

$$v_d = r i_d + p\Psi_d - \Psi_q \omega \qquad (4.58)$$

$$v_q = r i_q + p\Psi_q + \Psi_d \omega \qquad (4.59)$$

$$v_0 = r i_0 + p\Psi_0 \qquad (4.60)$$

$$v_f = r_f i_f + p\Psi_f \qquad (4.61)$$

where

$\omega = \dfrac{d\theta}{dt}$: Stator angular frequency in electrical degree per second.

$p\Psi_d$, $p\Psi_q$ are the transformer voltages; $\Psi_q\omega$, $\Psi_d\omega$ are the speed voltages.

In power system analysis, there are several type of models used to represent synchronous generator. These models are developed using some approximations to the basic machine equations (4.22-4.61). For load- flow and state estimation studies, the voltage of the synchronous terminals is accepted as a constant value, thus, it is assumed that transformer voltages are small compared to the speed voltages and therefore are neglected. Hence, the stator voltage equations expressed in per –unit notation are given by:

$$v_d = \frac{d\Psi_d}{dt} - \Psi_q \omega - r_s i_d$$
$$v_q = \frac{d\Psi_q}{dt} + \Psi_d \omega - r_s i_q \qquad (4.62)$$
$$v_0 = \frac{d\Psi_0}{dt} - r_s i_0$$

The corresponding flux linkage model of the machine is given by:

$$\Psi_d = -L_d i_d + L_{sf} i_{fd} + L_{skd} i_{kd}$$
$$\Psi_q = -L_q i_q + L_{skq} i_q \qquad (4.63)$$
$$\Psi_0 = -L_0 i_0$$

Classical representation of synchronous generator

Synchronous generator model expressed by equations (4.62) and (4.63) is often simplified by assuming that the electromotive force induced is constant and hence constant voltage supply. This assumption eliminates the differential equation associated with electrical characteristic of the machine [Momoh, 2009]. The second assumption is to ignore transient saliency by assuming that $x_d^{'} = x_q^{'}$ and also the flux linkage to remain constant throughout. With this assumption, the voltage behind the transient impedance $r_s + jx_d^{'}$ has constant magnitude. Therefore, the equivalent circuit of synchronous generator is shown in Figure 4.2, the terminal voltage phasor is represented by:

$$V_t = E\angle\delta - \left(r_s + jx_d^{'}\right)I_t \qquad (4.64)$$

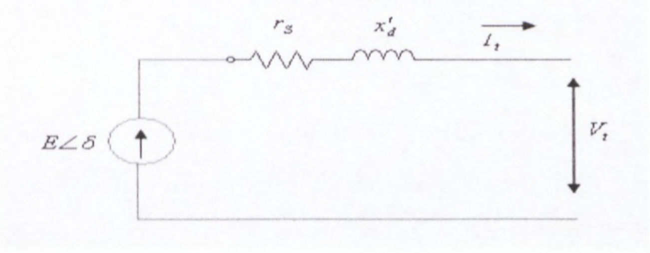


Figure 4.2: Equivalent circuit of AC synchronous generator

Equivalent circuit shown in Figure 4.2 is a classical representation of synchronous generator that suit stability studies. In load- flow and state estimation studies, the interest is in the steady-state power output of the synchronous generator; a model of a generator that supplies constant voltage source is needed. Hence, the model of Figure 4.2 is simplified to the model shown in Figure 4.3

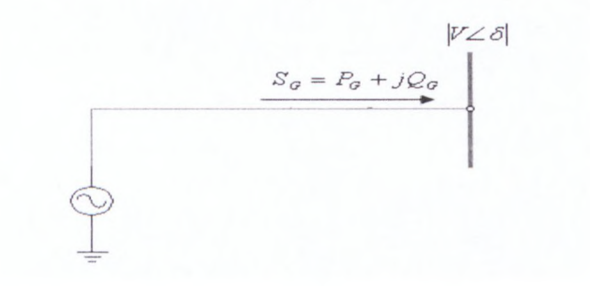


Figure 4.3: AC synchronous generator model

Synchronous generator reactive power limits

It is well known that generators have maximum and minimum real power capabilities. In addition, they also have maximum and minimum reactive power capabilities. The maximum reactive power capability corresponds to the maximum reactive power that the generator may produce when operating with a lagging power factor. The minimum reactive power capability corresponds to the maximum reactive power the generator may absorb when operating with a leading power factor. These limitations are a function of the real power output of the generator, that is, as the real power increases, the reactive power limitations move closer to zero. The solid curve in Figure 4.4 is a typical generator capability curve, which shows the lagging and leading reactive limitations as real power is varied. Most load flow programs model the generator reactive capabilities by assuming a somewhat conservative value for P_{max} (perhaps 95% of the actual value), and then fixing the reactive limits Q_{max} (for the lagging limit) and Q_{min} (for the leading limit) according to the dotted lines shown in Figure. 4.4

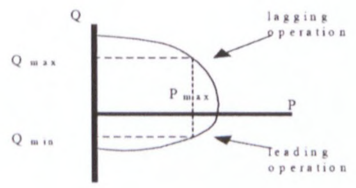


Fig 4.4: AC synchronous capability curve

4.4 Modelling of Transformers

Transformers are the link between generators and the power system and the transmission lines, and between transmission/distribution lines of different voltage levels [Grainger, J J & Stevenson, W D., 1994]. Efficient and effective transmission of electrical power over long distances requires much higher voltages. Transmission lines operate at nominal voltages of up to 765 kV line to line for really long distance transmission [Bergen, A R & Vittal, V., 2000]. Distribution of power takes a place at much lower voltages , typically at 33 or 11 kV and can be "stepped down" further to voltage of 440 V for industrial use or 220/240 V for commercial or residential use. "Stepping up" from generators for transmission purposes and "stepping down" for industrial, commercial or residential use is fulfilled by transformers.

Power transformers provide a trouble-free and efficient mean of changing from one voltage level to another. For this application, they have a fixed voltage ratio. Transformers can also be used for controlling voltage magnitude, phase angle or power flow under varying operating conditions. For this application they have a voltage ratio that can be varied in small increments, on command.

In this section, modelling of transformers for use in load flow and system state estimation studies is presented. Care is paid to transformers that regulate voltage magnitude, phase angle and control the real and reactive power flows.

4.4.1 Modelling of regulating transformers

In practice, all power transformers and many distribution transformers has taps in one or more windings for changing the turn ratio [Saadat, 2004]. The method is most popular since it can be used for controlling voltages at all levels. Tap-changing, by altering the voltage magnitude, affects the distribution of VArs and may therefore used to control the flow of reactive power. There are two types of tap-changing transformers in practice and are detailed in [Saadat, 2004; Grainger & Stevenson, 1994]

Regulating transformers are used to change the voltage magnitude and phase angle at certain point in the system by a small amount. These transformers add a small component of voltage typically less than 0.10 per unit [Bergen & Vital, 2000] to the line or phase voltages. Regulating transformers can be also used at any intermediate point in the system. RT and tap changing switch gears can be taken out of service for maintenance without affecting the system. RT can be used to control the real and reactive power flow in the system network. Therefore, it is absolutely essential to develop the bus admittance model (equations) to include such transformer in load flow and system state estimation.

4.4.2 Regulating transformer model

Figure 2 shows a detailed representation of a practical regulating transformer.

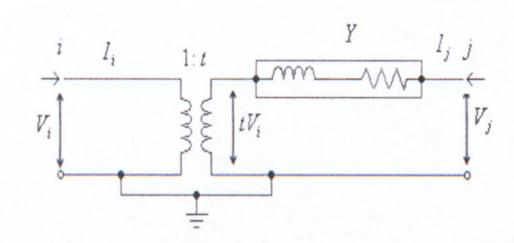


Figure 4.5: Detailed representation of RT

Figure 2 shows currents I_i and I_j entering the two buses, and the voltage at these buses are V_i and V_j referred to the reference bus. The complex expression for power into the ideal transformer with turns ratio *1: t* from bus *i* and bus *j* are, respectively

$$S_i = V_i I_i^* \tag{4.65}$$

$$S_j = tV_i I_j^* \tag{4.66}$$

Assuming the ideal transformer has no losses, the power $\boldsymbol{S_i}$ into the ideal transformer from bus *i* must equal the power $\boldsymbol{-S_j}$ out of the ideal transformer on the bus *j* side, so from (4.65) and (4.66)

$$\begin{aligned} S_i &= -S_j \\ V_i I_i^* &= -tV_i I_j^* \\ I_i &= -tI_j^* \end{aligned} \tag{4.67}$$

The current $\mathbf{I}_j$ can be expressed by

$$\begin{aligned} I_j &= (V_j - tV_i)Y \\ &= -tYV_i + V_jY \end{aligned} \tag{4.68}$$

Multiplying (8) by $-t^*$ and substituting $\mathbf{I}_i$ for $-t^*I_j$ yields:

$$I_i = tt^*YV_i - t^*YV_j \tag{4.69}$$

Setting $tt^* = /t/^2$ and re- arranging (4.68) and (4.69) into Ybus admittance matrix form, gives

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} |t|^2 Y & -t^*Y \\ -tY & Y \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{4.70}$$

The Π-equivalent model corresponding to (10) is shown in Figure 4.6.

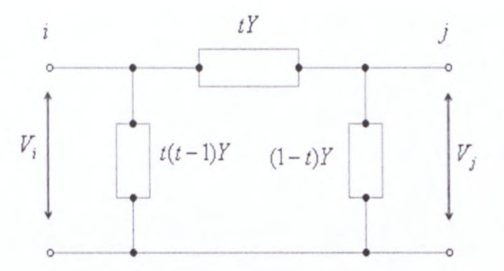


Figure 4.6: *π*-**circuit of RT**

4.5 Modelling of Transmission Lines

An electrical transmission line has 4 parameters which affect its ability to fulfil its function as part of a power system. These parameters are: resistance, inductance, capacitance and conductance. Inductance is the property of the circuit that relates the voltage induced by changing flux to the rate of change of current. Capacitance exists between the conductors and is the charge on the conductors per unit of potential difference between them. Conductance exists between conductors or conductors and the ground accounts for the leakage current at the insulators of the overhead transmission lines and the insulation of the conductors.

The resistance and inductance uniformly distributed along the transmission line form the series impedance. The conductance and capacitance existing between conductors of a single-phase line or from a conductor to neutral of three-phase line form the shunt admittance. Although the resistance, inductance and capacitance are distributed, the equivalent circuit of a transmission line is made up of lumped parameters.

The general equations relating voltages and current on a transmission line recognize that all the 4 parameters of the line are uniformly distributed along the line. But for short and medium length transmission lines lumped parameters give good accuracy [Stevenson, 1982]. If an overhead line is classified as short line (less than about 80 km long) shunt capacitance of this line is so small that it can be omitted with little loss of accuracy. Only the series resistance R and series inductance L are considered for the total length of the line. For transmission line classified as medium-length (with length between 80 km and 240 km long) this line can be represented well by series resistance R and series inductance L as lumped parameters, with half the capacitance to neutral of the line lumped at each end of the equivalent circuit. Transmission longer than 240 km require calculations in terms of distributed constants if a higher degree of accuracy is required and this is presented in detail in [Grainger& Stevenson, 1994; Stevenson, 1982].

In this section models representing short-length, medium-length and long -length transmission lines are considered. The models are presented and used for load flow and power system state estimation studies.

4.5.1 The short- length transmission line model

For a short transmission line model, shunt capacitance is so small and is neglected. The equivalent circuit (model) of a short transmission line is shown in Figure 4. 7

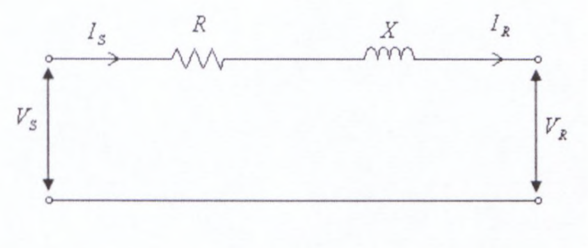


Figure 4.7: Short -length transmission line representation

I_S, I_R are the sending and receiving-end currents V_S and V_R are the sending and receiving end line- to-line neutral voltages.

Since there are no shunt arms in the circuit, then

$$I_S = I_R \tag{4.71}$$

The voltage at the sending end is

$$\begin{aligned} V_S &= V_R + I_R Z \\ &= V_R + I_R(R + j\omega L) \end{aligned} \tag{4.72}$$

z is *zl* , the series impedance of the transmission line.

4.5.2 The Medium-length transmission line model

The shunt admittance, usually pure capacitance is included in the model. The total shunt admittance is divided into two equal parts placed at the sending-end and receiving end of the transmission line. The circuit is usually called nominal π. Circuit representation of medium-lengt line is shown in Figure 4.8

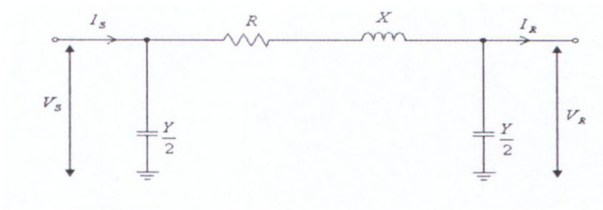


Figure 4.8: Medium-length transmission line representation

To obtain an expression for V_S, it is noted that the current in the capacitance at receiving-end is $V_R.\frac{Y}{2}$ and the current in the series arm is $I_R + V_R.\frac{Y}{2}$, then

$$V_S = (V_R.\frac{Y}{2} + I_R)Z + V_R \quad (4.73)$$

$$V_S = (Z.\frac{Y}{2} + 1)V_R + ZI_R \quad (4.74)$$

$$I_S = V_S.\frac{Y}{2} + V_R.\frac{Y}{2} + I_R \quad (4.75)$$

Substituting V_S as given by (4.74) in (4.75)

$$\begin{aligned} I_S &= [(\frac{ZY}{2} + 1)V_R + ZI_R].\frac{Y}{2} + V_R.\frac{Y}{2} + I_R \\ &= V_R Y(1 + \frac{ZY}{4}) + (\frac{ZY}{2} + 1)I_R \end{aligned} \quad (4.76)$$

Eqns (4.74) and (4.75) may be expressed in the general form:

$$\begin{aligned} V_S &= AV_R + BI_R \\ I_S &= CV_R + DI_R \end{aligned} \quad (4.77)$$

where

$$A = 1 + \frac{ZY}{2}, B = Z, C = Y(1 + \frac{ZY}{4}), D = 1 + \frac{ZY}{2}$$

It can be noted that ***A* = *B*.** **ABCD** are constants and are called the general circuit constants of a transmission line. In general these constants are complex numbers. Their physical meaning to medium-length transmission line is provided in [Grainger & Stevenson, 1994].

4.5.3 Modelling of long-length transmission line

The nominal π circuit representing medium-length transmission line (Figure 4.8) does not represent a transmission line exactly {Stevenson, 1982] because it does not account for parameters of the transmission line being uniformly distributed. The discrepancy between the nominal π circuit and the physical actual line becomes large as the length of the transmission line increases. Therefore, the discrepancy has to be corrected when working with long-length transmission lines which are part of power systems.

Assuming that a nominal π circuit of Figure 4.8 is the equivalent circuit of a long-length transmission line, Let us call the series arm of Figure 4.8 circuit Z' and the shunt arm $\frac{Y'}{2}$. The sending-end voltage (V_S) is given by Eqn (4.74). Substituting Z' and $\frac{Y'}{2}$ for Z and $\frac{Y}{2}$ in Eqn (4.74) yields

$$V_S = (\frac{Z'Y'}{2} + 1)V_R + Z'I_R \tag{4.78}$$

In order the proposed circuit to be equivalent to the long-length transmission line the coefficient of V_R and I_R must be identical to expression from [Saadat, 2004; Bergen, & Vittal, 2000] given by:

$$V_S = V_R \cosh \gamma l + I_R Z_c \sinh \gamma l \tag{4.79}$$

Equating the coefficient of I_R in the two equations yield

$$Z' = Z_c \sinh \gamma l \tag{4.80}$$

$$Z' = \sqrt{\frac{z}{y}} \sinh \gamma l = zl \frac{\sinh \gamma l}{\sqrt{zyl}} \tag{4.81}$$

$$Z' = Z \frac{\sinh \gamma l}{\gamma l} \tag{4.82}$$

where

$z = zl$, is the series impedance per length of the transmission line. $\frac{\sinh \gamma l}{\gamma l}$ is a factor by which the series impedance of the nominal π circuit must be multiplied to convert the circuit to equivalent of that of long-length transmission line.

Similarly, equating the coefficient V_R of Eqn (4.78) and Eqn (4.80) yields

$$\frac{Z'Y'}{2} + 1 = \cosh \gamma l \quad (4.83)$$

Substituting Z' from Eqn (4.82) gives

$$\frac{YZ_c \sinh \gamma l}{2} + 1 = \cosh \gamma l \quad (4.84)$$

$$\frac{Y'}{2} = \frac{1}{Z_c}\frac{\cosh \gamma l - 1}{\sinh \gamma l} \quad (4.85)$$

From mathematics (Geometry)

$$\tanh \gamma l = \frac{\cosh \gamma l - 1}{\sinh \gamma l} \quad (4.86)$$

Then

$$\frac{Y'}{2} = \frac{1}{Z_c}\tanh \gamma l = \frac{Y}{2}\frac{\tanh(\frac{\gamma l}{2})}{\frac{\gamma i}{2}} \quad (4.87)$$

where

$Y = yl$ is the total shunt admittance of the transmission line. $\frac{\tanh(\frac{\gamma l}{2})}{\frac{\gamma l}{2}}$ is a correction factor by which the shunt admittance of the nominal π circuit must be multiplied to convert the circuit to equivalent of the long-length transmission line. The long-length equivalent circuit after the transformation is given in Figure 4.9.

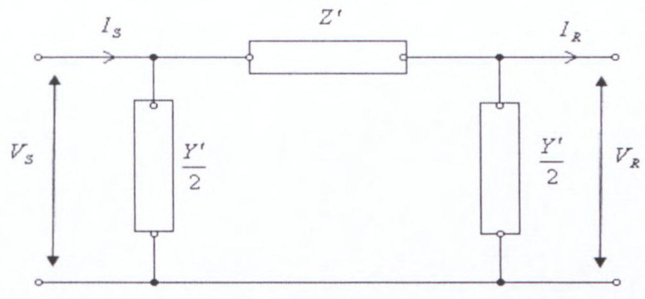


Figure 4.9: Equivalent π-circuit of a long-length transmission line

Practice indicates the following classification of transmission lines [Bergen & Vittal, 2000; Kothari & Nagrath, 2004] are reasonable and are adopted in this thesis. The classification is shown in Table 4.2

Table 4.2: Simplified transmission line circuits

Distance [km]	Description
$l < 80$	Use the Π-equivalent model, neglect ***Y/2***
$80 < l < 240$	Use the Π-equivalent model but replace ***Z*** by ***Z'*** and ***Y*** by ***Y'*** The Π-equivalent model consists of half the shunt admittance *Y/2*, at each end and the series impedance ***Z*** in the centre
$l > 240$	Use the exact model.

4.6 Modelling of FACTS devices

Flexible alternating current transmission systems (FACTS) device are installed in power systems to exercise continuous control over voltage profiles of power flow pattern [Hingorani & Gyugyi, 2000]. These devices enable the voltage profile and power flows to be changed in such a way that thermal limits of the transmission lines are not exceeded, stability margins are increased, losses minimized and contractual requirements fulfilled without violating the economic generation dispatch schedule [Noroozian et al, 1997]. However, the sole presence of these devices does not improve the overall damping of the system [Pal, B. & Chaudhuri, B., 2005]. FACTS devices discusses in this section are

- Thyristor Controlled Series Capacitor (TCSC)
- Static VAr Compensator (SVC)
- Thyristor Controlled phase Angle Regulator (TCPAR)

Thyristor Controlled series Capacitor (TCSC) is a capacitive reactance compensator which comprises of a series capacitor bank shunted by a thyristor controlled reactor

(TCR) in order to provide a smooth variation in series capacitive reactance The topology of TCSC is shown in Figure 4.10 and is normally connected in the line between bus *i and j*. The control action of the TCSC is expressed in terms of its percentage compensation k_c defined as $kc = X_C/X_L\ x100\%$. Where X_L is the reactance of the transmission line and X_C is the effective capacitive reactance offered by the TCSC.

Considering that TCSC is connected in the line between bus *i* and *j*. In this case the resistance of the transmission line is neglected for simplicity of calculation. If $\bar{I}$ is the current flowing through the transmission line, the TCSC having capacitive reactance X_C can be represented by a voltage source E_{se} as shown in Figure 4.11b. The voltage source is given by:

$$E_{se} = -jX_C\bar{I} \tag{4.88}$$

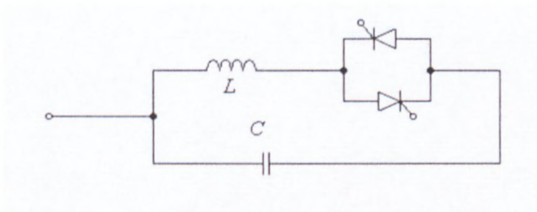


Figure 4.10: TCSC topology

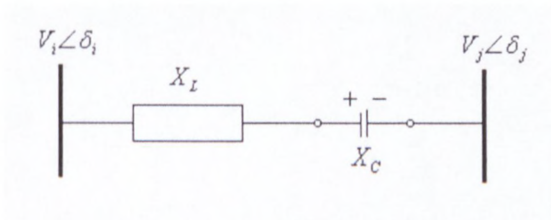


Figure 4.11a: Modified TCSC topology

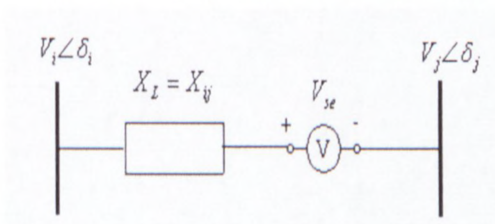


Figure 4.11b: TCSC voltage source model

The injection model is obtained by replacing the voltage source of Figure 4.11b by an equivalent current source $\bar{I}_{se}$ connected in parallel with the transmission line as shown in Figure 4.12. The current source $\bar{I}_{se}$ is given by:

$$\bar{I}_{se} = \frac{E_{se}}{X_{ij}} \tag{4.89}$$

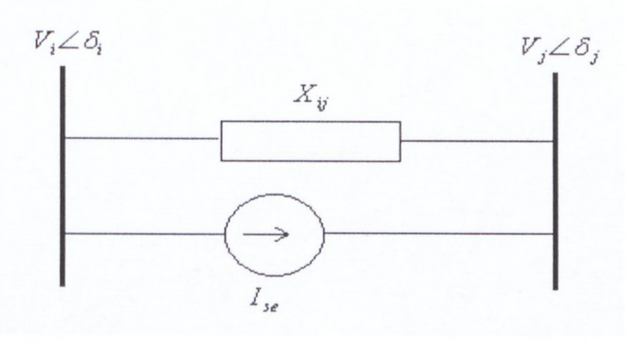


Figure 4.12: TCSC current source

The current source corresponds to the injection powers $\bar{S}_i$ and $\bar{S}_j$ which are given by:

$$\bar{S}_i = E_i(-\bar{I}_{se})^* \tag{4.90}$$

$$\bar{S}_j = E_j(\bar{I}_{se})^* \tag{4.91}$$

The real and reactive power injection equations of the TCSC connected between buses *i and j* can be obtained first by introducing a compensation percentage factor given by:

$$k_c = \frac{X_C}{X_L} \tag{4.92}$$

where

k_c is a percentage compensation factor

X_L is the reactance of the transmission line.

$$P_i = \frac{k_c}{(k_c - 1)} V_i V_j B_{ij} \sin(\delta_i - \delta_j) \tag{4.93}$$

$$Q_i = \frac{k_c}{(k_c - 1)} B_{ij} [V_i^2 - V_i V_j \cos(\delta_i - \delta_j) \tag{4.94}$$

$$P_j = \frac{k_c}{(k_c - 1)}.V_j V_i \sin(\delta_j - \delta_i) \tag{4.95}$$

$$Q_j = \frac{k_c}{(k_c - 1)}.B_{ji}[V_j^2 - V_j V_i \cos(\delta_j - \delta_i) \tag{4.96}$$

Figure 4.13 shows real and reactive power injection at bus *i* and *j*

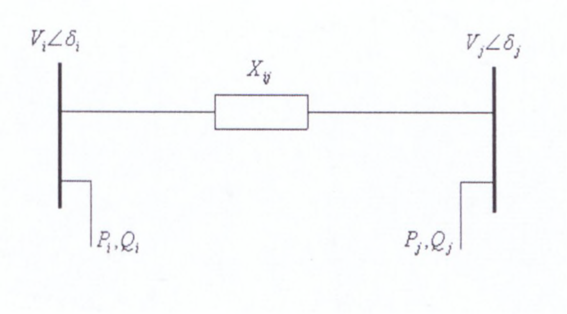


Figure 4.13: Power injection

Static VAr Compensator (SVC): SVC is a shunt connected static VAr generator or absorber whose output is adjusted to exchange capacitive or inductive current in order to maintain or control bus voltage magnitude [Song & Johns, 1999]. A typical topology of a SVC comprises a parallel combination of a thyristor controlled reactor and a fixed capacitor as shown in Figure 4.14

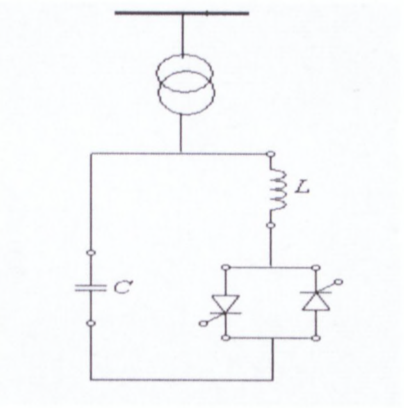


Figure 4.14: SVC topology

The reactive power injection of a SVC connected to bus *i* is given by:

$$Q_i = V_i^2 B_{SVC} \tag{4.97}$$

where

$B_{SVC} = B_C - B_L$

B_C is the susceptance of the fixed capacitor

B_L is the susceptance of thyristor controlled reactor

Thyristor Controlled Phase Angle Regulator (TCPAR): This device is basically a phase shifting transformer adjusted by thyristor switches to provide a rapidly phase angle [Hingorani & Gyugyi, 2000; Song & Johns, 1999]. A topology of a TCPAR connected in the line between buses *i* and *j* is shown in Figure 4.15 with its exciter transformer being fed from the bus *i* side.

The injected voltage can be modelled as an ideal source $\overline{V}_{se} = V_{se}\angle\varphi$ in series with line impedance $\overline{Z}_{ij}$. The injection model is obtained by replacing the voltage source with current source $\overline{I}_{se}$ in parallel with the transmission lines as shown in Figure 4.16b

Where currents $\overline{I}_{se}$ and $\overline{I}_{sh}$ are given by:

$$\overline{I}_{se} = \frac{E_{se}}{\overline{Z}_{ij}} \qquad \overline{I}_{sh} = \overline{I}_i - \overline{I}_s \tag{4.98}$$

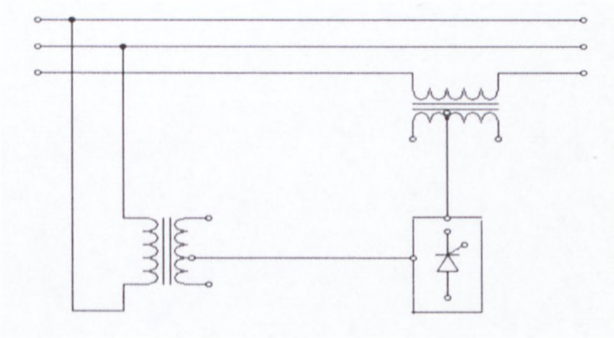


Figure 4.15: TCPAR approximate topology

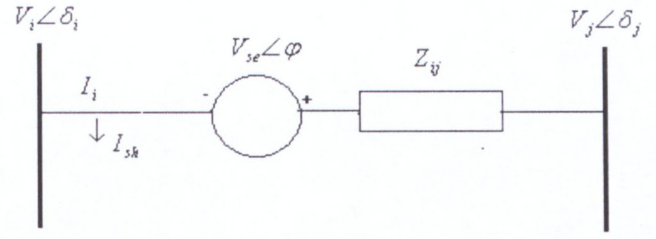


Figure 4.16a: TCPAR voltage source

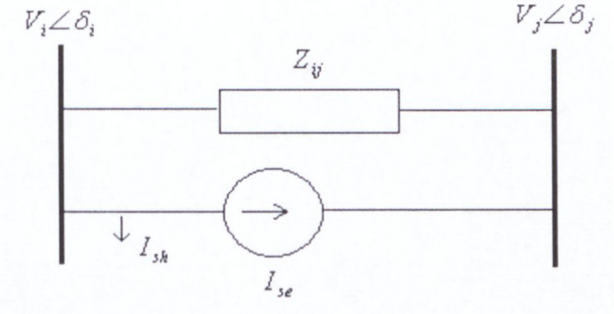


Figure 4.16b: TCPAR current source

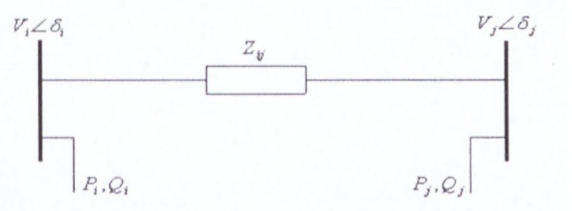


Figure 4.16c: Power injection

The series current source ($\bar{I}_s$) in addition to the shunt current $\bar{I}_{sh}$ corresponds to the injection powers $\bar{S}_i$ and $\bar{S}_j$ which are given by:

$$\bar{S}_i = E_i(-\bar{I}_{sh} - \bar{I}_s)^* \tag{4.99}$$

$$\bar{S}_j = E(\bar{I}_s)^* \tag{4.100}$$

The real and reactive power injection Eqns of the TCPAR connected between buses *i* and *j* can be obtained by algebraic simplification and are given by:

$$P_i = V_i V_j [G_{ij} \{\cos\delta_{ij} - \cos(\delta_{ij} + \varphi)\} + B_{ij} \{\sin\delta_{ij} - \sin(\delta_{ij} + \varphi)\}] \quad (4.101)$$

$$Q_i = V_i V_j [G_{ij} \{\sin\delta_{ij} - \sin(\delta_{ij} + \varphi)\} - B_{ij} \{\cos\delta_{ij} - \cos(\delta_{ij} + \varphi)\}] \quad (4.102)$$

$$P_j = V_j V_i [G_{ji} \{\cos\delta_{ji} - \cos(\delta_{ji} - \varphi)\} + B_{ji} \{\sin\delta ji - \sin(\delta_{ji} - \varphi)\}] \quad (4.103)$$

$$Q_j = V_j V_i [G_{ji} \{\sin\delta_{ji} - \sin(\delta_{ji} - \varphi)\} - B_{ji} \{\cos\delta_{ji} - \cos(\delta_{ji} - \varphi)\}] \quad (4.104)$$

4.7 Modelling of Loads

The term load means a device or a conglomeration of devices that taps energy from the power system network [Vadhera,1981]. Loads may be viewed as being divided into two classes namely static and dynamic. The static class consists of heating and lighting equipments. The dynamic (rotating) class of loads consists of synchronous motors, induction motors etc. The types of loads connected to a single bus (node) may vary over a wide range depending on the specific area (town, city, industrial, agricultural etc). The load does not remain constant but varies from time to time due to changing demand of the customers.

4.7.1 Characteristics of load elements

The characteristics of load elements are the real power or reactive power consumed by the load element, its torque or current as function of voltage and supply frequency. In practice, there are two types of characteristics:

- Static
- Dynamic

Static: is a relation between power, torque or current and the voltage (or frequency) defined for slow variations of the operating conditions; the following functions are defined as static:

$$P = P(|V|) \\ Q = Q(|V|) \quad (4.105)$$

Where *P*, *Q* and *V* represent the real power, reactive power and voltage respectively

Dynamic: is the same relation as for static but defined for fast variations of the operating conditions. The dynamic characteristics in some operating variable can be presented as a function of one or more operating variables and their derivatives as:

$$P = P(|V|, f, \frac{d|V|}{dt}, \frac{df}{dt}) \tag{4.106}$$

4.7.2 Composite load

Composite load at a bus varies with voltage and frequency. It may include some elements of lighting and heating, induction motors, rectifiers and inverters as well as transformer and cable losses. Composite load constitute the vast majority of loads in actual practice. Their real and reactive power may be expressed as:

$$\begin{aligned} P &= P(|V|, f) \\ Q &= Q(|V|, f) \end{aligned} \tag{4.107}$$

For small variations in supply frequency and voltage, the change in real power and reactive power can be expressed as:

$$\Delta P = \frac{\partial P}{\partial |V|}\Delta|V| + \frac{\partial P}{\partial f}\Delta f \tag{4.108}$$

$$\Delta Q = \frac{\partial Q}{\partial |V|}\Delta|V| + \frac{\partial Q}{\partial f}\Delta f \tag{4.109}$$

Eqns (4.113) and (4.114) can be written in matrix form as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial |V|} & \frac{\partial P}{\partial f} \\ \frac{\partial Q}{\partial |V|} & \frac{\partial Q}{\partial f} \end{bmatrix} \begin{bmatrix} \Delta |V| \\ \Delta f \end{bmatrix} \tag{4.110}$$

Eqn (4.115) is the basic mathematical load model of a composite load.

The mathematical model is developed based on the following assumptions:

- Real and reactive powers of the load are differentiable functions of voltage magnitude and frequency
- The changes in voltage and frequency is small

4.8 Conclusion

AC synchronous generator is an important component of an electrical power system. In this chapter, basic information of the synchronous generator is given. After that the general synchronous generator equations including the self and mutual inductances of the machine are derived. The general basic machine equations in *d, q, 0* frame of reference using Blondel's transformation have been developed. The AC synchronous model both in classical and model appropriate for power flow and power system state estimation studies are presented

A transformer is a static piece of equipment with the ability of transforming electrical power in one circuit to another with the same frequency. Tap-changing and regulating transformer (RT) models are derived. Transmission line is another key and critical component of the transmission network. Mathematical models as well as their circuit representation for the short, medium and long length transmission lines lines are derived and presented.

Synchronous generator; tap-changing and regulating transformers. Transmission line; FACTS devices; and Composite load models developed in this chapter are very useful, in forming a one-line diagram and network model that represent the Tanzanian power system network described in chapter 5. The model is used in load- flow and state estimation digital computer simulation studies.

Chapter 5 presents data acquisition, collection and classification methods and how to prepare database using MYSQL DBManager Professional Software; system network modeling and decomposition of the network model into block sub matrices.

CHAPTER FIVE: DEVELOPMENT OF SYSTEM NETWORK MODEL (TANZANIA)

5.1 Introduction

In chapter 4 mathematical models for a steady-state operation of a power system were developed. These models are very useful in load flow and power system state estimation studies. In this chapter, data acquisition and classification methods and how to prepare database using MYSQL software are presented. Generation schedule data, transmission and load demand of the Tanzanian Power system network are given.

System network modelling process that incorporates the models developed in chapter 4 is presented. The network model forms a basis for load flow analysis and power system state estimation studies presented in chapter 6. Also the model is used in decomposition process to form block sub matrices that will be applied in the solution of power system state estimation problem derived in chapter 7.

5.2 Data Acquisition Method

The high voltage (HV) of the Tanzania's power system network consists of several voltage levels: 220kV, 132kV and 66kV [http://www.tanesco.com]. The system has 38 main HV transmission and more than 100 distribution buses scattered all over the country. The thesis aims to develop a transmission power system network model, which can be used for load -flow, load forecasting, optimal power flow (OPF), stability analysis, and state estimation (SE) studies. So, data and information relating to model development were needed. Tanzania electricity Supply Company limited (TANESCO) and Ministry of Energy and Minerals [http://www.nishati.go.tz] were contacted to source for required information and data.

5.2.1 Data Classification

The data required to build the network model are divided into:

- Transmission network data (at 66kV, 132kV and 220kV level)
- Power plant locations.
- Type of fuels.
- Capacities of generating plants
- Load centre locations

- Load centre demand capacities

5.3 Data Collection

Tanzania Electricity Supply Company Limited (TANESCO), a government owned utility company owns majority of generating power plants except two thermal plants situated in Dar es Salaam, which are private owned. TANESCO owns high voltage and distribution transmission lines scattered all over the country.

The transmission lines are estimated to comprise of 2,624.4 km of system voltage 220kV; 1441.50 km of 132kV; and 486.0km of 66kV, totalling 4,551.86 kms by the end of December 2006. [http://www.tanesco.go.tz/2006]

TANESCO, in principle refused to make available load flow model and transmission capacity limits information citing sensitivity of the information. However, some information was made available such as:

- Generation/substation description.
- National generation, peak load
- Technical reports (various)
- TANESCO power system master plan (2003)
- Transmission grid schematic (various)
- Electrical parameters for 220kV, 132kV and 66kV
- TANESCO hydro generation optimization report
- Grid loading information
- All transmission substation schematic single-line diagrams.
- Operations Data (2003), Grid loading information
- TANESCO Transmission Equipment Nameplates
- TANESCO Annual Operation Report-2003
- TANESCO Power System Master Plan (2004)
- TANESCO Hydro Generation Optimization Report, January (2004)

The documents were used to extract the required information and data to achieve the thesis objective and create a working database. Some buses were merged to form equivalent bus. It was necessary to merge these buses because too-short transmission line in the network might cause ill-conditioning in load flow and state estimation studies. A database for 30 buses is prepared basing on documents.

5.4 Creating Database Using MySQL

My Structured Query Language (MYSQL) is a program that runs as a server providing multi-user access to number of databases. It is an interactive program that allows connecting to a MySQL server if available, run queries and view the results. MySQL may also be used in batch mode.

The aim of creating database using MySQL is to keep track of various types of information about the data, update, delete, and retrieving or make changes for future use of the data. The advantages of using MySQL are:

- Loading the database with the desired information
- Create tables to hold the data
- Answer different questions by retrieving data from the tables.

The process of creating and using the database include the following operations:

- Create a database
- Create a table
- Load data into the table
- Retrieve data from the table
- Use more than one table

Create Database

The MySQL administrator can create database for the user. Here a brief description is given how to create a database from well established principles. However, creation of database for implementing a state estimation of the Tanzanian Power System Network Model is done using DBManagement Professional software.

Create a Table

Before data can be entered (row) into a table, it is important to define what kind of data will be stored (column). The following MySQL code is used

```
<?php
// make a MySQL connection
mysql_conn("localhost","admin","ladmin") or die(mysql_erro( )' mysql_selected-db("test"
or die(mysql_error( ));
```

// Create a mysql table in the selected database mysql_query("CREATE TABLE" adam2009 (id INT NOT NUL AUTO_INCREMENT, PRIMARY KEY (id),

nameVARCHAR (25),

age INT)")

or die(mysql_error( ));

echo "Table Created";

?>

- INT –Stands for integer or whole number. "id" is defined as an integer.
- NOT NULL – These are two keywords, but they combine together to say that this column cannot be null (empty)
- AUTO_INCREMENT – Each time a new entry is added the value will be incremented by one (1)
- PRIMARY KEY (id) – is used as a unique identifier for rows
- name VARCHAR (25) – A new column is made with name VARCHAR: stands for "Variable character", "Character" means that any kind of typed information can be put in this column (letters, number, symbol etc). It is "variable" because it can adjust its size to store a little as 0 characters and up to a specified maximum number of characters.
- "Integer" – The third and final column, which store an integer.
- or die (mysql_error( )); - This command prints an error if there is a problem in table creation process.

Loading Data into a Table

After a table has been created, next is to populate it. The LOAD DATA and INSERT statements are used. Since the created table is empty, an easy way to populate it is to create a text file containing a row for each of the available data, and then the content of the file is loaded into the table with a single statement. To load the text file into the created table, the following command is used:

```
mysql>LOAD DATA LOCAL INFILE '/path/name.txt' INTO TABLE '....'
       -> LINE TERMINATOR BY 'r'n';
```

To add new records one at a time, the INSERT statement is used

```
Mysql > INSERT INTO '....'
       - > VALUES ('          ' , NULL);
```

Retrieving Information from a Table

The SELECT statement is used to retrieve information from a table. The general form of the statement is given by:

SELECT what_to_select

FROM which_table

WHERE condition_to_satisfy

What_to_select: indicates what to be selected from the table. This can be a list of columns.

Which_table (Where as optional if is present): indicates the table from which data is to be retrieved.

Conditions_to_satisfy: Specifies one or more conditions that rows must satisfy to qualify for retrieval.

The following actions relate to retrieving information from a table:

Selecting all data: This form of SELECT is useful when a review of an entire table is needed after loading with initial data set

Selecting Particular Rows: It is easy to retrieve an entire table. This is done by omitting the WHERE clause from the SELECT statement. When the table becomes large, it is not possible to see the entire table; instead particular rows can be selected from the table. The following statement is used to perform the action

mysql > SELECT * FROM '.....' WHERE name ='.....'

Selecting Particular Columns: If the entire rows from the created table are not needed, it is possible to name the columns in which the data are to be retrieved. This is done by naming the columns, separated by comma as follows:

mysql > SELECT name '...' FROM '....'

The WHERE clause can be used to combine row selection with column selection. The statement for this combination is:

mysql > SELECT name, '....','....' FROM '....'

- > WHERE data = ' ....' OR data ' ....'

Sorting Rows: It is important the output rows to be sorted and displayed in some meaningful way. Thus, ORDER BY clause is used in sorting and displaying the rows in a particular order.

```
mysql > SELECT name, ' ...' FROM ' ....' ORDER BY ' ....'
```

Using More Than one Table

The table created keeps track of the data. However, if new data are received, the information has to be recorded in a new table. The new table should contain the following information:

- Name of the table
- A date when the data was received
- A field such as transmission line data or generation data
- An event type field to categorize the event.

Hence, the CREATE TABLE statement for the new EVENT table is:

```
mysql > CREATE TABLE event (name VARCHAR (20), date DATE,
    - > type VARCHAR (15), remark VARCHAR (255));
```

The records are loaded in the event table using the following statement:

```
mysql > LOAD DATA LOCAL INFILE 'event.txt' INTO TABLE event
```

Source: The MYSQL 5.1 Reference Manual: Revision (16027). 2009.08.10

The following data are used in creating the database.

5.4.1 Generation schedule and spinning reserve

Power plant fuel types and capacities are related closely to their outputs, and therefore influence power flows. Generation data are commercially sensitive because they can reflect the marginal generation cost of generators. TANESCO and the two independent power producers (IPPs) i.e. Independent power Tanzania limited (IPTL) and SONGAS refused to make available minimum and maximum values of their generator's reactive power. However, they were eager to provide maximum real power output in MW. Due to this inconsistency, it is decided to use -9999 and +9999 values as minimum and maximum reactive powers for all generators operating in the system. Synchronous generators in the built model are simply classified as hydraulic or fossil thermal, so no sensitive information is included. To run a load-flow program and later a power system state estimation program, one needs the actual outputs of the power plants which could be obtained from a dispatch program. It was not possible TANESCO to run the dispatch program; instead generation schedule with spinning reserve from TANESCO is used. The data is given in Table 5.2. In addition, each power generating power plant is related to as a bus and identified by a name (which is a name of a place where the plant is located). Their abbreviated names are shown in Table 5.1 Table 5. 3 shows the generator capacities including their ratings MW, MVA, MVAr, minimum MW, maximum MW, minimum MVAr, and maximum MVAr. However, private generating plants were not eager to provide these data.

Table 5.1: Bus name and voltage level

Bus No.	Bus Name	Voltage [kV]	Bus No.	Bus Name	Voltage [kV]	Bus No.	Bus Name	Voltage [kV]
1	KIH	220	**11**	ZNZ	132	**21**	BAB	220
2	IRI	220	**12**	CHA	132	**22**	SGD	220
3	MUF	220	**13**	HALE	132	**23**	DOM	220
4	MBE	220	**14**	NPF	132	**24**	MTE	220
5	KID	220	**15**	TAG	132	**25**	SHY	220
6	MORO	220	**16**	KIY	132	**26**	MZA	220
7	MOROI	132	**17**	KIYI	66	**27**	MZAI	132
8	UBU	220	**18**	NYM	66	**28**	MSM	132
9	UBUI	132	**19**	ARU	66	**29**	SHYI	132
10	TEG	132	**20**	ARUI	220	**30**	TBR	132

Table 5.2: Generation Schedule with Spinning Reserve

Plant No.	Plant Name	Power Output [MW]	Spinning Reserve [MW}	Plant No.	Plant Name	Power Output [MW]	Sinning Reserve [MW]
1	KIH	60.00	120.00	**18**	NYM	03.60	00.00
4	MBE	14.00	00.00	**23**	DOM	00.00	07.44
5	KID	142.00	62.00	**24**	MTE	74.00	06.00
9	UBUI	259.00	00.00	**27**	MZAI	13.00	00.00
10	TEG	100.00	00.00	**28**	MSM	00.00	02.56
13	HALE	10.50	00.00	**30**	TBR	00.00	10.20
14	NPF	68.00	00.00				

5.4.2 Transmission Lines

Transmission line network comprises of high and medium voltage levels. Transmission line network especially high voltage is the backbone for transmitting power from the generation plants to demand centres as well as for load flow and state estimation studies. To develop a database and a model of the transmission line network is a time consuming activity. The electrical parameters of transmission lines are generally not available to the public; they can be calculated or estimated from the lengths and voltage levels of the lines; Tables 5.4 gives distances between buses and Table 5.5 gives Shunt devices installed at some of the buses of the system and their ratings. Resistance values per kilometre of overhead lines are available and structure of the transmission lines; they were to calculate the parameters of the lines. The following constants for transmission cables to prepare a database for transmission line network parameters were adopted.

Table 5.3: Generator capacities

Plant No.	Plant Name	Rating [MW	Rating [MVA]	Rating [MVAr]	pf	P_{Min} [MW]	P_{Max} [MW]	Q_{Min} [MVAr]	Q_{Max} [MVAr
1	KIH	180.00	211.50	111.05	0.85	39.00	180.00	-111.05	111.05
4	MBE	14.00	-	-	0.85	-	-	-	-
5	KID	204.00	240.00	126.43	0.85	96.00	204.00	-126.43	126.43
9	UBUI	259.00	-	-	0.85	-	-	-	-
10	TEG	100.00	-	-	0.85	-	-	-	-
13	HALE	21.00	24.70	13.00	0.85	10.00	21.00	-13.00	13.00
14	NPF	68.00	80.00	42.14	0.85	30.00	68.00	-42.14	42.14
18	NYM	08.00	10.00	06.00	0.85	04.00	08.00	-06.00	06.00
23	DOM	07.44	-	-	0.85	-	-	-	-
24	MTE	80.00	90.00	41.23	0.85	30.00	80.00	-41.23	41.23
27	MZAI	13.00	-	-	0.85	-	-	-	-
28	MSM	02.56	-	-	0.85	-	-	-	-
30	TBR	10.20	-	-	0.85	-	-	-	-

BLUE JAY: $R=0.05019\Omega/km$

RAIL: $R=0.05118\Omega/km$

HAWK: $R=0.1137\Omega/km$

BISON: $R=0.07254\Omega/km$

WOLF: $R=0.1694\Omega/km$

RABBIT: $R=0.2000\Omega/km$

Transmission line network data are instrumental in system network model development. The following assumptions were made before developing the model.

- Only those lines with voltage levels above and including 66kV are considered.
- The resistance and shunt admittance of some of transmission lines were neglected (chapter 4 Table 4.2).
- All circuit breakers are assumed to be closed under normal operation in order to make full utilization of the whole network.

Table 5.4: Transmission line distances

From Bus No.	To Bus No.	Distance [km]	From Bus No.	To Bus No.	Distance [km]	From Bus No.	To Bus No.	Distance [km]
1	2	97.00	10	11	67.50	22	23	210.00
2	3	130.00	12	13	175.00	23	24	130.00
3	4	220.00	13	14	11.30	22	25	200.00
2	5	160.00	13	15	60.00	25	26	140.00
1	5	180.00	14	15	64.00	27	28	205.00
2	24	107.00	13	16	275.00	29	30	200.00
5	6	128.00	16	19	78.00			
6	8	178.00	18	17	53.00			
7	12	82.00	20	21	162.00			
9	10	18.50	21	22	150.00			

Table 5.5: Shunt devices and their ratings

Bus No.	Bus Name	Voltage [kV]	No. of Devices	Rating [MVAr]
1	KIH	220	1	20.00
9	UBUI	132	1	20.00
20	ARUI	220	1	20.00
22	SGD	220	2	40.00
23	DOM	220	1	20.00

5.4.3 Installed Load Demand

Load centre location and capacities also have important influence on the two studies i.e. load-flow and SE. Unfortunately there was no actual sufficient information about loading of particular buses (substations). Data existing at TANESCO were in great proportion design and forecasting data. To have a full picture of demands of buses, design, forecasting data from TANESCO together with data from Ministry of Finance and Planning (MFP), and Ministry of Minerals and Energy (MEM) were used. The following web site was used;

(http://www.tanzania.go.tz/economicsurveyf.html/2008); from year 2002 to 2008. The data were analysed and demand for each substation established. Using the forecasting design and forecasting data alone was thought to be inappropriate. The obtained data corresponds approximately to demand of a particular area or region where the bus is located. Demand data are shown in Table 5.6. Transformers (power, tap-changing or regulating) are very important in both load flow and state estimation. Table 5.7 gives data for the installed transformer in the system.

Table 5.6: Installed load demand

Bus No.	Power Demand		Bus No.	Power Demand	
	MW	MVAr		MW	MVAr
2	6.20	1.50	**19**	22.00	5.00
3	20.00	7.00	**21**	6.50	1.20
4	27.20	7.80	**22**	5.00	1.40
6	18.00	9.10	**23**	6.20	1.60
8	233.10	45.10	**25**	21.70	9.00
11	17.60	9.00	**26**	29.70	9.60
12	12.00	2.50	**28**	11.50	5.00
15	21.00	8.30	**30**	5.40	1.50
16	23.09	9.00			

Table 5.7: Tap-changing transformers

Serial No.	Transformer Designation	Tap Setting Range [p.u.]	Setting Point
1	**5-6**	0.9000-1.1000	0.9000
2	**8-9**	0.8590-1.0846	0.8590
3	**16-17**	0.9000-1.1000	0.9000
4	**19-20**	0.8590-1.0750	0.8590
5	**25-29**	0.9000-1.1000	0.9000
6	**26-27**	0.9000-1.1000	0.9000

5.4.4 MYSQL Implementation

Creating database for the Tanzanian Power System Network is implemented using DBManager Professional software. It is the most powerful application for data management with built in support for MYSQL, PostgreSQL, Interbase/Firebird, SOLite, DBF Tables, MSAcess, MSSQL Server, Sybase, Oracle and ODBC database engines. Platform supported are Windows 2003 and Windows XP.

DBManagement Professional has the following features:

- Management for database, tables, domains, etc
- Query Editor with Query Editor, debugger, Planner, with multiple result set
- Many wizards to import and export data to and from a variety of sources, including: MSAcess, MSExcel, Test and XML files
- Database compares and database Control Version Systems
- Diagram Designer
- Form and Report Builder
- Monitor Server, Database and table activities.

Creating a database using DBManager Professional is the basis toward real-time implementation of state estimation

5.5 Integrated System Network Modelling

Systematic computation of currents and voltages in power system network when using digital computer may most conveniently be achieved by building a desirable mathematical model. System matrix equations provide a suited mathematical model and account both the characteristics of individual elements of the system network as well as their interconnections.

Analysis of a large power system similar to the Tanzanian power system network requires a primitive network first to be considered. The primitive network is a set of uncoupled elements of the system network [Vadhera, 1981]. Normally data available (section 5.2 to 5.4) of the Tanzania Electric Supply Company Limited (TANESCO) are in the form of primitive network matrix. This matrix, while adequately describes the characteristic of each element, does not provide any information concerning the network connections. Therefore, the primitive network matrix must be transformed into a network matrix that delineates the performance of the interconnected system network.

The first step in system network modelling is to prepare a single-line diagram of a 3-phase system. Since a balanced 3-phase system is solved as a single-phase circuit composed of one of 3 lines and neutral return, it is not necessary to show more than one phase and the neutral return when drawing the single-line diagram. The single-line diagram can be further simplified by omitting the complete circuit through the neutral by indicating each AC synchronous generator or transformer by appropriate symbol rather than by their equivalent circuit.

With all this in mind, a single-line diagram of the Tanzanian Power System Network is prepared and is depicted in appendix A5; this single-line diagram is used in developing system network model.

5.5.1 Integrated Network Equations

In order to develop system network equations, a portion of the one-line diagram shown in appendix A5 is used. The net power injected or currents injected in this portion are depicted in Figure 5.1. Using KCL, the following 3 equations are obtained:

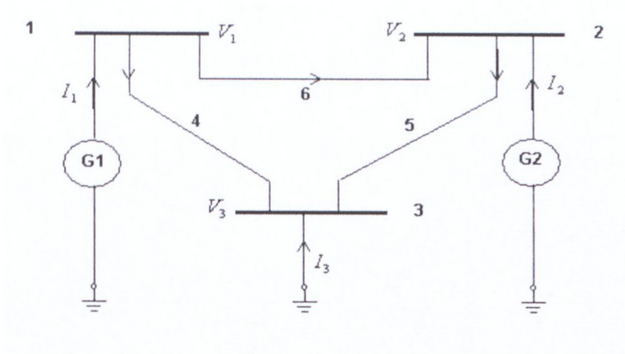


Figure 5.1: A network of 3 buses (a portion of appendix A5)

$$
\begin{aligned}
I_1 &= (Y_6+Y_4)V_1 - Y_6V_2 - Y_4V_3 \\
I_2 &= -Y_6V_1 + (Y_5+Y_6)V_2 - Y_5V_3 \\
I_3 &= -Y_4V_1 - Y_5V_2 + (Y_4+Y_5)V_3
\end{aligned} \qquad (5.1)
$$

Eqn (5.1) may be written as:

$$
\begin{aligned}
I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 \\
I_2 &= Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 \\
I_3 &= Y_{31}V_1 + Y_{31}V_2 + Y_{33}V_3
\end{aligned} \qquad (5.2)
$$

Eqn (5.2) can be extended to the whole system andcan be written in compact matrix form

$$I = YV \qquad (5.3)$$

where

$$I \underline{\underline{\Delta}} \begin{pmatrix} I_1 \\ \vdots \\ I_N \end{pmatrix} \text{ is the current vector} \qquad (5.4)$$

$$V \underline{\underline{\Delta}} \begin{pmatrix} V_1 \\ \vdots \\ V_N \end{pmatrix} \text{ is the voltage vector} \qquad (5.5)$$

$$Y_{BUS} \underline{\Delta} \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & & & \\ Y_{N1} & Y_{2N} & \cdots & Y_{NN} \end{pmatrix} \text{ is the bus admittance matrix} \tag{5.6}$$

Hence, in this thesis the system network model is given by:

$$I = YV \tag{5.7}$$

Bus admittance matrix (*Ybus*) of the model is given in appendix *E*

From Eqn (5.2), at bus i the injected current I_i for a general ***N-*** bus system (Case study: Tanzanian Power System)

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j \tag{5.8}$$

where

$$I_i = \frac{S_i^*}{V_i^*} \tag{5.9}$$

S_i^* : is the complex power at bus i

5.5.2 Incorporating Transformer model in the Ybus Matrix

Transformer discussed in this section is for a fixed tap-setting transformer. This transformer is used during load flow simulation in chapter 10.

A transformer with a fixed-tap setting is represented by impedance or admittance [Momoh, 2009; Vahdera, 1981; Granger & Stevenson, 1994] Y_{ij} in serial with an ideal autotransformer (Figure 5.2). For such transformer, at bus i in the line l_{ij}, the transformer ratio is given by:

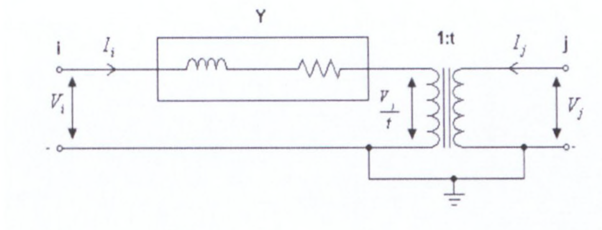


Figure 5.2: Fixed tap-setting transformer

$$\frac{V_i}{V_t} = \frac{I_{tj}}{I_i} = t \tag{5.10}$$

Where V_i is the voltage magnitude at bus i

Hence

$$I_i = \frac{I_{tj}}{t} = (V_t - V_j)\frac{Y_{ij}}{t} \tag{5.11}$$

But

$$V_t = \frac{V_i}{t} \tag{5.12}$$

Substituting Eqn (5.12) into Eqn (5.11) gives

$$I_i = (V_i - tV_j)\frac{Y_{ij}}{t^2} \tag{5.13}$$

Similarly

$$I_j = (V_j - V_t)Y_{ij} = (tV_j - V_i)\frac{Y_{ij}}{t} \tag{5.13}$$

Such a transformer model can be represented by an equivalent π-circuit shown in Figure 5.3. where ***A*** is the series impedance; ***B*** and ***C*** are the shunt admittances. The values of ***A***, ***B*** and ***C*** are given by:

$$A = \frac{Y_{ij}}{t}, B = \frac{1}{t}\left[\frac{1}{t} - 1\right]Y_{ij}, C = \left[1 - \frac{1}{t}\right]Y_{ij} \tag{5.14}$$

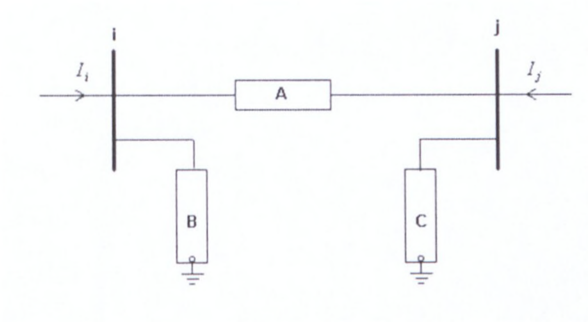


Figure 5.3: Equivalent circuit of a fixed -tap transformer

With the substitutions of ***A, B*** and ***C,*** Figure 5.3 takes the form of Figure 5.4 which is appropriate for load flow and state estimation studies.

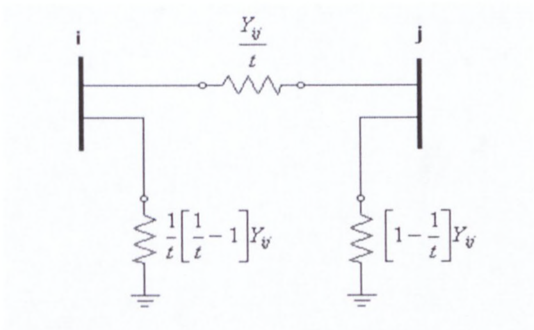


Figure 5.4: Equivalent π-circuit of a fixed tap- transformer

Thus, to represent the transformer shown in Figure 5.4 in the elements of the bus admittance matrix:

$$Y_{ii} = Y_{i1} + Y_{i2} + Y_{i3} + \cdots + \frac{Y_{ij}}{t} + \cdots + Y_{iN} + \frac{1}{t}\left[\frac{1}{t} - 1\right]Y_{ij}$$
$$Y_{ii} = Y_{i1} + Y_{i2} + Y_{i3} + \cdots + \frac{Y_{ij}}{t} + \cdots + Y_{iN} \tag{5.15}$$

Y_{ii} element in the bus admittance matrix changes when the fixed-tap transformer model is included. Moreover,

$$Y_{ij} = Y_{ji} = -\frac{Y_{ij}}{t} \tag{5.16}$$

Also

$$Y_{ji} = Y_{j1} + Y_{j2} + Y_{j3} + \cdots + \frac{Y_{ij}}{t} + \cdots + Y_{jN} + \left[1 - \frac{1}{t}\right] Y_{ij}$$
$$Y_{ji} = Y_{j1} + Y_{j2} + Y_{j3} + \cdots + Y_{ji} + \cdots + Y_{jN} \qquad (5.17)$$

Introducing fixed-tap transformer model does not change the Y_{ji} element in the bus admittance matrix. For a tap-changing under load transformer, the elements Y_{ii} and Y_{ij} are calculated with the changes in tappings.

5.6. Decomposing Ybus Matrix

The concept of matrix decomposition is based on decomposing large matrix into block matrices. Matrix decomposition is a process of transforming a given matrix into smaller matrices that simplifies the computing process. It is a fundamental theme in linear algebra as well as in applied statistics, which have scientific and engineering significance in a day to day of solving practical problems.

The objective of matrix decomposition involves two aspects: First, computational convenience when working with smaller matrices. Second; analytic simplicity; it is easier, comfortable and simple to make analysis on smaller matrix rather than a large matrix. Computing a matrix inverse of a large matrix or computing a determinant of a same matrix. It is cumbersome and time consuming. Thus, to convert a large matrix into block matrice became easier to work with. Computations will be simplified and time saved. Data matrices representing large numerical values such as Ybus matrix are hard to compute and analyse. Therefore, decomposing the data matrix into some lower-order or lower-rank standard form will reveal the inherent characteristic and structure of the matrix and help to compute or interpret its meaning.

The bus admittance matrix of the Tanzanian power system network is decomposed into 3 blocks. The decomposition of the matrix $\boldsymbol{Y} \in \Re^{30x30}$ is achieved on the basis of analysis of its structure and the following criteria:

- Selecting dimensions of the main sub matrices in such a way that the number of interconnections is small
- Selection of the interconnection between the submatrices to be of special kind

On the basis of the above, the dimensions of the considered three main submatrices are determined as: $n1 = 12$, $n2 = 9$, $n3 = 9$. Total $n1+n2+n3 = 30$ buses

$$Y = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \in \Re^{30x30} \tag{5.18}$$

where

$A_{11} \in \Re^{12x12}$, $A_{22} \in \Re^{9x9}$, $A_{33} \in \Re^{9x9}$

Are the block diagonal admittance submatrices, characterized with high density of buses and transmission lines; $A_{12} \in \Re^{12x9}$, $A_{13} \in \Re^{12x9}$, $A_{21} \in \Re^{9x12}$, $A_{23} \in \Re^{9x9}$, $A_{31} \in \Re^{9x12}$ $A_{32} \in \Re^{9x9}$ are the sub-matrices forming the interconnections with the main ones (**A11,A22,A33**).

The determined dimensions are used to decompose the vectors of voltage and currents in the following way

$$V = [V_1^T, V_2^T, V_3^T]^T \tag{5.19}$$

$$I = [I_1^T, I_2^T, I_3^T]^T \tag{5.20}$$

The model of the *N*-bus system is given by the equation

$$I = YV \tag{5.21}$$

This model can be represented as a set of interconnected sub models. Substitution of the corresponding decomposed vectors and admittance matrix gives:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{5.22}$$

On the basis of eqn (5.22), the expression for every separate interconnected subsystem of the model can be written in the following way:

$$\begin{aligned} I_1 &= A_{11}E_1 + A_{12}E_2 + A_{13}E_3 \\ I_2 &= A_{21}E_1 + A_{22}E_2 + A_{23}E_3 \\ I_3 &= A_{31}E_1 + A_{32}E_2 + A_{33}E_3 \end{aligned} \tag{5.23}$$

Each of the Eqns in (5.23) describes the behaviour of the interconnected sub-system. It depends on the following input inputs:

- Its own input voltage

- The sum of the currents coming from other interconnected subsystems to it's subsystem, which further will be noted with the letter **Z**

The two inputs have different meaning for the power subsystems:

- The first input can be considered as subsystem local load control signal.
- The second can be considered as a disturbance influencing the behaviour of the subsystem. If this disturbance is zero, then all subsystems will be independent.

The structure of the system considered as an interconnection of independent subsystems, where the interconnection influence is considered as a disturbance is shown in Figure 5.5.

On the basis of this representation, the mathematical model can be described in the following manner:

$$\begin{aligned} I_1 &= A_{11}E_1 + Z_1 \\ Z_1 &= A_{12}E_2 + A_{13}E_3 \end{aligned} \tag{5.24}$$

$$\begin{aligned} I_2 &= A_{22}E_2 + Z_2 \\ Z_2 &= A_{21}E_1 + A_{23}E_3 \end{aligned} \tag{5.25}$$

$$\begin{aligned} I_3 &= A_{33}E_3 + Z_3 \\ Z_3 &= A_{31}E_1 + A_{32}E_2 \end{aligned} \tag{5.26}$$

where

$\mathbf{Z_1} \in \Re^{n1}$, $\mathbf{Z_2} \in \Re^{n2}$, $\mathbf{Z_3} \in \Re^{n3}$ are the interconnection inputs.

The model given by eqns (5.24), (5.25) and (5.26) can be written in common notation as:

$$\begin{aligned} I_i &= A_{ii}E_i + Z_i \\ Z_i &= \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij}E_j \qquad i = \overline{1, N_S}, \quad j = \overline{1, N_S} \ , j \neq i \end{aligned} \tag{5.27}$$

Where N_S is the number of sub-systems

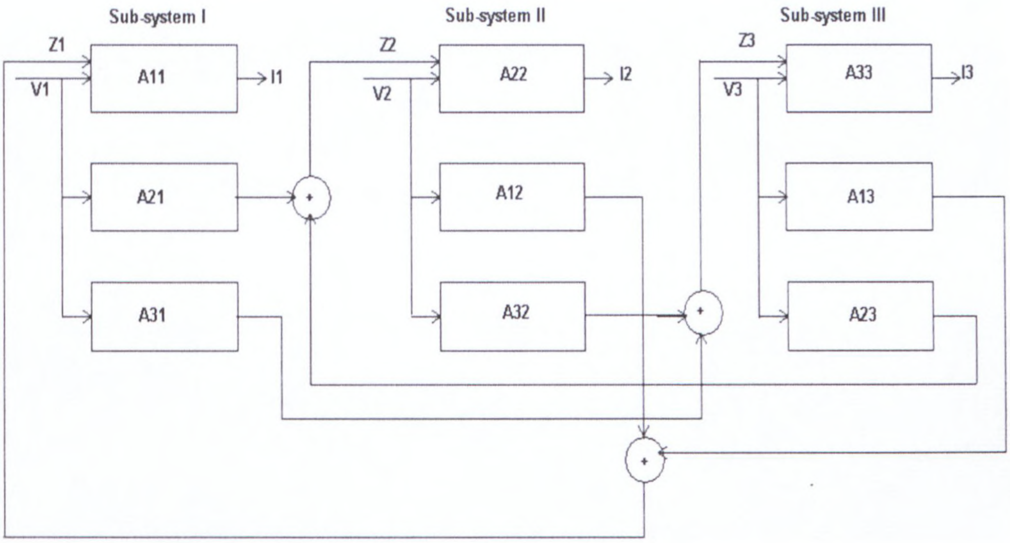


Figure 5.5: Decomposed model structure

Finally, matrices obtained after decomposition can be written as:

The block matrices (**A11, A22, and A33**) have well defined structure which makes it easy to be constructed. Their properties are:

- Square of order $n_i x n_i \qquad i = 1,2,3$
- Symmetrical
- Complex
- Each off-diagonal element is the negative of branch admittance between buses available in the sub-system
- Each diagonal element is the sum of the admittance of branches which terminate at bus *p*, including branches to ground
- The matrices **A11**, **A22** and **A33** are not sparse compared to the global bus admittance matrix.

As conclusion: The matrices situated on the main diagonal of the bus admittance matrix obtained after decomposition show all the features of the full admittance matrix, but are not sparse and are with smaller dimension

Let the ***ith*** sub-system from the formed sub-systems is considered. By assuming bus *p* and *q* as two neighbouring buses in this sub-system *i*, the current, voltage and interconnections from equation (5.27) can be expressed in a vector form as:

$$I_i \in R^{n_i} = \begin{bmatrix} I_1 \\ \vdots \\ I_p \\ \vdots \\ I_{n_i} \end{bmatrix} \qquad V_i = \begin{bmatrix} V_1 \\ \vdots \\ V_q \\ \vdots \\ V_{n_i} \end{bmatrix} \qquad Z_i = \begin{bmatrix} Z_1 \\ \vdots \\ Z_p \\ \vdots \\ Z_{n_i} \end{bmatrix} \tag{5.28}$$

where

$A_{ii} \in R^{n_i x n_i}$: the dimension of ***ith*** sub-system matrix

$p = 1, \cdots n_i$
$q = 1, \cdots n_i$

Are the ***ith*** sub-system admittance matrix indexes for the elements at each row and column

n_i :number of buses in the ***ith*** sub-system

For the ***ith*** isolated sub-system, the ***p-th*** component of the current I_i can be expressed as:

$$I_{i,p} = \sum_{q=1}^{n_i} A_{ii,pq} V_q \qquad p = \overline{1, n_i} \tag{5.29}$$

However, the ***ith*** sub-system is interconnected to a neighbouring sub-system j. Including the interconnection in (5.29) changes thise equation to

$$I_{i,p} = \sum_{q=1}^{n_i} A_{ii,pq} V_q + Z_{i,p} \qquad p = \overline{1, n_i} \tag{5.30}$$

But

***Z**$_i$ is* given by

$$Z_i = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} A_{ij} V_j \tag{5.31}$$

And the ***p-th*** element of the vector ***Z**$_i$* is given by the expression

$$Z_{i,p} = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \sum_{g=1}^{n_j} A_{ij,pg} V_{j,g} \tag{5.32}$$

where

$A_{ij,pg}$: is one element of the matrix $A_{ij} \in R^{n_i x n_j}$

Substituting $Z_{i,p}$ into (5.30) yields

$$I_{i,p} = \sum_{q=1}^{n_i} A_{ii,pq} V_q + \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \sum_{g=1}^{n_j} A_{ij,pg} V_{j,g} \tag{5.33}$$

where

N_s : number of sub-systems

$A_{ij} \in R^{n_i x n_j}$ $i = 1, \cdots, N_S, j = 1, \cdots, N_S, j \neq i$: coupling matrix between sub-system i and j

V_j : voltage from sub-system j linked with sub-system i

Eqn (5.33) can be used in formulating load flow non-linear equation of the sub-systems as well as decomposition-coordination method and algorithm for solution of multi-area (sub-system) state estimation problem presented in chapter 8.

5.6.2 Real and reactive power injection model of a sub-system

This section describes a process of developing a model of real and reactive power injections of a sub-system resulting from system decomposition. Let us consider ***ith*** isolated sub-system, and then current injected into bus ***p*** of this sub-system is given by the following equation

$$I_{i,p} = \sum_{q=1}^{n_i} A_{ii,pq} V_{i,q} + \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \sum_{g=1}^{n_j} A_{ij,pg} V_{j,g} \tag{5.34}$$

However, in power systems it is comfortable to work with real and reactive powers instead of currents. Working with powers can be easier in expressing the equations in polar coordinate form. Hence, eqn (5.34) must be transformed into expressions that include real and reactive powers. The transformation is normally carried out by considering current injected at bus p of the ***ith*** isolated sub-system.

The current injected into bus p of the ***ith*** sub-system is given by

$$I_{i,p} = \frac{P_{i,p} - jQ_{i,p}}{E_{i,p}} \tag{5.35}$$

Substituting eqn (5.34) into (5.35), the following eqn is obtained

$$\frac{P_{i,p} - jQ_{i,p}}{E_{i,p}} = \sum_{q=1}^{n_i} A_{ii,pq} E_{i,q} + \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \sum_{g=1}^{n_j} A_{ij,pg} E_{j,g} \tag{5.36}$$

Re-arranging eqn (5.36), yields

$$P_{i,p} - jQ_{i,p} = E_{i,p} \left(\sum_{q=1}^{n_i} A_{ii,pq} E_{i,q} + \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \sum_{g=1}^{n_j} AE_{j,g} \right) \tag{5.37}$$

It can be observed from equation (5.37), that real and reactive powers have two parts. The first part is power corresponding to the ***ith*** isolated sub-system and the second part is power corresponding to the interconnection.

Expressing the voltages $E_{i,p}, E_{i,q}$ and E_{jg} in a polar coordinate form

$$E = V_i e^{j\delta_{i,p}} \tag{5.38}$$

$$E = V_{i,q} e^{j\delta_{i,q}} \tag{5.39}$$

$$E = V_{j,g} e^{j\delta_{j,g}} \tag{5.40}$$

Expressing the bus admittance matrices $\mathbf{A}_{ii}$ and $\mathbf{A}_{ij}$ in terms conductance ($\mathbf{G}_{ii}$) and susceptance ($\mathbf{B}_{ii}$) as well as ($\mathbf{G}_{ij}$) and ($\mathbf{B}_{ij}$), substituting (5.38-5.40) and the corresponding conductance and susceptance matrices into (5.37), the following equations are obtained:

for the ***ith*** isolated sub-system

$$\begin{aligned} P_{i,p} - jQ_{i,p} &= V_{i,p} \sum_{q=1}^{n_i} \left(G_{ii,pq} - jB_{ii,pq} \right) V_{i,q} \left(\cos\delta_{i,pq} + j\sin\delta_{i,pq} \right) + \\ &+ V_{i,p} \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \sum_{g=1}^{n_j} \left(G_{ij,pg} - jB_{ij,pg} \right) V_{j,g} \left(\cos\delta_{j,pg} + j\sin\delta_{j,pg} \right) \end{aligned} \tag{5.41}$$

Expanding eqn (5.41) and separating real and imaginary parts, it is obtained

$$\begin{aligned} P_{i,p} &= V_{i,p} \sum_{q=1}^{n_i} V_{i,q} \left(G_{ii,pq} \cos\delta_{i,pq} + jB_{ii,pq} \sin\delta_{i,pq} \right) + \\ &+ V_{i,p} \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \sum_{g=1}^{n_j} V_{j,g} \left(G_{ij,pq} \cos\delta_{j,pg} + jB_{ij,pg} \sin\delta_{j,pg} \right) \end{aligned} \tag{5.42}$$

$$Q_{i,p} = -V_{i,p}\sum_{q=1}^{n_i} V_{i,q}\left(G_{ii,pq}\sin\delta_{i,pq} - jB_{ii,pq}\cos\delta_{i,pq}\right) + \\ -V_{i,p}\sum_{\substack{j=1 \\ j\neq i}}^{N_S}\sum_{g=1}^{n_j} V_{j,g}\left(G_{ij,pg}\sin\delta_{j,pg} - jB_{ij,pg}\cos\delta_{j,pg}\right) \tag{5.43}$$

Assuming that voltages $E_{i,p}$ and E_{jg} are of the same level i.e. they can either represent 132 kV, 220 kV or 330 kV; then, the angle $\delta_{j,pg}$ is zero. This means that $\cos\delta_{j,pg} = 1$ and $\sin\delta_{j,pg} = 0$. Substituting these values into Eqns (5.42) and (5.43) transforms the two equations into

$$P_{i,p} = \sum_{q=1}^{n_i} V_{i,p}V_{i,q}\left(G_{ii,pq}\cos\delta_{i,pq} + jB_{ii,pq}\sin\delta_{i,pq}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{N_S}\sum_{g=1}^{n_j} V_{i,p}V_{j,g}G_{ij,pg} \tag{5.44}$$

$$p = 1,\cdots,n_i$$

$$Q_{i,p} = -\sum_{q=1}^{n_i} V_{i,p}V_{i,q}\left(G_{ii,pq}\sin\delta_{i,pq} - jB_{ii,pq}\cos\delta_{i,pq}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{N_S}\sum_{g=1}^{n_j} V_{i,p}V_{j,g}B_{ij,pg} \tag{5.45}$$

$$p = 1,\cdots,n_i$$

Resulting load flow non-linear equations (5.44) and (5.45) can be described as follows:

At load buses (P-Q)

At load buses eqns (5.44) and (5.45) can be used in calculating the power mismatches $\Delta P_{i,p}$ and $\Delta Q_{i,p}$

At voltage-controlled buses (P-V)

Only eqn (5.44) can be used, since reactive power at voltage-controlled bus is not available

At slack bus (V-Q)

No equation

Eqns (5.44) and (5.45) represent the real and reactive power of the ***ith*** isolated sub-system interconnected to ***jth*** sub-system, expressed element by element in a vector matrix form

Eqns (5.44) and (5.45) are written for every bus $p = 1,\ldots,n_i$. They can be grouped by forming the following vectors

$$
P_i = \begin{bmatrix} P_{i,1} \\ \vdots \\ P_{i,p} \\ \vdots \\ P_{i,n_i} \end{bmatrix} \quad Q_i = \begin{bmatrix} Q_{i,1} \\ \vdots \\ Q_{i,p} \\ \vdots \\ Q_{i,n_i} \end{bmatrix} \quad V_i = \begin{bmatrix} V_{i,1} \\ \vdots \\ V_{i,p} \\ \vdots \\ V_{i,n_i} \end{bmatrix} \quad V_j = \begin{bmatrix} V_{j,1} \\ \vdots \\ V_{j,g} \\ \vdots \\ V_{j,n_j} \end{bmatrix} \tag{5.46}
$$

The procedure of how the process is implemented is described as follows:

Let assume that ***i =1*** and ***p =1***, Eqn (5.44) is transformed into

$$
\begin{aligned} P_{11} = V_{11}\{(G_{1111}\cos\delta_{1,11} + jB_{1111}\sin\delta_{1,11})\cdots(G_{11 1n_1}\cos\delta_{1,1n_1} + jB_{11 1n_1}\sin\delta_{1,1n_1})\}V_1 + \\ + \sum_{j=1}^{N_S} V_{11}\{G_{1j,11}, G_{1j,12}\cdots G_{1j,1n_j}\}V_j \end{aligned} \tag{5.47}
$$

When ***i = 1, p = p***

$$
\begin{aligned} P_{1p} = V_{1p}\{(G_{11 1p}\cos\delta_{1,1p} + jB_{11 1p}\sin\delta_{1,1p})\cdots(G_{1 1pn_1}\cos\delta_{1,pn_1} + jB_{1 1pn_1}\sin\delta_{1,pn_1})\}V_1 + \\ + \sum_{j=1}^{N_S} V_{1p}\{G_{1j,p1}, G_{1j,p2}\cdots G_{1j,pn_j}\} \end{aligned} \tag{5.48}
$$

When $i = 1$ and $p = n_1$

$$
\begin{aligned} P_{i,n_1} = V_{1,n_1}\{(G_{11 1n_1}\cos\delta_{1,1n_1} + jB_{11 1n_1}\sin\delta_{1,1n_1}) + \cdots (G_{1 1pn_1}\cos\delta_{1,pn_1} + jB_{1 1pn_1}\sin\delta_{1,pn_1})\}V_1 + \\ + \sum_{j=1}^{N_S} V_{1,p}\{G_{1j,pn_1}, G_{1j,p2}\cdots G_{1j,pn_1}\} \end{aligned} \tag{5.49}
$$

The row vectors in equations (5.47) and (5.48) can be grouped to form a matrix and then the equation for real power is written in a matrix form as

$$
P_i = G_{ii}^R V_i + \sum_{\substack{j=1 \\ j\neq i}}^{N_S} G_{ij}^R V_j \tag{5.50}
$$

where

$$
G_{ii}^R \in R^{n_i x n_i} \qquad G_{ij}^R \in R^{n_i x n_j}
$$

The steps used in deriving real power equations can be applied in deriving reactive power equations, and the resulting equation is given by

$$Q_i = G_{ii}^{im} V_i + \sum_{j=1}^{N_S} B_{ij}^{im} V_j \qquad (5.51)$$

The ***pq-th*** element of matrix G_{ii}^R is

$$G_{ii,pq}^R = \left[G_{ii,pq} \cos_{i,pq} + jB_{ii,pq} \sin \delta_{i,pq}\right] V_{i,p} \qquad (5.52)$$

And the ***pq-th*** element of matrix G_{ij}^R is

$$G_{ij,pq}^R = V_{i,p} . G_{ij,pq} \qquad (5.53)$$

The ***pg-th*** element of matrix $G_{ij,pq}^{im}$ is

$$G_{ij,pg}^{im} = -\left[G_{ii,pg} \sin \delta_{i,pg} - jB_{ii,pg} \cos \delta_{i,pg}\right] V_{i,p} \qquad (5.54)$$

And the *pg-th* element of $B_{ij,pg}^{im}$ is given by

$$B_{ij,pg}^{im} = V_{i,p.} B_{ij,pg} \qquad (5.55)$$

Let

$$y_i^R = \sum_{j=1}^{N_S} G_{ij}^R V_j \qquad (5.56)$$

And

$$y_i^{im} = \sum_{j=1}^{N_S} B_{ij}^{im} V_j \qquad (5.57)$$

Substituting Eqns (5.56), (5.57) into (5.50) and (5.51), yields

$$P_i = G_{ii}^R V_i + y_i^R \qquad (5.58)$$

$$G_i = G_{ii}^{im} V_i + y_i^{im} \qquad (5.59)$$

Equations (model) (5.58) and (5.59) are used for parallel calculation procedures.

5.6.3 Sub-matrices

Sub-matrices and their interconnections obtained after decomposing bus admittance matrix of the model are provided in Figures 5.6 to 5.23, respectively. These Figures represent matrix elements as well as their values. Values of these matrices are used in calculating conductance and susceptance which are important input in solving load flow as well as power system state estimation.

BUS No.	1	2	3	4	5	6	7	8	9	10	11	12
1	Y_{11}	Y_{12}	0	0	Y_{15}	0	0	0	0	0	0	0
2	Y_{21}	Y_{22}	Y_{23}	0	Y_{25}	0	0	0	0	0	0	0
3	0	Y_{32}	Y_{33}	Y_{34}	0	0	0	0	0	0	0	0
4	0	0	Y_{43}	Y_{44}	0	0	0	0	0	0	0	0
5	Y_{51}	Y_{52}	0	0	Y_{55}	Y_{56}	0	0	0	0	0	0
6	0	0	0	0	Y_{65}	Y_{66}	Y_{67}	Y_{68}	0	0	0	0
7	0	0	0	0	0	Y_{76}	Y_{77}	0	0	0	0	Y_{712}
8	0	0	0	0	0	Y_{86}	0	Y_{88}	Y_{89}	0	0	0
9	0	0	0	0	0	0	0	Y_{98}	Y_{99}	Y_{910}	0	Y_{912}
10	0	0	0	0	0	0	0	0	Y_{910}	Y_{1010}	Y_{1011}	0
11	0	0	0	0	0	0	0	0	0	Y_{1110}	Y_{1111}	0
12	0	0	0	0	0	0	Y_{127}	0	Y_{129}	0	0	Y_{1212}

Figure 5.6: A11 matrix elements

BUS No.	1	2	3	4	5	6	7	8	9	10	11	12
1	3.3- j18	-1.8 +j12	0	0	-1.5 +j6.1	0	0	0	0	0	0	0
2	-1.8- j12.1	7 -j38	-1.6 +j8.7	0	0	0	0	0	0	0	0	0
3	0	-1.57 +j8.7	24.0 -j21	-22 +j13	0	0	0	0	0	0	0	0
4	0	0	-22 +j13	22 +j12	0	0	0	0	0	0	0	0
5	-1.5 +j6.1	-1.3 +j7.1	0	0	47 -j48	-44 +j35	0	0	0	0	0	0
6	0	0	0	0	-44 +j35	77 -j65	0.0 +j3.7	-33 +j27	0	0	0	0
7	0	0	0	0	0	0.0 +j3.7	1.88 -j7.9	0	0	0	0	-1.88 +j4.3
8	0	0	0	0	0	-33 +j27	0	-33 +j27	0.0 +j16	0	0	0
9	0	0	0	0	0	0	0	0.0 +j16	3.94 -j28	18.2 -j17	0	-0.19 +j4.3
10	0	0	0	0	0	0	0	0	-3.75 +j8.6	18.2 -j17	-14 +j8.3	0
11	0	0	0	0	0	0	0	0	0	-14 +8.3	14 -j8.3	0
12	0	0	0	0	0	0	-1.88 +j4.3	0	-0.19 +j4.3	0	0	2.2 -j11

Figure 5.7: A11 matrix element values

BUS No.	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0
12	Y_{1213}	0	0	0	0	0	0	0	0

Figure 5.8: A12 matrix element

BUS No.	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0
12	-0.10 +j2.4	0	0	0	0	0	0	0	0

Figure 5.9: A12 matrix element values

BUS NO.	22	23	24	25	26	27	28	29	30
1	0	0	0	0	0	0	0	0	0
2	0	0	Y_{224}	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0

Figure 5.10: A13 matrix elements

BUS No.	22	23	24	25	26	27	28	29	30
1	0	0	0	0	0	0	0	0	0
2	0	0	-1.9 +j11	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0

Figure 5.11: A13 matrix element values

BUS No.	1	2	3	4	5	6	7	8	9	10	11	12
13	0	0	0	0	0	0	0	0	0	0	0	Y_{1312}
14	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5.12: A21 matrix elements

BUS No.	1	2	3	4	5	6	7	8	9	10	11	12
13	0	0	0	0	0	0	0	0	0	0	0	-0.1 +j2.4
14	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5.13: A21 matrix element values

BUS No.	13	14	15	16	17	18	19	20	21
13	Y_{1313}	Y_{1314}	Y_{1315}	Y_{1316}	0	0	0	0	0
14	Y_{1413}	Y_{1414}	Y_{1415}	0	0	0	0	0	0
15	Y_{1513}	Y_{1514}	Y_{1515}	0	0	0	0	0	0
16	Y_{1613}	0	0	Y_{1616}	Y_{1617}	0	Y_{1619}	Y_{1620}	0
17	0	0	0	Y_{1716}	Y_{1717}	Y_{1718}	Y_{1719}	0	0
18	0	0	0	0	Y_{1718}	Y_{1818}	0	0	0
19	0	0	0	Y_{1916}	Y_{1917}	0	Y_{1919}	Y_{1920}	0
20	0	0	0	0	0	0	Y_{2019}	Y_{2020}	Y_{2021}
21	0	0	0	0	0	0	0	Y_{2120}	Y_{2121}

Figure 5.14: A22 matrix elements

BUS No.	13	14	15	16	17	18	19	20	21
13	26 -j41	-11 +j33	-15 +j3.4	-0.07 +j1.7	0	0	0	0	0
14	-11 +j33	30 -j39	-19 +j5.4	0	0	0	0	0	0
15	-15 +3.4	-19 +j5.4	34 -j8.8	0	0	0	0	0	0
16	-0.07 +j1.2	0	0	0.07 -j3.0	0.0 +j1.4	0	0	0	0
17	0	0	0	0.00 +j1.4	10.7 -j25	-10.6 +j22	-0.07 +j1.4	0	0
18	0	0	0	0	-10.6 +j22	10.6 -j22	0	0	0
19	0	0	0		-0.07 +j1.4	0	0.07 -j8.5	0.0 +j7.1	0
20	0	0	0	0	0	0	0.0 +j7.1	32 -j26	-32 +j19
21	0	0	0	0	0	0	0	-32 +j19	33 -j27

Figure 5.15: A22 matrix element values

BUS No.	22	23	24	25	26	27	28	29	30
13	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0

Figure 5.16: A23 matrix elements

BUS No.	22	23	24	25	26	27	28	29	30
13	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0

Figure 5.17: A23 matrix element values

BUS No.	1	2	3	4	5	6	7	8	9	10	11	12
22	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0
24	0	Y_{242}	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5.18: A31 matrix elements

BUS No.	1	2	3	4	5	6	7	8	9	10	11	12
22	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0
24	0	-1.9 -j11	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5.19: A31 matrix element values

BUS No.	13	14	15	16	17	18	19	20	21
22	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0

Figure 5.20: A32 matrix elements

BUS No.	13	14	15	16	17	18	19	20	21
22	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0

Figure 5.21: A32 matrix element values

BUS No.	22	23	24	25	26	27	28	29	30
22	Y_{2222}	Y_{2223}	0	Y_{2225}	0	0	0	0	0
23	Y_{2322}	Y_{2323}	Y_{2324}	0	0	0	0	0	0
24	0	Y_{2423}	Y_{2424}	0	0	0	0	0	0
25	Y_{2522}	0	0	Y_{2525}	Y_{2526}	0	0	Y_{2529}	0
26	0	0	0	Y_{2625}	Y_{2626}	Y_{2627}	0	0	0
27	0	0	0	0	Y_{2726}	Y_{2727}	Y_{2728}	0	0
28	0	0	0	0	0	Y_{2827}	Y_{2828}	0	0
29	0	0	0	Y_{2925}	0	0	0	Y_{2929}	Y_{2930}
30	0	0	0	0	0	0	0	Y_{3029}	Y_{3030}

Figure 5.22: A33 matrix elements

BUS No.	22	23	24	25	26	27	28	29	30
22	26 -j25	-24 +j12	0	-0.9 +j5	0	0	0	0	0
23	-24 +j12	60 -j32	-36 +j21	0	0	0	0	0	0
24	0	-36 +j21	38 -j31	0	0	0	0	0	0
25	-0.9 +5	0	0	2.5 -j19	-1.5 +j8.2	0	0	0.0 +6.3	0
26	0	0	0	-1.5 +j8.2	1.5 -j14	0.0 +6.3	0	0	0
27	0	0	0	0	0.0 +6.3	0.62 -j7.6	-0.62 +1.4	0	0
28	0	0	0	0	0	-0.62 +1.4	0.62 -j1.3	0	0
29	0	0	0	0.0 +j6.3	0	0	0	0.09 -j8.7	-0.09 +j2.1
30	0	0	0	0	0	0	0	-0.09 +j2.1	0.09 -j2.0

Figure 5.23: A33 matrix element values

5.6 Conclusion

System network model and data are useful components in load flow and power system state estimaton studies. In this chapter, data acquisition and classification methods and how to prepare database using MYSQL software is presented.

Generation, transmission and load demand data from the Tanzanian power system network are prepared to be used in the load flow and power system state estimation (PSSE) studies. Integrated system network equations from one-line diagram are derived. Matrix decomposition method is employed in decomposing the bus admittance matrix from the network model to form 3 interconnecting subsystems. Block submatrices of the interconnected subsystems are calculated and presented. The subsystems developed in this chapter form a basis for developing a decomposition-coordination method and alogorithm which is applied in the solution of the multi-area power system state estimation.

The integrated system network model developed in this chapter is applied in developing Newton-Raphson algorithm and MATLAB program in chapter 6. The model is validated in chapter 10 by running the load flow analysis.

CHAPTER SIX: LOAD FLOW ANALYSIS

6.1 Introduction

Planning, design, and operation of electrical power systems require continuing and comprehensive analysis in order to determine system performance and to evaluate alternative system expansion plans. Because of the increasing cost of system additions and modifications, as well as soaring fuel costs, it is imperative that utilities consider a range of design options. In-depth analysis is necessary to determine the effectiveness of each alternative in alleviating operating conditions, for peak and off-peak loadings, and for both present and future power systems. As the size of the network grows, the determination of such system variables as voltage levels and power flows in the transmission lines, transformers, and generators becomes more and more difficult. A large volume of network data must be collected and handled accurately. This is aggravated by the dynamic nature of the system as it is configured to meet changing system conditions. Thus, as the electric system grows in size and the number of its interconnections increases, planning for future expansion becomes increasingly complex. The high costs of system operations and system expansions and modifications have paved the way for the use of computers in power systems.

Load flow calculations provide the power flows and voltages for a particular steady-state operating condition of the system. Power flow analysis is performed for small or large power systems, for high-and low-voltage systems, and for existing and future systems. During load flow analysis, per-unit representation is used. The circuit parameters are expressed in per unit, and all calculations are performed in per unit. The power base is chosen to be the rating of one of the major pieces of equipment or is set as a company policy. The power base is the same for all parts of the power system. The voltage bases are chosen as the nominal voltages in the various parts of the system or are selected to be the voltage ratings of the transformer windings in order to maintain a unity per-unit turns ratio of the transformer. The current and impedance bases are computed on the basis of the previous selected values of power base and voltage bases.

The mathematical formulation of load flow problems results in a system of non-linear equations. These equations are written in terms of either the bus admittance matrix, bus impedance matrix or using state variables. Writing the equations in terms of bus admittance matrix is more amenable to digital computer simulation and analysis, because of the case with the bus admittance matrix could be formed and modified for any network changes in subsequent cases. Furthermore, bus admittance matrix approach is the most economical from the point of view of computer time and memory requirement.

In this chapter both state variables and bus admittance matrix approaches are presented, latter bus admittance matrix approach is used in formulation of the load flow problem.

6.2 Bus Classification

Bulk high voltage transmission systems are comprised of three-phase circuits. However, under balanced conditions i.e. the currents in all three phases are equal in magnitude and phase separated by 120^0; therefore, the three-phase system is always analysed using a per-phase equivalent circuit consisting of single-phase and the neutral conductor. Per-unitization of a per-phase equivalent of a three-phase, balanced system results in the per-unit circuit. It is the per-unitized, per-phase equivalent circuit of the power system that is used to formulate and solve the power flow problem and state estimation problems.

For convenience, power system network is represented using single-line diagram, which can be thought of as the circuit diagram of the per-phase equivalent, but without the neutral conductor. First, nodes are classified depending on whether generator and/ or load are connected to them. Specifically, a bus may have generation only, load only or neither generation nor load. In some cases, a node may have both generation and load. This classification, which focuses on the generation and load, leads to the definition of the term "*node injection*" or more simply "injection"

An injection: is the power, either real or reactive that is being injected into or withdrawn from a bus by an element having its other terminal (in the per-phase equivalent circuit) connected to ground. Such an element would be either a generator or load. Positive injection is defined as one where electric power is flowing from the element into the bus (i.e. into the network), while negative injection is when electric power is flowing from the node (i.e. from the network) into the element. A generator normally have positive real power injection, although it may also be assigned negative real power injection in which case the generator is operating as a motor. A generator may have positive or negative reactive power injection: positive if the generator is operating lagging and delivering reactive power to the node. Negative if the generator is operating leading and absorbing reactive power from the node and zero if the generator is operating at unity power-factor [Lynn Powell., 2004]

Loads normally have negative real and reactive power injections, although they may also be assigned positive real power injections in case of a very special modelling needs as stated by Acha [Acha et al., 2004]. Although it is physically appealing to classify nodes basing on the generation or load mix connected to the node, it is needed to be more cautious and precise in order to analytically formulate the power flow or state estimation problems. For proper analytical formulation, it is appropriate to classify the nodes according to what information is known from the data collection about them before solving the power flow problem and later the state estimation problem. For each node, there are FOUR possible variables that characterize the bus electrical condition. For an arbitrary bus *i*. The four variables are: real and reactive power injection, P_i and Q_i, respectively, and voltage magnitude and voltage phase angle V_i and δ_i respectively. From this perspective, there are three basic types of nodes [Grainger & Stevenson., 1994; Hadi Saadat., 2004; Bergen & Vittal., 2000]

6.2.1 Slack Bus

Two other common names for this node are Slack bus and Reference node. There is only one swing node, and it can be designated by the power engineer to be any generator bus in the system. For the swing bus V and δ are known, the fact that δ is known is the reason why it is sometime called the reference bus. Physically, there is nothing special about the swing node; in fact, it is a mathematical artefact of the solution procedure. In case of the load flow problem, the swing node must supply both the load and the losses on the circuits. Before solving the load flow problem all injections at PQ nodes are known, but it is not known what the losses will be as losses are function of the flows on the circuits which are yet to be computed. So the real power injections may be set for at most, all but one of the generators. The one generator for which the real power injection is not set is the swing bus. Thus, this generator "swings" to compensate for the network losses or, one may say that it "takes up the slack". Therefore, rather than call this generator V, δ bus, the terminology *"swing"* or "slack" is selected as it helps to better remember its function. The voltage magnitude of the swing node is selected to correspond to the typical voltage setting of this generator. The voltage phase angle may be designated to be any angle, but normally it is designated as 0^0. At this bus the voltage magnitude and phase angle are set to reference values as flows:

$$dV_{SL} = V_{SL} - V_{SL}^{\mathrm{Reference}} = 0 \tag{6.1}$$

$$d\delta_{SL} = \delta_{SL} - \delta_{SL}^{\mathrm{Reference}} = 0 \tag{6.2}$$

6.2.2 Voltage –Controlled (PV) Buses

For *P-V* buses, the active power and voltage magnitude are known (P_i and V_i) but not the reactive power and the voltage angle (Q_i and δ_i.). These buses fall under what is known as voltage-controlled nodes because of the ability to specify (and therefore to know) the voltage magnitude of this node. Most generators in the system fall into this category, independent of whether it also has load; exceptions are nodes that have reactive power injection at either the generator's upper limits (Q_{Max}) or its lower limit (Q_{Min}). There are also special cases where a non-generator bus (i.e., either a busbar with load or a bus with neither generation nor load) may also be classified as *P-V*, and some examples of these special cases are buses having switched shunt capacitors or static Var Compensator system (SVCs). The real power injections of the *P-V* node are selected according to the system dispatch corresponding to the modelled loading conditions. The voltage magnitudes of *P-V* node are selected according to the expected terminal voltage settings, sometimes the generator "set points" of the units. The real power balance equation needs to be held and the voltages are set to reference value as:

$$dP = P_G - P_L - P = 0 \quad (6.3)$$

$$dV = V - V^{\text{Reference}} = 0 \quad (6.4)$$

6.2.3 Load (PQ) Buses

For PQ buses, P_i and Q_i are known but V_i and δ_i are not known. All load nodes fall into this class, including nodes that have neither load nor generation. The real power injections of the *P-Q* bus are selected according to the loading conditions. The reactive power injections of the *P-Q* bus are selected according to the expected power factor of the load. In this case, the real and reactive power balance equations have to be fulfilled as according to:

$$dP = P_G - P_D - P = 0 \quad (6.5)$$

$$dQ = Q_G - Q_D - Q = 0 \quad (6.6)$$

P and *Q* denominate real and reactive power injections into adjacent lines. P_G, Q_G, P_D, Q_D indicates the real and reactive power generation and demand (load) in the nodes.

Data from *P-V*, *P-Q* nodes and Q_{Max}, Q_{Min} are very important for load- flow problem. Table 6.1 shows the three types of buses and their respective variables.

Table 6.1: Bus types and their variables

Bus Type	Bus	P	Q	|V|	δ	Number of buses
Slack	Slack	Solve	Solve	Known	Known	1
PV	Generator	Known	Solve	Known	Solve	N_G
PQ	Load	Known	Known	Solve	Solve	$N-N_G-1$

6.3 System Variables

Before describing formulation and solution of the static power flow equations, it is important at this stage discussing about power system variables involved in static power flow and power system state estimation studies. The variables involved are classified into three group vectors:

- Dependent state variables;
- Independent control variables;
- Disturbance variables

Dependent State Variables

It is a clear fact that, with a change of active load power in the system there is appreciable change in phase angle, and with a change of reactive load power there is a change in the voltage magnitude. For a fixed load, the bus voltages are varied by change in reactive power generation, whereas the phase angles are varied by change in active power generation. Hence, these variables are dependent variables and are known as dependent state variables or in short; state variables. These variables are written as state vector *X* defined as follows:

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \triangleq \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_n \\ |V_1| \\ \vdots \\ |V_n| \end{bmatrix} \tag{6.7}$$

Independent Control Variables

Real and reactive powers generated are known as control variables. These variables can be changed or controlled. Real power generated is controlled by varying the turbine power, and reactive power is controlled by varying the excitation. These controrable variables are written as control vector U_c defined as:

$$U_c = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \stackrel{\Delta}{=} \begin{bmatrix} P_{G1} \\ \vdots \\ P_{Gn} \\ Q_{G1} \\ \vdots \\ Q_{Gn} \end{bmatrix} \tag{6.8}$$

where

P_G : is the real power generation

Q_G: is the reactive power generation

In practice tap-changing and phase-shifting transformers are incorporated in the power system network and their role and models have been discussed in the respective sections. If **T** is number of tap-changing transformers and **S** is number of phase-shifting transformers are included, there are additional **T+S** variables. These new variables are control variables and must be included in the control vector $\mathbf{U_c}$ of equation (6.8), which is now amended and is given by equation (6.9)

$$U_c = \begin{bmatrix} u_1 \\ \vdots \\ u_{2n-1} \\ u_{2n} \\ u_{2n+1} \\ \vdots \\ u_{2n+T} \\ u_{2n+T+1} \\ \vdots \\ u_{2n+T+S} \end{bmatrix} \stackrel{\Delta}{=} \begin{bmatrix} P_{G1} \\ \vdots \\ P_{G_n} \\ Q_{G1} \\ \vdots \\ Q_{Gn} \\ t_1 \\ \vdots \\ tT \\ \varphi_1 \\ \vdots \\ \varphi_s \end{bmatrix} \tag{6.9}$$

where

t : transformer ratio

T: number of tap-changing transformers

φ: phase shifting angle

S: number of phase-shifting transformers

Disturbance Variables

Power demands are disturbance variables. These variables are in the hand of the customers of power and the power system management has no control over these and that is why they are known as disturbance variables or uncontrorable variables. Disturbance variables are written as disturbance vector **D** defined by equation (6.10):

$$D = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} \triangleq \begin{bmatrix} P_{D_1} \\ \vdots \\ P_{D_n} \\ Q_{D_1} \\ \vdots \\ Q_{D_n} \end{bmatrix} \qquad (6.10)$$

where

P_{D}: is the real power demand

Q_D: is the reactive power demand

6.4 System Constraints

The constraints that any feasible solution of load flow equations must satisfy are divided into two types:

- Constraints that govern the current, voltage , and power flow relationship;
- Constraints that govern the operating limits of the system parameters.

The load flow must satisfy Kirchhoff's and Ohm's laws. This means that power injected into a node must be equal to power extracted from thebus. Thus, both summation of real and summation of reactive power at a node must be equal to zero. When deriving load flow equations of **N**-bus system the number of summation is **2N**. These equations are depicting equality constraint. By solving these equations, load flow solution is obtained. In addition, the power flow solution must satisfy the engineering design and operating limits. These limits are usually described in inequality constraints. These include: generator constraints; node constraints; transformer constraints and transmission line constraint. These constraints are explained in the following sections.

Generator Constraints

The real power limits of the generators are restricted to lie within given minimum and maximum limits that are the optimum solution P_{Gi} must fall within the following inequality constraints:

$$P_{Gk,\min} \leq P_{Gk,optimum} \leq P_{Gk,\max} \qquad k = 1, \cdots, N_G \qquad (6.11)$$

where N_G is the number of generators in the system

Similarly, reactive power limits of the generators are restricted to lie within given minimum and maximum limits and this limit is expressed as

$$Q_{Gk\min} \leq Q_{Gkoptimum} \leq Q_{Gk\max} \qquad k = 1, \cdots, N_G \qquad (6.12)$$

Where N_G is the number of generators in the system

Bus Constraint

In order to satisfy design limitations and statutory legal requirements, the voltage magnitude is restricted to limits, the magnitude limits are expressed as:

$$|V_k|_{\min} \leq |V_k| \leq |V_k|_{\max} \qquad k = 1, \cdots, N \qquad (6.13)$$

where N is the total number of buses

Transformer Constraints

Transformers working in power system are provided with taps that can be changed under load. System operator needs to know what tap position is required to adjust the magnitude of bus voltage. Taps of automatic tap-changing transformer must lie within given range. The tap limits are expressed as:

$$t_{k,\min} \leq t_k \leq t_{k,\max} \qquad k = 1, \cdots, T \qquad (6.14)$$

Where t_i represents tap setting and *"T"* is the total number of tap-changing transformers in the system.

Phase-shifters are also included in the system, thus, the angular position must not allow being outside of a given range. The constraint is expressed by:

$$\varphi_{k,\min} \leq \varphi_k \leq \varphi_{k,\max} \qquad k = 1, \cdots, S \qquad (6.15)$$

Where φ_s is angular position of the phase-shifter and S is the total number of phase-shifters.

Transmission Line Constraint

Limit on power transfer through the transmission line must not be exceeded. The constraint is expressed by:

$$|S_k| \le |S_k|_{\max} \qquad k = 1,\cdots,N_l \qquad (6.16)$$

Where S_k is the apparent power (MVA) of line *"k"* and *"N_l"* total number of transmission lines

6.5 Load Flow Equations of N-Bus System

From the injected current at a bus i, a general net injected current equation at bus i can be written as:

$$I_i = \sum_{j=1}^{N} \overline{Y}_{ij} E_j \qquad (6.17)$$

The net injected complex power at the same node is

$$S_i = P_i - jQ_i = E_i I_i^{*} \qquad i = 1,\cdots,N \qquad (6.18)$$

$$S_i = P_i - jQ_i = E_i \sum_{j=1}^{N} \overline{Y}_{ij}^{*} E_j^{*} \qquad i = 1,\cdots,N \qquad (6.19)$$

In a polar form

$$S = P_i - jQ_i = \sum_{j=1}^{N} V_i e^{j\delta_i} Y_{ij} e^{-j\delta_{ij}} V_j e^{-j\delta_j} \qquad (6.20)$$

$$S_i = P_i - jQ_i = \sum_{j=1}^{N} V_i Y_{ij} V_j e^{(\delta_i - \delta_{ij} - \delta_j)} \qquad (6.21)$$

Separating real and imaginary parts of (6.21), it is obtained

$$P_i = V^2 Y_{ii} \cos\delta_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{N} V_i Y_{ij} V_j \cos(\delta_{ij} + \delta_j - \delta_i) \qquad (6.22)$$

$$Q_i = -V^2 Y_{ii} \sin\delta_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{N} V_i Y_{ij} V_j \sin(\delta_{ij} + \delta_j - \delta_i) \qquad (6.23)$$

Re-arranging (6.21) and (6.23), the **2N** mismatch equations in a polar form are obtained as:

$$
\begin{aligned}
f_1 &= P_1 - \sum_{j=1}^{N} V_1 V_{1j} Y_{1j} \cos(\delta_{1j} + \delta_j - \delta_1) = 0 \\
&\vdots \\
f_N &= P_N - \sum_{j=1}^{N} V_N Y_{Nj} V_j \cos(\delta_{Nj} + \delta_j - \delta_N) = 0 \\
f_{N+1} &= Q_1 + \sum_{j=1}^{N} V_1 Y_{1j} V_j \sin(\delta_{1j} + \delta_j - \delta_1) = 0 \\
&\vdots \\
f_{2N} &= Q_N + \sum_{j=1}^{N} V_N V_j Y_{Nj} \sin(\delta_{Nj} + \delta_j - \delta_N) = 0
\end{aligned}
\qquad (6.24)
$$

In the extended (6.24), it should be noted that $\mathbf{Y}_{ii}$, is the summation of all admittances terminating on bus i and $\mathbf{Y}_{ij}$, is the negative of admittance between buse i and j.

The static power equations of (6.21), (6.23) and (6.24) can also be formulated using state variable models, the formulation is applicable in control field. The equations can be written in compact form as:

$$
f(X,U,D) \triangleq \begin{bmatrix} f_1(X,U,D) \\ f_2(X,U,D) \\ \vdots \\ f_{2N}(X,U,D) \end{bmatrix} = 0 \qquad (6.25)
$$

6.6 Load Flow Analysis

6.6.1 Iterative Methods

Iterative methods are the only methods of solving non-linear equations including those developed for load flow problem. Several methods of numerical iteration have proved to be successful in solving the power flow equations. These methods are named as:

- Gauss iterative method.
- Gauss-Seidel method.
- Newton-Raphson method
- Fast Decoupled method

Gauss; Gauss-Seidel; and Fast decoupled methods are exhaustively discussed in [Powel, 2004; Saadat, 2004; Tinney & Hart, 1967; Stott,1972; Stott &Alsac, 1974; Laughton et al, 1974; Britton, 1971]. In this chapter, Newton-Raphson iterative method will be discussed in detail and the method will be applied for solving the load-flow and validation of the Tanzania Power System Model built in chapter five.

6.7 The Newton-Raphson Iterative Method

Newton-Raphson (N-R) method is considered to be one of the best methods for solving load- flow equations. Many advantages are attributed to the method. Its convergence characteristics are relatively powerful compared to other methods, and low computing time is achieved when used in large and sparse bus admittance matrix. Its reliability is comparatively good since it can solve cases that lead to divergence with other methods, but the method is by no mean infallible. Failure of the method does occur on some ill-conditioned systems.

The Newton-Raphson method is often classified as root finding scheme because it's geared towards solving equations like $f(x) = 0$. The solution to such an equation, call it x^*, is a root of the function $f(x)$.The method is called iterative because it requires a series of successive approximations to the solutions. The procedure is generally as follows. First, guess a solution. So an update to the "old" solution is determined that moves to a "new" solution with the intention that the "new" solution is closer to the correct solution than was the "old" solution. A key aspect to this type of procedure is the way the update is obtained. If it can be guaranteed that the update is always improving the solution, such that the "new" solution is in fact always closer to the correct solution than the "old" solution, then such a procedure can be guaranteed to work if only enough updates are computed, i.e., if only enough times iterations are done (Rau , 2003).

The Newton-Raphson method has become the de-facto industry standard. The main reason for this is that the convergence properties of the Newton-Raphson scheme are very desirable when the initial, guessed solution is quite good, i.e., when it is chosen close to the correct solution. It is usually possible to make a good initial guess regarding the solution. One reason for this is that often, the solution of a particular set of conditions is actually known because we have already gone through the solution procedure, and the aim is to resolve for a set of conditions that are almost the same as the previous ones, e.g., maybe remove one circuit or change the load level a little. In this case, the previous solution is utilized as the initial guessed solution for the new conditions. This is sometimes referred to as a "**hot"** start.

There are basically two convergence properties of a root finding method: One is whether the method will converge. The second one is how fast the method will converge. For Newton-Raphson, whether the method will converge depends on two things: how close the guessed solution is to the correct solution and the nature of the function close to the correct solution. If the guessed solution is close, and if the function is reasonably "smooth" close to the correct solution, then the Newton-Raphson will converge. Not only that, but it will converge quadratically. Quadratic convergence means that each iteration increases the accuracy of the solution by two decimal places.

In the case of the power flow problem, alternative solutions are typically "*far away*" from initial guesses that have near-unity bus voltage magnitudes. On the other hand, it is possible for the solution to diverge, i.e., not to converge at all. This may occur if there is simply no solution, which is a case that engineers encounter frequently when studying highly stressed loading conditions which are served by weak transmission systems. It might also occur if the initial guessed solution is too far away from the correct solution. For this reason, "*flat*" starts encounter solution divergence more frequently than "***hot***" starts (Rau, 2003; Momoh, 2009).

6.7.1 Mathematical Background of the Method: Scalar Case

Assume that a solution $x^{(0)}$ to the problem $f(x)=0$ is guessed. Then $f(x^{(0)})\neq 0$ because $x^{(0)}$ is just a guess. But there must be some $\Delta x^{(0)}$ which will make $f(x^{(0)}+\Delta x^{(0)})=0$. One way to study this problem is to expand the function $f(x)$ in a Taylor series [Saadat, 2004] as follows:

$$f(x^{(0)}+\Delta x^{(0)})=f(x^{(0)})+f'(x^{(0)})\Delta x^{(0)}+\frac{1}{2}f''(x^{(0)})(\Delta x^{(0)})^2+...=0 \qquad (6.26)$$

If the guess is a good one, then $\Delta x^{(0)}$ will be small, and if this is true, then $(\Delta x^{(0)})^2$ will be very small, and any higher order terms in eqn (6.26), which will contain $\Delta x^{(0)}$ raised to even higher powers, will be infinitesimal. As a result, it is reasonable to approximate eqn. (6.26) as

$$f(x^{(0)}+\Delta x^{(0)})\approx f(x^{(0)})+f'(x^{(0)})\Delta x^{(0)}=0 \qquad (6.27)$$

Taking $f(x^{(0)})$ to the right hand side, it is obtained

$$-f'(x^{(0)})\Delta x^{(0)}=f(x^{(0)}) \qquad (6.28)$$

Eqn. (6.27) may be solved for $\Delta x^{(0)}$ according to:

$$\Delta x^{(0)}=-\{f'(x^{(0)})\}^{-1}f(x^{(0)}) \qquad (6.29)$$

Because $f'(x^{(0)})$ in eqn. (6.29) is scalar, its inverse is very easily evaluated using simple division so that:

$$\Delta x^{(0)}=\frac{-f(x^{(0)})}{f'(x^{(0)})} \qquad (6.30)$$

Eqn (6.30) provides the basis for the update formula to be used in the first iteration of the scalar Newton-Raphson method. This update formula is:

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = x^{(0)} + \frac{-f(x^{(0)})}{f'(x^{(0)})} \qquad (6.31)$$

and from eqn. (6.31), the updated formula for any particular iteration can be written as:

$$x^{(j+1)} = x^{(j)} + \Delta x^{(j)} = x^{(j)} + \frac{-f(x^{(j)})}{f'(x^{(j)})} \qquad (6.32)$$

6.7.2 Mathematical Background of the Method: Multidimensional Case

Assume having n nonlinear algebraic equations and n unknowns characterized by $\mathbf{f(x)=0}$, and that the guessed solution is $\mathbf{x^{(0)}}$. Then $\mathbf{f(x^{(0)})\neq 0}$ because $\mathbf{x^{(0)}}$ is just a guess. But there must be some $\Delta\mathbf{x^{(0)}}$ which will make $\mathbf{f(x^{(0)} + \Delta x^{(0)}) = 0}$. Again, expanding the function $\mathbf{f(x)}$ in Taylor series [Saadat, 2004], it follows:

$$f_1(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = f_1(\underline{x}^{(0)}) + f_1'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2}f_1''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + \ldots = 0$$

$$f_2(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = f_2(\underline{x}^{(0)}) + f_2'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2}f_2''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + \ldots = 0 \qquad (6.33)$$

$$\vdots$$

$$f_n(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = f_n(\underline{x}^{(0)}) + f_n'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2}f_n''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + \ldots = 0$$

Equations (6.32) may be written more compactly as

$$\underline{f}(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = \underline{f}(\underline{x}^{(0)}) + \underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2}\underline{f}''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + \ldots = \underline{0} \qquad (6.34)$$

Assuming the guess is a good one such that $\Delta\mathbf{x^{(0)}}$ is small, and then the higher order terms are also small and can be written

$$\underline{f}(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = \underline{f}(\underline{x}^{(0)}) + \underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} = \underline{0} \qquad (6.35)$$

Since there are ***n*** functions and ***n*** variables, it is possible to compute a derivative for each individual function with respect to each individual unknown, like $\partial f_k(x)/\partial x_j$, which gives the derivative of the k^{th} function with respect to the j^{th} unknown. Thus, there will be a number of such derivatives equal to the product of the number of functions by the number of unknowns, in this case, $n \times n$. Thus, it is convenient to store all of these derivatives in a matrix. This matrix has become quite well known as the Jacobian matrix, and it is often denoted using the letter ***J***.

The rows of *J* are ordered in the same order as the functions, that is, the k^{th} row contains the derivatives of the k^{th} function. In eqn. (6.35), since the product $f'(x^{(0)})\Delta x^{(0)}$ must provide a correction to the function $f(x^{(0)}+\Delta x^{(0)})$, i.e., since $\Delta f(x^{(0)}) = f'(x^{(0)})\Delta x^{(0)}$, then any row of the matrix ***J*** must be ordered so that the term in the j^{th} column contains a derivative with respect to the j^{th} unknown of the vector ***x***.

The reasoning in the last paragraph suggests that the Jacobian matrix be written as:

$$\underline{J} = \begin{bmatrix} \frac{\partial f_1(\underline{x}^{(0)})}{\partial x_1} & \frac{\partial f_1(\underline{x}^{(0)})}{\partial x_2} & \cdots & \frac{\partial f_1(\underline{x}^{(0)})}{\partial x_n} \\ \frac{\partial f_2(\underline{x}^{(0)})}{\partial x_1} & \frac{\partial f_2(\underline{x}^{(0)})}{\partial x_2} & \cdots & \frac{\partial f_2(\underline{x}^{(0)})}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n(\underline{x}^{(0)})}{\partial x_1} & \frac{\partial f_n(\underline{x}^{(0)})}{\partial x_2} & \cdots & \frac{\partial f_n(\underline{x}^{(0)})}{\partial x_n} \end{bmatrix} \tag{6.36}$$

In eqn. (6.36), taking $f(x^{(0)})$ to the right hand side, it is obtained

$$\underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} = -\underline{f}(\underline{x}^{(0)}) \tag{6.37}$$

or, in terms of the Jacobian matrix *J*, it is:

$$\underline{J}\Delta\underline{x}^{(0)} = -\underline{f}(\underline{x}^{(0)}) \tag{638}$$

Solving eqn. (6.38) for $\Delta x^{(0)}$, we have:

$$\Delta\underline{x}^{(0)} = -\left\{\underline{f}'(\underline{x}^{(0)})\right\}^{-1}\underline{f}(\underline{x}^{(0)}) = -\underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) \tag{6.39}$$

Eqn (6.39) provides the basis for the update formula to be used in the first iteration of the multi-dimensional case. This update formula is:

$$\underline{x}^{(1)} = \underline{x}^{(0)} + \Delta\underline{x}^{(0)} = \underline{x}^{(0)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) \tag{6.40}$$

and from eqn. (6.40), we may infer the update formula for any particular iteration as:

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} + \Delta\underline{x}^{(k)} = \underline{x}^{(k)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(k)}) \tag{6.41}$$

Where *k* is the iteration index

Update formula can be stated a little differently, so that one is prepared to utilize it for the general case using matrix factorization. To do this, eqn. (6.41) is written as

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} + \Delta\underline{x}^{(k)} \tag{6.42}$$

where $\Delta x^{(i)}$ is found from eqn. (6.48) using the matrix factorization technique.

$$-\underline{J}\Delta\underline{x}^{(k)} = \underline{f}(\underline{x}^{(k)}) \qquad (6.43)$$

Generally, it's needed to iterate several times in order to obtain a satisfactory solution. The Newton-Raphson algorithms employ a stopping criterion in order to determine when the solution is satisfactory. There are two ways to do this.

- **Type 1 stopping criterion:** Test the maximum change in the solution elements from one iteration to the next, and if this maximum change is smaller than a certain predefined tolerance, then stop. This means to compare the maximum absolute value of elements in Δx against a small positive number, call it $\varepsilon_1>0$
- **Type 2 stopping criterion:** Test the maximum value in the function elements of the most current iteration ***f(x),*** and if this maximum value of elements in ***f(x)*** is smaller than a certain predefined tolerance, then stop. This means to compare the maximum absolute value of elements in ***f(x)*** against a small positive number, call it $\varepsilon_1>0$ This is the most common stopping criterion for power flow solutions, and the value of each element in the function is referred to as the "power mismatch" for the bus corresponding to the function. For type *P-Q* buses, both real and reactive power mismatches are tested. For type *P-V* buses, only real power mismatches are tested.

6.8 The Newton-Raphson Load Flow Solution

The mathematical formulation of the load-flow problem is a set of non-linear sparse equations relating complex power to complex voltage. The problem is sparse due to the structure of the power system (bus admittance matrix), and non-linear because power is expressed as a function of the complex bus voltages, excluding the slack node since for this node it is assumed known quantities for the complex voltage magnitude and phase angle. Once the solution for the complex voltage values has been determined, the real and reactive power for the remaining nodes can be calculated and the load flow solution is determined. A review of various load flow calculation methods up to the mid 1970s is contained in (Stott, 1974).

Both the bus self- and mutual admittances which compose the bus admittance matrix or the driving-point and transfer impedances which compose Z_{bus} may be used in solving the power flow problem. The starting point in obtaining the data which must be furnished to the computer is the single-line diagram of the system. Transmission lines are represented by their per-phase nominal-π equivalent circuits. For each line numerical values for the series impedance $\boldsymbol{Z_l}$ and total line charging admittance Y are necessary so that the computer can determine all the elements of the bus admittance matrix of which the typical element $\boldsymbol{Y_{ij}}$ is

$$\overline{Y}_{ij} = Y_{ij}\angle\delta_{ij} = Y_{ij}\cos\delta_{ij} + jY_{ij}\sin\delta_{ij} = G_{ij} + jB_{ij} \tag{6.44}$$

where

$\boldsymbol{G_{ij}}$**:** is the series conductance between buses i and j

$\boldsymbol{B_{ij}}$**:** is the series susceptance between buses i and j

Other essential information includes transformer ratings and impedances, shunt capacitor ratings, and transformer tap settings. In advance of each power flow study certain bus voltages and power injections are given known values.

The voltage at a typical bus i of the system is given in a polar coordinate by

$$\overline{V}_i = V_i\angle\delta_i = V_i(\cos\delta_i + j\sin\delta_i) \tag{6.45}$$

The net current injected into the network at node i in terms of the elements Y_{ij} of Y_{bus} is given by

$$I_i = \overline{Y}_{i1}E_1 + \overline{Y}_{i2}E_2 + \cdots + \overline{Y}_{ij}E_j = \sum_{j=1}^{N}\overline{Y}_{ij}E_j \tag{6.46}$$

Where $\boldsymbol{N}$ is the number of nodes in the system

The complex conjugate of the power injected at bus i is

$$P_i - jQ_i = E_i^{\bullet}\sum_{j=1}^{N}\overline{Y}_{ij}E_j \tag{6.47}$$

In which by substituting from eqns (6.46) and (6.47) the following equation is obtained

$$P_i - jQ_i = \sum_{j=1}^{N}V_iY_{ij}V_j\angle(\delta_{ij} + \delta_j - \delta_i) \tag{6.48}$$

Expanding this equation and equating real and reactive parts, it is obtained

$$P_i = \sum_{j=1}^{N}V_iY_{ij}V_j\cos(\delta_{ij} + \delta_j - \delta_i) \tag{6.49}$$

$$Q_i = -\sum_{j=1}^{N}V_iY_{ij}V_j\sin(\delta_{ij} + \delta_j - \delta_i) \tag{6.50}$$

Eqns (6.49) and (6.50) constitute the polar form of the power flow equations; they provide calculated values for the net real power P_i and reactive power Q_i entering the network at typical bus i

Denoting the scheduled power being generated at node i by P_{Gi} and scheduled power demand of the load at that node by P_{Di}, then $P_{i,Sch} = P_{Gi} - P_{Di}$ is the net scheduled power being injected into the network at node i. Denoting the calculated value of P_i by $P_{i,Cal}$ leads to the definition of mismatch ΔP_i as the scheduled value $P_{i,Sch}$ minus the calculated value $P_{i,Cal}$,

$$\Delta P_i = P_{i,sch} - P_{i,calc} = (P_{gi} - P_{di}) - P_{i,calc} \tag{6.51}$$

Likewise, for reactive power at bus ***i*** we have

$$\Delta Q_i = Q_{i,sch} - Q_{i,calc} = (Q_{gi} - Q_{di}) - Q_{i,calc} \tag{6.52}$$

Mismatches occur when calculated values of P_i and Q_i do not coincide with the scheduled values. If the calculated values $P_{i,Cal}$, and $Q_{i,Cal}$ match the scheduled values $P_{i,Sch}$ and $Q_{i,Sc}$, perfectly, then the mismatches ΔP_i and ΔQ_i are zero at bus i and the power balance equations are

$$P_i - P_{i,sch} = P_i - (P_{gi} - P_{di}) = 0 \tag{6.53}$$

$$Q_i - Q_{i,sch} = Q_i - (Q_{gi} - Q_{di}) = 0 \tag{6.54}$$

6.8.1 The Newton-Raphson Algorithm

An essential step in applying Newton-Raphson to the load- flow problem is to enable calculation of the Jacobian elements, given for the general case by eqn. (6.36). Evaluation of these elements is facilitated by the recognition, that there are only two kinds of equations (real power equations and reactive power equations), and that there are only two kinds of unknowns (voltage angle unknowns and voltage magnitude unknowns). To apply the Newton-Raphson method to the solution of the power flow equations, node voltages and line admittances are expressed in a polar form. When the corresponding terms from equations for real and reactive power are separated from the summations, the following equations are obtained:

$$P_i = V_i^2 G_{ii} + \sum_{j=1}^{N} V_i Y_{ij} V_j \cos(\delta_{ij} + \delta_j - \delta_i) \tag{6.55}$$

$$Q_i = -V_i^2 B_{ii} - \sum_{j=1}^{N_S} V_i Y_{ij} V_j \sin(\delta_{ij} + \delta_j - \delta_i) \tag{6.56}$$

where

G_{ii} is the real part of the diagonal elements of the bus admittance matrix for the ***ith*** bus

B_{ii} is the imaginary part of the diagonal elements of the bus admittance matrix for the ***ith*** bus

These equations can be readily differentiated with respect to voltage angles and magnitudes and hence, mismatch equations can be written as follows:

For real power P_i:

$$\begin{aligned}\Delta P_i &= \frac{\partial P_i}{\partial \delta_2}\Delta\delta_2 + \frac{\partial P_i}{\partial \delta_3}\Delta\delta_3 + \cdots + \frac{\partial P_i}{\partial \delta_N}\Delta\delta_N + \cdots \\ &+ \frac{\partial P_i}{\partial V_2}\Delta V_2 + \frac{\partial P_i}{\partial V_3}\Delta V_3 + \cdots + \frac{\partial P_i}{\partial V_N}\Delta V_N\end{aligned} \tag{6.57}$$

Similarly, mismatch equation can be written for reactive power Q_i:

$$\begin{aligned}\Delta Q &= \frac{\partial Q_i}{\partial \delta_2}\Delta\delta_2 + \frac{\partial Q_i}{\partial \delta_3}\Delta\delta_3 + \cdots + \frac{\partial Q_i}{\partial \delta_N}\Delta\delta_N + \cdots + \\ &\frac{\partial Q_i}{\partial V_2}\frac{\Delta V_2}{} + \frac{\partial Q_i}{\partial V_3}\frac{\Delta V_3}{} + \cdots \\ &+ \frac{\partial Q_i}{\partial V_N}\frac{\Delta V_N}{}\end{aligned} \tag{6.58}$$

Each non-slack bus of the system has two equations like those for ΔP_i and ΔQ_i.

Collecting all the mismatch equations into a vector-matrix form yields

$$\left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_2}{\partial V_N} \\ \vdots & J_1 & \vdots & \vdots & J_2 & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_2} & \cdots & \frac{\partial P_N}{\partial V_N} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_2}{\partial V_N} \\ \vdots & J_3 & \vdots & \vdots & J_4 & \vdots \\ \frac{\partial Q_N}{\partial \delta_n} & \cdots & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_n}{\partial V_2} & \cdots & \frac{\partial Q_N}{\partial V_N} \end{array}\right] \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_n \\ \hline \Delta V_2 \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_N \\ \hline \Delta Q_2 \\ \vdots \\ \Delta Q_N \end{bmatrix} \tag{6.59}$$

In short form, (6.60) can be written in the form of Jacobian matrix as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \tag{6.60}$$

$J_1 \in \Re^{(N-1)(N-1)}$, $J_2 \in \Re^{(N-1)\times(N-1-m)}$, $J_3 \in \Re^{(N-1-m)\times(N-1)}$, $J_4 \in \Re^{(N-1-m)\times(N-1-m)}$

Where J_1, J_2, J_3 and J_4 constitute the Jacobian matrix

The Jacobian matrix gives the linearized relationship between small changes in voltage angle $\Delta\delta_i^{(k)}$ and voltage magnitude $\Delta V_i^{(k)}$ with the small changes in real and reactive power $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$. These changes can be obtained by the difference between the scheduled and calculated values. Elements of the Jacobian matrix are partial derivatives of eqns. (6.55) and (6.56) and they are developed as follows

The diagonal and the off-diagonal elements of J_1 are

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i}^{N} V_i Y_{ij} V_j \sin(\delta_{ij} + \delta_j - \delta_i) \qquad i = \overline{2.N} \tag{6.61}$$

$$\frac{\partial P_i}{\partial \delta_j} = -V_i Y_{ij} V_j \sin(\delta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.62}$$

The diagonal and the off-diagonal elements of J_2 are

$$\frac{\partial P_i}{\partial V_i} = 2V_i Y_{ij} \cos\theta_{ii} + \sum_{j \neq i} V_j Y_{ij} \cos(\delta_{ij} - \delta_i + \delta_j) \tag{6.63}$$

$$\frac{\partial P_i}{\partial V_j} = V_i Y_{ij} \cos(\delta_{ij} - \delta_i + \delta_j) \quad j \neq i \quad i = \overline{2.N} \tag{6.64}$$

the diagonal and off-diagonal elements of J_3 are

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} V_i Y_{ij} V_j \cos(\delta_{ij} - \delta_i + \delta_j) \tag{6.65}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -V_i Y_{ij} V_j \cos(\delta_{ij} - \delta_i + \delta_j) \qquad j \neq i \quad i = \overline{2.N} \tag{6.66}$$

the diagonal and off-diagonal elements of J_4 are

$$\frac{\partial Q_i}{\partial V_i} = -2V_i Y_{ij} \sin\theta_{ii} - \sum_{j \neq i} V_j Y_{ij} \sin(\delta_{ij} - \delta_i + \delta_j) \tag{6.67}$$

$$\frac{\partial Q_i}{\partial V_j} = -V_i Y_{ij} \sin(\delta_{ij} - \delta_i + \delta_j) \quad j \neq i \quad i = \overline{2.N},\ j = \overline{2.N} \tag{6.68}$$

The terms $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are the difference between the scheduled and calculated values, known as the power residuals or power mismatches.

The new estimates for node voltages are

$$\delta_i^{k+1} = \delta_i^k + \Delta\delta_i^k \quad (6.69)$$

$$V_i^{k+1} = V_i^k + \Delta V_i^k \quad (6.70)$$

Table 6.2 shows the contents of each of the four Jacobian sub matrices as represented in tabular form.

In compact form, these equations can be written in terms of *P, Q, Y,* and V parameters as follows:

$$Q_i = V_i \sum_{\substack{j=1 \\ j \neq i}}^{N} Y_{ij}.V_j \sin(\delta_i - \delta_j - \delta_{ij}) - |V_i|^2 B_{ii} \quad (6.71)$$

Which by observation, can be written as

$$Q_i = -\frac{\partial P_i}{\partial \delta_i} - V_i^2 B_{ii} \quad (6.72)$$

Or the diagonal elements of the matrix $\boldsymbol{J_1}$ can be expressed by the equation

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - V_i^2 B_{ii} \quad (6.73)$$

Which, for all *i* are the diagonal elements for $\boldsymbol{J_1}$

Similarly,

$\boldsymbol{J_2}$ diagonal elements may be written as $\frac{P_i}{V_i} + V_i G_{ii}$

$\boldsymbol{J_3}$ diagonal elements may be written as $P_i - V_i^2 G_{ii}$

$\boldsymbol{J_4}$ diagonal elements may be written as $\frac{Q_i}{V_i} - V_i B_{ii}$

Table 6.2: Contents of Jacobian sub matrices in polar coordinate

Sub matrix	Diagonal elements	Off-diagonal elements
J_1	$\frac{\partial P_i}{\partial \delta_i} = -V_i \sum_{\substack{j=1 \\ j\neq i}}^{N} V_j Y_{ij} \sin(\delta_{ij} + \delta_j - \delta_i)$	$\frac{\partial P_i}{\partial \delta_j} = V_i Y_{ij} V_j \sin(\delta_{ij} + \delta_j - \delta_i)$
J_2	$\frac{\partial P_i}{\partial V_i} = \sum_{\substack{j=1 \\ j\neq i}}^{N} V_j Y_{ij} \cos(\delta_{ij} + \delta_j - \delta_i) + 2G_{ii} V_i$	$\frac{\partial P_i}{\partial V_j} = V_i Y_{ij} \cos(\delta_{ij} + \delta_j - \delta_i)$
J_3	$\frac{\partial Q_i}{\partial \delta_i} = V_i \sum_{\substack{j=1 \\ j\neq i}}^{N} Y_{ij} V_j \cos(\delta_{ij} + \delta_j - \delta_i)$	$\frac{\partial Q_i}{\partial \delta_j} = -V_i Y_{ij} V_j \cos(\delta_{ij} + \delta_j - \delta_i)$
J_4	$\frac{\partial Q_i}{\partial V_i} = \sum_{\substack{j=1 \\ j\neq i}}^{N} V_j Y_{ij} \sin(\delta_{ij} + \delta_j - \delta_i) - 2B_{ii} V_i$	$\frac{\partial Q_i}{\partial V_j} = V_i Y_{ij} \sin(\delta_{ij} + \delta_j - \delta_i)$

For application to the load- flow problem, the Newton-Raphson algorithm therefore is given as follows:

I. Specify:

- All admittance data
- P_D and Q_D for all buses
- P_G and V for all P-V buses
- V for swing node, with $\delta = 0°$

II. Let the iteration counter ***k=1***. Use one of the following to guess the initial solution.

- Flat Start: $V_k = 1.0 \angle 0°$ for all nodes.
- Hot Start: Use the solution to a previously solved case for this network.

III. Compute the mismatch vector for $x^{(k)}$. In what follows, elements of the mismatch vector are denoted as ΔP_k and ΔQ_k corresponding to the real and reactive power mismatch, respectively, for the k^{th} node. This computation results in all necessary calculated real and reactive power injections. Perform the following stopping criterion tests:

If $\Delta P_k < \varepsilon_P$ for all type P-Q and P-V buses and

If $\Delta Q_k < \varepsilon_Q$ for all type P-Q buses,

Then go to step V

Otherwise, go to step IV.

IV. Find an improved solution as follows:

- Evaluate the Jacobian J at $x^{(k)}$. Denote this Jacobian as $J^{(k)}$
- Solve for $\Delta x^{(k)}$ from:

$$\underline{J}^{(k)}\Delta\underline{x}^{(k)} = -\begin{bmatrix} \Delta\underline{P} \\ ---- \\ \Delta\underline{Q} \end{bmatrix} \quad \text{or} \quad \Delta\underline{x}^{(k)} = -\left[\underline{J}^{(k)}\right]^{-1}\begin{bmatrix} \Delta\underline{P} \\ ---- \\ \Delta\underline{Q} \end{bmatrix} \tag{6.74}$$

Where factorization with the left equation is used if the system is large, but if the system is not large, the right hand equation may be used.

- Compute the updated solution vector as $\boldsymbol{x^{(k+1)} = x^{(k)} + \Delta x^{k}}$
- Return to step III with $\boldsymbol{k = k+1}$

V. Stop.

The above algorithm is applicable as long as all *P-V* buses remain within their reactive limits. To account for generator reactive limits, the algorithm is modified so that, at each iteration, it is checked to ensure *P-V* bus reactive generation is within its limits. In this case, steps I-IV remains exactly as given above, but new steps V and VI are needed:

VI. Check reactive limits for all generator buses as follows:

a. For all type PV buses, perform the following test:

- If $\boldsymbol{Q_{Gk} > Q_{Gk,\,max}}$ then $\boldsymbol{Q_{Gk} = Q_{Gk,\,max}}$ and change bus $\boldsymbol{k}$ to a type *P-Q* bus (see step VI-a)
- If $\boldsymbol{Q_{Gk} < Q_{Gk,min}}$, then $\boldsymbol{Q_{Gk} = Q_{Gk,\,min}}$ and change bus $\boldsymbol{k}$ to a type *P-Q* bus (see step VI-b)For all type PQ generator buses, perform the following test:
- If $\boldsymbol{Q_{Gk} = Q_{Gk,max}}$ and $\boldsymbol{V_k > V_{k,set}}$ or if $\boldsymbol{Q_{Gk} = Q_{Gk,\,min}}$ and $\boldsymbol{V_k < V_{k,se}}$, then change this bus back to a type *P-V* node (see step VI-b)

VII. If there were no changes in Step V, then stop. If there were one or more changes in step V, then modify the solution vector and the mismatch vector as follows:

For each change made in step V-a (changing a *P-V* bus to a *P-Q* bus):

- $\boldsymbol{N_G = N_G - 1}$
- Include the variable $\boldsymbol{\Delta V_k}$ to the vector $\boldsymbol{\Delta x}$ and the variable $\boldsymbol{V_k}$ to the vector $\boldsymbol{x}$.
- Include the reactive equation corresponding to node *k* to the vector $\boldsymbol{f(x)}$.
- Modify the Jacobian by including a column to $\boldsymbol{J_2\ (J^{PV})}$ and including a row to $\boldsymbol{J_3}$ $\boldsymbol{(J^{Q\theta})}$ and $\boldsymbol{J_4\ (J^{QV})}$.

For each change made in Step V-b (changing a *P-Q* bus back to a *P-V* bus):

- $\boldsymbol{N_G = N_G + 1}$
- Remove the variable $\boldsymbol{\Delta V_k}$ to the vector $\boldsymbol{\Delta x}$ and the variable $\boldsymbol{V_k}$ from the vector $\boldsymbol{x}$.
- Remove the reactive equation corresponding to bus $\boldsymbol{k}$ from the vector $\boldsymbol{f(x)}$.
- Modify the Jacobian by removing a column to $\boldsymbol{J_2}$ and removing a row from $\boldsymbol{J_3}$ and $\boldsymbol{J_4}$.

After modifications have been made for all changes, go back to Step IV.

When the algorithm stops, then all line flows may be computed using

$$S_{ij} = E_i I_{ij}^* = E_i [E_i - E_j] \bar{Y}_{ij}^* \qquad (6.75)$$

The solution strategy of Newton-Raphson method to the load flow problem can be summarized as follows:

I. Assign initial voltage magnitudes and zero voltage angles to all bus-bars

II. Using available bus voltages, calculate values for ΔP and ΔQ for all buses.

III. Check whether the values of ΔP and ΔQ are smaller than the predefined convergence tolerance

IV. If convergence has been achieved, output all busbar and line flow conditions and Terminate the procedure.

V. If convergence has not been achieved, then use Table 6.2 equations to derive the elements of the Jacobian matrix.

VI. Apply matrix inversion technique to solve for correction factors $\Delta\delta$ and $\frac{\Delta V}{V}$ from (6.74)

VII. Update values of δ and V in preparation for repeating the procedure, thus

$$\delta_i^{new} = \delta_i^{old} + \Delta\delta_i \qquad (6.76)$$

$$V_i^{New} = V_i^{Old} + \Delta\delta_i \qquad (6.77)$$

i.e. $$|V_i|^{new} = |V_i|^{old} \left\{ 1 + \frac{\Delta |V_i|}{|V_i|^{old}} \right\} \qquad (6.78)$$

Figure 6.1 gives the computation strategy (work flow) of the Newton-Raphson method when applied to load-flow problem [Powell, 2004].

6.9 Algorithm Evaluation Criteria

The load-flow cases and the acceptable performance under the given operating case have to be considered (Saadat, 2004; Glover et al., 2008).

- ***Base case:*** A base case is a design requirement case with all the equipment operating within the normal ratings. This is applicable for peak and off peak load conditions. In this thesis, the system voltage at all the buses is taken to be within ±5%.

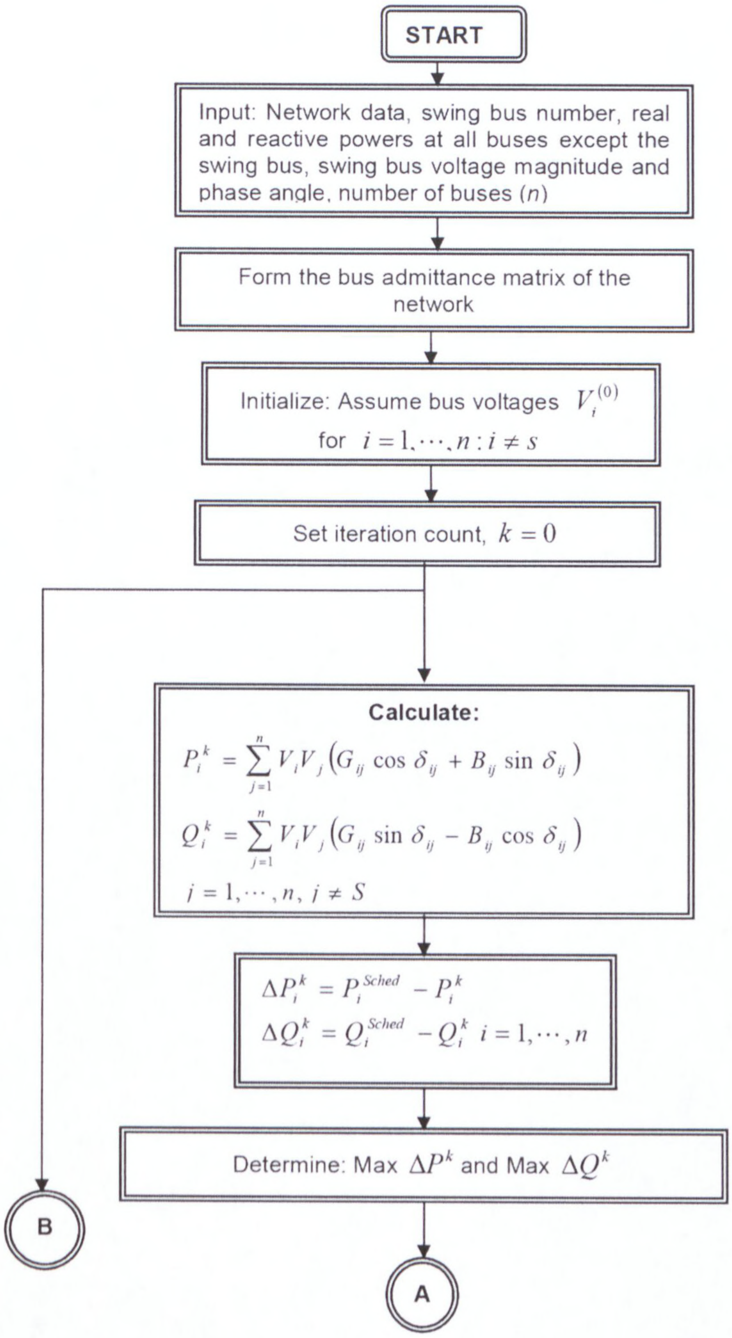
- ***Acceptable Cases:*** These are power flow cases, without any overloaded branches or under voltage or over voltage buses.
- ***Case with over loaded lines:*** If there are overloaded lines or transformers, then the line overloading can be brought to the normal ratings using transformer tap changing or other control actions.
- ***Cases with over voltage or under voltage:*** If there are over voltage or under voltage, then the line overloading can be brought to the normal ratings using transformer tap changing or other control actions
- ***Steady state voltage requirement:*** In case the voltage limit is not specified, it is a good practice to use a maximum and minimum voltage of +5% and –5% of the nominal voltage respectively. In extra high voltage systems, an upper voltage tolerance of +10% is often used.
- ***Loading levels:*** The loading levels of transmission lines, cable circuits and transformers are usually given as nominal ratings.

6.10 Computer Simulation Program

Due to large size and complexity of power system network, solving or analysing the set of non-linear power flow equations that make up the continuous component of the power system model becomes difficult. The analytical difficulties motivate the use of computer simulation to numerically solve the equations. Therefore, a simulation code have to be developed, expanded and refined in MATLAB environment.

The computer simulation of the approximate Tanzanian Power System Network Model serves as a basis for analysis process as well as to verify that the model is correct designed and implemented into the developed simulation code, and validate that the approximate model is accurately enough to represent the Tanzania Power System Network. In addition, the validation should prove:

- Simulation employs component-level model that justified through empirical data;
- Simulation yields high level results consistent with the type of model understudy.


START
Input: Network data, swing bus number, real and reactive powers at all buses except the swing bus, swing bus voltage magnitude and phase angle, number of buses (n)
Form the bus admittance matrix of the network
Initialize: Assume bus voltages $V_i^{(0)}$ for $i = 1,\cdots,n : i \neq s$
Set iteration count, $k = 0$
Calculate:
$P_i^k = \sum_{j=1}^{n} V_i V_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}\right)$
$Q_i^k = \sum_{j=1}^{n} V_i V_j \left(G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}\right)$
$i = 1,\cdots,n,\ i \neq S$
$\Delta P_i^k = P_i^{Sched} - P_i^k$
$\Delta Q_i^k = Q_i^{Sched} - Q_i^k \ i = 1,\cdots,n$
Determine: Max ΔP^k and Max ΔQ^k
B
A

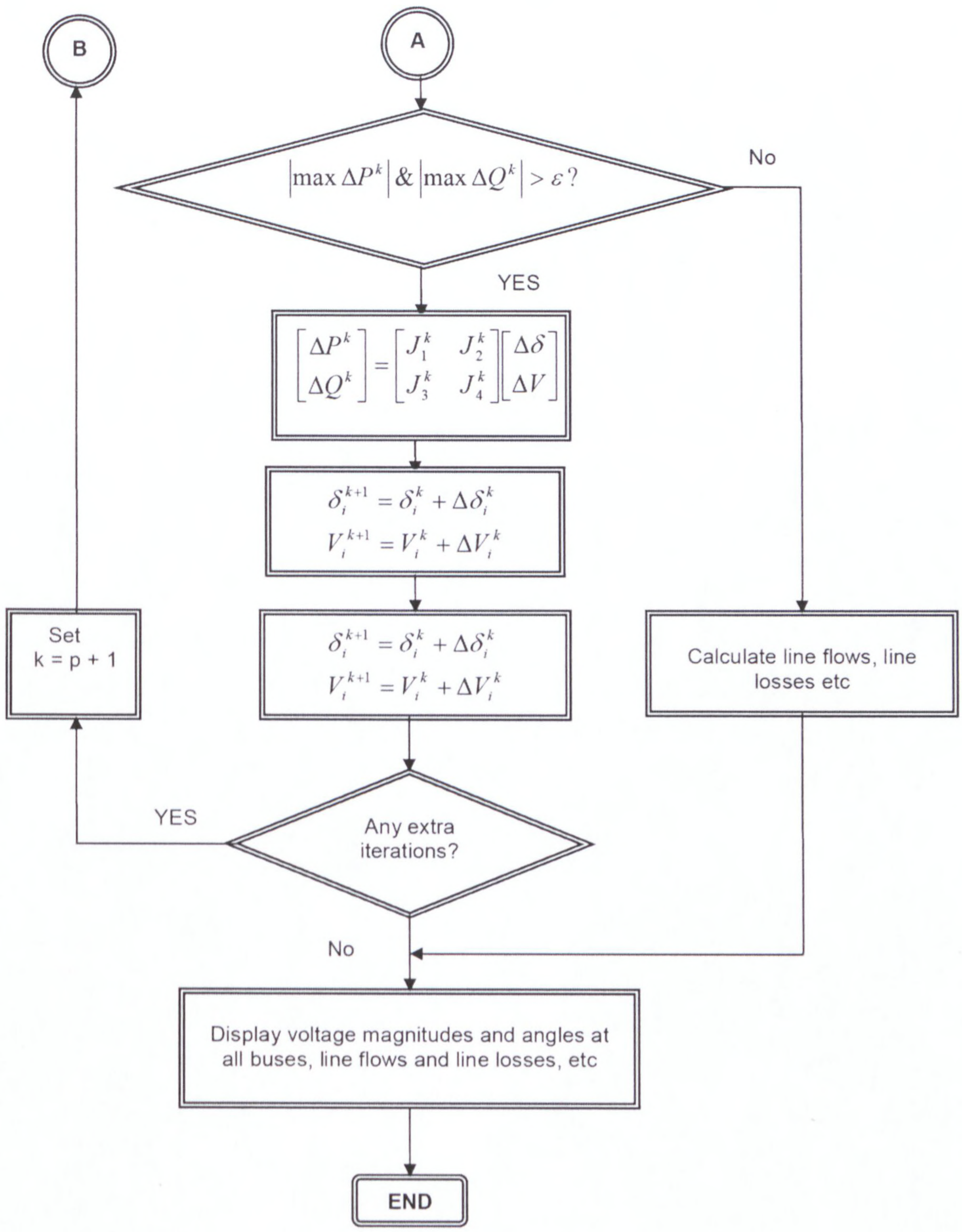


Figure 6.1: Flow chart for N-R load flow solution

Several computer programs have been developed for load-flow solutions for practical systems. In this thesis Load-flow program developed by H.Saadat is **MODIFIED** and used. The program is an open source. It allows some changes to be introduced or made according to the requirement of the problem.

The program consists of 4 parts. The program is preceded by *Lfybus*, the main program (*Lfnewton*), *busout* and *lineflow* programs. The program is given in Appendix C3. The 4 parts are explained as follows:

Lfybus: This program requires the line and transformer parameters and transformer tap settings specified in the input file named *linedata*. *Lfybus* converts impedances to admittances and obtains the bus admittance matrix.

Lfnewton: This program obtains the power flow solution by the Newton-Raphson method and requires the files named *busdata* and *linedata*. *Lfnewton* is designed for the direct use of load and generation in MW and MVAr, node voltages in per unit, and angle in degrees. Loads and generation are converted to per unit quantities on the base MVA selected. A provision is made to maintain the generator reactive power of the voltage-controlled buses within their specified limits. The violation of reactive power limit may occur if the specified voltage is either too high or too low. After a few iterations, the Var calculated at the generator buses are examined. If a limit is reached, the voltage magnitude is adjusted in steps of 0.5 percent to ±5 percent to bring the Var demand within the specified limits. The busdata and the line data files in this program have to be prepared such that the program can be able to identify them. In addition, nodes are coded. Codes are introduced in order to identify the type of a bus. The following codes are used.

3: This code is used for load nodes. The loads are entered positive in megawatts and Megavars. For this node, initial voltage estimate are scheduled (specified). In this thesis this is usually 1 and 0 for voltage magnitude and phase angle respectively.

1: This code is used to define the Slack bus. The only necessary information required from this node is the voltage magnitude and its phase angle. In this thesis the voltage magnitude is accepted as 1 and 0 for phase angle.

2: this code is used for voltage-controlled nodes (generation nodes). For this node, voltage magnitude, real power generation in Megawatts, and the minimum and maximum limits of the Megavars demand is scheduled.

Busout: This program produces the bus output result in a tabulated form. The bus output result includes the voltage magnitude and angle, real and reactive power of generators and loads, and the shunt capacitor/reactor MVar. Total generation and total load are also included.

Lineflow: This program prepares the line output data. "Lineflow" program is designed to display the active and reactive power flow entering the line terminals and line losses as well as the net power at each bus. Also included are the total real and reactive losses in the system.

Preparation of bus and transmission line data is carried out as follows:

Bus data preparation: The format of the node entry is selected in order to facilitate the required data for each node in a single row. The information required is included in a matrix form. Column 1 is the bus number. Column 2 is the bus code 3, 1 or 2. Column 3 and 4 are voltage magnitude in per-unit and phase angle in degree. Column 5 and 6 are load demands in MW and MVAr. Column 7, 8, 9 and 10 are MW, MVAR, minimum MVAr and maximum MVAr of generators. Column 11 is the injected MVAr of shunt/reactor injected at respective bus.

Transmission Line data preparation: Linedata are usually identified by the node-pair method. Linedata information is also includes in a matrix form. Column 1 and 2 are line numbers i.e. from sending end to receiving end node. Column 3, 4 and 5 contain the line resistance, line reactance and one-half of the total line charging susceptance in per-unit based on MVA base. Column 6 is usually used for transformer settings. For transmission line, 1 must be entered in this column. If the entry is a transformer, the left node number is assumed to be the tap side of the transformer and the number is entered. The number could be 1, or less than 1 depending on the information obtained from the transformer.

The developed program functions are shown in Table 6.3. The main MATLAB script to run the program is given below

Table 6.3: MATLAB files for calculating load flow solution

File	Description
LF30	M-file (to run the program)
LD30	Excel-file depicting line data of the system
BD30	Excel-file depicting bus data of the system
Lfybus.m	M-file, which calculates ybus admittance matrix
Lfnewton.m	M-file, which calculates load flow using N-R method
Lineflow.m	M-file, which calculates lineflow and losses
Busout.m	M-file, which prints the output on the screen in tabular form

LF30- M-File

```
Clear                              % Clear all variables...
clc                                % Clear command window...
tic;
basemva = 100;                     % Power system baseMVA...
```

```
accuracy = 1e-4;                  % Power Mismatch accuracy...
accel = 1.9;                      % Acceleration factor...
maxiter = 100;                    % Maximum number of iteration...
linedata = xlsread('LD30');       % Reads linedata file...
busdata = xlsread('BD30');        % Reads busdata file...
Lfybus        % Forms Ybus admittance matrix...
Lfnewton      % Calculates Load flow solution using N-R
method...
Lineflow      % Calculates and displays Lineflow and losses...
Busout        % Prints Load flow solution on the screen...
toc;
```

(Full version of the program is given in Appendix C3)

6.11 Line flows and losses equations

After the Newton-Raphson iterative solution of bus voltages and phase angles, the next step is computation of line flows i.e. real and reactive power and line losses of the transmission lines of the system. Real and reactive power flows are crucial as input data for the solution of system state estimation. They are components of the *zdata* file (chapter 10).

Consider a transmission line connecting two buses i and j in Figure 6.2. The line current I_{ij}, measured at bus i and defined positive in the direction from i to j is given by:

$$I_{ij} = I_l + I_{i0} = Y_{ij}(V_i - V_j) + Y_{i0}V_i \quad (6.79)$$

Similarly, the line current I_{ji} measured at bus j and defined positive in the direction from j to i is given by:

$$I_{ji} = -I_l + I_{j0} = Y_{ij}(V_j - V_i) + Y_{j0}V_j \quad (6.80)$$

The complex power S_{ij} from bus i to j and S_{ji} from bus j to i are:

$$S_{ij} = E_i I_{ij}^* \quad (6.81)$$

$$S_{ji} = E_j I_{ji}^* \quad (6.82)$$

The power loss in transmission line $i-j$ is the algebraic sum of the power flows determined by the two powers

$$S_{Lij} = S_{ij} + S_{ji} \quad (6.83)$$

Eqn (6.84) is used in developing a program coded in MATLAB in computing the losses in the entire system.

Transmission line model for computing line flows and losses is given in Figure 6.2

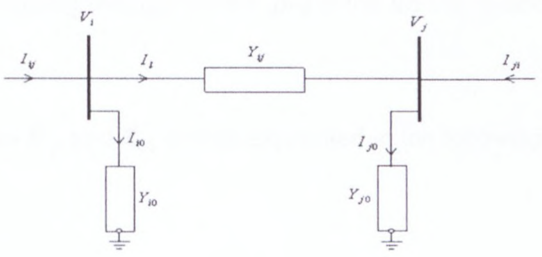


Figure 6.2: Transmission line representationl for calculating lineflows and losses

6.12 Real and reactive power flow model of tie-lines in a decomposed system

Section 6.10 has discussed and presented mathematical equations (model) and program pertaining to integrated power systems. Gauss-Seidel, Newton-Raphson or Fast-Decoupled method can be applied in solving the non-linear equations developed. It is apparently that electrical power system is expanding fast and becoming one of complex systems in the world; therefore, in order to obtain load flow solution of these systems new methods or innovations is required. One of the promising approaches is decomposition of a power system into overlapping sub-systems and then parallel processing procedures can be applied in calculating real and reactive power and later solving load flow problem. Nevertheless, before parallel computing can be applied it is necessary to have in place real and reactive power model that can be used to prepare a program (software) to perform the computation. Therefore, this section presents mathematical models of real and reactive power flows in tie-line(s) connecting the sub-systems.

Before considering interconnections between *ith* and *jth* sub-system, first derivation of power flows within *ith* sub-system is presented.

6.12.1 Real and reactive power flows in a line of a sub-system

Consider the complex current $I_{i,pq}$ flowing in a transmission line ***p-q*** in the ***ith*** sub-system. Let also consider $V_{i,p}$ and $V_{i,q}$ as the voltages at bus ***p*** and ***q*,** respectively. $Y_{ii,pq}$ let be the bus admittance of the line ***p-q*** of the sub-system. Then, the current flowing in the transmission line ***p-q*** is given by

$$I_{i,pq} = Y_{ii,pq}\left(E_{ip} - E_{iq}\right) + jb_{ipq}^{Sh}V_{ip} \tag{6.84}$$

where

$b_{i,pq}^{sh}$:is the shunt susceptance of a transmission line ***p-q***

The complex power flowing through the line ***p-q*** in the ***ith*** sub-system is given by

$$S_{ipq}^{*} = E_{ip}^{*}I_{i,pq} \tag{6.85}$$

The complex voltages $\boldsymbol{E_{i,p}}$ and $\boldsymbol{E_{i,q}}$ can be expressed in the following form

$$\begin{aligned} E_{i,p} &= V_{ip}e^{j\delta_{ip}} \\ E_{i,q} &= V_{iq}e^{j\delta_{iq}} \end{aligned} \tag{6.86}$$

Substituting Eqn (6.85) into (6.84), yields

$$S_{i,pq}^{*} = Y_{ii,pq}V_{i,p}e^{-j\delta_{i,p}}\left(V_{i,p}e^{\delta_{ip}} - V_{i,q}e^{\delta_{i,q}}\right) + jb_{i,pq}^{sh}V_{i,p}^{2} \tag{6.87}$$

The conjugate of the complex power flow of Eqn (6.86) is given by

$$S_{i,pq}^{*} = P_{i,pq} - jQ_{i,pq} \tag{6.88}$$

Then, substituting (6.88) into (6.87) transforms (6.87) into (6.89) as follows

$$P_{i,pq} - jQ_{i,pq} = V_{i,p}^{2}\left(G_{ii,pq} + jB_{ii,pq}\right) - V_{i,p}\left(G_{ii,pq} + jB_{ii,pq}\right)V_{qi,q}\left(\cos\delta_{i,pq} + j\sin\delta_{i,pq}\right) + jb_{i,pq}^{sh}V_{i,p}^{2} \tag{6.89}$$

$$\begin{aligned} P_{i,pq} - jQ_{i,pq} &= V_{i,p}^{2}G_{ii,pq} + jB_{ii,pq}V_{i,p}^{2} - V_{i,p}G_{ii,qg}V_{i,g}\left(\cos\delta_{i,q} + j\sin\delta_{i,pq}\right) - \\ &- V_{i,p}jB_{ii,pq}V_{i,q}\left(\cos\delta_{i,pq} + j\sin\delta_{i,pq}\right) + jb_{i,pq}^{2}V_{i,p}^{2} = \\ &= V_{i,p}^{2}G_{ii,pq} + jV_{i,p}^{2}B_{ii,pq} - V_{i,p}V_{i,q}G_{ii,pq}\cos\delta_{i,pq} - jV_{i,p}V_{i,q}G_{ii,pq}\sin\delta_{i,pq} - \\ &- jV_{i,p}V_{i,q}B_{ii,pq}\cos\delta_{i,pq} + V_{i,p}V_{i,q}B_{ii,pq}\sin\delta_{i,pq} + jb_{i,pq}^{sh}V_{i,p}^{2} \end{aligned} \tag{6.90}$$

Separating real and reactive parts and re-arranging eqn (6.90), yields

$$P_{i,pq} = V_{i,p}^{2}G_{ii,pq} - V_{i,p}V_{i,q}\left(\cos\delta_{i,pq} - B_{ii,pq}\sin\delta_{i,pq}\right) \tag{6.91}$$

$$Q_{i,pq} = -V_{i,p}^{2}\left(B_{ii,pq} + b_{i,pq}^{sh}\right) + V_{i,p}V_{i,q}\left(G_{ii,pq}\sin\delta_{i,pq} + B_{ii,pq}\cos\delta_{i,pq}\right) \tag{6.92}$$

Equations (6.91) and (6.92) represent the real and reactive power flows of the ***ith*** isolated sub-system.

6.12.2 Real and reactive power flows in the tie-line(s)

Consider a system decomposed into sub-systems and their tie-lines connecting the sub-systems shown in Figure 6.3. Let $N_{tl,ij}$ be the number of tie-line(s) connecting ***ith*** and ***jth*** sub-systems, also ***g*** is a bus in the border of ***jth*** sub-system that links with ***ith*** sub-system bus ***p.*** Therefore, the complex current $I_{ij,pg}$ flowing in the transmission line *p-g* is given by

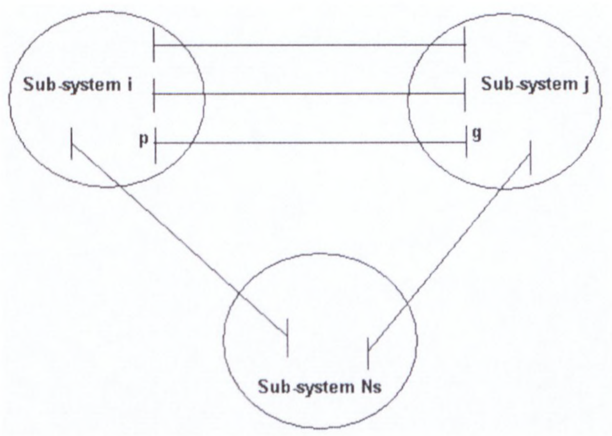


Figure 6.3: Tie-lines connecting sub-systems

$$I_{ij,pg} = Y_{ij,pg}\left(E_{i,p} - E_{jg}\right) + jb^{sh}_{ij,pg}E_{i,p} \qquad (6.93)$$

where

$N_{tl,ij}$: is the number of tie-lines connecting ***ith*** and ***jth*** sub-systems

$Y_{ij,pg}$: is the admittance of the line **p-g** between the ***ith*** and ***jth*** sub-system

$b^{sh}_{ij,pg}$:is the shunt susceptance of a transmission line **p-g**

$$p = \overline{1, N_S}, g = \overline{1, N_s} p \neq g$$

$$p = \overline{1, N_{tl,ij}}, g = \overline{1, N_{tl,ij}} \qquad p \neq g$$

The complex power flows through line **p-g** which is limited and fixed in case of power transfer from one country to another if the sub-systems define a power pool like east African power pool (EAPP), etc, is given by

$$S^*_{ij,pg} = E^*_{i,p} I_{ij,pg} \qquad (6.94)$$

The complex voltages for $E_{i,p}$ and E_{jg} are given by

$$E_{i,p} = V_{i,p} e^{j\delta_{i,p}}$$
$$E_{j,g} = V_{j,g} e^{j\delta_{jg}} \tag{6.95}$$

Therefore, the complex power flow can be given by

$$S^*_{ij,pg} = Y_{ij,pg} V_{i,p} e^{-j\delta_{i,p}} \left(V_{i,p} e^{j\delta_{i,p}} - V_{j,g} e^{j\delta_{j,g}} \right) + jb^{sh}_{ij,pg} V^2_{i,p}$$
$$p = \overline{1, N_{tl,ij}}, g = \overline{1, N_{tl,ij}} \tag{6.96}$$

The conjugate of the complex power flow can be expressed in terms of real and reactive power flows as follows

$$S^*_{ij,pg} = P_{ij,pg} - jQ_{ij,pg} \tag{6.97}$$

Substituting (6.97) into (6.96), yields

$$P_{ij,pg} - jQ_{ij,pg} = Y_{ij,pg} V_{i,p} e^{-j\delta_{i,p}} \left(V_{i,p} e^{j\delta_{i,p}} - V_{j,g} e^{j\delta_{j,g}} \right) + jb^{sh}_{ij,pg} V^2_{i,p} \tag{6.98}$$

Let

$$Y_{ij,pg} = G_{ij,pg} + jB_{ij,pg} \tag{6.99}$$

$$V_{i,p} e^{-j\delta_{i,p}} . V_{i,p} e^{j\delta_{i,p}} = V^2_{i,p} \tag{6.100}$$

$$V_{i,p} e^{-j\delta_{i,p}} . V_{j,g} e^{j\delta_{j,g}} = V_{i,p} . V_{j,g} . e^{j(\delta_{j,g} - \delta_{i,p})} = V_{i,p} V_{j,g} \left(\cos\delta_{ij,pg} + j \sin\delta_{ij,pg} \right) \tag{6.101}$$

Substituting (6.99), (6.100) and (6.101) into (6.98), gives

$$\begin{aligned}
P_{ij,pg} - jQ_{ij,pg} &= Y_{ij,pg} V_{i,p} e^{-j\delta_{i,p}} \left(V_{i,p} e^{j\delta_{i,p}} - V_{j,g} e^{j\delta_{j,g}} \right) + jb^{sh}_{ij,pg} V^2_{i,p} \\
&= Y_{ij,pg} V^2_{i,p} - Y_{ij,pg} V_{i,p} V_{j,g} e^{j(\delta_{j,g} - \delta_{i,p})} + jb^{sh}_{ij,pg} V^{sh}_{i,p} = \\
&= \left(G_{ij,pg} + jB_{ij,pg} \right) V^2_{i,p} - \left(b_{ij,pg} + jB_{ij,pg} \right) V_{i,p} V_{j,g} \left(\cos\delta_{ij,pg} + j \sin\delta_{ij,pg} \right) = \\
&= G_{ij,pg} V^2_{i,p} + jB_{ij,pg} V^2_{i,p} - G_{ij,pg} V_{i,p} V_{j,g} \left(\cos\delta_{ij,pg} + j \sin\delta_{ij,pg} \right) - \\
&- jB_{ij,pg} V_{i,p} V_{j,g} \left(\cos\delta_{ij,pg} + j \sin\delta_{ij,pg} \right) + jb^{sh}_{ij,pg} V^2_{i,p} = \\
&= G_{ij,pg} V^2_{i,p} + jB_{ij,pg} V^2_{i,p} - G_{ij,pg} V_{i,p} V_{j,g} \cos\delta_{ij,pg} - jG_{ij,pg} V_{i,p} V_{j,g} \sin\delta_{ij,pg} - \\
&- jB_{ij,pg} V_{i,p} V_{j,g} \cos\delta_{ij,pg} + B_{ij,pg} V_{i,p} V_{j,g} \sin\delta_{ij,pg} + jb^{sh}_{ij,pg} V^2_{i,p}
\end{aligned} \tag{6.102}$$

Separating real and imaginary parts, the following Eqns are obtained

$$P_{ij,pg} = G_{ij,pg} V^2_{i,p} - V_{i,p} V_{j,g} \left(G_{ij,pg} \cos\delta_{ij,pg} - B_{ij,pg} \sin\delta_{ij,pg} \right) \tag{6.103}$$

$$Q_{ij,pg} = -V^2_{i,p} \left(B_{ij,pg} + b^{sh}_{ij,pg} \right) + V_{i,p} V_{j,g} \left(G_{ij,pg} \sin\delta_{ij,pg} + B_{ij,pg} \cos\delta_{ij,pg} \right) \tag{6.104}$$

When $\delta_{ij,pg} = 0 \rightarrow \cos\delta_{ij,pg} = 1, \sin\delta_{ij,pg} = 0$, therefore, Eqns (6.104) and (6.104) are transformed into

$$P_{ij,pg} = G_{ij,pg}V_{i,p}^2 - V_{i,p}V_{j,g}G_{ij,pg} \tag{6.105}$$

$$Q_{ij,pg} = -V_{i,p}^2\left(B_{ij,pg} + b_{ij,pg}^{sh}\right) + V_{i,p}V_{j,g}B_{ij,pg} \tag{6.106}$$

The real and reactive power flows in the tie-line can be considered as equality constraints given by

$$P_{ij,pg} - V_{i,p}^2 G_{ij,pg} + V_{i,p}V_{j,g} = 0 \tag{6.107}$$

$$Q_{ij,pg} + V_{i,p}^2 B_{ij,pg} + jb_{ij,pg}^{sh}V_{i,p}^2 = 0 \tag{6.108}$$

The developed model of the power flows in the branches of the interconnected sub-systems and in the tie-lines are later used in chapter 8 for development of method for hierarchically solution of the problem for state estimation

6.13 Conclusion

Load-flow computations are performed in system planning, operational planning as well as in general operation of systems. In this chapter, the basic load-flow concepts and its related equations are presented. In a systematic way such that the developed equations can be used in developing a computer program employing the Newton-Raphson algorithm. Node classifications and their related equations existing in a real power system are developed. The variables and constrains for each node are described. These variables are divided into three groups: state variables, control variables and disturbance variables. For a feasible solution of a load-flow or state estimation problems, the constraints that govern the current, voltage and load flow have to be satisfied. These constraints are considered to avoid uncertainties existing in the system.

The power system comprises of conventional components discussed in chapter 4, these components are result of engineering design, therefore, engineering design and operational limits have to be considered both in the formulation of load-flow problems, and therefore, the mathematical inequality constraints of a generator, bus transformer, and transmission line are described

The load flow equations for a general *N*-bus system that describe the inter-relationship between the three groups of variables are formulated. The load-flow equations are non-linear in nature and iterative methods are used to solve these equations. Several methods have proved useful. The N-R method is considered to be the best in load-flow analysis, thus more emphasize is given to this iterative method and its algorithm is chosen to solve the load-flow problem and validate the integrated Tanzanian Power System Network Model developed in chapter 5.

Simulation of load flow is performed in chapter 10. Load flow results are important because they are assumed as true measurement values in this thesis and used for power system state estimation problem formulated in chapter 7.

Chapter 7 presents formulation of the problem for state estimation. System measurements, state variables, state estimation criteria, formulation of the problem, solution methods of non-linear optimization problem and solution of the problem for state estimation by primal-dual logarithmic barrier (PDLB) a class of path following interior point method is given. An algorithm and a MATLAB Program for solution of the state estimation are presented.

CHAPTER SEVEN: FORMULATION OF THE PROBLEM FOR STATE STIMATION

7.1 Introduction

In this chapter, power system state estimation problem is formulated. The weighted least absolute value (WLAV) criterion is used in the formulation and the primal-dual logarithmic barrier interior point method (PDLB)) algorithm is developed for the solution of the problem.

7.2 System Measurements

Successful power system operation under normal balanced three-phase steady-state conditions requires the following requirements to be observed:

- Node voltage magnitudes remain close to rated values normally 1 p.u
- Generators operate within specified real and reactive power limits
- Generation supplies the load plus losses, and
- Transmission lines and transformers are not overloaded.

Hence, power system monitoring is critical to make sure that any violation of the above mentioned factors is avoided. System monitoring provides the operators of the system with pertinent up-to-date information for the conditions of the system. System monitoring is the most important function for proper operation of the power systems.

Effective system monitoring requires that critical quantities be measured and the value of the measurements be transmitted to the central dispatch centre (CDC) for analysis. However, many problems are encountered in monitoring a transmitting system. These problems come primarily from the nature of measurement transducers and from communications problems in transmitting the measured values to the CDC. Transducers, like any measurement devices are subject to errors. If the errors are small they may be undetected and may cause misinterpretation by those reading the measured values. If transducers have gross measurement errors, this renders their value useless. In addition, the telemetry system or equipment often experience periods when communication channels are completely out; thus, depriving the operator at CDC of any information about other parts of the system network.

It is for these reasons that state estimation technique has been developed. A state estimator can "filter out" small random errors in meter readings, detect and identify gross measurement errors and "fills in" meter readings that have failed due to communication failure. State estimation is a mathematical procedure to process the set of real-time measurements using the topology determined by the topology processor, to come up with the "best" estimate of the current state of the power system. The result from state estimation provides the real-time database for other system applications, such as security assessment, control and optimal power flow (OPF). State estimation can also be used for data validation. In this section, measurements and model relating to state estimation process is presented.

7.2.1 Measurements

State estimation is a mathematical procedure to process the set of real-time measurements from the following source of information:

- Analog measurement
    - Voltage magnitude
    - Active power: branch flow power; branch group flow in a designated – group of branches; bus injection;
    - Reactive power: branch flow; branch –group flow in a designated group of branches; bus injection;
    - Current magnitude flow in a branch and injection;
    - Magnitude of transformer turns ratio;
    - Phase shift angle of transformer
    - Active power flow: in switches; in zero impedance branches; in branches of unknown impedance;
    - Reactive power flow: in switches; in zero impedance branches; in branches of unknown impedance
- Status Measurements: Switches/breakers on/off (digital measurements)
- Pseudo Measurements: Forecasted node load/generation (not measured, only available in database)
- Virtual Measurements (exact measurements): Zero injections

7.2.2 Measurement Model

A measurement vector *z* may be created which contains *m* measurements from the power system. Measurements include those given in 7.2.1. The ***2N-1*** state variables constitute the state vector x, which is related to the measurement z in a given model:

$$z = h(x) + r \qquad (7.1)$$

where

$r \in \Re^{mx1}$ is the residual vector that contains m variables.

Vector ε purpose is to account for the uncertainty in the measurements and the model. Hence, errors are random variables assumed to have a zero mean and a known diagonal covariance matrix $\boldsymbol{R_z}$

$$R_z = diag(\sigma_1^2, \sigma_2^2, \cdots, \sigma_m^2) \tag{7.2}$$

where

$\sigma_1^2, \ldots \sigma_m^2$ is the vector standard deviation of all measurements calculated to reflect the expected accuracy of the corresponding meters used.

The matrix $\boldsymbol{R_z}$ plays the role of weighting matrix which causes the calculated estimate to fit closer to the good measurements than to the bad measurements. The weighting is very important when meter readings of different accuracy are to be modeled together. However, relatively large errors in $\boldsymbol{R_z}$ usually have negligible effect on the actual estimate. Thus, in most cases $\boldsymbol{R_z}$ is assumed to be diagonal.

7.2.3 Measurement Function

The measured quantities are represented by the vector **z,** and have been discussed in subsection (7.2.1). ***h(x)*** represents a vector of non-linear functions relating the measurement and the state vectors. With exception of the voltage magnitude at the buses, these functions are usually non-linear and are used to calculate the estimated values corresponding to the measured values. For this thesis, only the bus voltage magnitude, the injected/demand real and reactive power as well as the real and reactive power flows measurements are obtained from the load flow analysis discussed in chapter 6.

The state vector has ***2N-1*** elements, ***N*** bus voltage magnitudes, and ***(N-1)*** voltage angles. The phase angle of one reference bus is set equal to an arbitrary value. The state vector x has the following form assuming bus1 chosen as the reference:

$$x^T = [\delta_2, \delta_3, \cdots, \delta_n; V_1, V_2, \cdots, V_n] \tag{7.3}$$

The mathematical expressions for the measurements are given below; assuming the general equivalent-π model for the network branches developed in chapter four is used:

- Real and reactive power injection/demand at bus i

$$P_i = V_i \sum_{j=1}^{N} V_j \left(G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}\right) \tag{7.4}$$

$$Q_i = V_i \sum_{j=1}^{N} V_j \left(G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}\right) \quad i, j \in N \tag{7.5}$$

- Real and reactive power flow from bus i to bus j

$$P_{ij} = V_i^2 (g_{ij}^{Sh} + g_{ij}) - V_i V_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) \tag{7.6}$$

$$Q_{ij} = -V_i^2 (b_{ij}^{Sh} + b_{ij}) - V_i V_j (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) \tag{7.7}$$

where

$V_i \angle \delta_i$ is the complex voltage at bus i

$G_{ij} + jB_{ij}$: are the *ijth* elements of the complex bus admittance matrix

$g_{ij} + jb_{ij}$: is the series admittance of the branch connecting nodes i and j

$g_{ij}^{Sh} + jb_{ij}^{Sh}$: is the shunt admittance of the branch connected to bus i

***N*:** is the number of nodes in the system

7.2.4 Measurement Errors

The measurement error ε is a random vector representing the normal difference between measurement **z** and ***h(x)***. Because of the presence of error ε, the dimension *m* of the measurement vector **z** must be larger than the dimension *n* of the unknown vector *x*. The error ε is normally caused by:

- Modelling errors
- Coding errors
- Random errors: related to the class of precision of the instrument.
- Intermittent errors: temporary failures in communication channels
- Systematic errors- introduced by:
  - The non-linearity of the current transformers (CT) and capacitor coupling voltage transformers (CCVT)
  - Deterioration of instrument with time, temperature, weather, and other environmental causes.

7.2.5 Choice of Measurement Model

Implementation of state estimation requires choice of numerical values for the network structure (bus admittance matrix developed in chapter 5) and the error structure. The choice or development of good network model is very important for satisfactory computation of state estimation. A precise error model, $\boldsymbol{R_z}$ may vary with load conditions, time of the day, and weather conditions. However, precise model is not needed, and approximate one is enough to conduct the estimation. The appropriate model of a state estimator is based on the following assumptions:

- Uncorrelated observation errors
- Small voltage difference across the lines
- Voltage magnitude close to the nominal one.

Uncorrelated errors are usually of interest to be investigated. The last two assumptions of the approximate model are also valid for use in normal and some abnormal operating conditions but may fail when the transmission system is stretched to its limits. In this thesis, the last two assumptions are also accommodated.

7.3 State Variables

Complex nodal voltages are the most commonly used as state variables. Turn ratios of transformers with taps that change under operating conditions are also treated as state variables. Power flows in branches that follow Ohm's law are usually dependent variables and can be determined from the state variables i.e. nodal voltages and turn ratios. Hence, the vector of state variable $\boldsymbol{x}$ normally includes the following states:

- Nodal voltage:
  - Voltage magnitude $\boldsymbol{V_i}$
  - Voltage angle δ_i
- Transformer turns ratio:
  - Turns ratio magnitude $\boldsymbol{t_{ij}}$
  - Phase shift angle φ_{ij}
- Complex power flow
  - Real power flow $\boldsymbol{P_{ij}}$ and $\boldsymbol{P_{ji}}$
  - Reactive power flow $\boldsymbol{Q_{ij}}$ and $\boldsymbol{Q_{ji}}$

7.4 State estimation criteria

There are two basic approaches for formulation of the problem for state estimation:

- Weighted Least squares (WLS) estimation formulation and

- Weighted Least Absolute value (WLAV) estimation formulation

Weighted Least squares formulation is used extensively for solution of power system state estimation. Least squares minimization gives maximum likelihood estimate when measurement error obeys the Gaussian distribution. The criterion is used to evaluate the state variable *x* stated as follows:

$$\min J(V\angle\delta)=\begin{cases}\sum_{j=1}^{M_V}\frac{(V_j^{meas}-V_j^{est})^2}{\sigma_{V_j}^2}+\sum_{j=1}^{M_{P_{inj}}}\frac{(P_j^{meas}-P_j^{est})^2}{\sigma_{P_j}^2}+\sum_{j=1}^{M_{Q_{inj}}}\frac{(Q_j^{meas}-Q_j^{est})^2}{\sigma_{Q_j}^2}\\+\sum_{j=1}^{M_{P_{Flow}}}\frac{(P_{flow_j}^{meas}-P_{flow_j}^{est})^2}{\sigma_{P_{floe,j}}^2}+\sum_{j=1}^{M_{Q_{flow}}}\frac{(Q_{flow_j}^{meas}-Q_{flow_j}^{est})^2}{\sigma_{Q_{flow,j}}^2}\end{cases} \quad (7.8)$$

where the variables are defined as follows:

M_V : is the number of voltage measurements

$M_{P_{inj}}$: is the number of real power injection measurements

$M_{Q_{inj}}$: is the number of reactive power injection measurements

$M_{P_{Flow}}$: is the number of real power flow measurements

$M_{Q_{flow}}$: is the number of reactive power flow measurements

σ_{Vi} : is the standard deviation of the meter installed at bus i

V_j^{meas} : is the j^{th} voltage measurement

V_j^{est} : is the j^{th} voltage estimation

P_j^{Meas} is the *jth* measured real power

Q_j^{Meas} is the *jth* measured reactive power

P_j^{Estim} is the *jth* estimated real power

Q_j^{Estim} is the *jth* estimated reactive power

P_{fl}^{Meas} is the *jth* measured real power flow

P_{fl}^{Estim} is the *jth* estimated real power flow

$Q_{fl,j}^{Meas}$ is the *jth* measured reactive power flow

$Q_{fl,j}^{Estim}$ is the *jth* estimated reactive power

σ_{Pj}^2 is the *jth* standard deviation of real power

σ_{Qj}^2 is the *jth* standard deviation of reactive power

$\sigma_{P_{Fl,j}}^2$ is the *jth* standard deviation of real power flow

$\sigma_{Qfl,j}^2$ is the *jth* standard deviation of reactive power

One of the problems encountered when using WLS estimator is that biased estimate may be obtained when bad data measurements are present in the data set; this occurs when there is communication or metering failures. Although several approaches have been studied how to detect, identify and remove bad data from the measurements, but the issue of bad data identification is still a challenge especially when working with large and interconnected power system. This is a motivation for researching and developing a robust state estimator that can reject bad data in the process of estimating the state of the system.

The weighted least absolute value (WLAV) estimator exhibits the property of rejecting bad data effectively compared to the WLS criterion. The difference between WLAV and WLS minimization techniques is that a best WLAV estimation is obtained by minimizing the sum of the absolute values of the errors whereas the best WLS estimation is obtained by minimizing the sum of the squares of the errors. Hence, this basic difference greatly affects the way that best estimations are obtained.

A WLAV estimation is obtained by interpolating at least *n* of *m* measurements (*n* is the number of state variables). This property is in contrast to the WLS estimation, which does not necessarily pass through any of the measurement points.

The criterion used to evaluate the state variable ***x*** using WLAV method is stated as follows:

$$\min J(V\angle\delta) = \begin{cases} \sum_{j=1}^{M_V} \frac{\left|V_j^{meas} - V_j^{est}\right|}{\sigma_{V_j}^2} + \sum_{j=1}^{M_{P_{inj}}} \frac{\left|P_j^{meas} - P_j^{est}\right|}{\sigma_{P_j}^2} + \sum_{j=1}^{M_{Q_{inj}}} \frac{\left|Q_j^{meas} - Q_j^{est}\right|}{\sigma_{Q_j}^2} \\ + \sum_{j=1}^{M_{P_{flow}}} \frac{\left|P_{flow_j}^{meas} - P_{flow_j}^{est}\right|}{\sigma_{P_{flow}}^2} + \sum_{j=1}^{M_{Q_{flow,j}}} \frac{\left|Q_{flow_j}^{meas} - Q_{flow_j}^{est}\right|}{\sigma_{Q_{flow,j}}^2} \end{cases} \quad (7.9)$$

As conclusion: The weighted least absolute value (WLAV) state estimation solution satisfies exactly a subset of *'**n**'* measurements while the remaining *'m'* measurements have non-zero residuals. This is the reason why the criterion is used to formulate the constrained state estimation problem for the solution of the Tanzanian Power System Network Model developed in chapter 5.

7.5 Formulation of the problem for state estimation

Formulation of the problem for state estimation is described in this sub section, at the end a mathematical model of the problem is presented.

The central dispatch centre (CDC) of a power system at any moment has three types of measurements.

- Unconstrained measurements (***z_u***)**,** which are telemetered to the centre from the substations scattered all-over the system. Due to measurement conventions (analog to digital), communication and transmission procedures, these measurements may contain gross errors. The mathematical model relating to these measurements and errors is given by;

$$z_u = h_u(x) + \varepsilon_u \quad (7.10)$$

- Exact measurements **(z_e),** which are not measured, but are known exactly a priori. These measurements include zero injection or virtual measurements from switching sub stations. The mathematical model relating to these measurements is given by

$$z_e = h_e(x) \Rightarrow g(x) = 0 \quad (7.11)$$

- Pseudo measurement, which are not measured but are known to remain in bounded interval. These measurements include bounds imposed on the reactive power injection at generator buses i.e. Var limits; transformer turns ratio limits and other measurements obtained from operational limits. The mathematical model of these measurements isgiven by

$$z_{\min} \leq h(x) \leq z_{\max} \quad (7.12)$$

The three measurements available at the central dispatch centre (CDC) and their related errors are summarized and shown in Table 7.1

Table 7.1: Summary of available measurements at CDC

Type	Related error
Unconstrained measurements	$\varepsilon \geq z - h(x)$ $\varepsilon \geq -z + h(x)$
Exact measurement	0
Pseudo Measurement	Bounded

The aim of power system state estimation is to minimize measurement error in order to obtain the "**best**" estimate of the state. The aim is attained by:

$$\begin{aligned} &\min \sum_{k=1}^{m} W^T \left|\varepsilon_k\right| \\ &s.t. \qquad \varepsilon \geq z - h(x) \\ &\qquad\qquad \varepsilon \geq -z + h(x) \\ &\qquad\qquad \varepsilon \geq 0 \end{aligned} \quad (7.13)$$

where

$W \in R^{mx1}$ is the weighting factor of the measurements

m: is the number of unconstrained measurements telemetered from the substations to the CDC.

In order to minimize the measurement errors in the entire system and make the estimate reliable, it is necessary to include all the measurement available in the system, i.e. to include all the constraints in the formulation of the problem. Therefore, the aim is accomplished by:

$$\begin{aligned}
&\min_x J(x) = \min \sum_{k=1}^{m} W^T |\varepsilon_k| && k = 1,\cdots,m \\
&s.t. \quad \sum_{p=1}^{m_E} g_p(x) = 0 && p = 1,\cdots,m_E \\
&\varepsilon_k \geq z_k - h_k(x) && \\
&\varepsilon_k \geq -z_k + h_k(x) && \\
&\varepsilon \geq 0 &&
\end{aligned} \tag{7.14}$$

where

m_E is the number of equality constraints existing in the system and are given by eqn (7.11). The equality constraints are expressed from the load flow equations given by:

$$P_i - V_i \sum_{j=1}^{N} V_j \left(G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}\right) = 0 \tag{7.15}$$

$$Q_i - V_i \sum_{j=1}^{N} V_j \left(G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}\right) = 0 \tag{7.16}$$

where

P_i and Q_i are real and reactive power injection at bus i

$\boldsymbol{x} \in \Re^n$, $\boldsymbol{n = 2N-1}$ is the state vector consisting of voltage magnitude and voltage phase angle of each bus of the system

$(\boldsymbol{\varepsilon, z, W}) \in \Re^{mx1}$

$\boldsymbol{h(x)} \in \Re^{mx1}$ is the non-linear function relating state vector to unconstrained measurements

$\boldsymbol{n < m}$

$\boldsymbol{n}$ is the dimension of state vector.

Errors are from injections and flows measurements i.e. real and reactive powers, real and reactive power flows as well as from voltage magnitude. It is apparaently that voltage magnitudes, real and reactive power injections and real and reactive power flows are measured quantities i.e. they are known. The unknown variable is the angle δ. One of the aims of the state estimation is to estimate the phase angle. Knowing the voltage magnitude and the phase angle, it is possible to determine the state of the power system.

Dropping the subscript in problem (7.14), yields

$$\begin{aligned} &\min W^T \varepsilon \\ &s.t. \quad z - h(x) - \varepsilon \leq 0 \\ &\quad -z + h(x) - \varepsilon \leq 0 \\ &\quad g(x) = 0 \\ &\quad \varepsilon \geq 0 \end{aligned} \qquad (7.16)$$

From measurement model given in Eqn (7.1), ***r*** can be written as

$$r = z - h(x) \qquad (7.17)$$

Substituting Eqn (7.17) into (7.16), yields

$$\begin{aligned} &\min W^T \varepsilon \\ &s.t. \quad -\varepsilon - r \leq 0 \\ &\quad -\varepsilon + r \leq 0 \\ &\quad g(x) = 0 \\ &\quad r - z + h(x) = 0 \\ &\quad (r, \varepsilon) \geq 0 \end{aligned} \qquad (7.18)$$

The estimator aims at estimating the state of the power system using problem (7.18). Critical analysis of problem (7.18) reveals to have characteristics shown by a non-linear problem. Thus, the problem is a non-linear in its formulation and is an optimization problem in its implementation. The problem aims at minimizing the measurement errors in order to obtain the **"best** "estimate of the state. This action is an optimization activity. Thence, solving problem (7.18) is equivalent to finding a solution of a non-linear optimization problem. In literature there are several approaches which have been developed and successfully used to find optimal solutions of optimization problems. Some of the methods are briefly discussed in the following subsection. The discussion of the methods is not complete; more information can be obtained from the cited references.

7.6 Solution methods of non-linear optimization problem

The following methods have been developed and successfully implemented in solving non-linear optimization problems.

Gradient descent method

Gradient descent method [Synman, 2005] is a first order optimization algorithm. It is also known as steepest descent or the method of steepest descent. The method is based on the observation that if the real function ***F(x)*** is defined and differentiable in a neighbourhood of a point *a*, the ***F(x)*** decreases fastest if one goes from **a** in the direction of the negative of ***F*** at ***a*** $-\nabla F(a)$. The weaknesses of gradient descent method are:

- The algorithm takes many iterations to converge towards a local minimum
- Finding the optimal step is time consuming

Non-linear conjugate gradient method

The non-linear conjugate gradient method is an extension of Conjugate gradient method. The non-linear conjugate gradient method is generalized for non-linear optimization. It is used to find the local minimum of a linear function using its gradient $\nabla_x f$ The method works perfectly when the function is approximately quadratic near the minimum which is the case when the function is twice differentiable at minimum.

Sub gradient methods

Sub gradient methods [Bertsekas, 1999; Shor, 1985] are algorithms for solving convex optimization problems. The methods can be used with a non-differentiable objective function. Sub gradient methods are much slower than Newton-Raphson method. However, they require much less computer memory. Moreover, by combining them with primal or dual decomposition technique, it is possible to develop a simple distributed algorithm for an optimization problem.

Newton-Raphson method

Newton-Raphson method [Kelly, 2003] can be used to solve system of non-linear equations to find the zeros of continuously differentiable functions $F: \Re^k \rightarrow \Re^k$. In the process, *F* has to be multiplied with the inverse of the ***k***-by-***k*** Jacobian matrix $J_F(x_n)$ instead of dividing by $f(x_n)$. The method only works if the initial value x^0 is close enough to the true zero. Newton-Raphson method has been discussed in detail in chapter six and used in solving the load flow problem

Interior Point methods (IPMs)

Interior Point Methods (IPM) also referred to as barrier methods are certain class of algorithms to solve linear and non-linear convex optimization problems. The methods have been discussed in detail in chapter two. In this sub section only the basic features are presented. The basic elements of the method consist of a self-concordant function $f:\Re\rightarrow\Re$ and a barrier functions. A barrier function is a continuous function whose value on a point increases to infinity as the point approaches the boundary of the feasible region [Nocedal & Wright, 1999]. Barrier function is used as a penalizing term for violation of constraints.

IPM reaches an optimal solution by traversing the interior of the feasible region instead of the vertices as in the case of simplex method. To date the following techniques have been developed and successfully implemented in solving power system optimization problems such as optimal power flow (OPF):

- The potential reduction algorithm which most closely embodies the construct of Karmarkar [Karmarkar,1984]
- The affine scaling algorithm
- Path following algorithms

The class of primal-dual path following interior point is considered the most successful. It operates simultaneously on the primal and dual problems.

As conclusion: Interior point methods (IPMs) are more efficient compared to other methods in solving non-linear problems. The methods work unitedly with Newton-Raphson method. Combinations of the two methods give more complexity and efficiency.

In this thesis, primal - dual logarithmic barrier (PDLB) a class of path following interior point method is used to solve the problem formulated in (7.18)

7.7 Solution of SE by PDLB interior point method

Path following (Logarithmic barrier) method is based on applying Newton-Raphson method to follow the central path of the feasible region. The central path is formed by the optimal solution of a family of problems defined by a logarithmic barrier function. The distinctive feature of path following method [Lustig, et al, 1994] comes from:

- Following the central path allows the algorithm to take a large step towards the optimal point;
- Applying Newton-Raphson direction produces quadratic convergence; and
- Using different step lengths in primal and dual spaces results in fast convergence.

In the following subsections, the procedure of solving the problem is presented:

- Transforming an inequality constrained optimization problem to equality constrained by using Logarithmic barrier method;
- Formulating the Lagrangian function using logarithmic barrier function;
- Setting the first-order necessary optimality conditions;
- Applying Newton-Raphson method to the set of equations coming from the first order optimality conditions; and
- Updating the variables and choosing step length, adjusting the barrier parameter and stopping the procedure.

7.7.1 Transforming inequality into equality constraints

The logarithmic barrier method is applied to problem (7.18) to eliminate the inequality constraints by incorporating into a logarithmic term that is appended to the objective function. By introducing slack variables l *and* u the original problem is transformed to a sequence of problems parameterized by the barrier parameter μ. Introduction of these slack variables and the barrier parameter transformed problem (7.18) to the following problem:

$$\begin{aligned}
&\min W^T\varepsilon - \mu\sum_{k=1}^{m}(\ln u_k + \ln l_k) \\
&s.t. \qquad -\varepsilon - r + u = 0 \\
&\qquad\qquad -\varepsilon + r + l = 0 \\
&\qquad\qquad g(x) = 0 \\
&\qquad\qquad r - z + h(x) = 0 \\
&\qquad\qquad (u, l, \varepsilon, r) \geq 0
\end{aligned} \tag{7.19}$$

where

μ is a scalar parameter called the barrier parameter. The value of μ is varied as the solution progress.

u and l are upper and lower slack variables used to convert the errors into equality.

k : is counting index

m: is the number of measurements

7.7.2 Formulating the Lagrangian function

The Lagrangian function associated with (7.19) can be written as:

$$L_{\wedge} = W^T\varepsilon - \mu\sum_{k=1}^{m}(\ln l_k + \ln u_k) - \lambda^T(-\varepsilon - r + u) - \beta^T(-\varepsilon + r + l) - v^T(r - z + h(x)) - \tau^T g(x) \tag{7.20}$$

where

$\lambda, \beta, \nu,$ and τ are vectors of Lagrange multipliers. Lagrange multipliers are optimization variables of an auxiliary optimization called "dual problem ". They are very important and assist when solving an optimization problem using interior point method.

7.7.3 Setting the first-order optimality conditions

The necessary conditions for an extreme value of the objective functions results when the partial derivatives of the Lagrangian function is taken respect to each variable and set these derivatives to zero. These conditions are known as Karush-Kuhn-Tucker (KKT) conditions. Convergence is attained when the KKT necessary conditions for optimality have been satisfied

Thus, the Karush-Kuhn-Tucker (KKT) first – order necessary optimality conditions for problem (7.20) are given by:

$$\nabla_l L_\wedge = -\mu U^{-1} e - \lambda = 0 \quad (7.21)$$

$$\nabla_u L_\wedge = -\mu L^{-1} e - \beta = 0 \quad (7.22)$$

$$\nabla_\varepsilon L_\wedge = W + \lambda + \beta = 0 \quad (7.23)$$

$$\nabla_\lambda L_\wedge = \varepsilon + r - u = 0 \quad (7.24)$$

$$\nabla_\pi L_\wedge = \varepsilon - r - l = 0 \quad (7.25)$$

$$\nabla_\nu L_\wedge = -r + z - h(x) = 0 \quad (7.26)$$

$$\nabla_\tau L_\wedge = -g(x) = 0 \quad (7.27)$$

$$\nabla_r L_\wedge = \lambda - \beta - \nu = 0 \quad (7.28)$$

$$\nabla_x L_\wedge = -H^T \nu - G^T \tau = 0 \quad (7.29)$$

$$(\lambda, \beta) \geq 0$$

where

G and **H** are Jacobian of ***g(x)*** and ***h(x)*** respectively.

L and **U** are diagonal matrices formed from slack vectors given by Eqns (7.30) and (7.31), respectively

$$U = \begin{bmatrix} u_1 & 0 & \cdots & 0 \\ 0 & u_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & u_m \end{bmatrix} \tag{7.30}$$

$$L = \begin{bmatrix} l_1 & 0 & \cdots & 0 \\ 0 & l_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & l_m \end{bmatrix} \tag{7.31}$$

Their ***kth*** diagonal elements are l_k and u_k

e* = *[1, 1, 1]^T : a vector with all its elements equals to one

W: a diagonal matrix whose elements are reciprocal of the error variance $\mathbf{R}_{ii}$. **W** is a matrix of weighting factors on the measurements. Normally, large weights are assigned to measurements that are more accurate.

7.7.4 Newton-Raphson method and the solution of KKT equations

The KKT non-linear equations can be solved using different methods. They can be solved either as all equations together or by reducing them by eliminating of variables. In this thesis, the equations are solved iteratively using the Newton-Raphson method. In this method, the following linearizing approximations are made at each iteration.

$$\begin{aligned} h(x) &\approx h(x^k) + H\Delta x \\ g(x) &\approx g(x^k) + G\Delta x \\ L^{-1}e &\approx (L^k)^{-1} - (L^k)^{-2} dl \\ U^{-1}e &\approx (U^k)^{-1} - (U^k)^{-2} du \end{aligned} \tag{7.32}$$

and

$$\begin{aligned} \lambda &= \lambda^k + d\lambda \\ \pi &= \beta^k + d\beta \\ \nu &= \nu^k + d\nu \\ \tau &= \tau^k + d\tau \end{aligned} \tag{7.33}$$

where

k: iteration index

Using approximations given in (7.32) and (7.33), Eqn (7.21) can be transformed into

$$-\mu U^{-1}e-\lambda=0$$
$$\mu U^{-1}e+\lambda=0$$
$$U^{-1}e+\frac{1}{\mu}\lambda=0$$
$$\left(U^{k}\right)^{-1}e-\left(U^{k}\right)^{-2}du+\frac{1}{\mu}\left(\lambda^{k}+d\lambda\right)=0$$
$$\frac{e}{U^{k}}-\frac{du}{\left(U^{k}\right)^{2}}+\frac{1}{\mu}\lambda=0$$
$$U^{k}e-du+\frac{1}{\mu}\left(U^{k}\right)^{2}.\lambda=0$$

$$\therefore du=\frac{\left(U^{k}\right)^{2}}{\mu}.\lambda+U^{k}e \quad (7.34)$$

Similarly,

$$-\mu L^{-1}e-\beta=0$$
$$\mu L^{-1}e+\beta=0$$
$$L^{-1}e+\frac{1}{\mu}\beta=0$$
$$\left(L^{k}\right)^{-1}e-\left(L^{k}\right)^{-1}dl+\frac{1}{\mu}\left(\beta^{k}+d\beta\right)=$$
$$\frac{e}{L^{k}}-\frac{dl}{\left(L^{k}\right)^{2}}+\frac{1}{\mu}\beta=0$$
$$L^{k}e-dl+\frac{\left(L^{k}\right)^{2}}{\mu}\beta=0$$

$$\therefore dl=\frac{\left(L^{k}\right)^{2}}{\mu}\beta+L^{k}.e \quad (7.35)$$

Eqn (7.23) can be used to express λ in terms of other variables. Hence,

$$\lambda=-W-\beta \quad (7.36)$$

Substituting λ into Eqn (7.34), gives

$$du=\frac{\left(U^{k}\right)^{2}}{\mu}\left(-W-\beta\right)+U^{k}e \quad (7.37)$$

Eqns (7.24) and (7.25) can be used to express ε and eliminate from the equations:

$$\varepsilon+r-u=0 \quad (7.38)$$

and

$$\varepsilon - r - l = 0 \tag{7.39}$$

Expressing $\boldsymbol{\varepsilon}$ in terms of ***r*** and ***u*** basing in Eqn (7.38) gives

$$\varepsilon = u - r \tag{7.40}$$

Substituting ε into eqn (7.39), yield

$$\begin{aligned} u - r - r - l &= 0 \\ u - 2r - l &= 0 \end{aligned} \tag{7.41}$$

$$\therefore r = 0.5l - 0.5u \tag{7.42}$$

Substituting ***r*** into eqn (7.26), gives

$$-(0.5l - 0.5u) + z - h(x) = 0 \tag{7.43}$$

Using approximations given in Eqn (7.32), Eqn (7.43) is transformed into

$$\begin{aligned} z - \left[h(x^k) + H\Delta x\right] + 0.5du - 0.5dl = 0 \\ z - h(x^k) - H\Delta x + 0.5du - 0.dl = 0 \end{aligned} \tag{7.44}$$

Let

$$r^k = z - h(x^k) \tag{7.45}$$

Replacing $\boldsymbol{r^k}$ with $\boldsymbol{z - h\ (x^k)}$, Eqn (7.44) is transformed into

$$r^k - H\Delta x + 0.5du - 0.5dl = 0 \tag{7.46}$$

Using approximations given in (7.32), Eqn (7.27) is transformed into

$$\begin{aligned} g(x) \approx g(x^k) + G\Delta x \\ -g(x^k) - G\Delta x = 0 \end{aligned} \tag{7.47}$$

$$\therefore G\Delta x = -g(x^k) \tag{7.48}$$

Eqn (7.28) can be used to eliminate $\boldsymbol{\nu}$

$$\begin{aligned} \lambda - \beta - \nu = 0 \\ \nu = \lambda - \beta \end{aligned} \tag{7.49}$$

However, $\boldsymbol{\lambda}$ is already expressed by Eqn (7.36), therefore,

$$\nu = -W - \beta - \beta$$

$$\therefore \nu = -W - 2\beta \tag{7.50}$$

Substituting ν into Eqn (7.29) gives

$$- G^T \tau - H^T(-W - 2\beta) = 0$$
$$- G^T \tau + 2H^T \beta + H^T W = 0$$

$$\therefore -G^T \tau + 2H^T \beta = -H^T W \qquad (7.51)$$

But

$W = R^{-1}$, hence, Eqn (7.51) is transformed into

$$- G^T \tau + 2H^T \beta = -H^T R^{-1} \qquad (7.52)$$

Let $U^k.e = u^k$,

$L^k e = l^k$

Substituting *du* and *dl* into Eqn (7.46), gives

$$\begin{aligned} & r^k - H\Delta x + 0.5\left(\frac{(U^k)^2}{\mu}(-W - \beta) + u^k\right) - 0.5\left(\frac{(L^k)^2}{\mu}\beta + l^k\right) = 0 \\ & r^k - H\Delta x - 0.5\frac{(U^k)^2}{\mu}W - 0.5\frac{(U^k)^2}{\mu}\beta + 0.5u^k - 0.5\frac{(L^k)^2}{\mu}\beta - 0.5l^k = 0 \end{aligned} \qquad (7.53)$$

Simplifying Eqn (7.53) gives

$$H\Delta x + 0.5.\frac{1}{\mu}\left\{(U^k)^2 + (L^k)^2\right\}\beta = r^k + 0.5u^k - 0.5l^k - 0.5\frac{(U^k)^2}{\mu}W \qquad (7.54)$$

Let

$$\begin{aligned} & W = R^{-1} \\ & D = \frac{0.5}{\mu}\left\{(U^k)^2 + (L^2)^2\right\} \end{aligned} \qquad (7.55)$$

D is a diagonal matrix representing the values of **U** and **L** at iteration index ***k***

Substituting (7.55) into (7.54), gives

$$D\beta + H\Delta x = r^k + 0.5u^k - 0.5l^k - \frac{0.5}{\mu}(U^k)^2 R^{-1} \qquad (7.56)$$

but

$$\begin{aligned} & r^k = 0.5l^k - 0.5u^k \\ & -r^k = -0.5l^k + u^k \end{aligned} \qquad (7.57)$$

Substituting eqn (7.57) into (7.56) gives

$$D\beta + H\Delta x = -\frac{0.5}{\mu}(U^k)^2 R^{-1} \qquad (7.58)$$

Re-arranging Eqns (7.58), (7.48) and (7.52), gives

$$\begin{aligned} D\beta + H\Delta x &= -\frac{0.5}{\mu}\left(U^k\right)^2 R^{-1} \\ G\Delta x &= -g\left(x^k\right) \\ -G^T\tau + 2H^T\beta &= -H^T R^{-1} \end{aligned} \tag{7.59}$$

Writing Eqn (7.59) in matrix form

$$\begin{bmatrix} D & 0 & H \\ 0 & 0 & G \\ 2H^T & -G^T & 0 \end{bmatrix}\begin{bmatrix} \beta \\ \tau \\ \Delta x \end{bmatrix} = \begin{bmatrix} -\frac{0.5}{\mu}\left(U^k\right)^2 R^{-1} \\ -g\left(x^k\right) \\ -H^T R^{-1} \end{bmatrix} \tag{7.60}$$

Let

$$T = \frac{0.5}{\mu}\left(U^k\right)^2 \tag{7.61}$$

Substituting **T** in Eqn (7.60)

$$\begin{bmatrix} D & 0 & H \\ 0 & 0 & G \\ 2H^T & -G^T & 0 \end{bmatrix}\begin{bmatrix} \beta \\ \tau \\ \Delta x \end{bmatrix} = \begin{bmatrix} -TR^{-1} \\ -g\left(x^k\right) \\ -H^T R^{-1} \end{bmatrix} \tag{7.62}$$

In physical systems the measurements from equality constraints i.e. measurements obtained from zero injections or virtual measurements at switching sub stations are error free. Therefore, errors from these measurements are eliminated from eqn (7.62). The action transforms Eqn (7.62) into:

$$\begin{bmatrix} D & H \\ 2H^T & 0 \end{bmatrix}\begin{bmatrix} \beta \\ \Delta x \end{bmatrix} = \begin{bmatrix} -TR^{-1} \\ -H^T R^{-1} \end{bmatrix} \tag{7.63}$$

Eqn (7.61) can be simplified into two separate equations

$$D\beta + H\Delta x = -TR^{-1} \tag{7.64}$$

$$2H^T\beta = -H^T R^{-1} \tag{7.65}$$

Expressing β in terms of other variables in Eqn (7.64)

$$\beta = D^{-1}\left\lfloor -TR^{-1} - H\Delta x \right\rfloor \tag{7.66}$$

Substituting β into (7.65)

$$2H^T\left\{D^{-1}\left\lfloor -TR^{-1} - H\Delta x \right\rfloor\right\} = -H^T R^{-1} \tag{7.67}$$

Re-arranging (7.67)

$$2H^T D^{-1} H\Delta x = H^T R^{-1} - 2H^T D^{-1} T R^{-1} \tag{7.68}$$

Solving for Δx

$$\Delta x = \left[2H^T D^{-1} H\right]^{-1} \cdot \left[H^T R^{-1} - 2H^T D^{-1} T R^{-1}\right] \tag{7.69}$$

Solving Eqn (7.67) for the updates Δx^{k+1} and iterating until the required accuracy ε_1 is reached: $\Delta x < \varepsilon_1$ will provide the solution of state estimation.

$$\Delta x^{k+1} = \left[2H^T(x^k)D^{-1}H(x^k)\right]^{-1}\left[H^T(x^k)R^{-1} - 2H^T(x^k)D^{-1}TR^{-1}\right] \tag{7.70}$$

When solving unconstrained state estimation problem using the weighted least squares (WLS) criterion: Δx is given by:

$$\Delta x^{k+1} = \left[H^T(x^k)R^{-1}H(x^k)\right]^{-1}\left[H^T(x^k)R^{-1}(z - h(x^k))\right] \tag{7.71}$$

where

$$G(x^k) = H^T(x^k)R^{-1}H(x^k) \tag{7.72}$$

Eqn (7.70) depicts the gain matrix.

When eqns (7.70) and (7.71) are compared, the two look similar. However, eqn (7.70) has two more matrices **D** and **T** which are missing in (7.71). The first right hand side of (7.68) is equal to the gain matrix of first right hand side of (7.71) with an additional diagonal matrix **D**. The second right hand side of (7.70) has two diagonal matrices **D** and **T** which are missing in the second hand side of (7.71).

As conclusion: Solving constrained WLAV state estimation problem using primal-dual logarithmic barrier interior point is equivalent to solving unconstrained WLS state estimation problem. The difference is on the additional two diagonal matrices **D** and **T**. This means, the software used in solving unconstrained WLS state estimation can be modified to include **D**, **T**, step length and barrier parameter and applied in solving the constrained WLAV estimation.

7.7.5 Updating of the variables

Primal and dual variables

New values of the primal and dual variables are usually calculated from:

Primal variables

$$
\begin{aligned}
x^{k+1} &= x^k + \alpha_p \Delta x \\
l^{k+1} &= l^k + \alpha_p dl \\
u^{k+1} &= u^k + \alpha_p du
\end{aligned}
\tag{7.73}
$$

Dual variables

$$
\begin{aligned}
\lambda^{k+1} &= \lambda^k + \alpha_D d\lambda \\
\beta^{k+1} &= \beta^k + \alpha_D d\beta
\end{aligned}
\tag{7.74}
$$

α_p, α_d are the scalar step size, they are chosen to preserve the non-negativity conditions.

Step length

Primal-dual interior point method allows separate step lengths in the primal and dual spaces [Torres et al., 1998]. This approach has proven efficiency in practice, significantly reducing the number of iterations to convergence in linear programming problems [Lustig et al., 1991]. The approach is adopted in this thesis. To incorporate this procedure, the step lengths for both primal and dual are given by:

$$
\alpha_p = 0.9995 \min\left\{ \min_k \left(-\frac{l_k}{dl_k} : dl_k < 0; -\frac{u_k}{du_i} : du_k < 0 \right), 1 \right\} \tag{7.75}
$$

$$
\alpha_d = 0.9995 \min\left\{ \min_k \left(-\frac{\lambda_k}{d\lambda_k} : d\lambda_k < 0; -\frac{\beta_k}{d\beta_k} : d\beta_k < 0 \right), 1 \right\} \tag{7.76}
$$

$$
k = 1, 2, \cdots, m
$$

Calculating the barrier parameter (μ)

A crucial step in applying primal-dual path following interior point algorithm is how to calculate the barrier parameter (μ). In solving linear programming (LP) problems, several schemes have been proposed to calculate (μ). The schemes are either based on duality gap proposed by Lustig et al [Lustig et al., 1991; McShane et al., 1989] or the complementary gap suggested by Lustig et al in 1994 [Lustig et al.,1994;Vanderbei,R.,1993]. The complementary gap approach is directly related to the barrier parameter (μ). Therefore, the complementary gap approach is adopted in the thesis. The complementary gap is calculated from:

$$C_{gap} \equiv \lambda^T l + \beta^T u \tag{7.77}$$

C_{gap}: Complementary gap.

Therefore, the barrier parameter is calculated from:

$$\mu = \frac{C_{gap}}{2m} = \frac{\lambda^T l + \beta^T u}{2m} \tag{7.78}$$

where

m: number of measurements

The theory behind the path following interior point method requires that barrier parameter must approach zero as the iteration progress; this means that the new value of barrier parameter should be substantially less than the current value. The new value can be obtained according to Vanderbei [Vanderbei, 1993]

$$\mu_{New} = k_d . \frac{\lambda^T l + \beta^T u}{2m} \tag{7.79}$$

where

$\boldsymbol{k_d}$: a constant, usually taken as 0.1 unless the primal objective value is less than the dual objective function.

In a situation when the primal and dual feasibility has not been achieved, the value of $\boldsymbol{k_d}$ is set to the range 2-10 [Vanderbei, 1993]. This large value of $\boldsymbol{k_d}$ helps to prevent the primal and dual feasibility not to be achieved.

Reducing the barrier parameter (μ)

For general non-linear programming (NLP) problems the choice of a good scheme to reduce the barrier parameter is far more complicated and often problem dependent. Some procedures in NLP simply decrease the barrier parameter by a fixed factor, according to Torres et al [Torres et al., 1998] up to a given lower bound. Usually, the decrease of barrier parameter is by:

$$\mu^{k+1} = \frac{\mu^k}{10} \tag{7.80}$$

Lasdon et al [Lasdon et al. 1995] proposed the procedure:

$$\hat{\mu} = \min\{0.95x\mu^k, 0.01x0.95^k x\|r\|_1\}$$
$$\mu^{k+1} = \max\{\hat{\mu}, 0.001x\|r\|_1\} \quad (7.81)$$

where

$\|r\|_1$ is the L_1-norm of the residual of the Karush-Kuhn-Tucker (KKT) conditions of the original problem. The term $0.95\mu^k$ in the above equation guarantees that $\hat{\mu}$ is smaller than μ^k, while 0.95^k is needed to guarantee super linear convergence of the problem. The lower bound is used to prevent μ^{k+1} from decreasing too fast, which may result in non-convergence of the problem. In this work reducing of the barrier parameter is achieved by a fixed factor.

Convergence criteria

For linear programming (LP) problems, stopping criteria are normally defined in terms of the relative gap according to Lustig et al [Lustig et al., 1991] For non-linear programming (NLP) problem the algorithm suggested by Wu et al [Wu et al.,1993] terminates as both the relative complementary duality gap and the mismatches of the Karush-Kuhn-Tucker (KKT) conditions are sufficiently small. Therefore, in this work the following criteria have to be satisfied:

$$\mu^k < 0.000001$$
$$\|\Delta x\| < 0.0001$$

7.8 Formulation of an algorithm and MATLAB Program

7.8.1 Algorithm formulation

Formulating an algorithm requires computation of Jacobian matrix **H,** and its transpose **H**^{T}**.** These matrices are computed from the structure of system network model developed in chapter 5 and are based on the measurements. They are referred to as measurement Jacobian matrices. The following is the outline on how the computation is carried out

The Measurement Jacobian matrices (H)

The structure of the measurement Jacobian **H** consists of voltage magnitude, real and reactive power injections, real and reactive power flows between the system transmission lines. The general structure of this matrix is given by:

$$H = \begin{bmatrix} 0 & \frac{\partial V_{mag}}{\partial V} \\ \frac{\partial P_{inj}}{\partial \delta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \delta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial P_{Flow}}{\partial \delta} & \frac{\partial P_{Flow}}{\partial V} \\ \frac{\partial Q_{Flow}}{\partial \delta} & \frac{\partial Q_{Flow}}{\partial V} \end{bmatrix} \in R^{mxn} \quad (7.82)$$

where

m : is the number of measurements

n ; is the state vector

Full expressions for each part of (7.82) for $i = \overline{1,N}, j = \overline{1,N}, j \neq i$ are: Elements corresponding to voltage magnitude measurements:

$$\frac{\partial V_{i,mag}}{\partial V_i} = 1; \frac{\partial V_{i,Mag}}{\partial V_j} = 0$$
$$\frac{\partial V_{i,Mag}}{\partial \delta_i} = 0; \frac{\partial V_i}{\partial \delta_j} = 0 \quad (7.83)$$

Elements corresponding to real power injection measurements:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{N} V_i V_j (-G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) - V_i^2 B_{ii}$$
$$\frac{\partial P_i}{\partial \delta_j} = V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \quad (7.84)$$

$$\frac{\partial P_i}{\partial V_i} = \sum_{j=1}^{N} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) + V_i B_{ii}$$
$$\frac{\partial P_i}{\partial V_j} = V_i (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \quad (7.85)$$

Elements corresponding to reactive power injection measurements:

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j=1}^{N} V_i V_j (G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}) - V_i^2 G_{ii}$$
$$\frac{\partial Q_i}{\partial \delta_j} = V_i V_j (-G_{ij} \cos\delta_{ij} - B_{ij} \sin\delta_{ij}) \qquad (7.86)$$

$$\frac{\partial Q_i}{\partial V_i} = \sum_{j=1}^{N} V_j (G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}) - V_i B_{ii}$$
$$\frac{\partial Q_i}{\partial V_j} = V_i (G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}) \qquad (7.87)$$

Elements corresponding to real power flow measurements:

$$\frac{\partial P_{ij}}{\partial \delta_i} = V_i V_j (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij})$$
$$\frac{\partial P_{ij}}{\partial \delta_j} = -V_i V_j (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) \qquad (7.88)$$

$$\frac{\partial P_{ij}}{\partial V_i} = -V_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) + 2V_i (g_{ij} + g_{ij}^{sh})$$
$$\frac{\partial P_{ij}}{\partial V_j} = -V_i (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) \qquad (7.89)$$

Elements corresponding to reactive power flow measurements:

$$\frac{\partial Q_{ij}}{\partial \delta_i} = -V_i V_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij})$$
$$\frac{\partial Q_{ij}}{\partial \delta_j} = V_i V_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) \qquad (7.90)$$

$$\frac{\partial Q_{ij}}{\partial V_i} = -V_j (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) - 2V_i (b_{ij} + b_i^{sh})$$
$$\frac{\partial Q_{ij}}{\partial V_j} = -V_i (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) \qquad (7.91)$$

The Measurement transposed Jacobian matrices ($\mathbf{H}^T$)

The measurement transpose Jacobian matrix $\mathbf{H}^T$ is obtained from eqn (7.82) and is given by eqns (7.90) as follows:

The transpose of Jacobian matrix $\mathbf{H}^T$:

$$H^T = \begin{bmatrix} 0 & \frac{\partial P_{inj}}{\partial \delta} & \frac{\partial Q_{inj}}{\partial \delta} & \frac{\partial P_{flo}}{\partial \delta} & \frac{\partial Q_{flo}}{\partial \delta} \\ \frac{\partial V_{mag}}{\partial V} & \frac{\partial P_{inj}}{\partial V} & \frac{\partial Q_{inj}}{\partial V} & \frac{\partial P_{flo}}{\partial V} & \frac{\partial Q_{flo}}{\partial V} \end{bmatrix} \tag{7.92}$$

7.8.2 Algorithm

The algorithm proposed to solve the state estimation problem applying primal-dual logarithmic barrier (PDLB) path following interior point is an iterative process consisting of two parts. Part I is concerned with initialization procedures. Part II is concerned with calculating of variables and implementation procedures. The algorithm is given as:

PART I: Initialize:

Initialize $\boldsymbol{k = 0}$.

$\boldsymbol{x^0}$ = flat start using values obtained from the load flow program

$\varepsilon^0 = /z\text{-}h(x^0)/$

$u^0 = \varepsilon^0 + r^0$

$l^0 = \varepsilon^0 - r^0$

$\lambda = 0;\ \beta = 0;\ v = 0;\ \tau = 0, \lambda = -\ 0.5\varepsilon,\ \beta = -0.5\varepsilon$

PART II: Calculation and implementation

I: Calculate the Jacobian matrix **H**

II. Calculate the transpose matrix $\mathbf{H^T}$

III. Calculate the complementary gap

IV. Calculate the barrier parameter μ

V. Calculate **D**

VI. Calculate *du*

VII. Calculate *dl*

VIII. Calculate l^k and $\boldsymbol{u^k}$

IX. Calculate $\boldsymbol{r^k = z\text{-}h(x^k)}$

X. Solve $\boldsymbol{\Delta x}$

XI. Calculate step length

XII. Update $\boldsymbol{x^{k+1} = x^k + \alpha_p \Delta x}$

XIII. Check if $/\boldsymbol{z\text{-}h\ (x^{k+1})}\ /< \boldsymbol{0.0001}$ if yes **STOP**. Otherwise go to **XIII**

XIV. Update λ and β

XV. Update $\boldsymbol{u}$ and l

XVI. Check convergence criteria

XVII. If optimum **TERMINATE** the procedure. Otherwise update $k = k+1$ and go to **III**

7.8.3 MATLAB program

A MATLAB program is developed basing on the algorithm presented in 7.8.2. The program has 10 files shown in Table 7.2 below. A full program is provided in Appendix C6.

Table 7.2: MATLAB files for calculating WLAV state estimation

File	Description
WLAVSE	M-file (to run the program)
LD30	Excel-file depicting line data of the system
BD30	Excel-file depicting bus data of the system
Lfybus.m	M-file, which calculates ybus admittance matrix
Bbusppg.m	M-file, which calculates shunt admittance matrix
Pol2rect.m	M-file, which converts polar to rectangular
Rect2pol.m	M-file, which converts rectangular to polar
Steplength.m	M-file, which calculates the steplength
Zdata.m	Excel-file depicting measurement data of the system
wlav	M-file calculates state estimation using WLAV method

WLVASE

```
clear                           % Clear all programs...
clc                             % Close command window...
tic;
basemva = 100;                  % Power System base MVA...
tolerance = 1e-5;               % Calculation tolerance...
maxiter = 20;                   % Maximum number of iterations...
linedata = xlsread('LD30');     % Reads linedata file...
busdata = xlsread('BD30');      % Reads busdata file...
Lfybus                          % Forms Ybus admittance matrix...
Bbus                            % Forms shunt admittance matrix...
Pol2rect                        %Converts polar to rectangular...
Rec2pol                         % Converts rect to polar...
zdata = xlsread('zdata30') % reads measurement data from file...
wlav    % Calculates state estimation solution using WLAV method...
toc;
```

7.9 Computational Implementation

The primal-dual method analysed in this chapter is characterized by the transformation of the original problem (7.14) into an unrestricted problem, through the Lagrangian function.

The conditions of optimality generate systems of non-linear equations which are treated by the Newton-Raphson method. Therefore, implementation procedure includes:

- Run load flow program and obtain initial bus voltage values, real and reactive power injections at each bus, and real and reactive power flows in the branches. These values are considered as true values of the system.
- Real and reactive power injections and real and reactive power flows in the branches are given in appendix B3. These data are used in simulating WLS and WLAV state estimation problems.
- An initial point is chosen by setting ***k =0,*** and then barrier parameter is defined. When choosing a starting point attention has to be paid to make sure that the choosen point statisfies the strict positivity condition.
- The Newton-Raphson system of equations is solved in order to determine the search direction and then calculating the step length in order to update the primal and dual variables.
- Test convergence, if convergence criterion is satisfied, stop. Otherwise set ***k = k+1*** and continue with the procedure.

The initial lower and upper slack variables are determined from;

$$l^0 = \varepsilon^0 - z - h(x^0) \tag{7.93}$$

$$u^0 = \varepsilon + z - h(x^0) \tag{7.94}$$

$$r^0 = z - h(x^0) \tag{7.95}$$

7.9.1 Simulation

The algorithm formulated in section 7.8.2 is applied in developing a MATLAB program presented in appendix C6. The simulation procedure is performed on a PC Pentium IV of 3.33 GHz and 0.99GB of RAM. Results from simulation are presented in chapter 10.

7.10 Multi-area state estimation

Sections (7.5) to (7.9) have addressed formulation of the problem for state estimation of a single area/system i.e. integrated power system and its implementation algorithm. Nowadays, electrical power systems span across continents, being interconnected among countries or areas strong enough for any area to be effected by incidence or action in other areas [Conejo et al., 2007]. As a consequence, state estimation process should recognize the influence of neighbouring areas to estimate the state of any given area. This approach advocates for implementing multi-area or multi-sub-system state estimation. Hence, in this section formulation of the problem for state estimation that requires just local (own area) and interconnecting variables (border variables) is introduced. In this way, the multi-area state estimation is achieved by proper and simple coordination among areas operators. Formulation of the problem is based on decomposition method described in chapter 5 where area is treated as a sub-system.

In general, the single-area state estimation problem can be formulated in a simplified way as follows:

$$\min_{x} J(x) \tag{7.96}$$

where

x: is the state vector

J(x): is a scalar function of the estimation error.

Eqn (7.94) can include equality and inequality constraints as it has been described in detail in sections (7.5) to (7.9)

In formulating state estimation problem of area i it is assumed that voltage magnitude and phase angle estimates of other areas that do affect area i are treated as measurements. It should be emphasized that these estimates are not measurements but treated as measurements. State estimation formulated in (7.96) is solved iteratively until convergence using the sum of single-area state estimation problems. The procedure is presented in chapter 8 under decomposition-coordination method and algorithm.

7.11 Conclusion

In this chapter, formulation of the problem for state estimation is presented. The problem includes both equality and inequality measurements. Equality constraints represent exact measurement such as zero injections at switching substations. Inequality constraints represent pseudo measurements which are not measured but are known to remain in bounded interval. These measurements include VAr limits on generators, transformer taps setting; and inter- area tie-line power flows. In this thesis only VAr limit on generators is considered.

The formulation of the problem follows the non-linear programming (NLP) approach. Primal-dual path following interior point method is used for the solution of the problem. A MATLAB algorithm for solution of the problem is developed. The MATLAB full program is given in Appendix D5. Simulation using the program is performed in chapter 10

Formulation of the problem for state estimation forms the basis of discussion of chapter 8, 9. Chapter 8 presents decomposition-coordination method and algorithm of a multi-area state estimation problem. First, reasons for system decomposition and decomposition methods are briefly discussed and then decomposition-coordination method and algorithms for solving a multi-area state estimation is presented. The algorithm provides means of applying parallel peocesing for solution of the problem.

CHAPTER EIGHT: DECOMPOSITION-COORDINATION METHOD FOR SOLUTION OF PROBLEM FOR STATE ESTIMATION

8.1 Introduction

Decomposition methods have been used in the solution of many operations, and planning problems in the past. The application of decomposition method provides potential gains in computational efficiency in solving large optimization problems.

In this chapter, decomposition-coordination method and algorithm is proposed. In implementation of the algorithm, a decentralized operation of the power system is preserved while at the same time attaining overall optimality by using decomposition-coordination algorithm. The proposed decomposition-coordination method and algorithms is used in solving a multi-area SE problem. The Ward hale 6-bus system and the Tanzanian Power System Network Model developed in chapter 5 are used as case studies.

8.2 Reasons for decomposition

Decomposition is a general method of solving a problem by breaking it into smaller sub- problems and solving each of the smaller sub- problem separately either in parallel or sequentially. Problems to which decomposition works in one step are called separable (block), or trivially parallelizable. A more interesting situation occurs when there is some coupling or interaction between the sub vectors, so the sub-problems cannot be solved independently. For these cases there are techniques that solve the overall problem by iteratively solving a sequence of smaller sub problems. This is the case considered in the thesis for which a solution is developed.

Majority of power utilities in the world are still regulated and have centralized control scheme, although the process of deregulation is gaining pace in some of the countries. In regulated systems, areas or regions which form the utility are not addresses as single entities but as one contiguous zone. Hence, one optimization problem such as SE, optimal power flow (OPF) is solved at the central dispatch centre (CDC). For the purpose of synchronization, all areas or regions are obliged to send their data to the CDC. After the CDC has obtained the system wide optimal solution, it re-distributes the calculated control variables to the areas/regions. Hence, the centralized computing is not robust because:

- In case of a transmission failure the central control is lost over all areas/regions;

- The region substations are spread over large distances and the communication links have limited transmission capacity. Hence, the execution of the control variables is slowed down considerably since a huge amount of information is exchanged and in addition, transferred over large distances; and
- Solving of a large optimization problem results in long computation times since large computational effort is required.

To palliate the drawbacks of the centralized control scheme, a de-centralized scheme is used in which the overall optimization problem is divided into sub problems according to areas or regions with some modifications. Each area or region has its own agent area dispatch control centre (ADCC), which solves its own optimization problem independently and specifically for that area or region. In order to obtain the global optimal solution, the subproblems are coordinated by a central agent (coordinator) who either calculates or updates information concerning the convergence of the problem or only distributes it.

Hence, the reasons to implement a de-centralized control scheme are:

- To speed up execution of control variables since a small amount of information is exchanged, i.e. only dedicated ones such as load flow of the border nodes;
- To delegate the goals to the corresponding areas/regions and its control agents;
- To shorten computation times; and
- To introduce and use distributed and parallel computing techniques.

There are computational and organizational advantages in using decomposition-coordination algorithms. From computational perspective, the advantage is; the sub problems are usually easier to solve than the original problems. The sub problems, are, smaller than the original problem. Moreover, the sub problems have special properties such as sparsity, or network constraints that enable using efficient specialized algorithm to solve them. By decomposing the original problem, an advantage of the efficient solution method available for the sub problem can be taken.

Thus, the main motivation of using decomposition-coordination algorithm is related to their organizational aspects. For example, most engineering design problems involve participation of different design groups who work is largely independent [Kroo, 1998]. Decomposition algorithms allow these problems to be solved in a distributed environment. The main point in designing a decomposition algorithm is that only limited communication between the subproblem and the master problem is required. The aim is, the local subproblems centres should solve only their sub problems and only a small amount of communication should be required from the coordinator.

8.3 Decomposition methods: An overview

Decomposition method is an old idea, and appears in early work on large-scale linear programming (LP) from the 1960s [Dantzig & Wolfe, 1960]. There are three main ideas encountered in literature. The rest are improvements from these three. The main are: Bender's, Dantzig-Wolfe, and Lagrangian Relaxation methods. The three methods and their corresponding improvements are briefly overviewd in the following subsection.

8.3.1 Bender's method

Benders decomposition was originally used for integer programming. The method is based on the idea of incorporating the complicating variables, meaning the integer variables. The complicating variables are the one which cause problem in solving the global problem. Because without complicating variables, the problem can be easily decomposed and solved. Hence, the problem is re-formulated by removing the complicating variables and replacing them with a large number of constraints, called feasibility constraints and the optimality constraints. A restricted master problem is optimized using only some of these constraints.

The non-complicating variables are placed in the sub problems. For the value of the complicating variables from the restricted master, the sub problem finds the best solution for the non-complicating (easy) variables. The dual value from this solution are used to create a new optimality constrain for feasibility constraints which is added to the restricted master. Benders decomposition is usually used for large IPs in which a nice structure allows the quick solution for continuous variables.

The classical Benders decomposition method can be applied to linear disjunctive programming problems, and the generalized Benders method to non-linear disjunctive problems. The beauty of Benders method is that one can often distinguish a few "hard" variables ***y*** from many "easy" variables ***x*** and when the hard variables are fixed, the problem becomes easy by decoupling it into several small sub problems that can be solved separately. The Benders decomposition method (BDM) algorithm [Bazaraa et al, 1990] is as follows:

$$\textbf{P:}\quad \begin{aligned} &\min cx \\ &s.t\quad Ax = b \\ &\qquad x \in X = \{x : Dx \geq d, x \geq 0\} \end{aligned} \qquad (8.1)$$

Usually $\boldsymbol{w_d}$ as the dual variables associated with the constraints ***Ax=b*** Then the dual problem for optimization of $\boldsymbol{w_d}$ is

$$\textbf{MP}: \begin{array}{l} \max z_d \\ s.t. \quad z_d \le w_d b + (c - w_d A)x_k \qquad k = 1,\ldots,n_p \end{array} \tag{8.2}$$

And the subproblems are

$$\overline{w}b + \underset{x \in X}{\min imum}\left\{c - \overline{w}A)x\right\} \tag{8.3}$$

Where **P** is the problem and **MP** is the master problem

(z'_d,w'_d) is an optimal solution for the relaxed master problem. If ***z'*** is less than or equal to the optimal objective value of the subproblem, then the problem is solved. Otherwise, assuming x_k solves the subproblem; the constraints ***$z_d \le w_d b + (c-w_d A)x_k$*** can be generated and added to the current relaxed master problem **(MP)** and re-optimize. Gomes et al [Gomez et al, 1991] extended the Benders decomposition method (BDM) to present normal and contingency operating conditions of a methodology that can provide a flexible, variable control of corrective and preventive actions.

Linear programming (LP), non-linear programming (NLP) and mixed integer programming (MIP), all treat the number and value of the capacitors as continuously differentiable. The favourite method is to decompose the problem into two optimization problems i.e. the master subproblem dealing with the investment decision of installing discredited new Var devices, and the slave problem dealing with the operation optimization. These techniques normally use what is known as Generalized Bender's Decomposition (GBD) method, which is used to decompose into a continuous problem and an integer problem. However, the GBD method does not always perform well in solving practical Var problems; the convergence can only be guaranteed under convexity assumptions of the objective functions of the operation problem, so the solution cannot always guaranteed.

Let the generalization of GBD as suggested by Bazaraa et al [Bazaraa et al, 1990] is considered for the following non-linear and/ or discrete problem

$$\textbf{P}: \begin{array}{l} \min cx + f(y) \\ s.t. \quad Ax + By = b \\ \qquad x \ge 0 \qquad y \in Y_{Set} \end{array} \tag{8.4}$$

where

a,b,x, y are vectors, **A, B** are matrices, ***f*** is an arbitrary function, and ***Y_{set}*** is an arbitrary set. The problem (8.4) can be decomposed in the following way

$$\textbf{MP}: \begin{array}{lll} \min z_d & & \\ s.t. & z_d \geq f(y) + w_k(b - By) & k = 1,...,n_p \\ & d_k(b - By) \leq 0 & k = 1,...,n_p \\ & f \in Y_{Set} & \end{array} \quad (8.5)$$

where

$w_1, ..., w_k$ **and** $d_1, ..., d_k$ are the extreme points and extreme directions generated

$$\text{Sub problem:} \begin{array}{ll} \max w(b - By) & \\ s.t. & wA \leq c \end{array} \quad (8.6)$$

The set Y_{set} can be discrete, and so the algorithm can be used for solving mixed integer problems.

Hong et al [Hong et al, 1990] integrated the Newton-OPF with the GBD method to solve the long term Var planning problem. Abdul-Rahman [Abdul Rahman & Shahidehpour, 1993] extended the GBD method and combined it with the Fuzzy set. Yorino et al [Yorino et al, 2003] applied GDB to decouple a mixed integer non-linear programming problem of large dimension taking into account the expected cost for voltage collapse and corrective control.

8.3.2 Dantzig-Wolfe method

Dantzig-Wolfe decomposition was originally used to a large-scale linear programming problem. It is based on the idea on how to treat complicating constraints. The easy constraints (non-complicating) must define the feasible region [Boyd et al, 2003] that is easy to optimize. The original problem is reformulated in terms of extreme points and extreme rays of the feasible region. A restricted master problem, which only has some of the column, is solved. The dual value of this solution is used to generate – one or more favourable columns in a columns generation sub problem. Dantzig-Wolfe decomposition is used for large LP where a subset of the constraints has a good structure (the non-complicating) constraint.

The Dantzig-Wolfe Decomposition Method (DWDM) was the first method developed to decompose a system's optimization problem into several subproblems corresponding to specific areas in the power system, so this method is very fast. The method partitions the problem into master and slave sub problems. The solution of the master sub problem depends on the dual solution provided by the sub problems. The master problem passes down a new set of cost coefficients to the sub problem and receives a new column based on these cost coefficients.

The DWDM algorithm suggested by Bazaraa et al [Bazaraa et al, 1990] is as follows: consider the linear program **P,** where ℵ is a polyhedral set representing constraint of a special structure. For convenience, it is assumed that ℵ is non-empty and bounded in the following decomposition algorithm, and then any point ***x*** ∈ℵ can be represented as a convex combination of the finite numbers of extreme point's $\boldsymbol{x_k}$ of ℵ as:

$$x = \sum_{k=1}^{n_p} \Gamma_k x_k$$
$$\sum_{k=1}^{n_p} \Gamma_k = 1 \qquad \Gamma_k \geq 0 \qquad k = 1, \cdots, n_p \tag{8.8}$$

Then, **P** is transformed into the master problem

$$\textbf{P}: \begin{array}{ll} & \min cx \\ s.t & Ax = b \\ & x \in \aleph \end{array} \tag{8.9}$$

$$\textbf{MP}: \begin{array}{l} \min \sum_{k=1}^{n_p} (cx_k) \\ s.t \ \sum_{k=1}^{n_p} (Ax_k)\Gamma_k = b \\ \sum_{k=1}^{n_p} \Gamma_k = 1 \qquad \Gamma_k \geq 0 \qquad k = 1, \ldots, n_p \end{array} \tag{8.10}$$

Given a basic feasible solution Γ_B, Γ_N with dual variables corresponding to the above derived two equality constraints by $\boldsymbol{w_d}$. The following sub problem is easier to solve than the master problem.

$$\text{Subproblem:} \begin{array}{l} \max(wA - c)x + \alpha_c \\ s.t. \qquad x \in \aleph \end{array} \tag{8.11}$$

where α_c is an equality constraint

Deeb and Shahidehpour [Deeb & Shahidehpour, 1990] applied the algorithm to a large-scale power system network. The proposed method shows robustness and is not sensitive to the feasibility of the starting point or to the type and size of variables in different areas.

8.3.3 Lagrangian Relaxation method

Lagrangian relaxation traces its origin to Lagrange multiplier from multivariable calculus. It is based on the idea of complicating constraints for a linear programming

(LP) or interior point (IP). A Lagrangian relaxation sub problem is formed for a particular set of Lagrange multipliers by moving the complicating constraints into the objective function. A solution to a Lagrangian relaxation gives a bound on the optimal objective value for the original problem. The problem of finding the best such bound is the Lagrangian one. For an IP, the value of the Lagrangian dual function is between the value of the optimal IP solution and the value of LP relaxation.

For an interior point (IP), the Lagrangian relaxation bounds are piece-wise linear, continuous, and convex (if maximizing) in the Lagrangian multipliers. Thus, it is theoretically possible to quickly solve the Lagrangian dual problem using techniques such as parameterized linear programming. In practice, however, it is more common to use a simpler and faster sub gradient approach that usually terminates without finding the best set of multipliers. The reason for this is that there is usually a gap between the true IP solution and the Lagrangian dual solution. Thus, the Lagrangian dual problem is just providing a bound, and finding a good bound quickly is usually better (and easier) than making the extra effort to find the best bound. Lagrangian relaxed method is usually applied to large IP in which a small number of complicating constraints exist.

8.3.4 Tammer's method

Bender's decomposition discussed in section 8.3.1 is a globally and fast locally convergent algorithm for the convex optimization problems with global variables (OPGV). Moreover, Benders computational procedure always finds a global minimizer. But the situation is complicated for the non-convex OPGV. For non-convex problems, one can only aspire to find local minimizers. Moreover, the global convergence can not be proven for a decomposition algorithm applied to the non-convex OPGV. However, the engineers always like to have fast locally convergent decomposition algorithms.

Non-convex duality theory proposed by Tind and Wolsey [Tind & Wolsey, 1981] leads to decomposition algorithms that require the solution of the problems in infinite dimensional spaces. Hence, if finite dimensionality is to be preserved, out of the two manipulations used to obtain the Benders master problem, only projection can be used for non-convex OPGV problems. Tammer [Tammer, 1988] proposed a decomposition algorithm, based on projection, for which he showed local convergence under rather restrictive non-degeneracy assumptions. Tammer proposed solving the following master problem:

$$\min_{x} \sum_{k=1}^{N_p} F_k^{\bullet}(x) \qquad (8.12)$$

where

$F_k^{\bullet}(x)$ is the optimal-value function corresponding to the *kth* sub problem

$$F_k^{\bullet}(x) = \min_{y_k} F_k(x, y_k) \quad s.t. \; c_k(x, y_k) \geq 0 \qquad (8.13)$$

The proposed master problem and sub problems are obtained by setting the global variables at fixed value. The constraints and the terms in the objective function corresponding to the ***kth*** system are kept within the kth sub problem. Thus, the master problem becomes unconstrained problem whose objective function is the summation of the subproblem optimal-value functions.

A major difficulty in implementing Tammer's method is that the algorithm developed from this method fails when it arrives at a value of the global variables for which one of the sub problems is infeasible. To rule out this possibility Tammer introduced the restrictive Strong Linear Independence Constraint Qualification (SLICQ).

SLICQ is defined when a little introduction notation in the optimization Problem with Global Variables (OPGV) Jacobian at a point ***(x, y_1, y_{Np})*** is given by

$$\begin{bmatrix} A_1 & B_1 & & \\ A_2 & & \ddots & \\ \vdots & & & B_{N_p} \\ A_{N_p} & & & \end{bmatrix} \qquad (8.14)$$

where

$$A_k = \nabla_x \hat{c}_k(x, y_k) \qquad (8.15)$$

$$B_k = \nabla_{y_k} \hat{c}_k(x, y_k) \qquad (8.16)$$

$\hat{c}_k$ are the active constraints

If SLICQ holds at a feasible point ***$(x, y_1, \ldots, y_k)$*** then, the subproblems defined by (8.13) are feasible for any value of the global variables in a neighbourhood of ***x***. Tammer assumed the SLICQ to ensure the sub problems are always feasible once the iterate are sufficiently close to the minimizer. However, the SLICQ does not hold for many optimization problems. Braun [Braun, 1996] proposed a decomposition algorithm known as Collaborative optimization.

8.3.5 Inexact penalty decomposition (IPD) method

In using IPD method the following master problem is defined

$$\min_{z_p} \sum_{k=1}^{N_p} F_k^{\bullet}(\varpi, z_p) \tag{8.17}$$

where

$F_k^{\bullet}(z_p)$ is the optimal-value function corresponding to the ***kth*** sub problem,

$$\begin{aligned} F_k^{\bullet}(\varpi, z_p) &= \min_{x_k, y_k} F_k(x_k, y_k) + \varpi \left\| x_k - z_p \right\|_2^2 \\ s.t.\quad & c_k(x_k, y_k) \geq 0 \end{aligned} \tag{8.18}$$

Unlike Co, IPD keeps the OPGV objective function term $F_k(x_k, y_k)$ within the ***kth*** sub problem. A penalty parameter ϖ is used to weight the quadratic penalty term $\| x_k - z_p \|_2^2$ with respect to $F_k(x, y_k)$.

Quadratic penalty functions are inexact penalty function in the sense that the exact solution $(x_k = z_p)$ is only retrieved for $\varpi = \infty$. The difference between CO and IPD is: IPD uses the sub problem optimal-value functions $F_k^{\bullet}(z_p)$ as a penalty term within the objective function of an unconstrained master problem.

The major problem encountered in implementing decomposition method is how to deal with the complicating variables. Complicating variables are the ones which cause problems in solving the global problem. Without complicating variables the problem can be easily decomposed and solved. The reviewed methods have introduced the procedure of breaking down the problem (decompose) into independent small problems (sub- problem), then a method of coordinating these small problems is proposed. The aim is to speed up computation procedure as well as to include the complicating variables in the solution of the global problem. Two structures are proposed; the lower level (slave) and the upper level (master). The coordination level is carried out at master level. Objective function and constrain function are defined using information gathered from the sub problem's solution. In some methods the use of quadratic penalty as sub problem objective function to force the variables to converge to the target variable or a penalty parameter to weigh the penalty term is used.

In this thesis the idea of decomposing a problem into sub problems and introducing a coordinator is adopted. The slight difference between the methods discussed in section 8.3 is on the approach towards the global solution. In the proposed method, a decomposition-coordination method and algorithm with two level structures is presented. The first level deals with the solutions of the sub-systems resulted from the decomposition while the second level take care the coordination issue, which is a solution of the global problem. The proposed method clearly defines the role of each level and is presented in section 8.4.

8.4 Formulation of decomposed solution of state estimation problem

8.4.1 Criterion

The criterion is considered as a sum of sub-criterion determined for everyone of the sub-system. In the common case it can be written as (Least absolute value)

$$\min J(V\angle\delta)=\sum_{i=1}^{N_S}\left\{\begin{array}{l}\sum_{k=1}^{M_{i,V}}\frac{\left|V_k^{meas}-V_k^{est}\right|}{\sigma_{Vk}^2}+\sum_{k=1}^{NM_{i,pinj}}\frac{\left|P_{k,inj}^{meas}-P_{k,inj}^{est}\right|}{\sigma_{Pinj}^2}+\sum_{k=1}^{M_{Qinj}}\frac{\left|Q_{k,inj}^{meas}-Q_{k,inj}^{est}\right|}{\sigma_{Qinj}^2}+\\+\sum_{k=1}^{M_{Pflow}}\frac{\left|P_{k,flow}^{meas}-P_{k,flow}^{est}\right|}{\sigma_{pkflow}^2}+\sum_{k=1}^{M_{Qflow}}\frac{\left|Q_{k,flow}^{mes}-Q_{k,flow}^{est}\right|}{\sigma_{Qkflow}^2}\end{array}\right\} \quad (8.19)$$

Equation (8.19) can be simplified and written as

$$\min J(V\angle\delta)=\sum_{i=1}^{N_S}J_i(V_i\angle\delta_i) \quad (8.20)$$

where

$M_{i,V}, M_{i,P}^{inj}, M_{i,Q}^{inj}, M_{i,P}^{Flow}, M_{i,Q}^{flow}$ are the dimensions of the corresponding vectors of measured variables. The type and number of measurements of different sub-system can be different. Simplified version of the criterion used in this thesis is given by:

$$\min J(V\angle\delta)=\sum_{i=1}^{N_S}\sum_{k=1}^{M_{i,V}}\frac{\left|V_k^{meas}-V_k^{est}\right|}{\sigma_{k,V}^2} \quad (8.21)$$

where

k **:** is iteration counter

8.4.2 Measurement model

The global model of the data measurement discussed in chapter 7 is given by the following equation

$$z=h(x)+r \quad (8.22)$$

where

r: is the measurement residual.

The measurement model has 4 parts determined by the type of the measurements available at the central dispatch centre. When power system model is considered to be used for the decomposed solution of the state estimation problem, these considered 4 parts of the measurements model can be determined in different ways as consisting of a model of the isolated sub-system and a model of the interconnection between the sub-systems. The following steps are used in determining the model.

Voltage magnitude measurement data model: This model for the entire power system is given by:

$$z_V = V \tag{8.23}$$

where

$z_V \in \Re^N$

N: is the number of the model buses

The decomposition of this model is direct and is determined by the selected number of buses in every sub-system. The sub-system model such as *ith* sub-system is given by

$$z_{i,V} = V_i + \varepsilon_{i,V} \qquad z_{i,V} \in \Re^{n_i} \quad i = \overline{i, N_S} \tag{8.24}$$

Where

n_i : is the number of buses in the ***ith*** sub-system

Real and reactive power injections data model: The real and reactive power injections model for the entire power system was decomposed in chapter 5. The obtained sub-system model is characterized with local for the sub-system state variables and with disturbance inputs from other sub-systems as follows:

$$P_{ii} = G_{ii}^R V_i + \sum_{\substack{j=1 \\ j \neq i}}^{N_S} G_{ij}^R V_j = G_{ii}^R V_i + y_i^R \tag{8.25}$$

$$y_i^R = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} G_{ij}^R V_j \qquad G_{ii}^R \in \Re^{n_i x n_i}, G_{ij}^R \in \Re^{n_i x n_j}, y_i^R \in \Re^{n_i}, i = \overline{1, N_{S,}}, j = \overline{1, N_S} j \neq i \tag{8.26}$$

$$Q_{ii} = G_{ii}^{im} + \sum_{g=1}^{N_S} B_{ij}^{im} V_j = G_{ii}^{im} V_i + y_i^{im} \qquad (8.27)$$

$$y_i^{im} = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} B_{ij}^{im} V_j \qquad G_{ii}^{im} \in \Re^{n_i x n_i}, B_{ij}^{im} \in \Re^{n_i x n_j}, y_i^{im} \in \Re^{n_i} \qquad (8.28)$$

The type and number of measurements in every sub-system can be different and independent. In the common case, the power injection data model can be written in matrix format as:

$$\begin{bmatrix} P_{ii} \\ Q_{ii} \end{bmatrix} = \begin{bmatrix} G_{ii}^R \\ G_{ii}^R \end{bmatrix} V_i + \begin{bmatrix} y_i^R \\ y_i^{im} \end{bmatrix} = G_{i,inj} V_i + y_{i,inj} \qquad G_{i,inj} \in \Re^{2n_i x n_i}, y_{i,inj} \in \Re^{2n_i} \qquad (8.29)$$

$$y_{i,inj} = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} \begin{bmatrix} G_{ij}^R \\ B_{ij}^{im} \end{bmatrix} V_j = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} h_{ij} \left(V_i \angle \delta_i, V_j \angle \delta_j \right) \qquad (8.30)$$

Equations (8.29) and (8.30) can be written in the notation of the data measurement model as follows:

$$z_{i,inj} = h_{i,inj} \left(V_i \angle \delta_i \right) + y_{i,inj} + \varepsilon_{i,inj} \qquad (8.31)$$

where

$h_{i,inj} \in \Re^{2n_i x n_i}$: is a non –linear matrix function of $V_i \angle \delta_i$

The dimension of the vector and matrices of Eqn (8.29) are the maximal possible ones but the number of measurements can be different.

Real and reactive power flow data model: The power flow data model is determined for every two interacting buses separately in the sub-systems. This means that this model can be directly decomposed according to the selected dimension of the sub-system i.e. $\boldsymbol{n_i}$**.** It is supposed for the power flows between the ***p-th*** and ***q-th*** bus in the ith sub-system that the real and reactive power flows are given from derivation in chapter 6.

$$P_{i,pq} = V_{i,p}^2 \left(g_{i,pq}^{sh} + g_{i,pq} \right) - V_{i,p} V_{i,q} \left[g_{i,pq} \cos\left(\delta_{i,p} - \delta_{i,q} \right) - b_{i,pq} \sin\left(\delta_{i,p} - \delta_{i,q} \right) \right] \qquad (8.32)$$

$$Q_{i,pq} = -V_{i,p}^2 \left(b_{i,pq}^{sh} + b_{i,pq} \right) + V_{i,p} V_{i,q} \left[g_{i,pq} \sin\left(\delta_{i,p} - \delta_{i,q} \right) + b_{i,pq} \cos\left(\delta_{i,p} - \delta_{i,q} \right) \right] \qquad (8.33)$$

The models of the flows from bus **q** to **p** are given by:

$$P_{i,qp} = V_{i,q}^2 \left(g_{i,qp}^{sh} + g_{i,qp} \right) - V_{i,q} V_{i,p} \left[g_{i,qp} \cos\left(\delta_{i,q} - \delta_{i,p} \right) - b_{i,qp} \sin\left(\delta_{i,q} - \delta_{i,p} \right) \right] \qquad (8.34)$$

$$Q_{i,qp} = -V_{i,q}^2\left(b_{i,qp}^{sh} + b_{i,qp}\right) + V_{i,q}V_{i,p}\left[g_{i,qp}\sin\left(\delta_{i,q} - \delta_{i,p}\right) + b_{i,q}\cos\left(\delta_{i,q} - \delta_{i,p}\right)\right] \quad (8.35)$$

In common case, the power flow data model can be written as:

$$z_{i,flow} = \left[P_{i,pq}, Q_{i,pq}, P_{i,qp}, Q_{i,qp}\right]^T = h_{i,flow}\left(V_i\angle\delta_i\right) + \varepsilon_{i,flow} \quad i = \overline{1, N_S} \quad (8.36)$$

$$z_{i,flow} \in \Re^{\left(M_{i,P}^{flow} + M_{i,Q}^{flow} + M_{i,O}^{flow} + M_{i,Q}^{flow}\right)}$$

The type of measurements and the number of measurements of the real and reactive power flows can be different for the different sub-systems.

Tie-line measurement data model: The N_s sub-systems are connected by tie-lines (that could be electrical transmission lines or transformers). The two ends of each tie-line are buses belonging to different sub-systems. The set of these boundary buses defines an ***(N_S+1) th*** sub-system called interconnection subsystem.

The measurement model for the tie-lines between the ***ith*** and ***jth*** subsystems, when the number of tie-lines is $N_{tl,ij}$ is given by equations (6.105) and (6.106) developed in chapter 6, as follows

$$P_{ij,pg} = G_{ij,pg}V_{i,p}^2 - V_{i,p}V_{j,g}G_{ij,pg} \qquad p = \overline{1, N_{tl,ij}} \quad p \neq g \quad (8.37)$$

$$Q_{ij,pg} = -V_{i,p}^2\left(B_{ij,pg} + b_{ij,pg}^{sh}\right) + V_{i,p}V_{j,g}B_{ij,pg}, \quad g = \overline{1, N_{tl,ij,}} \quad g \neq p, i = \overline{1, N_S}, j = \overline{1, N_S}, i \neq j \quad (8.38)$$

The vector of measurements for the *ith* sub-system can be formed as follows:

$$z_{tl,ij,pg} = \begin{bmatrix} P_{ij,pg} \\ Q_{ij,pg} \end{bmatrix} = \begin{bmatrix} G_{ij,pg} \\ -B_{ij,pg} - b_{ij,pg}^{sh} \end{bmatrix} V_{i,p}^2 + V_{i,p}\begin{bmatrix} G_{ij,pg} \\ B_{ij,pg} \end{bmatrix} V_{jg} \quad (8.39)$$

$\mathbf{Z}_{tl,ij}$ can be written in matrix form as:

$$z_{ij,tl} = \begin{bmatrix} P_{i1,11} & P_{i1,12} & \cdots & P_{i1,1g} & \cdots & \cdots & P_{i1,N_{tl,1}N_{tl1}} \\ P_{i2,11} & P_{i2,12} & \cdots & P_{i2,2g} & \cdots & \cdots & P_{i2,N_{tl,i}N_{tl2}} \\ \vdots & \vdots & & \vdots & & & \vdots \\ P_{ij,11} & P_{ij,12} & \cdots & P_{ij,jg} & \cdots & \cdots & P_{ij,N_{tli},N_{tlj}} \\ \vdots & & & \vdots & & & \vdots \\ P_{iN_S,11} & P_{iN_S,12} & \cdots & P_{iN_S,N_sg} & \cdots & \cdots & P_{iN_S,N_{tl},N_{tl},iN_S} \\ Q_{i1,11} & Q_{i1,12} & \cdots & Q_{i1,1g} & \cdots & \cdots & Q_{i1,N_{tl,i1}N_{tl1}} \\ Q_{i2,11} & Q_{i2,12} & \cdots & Q_{i2,2g} & \cdots & \cdots & Q_{i2,N_{tl,i2},N_{tl2}} \\ \vdots & \vdots & & \vdots & & & \vdots \\ Q_{ij,11} & Q_{ij,12} & \cdots & Q_{ij,jg} & \cdots & \cdots & Q_{ij,N_{tl,ij}N_{tl,j}} \\ \vdots & \vdots & & & & & \vdots \\ Q_{iN_S,11} & Q_{iN_2 12} & \cdots & Q_{iN_S N_S,g} & \cdots & \cdots & Q_{iN_S,N_{tl}i,N_S,N_{tl},iN_S} \end{bmatrix} \tag{8.40}$$

The vector for measurements for the interconnected system is given by

$$z_{tl} = \left\lfloor z_{tl,1}^T, z_{tl,2}^T, \cdots, z_{tl,N_S}^T \right\rfloor \tag{8.41}$$

The model of measurements is a non-linear one and can be written in the following way:

$$z_{tl} = h_{tl}\left(V_i \angle \delta_i, V_j \angle \delta_j\right) \quad i = \overline{1, N_S}, j = \overline{1, N_S} \quad i \neq j \tag{8.42}$$

It can be observed that the ***ijth*** sub-system tie-line flow measurement model has two components-one that depends on the ***ith*** sub-system voltages and the other that depends on the voltages of the ***jth*** sub-system. This means that the measurement model can be represented as a sum from a model of the ***ith*** separate sub-system and model of interconnections with other sub-systems. The first part depends on the voltages $\boldsymbol{V_{i,p}}$ of the ***ith*** sub-system and the second part depends on the voltages $\boldsymbol{V_{j,g}}$ of the jth sub-system. These voltages participate also in the injection models of the sub-system of the interconnected system. Included in the vector of voltages of the sub-systems are also the border buses end voltages. In this way it is not necessary to calculate again the state estimates of the border voltages as they are calculated using the injection data model.

On the basis of the above, the tie-line measurement model for the ***ith*** sub-system can be written in the following way:

$$z_{i,tl} = h_{i,tl}\left(V_i \angle \delta_i\right) + y_{i,tl} \tag{8.43}$$

$$y_{i,tl} = h_{ij,tl}\left(V_j \angle \delta_j\right) \tag{8.44}$$

The dimension of the measurement vector depends on the number of the tie-lines between the ith sub-system and other sub-system.

Finally, the model for the ***ith*** sub-system with measurements of the voltage, real and reactive power injections, real and reactive power flows, real and reactive power flows in the tie-lines can be written as follows:

$$z_{i,V} = V_i + \varepsilon_{i,V} \qquad z_{i,V} \in \Re^{n_i} \tag{8.45}$$

$$z_{i,inj} = h_{i,inj}\left(V_i \angle \delta_i\right) + y_{i,inj} + \varepsilon_{i,inj} \qquad z_{i,inj} \in \Re^{2n_i} \tag{8.46}$$

$$z_{i,flow} = h_{i,flow}\left(V_{i,} \angle \delta_i\right) + \varepsilon_{i,flow} \qquad z_{i,flow} \in \Re^{M_{i,flow}} \tag{8.47}$$

$$z_{i,tl} = h_{i,tl}\left(V_i \angle \delta_i\right) + y_{i,tl} + \varepsilon_{i,tl} \qquad z_{i,tl} \in \Re^{\sum_{\substack{j=1 \\ j \neq i}}^{N_S} N_{tl,ij}} \tag{8.48}$$

$$y_{i,inj} = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} h_{ij,inj}\left(V_i \angle \delta_i, V_j \angle \delta_j\right) \tag{8.49}$$

$$y_{i,tl} = h_{i,tl}\left(V_i \angle \delta_i, V_j \angle \delta_j\right) \tag{8.50}$$

This mode given by Eqns (8.45) to (8.50) is used for formulation and solution of the problem for state estimation.

8.5 Two level state estimation problem solution

The solution of state estimation problem is based on a Lagrangian function formulated in the following way for the interconnected part of the data model. It is accepted that $\boldsymbol{M_{i,V} = n_i}$

$$L = \sum_{i=1}^{N_S} \left\{ \begin{array}{l} \dfrac{\left|V_i^{mes} - \hat{V}_i\right|^2}{\sigma_{i,V}^2} + \rho_{i,inj}^T \left[y_{i,inj} - \sum\limits_{\substack{j=1 \\ j \neq i}}^{N_S} h_{i,inj}\left(V_i \angle \delta_i, V_j\right) \right] + \lambda_{i,inj}^T \left[z_{i,inj} - h_{i,inj}\left(V_i \angle \delta_i\right) - y_{i,inj} \right] + \\ + \rho_{i,tl}^T \left[y_{i,tl} - h_{ij,tl}\left(V_{i,} \angle \delta_i, V_j \angle \delta_j\right) \right] + \lambda_{i,tl}^T \left[z_{i,tl} - h_{i,tl}\left(V_i \angle \delta_i\right) - y_{i,tl} \right] \end{array} \right\} \tag{8.51}$$

Where

$\rho_{i,inj}, \rho_{i,tl}, \lambda_{i,inj}, \lambda_{i,tl}$ are vectors of Lagrange multipliers.

The Lagrangian function includes the criterion and the model of the interconnected equations of the sub-systems according to power injections and flows. It can be seen that if the interconnections $h_{i,inj}$ and $h_{i,tl}$ can be decomposed, then the Lagrangian function can be decomposed and the state estimation problem can be solved in a fully decentralized way.

Such type of decomposition of the Lagrangian function can be achieved if the problem for state estimation is solved in a two level structure using the principle of decomposition and coordination suggested in [Singh & Titli, 1978].

The mixed principle of prediction of the aims of the sub-systems represented by the Lagrange's variables $\rho_{i,inj}$ and $\rho_{i,tl}$ and of prediction of the interconnections of the sub-systems $y_{i,inj}$ and $y_{i,tl}$ is applied to the Lagrangian function (8.51). This principle is implemented by introducing a coordinator on the second level of the two level structures for solution of the problem. It predicts the values of the Lagrange's variables and interconnections as follows:

$$\begin{aligned} \rho_{i,inj} &= \rho_{i,inj} \\ \rho_{i,tl} &= \rho_{i,tl}^{c} \\ y_{i,inj} &= y_{i,inj} \\ y_{i,tl} &= y_{i,tl}^{c} \qquad i = \overline{1, N_S} \end{aligned} \tag{8.52}$$

Where c is the index of the coordination procedures; substitution of the coordinating variables given in (8.52) into the Lagrangian function allows the interconnection terms to be distributed between the sub-systems in the following way:

$$\sum_{i=1}^{N_S} \rho_{i,inj}^{T} \sum_{\substack{j=1 \\ j \neq i}}^{N_S} h_{ij,inj}\left(V_i \angle \delta_i, V_j \angle \delta_j\right) = \sum_{i=1}^{N_S} \sum_{\substack{j=1 \\ j \neq}}^{N_S} \rho_{j,inj}^{T} h_{ji,inj}\left(V_i \angle \delta_i\right) \tag{8.53}$$

$$\sum_{i=1}^{N_S} \rho_{i,tl}^{T} h_{ij,tl}\left(V_i \angle \delta_i, V_j \angle \delta_j\right) = \sum_{i=1}^{N_S} \rho_{j,tl}^{T} h_{ji,tl}\left(V_i \angle \delta_i\right) \tag{8.54}$$

The above transformation is possible on the basis of the connection between the primal variables V_j and the dual (Lagrange's) variables ρ_l. As the dual variables have known values, the voltages of the other sub-systems can be substituted by the voltages of the ith sub-system and the Lagrange's variables of the ***jth*** sub-system. In this way the Lagrangian' function is function only of the voltages of the ***ith*** sub-system and can be fully decomposed. This means that the problem for state estimation can be solved for every sub-system separately. The sub-system solutions in this case depend on the values of the coordinating variables, which mean that the optimal solutions will be obtained only when the values of the coordinating variables are the optimal ones. Improvement of the values of the coordinating variables is done by iterative process of coordination based on necessary conditions for optimality of the Lagrangian function towards the coordinating variables, and the solution of the separate sub-system's problems.

The necessary conditions for optimality of the Lagrangian function towards the coordinating variables are:

$$\frac{\partial L}{\partial \rho_{i,inj}} = y_{i,inj}^{c} - \sum_{\substack{j=1 \\ j \neq i}}^{N_S} h_{i,inj}\left(V_i \angle \delta_i\right) = 0 \qquad (8.55)$$

$$\frac{\partial L}{\partial y_{i,inj}} = \rho_{i,inj}^{c} - \lambda_{i,inj} = 0 \qquad (8.56)$$

$$\frac{\partial L}{\partial \rho_{i,tl}} = y_{i,tl}^{c} - h_{ij,tl}\left(V_i \angle \delta_i, V_j \angle \delta_j\right) = 0 \qquad (8.57)$$

$$\frac{\partial L}{\partial y_{i,tl}} = \rho_{i,tl}^{c} - \lambda_{i,tl} = 0 \qquad (8.58)$$

In the above equations, the values of the voltages and the values of the Lagrange's variables $\lambda_{i,inj}$ and $\lambda_{i,tl}$ can be obtained as solutions of the sub-system's state estimation problems.

Eqns (8.55) to (8.58) can be solved analytically using the solutions obtained from the first level sub-problems in the following way

$$y_{i,inj}^{c+1} = \sum_{\substack{j=1 \\ j \neq i}}^{N_S} h_{i,inj}\left(V_i^c \angle \delta_i^c, V_j^c \angle \delta_j^c\right) \qquad (8.59)$$

$$\rho_{i,inj}^{c+1} = \lambda_{i,inj}^{c} \qquad (8.60)$$

$$y_{i,tl}^{c+1} = h_{ij,tl}\left(V_i^c \angle \delta_i^c, V_j^c \angle \delta_j^c\right) \qquad (8.61)$$

$$\rho_{i,tl}^{c+1} = \lambda_{i,tl}^{c} \qquad (8.62)$$

The optimal solution of the initial state estimation problem is obtained if the necessary conditions for optimality according to the coordinating variables are fulfilled. This can be checked by calculation of the errors:

$$\begin{aligned} \varepsilon_1 &= y_{i,inj}^{c+1} - y_{i,inj}^{c} \\ \varepsilon_2 &= \rho_{i,inj}^{c+1} - \rho_{i,inj}^{c} \\ \varepsilon_3 &= y_{ij,tl}^{c+1} - y_{ij,tl}^{c} \\ \varepsilon_4 &= \rho_{i,tl}^{c+1} - \rho_{i,tl}^{c} \end{aligned} \qquad (8.63)$$

If $\varepsilon_1 \leq \varphi_1, \varepsilon_2 \leq \varphi_2, \varepsilon_3 \leq \varphi_3, \varepsilon_4 \leq \varphi_4$ where; $\varphi_1 > 0, \varphi_2 > 0, \varphi_3 > 0, \varphi_4 > 0$ are very small numbers, the optimal solutions of the coordinating problem and the sub-problems are obtained.

Different methods can be applied to obtain solution on the first level problems. The method developed in chapter 7 is considered and applied in the thesis. For every of the first level problem, a local angle reference bus has to be introduced. The solutions $V_i\angle\delta_i \quad i=\overline{1,N_S}$ are function of the coordinating variables.

The two level calculating structures is given in Figure 8.1

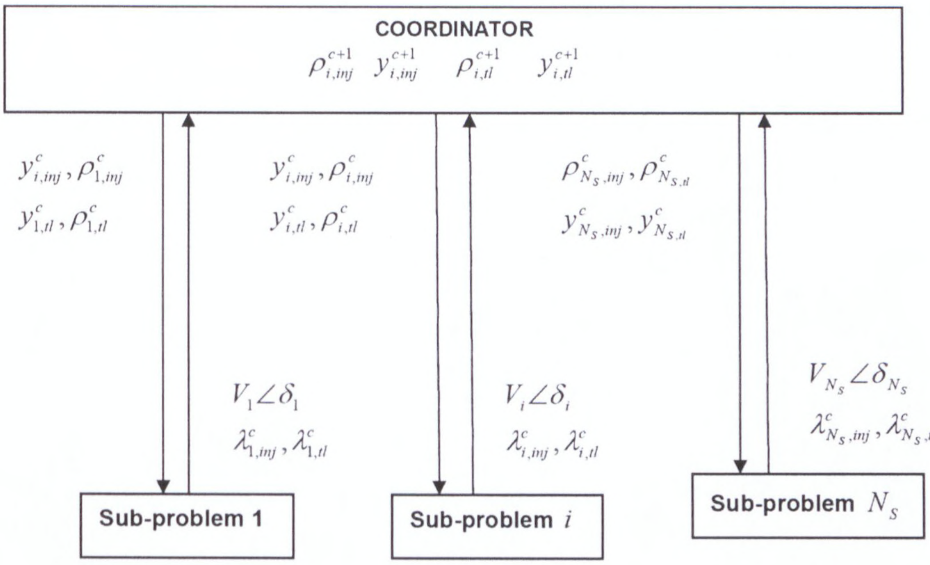


Figure 8.1: Two level structures for solution of state estimation problem

8.6 Formulation and Solution of the sub-problems on the first level

The state estimation problem for every isolated sub-problem is formulated using the decomposed Lagrangian function for sub-problem criterion

$$L_i = \frac{\left|V_i^{meas} - V_i\right|^2}{\sigma^2_{i,V}} - \sum_{\substack{j=1 \\ j\neq i}}^{N_S} \rho^c_{i,inj} y^c_{i,inj} \rho^c_{j,inj} h_{ij,inj}(V_i\angle\delta_i) + \lambda^T_{i,inj}[-y^c_{i,inj} + z_{i,inj} - h_{i,inj}(V_i\angle\delta_i)] + \\ + \rho^c_{i,tl} y^c_{i,tl} - \rho^c_{j,tl} h_{ji,tl}(V_i\angle\delta_i) + \lambda^T_{i,tl}[z_{i,tl} - h_{i,tl}(V_i\angle\delta_i)] \quad (8.64)$$

Have to be minimized for the rest of the measurement model equations

$$z_{i,V} = V_i + \varepsilon_{i,V} \\ z_{i,flow} = h_{i,flow}(V_i\angle\delta_i + \varepsilon_{i,flow}) \quad (8.65)$$

8.6.1 Solution of the First level Sub-problems

A Lagrangian function is formed for every sub-problem

$$L_i^i = \frac{\left|V_i^{meas} - V_i\right|^2}{\sigma_{i,V}^2} - \sum_{\substack{j=1 \\ j\neq i}}^{N_S} \rho_{i,inj}^c y_{i,inj}^c \rho_{j,inj}^c h_{i,inj}(V_i \angle \delta_i) + \lambda_{i,inj}^T[-y_{i,inj}^c + z_{i,inj} - h_{i,inj}(V_i \angle \delta_i)] +$$
$$+ \rho_{i,tl}^c y_{i,tl}^c - \rho_{j,tl}^c h_{ji,tl}(V_i \angle \delta_i) + \lambda_{i,tl}^T[z_{i,tl} - h_{i,tl}(V_i \angle \delta_i)] + \lambda_{i,V}^T(z_i - V_i) + \qquad (8.66)$$
$$+ \lambda_{i,flow}^T[z_{i,flow} - h_{i,flow}(V_i \angle \delta_i)]$$

The optimal solution is based on the necessary conditions for optimality suggested by Karush-Kuhn-Tucker (KKT) as follows:

$$\frac{\partial L_i^i}{\partial V_i} = -\frac{2(V_i^{meas} - V_i)}{\sigma_{i,V}^2} - \sum_{\substack{j=1 \\ j\neq i}}^{N_S} \left(\frac{\partial h_{ji,inj}(V_i \angle \delta_i)}{\partial V_i}\right)^T \cdot \rho_{j,inj}^c - \left(\frac{\partial h_{i,inj}(V_i \angle \delta_i)}{\partial V_i}\right)^T \lambda_{i,inj} -$$
$$-\left(\frac{\partial h_{ji,tl}(V_i \angle \delta_i)}{\partial V_i}\right)^T \cdot \rho_{j,tl}^c - \left(\frac{\partial h_{i,tl}(V_i \angle \delta_i)}{\partial V_i}\right)^T \lambda_{i,tl} - \lambda_{i,V} - \left(\frac{\partial h_{i,flow}(V_i \angle \delta_i)}{\partial V_i}\right)^T \lambda_{i,flow} = 0 \qquad (8.67)$$

$$\frac{\partial L_i^i}{\partial \delta_i} = -\sum_{\substack{j=1 \\ j\neq i}}^{N_S} \left(\frac{\partial h_{ji,inj}(V_i \angle \delta_i)}{\partial \delta_i}\right)^T \cdot \rho_{j,inj}^c - \left(\frac{\partial h_{i,inj}(V_i \angle \delta_i)}{\partial \delta_i}\right)^T \lambda_{i,inj} -$$
$$-\left(\frac{\partial h_{ji,tl}(V_i \angle \delta_i)}{\partial \delta_i}\right)^T \rho_{j,tl}^c - \left(\frac{\partial h_{i,tl}(V_i \angle \delta_i)}{\partial \delta_i}\right)^T \lambda_{i,tl} - \left(\frac{\partial h_{i,flow}(V_i \angle \delta_i)}{\partial \delta_i}\right)^T \lambda_{i,flow} = 0 \qquad (8.68)$$

$$\frac{\partial L_i}{\partial \lambda_{i,inj}} = z_{i,inj} - y_{i,inj}^c - h_{i,inj}(V_i \angle \delta_i) = 0 \qquad (8.69)$$

$$\frac{\partial L_i}{\partial \lambda_{i,tl}} = z_{i,tl} - h_{i,tl}(V_i \angle \delta_i) - y_{i,tl}^c = 0 \qquad (8.70)$$

$$\frac{\partial L_i}{\partial \lambda_{i,V}} = z_{i,V} - V_i = 0 \qquad (8.71)$$

$$\frac{\partial L_i}{\partial \lambda_{i,flow}} = z_{i,flow} - h_{i,flow}(V_i \angle \delta_i) = 0 \qquad (8.72)$$

The solution of Equations (8.67) to (8.72) which gives the necessary conditions for optimality determines the optimal solution of the ***ith*** sub-system. As the above system of equations is non-linear with many variables the analytical solution is not possible. Gradient procedures are used to calculate the values of the primal variables ***(V_i, δ_i)*** and of dual variables $\lambda_{i,inj}, \lambda_{i,tl}, \lambda_{i,flow}$ as follows:

$$
\begin{aligned}
V_{i,}^{t+1} &= V_i^t - \alpha_{i,V_i}\varepsilon_{i,V_i} \\
\delta^{t+1} &= \delta_i^t - \alpha_{i,\delta_i}\varepsilon_{i,\delta_i} \\
\lambda_i^{t+1} &= \lambda_{i,inj}^t + \alpha_{i,\lambda_{i,inj}}\varepsilon_{i,\lambda_{inj}} \\
\lambda_{i,tl}^{t+1} &= \lambda_{i,tl}^t + \alpha_{i,tl}\varepsilon_{i,tl} \\
\lambda_{i,V}^{t+1} &= \lambda_{i,v}^t + \alpha_{i,V}\varepsilon_{i,V} \\
\lambda_{i,flow}^{t+1} &= \lambda_{i,flow}^t + \alpha_{i,flow}\varepsilon_{i,flow}
\end{aligned} \tag{8.73}
$$

The calculations given by equation (8.73) are performed under the given by the second level values. If the gradient procedures continue until convergence on maximam number of iterations on first level is reached.

Check of convergence: Norms of the errors of every iteration at ***t =0, 1, 2,*** etc, are calculated as follows:

$$
\begin{aligned}
\varepsilon_1^i &= \left\|\varepsilon_{i,V_i}\right\| \\
\varepsilon_2^i &= \left\|\varepsilon_{i,\delta_i}\right\| \\
\varepsilon_3^i &= \left\|\varepsilon_{i,\lambda_i}\right\| \\
\varepsilon_4^i &= \left\|\varepsilon_{i,tl}\right\| \\
\varepsilon_5^i &= \left\|\varepsilon_{i,flow}\right\|
\end{aligned} \tag{8.74}
$$

Norms of the errors are compared with given small positive numbers

$\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5$ if $\varepsilon_1 \le \varphi_{1,}\varepsilon_2 \le \varphi_2, \varepsilon_3 \le \varphi_3, \varepsilon_4 \le \varphi_4, \varepsilon_5 \le \varphi_5$ optimal solution is attained and stop the calculation. When the calculations on first level are completed the values of $V_{i,}\delta_i, \lambda_{i,V}, \lambda_{i,tl}$ are sent to the second level and the new values of the coordinating variables are calculated, and so on. The derivation given above are presented in algorithm form given below

8.6.2 First level algorithm

At first level as has been observed, the optimal operating condition of each sub-system is determined by solving independently ***N*** power system state estimation sub-problems. At this level, the interconnection have not to be considered and gradient procedures are used to calculate the primal and dual variables of the isolated sub-systems. The following algorithm is used to calculate the solution of the first level sub-problem. Before, starting the algorithm, first the number of iteration is defined, in this case ***M1 = 100*** and this is given.

I: Initialize $t=0$,

Set initial values for $V_{i,}\delta_{i,}\lambda_{i,inj},\lambda_{i.flow},\lambda_{i,tl},\lambda_{i,V}$. Initial value of V_i is obtained from load flow program and δ_i is set equal to zero.

II. Obtain the values of the coordinating variables $\rho_{i,inj},\rho_{i,tl},y_{i,inj}y_{i,tl}$ from the second level

IIi: Improve values of set variables using Eqn (8.73)

IV. Check for conditions for convergence using (8.74). If conditions of convergence satisfy (8.74), stop the procedure and send the values of $V_{i,}\delta_{i,}\lambda_{i,inj},\lambda_{i,tl}$ to the second level. If not go to step **II**

First level algorithm is schematically given in Figure 8.2

8.6.3 Second level algorithm

The coordinator does not to know or need the detailed operating information of each sub-system. The coordinator executes the following functions:

- Predict the values of the Lagrange's variables and interconnection variables and send predicted variables ($\rho_{i,inj},\rho_{i,tl},y_{i,inj},y_{i,tl}$) to the first level
- Wait all calculation on the first level to be completed and receive the values of $V_{i,}\delta_{i,}\lambda_{i,inj},\lambda_{i,tl}$ from first leve sub-problems
- Compare $\rho_{i,inj},\rho_{i,tl,}$ sent by each sub-system and if the error between them is bigger than given tolerance calculate the improved values of the coordinating variables.
- If the error is smaller than the tolerance, stop the procedure.

It can be observed that the coordinator has little work to do. The coordinator does not calculate the state vector, but has to predict the Lagrange's variables and variables related to sub-system interconnections. The positive characteristic of this method is that of reducing computation work at coordinator level. The second level algorithm is schematically given in Figure 8.2

8.7 Application of the algorithms to power system state estimation problem

The developed decomposition-coordination method and algorithm in this chapter presents positive characteristic.

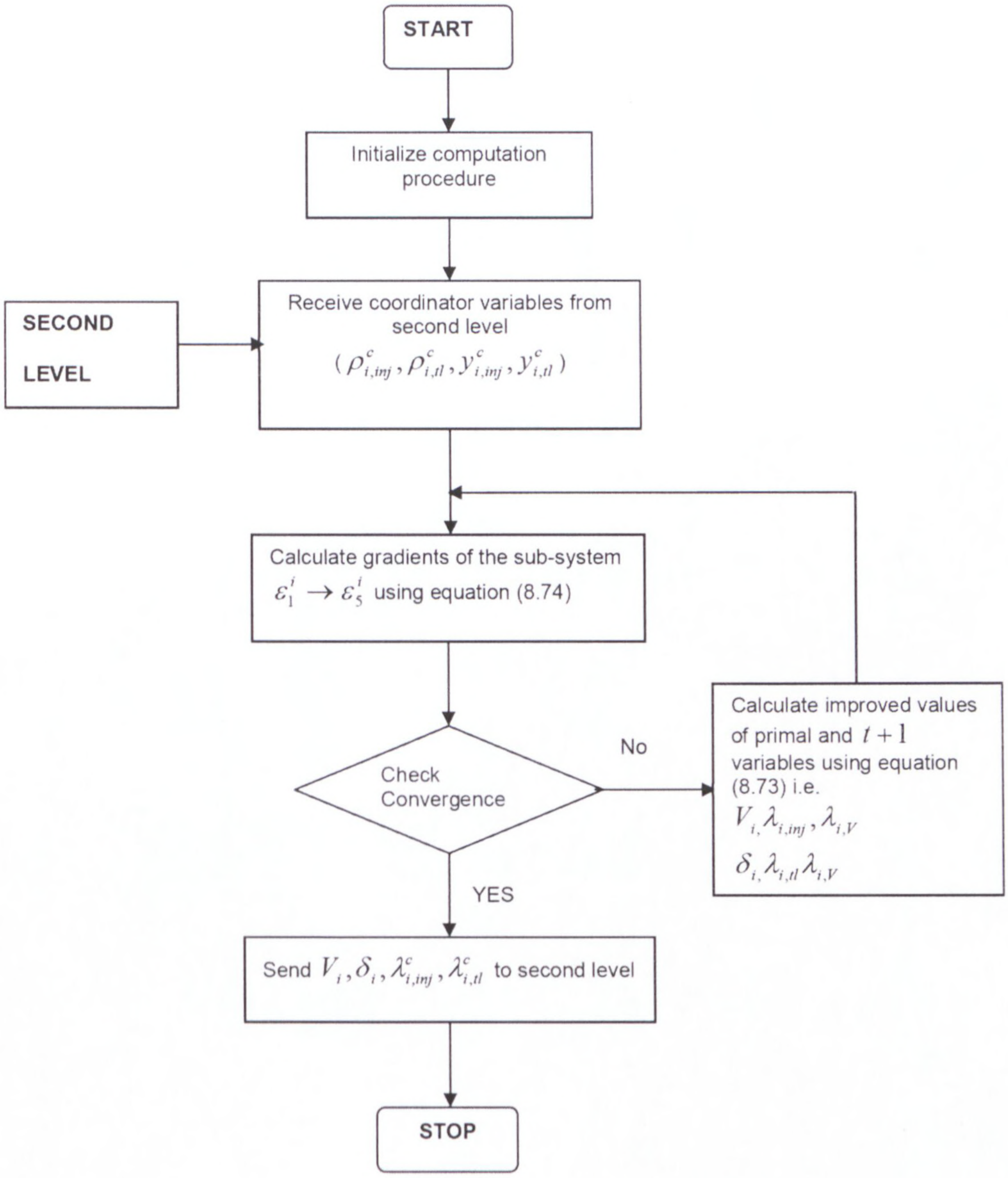


Figure 8.2: First level algorithm flowchart

First, the solution of the power system state estimation problem is reduced to solution of ***N+1*** independent sub-problem. Secondly, the method can be carried out by using parallel or distributed computing architectures. In this way the computational task corresponding to the decomposition work can be carried out by a unique parallel computer located in the power system control centre, using a cluster or by using a suitable distributed computing scheme with several processors located at the lower levels in the control hierarchy such as in the regional control centres.

First and second level algorithm can be applied in solving power system state estimation problem provided that the following steps are followed:

- The power system is decomposed into sub-systems.
- At least one generating bus is available in a sub-system.
- The sub-system is observable i.e. measurements from the sub-system are enough to conduct the state estimation.

Importance of using decomposition-coordination method and algorithm is to:

- Facilitate parallel processing of an optimization problem
- Decentralize measurement (data)
- Reduce the amount of data sent to central control centre
- Reduce computation time and complexity on solution.

Decomposition-coordination algorithm can be applied in:

- The Tanzanian power system network. The system is expanding rapidly and communication between the areas i.e. North to South-East or North -East to East is still a problem. Sending data from these areas to Ubungo National Grid Centre for processing could take a long time. Hence, use of the algorithm could alleviate the situation.
- Planned East African Power Pool (EAPP)
- South African Power Pool (SAPP)
- Other regional power blocks.

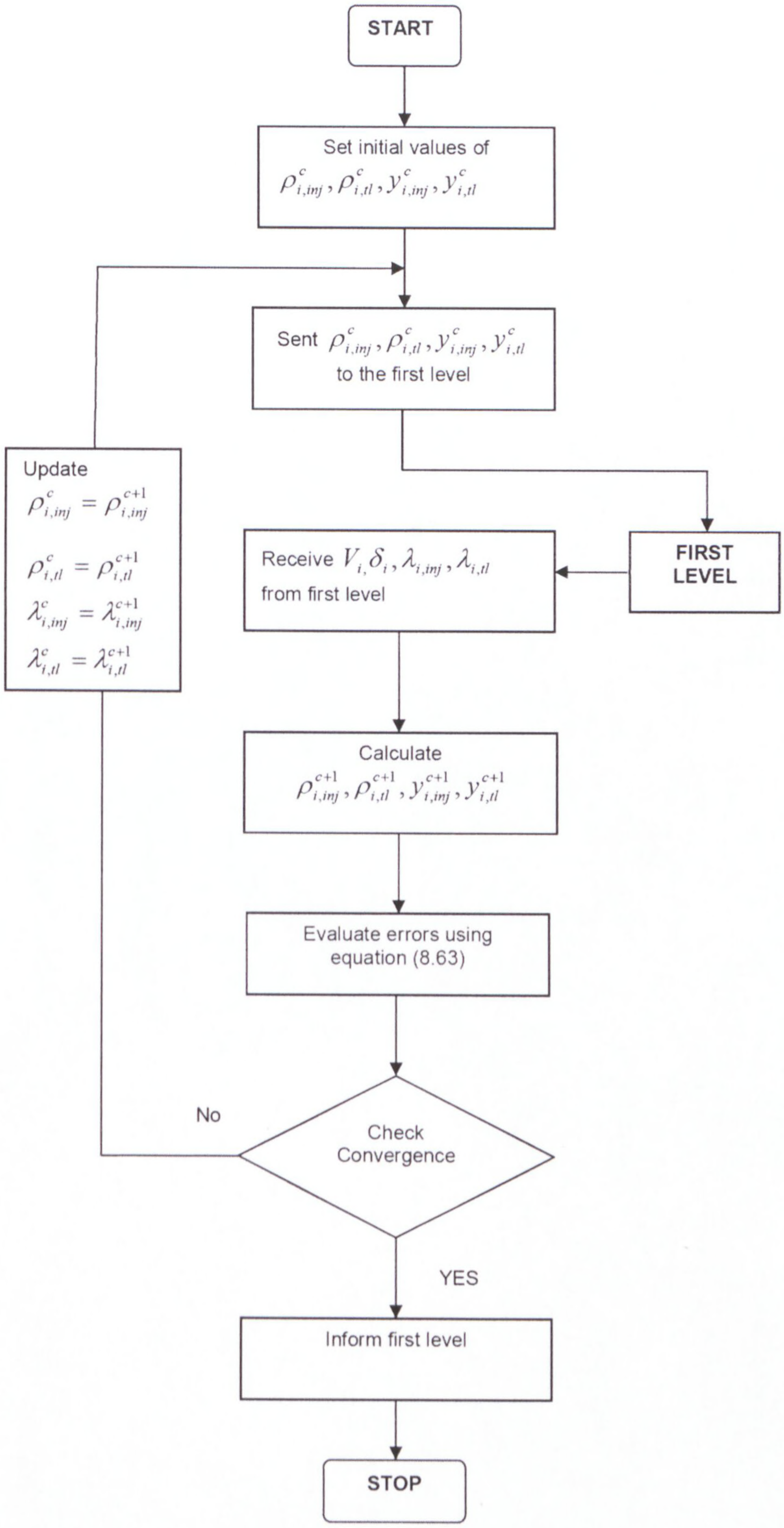


Figure 8.3: Second level algorithm flowcahrt

8.8 Conclusion

Decomposition is a general method of solving a large and complex problem by breaking it into small sub problems and solving each sub problem independently. Decomposition method appears in early works on solving large-scale linear programming problems since i1960s. It has gone through several modifications and improvements to arrive at the level it has today. The pioneers in this field are Bender, Dantzig, Wolfe and Tammer just to mention few and their works briefly have been reviewed in this chapter.

This chapter has addressed development of a decomposition-coordination method and algorithm for solution of a multi-area state estimation problem. Two level structures is proposed. The first level is responsible for solution of the sub problems resulted from decomposition method. The second level deals with coordination function that is solving the global problem. An algorithm for each level is proposed. The chapter gives a basis for preparation of a parallel processing program in solving the estimation problem.

Chapter 9 presents parallel processing of multi-area state estimation problem. Parallel and Distributed systems are overviewed, their advantages and disadvantages highlighted. Differences between distributed computing toolbox (DCT) and parallel computing toolbox (PCT) are provided. Programming Parallel Applications in MATLAB is presented.

CHAPTER NINE: PARALLEL PROCESSING METHOD

9.1 Introduction

Parallel and distributed processing are among the most promising methodologies of the new developments in processing large optimization problems [Tylavsky, 1992]; Bertsekas, 1999; Umar, 1993]. Parallel processing systems consist in the use of multiple hardware components to exploit concurrency in computation process. The advantage of using parallel processing application in power system and in particular SE is to speed up of computations in order to make viable the solution of the system intractable in a conventional computer. Distributed computer systems are defined as collection of independent computers joined together by high speed communication networks. Power system can and will benefit from this form of decentralized computer architecture due to its distributed nature, from its flexibility, scalability and cost reduction advantages.

This chapter presents parallel and distributed processing concepts. Initially, it presents a general description of parallel and distributed processing schemes overview, their advantages and disadvantages. Next, Distributed computing toolbox (DCT) and MATLAB distributed computing engine (MDCE) as well as parallel distributed computing toolbox (PCT) and MATLAB distributed computing server (MDCS) are presented.Finally, development of parallel algorithm for solution of SE problem is given.

9.2 Parallel and Distributed Processing: An Overview

Parallel and distributed processing is similar but relies on different information processing concepts. Distributed computer systems are collections of computers joined together with multiple objectives depending on the intended applications. Parallel computers are usually single frame units in which the idea of concurrent processing is exploited mainly to speed up computations. These two data processing approaches have achieved practical implementation because of breakthrough in science and technology and increase in microprocessor performance [Quinn, 1994]. Parallel and distributed processing has been an intense research area for several decades now. Although traditional supercomputers now in operation with a single processor are fast, but they are extremely expensive and their performance depends on their memory bandwidth. With rapid advancement in computer and communication technologies, traditional supercomputers with a single processor are being gradually replaced by less expensive and more powerful parallel and distributed processing architectures.

The main process for solving large-scale problem such as optimization problems with parallel and distributed approach is outlines as follows: First. The problems are formulated for parallel and distributed processing. Second, specific parallel and distributed algorithms are developed for the solution of these problems. Third, the complexity and the performance of the algorithms are evaluated for measuring the accuracy and reliability of the proposed solution.

9.2.1 Parallel Processing Systems

Parallel processing system is defined as utilization of multiple processing units to find a solution of one computational problem. There are two types of parallel systems. One is a parallel machine, and the other is a computer network. Both types of parallel processing systems comprise a number of processors that are closely coupled with a small physical space. When a network of computers is used for parallel computation, it represents a parallel virtual machine (PVM) with all the computers on the network as processors of this virtual machine (VM). Communication links between the processors of a parallel processing system are usually very short; for example, the processors of a parallel system is assumed to be very reliable, and communication delays, if considered, are predictable. The aim of using parallel processing system is to speed up computation by employing more than one processor at a time. That is, the sole purpose of employing parallel system is to obtain a fast solution by allowing several processors to function concurrently on a common task.

Parallel processing systems are classified according to their memory perspective, it can be either shared or distributed memory. Shared memory system, consists of:

- A single global address
- Symmetric multiprocessing
- Non-uniform memory access
- Multicore processor

Distributed memory system consists of:

- Node that has its own memory
- Massively parallel system
- Different type of clusters

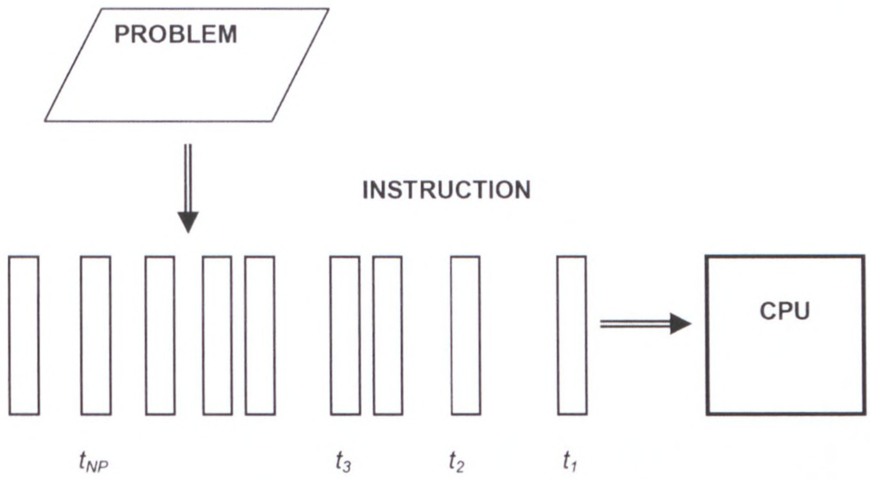


Figure 9.1: Parallel processing system

Shared memory (Figure 9.2) is user-friendly and its disadvantage is scalability. Distributed memory has an advantage in data locality i.e. no data interference and it is cost effective. However, its advantage lies in explicit communication and decomposition of data or tasks.

9.2.2 Distributed Processing Systems

Similar to parallel processing systems, a distributed system is a physical arrangement for distributed processing. But unlike a parallel system, a distributed system is usually a computer network that is geographically distributed over a large area. Distributed processing systems may include a few computers located in a single room (Laboratory), connected by a local area network using coaxial cables as interconnection media. In case of a large number of computers scattered hundred kilometres apart, they are interconnected by a wide area network (WAN) based on telephone lines, microwave channels, optical fibre etc. This type of decentralized processing approach is feasible due to availability of very powerful desk top computers (microcomputers and workstation), which can be interconnected by very fast network.

Some of the advantages of distributed processing systems over conventional sequential processing are:

- **Cost:** The average cost for million instructions per second (MIP) on a main frame is almost 100 times the one for workstation

- **Scalability:** distributed processing systems are expandable incrementally compared to sequential processing
- **Flexibility and Configurability:** Distributed processing systems have improved performance and reliability due to redundant data processing
- **Exploitation of special hardware:** Special graphic devices are easily added to the system.

Distributed processing applications are developed according to several different paradigms [Umar, 1993]. Among the paradigms are terminal emulator, which makes the computer look like a terminal, which is connected to another computer; a file transfer package that allows files to be transferred between different computers and the client – server system in which remotely located programs can exchange information in real-time. A distributed data and transaction management system is a sophisticated version of the client-server system which allows a user to store, access and manipulate data transparently from many computers. These types of system will eventually evolve a true distributed operating system to be employed in the solution of SE.

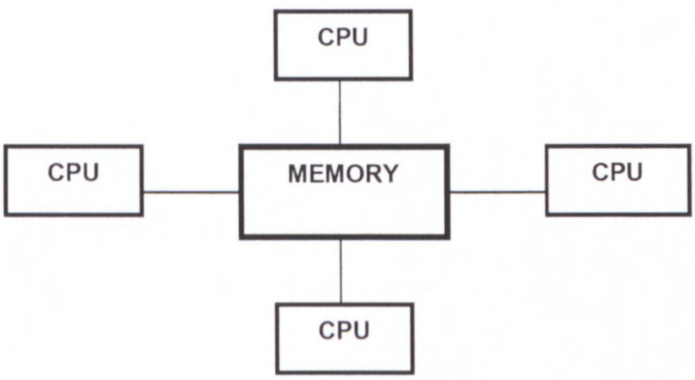


Figure 9.2: Shared memory

9.3 Design of Parallel Algorithms

Parallel and distributed algorithms are designed on the principle of concurrency and parallelism. As a strategy for processing large and complex tasks faster, concurrency and parallelism is done by first breaking up the task into smaller tasks, then assigning the smaller tasks to multiple processors to work on those tasks simultaneously, and finally coordinating the processors. Parallel and distributed algorithms follow design methodologies that are quite similar. However, different techniques and methodologies can be applied for creating parallel solution. The best parallel solution that can be developed is usually quite different from the sequential one. Foster in 1995 [Foster, 1995] introduced one of the many possible design methodologies now in application. One of the methodologies consists of four distinct stages that are performed one after another. They include: partitioning, communication, agglomeration and mapping. During the first and second stages the programmer focuses on concurrency and scalability, while during the third and fourth stages the attention of the programmer is on locality and other performance related issues. The four stages are explained as follows:

Partitioning: The first stage is partitioning of the problem. The parts on the problem that can be parallelized are identified and noted. This task is performed independently of the hardware type available, and issues such as number of processors available are not taken into consideration at this stage. The objective of performing partitioning step is to divide the problem in the smallest possible tasks, in order to obtain what is called a fine-grained decomposition of the problem. Partitioning is employed to both the whole computation job and the data associated to the process. Usually the first focus is on the data partitioning and then processes are associated to the partitioned data. This partitioning technique is known as domain decomposition. Another technique is to focus first in partitioning the global job in processes and to work out afterwards which is the best way to partition the data. This technique is called functional decomposition. These two techniques mentioned above are complementary, and they can be applied to different parts of a single problem or even both can be applied at the same time to the whole problem in order to obtain different parallel algorithms to solve a single problem. In this work the second technique is employed to partition the function i.e. process of problem solution decomposition.

Communication: The second stage is data communication. Data communication among processors becomes very critical especially when a task is completed through cooperation of a number of individual processors. Data communication in both parallel and distributed processing facilitates the processor's exchange of intermediate results. Data communication among processors of a parallel machine is predictable, while data communication in distributed processing is unpredictable. Hence, data communication among computers becomes more problematic, which must be taken into account during the design of parallel or distributed processing algorithms. Two schemes are commonly used for data communication in the two techniques.

Shared memory: In a shared memory block data will be stored in a designated common memory block. Figure 9.2 shows the set up of shared memory structure. The common memory block will be accessed by all processors. Data communication is realised instantly and reliably by accessing this common memory block. Shared memory is mostly applied for parallel processing. It is not applied in distributed processing because of technical implementation.

Message passing: Message passing interface as has been stated before, is a communication protocol for transferring data from one processor known as a sender to another (receiver). A message is transferred as packet of information over communication links or networks. MPI requires the cooperation of sending and receiving processes. A send operation of message passing requires the sending processors to specify the location, size, type, and the destination of data. In addition, the associated receive operation should match the corresponding send operation [Shahidehpour & Wang, 2003]. MPI has been widely used for parallel and distributed processing. MPI application package includes message passing library, which is a set of routines embedded in the application code to perform send, receive, and other message –passing operations.

Message passing mostly suits the distributed processing system, where each processor has its own local memory accessed directly by its own central processing unit (CPU). By using message passing, a distributed processor sends data to or receives data from all or some of the other processors. The data transfer is performed through data communications, which differ from a shared memory applied in the parallel processing system that permits multiple processors to directly access the same memory resource via a memory bus. The choice of message passing or shared memory depends on the system architecture and requirement for the design of algorithms.

Agglomeration: Agglomeration is the third stage in designing parallel and distributed algorithm. The stage focuses on performance requirements and implementation cost of the algorithm. Some of the processes are combined into larger groups in order to improve performance and implementation cost. Also, it focuses on the performance of the system that has been designed. Agglomeration stage moves from the theoretical design phase towards the realistic implementation, where the hardware resources available will be taken into account. The agglomeration is therefore, the opposite stage of partitioning phase, in which the fined grained solution is revised and the number of global tasks reduced. In addition, to do reduction in the number of processes, the replication of the data between different tasks is determined to be worthwhile or not in terms of efficiency. The objectives of agglomeration are two; first is to reduce the number of processes i.e. by combining them into large ones; second, to provide an appropriate number of them so that the processors can deal with, and also to reduce communication costs.

Mapping: The fourth and last stage is mapping. In mapping stage, the processes are organized according to the number of processors available. The main objective of this arrangement is to balance the work load of each processor and also to minimize communication costs. The mapping stage distributes the process among the available processors. This stage does not arise on single-processor or on shared-memory computers that provide automatic task scheduling. The goal of mapping algorithm is to minimize execution time. Two strategies are employed to achieve this goal

- Processes that are able to execute independently are executed in different processors in order to increase concurrency.
- Processes that communicate frequently are assigned to the same processor in order to increase locality.

Coordination becomes essential when more than one processor works jointly on a common task. Therefore, synchronization is important for the processors. Synchronization is a mechanism that allows processors to wait at some predetermined time points for the completion of other processor's computation. Synchronization provides an opportunity for cooperation, which is possible, can influence the solution and performance of parallel and distributed designed algorithms. Synchronization is based on the following three models:

Synchronous model: In synchronous model processors complete computation and communication tasks in synchronous rounds. Synchronous computation can be applied in circumstances where communication is reliable and communication time delay among the processors is predictable if not negligible. One advantage of the synchronous computation is that it can guarantee the convergence of computation.

Totally asynchronous model: In a totally asynchronous model, a processor does not need to wait for any data transfer from other processors and it can continue its computation with the existing information, though it might be outdated. The updated information will be used for computation immediately after it becomes available. Totally asynchronous algorithms are harder to program than synchronous algorithm because of the chaotic behaviour of the participating processors. Totally asynchronous algorithms are more general and portable and do not involve synchronization, and in some cases these algorithms cannot guarantee an efficient or even correct solution of the problem.

Partially synchronous model: A partially synchronous model usually imposes time restrictions on synchronization of events. In partially synchronous model, it is assumed that a processor's time step and message delivery time is between upper and lower bounds. Therefore, partially synchronous model lies properly between synchronous and totally asynchronous models. A partially synchronous model represents one of the realistic communication characteristics of distributed system [Shahidehpour & Wang, 2003]. The communication delays among power system control centres are hard to predict. However, in most cases, a communication delay has an upper bound, especially when data communications are performed on a dedicated wide area network (WAN) of a power system. A partially synchronous model has less uncertainty than a totally asynchronous model. However, partially synchronous model algorithm is usually more difficult to program due to extra complication that may rise from the timing.

A simple and practical way to design parallel and distributed algorithm is to parallelize their sequential programs to solve the problem that is executing too slow just because of specific bottlenecks at different stages of algorithm. Given a sequential program and its source code, then it is not necessary to apply the four stages described above. Instead, a common practice is to identify the parts in the code that represent the most important bottlenecks in the program and to apply parallelization mechanisms to these parts. Parallel algorithms are usually designed to be synchronous. This is because data communications among the processor of a parallel system are fast and reliable. However, it is more difficult to design distributed algorithms because of the existing uncertainties. The developed in chapter eight sequential programs developed using MATLAB code is employed in this work in order to avoid the four described stages.

9.4 Parallel Computer Architecture

There are several ways to build a parallel computer. On one end of the spectrum is the multiprocessor, in which multiple processors are connected via high performance communication media and are packaged into a single computer. Many modern "supercomputers" such as those made by Cray and IBM are multiprocessors composed of dozen to hundreds to even thousands of processors linked by high-performance interconnects. On the other end is the Beowulf cluster, which uses commodity network e.g.Gigabit Ethernet to connect commodity computers. The advantage of using Beowulf clusters is their lower costs and greater availability of components. Performance of Beowulf clusters has begun to rival that of traditional "supercomputers" MATLAB code supports many different operating systems and processor architecture; therefore, components for building a Beowulf cluster to run MATLAB in parallel and MATLAB MPI are relatively cheap and widely available. Beowulf cluster is used in solving the PSSE problem employing MATLAB in parallel and MATLAB MPI.

Parallel computer architecture is classified based on the number of processors and the arrangement of their central processing units (CPU), and memory within the different parallel systems. The following is a classification of the different parallel architectures:

Single Program Single Data (SPSD): This is an ordinary workstation system, where there is single CPU and a single address map accessible by the CPU. Usually this is not considered as a parallel system.

Single Program Multiple Data (SPMD): In atypical SPMD machine, there are hundreds of CPUs, even thousands, all of them with a small private memory space. They all execute at the same time the same program over different data. When a CPU requires data to be stored in another's memory address map an explicit communication procedure has to be executed before. In SPMD setup the same program runs on each processor (single program) but each program at instant processes different data (multiple data). Different instances of the program may follow different paths through the program e.g. if-else statements due to different input data, but the same basic program executes on all processors.

This thesis follows the SPMD model. The program is written by using if-else statements to execute different sections of MATLAB code on different processors. The scalability and flexibility of the SPMD model has made it the dominant parallel programming model and that is why it is chosen to be used in this thesis.

The two most common implementation of the single program multiple data model are the message passing interface (MPI) and OpenMP. In using MPI, the program is started on all processors. MPI program is responsible for distributing data among processors, then each processor processes its section of the data, communicating among them is needed.

Multiple Program Multiple Data (MPMD): In this architecture, there are more CPUs, all the processors have their particular memory space too, but the means of execution is very synchronous and each processor can execute a different program (multiple program), with the different programs working in tandem to process different data (multiple data). There are is a complete independence in execution between all the CPUs. MPMD system's performance shows a high dependence on the memory architecture, and depending on it, these systems are further classified as follows:

Shared memory: The system has a single memory space, so that any computer can access any local or remote memory in the system, independently of the process that is the owner. Having a single memory space allows having shared memory for communication between all the executing workers or tasks, which makes it to be an efficient communication mechanism. This is very convenient for parallel programs where the amount of data to process is very big, as the data require no copying in order for all the CPUs to access them at the same time. The disadvantage of using shared memory access is that shared variables lead to the existence of race conditions between parallel programs or routines, and therefore synchronization mechanisms have to be included in the parallel programs.

Distributed Memory: In this arrangement, each processor has a particular address map that the rest cannot access at all, these systems have the advantage of being very easily scalable, but lack of shared address map adds a time penalty for each inter-process communication

Shared Distributed memory: This is a hybrid model that has the goal of having the advantages of both systems i.e. the easy communication mechanism of shared memory and the scalability of distributed systems. These systems would contain their own memory address map, but the architecture is designed so that any processor can access the memory of another with slight time penalty.

9.5 Distributed Computing Toolbox (DCT)

The Distributed Computing Toolbox (DCT) and the MATLAB Distributed Computing Engine (MDCE) enable to execute coarse-grained MATLAB algorithms or Simulink applications (jobs) on computer cluster by dividing the applications into independent tasks. Each task evaluates a specified MATLAB function or Simulink model. The key features of DCT and MDCE include:

- Distributed execution of coarse-grained MATLAB algorithms and Simulink model on remote MATLAB sessions
- Control of distributed computing process via a function based or objects based-interface.
- Distributed computing on both homogeneous and heterogeneous platforms
- Support for synchronous and asynchronous operations
- Access to single or multiple clusters by single or multiple users

After an application has been submitted, MDCE executes each of its tasks. MDCE consists of a job manager who coordinates the distribution of the tasks and remote MATLAB sessions (Workers) that executes the tasks. Once the workers complete their tasks, they send results back to the job manager, where the user can access them using DCT.

The role of the DCT is to:

- Define and submit applications from the command line
- Divide the applications into tasks
- Send the applications to MDCE for execution
- Retrieving the results

The complete process includes the following steps:

- Finding a job manager
- Creating a job
- Creating tasks
- Submitting the job to the job queue
- Retrieving results.

The complete process mentioned above can be executed with a single command using the function based interface or using object-based interface to control each step.

9.5.1 Basic Terms

The basic terms encountered when applying DCT are given in Table 9.1

Table 9.1: Basic terms

Term	Description
Client	Computer on which MATLAB commands are entered
Job	Large MATLAB of Simulink operation to be performed on a computer cluster
Tasks	Segment of a job: the units that the workers receive for execution
Job Manager	Process that coordinates and asynchronously distributes tasks to the workers
Worker	Noninteractive MATLAB instance that processes tasks assigned by the job manager and sends the results back to the job manager

9.5.2 Configuration

The configuration of DCT/MDCE is shown in Figure 9.4. The interaction between the client machine, where the DCT is used to define jobs and tasks, and the MDCE is shown in Figure 9.3

9.6 Parallel Computing Toolbox (PCT)

Parallel computing Toolbox (PCT) enables to solve computationally and data-intensive problems using MATLAB and Simulink on multicore and multiprocessor computers. The key features of PCT include:

- Support for data –parallel and task –parallel application development
- Ability to annotate code segment with parfor (parallel for-loops) and spmd (single program multiple data) for implementing task-and data-parallel algorithms

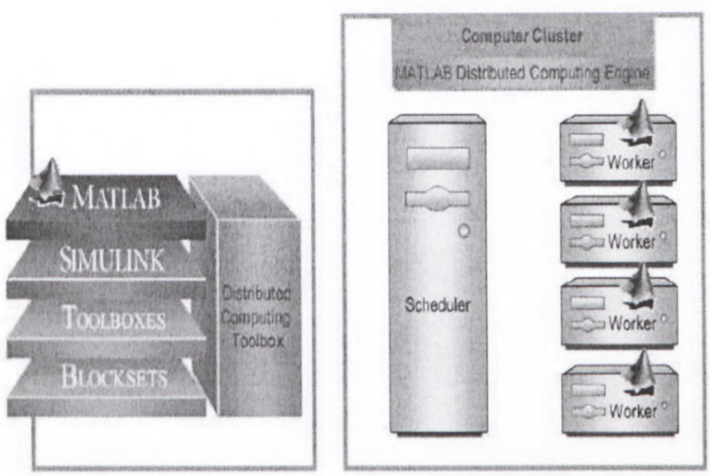


Figure 9.3: DCT/MDCE Configuration

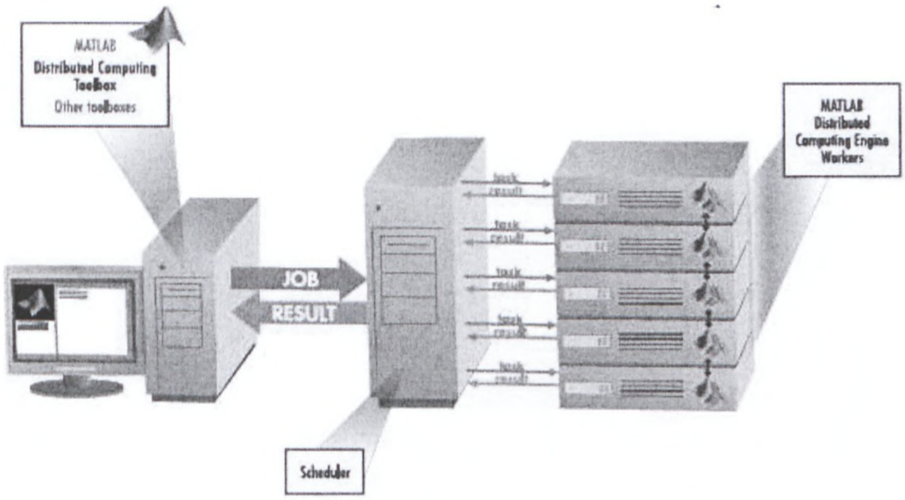


Figure 9.4: Interaction between DCT and MDCE

- High-level construct such as distributed arrays, parallel algorithms, and message-passing functions for processing large data set on multiple processors.
- Ability to run 4 workers 9lab) locally on a multicore desktop
- Integration with MATLAB Distributed Computing Server (MDCS) for cluster-based applications that use any scheduler or any number of workers (labs).
- Interactive and batch execution modes

The parallel processing construct enables:

- Parallel for-loop (parfor) and code blocks
- Distributed arrays
- Parallel numerical algorithms
- Message-passing functions

PCT enables to implement task-and data-parallel algorithms in MATLAB at a high level without for specific hardware and network architectures. Converting serial (sequential) MATLAB applications to parallel MATLAB applications requires few code modifications; an added advantage of PCT is: applications can be run interactively or offline, in batch environment.

PCT can be used to execute applications (jobs) on a single multicore or multiprocessor desktop. Without changing the code, it is possible to run the same application on a computer cluster using MATLAB Distributed computing Server (MDCS).

PCT provides several high-level programming construct that can convert serial (sequential) MATLAB code to run in parallel on several workers. These workers can run on client's (user) desktop or on a cluster (using) MDCS. The construct simplify parallel code development by abstracting away the complexity of managing coordination and distribution of calculations and data between a MATLAB client and workers, as well as between workers.

Advantages of using PCT and MDCS are on:

- Availability of the MATLAB pool and the parallel command window (pmode) which support interactive execution
- Parallel for-loops (parfor) offer one way to distributed tasks across multiple MATLAB workers
- Implementing Data-Parallel algorithms
- Working in an interactive parallel environment
- Ability to use up to 4 local workers on a multicore or multiprocessor computer using a single toolbox license.

9.6.1 Basic Terms

The basic terms encountered when applying PCT are the same as that of DCT except few which are given in Table 9.2

Table 9.2: Basic terms

Term	Description
parfor	Parallel for-loops
pmode	Parallel Command Window
spmd	Single program multiple data

9.6.2 Configuration

Configuration used in developing parallel applications with PCT, which enables application prototyping on the desktop with up to 4 worker (shown in the left of Figure 9.5), and with MDCS (right), the application can be scaled to multiple computers on a cluster.

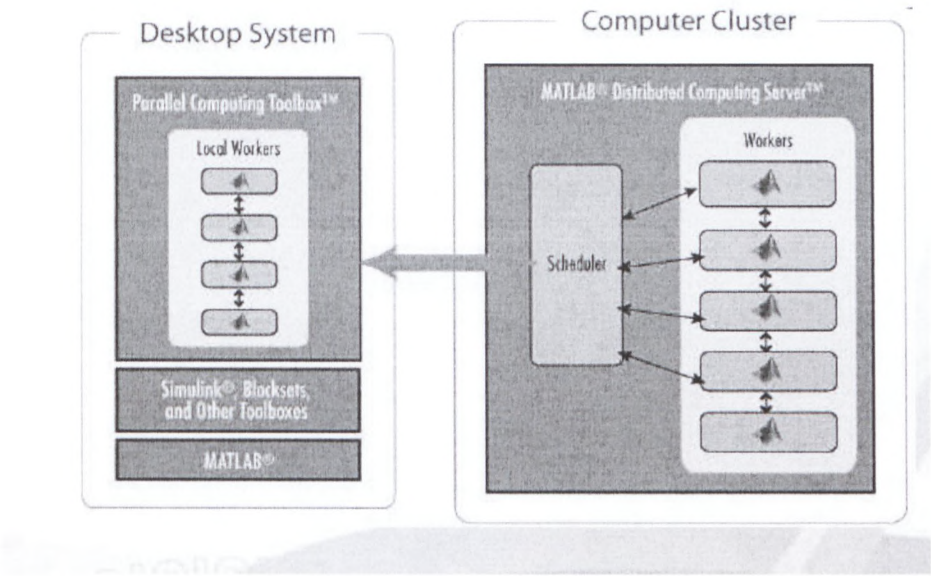


Figure 9.5: Configuration of PCT/MDCS

9.7 Comparison Analysis

Comparison analysis on using DCT/MDCE and PCT/MDCS is shown in Table 9.3

Table 9.3: Comparison analysis

DCT/MDCE	PCT/MDCS
MATLAB sessions, called workers, perform the task but do not communicate with each other.	MATLAB sessions, called labs, can communicate with each other during the running of their tasks
It is possible to define any number of tasks in a job	Only one task in a job is defined. Duplicates of that task run on all labs running the parallel job
Tasks need not run simultaneously. Tasks are distributed to workers as the workers become available, so a worker can perform several of the task in a job	Tasks run simultaneously, it is possible to run the job on as many labs as are available at run time. The start of the job might be delayed until the required number of labs is available

PCT/MDCS is suited for data-parallel and task-parallel application development. This thesis is working on power system state estimation problem (PSSE). PSSE problem involves data in order to compute the best estimate of the system. Therefore, use of PCT/MDCS is appropriate to the PSSE problem in particular when large and interconnected systems are involved.

Decomposition-coordination method and algorithm discussed in chapter 8 enhances the application of PCT/MDCS. The sub problems resulted from decomposition of the global problem can be assigned to the labs and solved in parallel. The application of PCT/MDCS will speed up computation by using multiple processors, utilize more memory than available on single machine and computation time may be reduced; hence, improves computation process of the estimation process. In this thesis PCT/MDCS is adopted and used in solving the state estimation problem.

9.8 PCT/MDCS and State Estimation Problem

To apply PCT/MDCS in solving power system state estimation problem needs the following steps:

- Programming Parallel Applications in MATLAB
- Implementing Task-Parallel Algorithms
- Implementing Data-Parallel Algorithms
- Scaling to a Cluster Using MATLAB Distributed Computing Server (MDCS)

9.8.1 Scaling to cluster using MDCS

The following are the steps which are applied in solving the state estimation problem using parallel processing cluster:

- Install parallel computing toolbox(PCT)
- Verify network connection
- Define a user configuration
- Run parallel MATLAB job

The four steps are critical for successful application of parallel processing of state estimation problem. The steps are described in detail in the following sub section.

Install parallel Computing Toolbox (PCT)

The installation instruction for PCT for window users can be found in the following PDF file: PCT windows installation instructions.

Verify the network connection

To verify the network connection, go to the directory where PCT has been installed by using: *cdmatlabroot\toolbox\distcom\bin*

matlabroot is the directory where MATLAB is installed e.g. *"C\ProgramFile\MATLAB\R2009b"*

Open a command window and run *nodestatus* to verify configuration and connection

Define a user configuration:

- Start MATLAB, then go to parallel->Manage Configuration
- Select file->New->Compute Cluster Server (ccs)
- Fill out the form shown in Figure 9.8 after the above action
    - Enter name for the configuration
    - Enter root directory of MATLAB installation for workers
    - Enter number of workers available to scheduler
    - Enter directory where job data is stored
    - Enter computer cluster scheduler host name

After the configuration, click the “start validation” button at the bottom right of Figure 9.7 to validate the configuration. It is advised to use the account used to log onto the compile node. If the configuration and authentication are correct, Figure 9.8 will show that all the tests are succeeded by indicating a green mark.

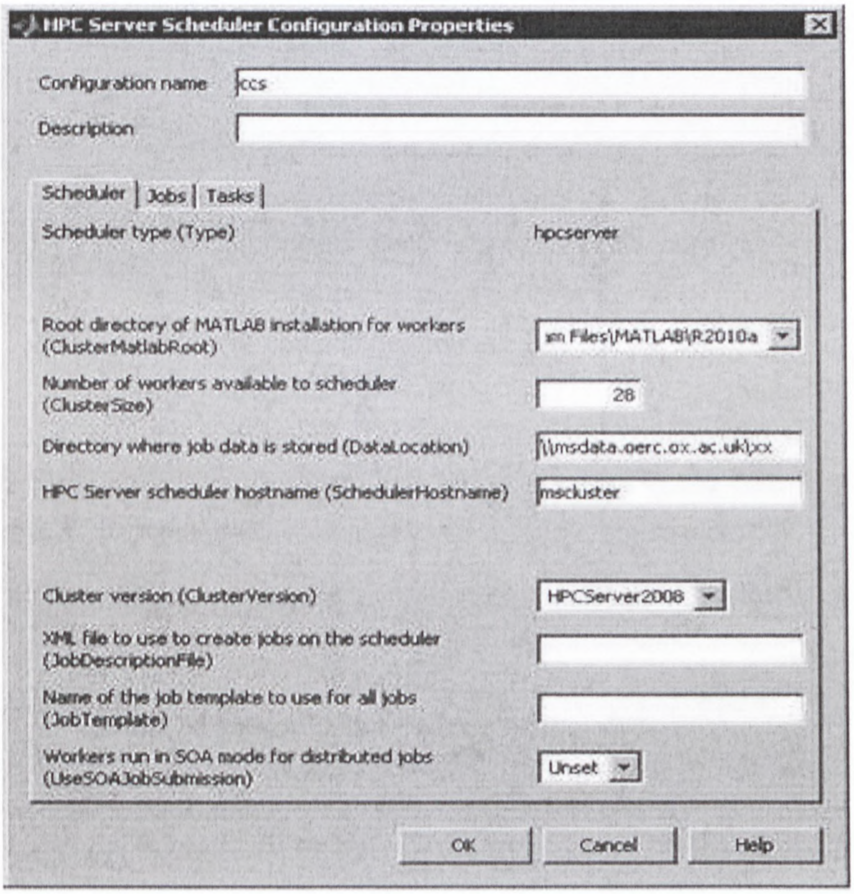


Figure 9.6: Job Manager Property window

(courtesy fromwww.oerc.ox.ac.uk/resources)

To enable the parallel language features of MATLAB be applicable, matlabpool command is used in parallelizing scripts or functions i.e. *Matlabpool open* <configuration>

<Configuration> is replaced with the name of configuration already defined is step one. To switch off parallel language features and use the local client instead, 'matlabpool' command is closed using "*matlabpool close*"

Run Parallel MATLAB jobs

Usually, parallel jobs are executed simultaneously on every worker/lab, this means every task of a job (i.e. state estimation) is running on each worker/lab at the same time. To make the workers communicate each other, message-passing facility is used. However, the facility is automatically accommodated in the PCT. Therefore, to run the script prepared in step tthree the following MATLAB job submission scripts is prepared.

```
% Create a local scheduler object
    sched = findResource('scheduler','Configuration','ccs');

% Create a parallel job object
    pjob = createParallelJob(sched);

% Set the maximum number of workers
% the maximum and default is 8
% this is the number of labs returned by
% the 'numlabs' command
    set(pjob,'MaximumNumberOfWorkers',3);
    set(pjob,'MinimumNumberOfWorkers',2);

% Create a task and assign it to the job
% in this case, this is 'paralleSE'
% it has two outputs
    createTask(pjob,@paralleSE,2,{});

% Submit the job to the local scheduler
    submit(pjob);

% Wait for the job to finish
    waitForState(pjob);

% Display all output parameters generated
% by @parallelSE
    results = getAllArguments(pjob);

% Destroy the job object
    destroy(pjob);
```

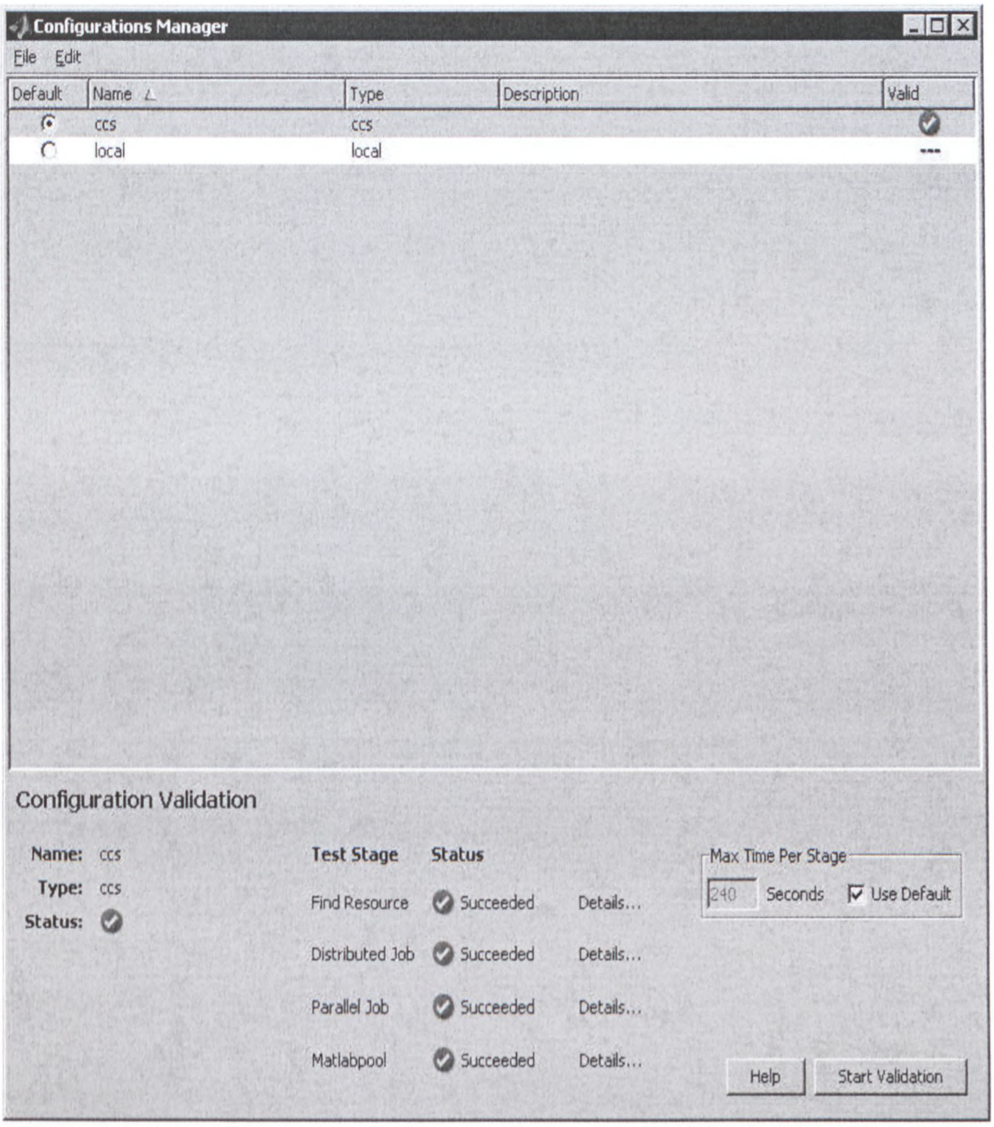


Figure 9.7: Validation set up (courtesy fromwww.oerc.ox.ac.uk/resources)

9.8.2 Programming Parallel Apllications in MATLAB

To start programming parallel applications in MATLAB, first, the tasks of a job have to be well defined. Then, the following steps are implemented:

- Identify a scheduler. findResource is used to identify a local scheduler and create the object sched, which represent the scheduler.

```
sched = findResource('scheduler', 'type','local')
```

- Create a job. Create job (*j*) on the scheduler

```
j = createJob(sched)
```

- Create task within the job (*j*)

- Submit the job to the scheduler queue for evaluation

  ```
  submit(job)
  ```

- Wait for the job to complete, then get the results from all the tasks of the job

  ```
  waitForState(job)
  results = getAllOutputArgument(job)
  ```

- Destroy the job.

  ```
  destroy(job)
  ```

Operations timing

It is important to time all operations separately allowing inspection of time taken in execution in detail. Detailed timings are essential in understanding where the time is spent, and to isolate the potential bottlenecks. All operations have to be written correctly and in vectorized task creation manner; *tic* and *toc* are used for measuring the elapsed time of all the operations instead of using the job and task properties *CreateTime*, *StartTime*, *FinishTime, etc*. The following activities are measured.

- **Job creation time**: the time taken in creating a job. For the job manager, this time involves a remote call, and the job manager allocates space in its database.
- **Task creation time:** the time taken in creating and saving the task information. The job manager saves this in its database.
- **Job submission time:** the time taken in submitting the job. For the job manager, this is the time to start executing the job it has in its database.
- **Job wait time:** this is the time for waiting after the job submission until job completion. It includes all activities that are taking place between job submissions and when the job has completes, such as: scheduler may need to start all workers and sending to the workers the task information; the workers reads the task information, and start executing the task function. In the case of the job manager, the workers then send the task results to the job manager, which in turn the job manager writes them to its database.
- **Task execution time:** the time spent in executing the task. The task function is instructed to accurately measure this task. This time is also included in the job wait time.
- **Result retrieval time:** the time taken in bringing the job results into the MATLAB client. For the job manager, the time is obtained from its database.
- **Job destruction time:** the time taken in deleting all the job and task information. The job manager deletes it from its database.
- **Total time:** the time taken in performing all the activities.

Operations timing is performed after the MATLAB client is checked and verified that is configured to the cluster configuration; if the client is not configured, then, first, it is configured using:

```
        out = findResource();
        out = findResource('scheduler',...'Configuration',...
    'ConfigurationName')
```

'**out**': Object or array of objects returned

'**Configuration**': Literal string to indicate usage of configuration

'**ConfigurationName**': Name of configuration to use

'**scheduler**': Literal string specifying how to find a scheduler, which can be a jobmanager.

Full timing of all activities is presented as:

```
function[times,description] = timeJob(sched,numTask)
% Measure how long it takes to create a job
timingStart = tic;
start = tic;
job = createJob(sched);
times.jobCreateTime = toc(start);
description.jobCreateTime = 'Job creation time';

% Create all the tasks in one call to createTask, and..
% measure how long that takes
start = tic;
times.taskCreateTime = toc(start);
description.taskCreateTime = 'Task creation time';

% Measure how long it takes to submit the job to the cluster
start = tic;
submit(job);
times.submitTime = toc(start);
description.submitTime = 'Job submission time';

% Once the job has been submitted, it is anticipated that...
% its task executes in parallel. Measure how long it takes...
% to start and run to completion
start = tic;
wait(job);
times.jobWaitTime = toc(start);
description.jobWaitTime = 'Job wait time';

% Tasks have been completed, measure how long it takes to ...
% retrieve all the job results
start = tic;
results = getAllOutputArguments(job);
times.resultsTime = toc(start);
description.resultsTime = 'Result retrieval time';

% Verify that the job ran without any error
if if ~all(cellfun(@isempty ,get(job.Tasks,{'ErrorMessage'})))
        taskErrorMsgs = script1_helper_getUniqueErrors(job);
        destroy(job);
        error('script1:JobErrored',...
            ['The following error(s) occured during task'...
            'execution: n n%s'],taskErrorMsgs;
    end
```

```
% Get the execution time of tasks. The task function...
% returns execution time as its second output argument
times.exeTime = max([results{:,2}]);
description.exeTime = 'Task execution time';


% Measure how long it takes to destroy the job and...
% all its tasks
start = tic;
destroy(job);
times.destroyTime = toc(start);
description.destrotTime = 'Job destruction time';

% Measure the total time elapsed from creating the job...
% up to destroying it
times.totalTime = toc(timingStart);
description.totalTime = 'Total time';

times.numTasks = numTasks;
description.numTasks = 'Number of tasks';
end
```

9.8.3 Implementing Parallel -Task Algorithm

The parallel problems are organized into independent tasks, and then implementing task-parallel-algorithm is started. Parallel for-loops (*parfor*) in the PCT are responsible for the organization. It offers one way to distribute tasks across a multiple MATLAB labs. It automatically distributes independent loop iteration to multiple MATLAB labs. Parallel for-loops (*parfor*) construct and manage data and code transfer between MATLAB client sessions and the labs. Parallel for-loops automatically detect the presence of labs and can revert to sequential execution if there is no lab.

9.8.4 Implementing Data-Parallel Algorithm

Power system state estimation problem involves data to calculate the best estimates of the system. For a large, interconnected power systems data are enormous. PCT provides distributed arrays, parallel functions and is able to comment sections of the code using single program multiple data (*spmd*) for parallel execution on several labs. These parallel construct handle the inter-lab communication and coordinate parallel computations. In this way, it facilitates implementation of data-parallel algorithm, which is critical to the solution of state estimation problem.

9.9 Parallel Processing of multi-area state estimation

Applying parallel processing in solving multi-area state estimation problem involves the following steps:

- Decomposing a system into sub-systems
- Developing hierarchical computing structure i.e. first level and second level
- Preparing first and second level algorithms
- Preparing MATLAB program and debbuging
- Scaling to cluster using MDCS
- Implementing parallel state estimation processing

9.10 Implementing parallel state estimation processing

Run MATLAB script as a batch job

j = batch ('aScript')

Argument

j The MATLAB pool job object

'aScript'. The script of MATLAB code to be evaluated by the MATLAB pool job

j = batch ('aScript) runs the script on a worker according to the scheduler defined in the default parallel configuration. The function returns **j**, a handle to the job object that runs the script. Usually the scrip file is added to the FileDependencies and copied to the worker. The following terminologies are used in running MATLAB script as a batch job.

- '**Configuration**'- A single string that is the name of a parallel configuration to be use and find the correct cluster. By default it is the string normally returned from defaultParallelConfig.

  The configuration is specified using: batch (...'Configuration', defaultParallelConfig)
- '**PathDependencies**'-A string or cell array of strings that defines paths to be added to the worker's (lab's) MATLAB path before the script is executed.
- '**FileDependencies**'-A string or cell array of strings. Each string in the list identifies either a file or a directory, which is transferred to the worker.
- '**CurrentDirectory**'-A string indicating in what folder the script executes. The default value for this property is the cwd of MATLAB when the batch command is executed, unless any FileDependencies are defined.
- '**CaptureDiary**'-A Boolean flag to indicate that the toolbox should collect the diary from the function call.

- '**Matlabpool**'-A positive scalar integer that defined the number of labs to make into a MATLAB pool for the job to run in addition to the worker running in the batch script. The script uses the pool for execution of statements such as *parfor* and *spmd.*

Batch script can be run on a worker without using a MATLAB pool, on MATLAB pool job on a remote cluster, using 8 workers for the MATLAB pool in addition to the worker running the batch script. Therefore, this job requires a total of 9 workers, and on a local worker, which employ two other local workers. The three options can be defined as:

j = batch ('script1', 'matlabpool', 0);

j = batch ('script1', 'matlabpool', 8);

j = batch ('script1', 'Configuration', 'local', 'matlabpool', 2);

MATLAB pool

MATLAB pool enables the parallel language features in the MATLAB language such as *parfor* by starting a parallel job that connects the MATLAB client with a number of labs (workers). MATLAB pool or MATLAB pool **open** starts a worker (lab) pool using the default parallel configuration, with the pool size specified by that configuration. The pool size can be specified using matlabpool **open** poolsize, but most schedulers have a maximum number of processes that they can start. If the configuration specifies a job manager as the scheduler, matlabpool reserves its workers (labs) from among those already running and available under the job manager. If the configuration specifies a third-party scheduler, matlabpool instructs the scheduler to start the worker.

Matlabpool **open** configname or matlabpool **open** configname poolsize starts a worker pool using the Parallel Computing Toolbox (PCT) user configuration identified by configname rather than the default configuration to locate a scheduler. If the pool size is specified, matlabpool overrides the maximum and minimum number of workers specified in the configuration and starts a pool of exactly that number of workers, even if it has to wait for them to be available. Without specifying **open** or **close**, the command default is **open**. Matlabpool **close** stops the worker pool, destroy the parallel job and makes all parallel language features revert to using the MATLAB client.

The following terminologies are used when operating matlabpool in parallel computing:

- Matlabpool **size** returns the size of the workerpool if it is open, or 0 if the pool is closed
- Matlabpool ('**open**'... '**FileDependencies**', filecell) starts a worker (lab) pool and allows to specifyfile dependencies so that necessary files can be passed to the workers in the pool. The cell array filecell is appended to the FileDependencies

specified in the configuration used for startup. The 'FileDependemcies' property is case sensitive and must appear as shown.

- Matlabpool ('**addfiledependencies**', filecell) allows adding extra file dependencies to an already running pool. filecell is a cell array of strings identical in form to those used when adding file dependencies to a job or when opening a matlabpool. Each string can specify either absolute or relative files, directories or a file on the MATLAB path. The command transfers the files to each worker (lab), placing the files in the dependencies directory.
- Matlabpool **updatefiledependencies** checks all the file dependencies of the current pool to see if they can be changed and replicates any changes to each of the labs in the pool. In this way code changes can be sent to remote labs.
- Matlabpool **close force** destroys all parallel jobs created by matlabpool for the current user under the scheduler specified by the default configuration
- Matlabpool **close force** configname destroys all parallel jobs being run under the scheduler specified in the configuration configname.

The parallel computing toolbox (PCT) enables to execute MATLAB programs on a cluster. A decision how to divide a large collection of MATLAB operations into smaller work units, called tasks, which the workers (labs) in the cluster can execute, has to be planned. One of the important advantages of PCT is that it builds very well on top of existing sequential code. Then, the focus is on developing an efficient sequential MATLAB code during the algorithm development, debugging and performance evaluation stages. During the development of sequential code, the computations from the pre-and post processing are separated ready to make the core of the computations as simple and independent from the rest of the code and afterward the computations are distributed.

On how to divide the computations into smaller tasks, attention is paid to:

- The number of function calls to be make
- The time it takes to execute each function call
- The number of workers that to be used in the cluster.

Implementation of the proposed algorithm and MATLAB program is given in chapter 10.

9.11 Conclusion

The chapter has presented the concept of parallel and distributed processing in solving a large power system optimization problem. Parallel and distributed processing systems are reviewed and presented. Advantages and disadvantages of the two systems basing on cost, scalability, flexibility and configurability, exploitation of special hardware and technical comparison is made. Both parallel and distributed processing is based on concurrency principle. A distinction between the two processing method is that: parallel processing employs a number of closely coupled processors, while distributed processing employs a number of loosely coupled geographically distributed computers. In addition, the distinction of the two also depends on how system components are physically organized.

The parallel computing toolbox (PCT) and the MATLAB distributed computing server (MDCS) are important for parallel processing. They give possibilities to coordinate and execute independent MATLAB operations on a cluster of computers. The chapter has presented in detail the basic distributed processing configuration, components represented in the client, and life cycle of a job.

To employ parallel for the solution of power system optimization problem, in particular state estimation (SE), a program that fulfills the requirement of the parallel computing toolbox (PCT) has to be developed. The chapter has presented a program for parallel processing method .The programs together with the WLAV state estimation code are used in solving the problem. The programs form the basis for parallel processing simulation of chapter 10.

Chapter 10 presents simulation results from load flow, WLS state estimation, WLAV state estimation for the integrated Tanzanian Power System Network Mode as well as State estimation results obtained by using a parallel processing method under decomposed model.

CHAPTER TEN: SIMULATION AND NUMERICAL RESULTS

10.1 Introduction

This chapter presents load-flow and state estimation simulation procedures and results. First, a load-flow program is run and the outputs from the simulation are used as input to the state estimation simulation. A MATLAB program developed using Newton-Raphson algorithm is employed in running the load flow simulation.

Next, MATLAB programs coded using WLS and WLAV state estimation criteria are used in the simulation using integrated network model of chapter 5. Finally, a MATLAB program developed based on the parallel processing method is used in the simulation using the decomposed model of chapter 5.

10.2 Computational results

10.2.1 Load-Flow (Integrated Model)

The purpose of implementing the load flow analysis is:

- To validate the built approximate Tanzanian Power System Network Model and the data collected from Tanzania Electric Supply Company Limited (TANESCO).
- To evaluate the performance of the improved MATLAB program
- To obtain bus injections (MW and MVAr), line flows (MW and MVAr) and line loading. TANESCO was not eager to provide these data which are essential input for power system state estimation studies.

Initial values are set at:

- 1.00 per-unit for all voltage-controlled and load demand buses
- 00.00 (Phase angle) for all buses

Data and information on the IEEE standard power systems and the 30- bus (Tanzanian Model) used are given as follows:

Ward Hale 6 bus system [Mariesa, 2003] represents a simple test system. The test system is shown in appendix A2. The test system consists of the following components:

- 2 Generators
- 7 Transmission lines
- 5 Demand centres
- 2 Transformers

Linedata and busdata for Ward Hale 6 bus system are given in appendices B4 and B5, respectively.

IEEE 14 bus test system [http://www.washington.edu.research/pstca] represents a portion of the American Electric Power System (AEP). The test system was taken form the University of Washington Power System Test System Archive. IEEE 14 bus test system is shown in appendix A3. The test system consists of the following components:

- 4 Generators
- 20 Transmission line
- 11 Load Centres
- 3 Transformers
- Does not have transmission line limits
- It had low base voltage
- Presented in ieee14cdf.txt

The linedata and busdata for IEEE 14 test system are presented in appendices B6 and B7, respectively.

IEEE 30 bus test system represents a portion of AEP, also taken from the University of Washington Power System Test System Archive. IEEE 30 bus test system is shown in appendix A4. The system consists of the following components:

- 6 Generators
- 41 Transmission lines
- 21 Load Centres
- 4 Transformers
- Total installed capacity of 435 MW
- Has maximum power demand of 283.40 MW
- Represents a portion of the American Electric Power System (in Midwestern US) as of December, 1961
- Does not have transmission line limit
- It had low base voltage
- Presented in ieee30cdf.txt

Linedata and busdata for IEEE 30 test system are presented in appendices B8 and B9, respectively.

30- Bus systems represent an approximate Tanzanian Power System Network Model. The approximate model is shown in appendix A5. The model consists of the following components:

- 13 Generators
- 33 Transmission lines
- 17 Load centres
- 6 Transformers.
- Approximate total installed capacity of 1.053.05 MW
- Has maximum power demand of more than 600 MW
- Represents high voltage transmission system comprising of 220, 132 and 66 kV
- Does not have transmission line limits (not disclosed by TANESCO)

Linedata and busdata of the system are presented in appendices B1 and B2, respectively.

Input Data: Input data were prepared according to IEEE Common Data Format found in the paper *"Common Data Format for the Exchange of Solved Load Flow Data", Working group on Transactions on Power Apparatus and Systems, Vol. PAS-92, No.6, November/December 1973, pp. 1916-1925.* For the 30 Bus systems, additional input data are provided in appendices B3, B4 and B5, respectively

Hardware: The modified and improved MATLAB program shown in appendix C3 was tested on a computer with CPU Pentium IV, 3.33 GHz and 0.99GB of RAM.

Computational Result: IEEE Standard power systems14, 30 and 30-bus Tanzanian Power System Model were used in implementing the simulation. Obtained computational results comprises of:

- Accepted voltage operating limits (Table 10.1).
- Voltage magnitude and phase angle profiles for 30 Bus in tabular form (Table 10.2).
- Voltage magnitude and phase angle profiles for 30 Bus in graphical form (Figure 10.1).
- Voltage magnitude and phase angle profiles for 30 Bus in kV and degrees (Table 10.3).
- Summary of load-flow results (Table 10.4)
- Comparison of voltage magnitude profiles (Figure 10.2)

Table 10.1: Accepted voltage level operating limit

Voltage Level [kV]	Operating Limit		Operating Limit	
	Lower [p.u.]	Upper [p.u.]	Lower [kV]	Upper [kV]
66	0.95	1.10	62.70	72.60
132	0.95	1.10	125.40	145.20
220	0.95	1.10	209.00	242.00

Table 10.2: Voltage magnitude and Angle profiles: 30-Bus

Bus No.	Voltage		Bus No.	Voltage	
	V_m [p.u.]	Angle [degree]		V_m [p.u.]	Angle [degree]
1	1.000	00.000	**16**	0.963	11.595
2	1.003	02.559	**17**	0.997	04.743
3	1.006	00.373	**18**	1.000	04.761
4	1.000	00.370	**19**	1.000	-03.155
5	1.000	11.197	**20**	1.010	-02.913
6	1.004	12.328	**21**	1.011	-02.843
7	0.991	17.351	**22**	0.982	-01.832
8	1.007	13.749	**23**	0.993	00.330
9	1.000	24.783	**24**	1.000	02.016
10	1.000	30.183	**25**	0.999	-08.438
11	0.988	30.148	**26**	0.996	-10.413
12	1.009	21.200	**27**	1.000	-10.309
13	1.000	25.460	**28**	0.971	-14.481
14	1.000	26.367	**29**	1.010	-08.929
15	0.994	26.013	**30**	1.022	-10.415

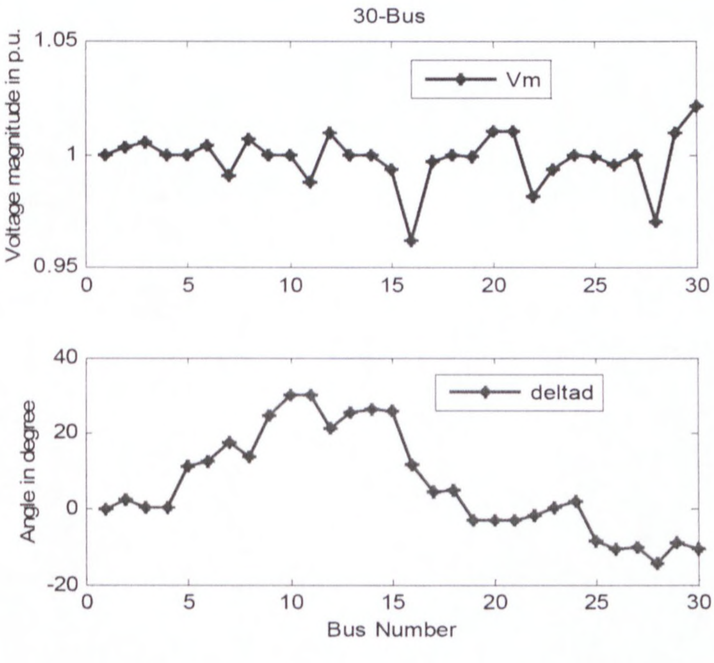


Figure 10.1: Voltage magnitude and Phase angle profile: 30- bus

Table 10.3: Voltage magnitude and Angle profiles in kV and degree: 30- Bus

Bus No.	Voltage Mag. [kV]	Angle [Degree]	Bus No.	Voltage Mag. [kV]	Angle [Degree]	Bus No.	Voltage Mag. [kV]	Angle [Degree]
1	217.78	00.000	**11**	130.20	30.148	**21**	218.88	-02.843
2	217.73	02.559	**12**	133.94	21.200	**22**	214.92	-01.832
3	218.06	00.373	**13**	133.94	25.460	**23**	215.62	00.330
4	216.77	00.370	**14**	133.89	26.367	**24**	214.28	02.016
5	218.80	11.197	**15**	133.07	26.013	**25**	223.15	-08.438
6	219.87	12.328	**16**	129.65	11.595	**26**	220.37	-10.413
7	131.18	17.351	**17**	66.50	04.743	**27**	131.83	-10.309
8	220.46	13.749	**18**	66.68	04.761	**28**	126.60	-14.481
9	132.35	24.783	**19**	65.31	-03.155	**29**	138.20	-08.929
10	130.57	30.183	**20**	218.90	-02.913	**30**	139.70	-10.415

Table 10.4: Summary of load flow results

Test System	IEEE14	IEEE30	30- Bus
Number of Line	20	41	33
Accuracy	0.0001	0.0001	0.0001
Trans. Tap setting	FIXED	FIXED	FIXED
Maximum Power Mismatch	$1.15x10^{-8}$	$1.2x10^{-8}$	$7.17x10^{-7}$
Number of iterations	10	9	4
CPU Time (Seconds)	2.16	3.46	3.16
Injected MVAr	0.00	0.00	0.00
Total System Loss (MW)	13.483	17.730	28.041
Total System Loss(MVAr)	31.618	23.611	-171.028

Table 10.5: Operating power factor (pf): 30- Bus

Bus No.	Power Facto (pf)	Bus No.	Power Factor (pf)	Bus No.	Power Factor (pf)
1	00.000	**11**	0.8790	**21**	0.9721
2	0.9973	**12**	0.9291	**22**	0.9502
3	0.9996	**13**	0.8972	**23**	0.9658
4	0.9995	**14**	0.8904	**24**	0.9070
5	0.9766	**15**	0.8931	**25**	0.8812
6	0.9721	**16**	0.9831	**26**	0.8790
7	0.9502	**17**	0.9973	**27**	0.9291
8	0.9658	**18**	0.9995	**28**	0.8972
9	0.9070	**19**	0.9995	**29**	0.8904
10	0.8812	**20**	0.9766	**30**	0.8931

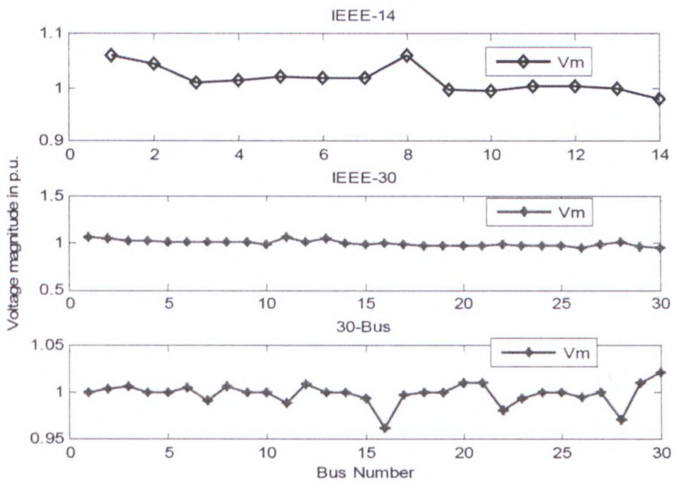


Figure 10.2: Comparison of Voltage magnitude profiles

Discussion:

- Computational results from IEEE standard power systems 14 and 30 compares favourably with the existing values from the archive.
- The voltage magnitude and phase angle profiles shown in Figure 10.1 and Table 10.2 reflect the steady-state acceptable value of the Tanzanian Electric Supply Company Limited (TANESCO). The voltage magnitudes in per unit at each bus is within the operating limit of 0.95 – 1.10. The power factor (pf) ranges between 0.864 and 0.910, which is the operating power factor (pf) of the system.
- The maximum power mismatch obtained is below the targeted value for both IEEE standard power systems as well as for the 30 Bus Tanzanian Power System Model.

The computational results demonstrate that; the selection of system components and the modelling process was appropriate and accurate. Also it shows that the high voltage transmission system at the time of simulation has the required nominal capacity to meet the present demand. Hence, the obtained computational results in particular bus injections (MW and MVAr) at all buses and line flows (MW and MVAr) from the 30 bus systems can be used for power system state estimation studies.

As conclusion: The improved MATLAB program developed using Newton-Raphson method, can be improved and used as an alternative load-flow software to PSS/E software now in application at TANESCO central dispatch centre.

10.2.2 WLS State Estimation (Integrated Model)

The weighted least squares (WLS) and the weighted least absolute value (WLAV) criteria are used to implement the computer simulation. The criteria are different in the objective functions as described in chapter 7 particularly in minimizing the measurement errors. But, the two criteria use Newton-Raphson method for linearization process. The measurement equations used by the criteria are non-linear functions derived from bus voltages and angles.

In both criterion, the measurement Jacobian (**H**) is obtained by taking the derivative of the measurement equations. The calculate measurements and measurement Jacobian (**H**) are used to solve the states based on the equations obtained for minimization of the objective function.

The common part for every criterion (WLS) and (WLAV) lies in calculating the measurements using measurement equation and solving for the measurement Jacobian matrix (**H**). The criterion differ only in the equation and the numerical method used for solving the system state.

The aim of conducting WLS state estimation simulation was:

- To evaluate the performance of the modified and improved version of WLS state estimation MATLAB program.
- To set up benchmark values for voltage magnitude and phase angle of each bus in particular for the Tanzanian Power System Network Model.

The WLS state estimation MATLAB program version was modified and improved on the basis of Deepark Krishnan's version [http://www.mathworks.com/matlab central/file exchange] developed in 2004. The old version is given in appendix C4. Krishnan's program is difficult to implement in large test system. It works well when applied to less than 10 bus test system. However, when applied to systems with more buses like the IEEE 14 bus test system the program took a long time to converge and even made the computer to freeze (stop working). In addition, the program is short of:

- Codes on the bus system. It was difficult to establish which bus is voltage-controlled or demand bus
- Format in preparation of linedata or busdata files.
- Value of MVA base

Thus, failure to solve large test systems, short of clear procedure/format in preparation of linedata and busdata, it was essential to make modifications and improvement on the program by introducing some of missing information. The following MATLAB files (*m*.files) were introduced and added to the program:

- *ybusppg.m* file
- *Bbppg .m* file
- *zdata.m* file
- random error

The modified/improved version of the program is shown in appendix C5. This program is used in the solution of the WLS state estimation. Measurements for the WLS simulation were obtained from Load-Flow results.

Measurements obtained from the remote terminal units (RTU) are not exact value. They are usually erroneous as stated in chapter 7. The errors are due to accuracy limits, bad calibration, aging of the RTU and also communication noise in transmission channels. The exact values of the measurements are obtained from a load- flow solution and these were accepted as true value. Since these measurements are true value, it is necessary to introduce a random error in order to represent as erroneous measurements.

A random error is introduced into the measurements using the following eqn:

$$z_k = A_k(1 + RND \times \sigma_k) \quad (10.1)$$

where

z_k is the kth simulated measured value, $k = 1, 2, m$

A_k is the actual value from the load-flow analysis

σ_k is the standard deviation of the kth measurement

m number of measurements

RND is a random number with normal distribution and zero mean which is between -1 to +1

The accepted values of the Standard deviations proposed in [Kamireddy, 2008; Abur & Celik, 1991] for different types of measurements are given in Table 10.6., and were used to compute the new measurements.

Table 10.6: Standard deviation values

Serial No.	Type of measurements	Value of standard deviation (σ) in per-unit
1	Voltage	0.004
2	Real power injection	0.01
3	Reactive power injection	0.01
4	Real power flow	0.008
5	Reactive power flow	0.008

After introduction of random error a new zdata30.m file was prepared. The file has given in appendix B6 has six columns; namely:

- number of measurements
- type of measurements
- value of measurements basing on MVA base
- From
- To
- R_{ii} (measurement covariance)

Before implementing the WLS state estimation simulation, initial values were set at:

- 1.00 per-units for voltage
- 00.00 degree for phase angle.
- A convergence tolerance was set at 10^{-4}

In the first iteration the measurements were calculated using the initial flat values of voltage and phase angle. After the first iteration, the measurements were calculated using the updated system state. The obtained error was compared against the tolerance in every iteration. The error is given by:

$$e = Maximum\left(absolutevalueof\ \left(H^T \times R^{-1} \times [z - h(x)]\right)\right) \qquad 10.2$$

When the error was less than a predefined tolerance, the iteration was terminated. The algorithm used is given in Figure 10.3

Input Data: Input data were prepared according to IEEE Data format. The linedata and busdata for all test systems are the same as those used in the load-flow simulation. Zdata30 file which include: real and reactive power injection, real and reactive power flows and voltage magnitude injection was an addition.

Hardware: The modified and improved MATLAB program shown in appendix C5 was tested on a computer with CPU Pentium IV, 3.33 GHz and 0.99GB of RAM.

MATLAB Program: A program to compute WLS state estimation solution for 30-bus was used. The program comprises of 9 files:

- SE30- M file (running program)
- LD30-Excel file
- BD30- Excel file
- Ybusppg – M file
- Bbusppg – M file

- Pol2rect – M file
- Rect2pol –M file
- Zdata30 – M file
- WLS- M file (main program)

Computational Result: The efficiency of the program was demonstrated by IEEE standard power systems: 14, 30 and the 30-bus system of the Tanzanian Power System Network Model. Obtained computational results comprises of:

- Voltage magnitude and phase angle profiles for 30- bus in Tabular form presented in Table 10.7
- Voltage magnitude and phase profiles for 30-bus in kV and degrees shown in Table 10.8
- Summary of WLS state estimation results are shown in Table 10.9
- Voltage magnitude measurement errors shown in Table 10.10
- Voltage magnitude and angle profiles for 30 Bus in graphical form presented in Figure 10.4
- Residual distribution for the 30- bus is given in Figure 10.5
- State estimator error distribution is shown in Figure 10.6
- Covariance values are given in graphical form in Figure 10.7

Table 10.7: Voltage magnitude and Angle profiles: 30-Bus

Bus No.	Voltage		Bus No.	Voltage	
	V_m [p.u.]	Angle [degree]		V_m [p.u.]	Angle [degree]
1	00.9614	00.0000	**16**	00.9350	14.4283
2	00.9594	04.2278	**17**	00.9646	09.9308
3	00.9608	01.7868	**18**	00.9675	09.9496
4	00.9548	01.7692	**19**	00.9705	02.4502
5	00.9636	13.1024	**20**	00.9696	02.9531
6	00.9687	14.3279	**21**	00.9697	02.9651
7	00.9579	19.2351	**22**	00.9494	03.4359
8	00.9716	15.8562	**23**	00.9533	05.2061
9	00.9675	26.6922	**24**	00.9450	05.8016
10	00.9545	30.7587	**25**	00.9989	-02.6612
11	00.9511	31.0538	**26**	00.9854	-04.5657
12	00.9775	23.1554	**27**	00.9823	-04.4707
13	00.9715	27.9832	**28**	00.9443	-08.6637
14	009715	28.9300	**29**	1.0458	-02.3242
15	00.9649	28.5558	**30**	1.0568	-03.7123

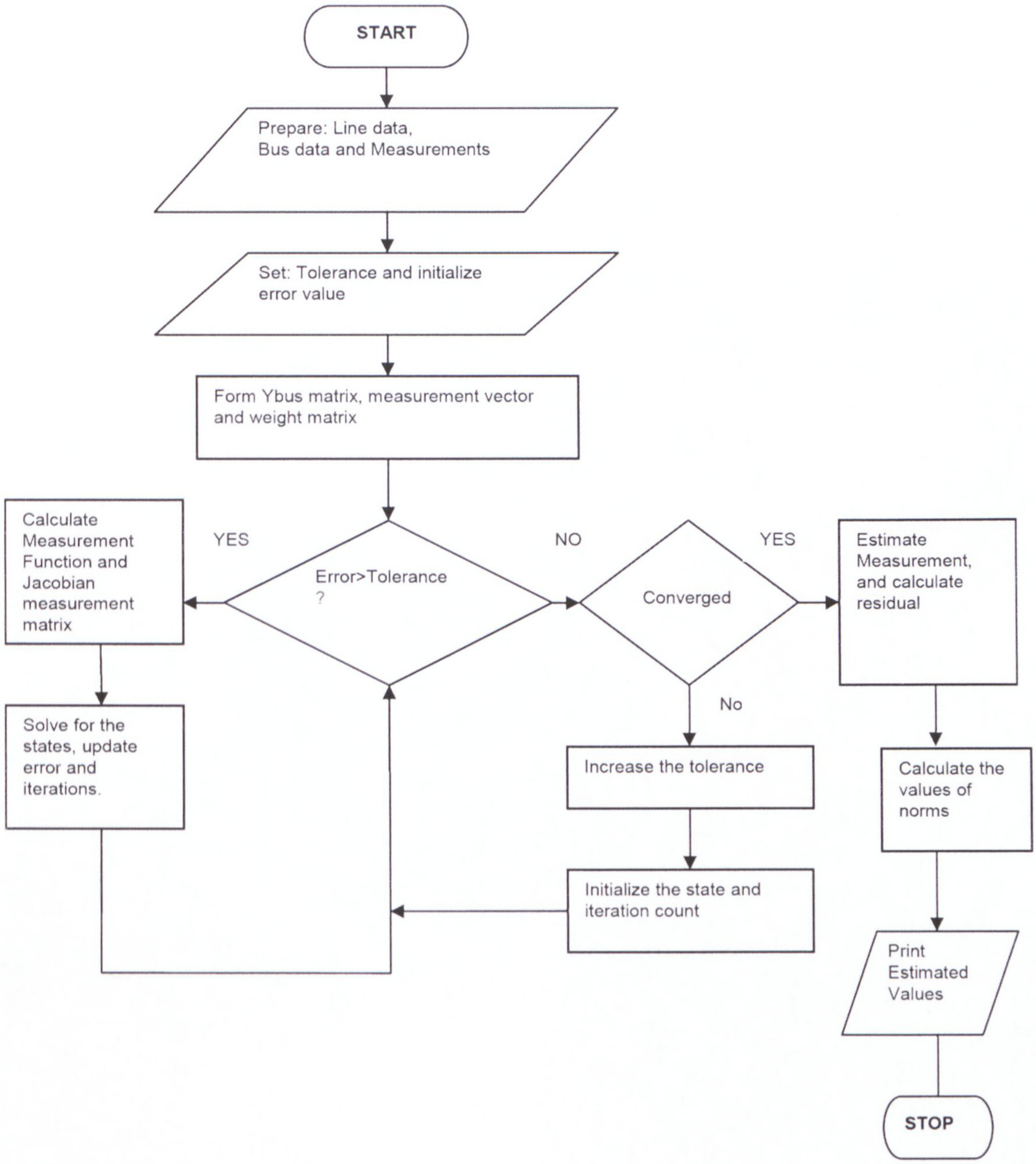


Figure10.3: General flow chart for SE program

Table 10.8: Voltage magnitude and Angle profiles in kV and degrees: 30- Bus

Bus No.	Voltage Mag. [p.u.]	Angle [Degree]	Bus No.	Voltage Mag. [p.u.]	Angle [Degree	Bus No.	Voltage Mag. [p.u.]	Angle [Degree
1	217.78	00.0000	**11**	130.20	31.0786	**21**	218.88	02.9676
2	217.73	04.2311	**12**	133.94	23.1738	**22**	214.92	03.4388
3	218.06	01.7882	**13**	133.90	28.0053	**23**	215.62	05.2101
4	216.77	01.7705	**14**	133.89	28.9528	**24**	214.28	05.8061
5	218.80	13.1128	**15**	133.07	28.5783	**25**	223.15	-02.6634
6	219.87	14.3393	**16**	129.65	14.4396	**26**	220.37	-04.3554
7	131.18	19.2503	**17**	66.50	09.9388	**27**	131.83	-03.5046
8	220.46	15.8688	**18**	66.68	09.9576	**28**	126.60	-03.0548
9	132.35	26.7135	**19**	65.31	02.4523	**29**	138.20	-02.5997
10	130.57	30.7833	**20**	218.90	02.9556	**30**	139.70	-02.3000

Table 10.9: Summary of WLS state estimation results

Test System	IEEE14	IEEE30	30 Bus
Number of Line	20	41	33
Number of Measurements	41	93	97
Redundancy (%)	152	158	164
Accuracy	0.0001	0.0001	0.0001
Number of States	27	59	59
Number of iterations	5	5	5
Tolerance	2.26E-007	9.54E-007	1.60E-005
CPU Time (Seconds)	0.165	0.231	0.274

Table 10-10: Voltage magnitude errors (WLS) - 30-Bus

Bus No.	True Meas. [p.u.]	Estimated Value [p.u.]	Error [p.u.]	Bus No.	True Meas. [p.u.]	Estimated Value [p.u.]	Error [p.u.]
1	01.0000	00.9610	00.0389	**16**	00.9630	00.9347	00.0283
2	01.0030	00.9590	00.0440	**17**	00.9970	00.9643	00.0327
3	01.0060	00.9604	00.0456	**18**	01.0000	00.9671	00.0383
4	01.0000	00.9544	00.0456	**19**	01.0000	00.9601	00.0398
5	01.0000	00.9632	00.0368	**20**	01.0100	00.9692	00.0408
6	01.0040	00.9683	00.0357	**21**	01.0110	00.9693	00.0417
7	00.9910	00.9575	00.0335	**22**	00.9820	00.9490	00.0330
8	01.0070	00.9713	00.0357	**23**	00.9930	00.9529	00.0401
9	01.0000	00.9671	00.0383	**24**	01.0000	00.9446	00.0554
10	01.0000	00.9541	00.0459	**25**	00.9990	00.9985	00.0005
11	00.9880	00.9507	00.0373	**26**	00.9960	00.9857	00.0103
12	01.0090	00.9771	00.0319	**27**	01.0000	00.9828	00.0172
13	01.0000	00.9711	00.0289	**28**	00.9710	00.9792	00.0082
14	01.0000	00.9711	00.0289	**29**	01.0100	01.0452	-00.0352
15	00.994	00.9646	-00.0206	**30**	01.0220	01.0515	-00.0355

Table 10-11: Voltage angle errors (WLS) - 30-Bus

Bus No.	True Meas. [Degree]	Estimated Value [Degree]	Error [Degree.]	Bus No.	True Meas. [Degree]	Estimated Value [Degree]	Error [Degree]
1	00.0000	00.0000	00.0000	**16**	11.5950	14.4396	-02.8446
2	02.5590	04.2311	-01.6721	**17**	04.7430	09.9388	-05.1958
3	00.3730	01.7882	-01.4152	**18**	04.7610	09.9576	-05.2146
4	00.3700	01.7705	-01.4005	**19**	-03.1550	02.4523	-05.6073
5	11.1970	13.1128	-01.9158	**20**	-02.9130	02.9556	-05.8686
6	12.3280	14.3393	-02.0113	**21**	-02.8430	02.9676	-05.8106
7	17.3510	19.2503	-01.8993	**22**	-01.8320	03.4388	-05.2708
8	13.7490	15.8688	-02.1198	**23**	00.3300	05.2101	-04.8801
9	24.7830	26.7135	-01.9305	**24**	02.0160	05.8061	-03.7901
10	30.1830	30.7833	-00.6003	**25**	-08.4380	-02.6634	-05.7746
11	30.1480	31.0786	-00.9306	**26**	-10.4130	-04.3554	-06.0576
12	21.2000	23.1738	-01.9738	**27**	-10.3090	-03.5046	-06.8044
13	25.4600	28.0053	-02.5453	**28**	-14.4810	-03.0548	-11.4262
14	26.3670	28.9528	-02.5858	**29**	-08.9290	-02.5997	-06.3293
15	26.0130	28.5783	-02.5653	**30**	-10.4150	-02.3000	-08.1150

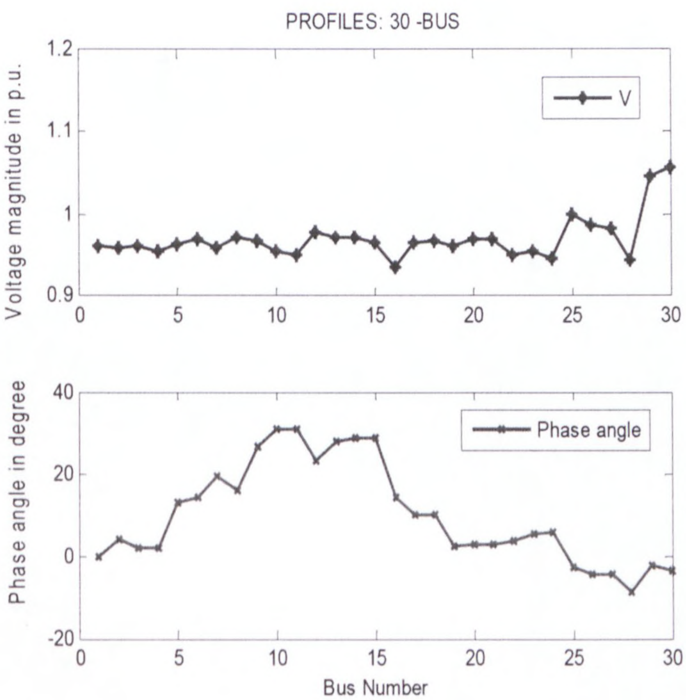


Figure 10.4: Voltage magnitude and Angle profiles: 30-Bus

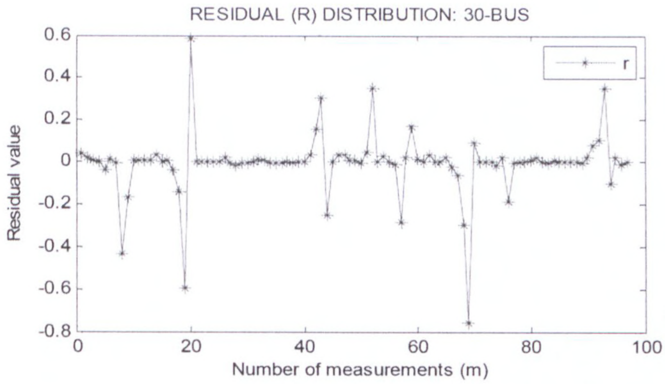


Figure 10.5: Residual distribution for 30-Bus

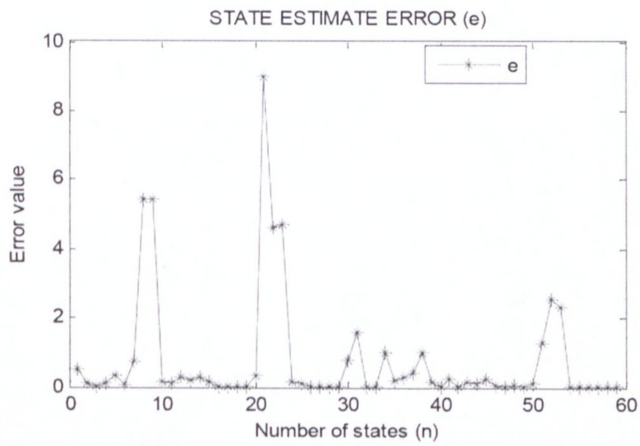


Figure 10.6: State estimate error distribution: 30-Bus

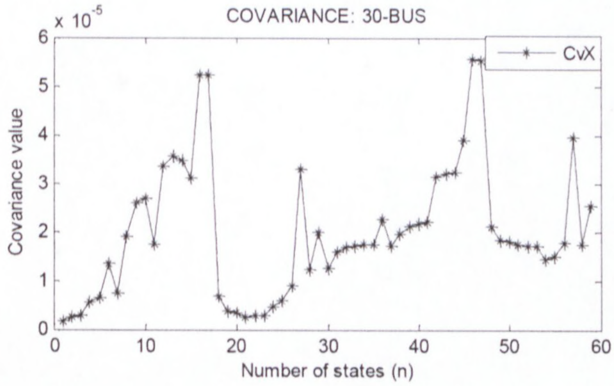


Figure 10.7: Covariance values: 30-Bus

Discussion:

- The voltage magnitude and angle profiles shown in Figure 10.4 and Table 10.7 reflect the steady-state acceptable value of the Tanzanian Electric Supply Company Limited (TANESCO). The voltage magnitudes in per unit at each bus is within the operating limit of 0.95 – 1.10. The power factor (pf) ranges between 0.86 and 0.910, which is the operating power factor (pf) of the system.
- The tolerance is below the targeted value for both IEEE standard power systems as well as for the 30- bus Tanzanian Power System Model.
- All the test system converged after 5 numbers of iterations
- Voltage magnitude and phase angle profiles of the 30-bBus obtained from the WLS state estimation simulation compares favourably with that of the load-flow simulation.
- Time taken for convergence increases as the number of test system buses increases

The computational results obtained from the WLS state estimation demonstrate that; the obtained computational results in particular bus injections (MW and MVAr) at all buses and line flows (MW and MVAr) from the 30- bus systems during load flow analysis reflects the approximate true value of the systems.

As conclusion: The improved WLS state estimation MATLAB program modified and improved can be further improved and used in the state estimation studies of the system. However, in order to make this implementable, the approximate Tanzanian Power System Network Model used in the simulation has to be improved and expanded.

10.2.3 WLAV State Estimation (Integrated Model)

The intention of performing the constrained weighted least absolute value (WLAV) state estimation is:

- To evaluate the efficiency of the algorithm developed in chapter 7.
- To evaluate effectiveness of adjusting the barrier parameter (μ)
- To obtain state vector solution which will be used as bench mark when working with parallel and distributed processing in the next subsection

Input Data: Almost all of the input data are the same as those used for WLS state estimation. Since the constrained WLAV algorithm included inequality constraints in its formulation as stated in chapter 7.

In all test cases, the generator reactive power (VAr) limit is modelled as inequality constraints. Numbers of inequality constraints reflect that every generator bus in the network has both an upper and lower VAr limit. Other form of inequality constrains existing in the network such as: transformer turn ratios and unmeasured load were not used.

MATLAB Program: A MATLAB program modified and improved from that used to compute the WLS state estimation solution for 30 Bus was used. The improved program is shown in appendix C6 and comprises of 10 files, namely:

- WLAV30- M file (running program)
- LD30-Excel file
- BD30- Excel file
- Ybusppg – M file
- Bbusppg – M file
- Pol2rect – M file
- Rect2pol –M file
- Zdata30 – M file
- WLVA - M file (main program)

In order the program to execute the intended task, all the files must be in the current directory. The WLAV30 can be used to run the whole program.

Hardware: MATLAB program shown in appendix C6 was tested on a computer with CPU Pentium IV, 3.33 GHz and 0.99GB of RAM.

Computational Result: It was observed that; when no inequality constraints become active, the WLAV state estimation converges to the WLS estimate. When the inequality constraints are introduced, the constrained WLAV estimation converges with extra iteration. Numbers of iterations was checked whenever the step size was modified to found if they increase or decrease. The iterations are terminated when the relative duality gap calculated using Eqn (7.68) is10^{-6}.

Obtained computational results comprises of:

- Table 10-11 presents voltage magnitude and angle profiles from WLAV state estimation
- Tables 10-12 and 10-13 Voltage magnitude and Voltage angle errors
- Figure 10-8 gives Voltage magnitude and Angle profiles of 30-Bus under WLAV criterion
- Figure 10-9 shows comparison of residual (r) and error (e) distribution
- Figure 10-10 shows distribution of upper (u) and lower (l) slack variables
- Figure 10-11 gives changes of barrier parameter (μ) against vector of Lagrange multipliers when are decreased from higher values to low values for a Ward Hale 6 bus test system.

- Figure 10-12 shows changes of barrier parameter(μ) with vector of Lagrange multipliers for low values of Lagrange multipliers

Table 10-12: Voltage magnitude and Angle profiles (WLAV): 30-Bus

Bus No.	Voltage Magnitude [p.u.]	Voltage Angle [Degree]	Bus No.	Voltage Magnitude [p.u.]	Voltage Angle [Degree]
1	00.9614	00.0000	**16**	00.9350	14.4283
2	00.9594	04.2278	**17**	00.9646	09.9308
3	00.9608	01.7868	**18**	00.9675	09.9496
4	00.9548	01.7692	**19**	00.9605	02.4502
5	00.9636	13.1024	**20**	00.9696	02.9531
6	00.9687	14.3279	**21**	00.9697	02.9651
7	00.9579	19.2351	**22**	00.9494	03.4359
8	00.9716	15.8562	**23**	00.9533	05.2061
9	00.9675	26.6922	**24**	00.9450	05.8016
10	00.9545	30.7587	**25**	00.9989	-02.6612
11	00.9511	31.0538	**26**	00.9854	-04.5657
12	00.9775	23.1554	**27**	00.9823	-04.4707
13	00.9715	27.9832	**28**	00.9443	-8.6637
14	00.9715	28.9300	**29**	01.0458	-02.3242
15	00.9649	28.5558	**30**	01.0568	-03.7123

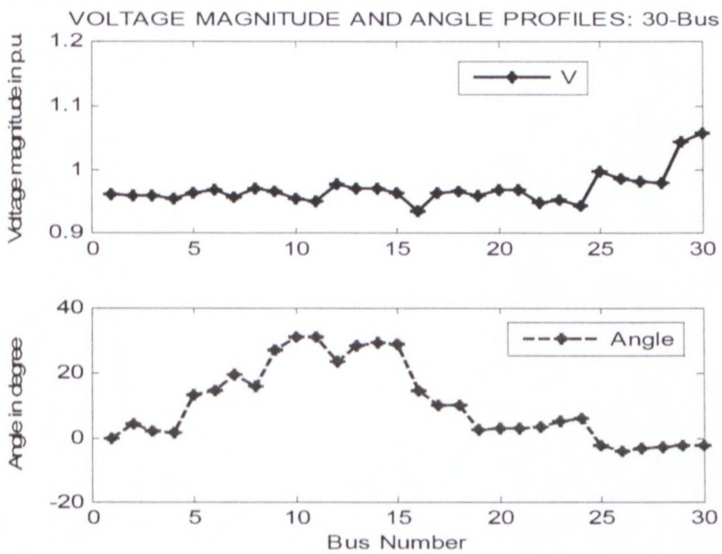


Figure 10-8: Voltage magnitude and Angle profiles- 30 Bus

Table 10-13: Voltage magnitude errors (WLAV) - 30-Bus

Bus No.	True Meas. [p.u.]	Estimated Value [p.u.]	Error [p.u.]	Bus No.	True Meas. [p.u.]	Estimated Value [p.u.]	Error [p.u.]
1	01.0000	00.9614	00.0386	**16**	00.9630	00.9350	00.0280
2	01.0030	00.9594	00.0406	**17**	00.9970	00.9646	00.0324
3	01.0060	00.9608	00.0452	**18**	01.0000	00.9675	00.0325
4	01.0000	00.9548	00.0452	**19**	01.0000	00.9605	00.0395
5	01.0000	00.9636	00.0364	**20**	01.0100	00.9696	00.0404
6	01.0040	00.9687	00.0353	**21**	01.0110	00.9697	00.0413
7	00.9910	00.9579	00.0331	**22**	00.9820	00.9494	00.0326
8	01.0070	00.9716	00.0354	**23**	00.9930	00.9533	00.0397
9	01.0000	00.9675	00.0325	**24**	01.0000	00.9450	00.0550
10	01.0000	00.9545	00.0455	**25**	00.9990	00.9989	00.0010
11	00.9880	00.9511	00.0369	**26**	00.9960	0.9854	00.0106
12	01.0090	00.9775	00.0315	**27**	01.0000	0.9823	00.0177
13	01.0000	00.9715	00.0285	**28**	00.9710	00.9443	00.0267
14	01.0000	00.9715	00.0285	**29**	01.0100	01.0458	-00.0358
15	00.994	00.9649	-00.0209	**30**	01.0220	01.0568	-00.0348

Table 10-14: Voltage angle errors (WLVA) - 30-Bus

Bus No.	True Meas. [Degree]	Estimated Value [Degree]	Error [Degree.]	Bus No.	True Meas. [Degree]	Estimated Value [Degree]	Error [Degree]
1	00.0000	00.0000	00.0000	**16**	11.5950	14.4283	-02.8333
2	02.5590	04.2278	-01.6688	**17**	04.7430	09.9308	-05.1878
3	00.3730	01.7868	-01.4138	**18**	04.7610	09.9496	-05.1886
4	00.3700	01.7692	-01.3992	**19**	-03.1550	02.4502	-05.6052
5	11.1970	13.1024	-01.9054	**20**	-02.9130	02.9531	-05.8440
6	12.3280	14.3279	-01.9999	**21**	-02.8430	02.9651	-05.8081
7	17.3510	19.2351	-01.8805	**22**	-01.8320	03.4359	-05.2679
8	13.7490	15.8562	-02.1072	**23**	00.3300	05.2061	-04.8761
9	24.7830	26.6922	-01.9092	**24**	02.0160	05.8016	-03.7856
10	30.1830	30.7587	-00.5757	**25**	-08.4380	-02.6612	-05.7768
11	30.1480	31.0538	-00.9058	**26**	-10.4130	-04.5657	-05.8473
12	21.2000	23.1554	-01.9554	**27**	-10.3090	-04.4707	-05.8383
13	25.4600	27.9832	-02.5232	**28**	-14.4810	-08.6637	-05.8173
14	26.3670	28.9300	-02.5630	**29**	-08.9290	-02.3242	-06.6048
15	26.0130	28.5558	-02.5428	**30**	-10.4150	-03.7123	-06.7027

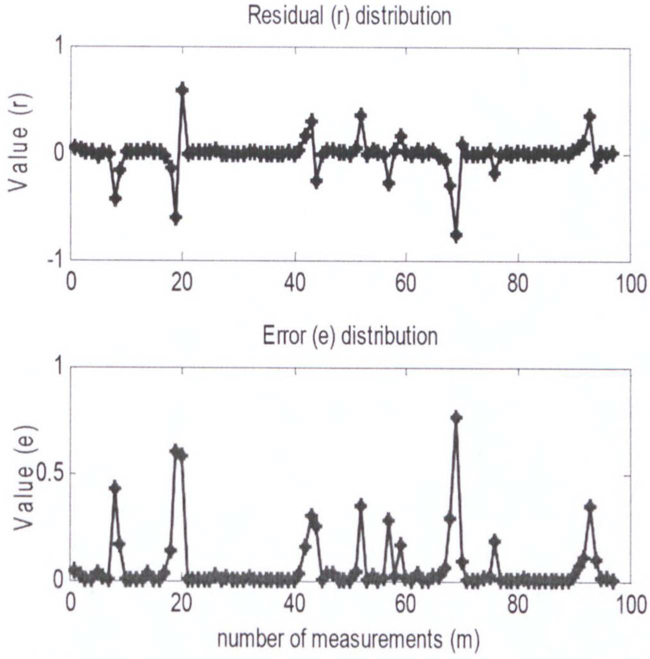


Figure 10-9: Comparison of residual and error distribution

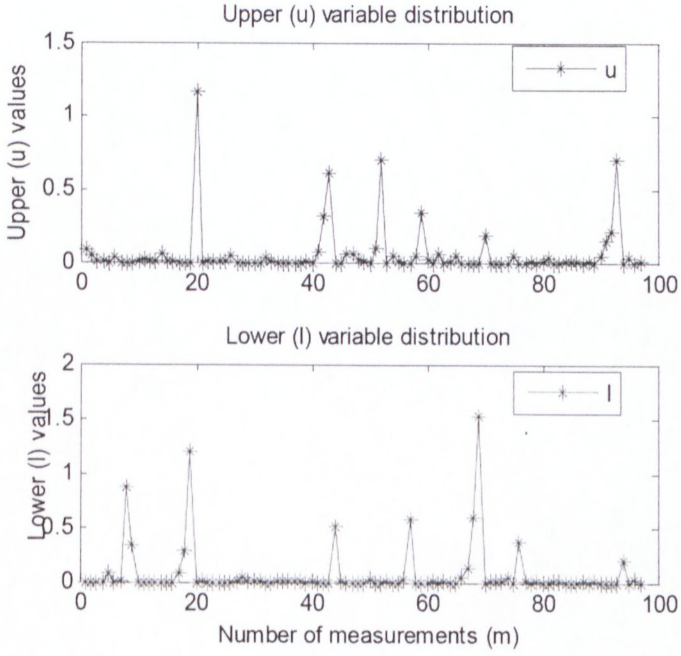


Figure 10-10: Upper (*u*) and lower (*l*) slack variable distribution

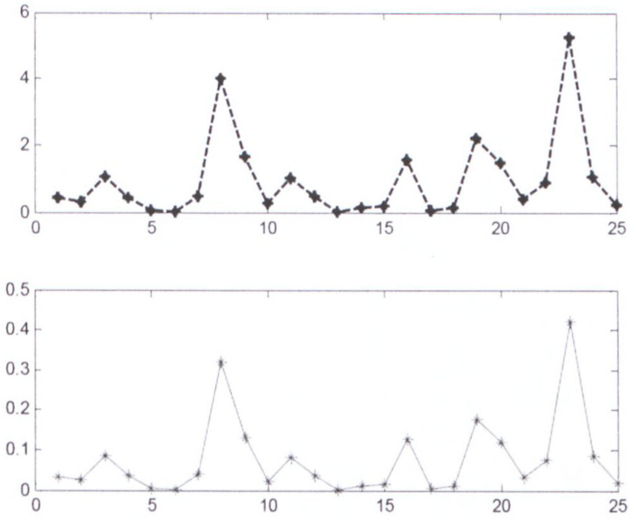


Figure 10-11: Changes of Vector of Lagrange with barrier parameter (μ)-high value

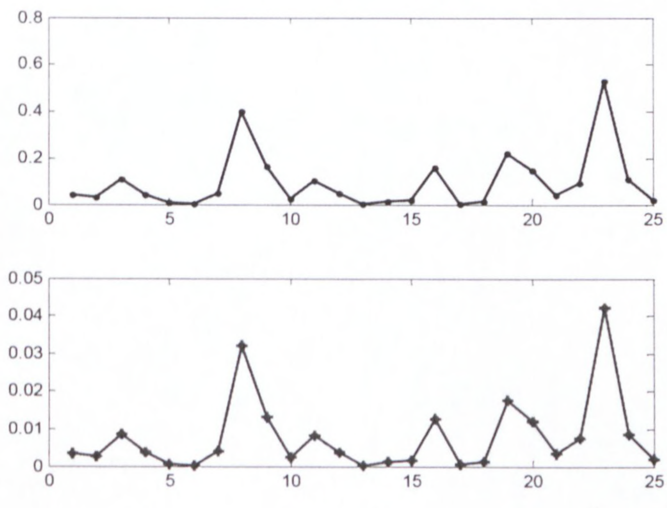


Figure 10-12: Change of Vector of Lagrange multipliers with barrier parameter (μ)-low value

Discussion:

- The voltage magnitude and angle profiles shown in Figure 10.8 and Table 10.12 reflect the steady-state acceptable value of the Tanzanian Electric Supply Company Limited (TANESCO). The voltage magnitudes in per unit at each bus is within the operating limit of 0.95 – 1.10. The power factor (pf) ranges between 0.86 and 0.910, which is the operating power factor (pf) of the system.
- Simulation converged after 6 numbers of iterations; time elapses till convergence is 3.07 second.

- Voltage magnitude and phase angle profiles of the 30-bBus obtained from the WLAV state estimation simulation compares favourably with that of the load-flow simulation. However, the erros for voltage magnitude and voltage angle is smaller compared to that obtained from WLS state estimation
- When obtained voltage magnitude values are converted and check against accepted operating voltage level limits given in Table 10.1, the results are within limits.
- Barrier parameter was obtained using the eqn (7.78) in chapter 7. Care was taken when changing the vectors of Lagrange multipliers given in (7.20). For higher values of Lagrange multiplier, barrier parameter becomes great than 1. Adjusting of complementary gap and barrier parameter was taken in such a way to make sure that the value of barrier parameter is not equal to zero but it approaches zero when the process progress. From the simulation it was observed that when the barrier parameter becomes zero it affects the diagonal matrix D given in Eqn (7.55) which makes the matrix singular. The optimal barrier parameter which gives optimal WLAV results was obtained at low vector of Lagrange multipliers.
- Step length was choosen so that the primal solution remains feasible and is interior to the constraints region. The non-linear constraints were checked to establish if there is violation after the step length is taken. If there is a violation, step length was reduced and a new solution point was re-calculated.
- Computational efficiency strongly depends how the correction equation (7.60) is structured. If the equation has more variables, the method takes more iteration to converge. When variables in the equation are reduced, number of iterations to convergence is reduced. It is therefore necessary to construct the correction equation with few variables i.e. with a reasonable matrix structure.
- The computational experience obtained after running the WLAV state estimation indicates that the number of iterations needed to converge to a solution is comparable to the weighted least squares (WLS) of section 10.2.2.
- It has been observed that WLAV state estimation solution satisfy a subset of (m) measurements exactly while the remaining (m-n) measurements have non-zero residual. In this thesis 38 measurements were identified to have zero residual. Therefore, computational procedure was reduced by terminating the iteration early by only solving the exactly determined system for the solution.

The computational results obtained from the WIAV state estimation demonstrate that; the obtained computational results in particular voltage magnitude and voltage angle at each bus of the system reflects the load flow result which was taken as the true values of the system.

As conclusion: The improved WLAV state estimation MATLAB program used in this thesis has a room for more improvement and used in the state estimation studies of the system. However, in order to make this implementable in future, the approximate Tanzanian Power System Network Model used in the simulation has also to be improved and expanded to include new buses.

10.2.4 Parallel Processing of SE (Decomposd Model)

The Tanzanian Power System Network Model proposed in chapter 5 is decomposed into a set of 3 interconnected sub-systems (Figure 5.3), which overlap each other. State vector of these sub-systems are used in the estimation process. There are 2 advantages to move from central (integrated) state estimation towards parallel and distributed approach:

- Local estimation in sub-systems eliminates the necessity for communication between subsystem and the central estimation unit.
- Local estimators' deals with much lower data (Tables 10.15 to 10. 17) compared to measurement vector used in central (integrated) estimation algorithms. Consequently, computational power requirement decreases with parallel processing estimation method.

Table 10.15: *zdata* for sub-system A11

Meas.	Type	Value	From Bus No.	To Bus No.	Meas.	Type	Value	From Bus No.	To Bus No.
1	1	01.000	1	0	**21**	4	01.082	5	2
2	2	-00.062	2	0	**22**	4	00.109	2	24
3	2	-00.200	3	0	**23**	4	00.131	3	4
4	2	-00.130	4	0	**24**	4	-00.877	5	6
5	2	01.420	5	0	**25**	4	-00.767	6	8
6	2	00.000	7	0	**26**	4	-00.318	7	12
7	2	02.590	9	0	**27**	4	-00.789	9	10
8	2	00.100	10	0	**28**	4	00.270	9	12
9	2	-00.176	11	0	**29**	4	00.178	10	11
10	2	-00.120	12	0	**30**	5	-00.162	2	3
11	3	-00.016	2	0	**31**	5	-0.149	5	2
12	3	-00.070	3	0	**32**	5	-00.51	2	24
13	3	-00.214	4	0	**33**	5	-00.073	3	4
14	3	00.185	5	0	**34**	5	00.640	5	6
15	3	00.000	7	0	**35**	5	00.640	6	8
16	3	00.511	9	0	**36**	5	00.036	7	12
17	3	-00.224	10	0	**37**	5	00.391	9	10
18	3	-00.090	11	0	**38**	5	-00.068	9	12
19	3	-00.025	12	0	**39**	5	00.091	10	11
20	4	00.333	2	3					

Nevertheless, the system is valid if it satisfied the following condition proposed by [Monticelli, A & Wu, F.F., 1985]: Each sub-system must obey the observability principle. i.e. its measurement Jacobian matrix must not have linearly dependent column.

Local measurements (N_S subsystems) are received from sensors (RTU) installed in the subsystem. After system decomposition, each sub-system is identified by its buses according to Tables 5.5 to 5.13. Buses in each sub-system are classified into two categories:

Table 10.16 *zdata* for subsystem A22

Meas.	Type	Value	From Bus No.	To Bus No.	Meas.	Type	Value	From Bus No.	To Bus No.
1	2	-00.120	12	0	**15**	3	-00.050	19	0
2	2	00.105	13	0	**16**	3	-00.012	21	0
3	2	00.680	14	0	**17**	4	00.063	13	15
4	2	-00.210	15	0	**18**	4	00.390	13	16
5	2	-00.231	16	0	**19**	4	00.151	14	15
6	2	00.036	18	0	**20**	4	00.036	18	17
7	2	-00.220	19	0	**21**	4	00.191	17	19
8	2	-00.065	21	0	**22**	4	-00.030	20	21
9	3	-00.025	12	0	**23**	5	00.166	13	15
10	3	00.310	13	0	**24**	5	00.031	13	16
11	3	-00.254	14	0	**25**	5	-00.082	14	15
12	3	-00.083	15	0	**26**	5	00.057	18	17
13	3	-00.090	16	0	**27**	5	00.000	17	19

Table 10.17: *zdata* for subsystem A33

Meas.	Type	Value	From Bus No.	To Bus No.	Meas.	Type	Value	From Bus No.	To Bus No.
1	2	-00.050	22	0	**16**	3	-00.015	30	0
2	2	-00.062	23	0	**17**	4	-00.095	21	22
3	2	00.740	24	0	**18**	4	-00.717	22	23
4	2	-00.217	25	0	**19**	4	-00.846	24	23
5	2	-00.297	26	0	**20**	4	00.571	22	25
6	2	001.30	27	0	**21**	4	00.288	25	26
7	2	-00.115	28	0	**22**	4	00.119	27	28
8	2	-00.054	30	0	**23**	4	00.054	29	30
9	3	-00.014	22	0	**24**	5	-00.095	21	22
10	3	-00.016	23	0	**25**	5	-00.717	22	23
11	3	-01.072	24	0	**26**	5	-00.846	24	23
12	3	-00.090	25	0	**27**	5	00.571	22	25
13	3	-00.096	26	0	**28**	5	00.288	25	26
14	3	-00.032	27	0	**29**	5	00.119	27	28
15	3	-00.050	28	0	**30**	5	00.054	29	30

Measurement type:

1: Voltage measurement 2: Real power injection 3: Reactive power injection

4: Real power flow 5: Reactive power flow

- Internal bus: a bus whose neighbouring buses are all within the same subsystem as itself
- Boundary buses: a bus that has some neighbouring buses within its own subsystem and some neighbours in other subsystems (buses 12, 13 for subsystem A11 and A22; buses 21 and 22).

Subsystems are connected to each other by tie lines between neighbouring boundary buses like $\boldsymbol{Y_{1213}}$ and $\boldsymbol{Y_{1312}}$ for **A11** and **A22**; $\boldsymbol{Y_{2122}}$ and $\boldsymbol{Y_{2221}}$ for **A22** and **A33.** Local estimates are calculated independently in the subsystems, tie-line measurements and local estimates are coordinated in a global estimator (Chapter 8, subsection 8.4) to provide a unified system-wide state estimator.

Tie-line measurements between (Table 10.17) any two neighbouring subsystems are sent to the global estimator. The global estimator considers these inputs as a new set of measurements.

Table 10.18: Tie-line measurement

Tie-Line	Measurement [MW]	Measurement [MVAr]
2-24	00.109	00.510
12-13	00.178	00.007
21-22	00.095	00.095

The purpose of performing parallel computing of constrained WLAV state estimation was:

- To evaluate efficiency of the proposed algorithm and the program developed using the algorithm
- To reduce computation time (CPU time)
- To establish database for future investigation of the Tanzanian Power System Network
- To improve efficiency of state estimation.

Input Data: The main input data is the zdata30 file. Data in this file are decomposed into 3 sub data files representing the subsystem of Figure 5.3. The decomposed data are given in Tables 10.14 to 10.16.

Hardware: A cluster of computers installed at department of electrical engineerig was applied used to compute the constrained WLAV state estimation. Since the system was decomposed in 3 subsystem, consequently, 3 workers from the cluster was assigned to perform the estimation task.

- **MATLAB program:** A MATLAB program shown in appendix C7 was developed and used in the simulation implementation.
- **Computational results:** computational results are the same with that of integrated WLAV state estimation; the results are shown both in Tabular and graphical form. Table 10-18 gives voltage magnitude and voltage angle profile in Tabular form while Figure 10-15 shows the same profiles in graphical form.

Table 10-19: Voltage magnitude and Angle profiles (Parallel): 30-Bus

Bus No.	Voltage Magnitude [p.u.]	Voltage Angle [Degree]	Bus No.	Voltage Magnitude [p.u.]	Voltage Angle [Degree]
1	00.9614	00.0000	**16**	00.9350	14.4283
2	00.9594	04.2278	**17**	00.9646	09.9308
3	00.9608	01.7868	**18**	00.9675	09.9496
4	00.9548	01.7692	**19**	00.9605	02.4502
5	00.9636	13.1024	**20**	00.9696	02.9531
6	00.9687	14.3279	**21**	00.9697	02.9651
7	00.9579	19.2351	**22**	00.9494	03.4359
8	00.9716	15.8562	**23**	00.9533	05.2061
9	00.9675	26.6922	**24**	00.9450	05.8016
10	00.9545	30.7587	**25**	00.9989	-02.6612
11	00.9511	31.0538	**26**	00.9854	-04.5657
12	00.9775	23.1554	**27**	00.9823	-04.4707
13	00.9715	27.9832	**28**	00.9443	-8.6637
14	00.9715	28.9300	**29**	01.0458	-02.3242
15	00.9649	28.5558	**30**	01.0568	-03.7123

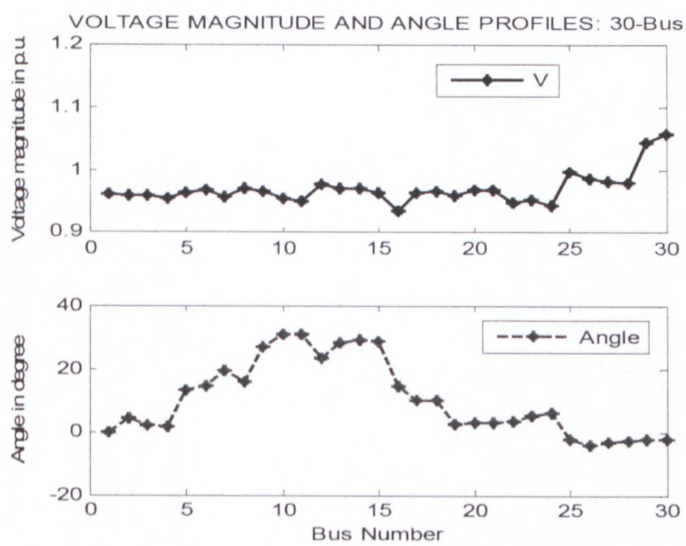


Figure 10-13: Voltage magnitude and Voltage angle profiles -30-Bus

Discussion

- The voltage magnitude and angle profiles shown in Figure 10.13 and Table 10.19 reflect the steady-state acceptable value of the Tanzanian Electric Supply Company Limited (TANESCO). The voltage magnitudes in per unit at each bus is within the operating limit of 0.95 – 1.10. The power factor (pf) ranges between 0.86 and 0.910, which is the operating power factor (pf) of the system.
- Simulation converged after 4 numbers of iterations; time elapses till convergence is 3.00 second.
- Results obtained by parallel processing do not differ much with that obtained from WLAV integrated model; instead the difference is on the computational time. The time taken to process the parallel computation is less compared to time take to process the integrated WLAV state estimation. In order to reduce time of computation, the MATLAB software must be improved.
- The computational results obtained from parallel processing method (using WLS and WLAV) show that it is possible to get a very high improvement in manner of time computation compared to integrated state estimation method. The only negative characteristic is the discrepancy in values of boundary and interconnecting buses and lines (tie-lines). This discrepancy needs more investigation in order to remove them

10.3 Conclusion

Load flow results from the integrated Tanzanian power system network model have been presented in this chapter. The obtained voltage magnitude and voltage angle of each bus in the system reflect the steady-state acceptable value of the Tanzania Electric Supply Company limited (TANESCO). The power factor (pf) and the voltage magnitude and voltage angle are within the operating limits required by TANESCO and falls within IEEE standards.

Results from WLS, WLAV and WLAV parallel computing criteria also refelects the steady-state acceptable values of the system. They correspond to acceptable IEEE standards. These computational results demonstrate that the obtained results from load flow (considered as true values) in particular bus injections (MW and MVAr) at load buses and line flows (MW and MVAr) reflects the approximate value of the system.MATLAB software for WLS, WLAV criteria can be further improved and used in state estimation studies of the system.

Chapter 11 presents conclusion of the thesis in which deriverables from the thesis are presented and explained, future research direction and issues to be investigated are provided.

CHAPTER ELEVEN: CONCLUSION

11.1 Introduction

The electrical power sector all around the world is undergoing significant and irreversible changes that are reshaping the industry into a new shape and outlook. Before, electrical power generation was once dominated by vertically integrated investor-owned utilities that owned most of the generation capacity, transmission and distribution facilities. The electrical industry now has many companies (private owned) that generate and market wholesale retail electricity especially in the United States of America. The significant feature of these changes is to allow for competition, which is seen as a necessary mean to increase efficiency of electrical energy generation and distribution and to offer a lower tariff, high quality and more secure electricity.

Several factors have motivated and contributed to the changes occurring in the electrical power industry to date. They include:

- Technological advances. Technological advances have altered the economic of electricity generation and have made possible the economic transmission of electricity over long distances so that customers in areas or countries residing far away from the generating facilities can access to electricity. Nevertheless, technological advances have enabled more and more interconnection of electric transmission lines.
- Lack of resources for investment and modernization of the vertically integrated systems has forced utilities/countries to seek financial assistance from the Breton Wood institution (IMF & World Bank) to finance investment and modernization of their systems. However, before a financial assistance is approved these institutions put forward the agenda of reforming or restructuring of the electricity industry.
- Legislative and regulatory mandates suggested by the Breton Wood institutions that require the unbundling of integrated service and creation of transmission and operation organizations.

With deregulation of the electrical power sector now a must in many countries, market participants such as independent power producers (IPPs), transmission and distribution companies (RTO and DITSCO) need to adjust the strategies of system operation, monitoring and control in order to reduce generation, transmission and distribution cost, enhance asset utilization, improve planning, better the system reliability and security and nurture customer retention.

Competitions in the power sector market forces the IPPs as well as system operators to operate the system with lower security margins compared to the previously vertically integrated operation in order to remain competitive with other electricity suppliers as well as to allow this competition to take wide area. This inclination, which already exists in some of deregulated systems, will continue to change power flow patterns, increase uncertainties and reduce security of the system if care is not taken, thus, in deregulated systems there are many issues which need to be addressed. In this thesis power system state estimation (PSSE) is one of the issues that have been addressed.

11.2 Aim of the Thesis

The operating state of a power system is determined by the state estimation function using a redundant set of real-time measurement data from the substations scattered all over the entire system. The state estimation is the basis of all advanced applications of an energy management system (EMS) such as stability analysis (SA), contingency analysis, optimal power flow (OPF), load flow and load forecasting. The results from SE are used to calculate various estimates for line flows, losses as well as power injections.

Deregulation of power sector industry is transforming state estimation function from an important application into a critical one. Many critical commercial issues in the power market such as congestion management need to be addressed and formulated on a more accurate model of power system, which is derived from the state estimation process. Failure to obtain state estimated quantities in real-time or miscalculating them should be avoided in order to ensure proper accounting of power transactions as well as security of the system.

This implies that the state estimation should be made robust against the topology changes and temporary loss of measurements or remote terminal unit (RTU). State estimation will not be possible if there are not enough measurements. A power system is determined to be observable if all the state variables can be estimated using available measurements. Various methods have been proposed to compute the state estimation problem. Some of the methods have been discussed in detail in chapter 2. The objective of the thesis has been focused on formulating a mathematical method, algorithm and modifying/improving a program using MATLAB code for the solution of constrained power system state estimation problem using parallel processing approach. This objective has been conducted in this thesis.

11.3 Deliverables

The following have been delivered. They include: development of models, methods and algorithms and improvement of software. The deliverables are presented in sections 11.4 and 11.5, respectively

11.4 Development

Development activities include:

11.4.1 Comparative analysis of the methods for state estimation

Comparative analysis on the existing state estimation method and algorithm was conducted in order to establish their strengths and weaknesses; finally a robust method was choosen for improvement.

11.4.2 Development of an approximate Tanzanian power system network model

An approximate Tanzanian Power System Network model comprising of 30 buses was developed. Bus admittance matrix of the model and its values was prepared, calculated and used in load flow analysis and state estimation.

11.4.3 Decomposition of bus admittance matrix

The bus admittance matrix of the Tanzanian Power System model was decomposed into 3 interconnected sub-systems

11.4.4 Development of decomposed mathematical model of power injection and power flows

Mathematical models for real and reactive power injections, real and reactive power flows in transmission lines and tie-lines of the interconnected sub-systems resulting from bus admittance matrix decomposition were developed. These models are important for future power systems studies such as optimal power flow, etc

11.4.5 Development of decomposed measurement model

Measurement model which has 4 parts determined by the type of measurement available in the system were developed. The models are for voltage magnitude measurement, real and reactive injection data, real and reactive power flows in transmission lines and in tie-lines. These models are applied in solving the power system state estimation problem under decentralized environment

11.4.6 Development of decomposition-coordination method and algorithm

Decomposition-coordination method and algorithm in order to solve the power system state estimation problem was developed. The method is implemented using two level structures. The first level is for solving PSSE of the sub-systems and the second level for coordination of the first level computation.

11.4.7 Formulation and solution of the problem for state estimation

Constrained weighted least absolute value (WLAV) state estimation problem using primal-dual logarithmic barrier interior point method (PDLB) was formulated and an algorithm and MATLAB program for solving the constrained WLAV state estimation problem is developed

11.5 Software improvement and development

Software improvement was concerned with improvement of Newton-Raphson MATLAB program developed by H.Saadat [Saadat, 2004], WLS state estimation software developed by Krishnan [Krishninan, 2004] as well as improvement on:

- Bus admittance matrix calculation software. This software is presented in appendix C2. Improvement was made in adding a minor decomposition code.
- Load flow analysis software. The software is presented in appendix C3. Improvement was made in adding timing (tic/toc), describing in full the variables in order to make it clear to the user, and the code for load buses was changed from '0' to '3' to remove some ambiguity when using the software.
- WLS state estimation software. The software is presented in appendix C5. The software was re-developed using the previous software depicted in appendix 4 as well as using assistance from http://www.mathworks.com/access/helpdesk/help/document/WLS state estimation.html services from mathwork.com.
- WLAV state estimation software. The software is depicted in appendix C6. This is new software and it needs more improvement. The software was developed using WLS state estimation software and assistance from http://www.mathwork.com/access/helpdesk/help/document/WLAV state estimation.html from mathwork.com
- ParallelSE software. The software is presented in appendix C7. This is new software and it is still under development and improvement. The software was developed using the following assistance from mathworks.com

  http://www.mathworks.com/access/helpdesk/help/toolbox/distcom/creatematlabpooljob.html

  http://www.mathworks.com/access/helpdesk/help/toolbox/distcomp/createparalleljob.html

  http://www.mathworks.com/access/helpdesk/help/toolbox/distcomp/filedependencies.html http://www.mathworks.com/access/helpdesk/help/toolbox/distcomp/document/labindex.html

http://www.mathworks.com/access/helpdesk/help/toolbox/distcomp/document/batch.html

A summary of software and their use for different purposes in this thesis is presented in Table 11.1

Table 11.1: Summary of software use

Model	Task	Software
Integrated-Tanzanian model	To calculate bus admittance matrix values	Ybusppg software. Presented in appendix C1
Integrated-Tanzanian model	To calculate decomposed bus admittance matrix values	Ybusppg software. (decomposed). Presented in appendix C2
Integrated-Tanzanian model	To compute load flow	Lf software. Presented in appendix C3
Integrated-Tanzanian model	To compute WLS state estimation	WLS state estimation software. Presented in appendix C5
Integrated-Tanzanian model	To compute WLAV state estimation	WLAV state estimation software. Presented in appendix C6
Decomposed model	To compute parallel state estimation (validation of WLSand WLAV criteria)	Parallel SE. Presented in appendix C7

11.6 Validation

Load flow software was used in validating the developed Tanzanian power system network model as well as the validity and accuracy of data collected from Tanzania Electric Supply Company limited (TANESCO)

11.7 Application of thesis results

Thesis results are grouped into two main categories. The first category is concerned with a result from developed mathematical model(s). The second category is concerned with results from simulation of load flow and state estimation procedures. The two categories are presented as follows.

11.7.1 Developed mathematical model(s) and software

- Developed Tanzanian power system network model can be applied at: TANESCO's research and development departments as benchmark for planning and future expanding of the system.

- Developed decomposed system model and algorithm can be applied in Institutions and Universities for teaching and research and development purposes.
- Software can be applied in solving load flow and state estimation problems in Institutions and Universities for training as well as research and development purposes. There is a need to improve the software for future training use. TANESCO's research and development departments can improve the software in particular for the WLS or WLAV version and use them for monitoring and control of the system.
- Parallel processing method and algorithm can be applied in Institutions and Universities for training and research purposes. In addition, institutions responsible for electricity supply such as the East African Power Pool (EAPP), etc can also apply the method.

11.7.2 Computational results

- Computational results from load flow and state estimation can be applied at TANESCO's research and development departmenst for system monitoring and control as well as planning and future expansion of the system
- Computational results can be applied at Institutions and Universities for training as well as research and development in solving the following system problems:
    - Optimal power flow (OPF)
    - Security constrained optimal power flow (SCOPF)
    - System stability analysis (SA)
    - Fault calculations
    - Contingency analysis (CA)
    - System monitoring and control

11.8 Future Research Direction

The work reported in this thesis is a basis for future research related to multi-area power system operation, monitoring and control. The case study of the Tanzanian Power System Network (HV transmission line) can be extended to include the East African Regional Power Pool (EARPP), Southern African Power Pool (SAPP) and other regional block cooperation.

Future research directions based on this thesis should be extended to:

- Real-time digital simulation of the developed model
- Improvement of ParallelSE software
- Improvement of WLAV state estimation criterion and software using decomposition-coordination method and algorithm proposed in the thesis

The idea of decomposition-coordination of a multi-area proposed should include:

- Load-flow analysis
- Optimal power flow analysis
- Stability analysis

11.9 Publications in connection with the thesis

This work (research) resulted in a number of publications, some of which are listed below:

Kusekwa, M. A and Tzoneva, R (2007) "Analysis on the algorithms for solution of the Weighted Least Absolute Value problem for power system state estimation" SAUPEC 2007, Cape Town, January 2007,pp.140-147 CD-ROM

Kusekwa, M .A and Tzoneva, R (2007) "Bad data Detection and Identification Algorithm in solving Power System State Estimation problem: A Comparison Review. The 4th International Scientific Symposium, 19-21 September, 2007, Kosice-Republic of Slovenia CD-ROM

Kusekwa, M. A and Tzoneva, R (2007) "Bad data Detection and Identification Algorithm in solving Power System State Estimation problem: A Review" Proceedings of the International Conference (AFRICON-2007) CD-ROM, Windhoek-Namibia, 25-28 September, 2007, CD-ROM.

Kusekwa, M. A and Tzoneva, R (2010) "A Primal-Dual Interior Point based algorithm for a solution of Constrained on-quadratic State Estimation" Submitted to SAIEE Africa Research Journal

Kusekwa, M. A and Tzoneva, R (2010) "Modelling of System Components for Load Flow Analysis: A Case study" Submitted to Energy Journal Paper

BIBLIOGRAPHY/REFERENCES

Abur, A. & Celik, M.K., 1992, Least absolute value state estimation with equality and inequality constraints ,IEEE Summer Power Meeting Paper 92 SM PWRS

Abur, A. & Celik, M.K., Feb.1991, A fast algorithm for the Weighted Least Absolute value State Estimation, IEEE Trans. Power systems,vol.6, no.1

Abur, A., & Exposito, A.G., 2004, Power System State Estimation: Theory and Implementation, Marcel Dekker, new York

Abur,A., 2005, Distributed state estimation for mega grids in Proc.15th Power Systems Computation Conference, Liege, Belgium, August 22-26

Acarnley, P. 2006. Power System Load Flow Analysis Using an Excel Workbook, International Journal of Electrical Engineering Education ,42(2), pp.186-202

Adler, I., Karmarkar, N., Resende, M.G.C, & VEIGA, G. 1989. An implementation of Karmarkar's algorithm for linear programming, Mathematical Programming 44. pp.297-335

Aguado, J.A., Prez-Molina,C.,& Quintana, V.H.,2001, Decentralized power system state estimation: a decomposition-coordination approach, in proc. Power Tech Proc., vol. 3, pp. 10-13

Alexandrov, N.M. & Lewis, R.M., 2000. Analytical and Computational aspects of collaborative optimization, Technical Report TM-2000-210104, NASA

Allemong,J.,June 2005, State Estimation Fundamentals for successful deployment, in Proc. IEEE PES General meeting, San Francisco, CA

Alsac, O., Vempati, N., Stott, B. & Monticelli,A. May 1997. Generalized State Estimation PICA Conference.

Alsac,O., Tinney,W.F., Stott,B. & Monticelli,A. August 1998, Generalized State Estimation, IEEE Trans. on Power Systems, vol.13,no.3, pp.1069-1075

Alvarado, F.L. & Tinney, W.F. July 1990. State Estimation using augmented block matrices. IEEE PES Summer Meeting, Paper 90 SM 273-3 PWRS, Minneapolis

Alvarado, F.L. 1999. Solving Power Flow Problem with a MATLAB Implementation of the Power System Applications Data Dictionary, Proceedings of the 32nd Hawaii International Conference on System Science, Hawaii, USA, pp. 1-7

Anstreicher, K.M. & Bosch,R.A., 1995, A new infinity-norm path following algorithm for Linear Programming, SIAM Journal of Optimization,vol.5,no.2, pp.236-246

Anstreicher, K.M., 1991, A combined phase I, phase II scaled potential algorithm for linear programming, Mathematical programming, vol.52, pp. 424-439

Arnold, M., Knopfili, S. & Andersson, G. 2003. Improvement of OPF Decomposition methods applied to Multi-area power systems. Power system Laboratory of Swiss Federal Institute of Technology (ETH) in Zurich, Physikstrasse 3, CH-8092 Zurich

Arrilaga, J. & Arnold, C.P. 1990. Computer Analysis of Power Systems. John Willey & Sons,Inc, West Sussex, England

ASchmoniet,F.C., Peterson,N.M.,& Adrian,E.C., May 1977, State Estimation with Equality Constraints in 10th PICA Conference Proc. Toronto Canada: pp.427-430

Baldick, R., Kim, B.H., Chase, C. & Luo, Y. 1999. A fast distributed implementation of optimal power flow. IEEE Trans on Power Systems, 14(3): 858- 864

Barnes,E.R., 1986, A variation on Karmarkar's algorithm for solving linear programming problems, Mathematical Programming, vol.36, pp.174-182

Barness, E.R. 1986. A variation on Karmarkar's algorithm for solving linear programming problems, Mathematical Programming 36. pp.174-182

Batut, J. & Renaud, A. August 1992, Daily generation scheduling optimization with transmission constraints: A new class of algorithm, IEE Trans. on Power App. & Systems, vol.7. no.3, pp. 982-989.

Batut, J. & Renaud, A. August 1992. Daily generation scheduling optimization with transmission constraints: A new class of algorithms. IEEE Trans on Power Apparatus & systems, 7(3): 982-989

Bayer, D.A. & Lagarais,J.C., 1989, The non-linear geometry of linear programming, Part I: affine and projective scaling trajectories; Part II: Legrendre transform coordinates; Part III: central trajectories; Part IV: Karmarkar's linear programming algorithm and Newton's method, Trans. American Mathematics society, vol.314, pp.499-581

Bazaraa, M.S., Jarvis, J.J. & Sherali, H.D., 1990. Linear Programming and Network Flows, John Wiley & Sons, Inc.

Bazaraa, M.S., Sherali, H.D. & Shetty, C.M., Nov. 1992. Non-linear programming: Theory and Algorithms, John Wiley & Sons,Inc.

Benders, J.F., 1962. Portioning Procedures for solving mixed variables programming problems, Numerische Mathematic,pp.238-252

Bernhard, P.,1976. Commande optimale, decentralisation et jeux dynamiques. Dunod Automatique, Paris

Bertsekas, D.P. and Tsitsikilis, J.N., 1989. Parallel and Distributed computation, Prentice-Hall, New York.

Bertsekas,D.P.,1999, Non-linear Programming, Belmont, MA: Athena Scientific

Bialeck, J. Oct.1986. Modified Algorithm of Revised Simplex Method Applied to Power System State Estimation, Electrical Power and Energy Systems, vol.8, no.4, pp.236-240

Bose, A. & Clements, K.A. Dec. 1987, Real-Time modelling of Power Network, Proc. IEEE,vol.75, no.12, pp.1607-1622

Brice, C.W. & Cavin, R.K., Feb. 1982, Multiprocessor Static state Estimation, IEEE Trans. on Power App.& Systems, vol.PAS-101,no.2, pp. 302-308

Brosilow, G.B, Lasdon, I.S. & Pearson, J.D. J.A.C.C. 1965. Feasible Optimization Methods for interconnected Systems

Celik, M.K. & Abur, A., Feb 1992, A robust WLAV State Estimator Using transformations. IEEE Trans. on Power Systems, vol.7, no.1, pp.106-113

Contreras, J., Losi, A., & Wu, F.F. February 2002. Simulation and Evaluation of Optimization Problem Solutions in Distributed Energy Management Systems. IEEE Trans on Power Systems, 17(1):5762

Celik, M.K. & Abur, A., May 1992, Use of Scaling in WLAV State Estimation of Power System States, IEEE, trans. on Power Systems, vol. 7, no. 2, pp.684-692

Chiite, K. & Swarup, K.S., 2003. Power System State Estimation Using Interior Point (IP) Barrier Method, Indian Institute of Technology, Madras, 600036, INDIA, pp.1-6

Clements, K.A, Woodzell, G.W. & Burchett, R.C. July 1990. A New method for solving equality constrained power system estimation. IEEE PES Summer Meeting, paper 89 SM 687-5 PWRS, Long Beach

Clements, K.A., & Wollenberg,B.F., 1975, An algorithm for observability determination in Power System State Estimation, in Proc. IEEE summer Power meeting paper A&447-3

Clements, K.A., April 1990, Observability methods and Optimal Meter placement, international Journal of Electrical Power and Energy, vol.12,no.2, pp.89-93

Clements, K.A., Davis, P.W., & Frey, K.D., 1989, An efficient algorithm for computing the weighted least absolute value estimate in power system static state estimation, in Proc. Of IFAC conference on Power Systems and Power Plant Control

Clements, K.A., Davis, P.W., &Frey, K.O., May 1995, Treatment of inequality constraints in Power System State Estimation, IEEE Trans. on Power Systems, vol. 10,no.2, pp. 567-574

Clements, K.A., Davis, P.W.,& Frey, K.D, 1991, An interior point algorithm for weighted least absolute value Power System State Estimation, IEEE Trans. on Power systems

Cohen, G. & Zhu, D.Li. 1984. Decomposition coordination methods in large scale optimization problems: The no differentiable case and the use of augmented Lagrangians.Advanced Large Scale Systems. 1:203-266

Cohen, G. November 1980. Auxiliary problem principle and decomposition of optimization problems. Journal of Optimization Theory applied. 32: 277-305

Cohen, G., 1980, Auxiliary Problem Principle and decomposition of Optimization Problems, Journal of Optimization Theory and Application, vol.32, no.3, pp. 277-305

Conejo, A., Nogales, F. & Prieto, F. 2002. A decomposition procedure based on approximate Newton directions. Mathematical Programming Ser.A 93: 495-515

Conejo, A.J., Nogales, F.J., & Prieto, F.J., 2002, A decomposition procedure based on approximation Newton directions, Math. Programming, vol.93, no.3, pp.495-515

Conejo, A.J., Torres, S.O.I., & Canas, M., Feb. 2007, An optimization approach to multi-area state estimation, IEEE Trans. on Power Systems., vol.22, no.1, pp.213-221

Crow, M., 2003. Computational methods for Electric Power Systems, CRC Press

Cutsem, T.V., Howard, J.L., & Ribbens-Pavella, M, 1981, A two-level static state estimator for electric power systems, IEEE Trans. on Power App. & Systems., vol, PAS-100,no.8, pp.3722-3732

Cutsem,T.V, & Ribbens-Pavella., Oct.1983, Critical survey of hierarchical static state estimation in large scale power systems, IEEE Trans. on Power App.& Systems., vol.PAS-102, no.10,pp.247-256

Dantzig, B.G, & Wolfe, P., 1960. Decomposition principle for linear program, Operation Research, vol.8, no.1, pp.101-111

Dantzig,B.G., 1963, Linear programming and extension, Princeton University Press, Princeton, New Jersey

Davis, P.W., Clements, K.A., & Frey, K.D., 1995, treatment of inequality constraints in Power System State Estimation, IEEE Trans. on Power Systems, vol.10, no.2, pp.567-574

Debs, A.S. 1992. Modern Power System Control and Operations, A book Published by Decisions Systems International

Deeb, N. I. & Shahidehpour, S.M., May 1990. Linear Reactive Power Optimization in a large Power Network Using Decomposition Approach, IEEE Trans. on Power App. & Systems, vol.5,no.2,pp.428-438

Deeb, N.I. & Shahidehpour, S.M., Nov.1993. Cross Decomposition for Multi-area optimal reactive power planning, IEEE Trans. on Power App.& Systems, vol.8, no.4, pp.1593-1544

Dikin,I.T.,1967, Iterative solution of Problems of linear and Quadratic programming, Soviet Mathematics Doklady,vol.8, pp.674-675

Distributed Computing Toolbox, User's Guide, Version 2, www.mathworks.com

Do Coutho Filho, M.B., Leite da Silva, A.M.& Falcao, D.M., 1991, Bibliography on Power System State Estimation (1968-1989), IEEE Trans. on Power Systems, vol.5, no.3, pp.511-513

Duff, I.S. & Reid, J.K. 1982. MA27-a set of Fortran subroutine for solving sparse symmetric linear equations. Report AERE R-10533, Computer Science and Systems Divisions, AERE Har-well, Harwell England

Dy Liacco, T.E., April 1990, The role and implementation of State Estimation in an Energy Management system, International Journal of Electrical Power and Energy, vol.12,no.2, pp.75-79

Ebrahimian, R & Baldick, R., Nov.2000, state estimation distributed processing for power systems, IEEEE Trans. on Power Systems, vol.15, no. 4, pp. 1240-1246

Ebrahimian, R.& Baldick, R., May 2001, State Estimator Number Analysis, IEEE Trans. on Power Systems, vol.16,no.2, pp.273-279

El-Keib, A.A., & Singh, H., May 1992. Fast Linear Programming State Estimation Using the Dual Formulation, IEEE Trans. on Power Systems, vol.7, no.2, pp.620-625

El-Keib,A.A., Nieplocha,J., Singh, H. & Maratukulam, D.J., August 1992, A Decomposed State Estimation Technique Suitable for Parallel Processor Implementation, IEEE Trans. on Power Systems, vol.7, no.3, pp.1088-1097

Falcao, D.M., Wu, F.F. & Murphy, L., July 1994. Parallel and Distributed State Estimation, presented at the 1994, IEEE PES Summer Meeting San Francisco, CA

Ferreira, C.A. & Da Costa, V.M 2005. Second Order Power Flow Based on Current Injection Equations. Electrical Power & Energy Systems, 27. pp.254-263

Frang, S.C, & Puthernpura, S., 1993, Linear optimization and extensions: Theory and algorithm. Prentice—Hall, Englewood Cliff, New Jersey

Fiacco, A.V. & McCormick, G.P., 1968, Non-linear programming. Sequential unconstrained minimization technique, Wiley

Frang,S.C. 1992. A new unconstrained convex programming approach to linear programming ZOR 36. pp. 149-161

Freund,R.,1991,Polynomial-time algorithms for linear programming based only on primal scaling and projected gradients of a potential function, Mathematical programming, vol.51, pp.203-222

Frisch, K.R., 1955,The logarithmic potential method of convex programming, technical report, University Institute of Economics, Oslo Norway

Geoffrion, A.M, A.M., 1974. Lagrangian Relaxation for Integer Programming, Mathematical Programming, vol.2, pp.82-113

Geoffrion, A.M., 1972. Generalized Benders Decomposition,JOTA,vol.10,no.4, pp.237-260

Gill, P.E. & Murray, W, 1974. Numerical methods for constrained optimization, Academic Press, London

Gill,P.E., Saunders, M.A., Tomlin,J.A. & Wright,M.H. 1986. On projected barrier methods for linear programming and an equivalence to Karmarkar's projective method, Mathematical Programming 36, pp. 183-209

Gjelsvik, A., Aam,S. & Holten,L. November 1985, Hactel's Augmented Matrix Method: a rapid method improving numerical stability in Power System State Estimation, IEEE Trans. on Power App.& Systems, vol.PAS-104, no.11, pp.2987-2993

Glav& Wenhenkel, L., Feb.2004. Interior Point Methods: A Survey, Short survey of Applications to Power Systems, and research opportunities, University of Liege, electrical Engineering and Computer Science Department, Sart Tilman B-28,400 Liege, BELGIUM, pp.1-39

Goldman, A.J., 1956. Resolution and separation theorems for polyhedral convex sets, in H.W. Kuhn and A.W. Tucker,editors,Linear Inequalities and Related Systems, volume 38 annals of Mathematics, 41-51, Princeton University Press, Princeton

Gonzaga, C.C., 1999, Complexity of Predictor-Corrector algorithm for LCP based on large neighbourhood of the central path, SIAM Journal of Optimization, vol.10, no.1, pp.183-194

Gonzaga,C.C., 1987, An algorithm for solving linear programming in $0(n^3L)$ operations, Technical report UCB/ERL M87/10, Electronic Research Laboratory, University of California, Berkeley, CA 94720

Gonzaga,C.C., 1997, The Largest step path following algorithm for monotone Linear Complementarity problems, Mathematical Programming, vol. 76, no. 2, pp.209-332

Granville,S., Feb. 1994, Optimal reactive dispatch through interior point methods, IEEE Trans. on Power Systems,vol.9,no.1, pp.136-142

Gu,J.W., Clements, K.A., Krumpholz, G.R., & Davis, P.W., Oct.1983, The solution of ill-conditioned Power System State estimation Problems via the method of Peters and Wilkinson, IEEE Trans. on Power App. & Systems,vol. 102, no.10, pp.3473-3480

Heydt, J. 1986. Computer Analysis Methods for Power Systems, A book Published by Macmillan Publishing Company

Holten,L., Gjelsvik,A.,Aaam,S. & Wu, F.F.,Nov.1988, Comparison of different methods for State Estimation, IEEE, Trans. on Power Systems, vol.3, no.4, pp.1798-1806

Hur, D., Park, J.K, & Kim, B.H. 2003. On the convergence rate improvement of mathematical decomposition technique on distributed optimal power flow. Electrical Power and Energy Systems. 25: 31-39

IEEE Committee Report. 1973. Common format for exchange of solved Load Flow data. IEEE Trans. on Power Apparatus and Systems, PAS-92,6.pp.1916-1925

IEEE Spectrum, September 1992. Special Issue: Supercomputer.

Irving, M.R., Owen, R.C. & Sterling, M.J.H., September 1978, Power System State estimation Using Linear Programming Proc. Of the Institute of Electrical Engineers (Part C) 125: 978-985

Jabr, R.A., 2005. Power System Huber M-Estimation with equality and Inequality Constrains, Electrical Power System research,no.74, pp.239-246

Jabr, R.A., 2006. Power System State Estimation using Iteratively Re-weighted Least Squares Method for Sequential l_1-regression, Electrical Power and Energy System, no.28, pp.86-92

Karmarkar, N., 1984, A polynomial-time algorithm for linear programming, Combinatorica, vol.4, pp.373-395

Khachiyan,L.G.,1979, A polynomial algorithm in linear programming, Soviet Mathematics Doklady, vol.20, pp.191-194

Kim, B., 1997. Modification to interior point OPF code. Technical report, University of Texas at Austin

Kim, B., 1997. Multi-area distributed OPF user's guide. Technical report, University of Texas at Austin.

Kim, B.H. & Baldick, R. May 1997. Coarse-grained distributed optimal power flow. IEEE Trans Power Systems, 12: 932-939

Klee, V. & Minty, J.G., 1972, How good is the simplex algorithm? In O.Shisha, editor, inequalities III, Academic press, New York, pp. 159-175

Kojima,M., Mizuno,S. & Yoshise,A.A.,1989, A primal-dual interior point algorithm for linear programming: Progress in Mathematical Programming 29-47,N.Megiddo ed., Springer-Velag,New York

Kontoghiorghes, E.J., 2006. Handbook of Parallel Computing and Statistics, Chapman &

Hall/CRC

Kotiuga, W.W. & Vidyasagar, M., April 1982, Bad data Rejection Properties of Weighted least Absolute Value Technique Applied to Static State Estimation, IEEE Trans. on Power App. & Systems., vol.PAS-101,no.4, pp.844-851

Kroo, I.M., 1997. MDO for large –scale design in N.M. Alexandrov, N.M and M.Y. Hussaini, editors, multidisciplinary Design Optimization: State of the Art.

Krumpholz, G.R., Clements, K.A., & Davis, P.W., July/August 1980, Power System Observability: A Practical algorithm using network topology, IEEE Trans. on Power App.& Systems, vol.PAS-99,no.4, pp.1534-1542

Kuhn, H.W., & Tucker, A.W., 1951, Non-linear programming, in J.Neyman, editor, Proc. Of second Berkeley Symposium on Mathematical Statistics and Probability,481-493, University of California Press

Larson, R., Tinney, W. & Peschon, J. March 1970. State Estimation in Power System, PartI: Theory and Feasibility, IEEE Trans. Power App.& System, vol.PAS-89,no.3, pp.345-352

Larson., Tinney, W., Hadju, L. & Piercy,O., March 1970, State estimation in Power Systems, Part II: Implementation and Applications, IEEE Trans. on Power App.& Systems, vol. PAS-89,no.3, pp.353-362

Leopold, C., 2001. Parallel and Distributed Computing: A survey of Models, Paradigms, and Approaches, John Wiley & Sons, Inc.

Lewis, R.M. & El-Rewin, H., 1992. Introduction to Parallel Computing, Prentice Hall, New York

Lin, S.Y., May 1992, A Distributed State Estimator for Electric Power Systems, IEEE Trans. on Power Systems, vol. 7, no.2, pp.551-517

Lo, K. & Mahmoud, Y., Feb. 1986. A Decoupled Linear Programming Technique for Power System State Estimation, IEEE Trans. on Power Systems, vol. PWRS-1, no.1, pp.154-160

Lustig,I.J., Marsten,R.E. & Shanno, D.F., 1992, On implementing Mehrotra's predictor-corrector interior point method for linear programming, SIAM, Journal of Optimization,Vol.2,no.3, pp.435-449

Lustig,I.J., Marsten,R.E. & Shanno,D.F.,1991, Computational experience with a primal-dual interior point method for linear programming, linear algebra and its applications,vol.152, pp.191-222

MATLAB User's guide: The MathWorks, Inc, 2009

Mcshane,K.A.,Monma,C.L. & Shanno, D.F.,1989, an implementation of a primal-dual interior point method for linear programming,ORSA Journal on Computing,vol.1, pp.70-83

Megiddo, N. 1989. Pathways to the optimal set of linear programming, Progress in Mathematical Programming: Interior –Point and Related Methods, Springer-Verlag, pp. 131-158

Megiddo, N. 1989. Progress in Mathematical Programming: Interior-Point and Related Methods, Springer-Verlag

Megiddo. & Shub,M., 1986, Boundary behaviour of interior point algorithms in linear programming technical report RJ 5319,IBM Thomas J.Watson Research Centre, Yorktown Heights, New York

Mehrotra,S., 1992, On the implementation of a primal-dual interior point method, SIAM journal on Optimization, vol.2, pp.575-601

Mittsche, J.V. Jan. 1993. Stretching the limits of Power System Analysis, IEEE Computer Application in Power,vol.6, no.1, pp.16-21

Mizuno, S. & Nagasawa, A. 1993, A primal-dual affine scaling potential reduction algorithm for Linear programming, Mathematical Programming, vol.44, pp.27-41

Momoh, J.A., 2006, Interior Point Methods and variants for OPF: A tutorial IEEE

Monteiro,R.C. & Adler, I., 1989, Interior path-following primal-dual algorithms, Part I: Linear Programming, Mathematical Programming, vol.44, pp.27-41

Monticelli, A. & Garcia, A., November 1991, Modelling Zero Impedance Branches in Power System State Estimation, IEEE Trans. on PWRS,vol.6

Monticelli, A., 1999, State Estimation in Electric Power System: A General Approach, Boston: Kluwer Academic Publishers

Monticelli, A., 2000, Electric Power System State Estimation proc. IEEE,vol.88,no.2, pp.262-282

Monticelli, A., Murari, C.A.F. & Wu, F.F., Dec.1985. A Hybrid State Estimation: Solving Normal Equation by Orthogonal Transformation, IEEE Trans. on Power Systems, vol.105, pp.3460-3468

Mukai, H. Jan.1982, Parallel Multi-area State Estimation, EPRI, Report no. 1764-1, Electric Power Re.Inst. Palo Alto, CA

Nesterov,Y. & Nemirovskii,A. 1994, Interior point polynomial methods in convex programming, SIAM, Philadelphia

Nima, A. May 2006. Day-Ahead price forecasting of Electricity Markets by new Fuzzy Neural Network, IEEE Trans. on Power Systems, vol.21, no.2, pp.887-896

Nogales, F., Prieto, F. & Conejo, A. 2003. A decomposition methodology applied to multi-area optimal power flow problem. Annals of Operation Research, 120: 99-116

Optimization Toolbox, User's Guide, Version 1, www.mathworks.com

Padilla, A. & Morelato, A., August 1992. A W-Matrix Methodology for solving Sparse Network Equations on Multiprocessor Computers, IEEE Trans. on Power Systems, vol.7, no.3, pp.1023-1030

Pajic,S., & Clements, K.A. June 23-26 2003, Globally convergent State Estimation via the trust region Method in Proc. IEEE Power Tech. Conference, Bologna, Italy

Pajic,S., Nov.2005, Power System State Estimation via globally convergent methods, IEEE Trans on Power system, vol.20, no.4, pp. 1683-1689

Palm, W.J. 2005. Introduction to MATLAB 7 for Engineers. McGraw Hill, New York

Pandian, A. & Soman, A.S., August 1999. The faster Givens rotational based Power system State estimation, IEEE Trans. on Power Systems, vol.14, no.3, pp.837-843

Power Systems Test Case Archive, http://www.ee.washington.edu/research/pstca

Pshenichny, B.N, & Danilin, Y.M. 1978. Numerical Methods in external problems, Mir Publishers, Moscow

PVM-Parallel Visual Machine, home page. http://www.epm.ornl.gov/pvm/pvm_home.html.

Quinn, M.J., 1994. Parallel Computing: Theory and Practice,McGraw-Hill,New York

Rakpenthai., Premrudeepreechacharn, S., Uatrongjit,S. & Watson, N.R. 2005, Measurement Placement for Power system Estimation using Decomposition Technique, Electrical Power System Research, vol.75, pp.41-49

Ramirez, J.M. & Barocio, E.E., 2000. Solving State Estimation in Power System by Interior Point Method, Electrical Power and Energy System, no.22, pp.355-365

Schweppe,F.C, Wildes, J., & Rom,D.B., Jan. 1970, Power System Static state Estimation, part I,II,III, IEEE Trans. on Power App. & Systems., vol. PAS-89, pp. 120-135

Shahidepour, M. & Wang, Y., 2003. Communication and Control in Electric Power Systems, Application of Parallel and Distributed Processing, John Wiley & Sons, Inc.

Shanno, D.F. & BaGchi, A. 1988. A unified view of interior point algorithm for linear programming.Ructor Research Report No. 35-88

Sheu, R.L. 1992. Insight into the interior Point Methods, ZOR 36, pp.227-257

Shinha, A.K. & Mandal, J.K. Nov.1999. Hierarchical Dynamic State Estimator Using ANN bases dynamic load prediction,IEE Proc. Generation Trans. Distribution, vol. 146, no. 6, pp.541-549

Shirmohammadi,O., Nov.1998. Transmission dispatch and Congestion Management in the Emerging Energy Market Structure, IEEE Trans. on Power Systems, vol.13, no.4, pp.1466-1474

Simoes-Costa, A. & Quintana, V.H., Feb.1981, A robust numerical technique of Power System State Estimation, IEEE Trans. on Power App. & Systems,vol.PAS-100,no.2, pp.691-698

Simoes-Costa,A., & Quintana,V.H., August 1981, an orthogonal row processing algorithm for Power System Sequential State Estimation, IEEE Trans. on Power App.& Systems, vol.PAS-100, no.8, pp.3791-3800

Singh, H. & Alvarado, F.L., August 1994, WLAV State Estimation Using Interior Point Methods, IEEE Trans. on Power Systems, vol.9, no.3, pp.1478-1484

Singh, H., Alvarado, F.L.,& Liu, W.H.E., Feb. 1997, Constrained LAV State Estimation using penalty functions, IEEE Trans. on Power Systems, vol.12, pp.383-388

Stott, B. & Alsac, O., May/June 1974, Fast-Decoupled Load Flow, IEEE Trans. on Power App.& Systems,vol. PAS-93, pp.859-869

Stott, B., Alsac, O. & Monticelli, A., Dec. 1987, Security Analysis and optimization, Proc. IEEE,vol.75,no.12, pp.1623-1644

Tinney,W.F. & Hart,C.E., November 1967, Power Flow solution by Newton's Method, IEEE Trans. on Power App.& Systems, Vol. PAS-86, no. 11, pp.1447-1467

Todd, M.J. & Ye, Y., 1990, A centred projective algorithm for linear programming, Mathematical Operation Research, vol. 15, pp.508-529

Todd,M.J. & Tuncel,L., 1996,Asmptotic behaviour of interior point methods: a view from semi-infinite programming, Mathematical operation Research,vol.21,no.2, pp.354-381

Todd,M.J.,1994, Interior Point algorithms for Semi-definite programming, Mathematical Programming, vol, 65, pp.217-245

Tutorial course 96TP111-0. Optimal Power Flow: Solution techniques, requirements and challenges

Tylavsky, D.J. & Bose, A., May 1992. Parallel Processing in Power System Computation, IEEE Trans. on Power Systems, vol.7, no.2, pp.629-637

Umar, A., 1993. Distributed Computing: A Practical Synthesis, Prentice-Hall, New Jersey

Vale, M.H.M., Falcao, D.M., & Kaszkurewicz,E.E, E.E., May 1992. Electrical Power Network Decomposition for Parallel Computations, IEEE Symposium on Circuits and Systems, San Diego, CA: 2761-2764

Van Amerongen, R.A.M, November 1995, On convergence analysis enhancement of Power system Least Squares state Estimation, IEEE Trans. on Power Systems, vol.10,no.4, pp.2038-2044

Vandebei, R.J., 1995, Affine-Scaling trajectories associated with a linear semi-finite program. Mathematical Operation Research, vol.20, pp.163-175

Vempati,N., slutsker,I.W. & Tinney,W.F., 1991, enhancement to givens rotations for Power System State Estimation, IEEE Trans. on Power Systems, vol.6,no.4, pp.842-849

Wallach, Y. 1986. Calculations and Programs for Power System networks, A book Published by Macmillan Publishing Company

Wei, H.., Sasaki,H., Kubokwa,J. & Yokoyama,R., 1998, An interior point method for Power System weighted non-linear l_1 norm Static State Estimation, IEEE Trans. on Power Systems, vol.13, no.2, pp.617-623

Wood, A.J. & Wollenberg, B.F. 1996. Power Generation, Operation and Control, Wiley, New York, second edition

Wood, A.J. & Wollenberg, B.F.,1996, Power Generation, Operation and Control 2nd Edition New York, Wiley

Wright, S.J., 1997, Primal-Dual Interior point, SIAM, Philadelphia

Wu, Y.C, Debs, A.S. & Marsten, R.E. May 1994. A direct nonlinear predictor-corrector primal-dual interior point algorithm for optimal power flows. IEEE Trans on Power Systems, 9(2): 876-883

Wu,F.F., 1990, Power system State Estimation: a survey., Int. Journal of Electrical Power and Energy Systems, vol.12, no.2, pp.80-87

Ye, Y., Guler,O., Tapia,R.A.,& Zhang,Y.,1993, A Quadratically convergent (√n L0-iteration algorithm for Linear Programming, Mathematical Programming,vol.59, pp.151-162

Ye,Y.,1997, Interior Point Algorithm: Theory and Analysis.Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wile &Sons

Zhao, L. & Abur, A., May 2005, Multi-area state estimation using Synchronized phasor Measurements, IEEE Trans. on Power Systems., vol. 20, no. 2, pp.611-617

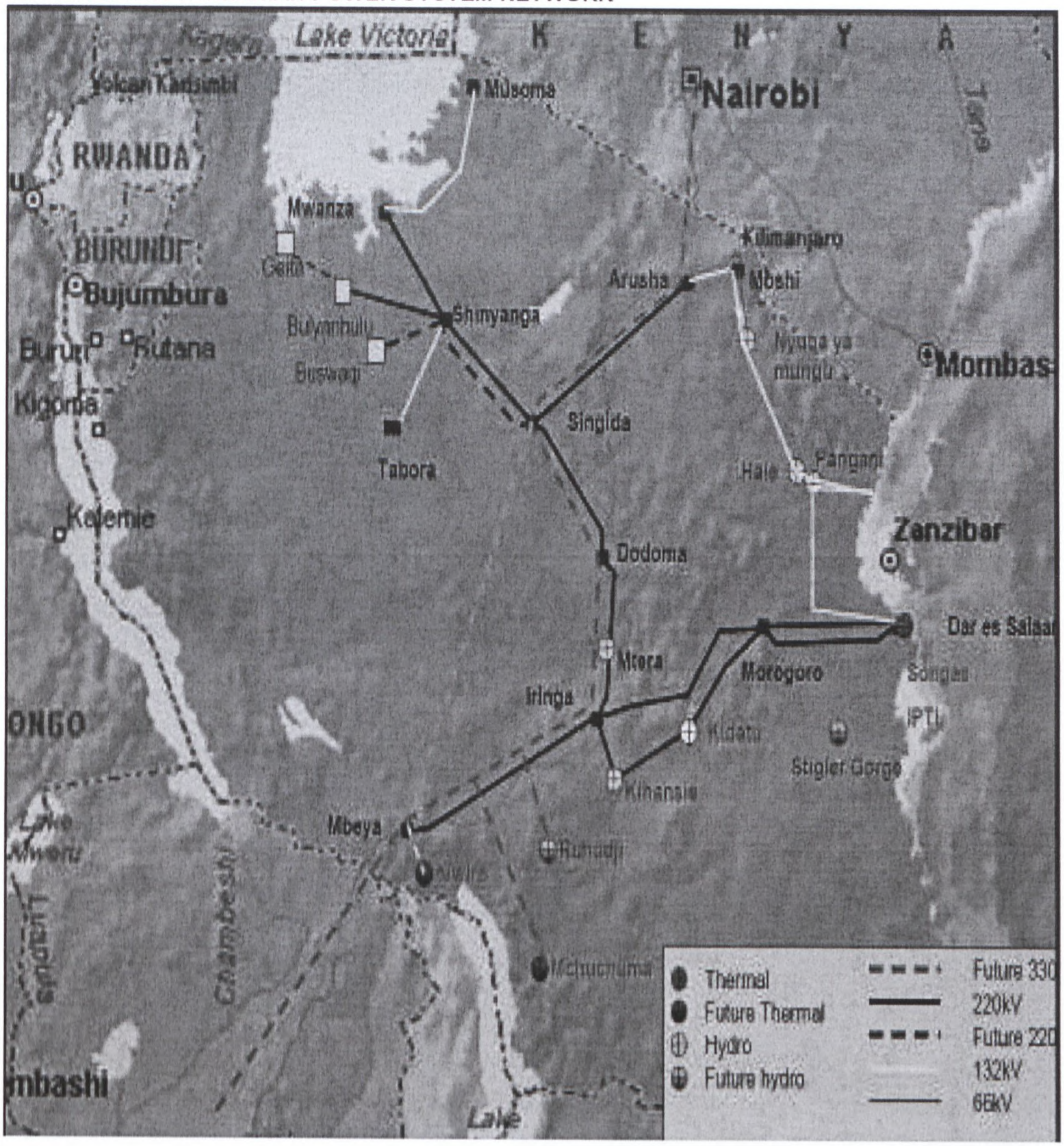
Zhengchun, Zhenyong, N. & Wanliang, F., 2005. Block QR Decomposition based Power System State Estimation Algorithm, Electrical Power System research, no.76, pp.86-92

Zhou, Q. & Bialeck, J.W. May 2005. Approximate Model of European Interconnected System as a Benchmark System to Study Effects of Cross-Border Trades. IEEE Trans on Power Systems, 20(2). Pp. 782-788

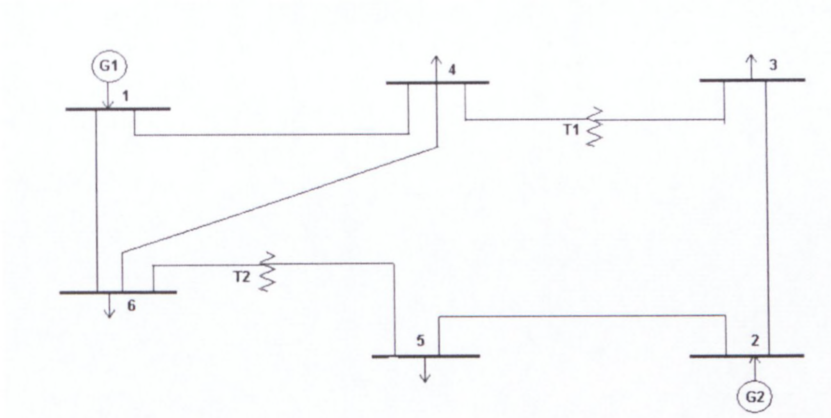
APPENDICES

APPENDIX A: POWER SYSTEMS

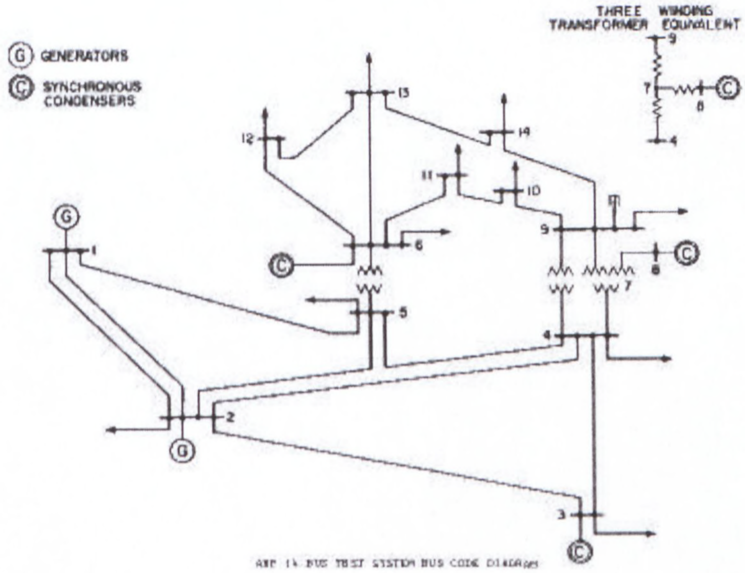
APPENDIX A1: TANZANIAN POWER SYSTEM NETWORK



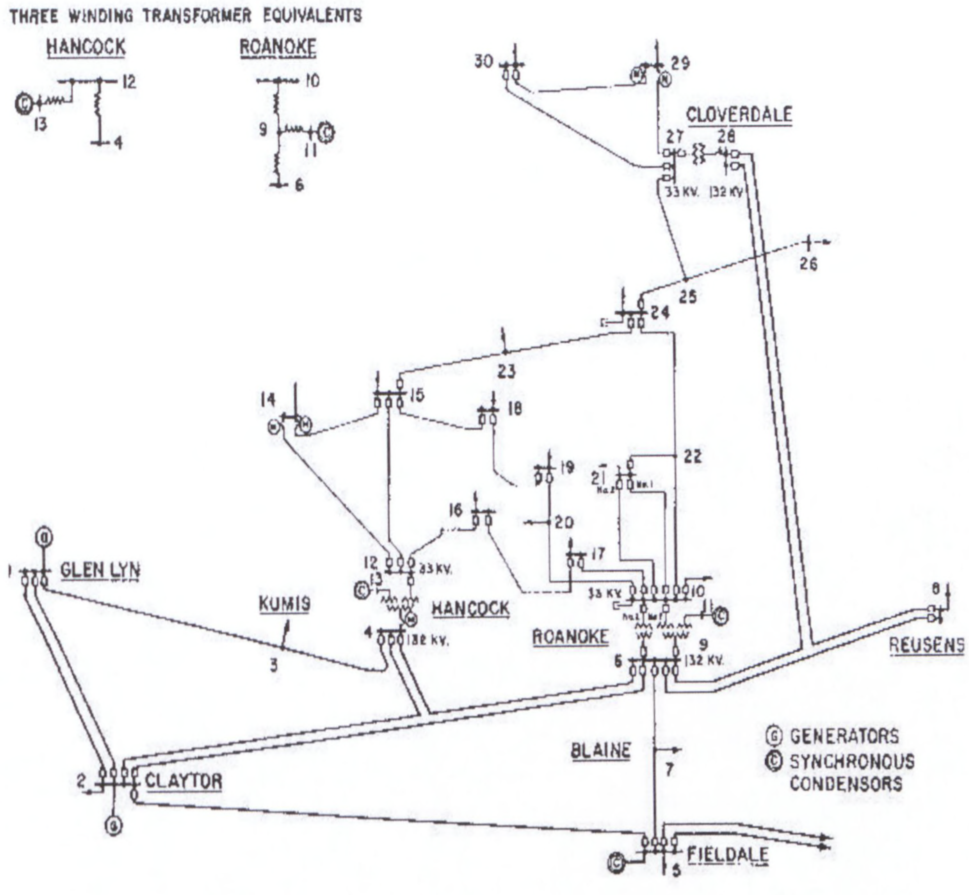
APPENDIX A2: WARD HALE 6 BUS SYSTEM



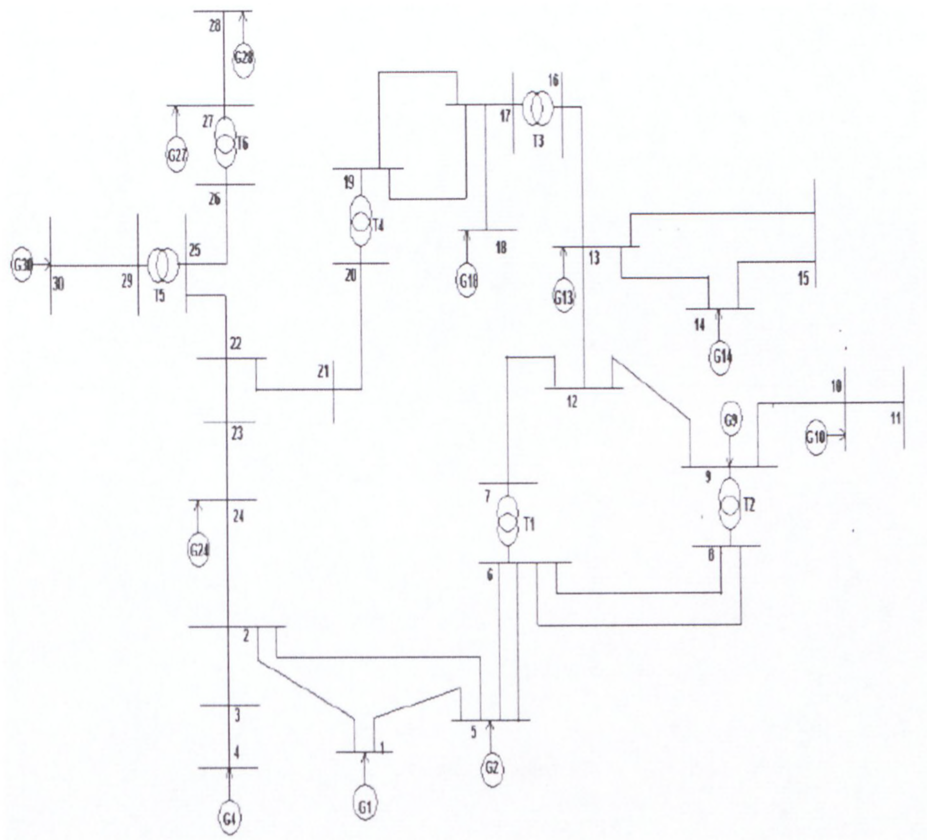
APPENDIX A3: IEEE 14 BUS SYSTEMS



APPENDIX A4: IEEE 30 BUS SYSTEMS



APPENDIX A5: 30-Bus (TANZANIA)



KEY

G: Generator

T: Two winding transformer

APPENDIX B: INPUT DATA

APPENDIX B1: LINEDATA 30-BUS for integrated system

From	To	Impedance	Half of line charging	Tap ratio setting	From	To	Impedance	Half of line charging	Tap ratio setting
1	2	0.012+j0.081	0.00+j0.065	1.00	14	15	0.049+j0.014	0.00+j0.00	1.00
2	3	0.020+j0.111	0.00+j0.085	1.00	13	16	0.026+j0.597	0.00+j0.062	1.00
1	5	0.039+j0.154	0.00+j0.122	1.00	16	17	0.00+j0.7373	0.00+j0.00	1.00
2	5	0.025+j0.136	0.00+j0.010	1.00	17	19	0.036+j0.716	0.00+j0.00	1.00
2	24	0.016+j0.090	0.00+j0.068	1.00	18	17	0.018+j0.037	0.00+j0.00	1.00
3	4	0.034+j0.019	0.00+j0.143	1.00	20	19	0.00+j0.1416	0.00+j0.00	1.00
5	6	0.014+j0.011	0.00+j0.087	1.00	20	21	0.023+j0.014	0.00+j0.111	1.00
6	7	0.00+j0.274	0.00+j0.00	1.00	21	22	0.021+j0.131	0.00+j0.100	1.00
6	8	0.018+j0.015	0.00+j0.117	1.00	22	23	0.033+j0.017	0.00+j0.137	1.00
7	12	0.086+j0.196	0.00+j0.020	1.00	23	24	0.021+j0.012	0.00+0.081	1.00
8	9	0.00+j0.062	0.00+j0.00	1.00	22	25	0.034+j0.188	0.00+j0.143	1.00
9	10	0.043+j0.098	0.00+j0.00	1.00	25	26	0.022+j0.118	0.00+j0.095	1.00
9	12	0.010+j0.232	0.00+j0.024	1.00	25	29	0.00+j0.160	0.00+j0.00	1.00
10	11	0.052+j0.030	0.00+j0.00	1.00	26	27	0.00+j0.160	0.00+j0.00	1.00
12	13	0.018+j0.418	0.00+j0.043	1.00	27	28	0.263+j0.597	0.00+j0.061	1.00
13	14	0.009+j0.027	0.00+j0.00	1.00	29	30	0.021+j0.485	0.00+j041	1.00
13	15	0.063+j0.014	0.00+j0.00	1.00					

APPENDIX B2: BUSDATA-30 BUS for integrated system

Bus No.	Load demand		Generation		Bus No	Load demand		Generation	
	MW	MVAr	MW	MVAr		MW	MVAr	MW	MVAr
1	-	-	-	-	16	23.10	09.00	-	-
2	06.20	01.60	-	-	17	00.00	00.00	-	-
3	20.00	07.00	-	-	18	-	-	03.60	-
4	27.00	07.80	14	-	19	22.00	05.00	-	-
5	-	-	142.00	-	20	00.00	00.00	-	-
6	18.00	09.10	-	-	21	06.50	01.20	-	-
7	00.00	00.00	-	-	22	05.00	01.40	-	-
8	233.10	45.10	-	-	23	06.20	01.60	-	-
9	-	-	259.00	-	24	-	-	74.00	-
10	-	-	100.00	-	25	21.70	09.00	-	-
11	17.60	09.00	-	-	26	29.70	09.60	-	-
12	12.00	02.50	-	-	27	-	-	13.00	-
13	-	-	10.50	-	28	11.50	05.00	-	-
14	-	-	68.00	-	29	00.00	00.00	-	-
15	21.00	08.30	-	-	30	05.40	01.50	-	-

APPENDIX B3: zdata30 (30- Bus) for integrated system (WLS & WLAV) SE

Meas.	Type	Value	From	TO	Meas.	Type	Value	From	To
1	1	01.000	1	0	**50**	4	00.333	2	3
2	2	-00.062	2	0	**51**	4	01.082	5	2
3	2	-00.200	3	0	**52**	4	00.109	2	24
4	2	-00.130	4	0	**53**	4	00.131	3	4
5	2	01.420	5	0	**54**	4	-00.877	5	6
6	2	00.000	7	0	**55**	4	-00.767	6	8
7	2	02.590	9	0	**56**	4	-00.318	7	12
8	2	00.100	10	0	**57**	4	-00.789	9	10
9	2	-00.176	11	0	**58**	4	00.270	9	12
10	2	-00.120	12	0	**59**	4	00.178	10	11
11	2	00.105	13	0	**60**	4	-00.178	12	13
12	2	00.680	14	0	**61**	4	00.063	13	15
13	2	-00.210	15	0	**62**	4	00.390	13	16
14	2	-00.231	16	0	**63**	4	00.151	14	15
15	2	00.036	18	0	**64**	4	00.036	18	17
16	2	-00.220	19	0	**65**	4	00.191	17	19
17	2	-00.065	21	0	**66**	4	-00.030	20	21
18	2	-00.050	22	0	**67**	4	-00.095	21	22
19	2	-00.062	23	0	**68**	4	-00.717	22	23
20	2	00.740	24	0	**69**	4	-00.846	24	23
21	2	-00.217	25	0	**70**	4	00.571	22	25
22	2	-00.297	26	0	**71**	4	00.288	25	26
23	2	001.30	27	0	**72**	4	00.119	27	28
24	2	-00.115	28	0	**73**	4	00.054	29	30
25	2	-00.054	30	0	**74**	5	-00.162	2	3
26	3	-00.016	2	0	**75**	5	-0.149	5	2
27	3	-00.070	3	0	**76**	5	-00.51	2	24
28	3	-00.214	4	0	**77**	5	-00.073	3	4
29	3	00.185	5	0	**78**	5	00.640	5	6
30	3	00.000	7	0	**79**	5	00.640	6	8
31	3	00.511	9	0	**80**	5	00.036	7	12
32	3	-00.224	10	0	**81**	5	00.391	9	10
33	3	-00.090	11	0	**82**	5	-00.068	9	12
34	3	-00.025	12	0	**83**	5	00.091	10	11
35	3	00.310	13	0	**84**	5	-00.007	12	13
36	3	-00.254	14	0	**85**	5	00.166	13	15
37	3	-00.083	15	0	**86**	5	00.031	13	16
38	3	-00.090	16	0	**87**	5	-00.082	14	15
39	3	00.057	18	0	**88**	5	00.057	18	17
40	3	-00.050	19	0	**89**	5	00.000	17	19
41	3	-00.012	21	0	**90**	5	-00.077	20	21
42	3	-00.014	22	0	**91**	5	-00.095	21	22
43	3	-00.016	23	0	**92**	5	-00.717	22	23
44	3	-01.072	24	0	**93**	5	-00.846	24	23
45	3	-00.090	25	0	**94**	5	00.571	22	25
46	3	-00.096	26	0	**95**	5	00.288	25	26
47	3	-00.032	27	0	**96**	5	00.119	27	28
48	3	-00.050	28	0	**97**	5	00.054	29	30
49	3	-00.015	30	0					

1: Voltage magnitude measurement.

2: Real power Injection measurement

3: Reactive Power Injection measurement

4: Real Power Flow Measurement

5: Reactive Power Flow measurement

APPENDIX B4: LINEDATA (LD6) for integrated system

From	To	Impedance	Half of line charging	Tap ratio setting
1	4	00.080+j00.370	00.000+j00.015	00.000
1	6	00.123+j00.158	00.000+j00.021	00.000
2	3	00.723+j01.050	00.000+j00.000	00.000
2	5	00.282+j00.640	00.000+j00.000	00.000
3	4	00.000+j00.133	00.000+j00.000	01.100
4	6	00.097+j00.407	00.000+j00.015	00.000
5	6	00.000+j00.300	00.000+j00.000	01.025

APPENDIX B5: BUS DATA (BD6) for integrated system

Bus No.	Voltage		Load demand		Generation	
	V_m [p.u.]	δ [degree]	[MW]	[MVAr]	[MW]	[MVAr]
1	1.050	00.000	-	-	-	-
2	1.100	-	00.000	00.000	50.1000	00.000
3	1.000	-	55.000	13.000	-	-
4	1.000	-	00.000	00.000	-	-
5	1.000	-	30.000	18.000	-	-
6	1.000	-	50.000	05.000	-	-

APPENDIX B6: LINEDATA (LD 14) for integrated system

From	To	Impedance	Half of line charging	Tap ratio setting	From	To	Impedance	Half of line charging	Tap ratio setting
1	2	0.0194+j0.0592	0.00+j0.0264	1.00	6	11	0.095+j0.1989	0.00+j0.00	1.00
1	5	0.054+j0.223	0.00+j0.0246	1.00	6	12	0.1229+j0.2258	0.00+j0.00	1.00
2	3	0.047+j0.198	0.00+j0.0219	1.00	6	13	0.0662+j0.1303	0.00+j0.00	1.00
2	4	0.0581+j0.1763	0.00+j0.0170	1.00	7	8	0.00+j0.1762	0.00+j0.00	1.00
2	5	0.057+j0.1739	0.00+j0.0064	1.00	7	9	0.00+j0.1100	0.00+j0.00	1.00
3	4	0.0134+j0.0421	0.00+j0.00	1.00	9	10	0.0318+j0.0845	0.00+j0.00	1.00
4	5	0.0134+j0.0421	0.00+j0.00	1.00	9	14	0.1271+j0.2704	0.00+j0.00	1.00
4	7	0.00+j0.2091	0.00+j0.00	1.00	10	11	0.0821+j0.1921	0.00+j0.00	1.00
4	9	0.00+j0.5562	0.00+j0.00	1.00	12	13	0.2209+j0.2000	0.00+j0.00	1.00
5	6	0.00+j0.2520	0.00+j0.00	1.00	13	14	0.1709+j0.3480	0.00+j0.00	1.00

APPENDIX B7: BUSDATA-IEEE14 for integrated system

Bus No.	Load demand		Generation		Bus No	Load demand		Generation	
	MW	MVAr	MW	MVAr		MW	MVAr	MW	MVAr
1	-	-	-	-	8	-	-	-	17.40
2	21.70	12.70	40.00	42.40	9	29.50	16.60	-	-
3	94.20	19.00	-	23.40	10	09.00	05.80	-	-
4	47.80	03.90	-	-	11	03.50	01.80	-	-
5	07.60	01.60	-	-	12	06.10	01.60	-	-
6	11.20	07.50	-	12.20	13	13.50	05.80	-	-
7	00.00	00.00	-	-	14	14.90	05.00	-	-

APPENDIX B8: LINEDATA (LD 30) for integrated system

From	To	Impedance	Half of line charging	Tap ratio setting	From	To	Impedance	Half of line charging	Tap ratio setting
1	2	0.0192+j0.0575	0.00+j0.0264	1.00	15	18	0.1073+j0.2185	0.00+j0.00	1.00
1	3	0.0452+j0.1652	0.00+j0.0204	1.00	18	19	0.0639+j0.1292	0.00+j0.00	1.00
2	4	0.057+j0.1737	0.00+j0.0184	1.00	19	20	0.034+j0.068	0.00+j0.00	1.00
3	4	0.0132+j0.0379	0.00+j0.0042	1.00	10	20	0.0936+j0.2090	0.00+j0.00	1.00
2	5	0.0472+j0.1983	0.00+j0.0209	1.00	10	17	0.0324+j0.0845	0.00+j0.00	1.00
2	6	0.0581+j0.1763	0.00+j0.0187	1.00	10	21	0.0348+j0.0749	0.00+j0.00	1.00
4	6	0.0119+j0.0414	0.00+j0.0045	1.00	10	22	0.0727+j0.1499	0.00+j0.00	1.00
5	7	0.046+j0.1160	0.00+j0.0102	1.00	21	23	0.0116+j0.0236	0.00+j0.00	1.00
6	7	0.0267+j0.0820	0.00+j0.0085	1.00	15	23	0.1000+j0.2020	0.00+j0.00	1.00
6	8	0.0120+j0.0420	0.00+j0.0045	1.00	22	24	0.1150+j0.1790	0.00+j0.00	1.00
6	9	0.00+j0.2080	0.00+j0.00	1.00	23	24	0.1320+j0.2700	0.00+j0.00	1.00
6	10	0.00+j0.5560	0.00+j0.00	1.00	24	25	0.1885+j0.3292	0.00+j0.00	1.00
9	11	0.00+j0.2080	0.00+j0.00	1.00	25	26	0.2544+j0.3800	0.00+j0.00	1.00
9	10	0.00+j0.1100	0.00+j0.00	1.00	25	27	0.1093+j0.2087	0.00+j0.00	1.00
4	12	0.00+j0.2560	0.00+j0.00	1.00	28	27	0.00+j0.3960	0.00+j0.00	1.00
12	13	0.00+j0.1400	0.00+j0.00	1.00	27	29	0.2198+j0.4153	0.00+j0.00	1.00
12	14	0.1231+j0.2559	0.00+j0.00	1.00	27	30	0.3202+j0.6027	0.00+j0.00	1.00
12	15	0.0662+j0.1304	0.00+j0.00	1.00	29	30	0.2399+j0.4533	0.00+j0.00	1.00
12	16	0.0945+j0.1987	0.00+j0.00	1.00	8	28	0.0636+j0.2000	0.00+j0.0214	1.00
14	15	0.221+j0.1997	0.00+j0.00	1.00	6	28	0.0169+j0.0599	0.00+j0.0650	1.00
16	17	0.0824+j0.1923	0.00+j0.00	1.00					

APPENDIX B9: BUSDATA-IEEE30 for integrated system

Bus No.	Load demand		Generation		Bus No	Load demand		Generation	
	MW	MVAr	MW	MVAr		MW	MVAr	MW	MVAr
1	-	-	-	-	16	03.50	01.80	-	-
2	21.70	12.70	40.00	50.00	17	09.00	05.80	-	-
3	02.40	01.00	-	--	18	03.20	00.90	-	-
4	07.60	01.60	-	-	19	09.50	03.40	-	-
5	94.20	19.00	-	37.00	20	02.20	00.70	-	-
6	00.00	00.00	-	-	21	17.50	11.20	-	-
7	22.80	10.90	-	-	22	0.00	00.00	-	-
8	30.00	30.00	-	37.30	23	03.20	01.60	-	-
9	00.00	00.00	-	-	24	08.70	06.70	-	-
10	05.80	02.00	-	-	25	00.00	00.00	-	-
11	00.00	00.00	-	-	26	03.50	02.30	-	-
12	11.20	07.50	-	-	27	00.00	00.00	-	-
13	00.00	00.00	-	10.20	28	00.00	00.00	-	-
14	06.20	01.60	-	-	29	02.40	00.90		
15	08.20	02.50	-	-	30	10.60	01.90	-	-

APPENDIX C: MATLAB PROGRAMS

APPENDIX C1: A program to obtain bus admittance matrix of an integrated system

Ybusppg.m

```
%% A Program to obtain Ybus admittance matrix
clear                       % Clear all variables...
clC                         % Close all programs...
i = sqrt(-1);               % Catering for complex components...
fbus = linedata(:,1);       % From bus number...
tbus = linedata(:,2);       % To bus number...
r =    linedata(:,3);       % Line resistance...
x =    linedata(:,4);       % Line reactance...
b =    linedata(:,5);       % Ground admittance...
a =    linedata(:,6);       % Transformer tap setting...
z = r+i*x;                  % Calculate line impedance...
y = 1./z;                   % Calculate line admittance...
B = i*b;                    % Make B imaginary...
%% Calculate number of buses and branches...
nbus = max(max(fbus),max(tbus)); % Number of buses in the system...
nbranch = length(tbus);          % Number of branches in the system...
Ybus = zeros(nbus,nbus);         % Initialize Ybus admittance matrix...
%% Formation of the OFF-diagonal elements in Ybus admittance matrix...
for k = 1:nbranch
    Ybus(fbus(k),tbus(k)) = Ybus(fbus(k),tbus(k))-y(k)/a(k);
    Ybus(tbus(k),fbus(k) = Ybus(fbus(k),tbus(k));
end
%% Formation of DIAGONAL elements in Ybus admittance matrix...
for m = 1:nbus
    for n = 1:branch
        if fbus(n) == m
            Ybus(m,m) = Ybus(m,m)+y(n)/(a(n)^2+b(n);
        elseif tbus(n) == m
            Ybus (m, m) = Ybus(m,m)+y(n)+b(n);
        end
    end
end
Ybus                                % Ybus admittance matrix...
```

APPENDIX C2: A program to decompose bus admittance matrix to block matrices

Ybusppg.m (Decomposition)

```
%% A Program to obtain Ybus admittance matrix and decompose the matrix
clear                        % Clear all variables...
clc                          % Close command window...
tic;
i = sqrt(-1);                % Catering for complex components...
fbus = linedata(:,1);        % From bus number...
tbus = linedata(:,2);        % To bus number...
r =    linedata(:,3);        % Line resistance...
x =    linedata(:,4);        % Line reactance...
b =    linedata(:,5);        % Ground admittance...
a =    linedata(:,6);        % Transformer tap setting...
z = r+i*x;                   % Calculate line impedance...
y = 1./z;                    % Calculate line admittance...
B = i*b;                     % Make B imaginary...
%% Calculate number of buses and branches...
nbus = max(max(fbus),max(tbus));  % Number of buses in the system...
nbranch = length(tbus);           % Number of branches in the system...
Ybus = zeros(nbus,nbus);          % Initialize Ybus admittance matrix...
%% Formation of the OFF-diagonal elements in Ybus admittance matrix...
for k = 1:nbranch
    Ybus(fbus(k),tbus(k)) = Ybus(fbus(k),tbus(k))-y(k)/a(k);
    Ybus(tbus(k),fbus(k) = Ybus(fbus(k),tbus(k));
end
%% Formation of DIAGONAL elements in Ybus admittance matrix...
for m = 1:nbus
    for n = 1:branch
        if fbus(n) == m
            Ybus(m,m) = Ybus(m,m)+y(n)/(a(n)^2+b(n);
        elseif tbus(n) == m
            Ybus(m,m) = Ybus(m,m)+y(n)+b(n);
        end
    end
end
Ybus                                % Ybus admittance matrix...
%% Decomposition of the Ybus admittance matrix into block sub matrices...
A = Ybus
A11 = A(1:12,1:12)
A12 = A(1:12,13:21)
A13 = A(1:12,22:30)

A21 = A(13:21,1:12)
A22 = A(13:21,13:21)
A23 = A(13:21,22:30)

A31 = A(22:30,1:12)
A32 = A(22:30,13:21)
A33 = A(22:30,22:30)
toc;
```

APPENDIX C3: A program to calculate load-flow using Newton-Raphson method
All files including the data files must be in the current directory in order to execute the program

Lfybus

```
% A program to obtain ybus matrix for load flow analysis...

 i = sqrt (-1);                    % Cater for complex value...
fb = linedata(:,1); tb = linedata(:,2); r = linedata(:,3);
x = linedata (:, 4); Bc = j*linedata (:, 5); a = linedata (:, 6);
nbranch =length(linedata(:,1));  % Number of branches...
nbus = max(max(fb), max(tb));    % Number of buses...
z = r + i*x;                     % Line impedance...
y= ones(nbr,1)./z;               %Line admittance...
for n = 1:nbr
if a(n) <= 0  a(n) = 1; else end
Ybus=zeros(nbus,nbus);           % Initialize Ybus ...

% Formation of the off-diagonal elements...
for k=1:nbranch;
       Ybus(fb(k),tb(k))=Ybus(fb(k),tb(k))-y(k)/a(k);
       Ybus(tb(k),fb(k))=Ybus(fb(k),tb(k));
    end
end
% Formation of the diagonal elements...
for  n=1:nbus
     for k=1:nbranch
         if fb(k)==n
         Ybus(n,n) = Ybus(n,n)+y(k)/(a(k)^2) + Bc(k);
         elseif tb(k)==n
         Ybus(n,n) = Ybus(n,n)+y(k) +Bc(k);
         else, end
     end
end
```

Lfnewton

```
% A program for load flow analysis by Newton-Raphson method
% developed by H. Saadat (Copyright (c) 1998) modified in 2010 by M.A.
Kusekwa

% to include two or more parallel lines
close all
clc;                     % Close commanding window
tic;                     % Start computation timing
% Initialize variables...
ns=0;                    % Number of slack bus
ng=0;                    % Number of generators
Vm=0;                    % Voltage magnitude
delta=0;                 % Initial angle
yload=0;                 % Power constant
deltad=0;                % Phase angle
nbus = length(busdata(:,1));     % Number of system buses...

% Initialize system values
kb=[];                   % Type of bus
Vm=[];                   % Voltage magnitude
delta=[];                % Initial phase angle
Pd=[];                   % Real demand power
```

```
Qd=[];                  % Reactive demand power
Pg=[];                  % Real generation power
Qg=[];                  % Reactive generation power
Qmin=[];                % Minimum reactive power
Qmax=[];                % Maximum reactive power

% Initialize scheduled powers
Pk=[];                  % Real scheduled power at bus k
P=[];                   % Real scheduled power
Qk=[];                  % Reactive scheduled power at bus k
Q=[];                   % Reactive scheduled power
S=[];                   % Scheduled apparent power
V=[];                   % Scheduled voltage

% Matrix column...
for k=1:nbus
n=busdata(k,1);             % First column number of buses
kb(n)=busdata(k,2);         % Second column, type of bus
Vm(n)=busdata(k,3);         % Third column, voltage magnitude in p.u.
delta(n)=busdata(k, 4);     % Fourth column, phase angle in degree
Pd(n)=busdata(k,5);         % Fifth column, real demand power
Qd(n)=busdata(k,6);         % Sixth column, reactive demand power
Pg(n)=busdata(k,7);         % Seventh column, real generation power
Qg(n) = busdata(k,8);       % Eighth column, reactive generation power
Qmin(n)=busdata(k, 9);      % Ninth column, minimum reactive power
Qmax(n)=busdata(k, 10);     % Tenth column, maximum reactive power
Qsh(n)=busdata(k, 11);      % Eleventh column, injected reactive power

% Check computation values...
    if Vm(n) <= 0  Vm(n) = 1.0; V(n) = 1 + i*0;
    else delta(n) = pi/180*delta(n);
         V(n) = Vm(n)*(cos(delta(n)) + i*sin(delta(n)));
         P(n)=(Pg(n)-Pd(n))/basemva;
         Q(n)=(Qg(n)-Qd(n)+ Qsh(n))/basemva;
         S(n) = P(n) + i*Q(n);
    end
end
for k=1:nbus
if kb(k) == 1, ns = ns+1; else, end
if kb(k) == 2 ng = ng+1; else, end
ngs(k) = ng;
nss(k) = ns;
end

% Define ybus angles
Ym=abs(Ybus); t = angle(Ybus);
m=2*nbus-ng-2*ns;
maxerror = 1; converge=1;
iter = 0;

% Add parallel lines
mline=ones(nbranch,1);
for k=1:nbr
      for m=k+1:nbranc
         if((fb(k)==fb(m)) & (tb(k)==tb(m)));
            mline(m)=2;
         elseif ((fb(k)==tb(m)) & (tb(k)==fb(m)));
         mline(m)=2;
         else, end
      end
   end
% End of statements for parallel lines

% Start of iterations
```

```
clear A  DC   J  DX
while maxerror >= accuracy & iter <= maxiter % Test for max. power mismatch
for ii=1:m
for k=1:m
   A(ii,k)=0;        %Initializing Jacobian matrix
end, end
% Initialize iteration
iter = iter+1;
for n=1:nbus
nn=n-nss(n);
lm=nbus+n-ngs(n)-nss(n)-ns;
J11=0; J22=0; J33=0; J44=0;
   for ii=1:nbr
    if mline(ii)==1   % Added to include parallel lines
        if fb(ii) == n | tb(ii) == n
            if fb(ii) == n ,  l = tb(ii); end
            if tb(ii) == n ,  l = fb(ii); end
         J11=J11+ Vm(n)*Vm(l)*Ym(n,l)*sin(t(n,l)- delta(n) + delta(l));
         J33=J33+ Vm(n)*Vm(l)*Ym(n,l)*cos(t(n,l)- delta(n) + delta(l));
                if kb(n)~=1
                J22=J22+ Vm(l)*Ym(n,l)*cos(t(n,l)- delta(n) + delta(l));
                J44=J44+ Vm(l)*Ym(n,l)*sin(t(n,l)- delta(n) + delta(l));
                else, end
                if kb(n) ~= 1  & kb(l) ~=1
                    lk = nbus+l-ngs(l)-nss(l)-ns;
                    ll = l -nss(l);
                % Off diagonalelements of J1
                    A(nn, ll) =-Vm(n)*Vm(l)*Ym(n,l)*sin(t(n,l)- delta(n) +
delta(l));
                    if kb(l) == 3        % off diagonal elements of J2
                    A(nn, lk) =Vm(n)*Ym(n,l)*cos(t(n,l)- delta(n) +
delta(l));end
                    if kb(n) == 3        % off diagonal elements of J3
                    A(lm, ll) =-Vm(n)*Vm(l)*Ym(n,l)*cos(t(n,l)-
delta(n)+delta(l)); end
                    if kb(n) == 3 & kb(l) == 3  % off diagonal elements of
J4
                    A(lm, lk) =-Vm(n)*Ym(n,l)*sin(t(n,l)- delta(n) +
delta(l));end
               else end
            else , end
      else, end
   end
   Pk = Vm(n)^2*Ym(n,n)*cos(t(n,n))+J33;
   Qk = -Vm(n)^2*Ym(n,n)*sin(t(n,n))-J11;
   if kb(n) == 1 P(n)=Pk; Q(n) = Qk; end     % Swing bus P
     if kb(n) == 2  Q(n)=Qk;
         if Qmax(n) ~= 0
           Qgc = Q(n)*basemva + Qd(n) - Qsh(n);
           if iter <= 7            % Between the 2th & 6th iterations
              if iter > 2          % the Mvar of generator buses are
                if Qgc  < Qmin(n), % tested. If not within limits Vm(n)
                   Vm(n) = Vm(n) + 0.01;  % is changed in steps of
                                             0.01... pu to
                elseif Qgc  > Qmax(n), % bring the generator Mvar…
                                            within
                Vm(n) = Vm(n) - 0.01;end % the specified limits.
              else, end
           else,end
         else,end
     end
   if kb(n) ~= 1
     A(nn,nn) = J11;  %diagonal elements of J1
     DC(nn) = P(n)-Pk;
```

```
    end
    if kb(n) == 3
      A(nn,lm) = 2*Vm(n)*Ym(n,n)*cos(t(n,n))+J22;  %diagonal elements of J2
      A(lm,nn)= J33;         %diagonal elements of J3
      A(lm,lm) =-2*Vm(n)*Ym(n,n)*sin(t(n,n))-J44;  %diagonal of elements of
J4
      DC(lm) = Q(n)-Qk;
    end
end
% Calculate DX
DX=A\DC';
for n=1:nbus
  nn=n-nss(n);
  lm=nbus+n-ngs(n)-nss(n)-ns;
    if kb(n) ~= 1
    delta(n) = delta(n)+DX(nn); end
    if kb(n) == 3
    Vm(n)=Vm(n)+DX(lm); end
 end
  maxerror=max(abs(DC));
     if iter == maxiter & maxerror > accuracy
   fprintf('\nWARNING: Iterative solution did not converged after ')
   fprintf('%g', iter), fprintf(' iterations.\n\n')
   fprintf('Press Enter to terminate the iterations and print the results
\n')
   converge = 0; pause, else, end

end

if converge ~= 1
   tech= ('                      ITERATIVE SOLUTION DID NOT CONVERGE');
else,
   tech=('                    Power Flow Solution by Newton-Raphson
Method');
end
% Calculate V
V = Vm.*cos(delta)+j*Vm.*sin(delta);
deltad=180/pi*delta;
i=sqrt(-1);
k=0;
for n = 1:nbus
     if kb(n) == 1
     k=k+1;
     S(n)= P(n)+j*Q(n);
     Pg(n) = P(n)*basemva + Pd(n);
     Qg(n) = Q(n)*basemva + Qd(n) - Qsh(n);
     Pgg(k)=Pg(n);
     Qgg(k)=Qg(n);
     elseif  kb(n) ==2
     k=k+1;
     S(n)=P(n)+j*Q(n);
     Qg(n) = Q(n)*basemva + Qd(n) - Qsh(n);
     Pgg(k)=Pg(n);
     Qgg(k)=Qg(n);
  end
yload(n) = (Pd(n)- j*Qd(n)+j*Qsh(n))/(basemva*Vm(n)^2);
end
busdata(:,3)=Vm'; busdata(:,4)=deltad';
% Obtain outputs
Pgt = sum(Pg);  Qgt = sum(Qg); Pdt = sum(Pd); Qdt = sum(Qd); Qsht =
sum(Qsh);

%clear A DC DX J11 J22 J33 J44 Qk delta lk ll lm
%clear A DC DX J11 J22 J33 Qk delta lk ll lm
```

Lineflow.m

```
% This program is used in conjunction with Lfnewton
% for the calculation of line flow and line losses
% developed by H.Saadat (Copyright (c))
% modified in 2010
% Start of the program
SLT = 0;
fprintf('\n')
fprintf('                              Line Flow and Losses \n\n')
fprintf('      --Line--  Power at bus & line flow    --Line loss--
Transformer\n')
fprintf('      from  to    MW      Mvar     MVA       MW      Mvar
tap\n')

for n = 1:nbus
busprt = 0;
   for L = 1:nbranch;
       if busprt == 0
       fprintf('   \n'), fprintf('%6g', n), fprintf('      %9.3f',
P(n)*basemva)
       fprintf('%9.3f', Q(n)*basemva), fprintf('%9.3f\n',
abs(S(n)*basemva))

       busprt = 1;
       else, end
       if fb(L)==n      k = tb(L);
       In = (V(n) - a(L)*V(k))*y(L)/a(L)^2 + Bc(L)/a(L)^2*V(n);
       Ik = (V(k) - V(n)/a(L))*y(L) + Bc(L)*V(k);
       Snk = V(n)*conj(In)*basemva;
       Skn = V(k)*conj(Ik)*basemva;
       SL  = Snk + Skn;
       SLT = SLT + SL;
       elseif tb(L)==n  k = fb(L);
       In = (V(n) - V(k)/a(L))*y(L) + Bc(L)*V(n);
       Ik = (V(k) - a(L)*V(n))*y(L)/a(L)^2 + Bc(L)/a(L)^2*V(k);
       Snk = V(n)*conj(In)*basemva;
       Skn = V(k)*conj(Ik)*basemva;
       SL  = Snk + Skn;
       SLT = SLT + SL;
       else, end
         if fb(L)==n | tb(L)==n
         fprintf('%12g', k),
         fprintf('%9.3f', real(Snk)), fprintf('%9.3f', imag(Snk))
         fprintf('%9.3f', abs(Snk)),
         fprintf('%9.3f', real(SL)),
             if fb(L) ==n & a(L) ~= 1
             fprintf('%9.3f', imag(SL)), fprintf('%9.3f\n', a(L))
             else, fprintf('%9.3f\n', imag(SL))
             end
         else, end
  end
end
SLT = SLT/2;
fprintf('   \n'), fprintf('    Total loss                       ')
fprintf('%9.3f', real(SLT)), fprintf('%9.3f\n', imag(SLT))
clear Ik In SL SLT Skn Snk
```

Busout

```
% This program prints the power flow solution
% in a tabulated form on the screen
```

```
% developed by H. Saadat (Copyright (c) 1998)
% modified in 2010 by M.A. Kusekwa
% Start the program
disp(tech)
fprintf('                          Maximum Power Mismatch = %g \n', maxerror)
fprintf('                                  No. of Iterations = %g \n\n', iter)
head =['    Bus  Voltage  Angle    ------Load------    ---Generation---
Injected'
       '    No.  Mag.     Degree     MW       Mvar       MW       Mvar
Mvar '
       '
'];
disp(head)
for n=1:nbus
     fprintf(' %5g', n), fprintf(' %7.3f', Vm(n)),
     fprintf(' %8.3f', deltad(n)), fprintf(' %9.3f', Pd(n)),
     fprintf(' %9.3f', Qd(n)),  fprintf(' %9.3f', Pg(n)),
     fprintf(' %9.3f ', Qg(n)), fprintf(' %8.3f\n', Qsh(n))
end
    fprintf('      \n'), fprintf('    Total              ')
    fprintf(' %9.3f', Pdt), fprintf(' %9.3f', Qdt),
    fprintf(' %9.3f', Pgt), fprintf(' %9.3f', Qgt), fprintf(' %9.3f\n\n',
Qsht)
```

APPENDIX C4: A program to calculate SE using WLS criterion proposed by Deepark Krishnan (2004)

```
clear all;
clc;
global nbus;
global nlines;
global nshunts;
global tolerance;

nbus=input('enter the number of buses.');
nlines=input('enter the number of lines.');
nshunts=input('enter the number of shunts.');
tolerance=input('enter the tolerance.');

disp('1:input data 2:run program\n');
choice=input('enter whether you want to enter data or run the
program.\n');
if choice==1
    nbus=input('enter the number of buses.');
    nlines=input('enter the number of lines.');
    nshunts=input('enter the number of shunts.');
    tolerance=input('enter the tolerance.');
    dataentry(choice,nbus,nlines,nshunts,tolerance);
    %nbus=x(1);
    %nlines=x(2);
    %nshunts=x(3);
    %tolerance=x(4);
end
if choice==2
    programengine(choice,nbus,nlines,nshunts,tolerance);
end
%if choice==3
 %   viewoutput(choice);
 %end

function x1=programengine(choice,nbus,nlines,nshunts,tolerance);
nbus=nbus;
nlines=nlines;
nshunts=nshunts;
tolerance=tolerance;

%reading data related to buses
fid=fopen('busdata.txt','r');
a=textread('busdata.txt');
fclose(fid);

%reading data pertaining to lines
fid=fopen('linedata.txt','r');
b=textread('linedata.txt');
fclose(fid);

%reading data pertaining to shunts
fid=fopen('shuntdata.txt','r');
c=textread('shuntdata.txt');
fclose(fid);

%reading data pertaining to weights
fid=fopen('busweightsdata.txt','r');
buswts=textread('busweightsdata.txt');
fclose(fid);

fid=fopen('lineweightsdata.txt','r');
linewts=textread('lineweightsdata.txt');
```

```
fclose(fid);

v_wts=buswts(:,1:nbus);
p_wts=buswts(:,nbus+1:2*nbus);
q_wts=buswts(:,2*nbus+1:3*nbus);
pij_wts=linewts(:,1:nlines);
qij_wts=linewts(:,nlines+1:2*nlines);
%pji_wts=linewts(:,2*nlines+1:3*nlines);
%qji_wts=linewts(:,3*nlines+1:4*nlines);

%STATE ESTIMATION PROGRAM

%Formation of Ybus
Ybus=zeros(nbus);
tap=b(:,6);
x=b(:,1);
y=b(:,2);
zreal=b(:,3);
zimag=b(:,4);
y1=b(:,5);

if size(c)~=0
    shun_bus=c(:,1);
    shun_val=c(:,2);
end

%effect of line impedances and shunt admittances
for i=1:length(x)
    tap_effect11=(1/(zreal(i)+zimag(i)*j))/(tap(i)^2);
    tap_effect12=(-1/(zreal(i)+zimag(i)*j))/(tap(i));
    tap_effect21=(-1/(zreal(i)+zimag(i)*j))/(tap(i));
    tap_effect22=(1/(zreal(i)+zimag(i)*j));
    Ybus(x(i),x(i))=Ybus(x(i),x(i))+tap_effect11+y1(i);
    Ybus(y(i),y(i))=Ybus(y(i),y(i))+tap_effect22+y1(i);
    Ybus(x(i),y(i))=Ybus(x(i),y(i))+tap_effect12;
    Ybus(y(i),x(i))=Ybus(y(i),x(i))+tap_effect21;
end
Ybus;

%inclusion of shunts
if size(c)~=0
    for i=1:length(shun_bus)

Ybus(shun_bus(i),shun_bus(i))=Ybus(shun_bus(i),shun_bus(i))+shun_val(
i);
    end
end

%separation into G and B
G=real(Ybus);
B=imag(Ybus);

%computation of measurement mismatch
%the measurement mismatch vector is of the form [V Pi Qi Pij Qij Pji
Qji]' where each
%term represents a subvector

%state mismatch vector
%state mismatch vector is of the form [theta V] where term represents
a
%subvector and the theta subvector excludes the slack bus angle


v_bus=a(:,2);
```

```
ang_bus=a(:,3);
v_meas=a(:,4);
p_meas=a(:,5);
q_meas=a(:,6);

pij_flow=b(:,7);
qij_flow=b(:,8);
%pji_flow=b(:,9);
%qji_flow=b(:,10);

count=0;
flag=1;
while flag>0
%for count=0:5
 %voltage calculation
 v_calc=v_bus;

%Real power injection calculation
p_calc=[];
for i=1:nbus
    p_calc(i)=0;
    for j=1:nbus
        p_calc(i)=p_calc(i)+v_bus(i)*v_bus(j)*(G(i,j)*cos(ang_bus(i)-
ang_bus(j))+B(i,j)*sin(ang_bus(i)-ang_bus(j)));
    end
end

%reactive power injection calculations
q_calc=[];
for i=1:nbus
    q_calc(i)=0;
    for j=1:nbus
        q_calc(i)=q_calc(i)+v_bus(i)*v_bus(j)*(G(i,j)*sin(ang_bus(i)-
ang_bus(j))-B(i,j)*cos(ang_bus(i)-ang_bus(j)));
    end
end

start_bus=x;
end_bus=y;

%i to j real flows calculation
pij_calc=[];
length(x)
for i=1:length(x)
    q=start_bus(i);
    r=end_bus(i);
    pij_calc(i)=-
((v_bus(q))^2)*G(q,r)+v_bus(q)*v_bus(r)*(cos(ang_bus(q)-
ang_bus(r))*G(q,r)+sin(ang_bus(q)-ang_bus(r))*B(q,r));
end

%i to j reactive flows calculation
qij_calc=[];
for i=1:length(x)
    q=start_bus(i);
    r=end_bus(i);

qij_calc(i)=((v_bus(q))^2)*B(q,r)+v_bus(q)*v_bus(r)*(sin(ang_bus(q)-
ang_bus(r))*G(q,r)-cos(ang_bus(q)-ang_bus(r))*B(q,r));
end


%the data for flows from bus j to bus i is not computed in this
algorithm
```

```
%because the transformer taps are taken wrt bus i. hence since no
separate
%calculations are performed for the tapping j to i flows are not
considered
%j to i real flows calculation
%pji_calc=[];
%for i=1:length(x)
 %    q=start_bus(i);
  %   r=end_bus(i);
  %   pji_calc(i)=-
((v_bus(r))^2)*G(q,r)+v_bus(q)*v_bus(r)*(cos(ang_bus(r)-
ang_bus(q))*G(q,r)+sin(ang_bus(r)-ang_bus(q))*B(q,r));
  %end

%j to i reactive flows calculation
%qji_calc=[];
%for i=1:length(x)
 %    q=start_bus(i);
  %   r=end_bus(i);
   %
qji_calc(i)=((v_bus(r))^2)*B(q,r)+v_bus(q)*v_bus(r)*(sin(ang_bus(r)-
ang_bus(q))*G(q,r)-cos(ang_bus(r)-ang_bus(q))*B(q,r));
   %end

%DETERMINING MISMATCH VECTORS
%voltage mismatch
%v_meas
%v_calc
mmv=v_meas-v_calc;
%mmv
%real injections
%p_meas
%p_calc
mmp=p_meas'-p_calc;
%mmp
%reactive injections
%q_meas
%q_calc
mmq=q_meas'-q_calc;
%mmq
%real ij flows
%pij_flow
%pij_calc
mmpij=pij_flow'-pij_calc;
%mmpij
%reactive ij flows
%qij_flow
%qij_calc
mmqij=qij_flow'-qij_calc;
%mmqij
%real ji flows
%pji_flow
%pji_calc
%mmpji=pji_flow'-pji_calc;
%mmpji
%reactive ji flows
%qji_flow
%qji_calc
%mmqji=qji_flow'-qji_calc;
%mmqji

%FINAL MISMATCH VECTOR
%mm=[mmv' mmp mmq mmpij mmqij mmpji mmqji]'
mm=[mmv' mmp mmq mmpij mmqij]';
```

```
nmeas=length(mm);

matrixstate=zeros(2*nbus);
state=(matrixstate(1,:))';
nstate=length(state);

%formation of the Jacobian
Hv=zeros(nbus,nstate);
Hp=zeros(nbus,nstate);
Hq=zeros(nbus,nstate);
Hpij=zeros(nlines,nstate);
Hqij=zeros(nlines,nstate);
%Hpji=zeros(nlines,nstate);
%Hqji=zeros(nlines,nstate);

bus_number=[];
for i=1:nbus
    bus_number=[bus_number i];
end

%The complete Jacobian H will be an agglomeration of these
submatrices
line_tracker=[x';y'];
bus_number;
%formation of Hv submatrix corresponding to voltage mismatches
for i=1:length(mmv)
    for j=1:nstate
        if j~=(nbus+i)
            Hv(i,j)=0;
        end
        if j==(nbus+i)
            Hv(i,j)=1;
        end
    end
end
%Hv

%formation of Hp submatrix corresponding to real power injection
mismatches
for i=1:length(mmp)
    for j=1:nstate
        if i==j
            Hp(i,j)=-q_calc(i)-B(i,i)*v_bus(i)*v_bus(i);
        end
        if j~=i & j<=nbus
            Hp(i,j)=v_bus(i)*v_bus(j)*(G(i,j)*sin(ang_bus(i)-
ang_bus(j))-B(i,j)*cos(ang_bus(i)-ang_bus(j)));
        end
        if j==i+nbus
            Hp(i,j)=p_calc(i)/v_bus(i)+G(i,i)*v_bus(i);
        end
        if j~=i+nbus & j>nbus
            Hp(i,j)=v_bus(i)*v_bus(j-nbus)*(G(i,j-
nbus)*cos(ang_bus(i)-ang_bus(j-nbus))+B(i,j-nbus)*sin(ang_bus(i)-
ang_bus(j-nbus)));
        end
    end
end
%Hp

%formation of Hq submatrix corresponding to reactive power injection
%mismatches
for i=1:length(mmq)
    for j=1:nstate
```

```
        if i==j
            Hq(i,j)=p_calc(i)-G(i,i)*v_bus(i)*v_bus(i);
        end
        if j~=i & j<=nbus
            Hq(i,j)=-v_bus(i)*v_bus(j)*(G(i,j)*cos(ang_bus(i)-
ang_bus(j))+B(i,j)*sin(ang_bus(i)-ang_bus(j)));
        end
        if j==i+nbus
            Hq(i,j)=q_calc(i)/v_bus(i)-B(i,i)*v_bus(i);
        end
        if j~=i+nbus & j>nbus
            Hq(i,j)=v_bus(i)*v_bus(j-nbus)*(G(i,j-
nbus)*sin(ang_bus(i)-ang_bus(j-nbus))-B(i,j-nbus)*cos(ang_bus(i)-
ang_bus(j-nbus)));
        end
    end
end
%Hq

%formation of Hpij matrix corresponding to real power flow between
bus i and j
%mismatches
for i=1:length(mmpij)
    for j=1:nstate
        temp=line_tracker(:,i);
        Hpij(i,j)=0;
        if j==temp(1)
            Hpij(i,j)=v_bus(temp(1))*v_bus(temp(2))*(-
G(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*cos(ang_bus(temp(1))-
ang_bus(temp(2))));
        end
        if j==temp(2)

Hpij(i,j)=v_bus(temp(1))*v_bus(temp(2))*(G(temp(1),temp(2))*sin(ang_b
us(temp(1))-ang_bus(temp(2)))-
B(temp(1),temp(2))*cos(ang_bus(temp(1))-ang_bus(temp(2))));
        end
        if j==temp(1)+nbus
            Hpij(i,j)=-
2*v_bus(temp(1))*G(temp(1),temp(2))+v_bus(temp(2))*(G(temp(1),temp(2)
)*cos(ang_bus(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2))));
        end
        if j==temp(2)+nbus

Hpij(i,j)=v_bus(temp(1))*(G(temp(1),temp(2))*cos(ang_bus(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2))));
        end
    end
end
%Hpij

%formation of Hqij matrix corresponding to reactive power flow
between bus i and j
%mismatches
for i=1:length(mmqij)
    for j=1:nstate
        temp=line_tracker(:,i);
        Hqij(i,j)=0;
        if j==temp(1)
```

```
Hqij(i,j)=v_bus(temp(1))*v_bus(temp(2))*(G(temp(1),temp(2))*cos(ang_b
us(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2))));
        end
        if j==temp(2)
            Hqij(i,j)=v_bus(temp(1))*v_bus(temp(2))*(-
G(temp(1),temp(2))*cos(ang_bus(temp(1))-ang_bus(temp(2)))-
B(temp(1),temp(2))*sin(ang_bus(temp(1))-ang_bus(temp(2))));
        end
        if j==temp(1)+nbus

Hqij(i,j)=2*v_bus(temp(1))*B(temp(1),temp(2))+v_bus(temp(2))*(G(temp(
1),temp(2))*sin(ang_bus(temp(1))-ang_bus(temp(2)))-
B(temp(1),temp(2))*cos(ang_bus(temp(1))-ang_bus(temp(2))));
        end
        if j==temp(2)+nbus

Hqij(i,j)=v_bus(temp(1))*(G(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2)))-B(temp(1),temp(2))*cos(ang_bus(temp(1))-
ang_bus(temp(2))));
        end
    end
end
%Hqij

%formation of Hpji matrix corresponding to real power flow between
bus j
%and i mismatches
%for i=1:length(mmpji)
 %   for j=1:nstate
  %      temp=line_tracker(:,i);
   %     trash=temp(1);
    %    temp(1)=temp(2);
     %   temp(2)=trash;
      %  Hpji(i,j)=0;
       % if j==temp(1)
        %     Hpji(i,j)=v_bus(temp(1))*v_bus(temp(2))*(-
G(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*cos(ang_bus(temp(1))-
ang_bus(temp(2))));
        %end
       % if j==temp(2)
            %
Hpji(i,j)=v_bus(temp(1))*v_bus(temp(2))*(G(temp(1),temp(2))*sin(ang_b
us(temp(1))-ang_bus(temp(2)))-
B(temp(1),temp(2))*cos(ang_bus(temp(1))-ang_bus(temp(2))));
            %end
        %if j==temp(1)+nbus
         %   Hpji(i,j)=-
2*v_bus(temp(1))*G(temp(1),temp(2))+v_bus(temp(2))*(G(temp(1),temp(2)
)*cos(ang_bus(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2))));
         %end
        %if j==temp(2)+nbus
        %
Hpji(i,j)=v_bus(temp(1))*(G(temp(1),temp(2))*cos(ang_bus(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2))));
        %end
        %end
        %end
```

```
%Hpji

%formation of Hqji matrix corresponding to reactive power flow
between bus
%j and i mismatches
%for i=1:length(mmqji)
 %    for j=1:nstate
  %       temp=line_tracker(:,i);
   %      trash=temp(1);
   %      temp(1)=temp(2);
   %      temp(2)=trash;
   %      Hqji(i,j)=0;
   %      if j==temp(1)
   %
Hqji(i,j)=v_bus(temp(1))*v_bus(temp(2))*(G(temp(1),temp(2))*cos(ang_b
us(temp(1))-
ang_bus(temp(2)))+B(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2))));
   %     end
  %      if j==temp(2)
  %          Hqji(i,j)=v_bus(temp(1))*v_bus(temp(2))*(-
G(temp(1),temp(2))*cos(ang_bus(temp(1))-ang_bus(temp(2)))-
B(temp(1),temp(2))*sin(ang_bus(temp(1))-ang_bus(temp(2))));
  %      end
 %       if j==temp(1)+nbus
 %
Hqji(i,j)=2*v_bus(temp(1))*B(temp(1),temp(2))+v_bus(temp(2))*(G(temp(
1),temp(2))*sin(ang_bus(temp(1))-ang_bus(temp(2)))-
B(temp(1),temp(2))*cos(ang_bus(temp(1))-ang_bus(temp(2))));
 %       end
%        if j==temp(2)+nbus
%
Hqji(i,j)=v_bus(temp(1))*(G(temp(1),temp(2))*sin(ang_bus(temp(1))-
ang_bus(temp(2)))-B(temp(1),temp(2))*cos(ang_bus(temp(1))-
ang_bus(temp(2))));
%        end
%    end
%end
%Hqji

%birth of Jacobian
%H=[Hv; Hp; Hq; Hpij; Hqij; Hpji; Hqji]
H=[Hv; Hp; Hq; Hpij; Hqij];
hsize=size(H);
if count==0
   fid=fopen('output.txt','w');
   fprintf(fid,'Iteration number %d\n\n',count);
   fprintf(fid,'The Jacobian\n\n');
    for i=1:hsize(1)
    fprintf(fid,'%10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f
%10.2f %10.2f %10.2f\n',H(i,:));
end
fclose(fid);
end
if count>0
    fid=fopen('output.txt','a');
    fprintf(fid,'Iteration number %d\n\n',count);
    fprintf(fid,'The Jacobian\n\n');
    for i=1:hsize(1)
        fprintf(fid,'%10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f
%10.2f %10.2f %10.2f\n',H(i,:));
    end
    fclose(fid);
end
```

```
%determination of weight matrix
%v_wts
%p_wts
%q_wts
%pij_wts
%qij_wts
%pji_wts
%qji_wts
%W=eye(nmeas,nmeas);
W=zeros(nmeas,nmeas);
%W
for i=1:nbus
    %i
    for j=1:nmeas
     %   j
        W(i,j)=0;
        if j==i
            W(i,j)=v_wts(i);
        end
    end
    %W
end
i_temp=i;

for i=i_temp+1:i_temp+nbus
    %i
    for j=1:nmeas
        W(i,j)=0;
        if j==i
            W(i,j)=p_wts(i-i_temp);
        end
    end
    %W
end
i_temp=i;
for i=i_temp+1:i_temp+nbus
    %i
    for j=1:nmeas
        W(i,j)=0;
        if j==i
            W(i,j)=q_wts(i-i_temp);
        end
    end
    %W
end
i_temp=i;
for i=i_temp+1:i_temp+nlines
    for j=1:nmeas
        W(i,j)=0;
        if j==i
            W(i,j)=pij_wts(i-i_temp);
        end
    end
    %W
end
i_temp=i;
for i=i_temp+1:i_temp+nlines
    for j=1:nmeas
        W(i,j)=0;
        if j==i
            W(i,j)=qij_wts(i-i_temp);
        end
    end
    %W
```

```
end
%i_temp=i;
%for i=i_temp+1:i_temp+nlines
 %    for j=1:nmeas
  %       W(i,j)=0;
   %      if j==i
    %          W(i,j)=pji_wts(i-i_temp);
    %    end
    %end
   % W
   %end
%i_temp=i;
%for i=i_temp+1:i_temp+nlines
 %    for j=1:nmeas
  %       W(i,j)=0;
   %      if j==i
    %          W(i,j)=qji_wts(i-i_temp);
    %    end
    %end
    %W
    %end
 %    size(H)
  %   size(H')
   % size(W)

%determination of state mismatch vector
%W;
Gain=H'*W*H;
%Gain
Gain(1,1)=10000000;
gsize=size(Gain);
fid=fopen('output.txt','a');
fprintf(fid,'\n\nThe following is the Gain matrix.\n\n');
for i=1:gsize(1);
    fprintf(fid,'%10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f
%10.2f %10.2f %10.2f\n',Gain(i,:));
end
fclose(fid);
niag=inv(Gain);
smm=inv(Gain)*H'*W*mm;
%test=smm*1000
count=count+1;
%v_bus
%ang_bus
smm(1)=0; %to ensure that bus angle 1 remains as reference. no change
in it
%smm
smma=smm(1:nbus,:);
smmv=smm(nbus+1:2*nbus,:);
%determination of tolerance
plz_giveta=[];
for i=1:length(smm)
    if smm(i)>tolerance
        plz_giveta(i)=1;
    end
end
if length(plz_giveta)>0
   flag=1;
    v_bus=v_bus+smmv;
   ang_bus=ang_bus+smma;
end
%v_bus
%ang_bus
if length(plz_giveta)==0
```

```
     flag=0;
    % clc;
    fid=fopen('output.txt','a');
     fprintf(fid,'\n\nConvergence has occured in %d
iterations.\n',count);
     fprintf(fid,'Estimated bus voltages.\n');
     fprintf(fid,'Bus number  Voltage\n');
     for i=1:nbus
         fprintf(fid,'%f %f\n',bus_number(i),v_bus(i));
     end
     fprintf(fid,'Estimated bus angles\n.');
     fprintf(fid,'Bus number  Angle\n');
     for i=1:nbus
         fprintf(fid,'%f %f\n',bus_number(i),ang_bus(i));
     end
     %Real power injection calculation
p_calc=[];
for i=1:nbus
     p_calc(i)=0;
     for j=1:nbus
         p_calc(i)=p_calc(i)+v_bus(i)*v_bus(j)*(G(i,j)*cos(ang_bus(i)-
ang_bus(j))+B(i,j)*sin(ang_bus(i)-ang_bus(j)));
     end
end
fprintf(fid,'Estimated real power bus injections.\n');
     fprintf(fid,'Bus number  P\n');
     for i=1:nbus
         fprintf(fid,'%f %f\n',bus_number(i),p_calc(i));
     end

%reactive power injection calculations
q_calc=[];
for i=1:nbus
     q_calc(i)=0;
     for j=1:nbus
         q_calc(i)=q_calc(i)+v_bus(i)*v_bus(j)*(G(i,j)*sin(ang_bus(i)-
ang_bus(j))-B(i,j)*cos(ang_bus(i)-ang_bus(j)));
     end
end
fprintf(fid,'Estimated reactive power bus injections.\n');
     fprintf(fid,'Bus number  Q\n');
     for i=1:nbus
         fprintf(fid,'%f %f\n',bus_number(i),q_calc(i));
     end

start_bus=x;
end_bus=y;

%i to j real flows calculation
pij_calc=[];
length(x);
for i=1:length(x)
     q=start_bus(i);
     r=end_bus(i);
     pij_calc(i)=-
((v_bus(q))^2)*G(q,r)+v_bus(q)*v_bus(r)*(cos(ang_bus(q)-
ang_bus(r))*G(q,r)+sin(ang_bus(q)-ang_bus(r))*B(q,r));
end
fprintf(fid,'Real line flows\n.');
fprintf(fid,'From_bus   To_bus  Real power flow\n');
for i=1:length(start_bus)
fprintf(fid,'%f %f  %f\n',start_bus(i),end_bus(i),pij_calc(i));

end
```

```
%i to j reactive flows calculation
qij_calc=[];
for i=1:length(x)
    q=start_bus(i);
    r=end_bus(i);

qij_calc(i)=((v_bus(q))^2)*B(q,r)+v_bus(q)*v_bus(r)*(sin(ang_bus(q)-
ang_bus(r))*G(q,r)-cos(ang_bus(q)-ang_bus(r))*B(q,r));

end
fprintf(fid,'Reactive line flows\n.');
fprintf(fid,'From_bus   To_bus  Reactive power flow\n');
for i=1:length(start_bus)
fprintf(fid,'%f %f  %f\n',start_bus(i),end_bus(i),qij_calc(i));

end
fclose(fid);
  break;
end
%if count==1
 %   fid=fopen('output.txt','w');
  %  fprintf(fid,'%6.2f %6.ff %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f
%6.2f %6.2f\n',smm');
   %fclose(fid);
   %end
%if count>1
 %  fid=fopen('output.txt','a');
  %fprintf(fid,'%6.2f %6.ff %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f
%6.2f\n',smm');
 %fclose(fid);
 %end
%count
End
function x=dataentry(choice,nbus,nlines,nshunts,tolerance);
%nbus=input('enter the number of buses.');
%nlines=input('enter the number of lines.');
%nshunts=input('enter the number of shunts.');
%tolerance=input('enter the tolerance.');
nbus=nbus;
nlines=nlines;
nshunts=nshunts;
tolerance=tolerance;
%Entering bus data
disp('The following code is employed for the status of the buses.');
disp('0-Slack Bus');
disp('1-Buses of PV or PQ type');
disp('You will now be asked data pertaining to initial conditions of
each bus.');
bus_status=[];
v_bus=[];
ang_bus=[];
%weight variable for voltage magnitudes and power injections
%funda of wts: if the value is somewhat near desired then wts are
low, else
%if values drop very low then wts have to be very high. no measure
implies
%very high weights
v_wts=[];
p_wts=[];
q_wts=[];
bus_number=[];
for i=1:nbus
```

```
    bus_number=[bus_number i];
    disp('This is the data for bus');
    disp(i);
    bus_status(i)=input('Bus status:');
    disp('Please enter the starting conditions.');
    v_bus(i)=input('Initial voltage:');
    ang_bus(i)=input('Initial angle:');
    disp('Please enter the measured values.');
    v_meas(i)=input('Please enter the measured value of voltage.');
    v_wts(i)=input('Enter the weight for this voltage measure.');
    p_meas(i)=input('Please enter the measured value of real power
injection.');
    p_wts(i)=input('Enter the weight for this real power injection
measure.');
    q_meas(i)=input('Please enter the measured value of reactive
power injection.');
    q_wts(i)=input('Enter the weight for this reactive power
injection measure.');
end
a=[bus_status' v_bus' ang_bus' v_meas' p_meas' q_meas'];
a=a';
fid=fopen('busdata.txt','w');
fprintf(fid,'%6.2f %6.2f %6.2f %6.2f %6.2f %6.2f\n',a);
fclose(fid);
%Entering line data
disp('You will now be asked to enter line data.');
start_bus=[];
end_bus=[];
resistance=[];
reactance=[];
shunt_admit=[];
tap=[]; %This refers only to the tap magnitude
%line flows
pij_flow=[];
qij_flow=[];
pji_flow=[];
qji_flow=[];
%weights for line flows..same fundas apply
pij_wts=[];
qij_wts=[];
pji_wts=[];
qji_wts=[];
for i=1:nlines
    fprintf('This is data for line %d',i);
    start_bus(i)=input('Enter the starting bus:');
    end_bus(i)=input('Enter the ending bus:');
    resistance(i)=input('Enter the resistance of the line:');
    reactance(i)=input('Enter the reactance of the line:');
    shunt_admit(i)=input('Enter the shunt admittance of the line:');
    tap(i)=input('Enter the tap of the line (only magnitude)');
    fprintf('Line flow from bus %d to bus
%d',start_bus(i),end_bus(i));
    pij_flow(i)=input('Enter the real power flow on the line.');
    pij_wts(i)=input('Enter the weights for this measure.');
    qij_flow(i)=input('Enter the reactive power flow on the line.');
    qij_wts(i)=input('Enter the weights for this measure.');
    %fprintf('Line flow from bus %d to bus
%d',end_bus(i),start_bus(i));
    %pji_flow(i)=input('Enter the real power flow on the line.');
    %pji_wts(i)=input('Enter the weights for this measure.');
    %qji_flow(i)=input('Enter the reactive power flow on the line.');
    %qji_wts(i)=input('Enter the weights for this measure.');
end
```

```
b=[start_bus' end_bus' resistance' reactance' shunt_admit' tap'
pij_flow' qij_flow'];
b=b';
fid=fopen('linedata.txt','w');
fprintf(fid,'%6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f\n',b);
fclose(fid);
%Entering shunt data
shunts=[];
bus_shunt=[];
for i=1:nshunts
    bus_shunt(i)=input('Enter the bus number of the shunt.');
    shunts(i)=input('Enter the shunt magnitude.');
end
c=[bus_shunt' shunts'];
c=c';
fid=fopen('shuntdata.txt','w');
fprintf(fid,'%6.2f %6.2f\n',c);
fclose(fid);

%writing weights data into file
buswts=[v_wts' p_wts' q_wts'];
%linewts=[pij_wts' qij_wts' pji_wts' qji_wts'];
linewts=[pij_wts' qij_wts'];
buswts=buswts';
linewts=linewts';
fid=fopen('busweightsdata.txt','w');
fprintf(fid,'%6.2f %6.2f %6.2f',buswts);
fclose(fid);
fid=fopen('lineweightsdata.txt','w');
fprintf(fid,'%6.2f %6.2f',linewts);
fclose(fid);


%x=[nbus nlines nshunts tolerance];
```

APPENDIX C5: A program to calculate SE using WLS criterion (improved version by M.A. Kusekwa-2009 using http://www.mathworks/acces/helpdesk/help/state estimation.html)
All files including the data files must be in the current directory in order to execute the program

ybusppg.m

```
% Program to obtain ybus admittance matrix…


function ybus = ybusppg(num)    % Returns ybus...

linedata = linedatas(num);      % Calling  Line Data...
fb = linedata(:,1);             % From bus number...
tb = linedata(:,2);             % To bus number...
r = linedata(:,3);              % Resistance, R...
x = linedata(:,4);              % Reactance, X...
b = linedata(:,5);              % Ground Admittance, B/2...
a = linedata(:,6);              % Tap setting value..
z = r + i*x;                    % Line impedance...
y = 1./z;                       % Inverse of line impedance...
B = i*b;                        % Make B imaginary...

nbus = max(max(fb),max(tb));     % Number of buses...
nbranch = length(fb);            % Number of branches ...
ybus = zeros(nbus,nbus);         % Initialize ybus matrix..

 % Formation of the Off Diagonal Elements...
 for k=1:nbranch
     ybus(fb(k),tb(k)) = ybus(fb(k),tb(k))-y(k)/a(k);
     ybus(tb(k),fb(k)) = ybus(fb(k),tb(k));
 end

 % Formation of Diagonal Elements....
 for m =1:nbus
     for n =1:nbranch
         if fb(n) == m
             ybus(m,m) = ybus(m,m) + y(n)/(a(n)^2) + b(n);
         elseif tb(n) == m
             ybus(m,m) = ybus(m,m) + y(n) + b(n);
         end
     end
 end
 %ybus;                          % Bus Admittance Matrix
 %zbus = inv(ybus);              % Bus Impedance Matrix
```

pol2rec.m

```
% Polar to Rectangular Conversion
% RECT - Complex matrix or number,
% RECT = A + iB, A = Real, B = Imag.
% RHO - Magnitude
% THETA - Angle in radians,…

function rect = pol2rect(rho,theta)
rect = rho.*cos(theta) + j*rho.*sin(theta);
```

rec2pol.m

```
% Polar to Rectangular Conversion
% [RHO THETA] = RECT2POL(X)
% X - Complex matrix or number, X = A + iB
% RHO - Magnitude
% THETA - Angle in radians

function [rho theta] = rect2pol(x)
rho = sqrt(real(x).^2 + imag(x).^2);
theta = atan(imag(x)./real(x));
```

bbusppg.m

```
% A program to obtain shunt admittance matrix…

function bbus = bbusppg(num)      % Returns b-bus..

linedata = linedatas(num);
fb = linedata(:,1);
tb = linedata(:,2);
b = linedata(:,5);
nbus = max(max(fb),max(tb));     % Number of buses…
nbranch = length(fb);            % Number  of branches...
bbus = zeros(nbus,nbus);

 for k=1:nbranch
     bbus(fb(k),tb(k)) = b(k);
     bbus(tb(k),fb(k)) = bbus(fb(k),tb(k));
 end
```

wls.m

```
% A program to obtain power system State estimation using WLS Method...

clear                        % Clear all variables...
clc;                         % Clear commanding window...
tic,
num = (num;                  % Number of bus system....
ybus = ybusppg(num);         % Get YBus...
zdata = zdatas(num);         % Get Measurement data...
bpq = bbusppg(num);          % Get shunt data...
nbus = max(max(zdata(:,4)),max(zdata(:,5))); % Get number of buses...
type = zdata(:,2);           % Type of measurement...
z = zdata(:,3);              % Measurement values...
fbus = zdata(:,4);           % From bus...
tbus = zdata(:,5);           % To bus...
Rii = diag(zdata(:,6));      % Measurement Error...
V = ones(nbus,1);            % Initialize the bus voltages...
delta = zeros(nbus,1);       % Initialize the bus angles...
X = [delta(2:end); V];       % State Vector...
G = real(ybus);
B = imag(ybus);

% Measurement index...
vi = find(type == 1); % Index of voltage magnitude measurements...
ppi = find(type == 2); % Index of real power injection measurements...
qi = find(type == 3); % Index of reactive power injection measurements...
pf = find(type == 4); % Index of real powerflow measurements...
qf = find(type == 5); % Index of reactive powerflow measurements...

% Number of measurement...
nvi = length(vi); % Number of Voltage measurements...
npi = length(ppi); % Number of Real Power Injection measurements...
nqi = length(qi); % Number of Reactive Power Injection measurements...
```

```
npf = length(pf); % Number of Real Power Flow measurements...
nqf = length(qf); % Number of Reactive Power Flow measurements...

% Initialize iteration...
iter = 1;
tol = 5;

while(tol > 1e-4)

%Measurement Function, h...
    h1 = V(fbus(vi),1);
    h2 = zeros(npi,1);
    h3 = zeros(nqi,1);
    h4 = zeros(npf,1);
    h5 = zeros(nqf,1);

    for i = 1:npi
        m = fbus(ppi(i));
        for k = 1:nbus
            h2(i) = h2(i) + V(m)*V(k)*(G(m,k)*cos(delta(m)-delta(k)) +
B(m,k)*sin(delta(m)-delta(k)));
        end
    end

    for i = 1:nqi
        m = fbus(qi(i));
        for k = 1:nbus
            h3(i) = h3(i) + V(m)*V(k)*(G(m,k)*sin(delta(m)-delta(k)) -
B(m,k)*cos(delta(m)-delta(k)));
        end
    end

    for i = 1:npf
        m = fbus(pf(i));
        n = tbus(pf(i));
        h4(i) = -V(m)^2*G(m,n) - V(m)*V(n)*(-G(m,n)*cos(delta(m)-delta(n))
- B(m,n)*sin(delta(m)-delta(n)));
    end

    for i = 1:nqf
        m = fbus(qf(i));
        n = tbus(qf(i));
        h5(i) = -V(m)^2*(-B(m,n)+bpq(m,n)) - V(m)*V(n)*(-
G(m,n)*sin(delta(m)-delta(n)) + B(m,n)*cos(delta(m)-delta(n)));
    end

    h = [h1; h2; h3; h4; h5];

  % Measurement residual...
    r = z - h;
    e = abs(r);
    l = e-r;
    u = e+r;

  %Calculate Jacobian matrix, H...
    % H11 - Derivative of V with respect to angles.. All Zeros
    H11 = zeros(nvi,nbus-1);

    % H12 - Derivative of V with respect to V...
    H12 = zeros(nvi,nbus);
    for k = 1:nvi
        for n = 1:nbus
            if n == k
                H12(k,n) = 1;
```

```
            end
        end
    end

    % H21 - Derivative of Real Power Injections with Angles...
    H21 = zeros(npi,nbus-1);
    for i = 1:npi
        m = fbus(ppi(i));
        for k = 1:(nbus-1)
            if k+1 == m
                for n = 1:nbus
                    H21(i,k) = H21(i,k) + V(m)* V(n)*(-G(m,n)*sin(delta(m)-
delta(n)) + B(m,n)*cos(delta(m)-delta(n)));
                end
                H21(i,k) = H21(i,k) - V(m)^2*B(m,m);
            else
                H21(i,k) = V(m)* V(k+1)*(G(m,k+1)*sin(delta(m)-delta(k+1))
- B(m,k+1)*cos(delta(m)-delta(k+1)));
            end
        end
    end

    % H22 - Derivative of Real Power Injections with V...
    H22 = zeros(npi,nbus);
    for i = 1:npi
        m = fbus(ppi(i));
        for k = 1:(nbus)
            if k == m
                for n = 1:nbus
                    H22(i,k) = H22(i,k) + V(n)*(G(m,n)*cos(delta(m)-
delta(n)) + B(m,n)*sin(delta(m)-delta(n)));
                end
                H22(i,k) = H22(i,k) + V(m)*G(m,m);
            else
                H22(i,k) = V(m)*(G(m,k)*cos(delta(m)-delta(k)) +
B(m,k)*sin(delta(m)-delta(k)));
            end
        end
    end

    % H31 - Derivative of Reactive Power Injections with Angles...
    H31 = zeros(nqi,nbus-1);
    for i = 1:nqi
        m = fbus(qi(i));
        for k = 1:(nbus-1)
            if k+1 == m
                for n = 1:nbus
                    H31(i,k) = H31(i,k) + V(m)* V(n)*(G(m,n)*cos(delta(m)-
delta(n)) + B(m,n)*sin(delta(m)-delta(n)));
                end
                H31(i,k) = H31(i,k) - V(m)^2*G(m,m);
            else
                H31(i,k) = V(m)* V(k+1)*(-G(m,k+1)*cos(delta(m)-delta(k+1))
- B(m,k+1)*sin(delta(m)-delta(k+1)));
            end
        end
    end

    % H32 - Derivative of Reactive Power Injections with V...
    H32 = zeros(nqi,nbus);
    for i = 1:nqi
        m = fbus(qi(i));
        for k = 1:(nbus)
            if k == m
```

```
                for n = 1:nbus
                    H32(i,k) = H32(i,k) + V(n)*(G(m,n)*sin(delta(m)-
delta(n)) - B(m,n)*cos(delta(m)-delta(n)));
                end
                H32(i,k) = H32(i,k) - V(m)*B(m,m);
            else
                H32(i,k) = V(m)*(G(m,k)*sin(delta(m)-delta(k)) -
B(m,k)*cos(delta(m)-delta(k)));
            end
        end
    end

    % H41 - Derivative of Real Power Flows with Angles...
    H41 = zeros(npf,nbus-1);
    for i = 1:npf
        m = fbus(pf(i));
        n = tbus(pf(i));
        for k = 1:(nbus-1)
            if k+1 == m
                H41(i,k) = V(m)* V(n)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n)));
            else if k+1 == n
                H41(i,k) = -V(m)* V(n)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n)));
                else
                    H41(i,k) = 0;
                end
            end
        end
    end
        % H42 - Derivative of Real Power Flows with V...
    H42 = zeros(npf,nbus);
    for i = 1:npf
        m = fbus(pf(i));
        n = tbus(pf(i));
        for k = 1:nbus
            if k == m
                H42(i,k) = -V(n)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n))) - 2*G(m,n)*V(m);
            else if k == n
                H42(i,k) = -V(m)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n)));
                else
                    H42(i,k) = 0;
                end
            end
        end
    end

    % H51 - Derivative of Reactive Power Flows with Angles...
    H51 = zeros(nqf,nbus-1);
    for i = 1:nqf
        m = fbus(qf(i));
        n = tbus(qf(i));
        for k = 1:(nbus-1)
            if k+1 == m
                H51(i,k) = -V(m)* V(n)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n)));
            else if k+1 == n
                H51(i,k) = V(m)* V(n)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n)));
                else
                    H51(i,k) = 0;
                end
```

```
                end
            end
        end

        % H52 - Derivative of Reactive Power Flows with V...
        H52 = zeros(nqf,nbus);
        for i = 1:nqf
            m = fbus(qf(i));
            n = tbus(qf(i));
            for k = 1:nbus
                if k == m
                    H52(i,k) = -V(n)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n))) - 2*V(m)*(-B(m,n)+ bpq(m,n));
                else if k == n
                    H52(i,k) = -V(m)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n)));
                    else
                        H52(i,k) = 0;
                    end
                end
            end
        end

    % Measurement Jacobian, H…
        H = [H11 H12; H21 H22; H31 H32; H41 H42; H51 H52];

    % Gain Matrix, Gm...
        Gm = H'*inv (Rii)*H;

    %Objective Function…
        J = sum (inv(Rii)*r.^2);

    % State Vector…
        dX = inv (Gm)*(H'*inv(Rii)*r);
        X = X + dX;
        delta(2:end) = X(1:nbus-1);
        V = X(nbus:end);
        iter = iter + 1;
        tol = max(abs(dX));
end

% Covariance matrix...
CvE = diag (inv (H'*inv (Rii)*H));

Delta = 180/pi*delta;

  % Bus Voltages and angles...
X2 = [V Delta];
disp('-------- State Estimation ------------------');
disp('--------------------------');
disp('| Bus |    V    |  Angle  | ');
disp('| No  |    pu   |  Degree | ');
disp('--------------------------');
for m = 1:n
    fprintf('%4g', m); fprintf('  %8.4f', V(m)); fprintf('   %8.4f',
Delta(m)); fprintf('\n');
end
disp('-------------------------------------------------');
toc;
```

APPENDIX C6: **A program to calculate state estimation using WLAV criterion developed by M.A. Kusekwa-2010 with assistance from http://www.mathworks.com/access/helpdesk/help/document.html)**

All files including the data files must be in the current directory in order to execute the program

ybusppg.m

```
% Program to obtain ybus admittance matrix…


function ybus = ybusppg(num)    % Returns ybus...

linedata = linedatas(num);      % Calling  Line Data...
fb = linedata(:,1);             % From bus number...
tb = linedata(:,2);             % To bus number...
r = linedata(:,3);              % Resistance, R...
x = linedata(:,4);              % Reactance, X...
b = linedata(:,5);              % Ground Admittance, B/2...
a = linedata(:,6);              % Tap setting value..
z = r + i*x;                    % Line impedance...
y = 1./z;                       % Inverse of line impedance...
B = i*b;                        % Make B imaginary...

nbus = max(max(fb),max(tb));     % Number of buses...
nbranch = length(fb);            % Number of branches ...
ybus = zeros(nbus,nbus);         % Initialize ybus matrix..

 % Formation of the Off Diagonal Elements...
 for k=1:nbranch
     ybus(fb(k),tb(k)) = ybus(fb(k),tb(k))-y(k)/a(k);
     ybus(tb(k),fb(k)) = ybus(fb(k),tb(k));
 end

 % Formation of Diagonal Elements....
 for m =1:nbus
     for n =1:nbranch
         if fb(n) == m
             ybus(m,m) = ybus(m,m) + y(n)/(a(n)^2) + b(n);
         elseif tb(n) == m
             ybus(m,m) = ybus(m,m) + y(n) + b(n);
         end
     end
 end
 %ybus;                          % Bus Admittance Matrix
 %zbus = inv(ybus);              % Bus Impedance Matrix
```

pol2rec.m

```
% Polar to Rectangular Conversion
% RECT - Complex matrix or number,
% RECT = A + iB, A = Real, B = Imag.
% RHO - Magnitude
% THETA - Angle in radians,…

function rect = pol2rect(rho,theta)
rect = rho.*cos(theta) + j*rho.*sin(theta);
```

rec2pol.m

```
% Polar to Rectangular Conversion
% [RHO THETA] = RECT2POL(X)
% X - Complex matrix or number, X = A + iB
% RHO - Magnitude
% THETA - Angle in radians

function [rho theta] = rect2pol(x)
rho = sqrt(real(x).^2 + imag(x).^2);
theta = atan(imag(x)./real(x));
```

bbusppg.m

```
% A program to obtain shunt admittance matrix…

function bbus = bbusppg(num)     % Returns b-bus..

linedata = linedatas(num);
fb = linedata(:,1);
tb = linedata(:,2);
b = linedata(:,5);
nbus = max(max(fb),max(tb));     % Number of buses…
nbranch = length(fb);            % Number  of branches...
bbus = zeros(nbus,nbus);

 for k=1:nbranch
     bbus(fb(k),tb(k)) = b(k);
     bbus(tb(k),fb(k)) = bbus(fb(k),tb(k));
 end
```

Step length

```
% syntax: [alpha, alphax, alphau, alphal alphalambda, alphabeta]=...
%         = steplength(x,u,l lambda,beta Dx,Du,Dl Dlambda,Dbeta eta)
%
% PRIMAL VARIABLE
% Given current iterate (x, u,l) and steps (Dx,Du,Dl), calculate
% steplengths that ensure that x+ alphax*Dx>0, u +alphau*Du>0,
% l+alphal*Dl>0 and alpha = min(alphax, alphau,alphal).eta indicates the
% maximum fraction of step to the boundary 9typical value of eta: eta =
% 0.9995)
    alphax = -1/min(min(Dx./x),-1); alphax = min(1,eta*alphax);
    alphau = -1/min(min(Du./u),-1); alphau = min(1,eta*alphau);
    alphal = -1/min(min(Dl./l),-1);alphal = min(1,eta*alphal);
    alpha = min (alphax,alphau,alphal);

 % DUAL VARIABLE
% Given current iterate (lambda,beta) and steps (Dlambda,Dbeta), compute
steplengths
% that ensure that  lambda + alphalambda*Dlambda>0,beta+ alphabeta*Dbeta,
and
% alpha = min(alphalambda,alphabeta). eta indicates the maximum fraction of
% step to the boundary (typical value: eta=.9995)

  alphalambda = -1/min(min(Dlambda./lambda),-1); alphalambda = min(1, eta *
alphalambda);
  alphabeta = -1/min(min(Dbeta./beta),-1); alphabeta = min(1, eta
*alphabeta);
  alpha = min(alphalambda, alphabeta);
```

```
% WLAV main program

close all              % Close all programs
clear                  % Clear all variables
clc;                   % Clear commanding window
tic,
num = [];               % Ward Hale bus system
ybus = ybusppg(num);   % Get yBus
zdata = zdatas(num);   % Get Measurement data
bpq = bbusppg(num);    % Get B data
nbus = max(max(zdata(:,4)),max(zdata(:,5))); % Get number of buses
type = zdata(:,2);     % Type of measurement
nm = [];               % Number of measurement
z = zdata(:,3);        % Measuement values
fbus = zdata(:,4);     % From bus
tbus = zdata(:,5);     % To bus
Rii = diag(zdata(:,6)); % Measurement Error
W =inv(Rii);            % Weighting factor
V = ones(nbus,1);       % Initialize the bus voltages
delta = zeros(nbus,1);  % Initialize the bus angles
X = [delta(2:end); V];   % State Vector
G = real(ybus);
B = imag(ybus);

% Measurement index...
vi = find(type == 1);  % Index of voltage magnitude measurements
ppi = find(type == 2); % Index of real power injection measurements
qi = find(type == 3);  % Index of reactive power injection measurements
pf = find(type == 4);  % Index of real powerflow measurements
qf = find(type == 5);  % Index of reactive powerflow measurements

% Number of measurement
nvi = length(vi); % Number of Voltage measurements
npi = length(ppi); % Number of Real Power Injection measurements
nqi = length(qi); % Number of Reactive Power Injection measurements
npf = length(pf); % Number of Real Power Flow measurements
nqf = length(qf); % Number of Reactive Power Flow measurements

% Initialize iteration
iter = 1;
tol = 5;

while(tol > 1e-4)

%Measurement Function, h
    h1 = V(fbus(vi),1);
    h2 = zeros(npi,1);
    h3 = zeros(nqi,1);
    h4 = zeros(npf,1);
    h5 = zeros(nqf,1);

    for i = 1:npi
        m = fbus(ppi(i));
        for k = 1:nbus
            h2(i) = h2(i) + V(m)*V(k)*(G(m,k)*cos(delta(m)-delta(k)) +
B(m,k)*sin(delta(m)-delta(k)));
        end
    end
```

```
    for i = 1:nqi
        m = fbus(qi(i));
        for k = 1:nbus
            h3(i) = h3(i) + V(m)*V(k)*(G(m,k)*sin(delta(m)-delta(k)) -
B(m,k)*cos(delta(m)-delta(k)));
        end
    end

    for i = 1:npf
        m = fbus(pf(i));
        n = tbus(pf(i));
        h4(i) = -V(m)^2*G(m,n) - V(m)*V(n)*(-G(m,n)*cos(delta(m)-delta(n))
- B(m,n)*sin(delta(m)-delta(n)));
    end

    for i = 1:nqf
        m = fbus(qf(i));
        n = tbus(qf(i));
        h5(i) = -V(m)^2*(-B(m,n)+bpq(m,n)) - V(m)*V(n)*(-
G(m,n)*sin(delta(m)-delta(n)) + B(m,n)*cos(delta(m)-delta(n)));
    end

    h = [h1; h2; h3; h4; h5];

 % Measurement residual and error
    r = z - h;
    e = abs(r);

 % Initialize calculation procedure
    k      = 0;
    lambda = 0;       % Vector ofLagrange multiplier
    beta   = 0;       % Vector of Lagrange multiplier

  %Calculate Jacobian  matrix,H
    % H11 - Derivative of V with respect to angles.. All Zeros
    H11 = zeros(nvi,nbus-1);

    % H12 - Derivative of V with respect to V
    H12 = zeros(nvi,nbus);
    for k = 1:nvi
        for n = 1:nbus
            if n == k
                H12(k,n) = 1;
            end
        end
    end

    % H21 - Derivative of Real Power Injections with Angles
    H21 = zeros(npi,nbus-1);
    for i = 1:npi
        m = fbus(ppi(i));
        for k = 1:(nbus-1)
            if k+1 == m
                for n = 1:nbus
                    H21(i,k) = H21(i,k) + V(m)* V(n)*(-G(m,n)*sin(delta(m)-
delta(n)) + B(m,n)*cos(delta(m)-delta(n)));
                end
                H21(i,k) = H21(i,k) - V(m)^2*B(m,m);
            else
                H21(i,k) = V(m)* V(k+1)*(G(m,k+1)*sin(delta(m)-delta(k+1))
- B(m,k+1)*cos(delta(m)-delta(k+1)));
            end
        end
```

```
    end

    % H22 - Derivative of Real Power Injections with V
    H22 = zeros(npi,nbus);
    for i = 1:npi
        m = fbus(ppi(i));
        for k = 1:(nbus)
            if k == m
                for n = 1:nbus
                    H22(i,k) = H22(i,k) + V(n)*(G(m,n)*cos(delta(m)-
delta(n)) + B(m,n)*sin(delta(m)-delta(n)));
                end
                H22(i,k) = H22(i,k) + V(m)*G(m,m);
            else
                H22(i,k) = V(m)*(G(m,k)*cos(delta(m)-delta(k)) +
B(m,k)*sin(delta(m)-delta(k)));
            end
        end
    end

    % H31 - Derivative of Reactive Power Injections with Angles
    H31 = zeros(nqi,nbus-1);
    for i = 1:nqi
        m = fbus(qi(i));
        for k = 1:(nbus-1)
            if k+1 == m
                for n = 1:nbus
                    H31(i,k) = H31(i,k) + V(m)* V(n)*(G(m,n)*cos(delta(m)-
delta(n)) + B(m,n)*sin(delta(m)-delta(n)));
                end
                H31(i,k) = H31(i,k) - V(m)^2*G(m,m);
            else
                H31(i,k) = V(m)* V(k+1)*(-G(m,k+1)*cos(delta(m)-delta(k+1))
- B(m,k+1)*sin(delta(m)-delta(k+1)));
            end
        end
    end

    % H32 - Derivative of Reactive Power Injections with V
    H32 = zeros(nqi,nbus);
    for i = 1:nqi
        m = fbus(qi(i));
        for k = 1:(nbus)
            if k == m
                for n = 1:nbus
                    H32(i,k) = H32(i,k) + V(n)*(G(m,n)*sin(delta(m)-
delta(n)) - B(m,n)*cos(delta(m)-delta(n)));
                end
                H32(i,k) = H32(i,k) - V(m)*B(m,m);
            else
                H32(i,k) = V(m)*(G(m,k)*sin(delta(m)-delta(k)) -
B(m,k)*cos(delta(m)-delta(k)));
            end
        end
    end

    % H41 - Derivative of Real Power Flows with Angles
    H41 = zeros(npf,nbus-1);
    for i = 1:npf
        m = fbus(pf(i));
        n = tbus(pf(i));
        for k = 1:(nbus-1)
            if k+1 == m
```

```
                H41(i,k) = V(m)* V(n)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n)));
            else if k+1 == n
                H41(i,k) = -V(m)* V(n)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n)));
                else
                    H41(i,k) = 0;
                end
            end
        end
    end

    % H42 - Derivative of Real Power Flows with V
    H42 = zeros(npf,nbus);
    for i = 1:npf
        m = fbus(pf(i));
        n = tbus(pf(i));
        for k = 1:nbus
            if k == m
                H42(i,k) = -V(n)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n))) - 2*G(m,n)*V(m);
            else if k == n
                H42(i,k) = -V(m)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n)));
                else
                    H42(i,k) = 0;
                end
            end
        end
    end

    % H51 - Derivative of Reactive Power Flows with Angles
    H51 = zeros(nqf,nbus-1);
    for i = 1:nqf
        m = fbus(qf(i));
        n = tbus(qf(i));
        for k = 1:(nbus-1)
            if k+1 == m
                H51(i,k) = -V(m)* V(n)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n)));
            else if k+1 == n
                H51(i,k) = V(m)* V(n)*(-G(m,n)*cos(delta(m)-delta(n)) -
B(m,n)*sin(delta(m)-delta(n)));
                else
                    H51(i,k) = 0;
                end
            end
        end
    end

    % H52 - Derivative of Reactive Power Flows with V
    H52 = zeros(nqf,nbus);
    for i = 1:nqf
        m = fbus(qf(i));
        n = tbus(qf(i));
        for k = 1:nbus
            if k == m
                H52(i,k) = -V(n)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n))) - 2*V(m)*(-B(m,n)+ bpq(m,n));
            else if k == n
                H52(i,k) = -V(m)*(-G(m,n)*sin(delta(m)-delta(n)) +
B(m,n)*cos(delta(m)-delta(n)));
                else
                    H52(i,k) = 0;
```

```
                end
            end
        end
    end

 % Form measurement Jacobian, H
    H = [H11 H12; H21 H22; H31 H32; H41 H42; H51 H52];

    % Define upper and lower slack variables
    u = e+r;
    l = e-r;
 %  Form diagonal  matrix U,L

    U = diag(u);
    L = diag(l);

 % Calculate complementary gap, Cgap
    Cgap = (lambda)'*u + (beta)'*l;

 % Display comlementary gap
 disp('---------- complementary gap ----------')
 disp('---------------------------------------');

 % Calculate barrier parameter,myu
 myu = Cgap/2*nm;
 disp('--------------- barrier parameter-------')
 disp(----------------------------------------')

% Calculate matrix, D
    D = 0.5*((U).^k).^2 + 0.5*((L).^k).^2;

 % Calculate T
    T = ).5*((U).^k).^2;

 % Calculate K,
    K = 0.5*W*((U).^k).^2;

 % Calculate M,
    M = H'*W;

  % Calculate N,
  N = T*W;
  disp('---------T,K,M,N-----------------------')
  disp('---------------------------------------');


 %Objective Function
    J = sum(W'*e);

% Calculate Gm
    Gm = 2*H'*inv(D)*H;
 disp('------------- Gm -----------')
 disp('----------------------------');
 % State Vector
    dX = inv(Gm)*[H'*Inv(Rii)-2*H'*inv(D)*T*inv(Rii)];
    X = X + dX;
    delta(2:end) = X(1:nbus-1);
    V = X(nbus:end);
    iter = iter + 1;
    tol = max(abs(dX));
end


Delta = 180/pi*delta;
```

```
% Bus Voltages and angles
X2 = [V Delta];
disp('-------- State Estimation ------------------');
disp('---------------------------');
disp('| Bus |    V    |  Angle   | ');
disp('| No  |   pu    |  Degree | ');
disp('---------------------------');
for m = 1:n
    fprintf('%4g', m); fprintf('  %8.4f', V(m)); fprintf('   %8.4f',
Delta(m)); fprintf('\n');
end
disp('---------------------------------------------');
toc;
```

APPENDIX C7 (ParallelSE): **A program to calculate se using WLAV criterion under Parallel computing (under development and to be improved) developed by M.A. Kusekwa**
All files including the data files must be in the current directory in order to execute the program

```
%============ RUN A MATLAB SCRIPT AS BATCH JOB ============%

% A MATLAB script ParallelSE.m is on the client path, but it...
% has to be available to the workers. One way to do this is to use...
% the FileDependencies property of the pjob...
sz = matlabpool('size');
if sz>0
    disp({'a matlabpool of size'int2str(sz)'is open.']
    val = input('Do you want to close the matlabpool(y/n?','s');
    if val == 'n'    val == 'N'
        return
    else
        matlabpool close
    end
end

% Start clock...
t0 = clock;

% ============ START COORDINATING PROCEDURE ============%
j = 1;
while j <= M2   % Second level iteration
diary parallelWLAV

% Find a scheduler and create a scheduler object in MATLAB client...
% jm = findResource('scheduler','configuration',jobmanagerconfg4');
jm = findResource('scheduler','configuration','local');
disp(get(jm));            % Check the status of the jobmanager...


% ------------ TIMING ------------%
% How long it takes to create a job...
timingStart = tic;
start = tic;

% Create parallel job...
% pjob = CreateParallelJob(jm,'Configuration',jobmanagerconfig4');
pjob = createParallelJob(jm,'Configuration','local');

% RestartWorker...
% set(pjob,'RestartWorker',true);
get(pjob)

% Timing time to create job
times.jobCreateTime = toc(start);
description.jobCreateTime = 'Job creation time';

% pjob = createParallelJob(jm)
Tasks = findTask(pjob);
[pending running finished] = findTask(pjob);
% set(pjob,'Configuration','jobmanagerconfg4');
set(pjob,'Configuration','local'
set(pjob,'MinimumNumberOfWorkers',3);
set(pjob,'MaximumNumberOfWorkers',3);
set(pjob,'FileDependencies',{WLAV.m'})
get(jm)
```

```
disp('----------------------job---------')
disp(get(pjob))
disp('busy workers')
disp(get(jm,'NumberOfbusyWorkers'))


%============ START PARALLEL COMPUTING ============%
% Create one task with 2 output arguments and not input arguments...
% measure how long that takes and measure how long it takes to submit...
% the job to the cluster...
task = createTask(pjob,@WLAV.m,2,{});
get(pjob,'Tasks')
disp('------------task times------------')
tstime = get(pjob,'StartTime');
tetime = get(pjob,'FinishTime');
times.taskCreateTime = toc(start);
description.taskCreateTime = 'Task creation time';
get(pjob)
get(jm)
submit(pjob)
time.submitTime = toc(start);
description.submitTime = 'Job submission time';

% Once the job has been submitted, it is anticipated that all its...
% task execute in parallel. Meassure how long it takes for all tasks to ...
% start and to run to completion
waitForState(pjob),'finished')
%task = findTask(pjob,'ID',1);
times.jobWaitTime = toc(start);
description.pjobWaitTime = 'Job wait time';
get(jm)
disp('------------job------------')
disp('State')
disp(get(pjob,'State'))
disp('busy workers')
% disp(get(jm,'NumberOfbusyWorkers'))
disp('------------job------------')

% Tasks have now completed,so the code is beeing executed in the matlab
% client. Measure how long it takes to retrieve all the job result...
reults = getAllOutputArguments(pjob);
times.resultsTime = toc(start);
description.resultsTime = 'Results retrieval time';
get(pjob,'StartTime');
get(pjob,'FinishTime');

% Verify if the job ran without errors...
errmsgs = get(pjob.Tasks, {'ErrorMessage'}) %ok<NOPTS>
nonempty = cellfun(@isempty,errmsg) %ok<NOPTS>
celldisp(errmsgs(nonempty))

% Measure the total time elapsed from creating the job up to the
% results
times.totalTime = toc(start);
disp(sprintf('parallel time for %workers: %fseconds',numlabs,toc))
disp('State')
disp(get(pjob,'State'))
disp('starttime')
disp(get(pjob,'StartTime'))

operationtime = etime(clock,t0)
 disp('jobmanager')
 disp(get(jm))
 fin_jobs = findJob(jm,'State',finished')
```

```
 results = getAllOutputArguments(fin_jobs(length(fin_jobs)))
destroy(pjob)
times.destroyTime = toc(start);
description.destroyTime = 'Job destruction time';

disp('jobmanager')
disp(get(jm))
fin_jobs = findJob(jm,'State','finished')
results = getAllOutputArguments(fin_jobs(length(fin_jobs)))
```

APPENDIX D: BUS ADMITTANCE MATRIC (30-BUS)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	Y11	Y12	0	0	Y15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	Y21	Y22	Y23	0	Y25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y224	0	0	0	0	0	0
3	0	Y32	Y33	Y34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	Y43	Y44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	Y51	Y52	0	0	Y55	Y56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	Y65	Y66	Y67	Y68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	Y76	Y77	0	0	0	0	Y712	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	Y86	0	Y88	Y89	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	Y98	Y99	Y910	0	Y912	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	Y109	Y1010	Y1011	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	Y1110	Y1111	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	Y127	0	Y129	0	0	Y1212	Y1213	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	Y1312	Y1313	Y1314	Y1315	Y1316	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	Y1413	Y1414	Y1415	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	Y1513	Y1514	Y1515	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	Y1613	0	0	Y1616	Y1617	0	Y1619	Y1620	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y1716	Y1717	Y1718	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y1817	Y1818	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y1916	0	0	Y1919	Y1920	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2016	0	0	Y2019	Y2020	Y2021	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2120	Y2121	Y2122	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2221	Y2222	Y2223	0	Y2225	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2322	Y2323	Y2324	0	0	0	0	0	0
24	0	Y242	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2423	Y2424	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2522	0	0	Y2525	Y2526	0	0	Y2529	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2625	Y2626	Y2627	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2726	Y2727	Y2728	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2827	Y2828	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y2925	0	0	0	Y2929	Y2930
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Y3029	Y3030