Cape Peninsula
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ANALYSIS OF ERRORS PRODUCED BY NCV-LEVEL 2 STUDENTS WHEN SOLVING LINEAR EQUATIONS WITH FRACTIONAL COEFFICIENTS

by

NIKEZWA TRIPHINA MEHLO

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Supervisor: Dr S. McAuliffe

Co-Supervisor: Dr Mark Jacobs

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Declaration

I, Nikezwa Triphina Mehlo, declare that the contents of this thesis represent my own unaided work and that the thesis/dissertation has not previously been submitted for academic examination towards any qualification. Furthermore, it represents my own opinions and not necessarily those of the Cape Peninsula University of Technology. I acknowledge that portions of this thesis were developed with the assistance of AI tools (ChatGPT, OpenAI, 2025) used solely for language refinement and formatting suggestions. All intellectual content, analysis, and interpretations are my own.

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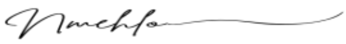

Abstract

There is growing concern and awareness of the low levels of mathematics achievement in the Technical Vocational Education and Training (TVET) sector. Students perform particularly poorly in algebra related activities. This study focuses on National Certificate Vocational (NCV) Level 2 students and the common errors they make when solving linear equations with fraction coefficients and the causes of these errors. Teachers should understand the conceptual difficulties students face when working with algebra and try to find ways to support their development. This is a qualitative case study that seeks to understand how students think and communicate when solving linear equations interpreted through the lens of Sfard's (2008) commognitive theory. The research question is why do NCV Level 2 students perform poorly when solving linear equations with fractional coefficients. It involves the use of student errors to understand their cognitive and discursive processes. Data were collected from a written test containing twelve linear equations (with and without fractions) completed by all students and individual interviews with a select group of students, based on their errors, to probe their algebraic reasoning. Data analysis included: identifying incorrect responses, classifying error types, categorising error patterns, and interpreting the explanations of students through the commognitive lens. The findings show that the students had difficulty with conceptual, procedural, and operational errors, as well as inverse operations, fractional coefficients, and the concept of equality. The study contributes to growing research on mathematics learning in the TVET field and highlights what student discourse can show about their reasoning practices and misconceptions. Mathematics lecturers must encourage more algebraic discourse and interactions in classrooms to make use of different representations and teaching strategies, and promote error-based learning approaches. These findings have important consequences for teaching practice, curriculum, and teacher professional development which is aimed at building student conceptual understanding and problem-solving skills in algebra. One of recommendations for future research would be to have a larger and more diverse sample of NCV Level 2 mathematics students to better understand how students' educational backgrounds might influence their conceptual understanding of linear equations with fraction coefficients..

Keywords: TVET, NCV Level 2, linear equations, fractional coefficients, student errors, commognition theory, and algebraic reasoning.

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Dedication

I dedicate this thesis to my beloved daughter, Kunenjabulo Mehlo, and my son, Sisongo Mehlo. Your presence has been my greatest source of strength, inspiration, and purpose throughout this journey.

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Glossary

CAPS	Curriculum and Assessment Policy Statement
WCED	Western Cape Education Department
TVET	Technical Vocational Education and Training
NCV	National Certiifcate Vocational

1 INTRODUCTION TO THE STUDY

1.1 Introduction

The focus of this study is to investigate the types of errors made by the National Certificate Vocational (NCV) Level 2 Mathematics students when solving linear equations, especially those with fractional coefficients, and the reasons for these errors. The study was conducted at a Technical, Vocational Education and Training (TVET) college in the Engineering department. The students were completing a mathematics course as part of their qualification for NCV Level 2, which is part of a programme that prepares the student for a particular career. Mathematics is one of the subjects in the programme, and many students struggle to pass the course. Students often do not perform well because their lack of understanding in mathematics causes difficulty in problem solving due to poor concept development (Ngoveni & Mofolo-Mbokane, 2019).

The minimum entrance requirements for NCV Level 2 are a Grade 9 qualification, but can also include Grade 9 and 12 students who drop out of the formal school sector. Some of the students have studied mathematics literacy from Grade 10 to Grade 12 instead of taking mathematics in high school. The NCV programme is intended to equip students with mathematical knowledge and skills that are directly applicable to engineering contexts and real-world technical problems. However, NCV Level 2 students struggle with mathematics and more especially with algebraic knowledge and skills. Often students' critical thinking in algebraic skills is hampered by the challenges they faced in the lower grades. McAuliffe, Tambara, and Simsek (2020) claim that students face challenges with the equivalence concept in primary schools which impacts their understanding of algebra and equations in the later grades. Many students do poorly in solving linear equations, which is one of the most important concepts in algebra. It is a fundamental core concept with broad applications and plays a crucial role in building more complex mathematical understanding. Equations enable modelling of real-life situations and help develop an understanding of more advanced mathematical concepts.

Many countries are affected by the quality of education and the success rates in vocational skills, knowledge, attitudes, and experiences (Ngoveni & Mofolo-Mbokane, 2019). In TVET colleges, students' difficulties with linear equations involving fractional coefficients are a global and national issue, as in the context of South Africa. This has affected the performance of students in mathematics. The Department of Higher Education and Training (2016) addressed the issue of low academic achievement in TVET colleges, and a report by the National Treasury (2016) in the same year showed that only 2% of students finish their programmes in the expected three-year period, mostly due to struggling with mathematics. The lack of algebraic knowledge and skills at NCV Level 2 affects students' learning in Levels 3 and 4, as these levels build on basic skills from Level 2, including solving fractional equations. This topic fits well in engineering fields such as engineering drawing, where students use algebraic and mathematical skills to produce patterns and visual designs. These basic skills are important in engineering production.

As a TVET lecturer, I observed that NCV Level 2 students tend to perform poorly in solving linear equations. Students demonstrate a lack of understanding of both procedural (how) and conceptual (why) knowledge when solving linear equations, especially those with fractional coefficients. They cannot explain their reasoning in the problem-solving processes, and they cannot communicate their mathematical understanding in solving algebraic problems, which could help reduce their number of errors. Nisa and Lukito (2021), Roberts and Le Roux (2019) as well as Lestari, Nusantara, Chandra, and Irfan (2020) noticed similar results in their research which highlighted the crucial role of mathematical thinking, communication, and reasoning in solving linear equations.

1.2 Background to the study

The continuous underperformance of National Certificate Vocational (NCV) students in mathematics within TVET colleges has raised concerns for educators and education departments across the country. This has led to various strategies to solve the issue. Current and previous studies highlight several errors that students make when solving mathematical problems. These errors impact students' ability to understand critical mathematical concepts, particularly algebra, when solving linear equations and continue to be a common struggle for them (Pournara, 2020; Samuel, Mulenga, & Angel, 2016; Wati & Fitriana, 2018). Although poor student performance may be affected by errors they make, Ngoveni (2018) suggests that basic mathematical knowledge and skills are lacking, and these are essential in algebra because they provide the foundation for solving more complex problems. It is important for students to have a good knowledge of the basic operations when needing to simplify expressions and solve equations; students often lack these skills and make errors based on these poor foundations. When solving equations, students can also make errors based on their lack of knowledge and understanding of variables, symbols, and coefficients in equations (Samuel, Mulenga, & Angel, 2016).

The same problem was observed among NCV Level 2 mathematics students. Students experienced difficulties in algebra and demonstrated poor performance in solving linear equations, particularly those involving fractional coefficients. Various errors were identified, including operational errors, computational errors, and conceptual misunderstandings when solving linear equations. Students also struggled to apply algebraic rules correctly. These observations are consistent with the findings of Pournara (2020).Algebraic thinking and algebra are taught in primary and high school, but many students, including those in TVET colleges, find it difficult to understand the topic because it feels more abstract than other areas of mathematics (Marpa, 2019). Two previous studies conducted in South African TVET colleges show poor mathematics performance and highlight a gap in the approach to teaching algebra that lacks real-life and practical application (Ngoveni & Mofolo-Mbokane, 2019; Vimbelo & Bayaga, 2024). This often leads to poor engagement in the learning process for students. Teachers also have a role to play in the strategies they use to teach equations in the mathematics classroom. There are three different approaches that have been researched and shown to be effective in the teaching of equations. Firstly, there is the innovative strategy highlighted by Roberts and Le Roux (2019), which can be used to develop student's problem-solving skills and conceptual understanding. Secondly, there is the balance method, which focuses on the concept of equality of equations (Anthony & Burgess, 2014; Otten, Van den Heuvel-Panhuizen, & Veldhuis, 2019). And finally, the transposition method was used to help students develop the ability to manipulate and solve algebraic equations.

1.3 Background of the researcher

Through my experience as a lecturer at a TVET college, I became aware of the many difficulties TVET students face during their mathematics learning, especially when it comes to algebra. These difficulties prevent them from successfully completing their qualifications, because they fail to pass mathematics, which is the final subject in their last year of study. I was interested to know why students struggle so much with mathematics and how it impacts their ability to progress in college. I also wanted to investigate the role of communication and its purpose in the mathematics classroom, particularly the role of discourse to help students better understand mathematics concepts. By analysing both the errors students make and the ways they talk about their errors, I hoped to gain insight into their thinking about the errors they make and how instruction can be improved to better support their learning. This drew me to the work of Sfard

(2008) and her *Theory on Commognition,* as I wanted to learn more about this theory and its implications mathematics teaching and learning.

1.4 Rationale for the study

As mentioned previously, NCV Level 2 students perform poorly in mathematics, and this can impact their future in mathematics related studies such as engineering and applied science studies. Student errors in mathematics can be a contributing factor to poor results (Ngoveni, 2018) and the aim of this research is to add to the study of errors students make when solving linear equations, particularly those involving fractional coefficients.

I am researching equations with fractional coefficients first because of the difficulty students have in transitioning from arithmetic to algebraic reasoning and the impact this can have in solving equations. Secondly, students struggle with conceptual understanding of fraction operations, which can add to difficulties when working with equations. And finally, the more we understand the reasons for student errors, the better we can adapt our teaching to support mathematical understanding.

The teachers need to continuously identify and address students' conceptual misunderstandings, procedural errors, and gaps in foundational knowledge to improve mathematical learning. We must work with the students' errors when solving problems and help them recognise, understand, and correct their solutions. This can help to boost student confidence and to understand that making errors is crucial in learning. According to Gardee (2015), errors are valuable contributors in the learning process and enhance deeper understanding. They provide opportunities for teachers to understand why students make errors and to implement teaching strategies that could be used efficiently to reduce future errors. The purpose of the study is to investigate the types of errors produced by NCV Level 2 students when solving linear equations, especially those with fractional coefficients, and to find possible causes of these errors. The findings are intended to inform teaching practices, and the mathematics curriculum and to suggest intervention strategies for NCV Level 2 mathematics lecturers to develop students' conceptual understanding and problem-solving skills in algebra.

1.5 The research problem

The learning progress and overall mathematics success of students can be impacted by the errors students make in the learning and understanding of algebra (Pournara, Sanders, & Takker, 2022). Previous studies in this area indicate that students are affected by errors of different types, such as conceptual errors, computation error, procedural error, equal sign error, and overgeneralisation, particularly with linear equations and fraction coefficients (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014; Msomi & Bansilal, 2022; Qetrani, Ouailal, & Achtaich, 2021). Linear equations are considered the foundation of algebra and part of the future development of problem solving and mathematical thinking. The focus of this study is to understand the nature of NCV Level 2 student errors and the causes of errors when solving algebraic equations with fraction coefficients.

1.6 Research questions

This study addresses the following research questions:

1.6.1 Main research question

Why do NCV Level 2 students perform poorly when solving linear equations with fractional coefficients?

1.6.2 Research sub-questions

What kinds of errors are demonstrated by students when solving linear equations with fractional coefficients?

What are the possible causes of the errors that students demonstrate when solving linear equations with fractional coefficients?

1.7 Purpose of the study

The purpose of the study is to investigate the types of errors produced by NCV-Level 2 students and the reasons for these errors when solving linear equations with fractional coefficients.

1.8 Literature review and Theoretical framework

1.8.1 Literature review

The purpose of this literature review is to provide an overview of the existing literature related to the errors students make when solving linear equations. The review considers effective strategies arising from the studies that can benefit students by improving their learning and enhancing their cognition connections in algebra. These effective strategies are presented and expanded in Chapter 2.

Some studies highlight the importance of understanding errors for effective teaching and learning (Gumpo, 2014; Ncube, 2016; Msomi & Bansilal, 2022; Pournara, 2020). These studies contribute to the learning process of mathematics and to understanding student thinking when solving linear equations. Students' thinking as characterised through the analysis of types of errors made allows for a deeper understanding of students' engagement with the algebraic concepts and procedures when solving equations.

Chow (2011), Jupri, Drijvers, and van den Heuvel-Panhuizen (2014), and Wati and Fitriana (2018) highlight students' weaknesses and misconceptions when solving equations. While other studies focus on the types of errors students make when solving linear equations such as conceptual and procedural errors (Gardee, 2015; Msomi & Bansilal, 2022; Ncube, 2016). Procedural errors occur when a student knows the correct concept but follows the wrong steps in applying it or uses an inappropriate strategy to solve a problem. Msomi and Bansilal (2022) found that students struggle to apply inverse operation correctly and to work with variables on both sides of the equation, which indicates gaps in procedural fluency. Conceptual errors arise when a student has an incomplete or incorrect understanding of the underlying mathematical concepts when solving an equation. Ncube (2016) found that students lack conceptual understanding, leading to difficulties in simplifying algebraic expressions. A brief discussion of the types of errors that students produce and the reasons for these errors when solving linear equations is presented and expanded in Chapter 2. The most frequent errors observed among students are those such as: misapplication of the equal sign, right-to-left reasoning, inverse errors, errors from misinterpretation of commutative and distributive properties, errors in generalising operations, and errors in the application of the minus sign.

Although previous studies highlighted, classified, and discussed the errors students made when solving linear equations with fractional coefficients, there is limited focus on NCV Level 2 (TVET) students. These students face challenges in solving linear equations, and these challenges remain underexplored. This study aims to address this gap by investigating the type of errors produced by NCV Level 2 students when solving linear equations with fractional coefficient with the aim of contributing to more targeted interventions for students.

1.8.2 Theoretical framework

The Commognition framework underpins this research. Commognition is the combination of two concepts, communication and cognition, and emphasises cognitive processing as a means of conveying ideas. The purpose of the study is to investigate the types of errors produced by NCV-Level 2 students and the reasons for these errors when solving linear equations with fractional coefficients. According to Sfard's (2008) theory, mathematical thinking is connected to the idea that learning mathematics is a process of discourse rather than simply acquiring knowledge. It is the theory of commognition that regards thinking as a kind of inner communication with oneself and that this inner dialogue is what happens during the learning process (Sfard, 2012). This theory describes mathematical thinking as a special type of communication, including four elementary aspects: Words in use (terminology), visual mediators (symbolic), routines (problem solving steps by steps or operation and procedure) and endorsed narratives (accepted explanations and justifications).

The commognition framework highlights the key conflicts that arise when students think or communicate based on their personal understanding or the internal logic of mathematics. It also considers how their use of terminology, symbols, procedures, and reasoning may differ from accepted mathematical conventions, often leading to errors or misunderstandings when engaging in problem solving.

Commognition is used in this study to analyse students' thinking and communication (talk, write, and think) when solving linear equations with fractional coefficients. It plays an essential role in the research to explore the conceptual and procedural understanding of students through the descriptions they give about their reasoning when solving problems.The purpose of the study is to investigate the types of errors produced by NCV-Level 2 students and the reasons for these errors when solving linear equations with fractional coefficients. Commognition theory helps shift the focus from *what* errors students make, to *why the errors* occur by analysing the discourse that accompany the problem-solving processes of the students.

1.9 Research methodology

This is a qualitative case study guided by an interpretive paradigm. The paradigm is appropriate because it seeks to understand the actual experiences of the participants and the meaning of their words about how they understand and approach solving linear equations. Using a qualitative approach in this research study helps to provide an in-depth understanding of how students solve linear equations with fraction coefficients. A case study design helped me explore the real-life situation of NCV Level 2 students within a TVET college environment (Creswell, 2012).

1.9.1 Site selection

The study was conducted at a public Technical and Vocational Education and Training (TVET) college in Cape Town, where I was employed as a Level 2 mathematics lecturer. I purposely selected this site for its accessibility and proximity to the participants involved in the study. TVET focuses on providing practical, career-orientated education and training, equipping students with skills and knowledge for specific trades and industries. The college was established in 2002. The study was conducted on the Engineering campus that offers various skills training programmes, particularly in the technical and engineering fields. The NCV Level 2 programme includes mathematics as a fundamental subject.

1.9.2 Sampling procedures

I used convenient and purposeful sampling methods to select participants for my study. I chose to study the students with whom I was working with in my class so that I could better understand their mathematical reasoning when solving equations. I wanted to work with students who were studying mathematics for NCV Level 2 because I knew that students had struggled with the course content in previous years.

1.9.3 Data collection

I designed a written test consisting of twelve different types of linear equation questions based on the curriculum requirements for Level 2 mathematics (refer to p.47). The questions included equations with fractions and non-fraction coefficients, and the test was given to all students in my class. I first graded the question answers and analysed the errors students made in the written responses. Secondly, I selected a group of learners based on their overall test performance and conducted individual interviews to discuss the errors they had made and the possible causes of the errors. The criterion I used for selecting students for individual interviews was their overall test performance. I chose the written test and interviews as my data collection methods as they provided rich and in-depth insights into the reasoning of students when solving equations.

1.9.4 Data Analysis

I conducted the data analysis process by collecting, organising and interpreting my qualitative data (test responses and interviews), to understand its meaning and significance. I used four levels of analysis to have a very detailed and thorough understanding of the data needed to answer my research questions. First, I evaluated the responses to the test questions to identify correct and incorrect answers. Secondly, I classified students' errors on a question-by-question basis. Third, I categorised the various types of error students made to detect either problems or problems in their solutions. Finally, I analysed student interview responses, using Sfard's commognition framework, to investigate possible causes of their errors when solving linear equations.

1.9.5 Trustworthiness of data

Specific evaluation criteria are used in different research approaches to maintain rigour and to strengthen the credibility and trustworthiness of the study (Korstjens & Moser, 2018). The trustworthiness of the data analysis is determined by considering 'credibility, transferability, dependability, and confirmability. I checked with the students about their understanding when solving equations asking them to explain their test solutions (through interviews). I requested outside researchers, that is, my supervisors, to evaluate and examine the research process and data analysis to ensure that study findings were consistent and could be replicated. I kept detailed records of the students' work and shared their solutions as evidence in my findings and discussions. And I acknowledge my dual role as teacher and researcher in the study and the biases that may arise due to this positioning.

1.9.6 Research ethical considerations

Formal permission was obtained from the Cape Peninsula University of Technology Faculty Ethics Committee. The letters of consent were completed by the college principal, the parents of the participants under 18 years old, and the NCV Level 2 student who participated in the study before proceeding with data collection. (See ethics document CPUT EFEC 4-06/2022 (Appendix B).

1.10 Outline of the thesis chapters

This thesis is structured into five chapters, outlined as follows:

Chapter 1: Introduction to the Study

In this chapter, I explore the challenges experienced by National Certificate Vocational (NCV) Level 2 students in South Africa, and the consistently poor performance in national mathematics assessments. I discuss various scholars who have examined and discussed the issue of poor performance in mathematics within the NCV framework.

Chapter 2: Literature Review and Theoretical Framework

This chapter provides an analysis of the existing literature and the underlying theoretical foundation for the study. It outlines key ideas and prior studies related to the topic and offers insights into patterns and gaps in the literature. The theoretical framework guiding the research is discussed in some detail and shows relevance and application to the research questions.

Chapter 3: Methodology

The main goal of this chapter is to outline the research methodology and the research design of the study and to state the aim of collecting and analysing data. It explains the procedures taken to choose and sample participants, ethical issues, confidentiality, informed consent, and the trustworthiness of the study.

Chapter 4: Data Analysis and Findings

The chapter presents the data and discussions of the students' responses to the test and the interviews. It gives a detailed account of the various levels of analysis with illustrations of the student' test solutions and interview transcripts. The connections to the literature review and the theoretical framework used to guide the analysis of the results are also highlighted.

Chapter 5: Discussion, conclusion, and recommendations

The final chapter deals with the interpretation and discussion of the findings related to the research questions. It highlights patterns that emerged within the data and compares these with previous studies showing commonalities and differences between different contexts. The research highlights students' difficulties with linear equations and fraction coeffcients which stem from limited conceptual understanding, weak use of mathematical language and representations, and require explicit teacher scaffolding to support effective problem solving.Future research in terms of practice and policy explored and the limitations are acknowledged to provide evidence of reflection of the process and trustworthiness of the data. Additional research opportunities are identified, and recommendations are made to extend and improve the study further.

2 LITERATURE REVIEW

2.1 Introduction

This chapter comprises a summary of literature related to research focusing on errors in solving linear equations with fraction coefficients and connected topics. The data were collated from conference papers, journal articles, dissertations, and books that cover a range of different research over the past few years. Consequently, this chapter first presents an overview of the literature gathered from various studies that investigate the difficulties students face in solving linear equations. Second, it examines the instructional strategies that can be employed to improve the comprehension of students, and thirdly, it identifies prevalent misconceptions in solving equations, and lastly, specific errors related to solving linear equations and algebraic fractions as reported in the literature are presented in some detail.

Additionally, this chapter includes an outline of Sfard's (2002) discursive approach to learning and thinking as a theoretical lens to understand the educational context under consideration. In conclusion, the reviewed literature is synthesised, culminating in the development of a provisional framework explaining the possible reasons behind the poor performance of students in solving linear equations with fractional coefficients.

2.2 Students' difficulties in solving linear equations and algebraic fractions

Wati and Fitriana (2018) show that one of the reasons why students have trouble with linear equation problems could be the misunderstanding of mathematical elements such as facts, concepts, operations, and principles. In contrast, Jupri, Drijvers, and van den Heuvel-Panhuizen (2014) state that students' difficulties in linear equation solving arise from both a poor understanding of arithmetic operations in numerical and algebraic expressions and confusion about the role of variables, misunderstandings about algebraic expressions, and different interpretations of the equals sign and the whole process of mathematisation. In this study, mathematisation is understood as the process by which students convert mathematical problem situations into algebraic equations and use algebraic reasoning to solve them (Jupri and Drijvers, 2016).

Furthermore, Wati and Fitriana (2018) highlight that students have difficulties with algebra because they do not fully understand the meaning of symbols, signs, or notationt. Syam (2019) states that knowing linear equations as a fact means being able to change from one form into another. Factual knowledge involves recognising different mathematical forms, interpreting them, and being able to translate between symbolic and verbal representations. This knowledge includes both understanding (conceptual) and skills (procedural). Syam (2019: 121) reports in his research, that students expressed factual knowledge as follows:

1. $Q - 007$ asked $LS\ 00$, what is the coefficient? The learner answered the number in front of x
2. Q-008 $Q - 008$ asked $LS - 008$, what about the variables? The learner responded that they are letters or symbols used, such as $x\ and\ y$.
3. $Q - 009$ asked $S - 009$, what does the word constant mean? The learner answered that it is a number with no variables.

Jupri, Drijvers, and van den Heuvel-Panhuizen (2014) emphasise the importance of students acquiring a solid understanding of algebraic equations. Consequently, to solve an equation such as $2\,[(3x - 4) + (x + 1)] = 34$, students must identify the equation and employ efficient strategies to manipulate the expression. For example, in $2[(3x - 4) + (x + 1)] = 34$, the students could demonstrate proficiency in the order of operations, opting to address the multiplication by 2 first by dividing both sides of the equation by 2. Then, the equation can be simplified, resulting in a more manageable form, as illustrated by the transformation $3x - 4 + x + 1 = 17$.

Line 1: $3x + x = 17 + 4 - 1$ (collect like terms)

Line 2: $4x = 20$ (simplify)

Line 3: $x = 5$ (divide both sides of the equation by 4)

The student can validate the equality or balance of the equation by substituting $x\ with\ 5$. The inability to apply the order of operations when solving linear and fractional equations reflects a lack of conceptual understanding. Wati and Fitriana (2018) support this notion, stating that a lack of conceptual understanding arises when students struggle to integrate ideas with existing knowledge to derive solutions to specific problems.

Additionally, Qetrani et al. (2021) argue that a lack of conceptual knowledge manifests itself as an explicit or implicit understanding of concepts, operations, and relationships encompassing the manipulation of algebraic expressions. For instance, when students learn by gathering like terms, they should also learn to apply the order of operations a set of guidelines for solving equations to combine the components. Operational understanding, defined as the challenge faced by students when counting without applying algebraic or arithmetic skills (Jupri et al., 2014), highlights the importance of integrating algebraic and arithmetic skills during problem-solving. Samuel, Mulenga, and Angel (2016) agree, highlighting students' struggles to manipulate algebraic expressions and comprehend concepts related to fundamental arithmetic operations such as addition, subtraction, multiplication, and division. Having the relevant knowledge and skills, particularly knowing how to approach a problem before starting, is considered essential.

Chow (2011) claims that operational challenges stem from conceptual difficulties hindering students from mastering a topic, particularly in relation to abstract concepts used for problem-solving. Therefore, challenges to mastering the counting and algebraic craftsmanship are categorised as operational difficulties. In addition, Chow (2011) highlights the importance of enriching learning by comprehending mathematics through the implementation of arithmetic laws of operation (commutative, associative, and distributive) and adhering to the order of convention for operations in algebra. For example, students who use the order of operations to evaluate algebraic expressions are assured of obtaining consistent results.

The difficulties that students face when learning concepts related to axioms, theorems, relationships between various basic mathematical objects and mathematical formulas are referred to as "principal difficulties" (Wati & Fitriana, 2018). In agreement, Shaker and Berger (2016) claim that students' inability to properly interpret and implement the definition causes them to struggle to provide the overall structure and proof method for a given proof and the reasoning for the construction.

Several studies emphasise the importance of improving the ability of students to solve linear equations by improving procedural flexibility and employing multiple strategies (Maciejewski, 2022; Qetrani et al., 2021). Star, Rittle-Johnson, Durkin, Shero, and Sommer (2020) highlight that the concept of procedural flexibility has gained prominence in mathematics education. This prominence is also reflected in educational policies and practices, such as those outlined by the National Council of Teachers of Mathematics (2014). Similarly, Star, Tuomela, Prieto, Hästö, Palkki, Abánades, Pejlare, Jiang, and Liu (2022) define procedural flexibility as the ability to access various strategies and choose the most suitable one for a specific problem. They emphasise the need for all students to have deep and flexible knowledge of diverse procedures, coupled with the ability to make informed judgments about the suitability of procedures and strategies in specific situations.

Altindis and Fonger (2019) argue that the use of a range of instructional strategies in mathematics education contributes to equity, opportunity, and asset-based learning within the educational system. Furthermore, procedural flexibility is seen to provide ‘opportunities for diverse learners to bring their culture, identity, and thinking into mathematics’ (Altindis & Fonger, 2019: 123). In addition, Altindis and Fonger (2019) report that when students are exposed to various strategies, they gain confidence, strengthen their thinking, and become adept at using high-quality strategies for teaching and learning, leading to overall competence. Consequently, a student who embraces diverse learning strategies opens numerous opportunities across varied career paths, and conceptual understanding supports logical and analytical thinking.

This section of the chapter dealt with the problems faced by students in the process of solving linear equations. These problems included misunderstandings of mathematical concepts, facts, operations, and principles. Subsequently, in Section 2.3, the analysis moves to a group of three strategies used to improve the students’ comprehension in solving linear equations. Section 2.3 aims to develop the research process by discussing the strategies that were used to improve the students’ understanding of the solution of linear equations.

2.3 Strategies used to develop students’ understanding in solving linear equations

Various strategies such as innovative strategy, the balance method, and the transposition method have been demonstrated to be highly effective in learning to solve linear equations. Roberts and Le Roux (2019) have emphasised the structural effectiveness of these strategies in fostering enhanced learning capabilities and promoting instrumental thinking among learners. It is widely acknowledged within the field of Mathematics education that relational thinking plays a central role in facilitating the transition from arithmetic to algebra (Roberts & Le Roux, 2019). These strategies can be broadly classified into three main categories, all of which contribute to the mentioned benefits of improved learning and enhanced cognitive connections for students. Sub-section 2.3.1 discusses the first strategy namely the “innovative strategy”.

2.3.1 Innovative strategy

The innovative strategy stands out because of its focus on structure sense, which is the ability to recognise the relationships between the different structures and to realise the useful manipulations. Roberts and Le Roux (2019) assert that this technique is particularly good for students as it helps them acquire learning in a more profound and lasting way. They claim that when students lack structure sense, their errors will be increased, and hence the need to comprehend the 'why' and 'when' of problem-solving is highlighted. Agommuoh and Ifeanacho (2013: 126) support this perspective by stating that the innovative techniques like 'brainstorming and peer tutoring' are vital in making the students professionally equipped with the basic 'skills, abilities and competences' to solve problems in an effective manner. Therefore, learners should be trained to formulate and apply different contexts, thus changing their knowledge to various problems (Agommuoh & Ifeanacho, 2013).

Roberts and Le Roux (2019) caution that when students lack a conceptual structure, they may rely solely on their own decisions when solving problems. Xu, Liu, Star, Wang, Liu, and Zhen (2017) further argue that strategic flexibility, including knowledge of multiple approaches and the capacity to apply mathematics, is essential for the success of the innovative strategy. It is imperative for students to develop conceptual understanding to progress toward becoming effective problem solvers.

Maass, Cobb, Krainer, and Potari (2019: 315) state that 'innovative teaching approaches often conflict with national assessments, which prioritise procedural competences over conceptual understanding, problem-solving, and mathematical communication'. There are also contextual variations, such as cultural and national priorities in mathematics education, that can influence whether an improvement observed in one context can hold for another. The authors emphasise that

> *the implementation of innovation is crucially adapted to the local school and classroom context, involving a mutual adaptation between the innovation and the local context. (Maass, Cobb, Krainer, & Potari, 2019: 315)*

They conclude that, even though there are different teaching methods, the transformation of these methods from design to actual use in professional development and classroom practice remains a difficult process. The findings further show that context is a major factor influencing the success of professional development programmes The next sub-section discusses the balance method.

2.3.2 The balance method

The balance method is a necessary step in algebraic equation manipulation, where the equality of both sides must be maintained. It is a strategy that investigates the use of tools in thinking as a means of expressing the students' mental states during the linear equation solving. Roberts and Le Roux (2019) agree that it is associated with relationship-rich, flexible, independent, and more accurate problem-solving. The use of the balance method helps to provide students with a better conceptual understanding view of linear equations Thus, the balance model can be used to explore the equality of the left and right sides of an equation (Otten, Van den Heuvel-

Panhuizen, & Veldhuis, 2019). Sanders (2017) notes that the balance method is the most conceptually beneficial. Likewise, Anthony and Burgess (2014) say that the balance model's usage is the same as that of the original problem, which is the equivalence between the expressions on both sides of an equation. The balance model operates independently in dealing with linear equations (Otten et al., 2019).

Atteh, Andam, Amoako, Obeng–Denteh, and Wiafe (2017) consider the balance method as a valuable educational tool for establishing connections between "seesaws". Linear equations in one variable create a robust link between what is taught in school and everyday life activities of children. However, the balance model has limitations. For instance, Otten et al. (2019) note that although students have been taught formal linear equation solving using the balance model and while it was helpful for the operative mental image of implementing strategies, this model also had shortcomings. It becomes evident to students that the procedures are less effective, particularly when solving equations involving negative quantities, subtraction, or other equations detached from the model and no longer referring to a concrete representation (Otten et al., 2019). For example, when negative values appear, as in equation $x + 5 = 3$, or equations with subtraction, such as $2x - 3 = 5$, expressing the solution in terms of physical weight becomes challenging, making it difficult to construct meaning for these equations. The following sub-section will discuss the transposition method.

2.3.3 Transposition method

The transposition method is the process of shifting a number from one side of an equation to the other. Gumpo (2014) explains that the transposition method in algebra involves performing the same mathematical operation on both sides of an equation, with the objective of rearranging the terms and isolating the variable on one side. For example, in the equation: $a + b = c$. If we subtract b from both sides, the equation becomes $a = c - b$. This implies that the **b** has been transposed. The transposition method is commonly used to help students develop a conceptual understanding of solving linear equations, as highlighted by Gumpo (2014). It is praised for its efficiency and ability to reduce errors. Sanders (2017) also emphasises that transposition gives students procedural mastery and helps them realise important changes in equations. For instance, if there is an addition of three for a variable on one side of the equal sign, it will be equal to subtracting three from both sides of the equal sign (Gumpo, 2014).

According to Powell (2012), it is essential for students to understand the equal sign as relational, indicating a connection between the expressions or numbers on either side. Consequently, a mathematical statement that uses the equal sign to denote the equivalence of a number or expression on both sides is called an equation (Powell, 2012). Understanding the relational aspect of the equal sign is crucial in effectively using the transposition method to solve equations and work with mathematical expressions.

In summary, this section (2.3.1–2.3.3) discussed three strategies to teach linear equations: the innovative strategy, the balance method, and the transposition method. Each approach supports the development of both conceptual understanding and procedural fluency in distinct ways. Collectively, these strategies highlight the need for balanced instructional practices that integrate conceptual reasoning with procedural competence to enhance the algebraic problem-solving skills of students.

Sections 2.4 and 2.5 introduce the discussion of misconceptions and errors in learning mathematics, highlighting the common misunderstandings that teachers and students experience during the process of teaching and learning mathematical concepts.

2.4 Misconceptions in solving mathematics problems

The process of learning mathematics involves making mistakes and misunderstandings, but mathematical errors and misconceptions are not the same thing. Their causes and characteristics are different. According to Luneta and Makonye (2010) errors are defined as mistakes made by students that can occur due to several factors such as inaccurate data entry, computation, or a lack of conceptual understanding. Hence, that errors can serve as indicators of underlying misconceptions of misconceptions (Luneta & Makonye, 2010). Matindike and Makonye (2023) defined error as a mistake, slip, blunder, or inaccuracy that is evident in students' written work and prevents them from receiving a perfect score on assignments or tests. According to Luneta and Makonye (2010: 44) misconceptions refer to ‘false interpretations’ that result from ‘misunderstandings and misinterpretation’. They conclude that misconceptions manifest themselves through errors. Misconceptions are more deeply rooted and structural in that they stem from underlying mental structures (Matindike & Makonye, 2023).

Making mistakes (errors) is an essential component of learning Mathematics, both teachers and students can embrace and learn from these errors (Gardee, 2015). There is also evidence to suggest that encountering obstacles and making errors during the learning process is beneficial and acceptable for students’ understanding (Sanders, 2017). This means that students should be encouraged to learn and understand the concept of learning from mistakes and to accept mistakes and be corrected. However, some students may fail to find the correct solution to a mathematical problem due to various reasons, such as lack of knowledge or careless computations/errors (Gardee, 2015; Luneta and Makonye , 2010).

As discussed above, errors are an important component of learning and can be caused by carelessness which Gardee (2015) describes as “slips”. Sarwadi and Shahrill (2014) highlight the cumulative nature of mathematics learning and the importance of the balance process in understanding concepts and avoiding misconceptions. Misconceptions are seen as a natural part of the learning process (Sarwadi & Shahrill, 2014). They agree that mathematics learning is cumulative, meaning that the new information you learn is connected to what you already know. This is called cognitive structure. As a result, if the students failed to implement the balance process, it could lead to a void in the understanding of the concept, resulting in mathematical slips or misconceptions (Sarwadi & Shahrill, 2014). They further discuss that for students it is not entirely bad to have misconceptions; it is sometimes part of the process for understanding new concepts. The results of the Roberts and Le Roux (2019) study on Grade 6 and 9 students’ thinking about linear equations indicated that the failure to implement a systematic approach when teaching linear equations and algebraic expressions led studentsfrom various geographic and socioeconomic contexts to demonstrate the same misconceptions in these topics. Sarwadi and Shahrill (2014: 2) explain that misconceptions are a significant ‘part of the learning process’, and it would be helpful if these misconceptions were dealt with in a diagnostic manner. Siyepu and Ralarala (2014: 578) argue that a ‘misconception is part of a conceptual framework that a learner develops that makes sense in the context of their prior knowledge but is incompatible with conventional mathematical knowledge. They further define slips as incorrect responses due to processing; they are not deliberate mistakes, but rather careless ones that are made by both experts and novices. In a study done by Yansa, Retnawati, and Janna (2021) results show that students worldwide experienced misconception through, struggling with algebraic notation and symbolism.

2.5 Errors in solving linear equations

Students face difficulties when manipulating an algebraic equation due to several errors**.** It might result from general types of errors such as conceptual, procedural, application, and language when manipulating algebraic expressions and equations. According to Ncube (2016) when simplifying algebraic expressions, students make mistakes because they lack conceptual understanding. However, Msomi and Bansilal (2022) argue that conceptual errors are those that occur because of not achieving learning that entails comprehending and interpreting concepts as well as the relationships between them. Based on that, they conclude that misunderstanding and mis-interpretation of concepts is the cause of these errors. Msomi and Bansilal (2022) recommend that to reduce the number of errors made by students, more effort should be made when introducing the topic by emphasising the connections to other concepts, so that students have a better chance of understanding the concepts immediately after the teaching. In addition, errors in procedure are those that occur when attempting to perform a procedure after comprehending the fundamental ideas underlying the issue (Msomi & Bansilal, 2022). Language errors arise when students lacking mathematical terminology find it difficult to translate a given expression into algebraic equations using symbols, numerals, signs, and variables (Sa'dijah & Muksar, 2021). This study was designed to investigate the types of errors produced by NCV Level 2 students when solving linear equations.

According to the findings of Gardee (2015), errors in the context of mathematical problem-solving exhibit systematic characteristics, manifesting regularly and pervasively across diverse situations, thereby displaying a persistent nature. Conceptual difficulties encountered by students often manifest themselves as errors in expression. Insufficient comprehension of mathematical concepts leads to the formulation of strategic errors during the process of simplifying an expression. The ensuing discussion delineates typical errors observed among students, encompassing misunderstanding of the equal sign, right-to-left reasoning, inverse errors, errors stemming from misinterpretation of commutative and distributive properties, errors in generalising operations, and errors associated with the application of the minus sign.

2.5.1 Misunderstanding of the equal sign

The equal sign is an innovative mathematics tool for developing algebraic mathematical thinking (Hibi & Assadi, 2022). Algebraic thinking is based on the basic notion of equivalency, which is required at all levels of mathematical accomplishment (McAuliffe, Tambara, & Simsek, 2020). It is a symbol that represents the proportional connection between two equal sides which means that if $a = b$, then, b is also equal to a $(b = a)$, both sides of equation have the same value. In this regard, understanding the relational aspect of the equal sign is crucial to solving equations and working with mathematical expressions.

McAuliffe, Tambara, and Simsek (2020) state that the concept of equivalence is widely recognised as one of the greatest challenges for primary school learners. Research further shows that difficulties with equivalence persist beyond the primary level (Bush & Karp, 2013; Chan, Lee, Mason, Sawrey, & Ottmar, 2022). According to McAuliffe et al. (2020), the early exposure of learners to traditional equation formats can shape their later understanding of mathematical equivalence. Sanders (2017) similarly notes that, in the early years of learning, students tend to interpret the equal sign (=) as a signal to perform an operation to find an answer. As learners progress to higher grades, they must extend this view to understand the equal sign as a *relational* rather than merely *operational* symbol. Both Sanders (2017) and

McAuliffe et al. (2020) emphasise that grasping the relational meaning of equivalence is essential for learning algebra successfully in high school. McAuliffe et al. (2020: 1) conclude that 'difficulties with equivalence are linked to inappropriate generalisation of knowledge constructed from overly narrow experience with arithmetic.'

Vermeulen and Meyer (2017) concur that students do not understand the meaning of the equal sign, hence they commit misconceptions. The consideration of the equal sign by students is operational and is interpreted as an arithmetic calculation (Mc Auliffe et al., 2020). In addition, many studies report that the possible reason for learners struggling when solving equations is that they do not have a clear understanding of the relational view of the equal sign but instead only an operational view (Eichhorn, Perry, & Brombacher, 2018; McAuliffe et al., 2020; Sanders, 2017). Machaba (2017) contends that teachers should strive to emphasise mathematical equivalency in early grades by using different examples with different learning materials. Right-to-Left reasoning is the subject of the following discussion.

2.5.2 Right-to-Left reasoning

Right-to-left reasoning is a situation where a student performs operations in reverse order – working from right to left instead of following the conventional left-to-right approach (Gumpo, 2014). In an investigation conducted by Pournara (2020), the analysis of didactic approaches and student errors in solving linear equations highlights the necessity for explicit instructional guidance when addressing equations in the form $ax + b = cx + d$. In the general linear equation $ax + b = cx + d$, the symbols $a \text{ and } c$ represent the coefficients of the variable x, meaning they are the numerical values that multiply the variable. The symbols $b \text{ and } d$ denote constants, which are fixed numerical terms that do not contain the variable. The letter x represents the variable, that is, the unknown value to be determined through the process of solving the equation (Lial, Hornsby, Schneider, & Daniels, 2016). Pournara (2020) observed that many students struggle with equations containing variables on both sides, such as $ax + b$ = $cx + d$, when multiple variable terms appear on one side. For example, a student who solved the equation incorectly $3x - 2 = 4 + x$ as follows:

$$3x - 2 = 4 + x$$

$$3x - 2 + 2 = 4 + 2 + x$$

$$\frac{3x}{3} = \frac{6 + x}{3}$$

$$x = 2x$$

The correct step should be:

$$3x - 2 = 4 + x$$

$3x - 2 - x = 4 + x$ (subtracting x on both sides)

$2x - 2 + 2 = 4 + 2$ (add 2 to both sides of the equation)

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

Pournara (2020) emphasises that teachers play a pivotal role in helping students make cognitive shifts when solving equation variables on both sides. Sanders (2017) supports this, noting that students often interpret the equal sign unidirectionally, which leads to both right-to-left and left-to-right reasoning errors. The main issue is not the direction of operations, but the one-sided nature of a student's reasoning.

Aydin-Güç and Aygün (2021) found that such misconceptions often result in broken equality, where operations are performed on one side of the equation only. For example, when solving $x = 37 + 150$, students may know that they must subtract 37 but often do so only on the left-hand side, failing to maintain balance by performing the same operation on the right-hand side.

Halley (2019) describes this as a *didactic cut* that occurs when students fail to transition from arithmetic to algebraic thinking. For example, a simple arithmetic equation such as $3x + 2 = 17$ can be solved numerically as $3x + \square + 2 = 17$, using a purely arithmetic approach. However, when confronted with an algebraic equation involving variables on both sides, such as $3x + 5 = 2x + 14$, students must manipulate symbolic expressions.

The correct procedure is the following.

$3x + 5 = 2x + 14$

$3x - 2x + 5 = 2x - 2x + 14$ (subtract 2x from both sides)

$x + 5 = 14$ (simplify)

$x + 5 – 5 = 14 – 5$ (subtract 5 from both sides)

$= 9$

Roberts and Le Roux (2019) observed that students respond differently when variables appear on one side of the equation compared to when they appear on both sides, indicating a gap in conceptual understanding.

A study by Andrews (2011: 6-7) shows how a teacher plans a lesson with real-world examples to support conceptual understanding, for example:

> *On two consecutive days the same weight of potatoes was delivered to the school's kitchen. On the first day 3 large bags and 2 bags of 10kg were delivered. On the second day 2 large bags and 7 bags of 10kg were delivered. If the weight of each large bag was the same, what weight of potatoes was in the large bag? (Andrews, 2011: 6-7)*

The student volunteer correctly wrote $3x + 20 = 2x + 70$. The teacher then drew a picture of a scale balance to represent the equality and gradually simplified:

$3x + 20 = 2x + 70 \mid -20$ (subtract 20 both sides)

$3x = 2x + 50 \mid -2x$ (subtract 2 both sides)

$x = 50$ kg

Finally, the teacher reminds the students of the need to check the answer to make that both sides of the equation remain balanced after finding the solution.

The following sub-section discusses inverse errors.

2.5.3 Inverse error

An inverse error is an error that students make when performing operations with additive or multiplicative inverses (Jupri et al., 2014). Inverse errors occur because of a lack of understanding of the structure of the algebraic expressions involved, such as mixing up multiplication, and addition of sub-expressions (Jupri et al., 2014). In an investigation conducted by Halley (2019) on the progress of Grade 9 learners and their difficulties in working with linear equations, the study revealed that the students experienced difficulties when faced with letters on both sides. According to the conventional methodology of 'same operations on both sides', this was considered premature inverse operations (Halley, 2019: 135). Halley (2019) recommends that the more necessary effort in the use of 'extreme' cases, such as ax + c = bx + d can help to increase student engagement with complex mathematical concepts.

Example: $\mathbf{5x + 4 = 2x + 1}$

$\mathbf{5x + 2x = 1 + 4}$ (Incorrect, the inverse of positive is negative).

$\mathbf{7x = 5}$

$\mathbf{x = 5/7}$

In this regard, the students used the transportation method but did not maintain the sign changes. Students must be reminded that to solve a linear equation, one needs to execute the same method on both sides of the equation to obtain the variable itself. The emphasis is that to do this, one should do the opposite of what was done with that variable, but to keep the order of operations in mind.

For example, solving this equation $\mathbf{5x + 4 = 2x + 1}$ requires the use of additive and multiplicative inverses. $\mathbf{5x - 2x + 4 - 4 = 2x - 2x + 1 - 4}$ (use the inverses of 4 and $2x$ on both side), then $\mathbf{3x = -3}$ (divide it by 3 both sides, use the multiplicative inverse that is division). Finally, $\mathbf{x = -1}$ is the solution because $\mathbf{5(-1) + 4 = 2(-1) + 1{:}\ -1 = -1.}$

From the student solutions, they fail to use the multiplicative inverse of positive $2x$ and positive 4. Students confused the additive inverse with the multiplicative inverse. Sanders (2017) suggests that in mathematics, students should have a clear understanding of operational inverses. Additive, subtractive, multiplicative, and division inverses are examples of inverse operations that reverse the effect of another operation." For example, addition is the inverse operation of subtraction, and multiplication is the inverse operation of division.

Vlassis (2002) suggests that in equations with one unknown, the cover-up and substitution strategies are effective teaching strategies to improve learning how to solve linear equations and to help students avoid errors. Hall (2002) argues that 'trial and improvement' could be used efficiently and that it allows students to solve all types of linear equations with one unknown. In addition, Pournara, Sanders, and Takker (2022) suggest that for students to be able to apply inverses, they should learn to start with the constants and then move to the variables or start with the terms containing the variable. They can also choose to start on the right, or on the left. The discussion that follows will centre on errors due to commutative and distributive properties.

2.5.4 Errors due to commutative and distributive properties

The distributive law is one of the most important properties of algebra in the arithmetic of numbers and the manipulation of expressions (Mok, 2010). Some studies claim that by the third grade, students are applying the commutative and associative properties of multiplication and learning the distributive property, despite the fact that errors resulting from these properties originate in lower grades (Booth, Barbieri, Eyer, & Paré-Blagoev, 2014; Bush & Karp, 2013; Chan, Lee, Mason, Sawrey, & Ottmar, 2022). Fatmiyati and Fitriana (2020) state that students' failure to comprehend definitions is the most common conceptual error that arises when they attempt to solve the distributive and commutative properties. They recommend that teachers provide students with a context in which they can apply definitions to the procedures and concepts they learn in mathematics.

As noted by El-Khateeb (2016), mistakes frequently occur in arithmetic when students incorrectly apply the commutative and distributive properties, especially in operations such as subtraction and division where these properties are not valid. These mistakes result in erroneous calculations and ultimately, confusion. Students tend to make errors in expressions with different terms because they fail to identify the differences between the mixed basic number operations. They make a distributive error when expanding the bracket (Seng, 2010). For example, a student solved the equation as follows:

$$\mathbf{3x + 7 = x + 3 + 2(x + 1)}$$

$$\mathbf{8x = x^2 + 4 + 2} \text{ (incorrect)}$$

$$\mathbf{8x = x^2 + \ 6} \text{ as an incorrect solution.}$$

In the above equation, the student fails to remove brackets correctly. The distributive property affects the solution of an equation when students cannot correctly remove parentheses (Gumpo, 2014).

The order of operation commonly referred to as BODMAS, which stands for parentheses/brackets, orders, division, multiplication, addition, and subtraction, is required to solve the expression on either side and guides the sequence in which operations should be performed (Burger & Starbird, 2019). Ncube (2016) point out that students encountered difficulties with bracket expansion because of applying the distributive property incorrectly in multiple contexts. Booth, Barbieri, Eyer, and Paré-Blagoev (2014) recommend that addressing these kinds of errors that are due to misunderstanding the commutative and distributive properties, would be possible if elementary school teachers could teach their students to justify their use of properties of numbers in the same way that they do in geometry proofs. The errors related to the minus sign is covered in the next subsection.

2.5.5 Minus sign

The minus sign in a subtraction is the most typical source of confusion in the context of an incomplete distribution. Incomplete distribution means that the integers with negative sign are touched and not by a subtraction operation. These are common errors caused by a lack of mathematical discourse (routines), such as basic rules for positive and negative numbers. A routine is a discursive framework that displays guidelines that should be followed to be well-informed (Sfard, 2018). Students should be governed by strict rules that are authorised by the teacher or textbook. To facilitate communication, those rules must become ritualised.

Fuadiah and Suryadi (2017) reveal that the 7th grade students have epistemological obstacles to negative numbers. For example, for $+(-\mathbf{6}) = -\mathbf{25}$, the students add numbers without considering the signs. The expression should be written as $\mathbf{19 - 6 = 13.}$ In addition, as in the example $-\mathbf{31} + (-\mathbf{8}) = \mathbf{39}$, the minus sign before the numbers was not considered. The expression should be solved as $-\mathbf{31} - \mathbf{8} = -\mathbf{39}.$ Therefore, since results shows negative numbers to be an abstract concept, Fuadiah and Suryadi (2017) suggest that students need to be taught how to do this in order to avoid epistemological obstacles through phenomenological guidance.

Vlassis and Demonty (2022) also found that Grade 6 students showed an inability to see the subtraction operation in operations with negatives. For example: $\mathbf{237 + 89 - 89 + 67 - 92 + 92}$ was simplified to $\mathbf{237 + 89 - 89 + 67 - 184}$ as if the minus sign before **92** was not attached to the number. This highlights that a student might lack a sense of number structure. Therefore, for students to solve the expression, they should use the relationship to cancel the opposites and rewrite the expression as $\mathbf{237 + 67}$. Vlassis and Demonty (2022) conclude that one of the reasons students ignored the minus sign is due to “Detachment from the Minus Sign” (DFMS), which consisted of ignoring the minus sign before a number and then focusing on simplifying the expression. The next sub-section covers the process of ignoring the minus sign.

2.5.6 Minus sign ignored

The process of detaching from the negative sign integer is known as ignoring the minus sign (Seng, 2010). The results of the study by Fuadiah and Suryadi (2017) of Grade 7 students’ difficulties understanding negative numbers indicate that the reason students missed the minus sign is the lack of the epistemological concept. This means that students did not just make careless mistakes, the problem is deeper and related to the lack of conceptual understanding of what negative numbers are. They do not understand the nature and role of negative numbers in mathematics, which means that they often ignore or overlook the minus sign. This shows the lack of implementation of the reverse operation by the students. As a result, incorrect solution methods are used. For example, in a study conducted by Aydin-Güç and Aygün (2021), the results show that students overlook negatives when solving equations, which often leads them to make incorrect solutions.

For example:

$$-3x + 6 = 2x + 16$$

$$-3x - 2x + 6 = 3x - 2x + 16$$

$$1x + 6 = 16.$$

In these cases, the student ignored the minus sign leading to the incorrect solution instead of seeing $-3x - 2x$ as $-5x$, the sum of two negative numbers is a negative number (the signs are the same).

In support, Bofferding, Aqazade and Farmer (2017) agree that negative signs are easily ignored when students are unfamiliar with negative numbers in integers and are used to working with positive whole numbers when solving problems (e.g., $-\mathbf{4} + \mathbf{5} = \mathbf{9}$). Gumpo (2014) confirms that the most typical error in solving algebraic equations is ignoring the operation sign. The minus sign is overlooked or interpreted positively as in the equation above, in contrast to "right to left" reasoning in solving equations. Furthermore, Vlassis and Demonty (2022) emphasise that when students first learn about negative integers, it becomes a barrier for students to accept that the answer in an equation can be negative. A negative solution seems impossible to them because they are unable to give it a concrete interpretation. The lack of understanding about the multiplication of negative integers also causes problems when trying to solve additional problems, or solve problems involving integers (Mathunya, 2023). The context used in the teaching and learning of negative numbers can negatively impact student performance when solving algebraic equations. The following sub-section discusses the generalisation over operations.

2.5.7 Generalisation over operations

Generalisation is a ‘common attribute of human thinking and consequently cannot capture the specificity of algebraic thinking’ (Vlassis & Demonty, 2022: 2). Overgeneralisation is an error students make in operation when they need to balance equations by implementing the inverse. This means that students generalise the priority of operations in arithmetic to algebraic expressions, for example:

$$\frac{x}{4} + 22 = 182$$

$$\frac{x}{4} + 22 = 182 + 22$$

$$\frac{x}{4} = 204$$

$$x = 204 \times 4$$

$$= 816$$

This solution comes from a study done by Aydin-Güç and Aygün (2021) of Grade 8 students on operations with algebraic expression. This indicated a student overgeneralisation error occurs due to an addition operation in the equation by implementing an existing operation in the equation to the other side. When considering the addition operation, the students did not change the sign in the equation to the other side. This indicates a confusion of working with addition operations in algebra. This overgeneralised rule appears to eliminate the ‘old rule’ of the change sign rule.

This section describes seven typical errors students make when solving arithmetic and algebraic equations. The next section will look at errors related to algebraic fractions.

2.6 Errors in solving algebraic fractions

This section is included in the literature review to help identify the types of errors students make when working with algebraic fractions to help inform the analysis of students solutions. Algebraic fractions involve expressions with variables as numerators and denominators. According to Ncube (2016) and Hayati and Setyaningrum (2019), students often make errors when simplifying algebraic expressions and operating with the addition or subtraction of fractions due to a lack of conceptual understanding. Engaging students in discussions, using visual aids, and relating algebraic concepts to real-world scenarios can contribute to a deeper understanding of algebraic expressions and their simplification (Ncube, 2016). Some studies have been conducted in the Northern countries that reveal that students have difficulties in understanding algebraic expressions and applying arithmetic operations in numerical and algebraic expressions (Hariyani, Herman, Suryad, & Prabawanto, 2022; Zulfa, Suryadi, Fatimah & Jupri, 2020).

There are studies in South Africa that show that students also have difficulties solving algebraic fractions. For example, the study by Makonye and Khanyile (2015) with Grade 10 students about their mathematical errors in simplifying algebraic fractions that found students had the highest frequency of error which finding the lowest common denominator. They suggest that teachers need to spend more time reinforcing the concept of understanding the purpose of the lowest common denominator and the importance of simplifying fractions. They recommend that instead of teachers avoiding or dismissing students' errors, teachers should embrace these errors as valuable resources in the teaching of mathematics.

The results of Mhakure, Jacobs, and Julie's study (2014) of Grade 10 students' facility with algebraic fractions indicated that the students made cancellation errors due to the failure to consider the fraction. According to these researchers, students demonstrate a lack of conceptual understanding when solving algebraic fractions and need to build a solid foundation of rational number before tackling more challenging algebraic fraction problems.

Baidoo (2019) revealed that Grade 10 students make misconceptions and errors when simplifying algebraic fractions and indicated several types of errors such as conceptual, mathematical language, procedural and application errors, which hinder students' appropriate understanding as well as the application. He claimed that mathematical errors and misconceptions are both aspects of learning mathematics that involve mistakes or misunderstandings, but they differ in their nature and causes.

Mathematical errors

- Nature: Mathematical errors refer to mistakes made during the computation or manipulation of numbers and symbols.
- Causes: Computational mistakes, typos, misalignment of digits, or miscalculations.

Mathematical Misconceptions

- Nature: Mathematical misconceptions involve a deeper level of misunderstanding or flawed conceptualisation of mathematical ideas or principles.
- Causes: Misconceptions can arise from a variety of sources, including incorrect prior learning, incomplete understanding of foundational concepts, or misinterpretation of mathematical symbols and operations

Baidoo (2019) recommends that teachers use effective instruction to reinforce the concept image and improve general understanding. He concludes that when instructions are well-integrated, students are more likely to acquire and retain conceptual knowledge. The results of the Baidoo, Adane, and Luneta study (2020) on student's error analysis in solving algebraic fractions in high schools indicate that students encounter challenges in simplifying algebraic fractions due to lack of knowledge of the arithmetic structure. However, there are also studies that highlight the fear that students have towards algebraic fractions, and it is evident that this topic poses a significant challenge for many students (Baidoo, Adane, & Luneta, 2020; Mhakure et al., 2014). Studies on the teaching of algebraic fractions in elementary and middle schools view the algebraic fraction as an important foundation for learning more advanced mathematics (Baidoo, Adane, & Luneta, 2020; Lamon, 2020). Therefore, before formal work with fractions begins, it is necessary to establish a solid basis for number sense, and a deeper understanding of the algorithms for operations should be developed and nurtured in all students (Baidoo, Adane, & Luneta, 2020). The synthesis of the reviewed literature is covered in the section below.

2.7 Synthesis of the reviewed literature

In this chapter, an extensive review of the literature on students' difficulties in solving linear equations and algebraic fractions is given. Several studies have recognised and classified the different factors that hinder the learning of this topic, among which are lack of procedural knowledge, incorrect ideas about the relationship between equation solutions and inverses, and incorrect actions when operating with negative integers using the four basic operations (Jupri et al., 2014; Pournara et al., 2022; Wati & Fitriana, 2018).

In addition, the chapter examines various teaching methods that are geared towards developing students' capacity to solve linear equations and that particularly highlight transposition, balance, and creative techniques. The instructional strategies suggested by the present research are considered as potential answers to the hurdles that learners encounter with linear equations. Numerous researchers such as El-khateeb (2016), Gumpo (2014), Pournara (2020), Pulungan (2019), Sanders (2017), Syam (2019), Adu, Assuah, and Asiedu-Addo (2015), Anthony and Burgess (2014), and Chow (2011) highlight the important factors causing students' struggles in the solving of linear equations, and highlighting the lack of conceptual understanding as one of the causes. They stress the need to foster students' skills and improve their learning experiences.

Furthermore, this literature review investigates misconceptions and errors in the process of solving linear equations. Siyepu (2013) maintains that although errors and misconceptions may be different, they still have common factors that cause them such as lack of attention, carelessness, and misunderstanding of a question. As a result, these errors become very evident in the writings and discourse of the students. The chapter illustrates the errors that students often commit when solving linear equations, as revealed by the literature review. I compiled a wide list of errors for students to keep in mind while working on linear equations to assist in the analysis of students' solutions.

This chapter reveals that students in general face numerous difficulties because of a lack of structural understanding, misconceptions, and errors in learning linear equations. It explains of the possible reasons for the poor performance of students in solving linear equations with fractional coefficients.

TABLE 2.1: PROVISIONAL FRAMEWORK LINEAR EQUATIONS

Student's difficulties in solving linear equations	Students' struggles with solving linear equations because of their incapacity to comprehend mathematical concepts, revealing a lack of factual, conceptual, and procedural knowledge (Syam, 2019).
Strategies used to develop learners' understanding in solving linear equations.	When applying multiple strategies to solve linear equations, students encounter many difficulties that should considered when studying (Roberts & Le Roux, 2019). Those difficulties can be factual, conceptual, operational, and principle difficulties (Wati & Fitriana, 2018).
Misconceptions	When students are exposed to new content, particularly when it appears to be quite different from what they already know, several of misconceptions may arise (Barbieri & Devlin, 2024).
Errors	Students made errors due to several of factors, suc as inaccurate data entry, computation, or lack of conceptual understanding (Moru & Mathunya, 2022).

2.8 The theoretical framework

This study is guided by the commognition framework developed by Sfard (2008). It combines the ideas of communication and cognition and says that learning mathematics is not only about what happens in your head, but also about how you speak, think, write and use symbols or pictures when working with mathematics. This framework clarifies what students say and do when solving linear equations with algebraic fractions. It was used for the analysis of student interviews about their solution strategies by looking at four key ideas from the framework: words used, the visual mediators, routines, and narrative endorsement. This is explored in more detail in the following sections and includes a summary of some of the key findings from empirical studies that have used the commognition framework.

This study uses the commognitive framework which combines the ideas of communication and thinking and says that learning mathematics is not only about thinking in your head, but also about how you speak, write, and use symbols or pictures when working with mathematics. The framework helps to understand the important ideas students need for learning algebra and guides how I look at what students say and do when solving linear equations.

Four main features of the framework are used:

1. **Words in use** – the mathematics words students use when talking about equations.
2. **Visual mediators** – such as diagrams, graphs, or symbols that help explain the maths.
3. **Routines** – the usual steps or methods students use to solve a problem.
4. **Endorsement narratives** – how students decide or explain that an answer is correct.

I used this framework to interview and listen to students as they solve mathematics problems and note the words, visuals, steps, and explanations they use. This helped me to understand how students think and talk about mathematics while solving linear equations involving fractional coefficients.

2.9 Commognition theory

Sfard (2020) introduces the concept of commognition, defining it as the merging of two fundamental components: communication and thinking. She views thinking as an internal dialogue, depicting its nature as a system that employs objects to articulate thoughts (Sfard, 2020). This perspective has its basis in the commonly accepted principle that emphasises the power of learning and thinking through interpersonal communication.

In this theory, thinking is regarded as the act of talking to yourself. Sfard (2015) claims that thinking is to communicate and thus presents it as a sort of internal process where the communication of mathematical concepts is the major factor. Therefore, the theory proclaims that the best communication happens when students are closely tied to the mathematics problem, try different solutions, and apply various strategies.

The essence of this theory lies in the idea that communication thrives when individuals actively participate in mathematical activities. This concept aligns with the notion presented by Sfard (2008: 44), who describe it as ‘a system that contains the objects of talk along with the talk itself’. For example, individuals naturally engage in self-dialogue, exemplifying seamless interchange within this system.

In brief, Sfard’s commognitive framework underscores the interconnectedness of communication and thinking, asserting that meaningful communication occurs when individuals actively engage in mathematical processes and participate in a diverse range of problem-solving methods.

2.10 Discourse

Through the framework of discourse analysis, an interpersonal communication situation is revealed in which teachers and students relate to each other. For example, in the cognitive process, it encourages teachers and students to engage in meaningful discourse through participation. Discourse is viewed as the mathematical thinking involved in communication (Sfard, 2012). This means that discourse is communication that is achieved through speaking or writing with oneself. Sfard (2020) explores the notion of discourse as a dialogue created using communication methods such as speech or writing. The deliberate implementation of ideas occurs in discourse. Discourse analysis shows how communication is used in actual circumstances.

The identification of key concepts in student interactions becomes clearer when students' active use of specialised keywords for the given activity is taken into account (Sfard, 2020). According to Sfard (2012), the terms of discourse can be grouped into two main categories that are key concepts of colloquial and literate discourses, which have their own distinct traits. These concepts are the basic elements that help to build up the students' understanding of the topic better.

2.10.1 Colloquial discourses

Colloquial discourse refers to words that arise from everyday conversation. For instance, in linear equations integers (whole numbers), variables, and algebraic terms. The colloquial discourse in the setting improves collaboration in the learning environment through interpersonal communication. Colloquial words are described by Sfard (2008:80) as “everyday communicative practices,”” in the context of mathematical development. Moreover, colloquial words are naturally produced and are known as words that make it difficult to see clearly. In a sense that during mathematics discourse, colloquial language is evident. To improve learning, familiar words are introduced for the development of mathematical discourse. However, it is more challenging to understand the meaning of these words. As a result, students struggle to comprehend the material. In contrast, observing the use of everyday language could make learning easier and understandable, but because the meaning of words does not necessarily represent mathematical usage, learning becomes difficult. The subsequent sub-section discusses the literate discourses.

2.10.2 Literate discourses

Discourses that are specialised are referred to as literary discourses. Sfard (2014) defines literate words as Literacy. Literacy refers to subjects that require students to read and write. Sfard's own contribution helps to make the concept of literacy more understandable by relating it to another theme that is just as important to educators for mathematics and mathematicians: the concept of communication.

These two key features (Literate and Colloquial) of mathematical discourse are deliberate and intended to bring about change. Therefore, to make the use of these key features meaningful, mathematical understanding will be required.

2.10.3 Mathematical Discourses

The identification of a mathematical discourse is accomplished through connection. The success of mathematical discourse is achieved through the participation of the students. Thus, Sfard (2012) discovers that thinking is the activity of communicating with oneself. She separates words in use (vocabulary), visual mediators, routines, and endorsed narratives into four categories that make up mathematical discourse. The discourse of the learner is identified by the key concepts above, which are discussed in more detail below:

Words in use

Keywords in mathematical discourse play an important role in providing a clear understanding of content and strategies useful in learning and teaching mathematics. According to Sfard (2021), when communicating, certain words are used for a specific purpose. Then, in mathematics, the words should be the vocabulary to learn. These words should be communicated in the same language with substantive meaning (Sfard, 2021). For example, if teachers integrate words based on topic, then conceptual understanding can be enhanced. Tasara (2017) argues that typical words used in discourse are mathematical terminologies. This facilitates comprehension of the content by the participants. There are words for students that sound novel in mathematics. Then, these words are used in different contexts and mean numbers, variables, and functions. For example, the word 'variable.' In mathematics, a variable refers to a symbol standing for an unknown numerical value. Another word *'coefficient'.* In mathematics, the word coefficient is the numerical factor that multiplies a variable. In a linear equation, a constant' represents a term that does not contain a variable.

Visual mediators

Symbolic artefacts such as numbers, tables, algebraic expressions, equations, and graphs are referred to as visual mediators. Even though, when the mathematical discourse is represented in a symbolic artefact, the sequence explanations can sound clear and conceptually understandable. In the context of mathematical discourse analysis, visual mediators provide clear, detailed, and understandable explanations of sequences (Sfard, 2018). They help students actively participate and devote energy to conceptual understanding. Putting effort into working with numbers and algebraic expressions by implementing visual mediators helps students. In a discursive process, visual objects are formed to enhance the visual mediator (Sfard, 2015). In other words, it improves students' understanding of complex concepts, responds to their needs, encourages logical and critical thinking, and helps develop communication skills. They help by giving students useful details about the dimensions, composition, and appearance of an object that is being studied. Ryve, Nilsson, and Pettersson (2013) emphasise the point that students' creativity can help make mathematical communication effective by incorporating visual mediators from both inside and outside of mathematics. Sfard (2021: 4) states that ‘the set of visual mediators of physical entities, with which participants make clearer what they are talking about, is another important discourse-defining feature’.

Routines

Routines are groups of metarules that describe repeated discursive action. Routines allow participants to successfully complete communication objectives and improve conceptual understanding. According to Sfard (2012: 2) routines are ‘predetermined patterns for carrying out mathematical tasks’. In the context of mathematical discourse analysis, they help students develop the habit of connecting. As a result, understanding can be attained. A routine is a discursive framework that displays guidelines that should be followed to be well-informed (Sfard, 2018). For example, through repeated discursive action, teaching and learning become successful. Routines are classified into two types: rituals and explorations.

Ritual (a type of routine)

Rituals are determined by action, which substantiates the words ‘what and how’. For example, students perform the task assigned to them by following the procedures. The first ‘prerequisite for effective communication is illustrating and engaging the researcher's vocabulary’ (Sfard, 2012: 8). In support, Heyd-Metzuyanim and Graven (2019) argue that rituals facilitate close connections. The participants' interest in and habit of learning mathematics can significantly increase because mathematics will be easier to understand if they know why things are done or what strategies are used.

Exploration (a type of routine)

Exploration is a type of routine that aims to create endorsed narratives about mathematical objects (Sfard, 2012). In exploratory routines, students deeply engage in a task, examining relationships and details to enhance their learning. The main goal of exploration is to increase cognitive development by encouraging students to construct, test, and justify their mathematical claims. According to Heyd-Metzuyanim and Graven (2019), exploratory learning is more adaptable, coherent, and connected, when students develop the habit of making repeated attempts to understand and solve problems. Students begin to view mathematics not only as a set of procedures, but also as a reflection of their own intellectual growth and reasoning ability (Rahman & Ahmar, 2016). It also exposes students to meta-level learning, where they reflect on mathematical practices and develop strategies for approaching problems in flexible ways. This exposure can lead to communicational conflict, as different teachers use language, interpret visual cues, or apply discursive techniques differently (Sfard, 2015). These conflicts play an important role in developing communication skills of students, and it helps them to form habits of seeking discursive solutions to mathematical problems and enhancing their conceptual understanding.

Endorsed Narratives

Learning theorems, definitions, and computational rules form the basis of narratives, which provide context and are 'particularly effective in their role of sense-making' (Sfard, 2015: 5). Within this perspective, narratives give voice to mathematical objects, processes, or relationships and can vary from personal conjectures to formalised rules. When the mathematical community accepts a narrative as valid, it turns into an endorsed narrative, becoming part of the socially justified body of mathematical knowledge that supports reasoning and problem-solving. For example, if the equal sign represents the link between two expressions, the students need to understand the concept of the equal sign to distinguish between an equation and solve it. When students grasp the balance method for solving linear equations, it empowers them to use diverse strategies, and their input in the classroom discussion is what forms these narratives.

According to Gcasamba (2014), citing Sfard (2008), mathematical discourse comprises narratives of construction, recall, and substantiation. Students construct knowledge through creativity, critical thinking, communication and collaboration, recall it efficiently, and substantiate reasoning based on what, how, how well, and why they learn. Research shows that commognition theory is a useful framework for understanding mathematical discourse, with an emphasis on student thinking and learning (Lavie, Steiner & Sfard, 2019; Sfard, 2020).

2.10.4 Existing Literature on Commognition

Numerous studies based on the commognition perspective have also been conducted in various grades in secondary schools, and in tertiary institutions in different contexts (Gcasamba, 2014; Heyd-Metzuyanim & Graven, 2019; Roberts & Le Roux, 2019; Ryve et al., 2013; Nisa & Lukito, 2021). The purpose of the research by Gcasamba (2014: 32) was to analyse the mathematical thinking of Grade 11 learners based on features of 'mathematical discourse such as word use, visual mediators, narratives, and routines as they work with the function concept. The results show that learners lack mathematical discourse means that they face challenges in learning the four major mathematical functions**.**

The Heyd-Metzuyanim and Graven's study (2019: 150) about 'rituals and explorations in mathematical teaching and learning' show findings indicating that the proficiency of English language learners was limited, making it difficult for them to communicate effectively in the classroom, using the specialised language of mathematics. As a result, and in part due to the learners limited outside-of-classroom exposure to English, it means that mathematics conversations in the classroom are destined to remain at the level of imitative repetition of pre-established mathematics narratives.

The study by Roberts and Le Roux (2019) investigated the nature of Grade 8 and Grade 9 learners' thinking about linear equations, with particular attention to the essential elements of *words in use,* as well as *visual mediators* to replicate routines and narratives. Their findings show that learners often rely on routine-driven strategies when working with mathematical tasks. And when learners work with words such as positive and negative integers, evidence suggests that they struggle with the objectification of mind perceptions. In this case, learners are trying to conceptualise the properties and relationships between abstract entities such as numbers and symbols. This suggests that learners are in a stage of transition, moving from ritualised discourse toward exploratory discourse.

Ryve, Nilsson, and Pettersson (2013) conducted a study on how to effectively communicate mathematical ideas to sixth graders (12–13 years) and university students by using *visual mediators* and technical terms, while engaging with both *ritualised practices and exploratory formulations.* The findings show that the objective was reached and the student interactions became more conceptualised and elaborated while engaged in 'guesswork' together, as their goal was to improve their ability to communicate effectively. To give an example in terms of mathematics discourse, their participation was mutual and cooperative, and everyone developed a conceptual understanding, and thus the teaching and learning were being effective. As a result, this research underlines the important need to further explore visual mediators in mathematical discourse.

The research of Nisa and Lukito (2021) aimed to analyse the mathematical communication of 10th-grade students while solving absolute value equations. The study investigated four features of students' communication: vocabulary, use of drawings and other visual aids, storytelling, and routines. The communication of the students was then classified into two categories: ritualised or exploratory. The results indicated that the discourse of the students had evolved to the exploratory level, indicating a transition from ritualised methods to more conceptual understanding and the use of mathematical ideas in their storytelling. It indicates a higher ability of students for making sense of and engage in exploratory activities through their communication.

These studies show us that the quality and nature of mathematical discourse can have a strong influence on how learners make sense of mathematics. This indicates that structured support provided by the teacher is needed to move learners from ritualised practices toward exploratory formulation and new ways of engaging.

2.11 Summary

In earlier sections, I have explored the various factors that contribute to the challenges of students in solving linear equations and algebraic fractions. Strategies designed to aid in the resolution of linear equations were also deliberated upon. In addition, I have explored common mistakes made by students in this context and offered insights into how these errors could be mitigated. This was followed by the theoretical framework that guided the study: commognitive framework integrating communication and cognition (Sfard, 2008, 2012, 2020). Learning mathematics is seen as both a cognitive and a discursive activity in which thinking is conceptualised as internalised communication, and discourse involves the words, visual mediators, routines, and narratives through which learners engage with mathematics. Research involving commognitive theory is discussed in different studies and the findings highlight their value in analysing the mathematical thinking of learners across various contexts.

2.12 Conclusion

The literature review provides a critical analysis and summary of existing research related to student errors and misconceptions when solving linear equations and algebraic fractions. It explores studies related to strategies to help students overcome these challenges and typical mistakes that can be mitigated in the teaching process. This is followed by an explanation of the theoretical framework used to guide this study in the collection, analysis, and interpretation of the data collected. Although the literature review helps to understand what research has been done before and locates this study in that context, the theoretical framework provides the tools for the research itself.

3 METHODOLOGY

3.1 Introduction

The preceding chapter presented the theoretical framework, which provided a conceptualisation of the research problem, and laid the foundation for this study. The present chapter builds on this foundation and explains the methodology used in the study, including the research paradigm, the research approach, the research design, and the procedures followed, and is a guide to address the research questions given in the introduction chapter. This chapter outlines the qualitative research approach adopted and provides detailed descriptions of site selection, participant selection and sampling, data collection, data analysis, considerations of trustworthiness, researcher positioning, and ethical aspects.

3.2 Research paradigm

There are several of research paradigms that are commonly used in the research, namely positivist, interpretivist, constructivism, advocacy/participatory, and pragmatism (Creswell & Poth, 2018). This study adopted an interpretivist research paradigm that focuses on understanding the subjective experiences of the participants (Creswell & Poth, 2016). Using this approach, the study seeks to interpret the errors of NCV students when solving linear equations with fraction coefficients and to capture their experiences in their own words to produce rich and meaningful data for analysis.

3.3 Research approach

There are different approaches to research inquiry, and these include quantitative, qualitative, and mixed methods (Creswell , 2014); Taherdoost, 2021). This study involved a qualitative approach aiming to explore, generate, and share lived experiences (Creswell & Poth, 2016). It was well suited for this research because it provided a more in-depth study of NCV students' experiences of solving linear equations with fractional coefficients. It also included the use of interviews to better understand student perspectives and the meaning they make of the learning process (Hennink, Bailey, & Hutter, 2020).

3.4 Research design

The qualitative approach can employ the use of case studies, grounded theory, phenomenology, and participatory action research. In the context of this study, a case study research design was adopted, which is an in-depth examination of individuals or small groups in each environment (Creswell, 2012). This study sought to examine a group of Level 2 Mathematics students studying at a TVET public college in the Western Cape, aiming to investigate the participants' experiences and to gather rich insights into the research problem.

3.5 Research methods

According to Ellis and Levy (2009), research methodology refers to the systematic approach that is characterised by the type of study being conducted and methods employed. In this study, the methodology describes the specific methods used to answer research questions. It provides details on site selection, participant selection and sampling, data collection, data analysis, trustworthiness, the position of the researcher, and ethical considerations.

3.5.1 Site selection

The present investigation took place at a TVET public college situated in a suburban setting within the Western Cape region of South Africa. The selection of this educational institution as the focal point for the study came from the dual role of me, the researcher, who concurrently served as a mathematics Level 2 lecturer within the college.. The primary objective of TVET colleges is to impart specialised education geared towards the cultivation of skills, knowledge, and attitudes essential for effective participation in the contemporary labour market. Therefore, this study focuses on the fundamental aspects of accessibility within the context of TVET education, exploring the dynamics and nuances that influence the accessibility of educational resources and opportunities within the specified academic setting.

3.5.2 Participant selection and sampling

The study focused on a Level 2 Mathematics class consisting of 35 students, twenty-eight males and seven females, registered for the course in 2022. The participants, aged 16 to 28, were purposefully selected to investigate errors in solving linear equations with fractional coefficients. Purposive and convenience sampling methods were used for participant and site selection. Purposive sampling involved the deliberate selection of 35 students based on their performance ratings, while convenience sampling utilised the college, where I was the Level 2 mathematics lecturer and had access to the students. The four criteria used to guide the selection process were accessibility, geographical proximity, time proximity, and availability.

3.5.3 Data collection

Some of the data collection methods used in qualitative research are participant observation and interviews (Moser & Korstjens, 2017). In the context of this study, two methods were used for data collection: a written test and individual interviews. The method of data collection through interviews involved asking questions directly to the interviewee(s), face-to-face (Moser & Korstjens, 2017). The purpose of conducting the test and interviews was to gather qualitative data that would assist me in gaining an in-depth understanding of the student thinking when solving equations with fractional coefficients. The test answers were used to gather information related to the students' errors and the interviews provided information about the reasons for the errors made.

The instruments were designed based on the research questions that were being investigated. I designed a written test consisting of twelve different types of linear equation items based on the curriculum requirements for Level 2 mathematics. The questions included equations with fractions and non-fraction co-efficients. After the students have completed the test, I purposely selected 12 students based on the errors noted in their written responses, and categorised the type of erros they made. I then conducted face-to-face interviews with the students to explore the reasons behind their errors.

TABLE 3.1: DATA COLLECTION PLAN

Research sub-questions	**Research instruments**

1. What kinds of errors are demonstrated by students when solving linear equations with fractional coefficients?	Test
2. What are the possible causes of the errors that students demonstrate when solving linear equations with fractional coefficients?	Interviews

Designing the test

First, I designed a written test to collect data on how students make errors. The test was investigated and analysed in full detail. In this regard, a twelve- questions test was developed to assess proficiency with linear equations and linear equations with fractions. There were three versions of the tests designed to select the most suitable items for Level 2 mathematics. The test was designed to cover the content indicated in the curriculum and to be completed in the stipulated time. Each question was created in a way to explore the errors that students typically make when answering equation problems, including the use of signs, order of variation changes, different denominators, and expanding brackets, operation of the equal sign, and balance equations.

The strategy approach used to create the test ranged from Easy to Medium in terms of difficulty. The easiest questions came first, and the most difficult questions followed later. The goal of the strategy was to encourage the students to answer all the questions. When students participate in lessons, they are encouraged to answer all questions, to take risks, and to understand and learn from their mistakes. Boaler (2014) argues that to actively engage in learning mathematics, students should recognise that making mistakes is an integral aspect of the learning process and that mathematics is a subject that promotes growth.

The tests were completed in Term 3 with the intention to assess students recall of the content taught in the previous two terms related to solving equations. The test questions were designed and selected using two principles:

1. Learners should be familiar with the type of test questions covered in class.
2. The test questions should include variation (both in the types of equations and levels of difficulty) and familiar contexts (guided by the "*National Certificate Vocational (NCV), National Curriculum Statement and the standards"*).

The factors considered in the test design include test structure, item suitability, and trustworthiness, and this was moderated by the mathematics lecturer who teaches at the same level. The written test was validated and standardised to ensure its suitability for Level 2 mathematics students. It was designed based on the curriculum requirements and reviewed by a mathematics lecturer teaching at the same level to confirm the clarity, relevance, and ability of the questions to capture common student errors. Multiple versions of the test were analysed to select items with appropriate difficulty, variation, and familiar contexts, ensuring that students could attempt all questions and that the test reliably assessed their understanding of linear equations with fractional coefficients.

The test questions

The test consists of 12 questions sourced from the National Certificate Vocational (NCV) textbook and the National Curriculum Statement and Standards. There were four questions related to linear equations with unknown variables and eight questions with fraction co-efficients. The questions without fractional coefficients were presented first to capture students' attention and encourage them to attempt all items. It was anticipated that encountering fractional equations at the start might lead to disengagement. The aim was to guide students to begin with simpler questions and gradually progress to more challenging ones, enabling them to feel confident in attempting the more difficult problems.The students were encouraged to use different solution strategies (Trial and Error method, Systematic method, and Transposition method) to solve the problem and to represent their thinking and communication.

All questions were created with due consideration of the literature related to factors that influence students' struggle and the unsuccessful use of strategies to solve linear equations, as discussed in Section 2.2 of Chapter 2. Each question was created to help categorise the common errors that emerged from the literature review. The following is a discussion of the individual questions and the typical mistakes associated with these types of questions.

Discussion of the twelve questions

These are the twelve test questions that were used to design the test, the assessment criteria for each test item are also given:

TABLE 3.2: THE TWELVE QUESTIONS

Type of Questions	**Assessment criteria for each question:**
$3x - 6 = 9$	Assess students' ability to identify inverse operations and balance equations.

$13 = 2(\mathrm{k} - 4) + 5$	Assess ability to expand brackets, apply multiplication, and work with the negative sign.
$d - 9 - 2d = 7$	Assess the ability to combine similar terms and manage negative signs.
$13 = 5 - 2(\mathrm{k} - 4)$	Assess the ability to work with order of operations and apply distributive property with negative numbers.
$\frac{x}{4} + 1 = 6$	Assess the ability to solve equations involving fractions and isolate the variables.
$1 - \frac{1y}{3} = 6$	Assess the ability to solve equations involving fractional coefficients and negative terms.
$\frac{(3x-5)}{5} = \frac{(2x-8)}{4}$	Assess the ability to find the lowest common denominator and apply to both sides.
$\frac{y+4}{3} - \frac{2y+2}{5} = \frac{y}{15}$	Assess ability to manage subtraction with fractions, find the LCM and maintain equality.
$\frac{x}{6} = 12(1 - \frac{x}{3})$	Assess the ability to apply the order of operations and isolate the variables.
$\frac{6}{4}(x+4) - \frac{2x}{6} = 2 - \frac{4x-3}{12}$	Assess the ability to find the LCM of multiple denominators and work with distributive property and negative numbers.
$\frac{5}{2d-20} = 9$	Assess the ability to solve rational equations using cross-multiplication and simplification.
$\frac{3}{x} = \frac{4}{x-4}$	Assess the ability to apply cross-multiplication with variables in denominators.

Interviews

According to Gill, Stewart, Treasure, and Chadwick (2008), interviews are conducted to explore the views, experiences, beliefs, and/or motivations of individuals on specific matters. Sharma

(2013) argues that the interview method takes the form of a dialogue in which the researcher seeks to elicit information from the subject about their thoughts. Interviews enable the interviewer to investigate deeper by asking follow-up questions to gain better understanding. Before the interviews, I clarified the purpose of the interviews with the students to help them understand what they can expect from the process. I purposively selected a group of 12 students based on the types of errors they made in the test questions. The aim was to be able to obtain rich data on the types of errors they made. The interviews were semi-structured, consisting of four different questions to explore the students' thinking about their solution strategies. Those four questions are as follows:

1. How did you solve this problem?
2. Is there any other approach you can use to solve this problem? If so, which approach?
3. Can you identify the error?
4. Could you correct the error?

Students were encouraged to freely express their opinions, share their experiences and discuss the reasons behind their errors (Kallio, Pietilä, Johnson, & Kangasniemi, 2016). The interviews were recorded with permission from the students and then transcribed.

3.5.4 Data analysis

Qualitative data analysis in this study involved the process of organising, interpreting and making sense of what emerged from the written tests and interviews (Akinyode & Khan, 2018). Two corresponding processes took place involving content analysis and discourse analysis, and these were aligned with the research questions and the commognition framework (Sfard, 2020).

Content analysis

In analysing the written test responses, I used a directed content analysis approach which is a combination of deductive and inductive reasoning (Hsieh & Shannon, 2005). Initially, the 35 student tests were marked for correct, incorrect, and not attempted responses. Incorrect responses were then analysed to find patterns and recurring errors. The deductive process informed the classification of the errors into categories guided by the literature and patterns that emerged from the data, these errors included procedure, conceptual, computation, conjoining, inverse, distributive, minus ignored, and equal sign. Simultaneously, I used an inductive approach for student' errors that did not fall into the predefined categories, and I added new categories such as careless errors, balancing errors, and over-generalisation. This combined use of deductive and inductive analysis is consistent with qualitative research practices, where a priori categories drawn from existing literature are first applied to the data, and then new codes are generated from the data itself as they emerge during analysis (Creswell & Creswell, 2018). This hybrid approach allowed me to connect the analysis with existing research while also allowing me to respond to the specific errors shown by the students in this study. These results informed the purposive selection of 12 participants for follow-up interviews to explore their reasoning in more detail.

Discourse analysis

Discourse analysis was used to explore students' thinking, reasoning, and communication about their mathematical solutions (Kovalainen & Eriksson, 2015; Stainton & Willig, 2017). This involved semi-structured interviews with the 12 purposively selected students which were transcribed and analysed based on the four key features of mathematical discourse in the commognition framework (Sfard, 2012).

- Word use: Students' use of vocabulary when explaining their solutions.
- Visual mediators: How students used symbols and diagrams when representing mathematical operations.
- Routines: Student strategies and procedures used in problem-solving.
- Endorsed narratives: How students explained and justified their solutions and the correctness of their answers.

The findings of the content and discourse analyses were integrated and helped identify the types of errors students made and the underlying reasons that contributed to these errors. This dual approach helped to provide a deeper understanding of students' mathematical thinking, the challenges they faced, and the ways their discourse shapes learning and the problem-solving process.

3.5.5 Trustworthiness

Qualitative researchers utilise various criteria to evaluate the trustworthiness of qualitative research, including credibility, dependability, confirmability, and transferability (Elo, Kääriäinen, Kanste, Pölkki, Utriainen, & Kyngäs, 2014; Korstjens & Moser, 2018). The study's findings were determined by considering credibility, transferability, dependability, confirmability, and member (student) checks. Forero, Nahidi, De Costa, Mohsin, Fitzgerald, Gibson, McCarthy, and Aboagye-Sarfo (2018: 11) affirm that credibility refers to the establishment of confidence that the results are true, credible, and believable. I collected different sources of data (test answers and interviews) to triangulate the results. I also used member checks to achieve credibility by requesting the participants to verify that the data collected accurately reflected their comments. Member checking provided a way for me to ensure accurate representation of the voices of participants by allowing the participants to confirm or deny the accuracy and interpretations of the data, thus adding credibility to the qualitative study (Candela, 2019).

Transferability refers to the extent to which qualitative findings may be applied to other similar contexts through the provision of rich, thick description (Creswell, 2014).To achieve transferability, I evaluated whether the findings of the study can be applicable to other contexts in a similar situation. Transferability in this study was achieved by giving a detailed explanation of the research context, the NCV Level 2 students who participated, and the procedures followed during data collection and analysis. I provided examples of students' responses and the types of errors they made when solving linear equations with fractional coefficients. By describing the college setting and the nature of the tasks in detail, readers are able to judge whether the findings may apply to similar vocational education contexts. Elo, Kääriäinen, Kanste, Pölkki, Utriainen, and Kyngäs (2014) affirm that dependability refers to the stability of data over time and under different conditions. Therefore, confirmability refers to the degree to which the findings of the research study could be confirmed by other researchers (Korstjens & Moser, 2018) To achieve dependability and confirmability, I requested external researchers (my supervisors) to review and examine the research process and data analysis to ensure that the results of the study were consistent and could be replicated. I also kept detailed records of the research process, including data collection, coding, and analysis decisions. This provides for transparency and the ability to track and check analytical choices.

3.5.6 Position of the researcher

As the researcher, I assumed a facilitator role during data collection. I was responsible for the development of the study, including the research instruments, their implementation, analysis, and the development of the entire research project. In addition to being the researcher, I was also a staff member of the college, serving as the class teacher for this sample of students. I selected this college and group of students because I wanted to understand more about the difficulties my students were having in working with equations. I hoped to use the results of this study to improve and adapt my teaching in the future and share with my mathematics colleagues .

Although every effort was made to maintain objectivity in the study, it is important to note that subjectivity cannot be completely avoided. Objectivity, in this context, refers to the researcher not allowing personal beliefs or biases to interfere with the ability of the facts to speak for themselves (Letherby, Scott, & Williams, 2012). However, subjectivity is necessary for the researcher to seek out, understand, and interpret how the social world is experienced and produced (May & Perry, 2022). Connelly (2016) advises that the findings should be reviewed by a colleague to prevent biases arising from the perspective of a single person on the research. I tried to manage biases as much as possible by involving outside independent researchers (my supervisors) to review and confirm the accuracy and interpretation of my results.

3.5.7 Ethical considerations

According to Coffelt (2017), confidentiality and anonymity are ethical practices designed to protect the privacy of human subjects while collecting, analysing, and reporting data. Confidentiality refers to 'separating or modifying any personal, identifiable information provided by participants from the data' (Coffelt, 2017: 227). By contrast, anonymity refers to collecting data without obtaining any personal identifying information provided by participants from the data. Confidentiality and anonymity were used to conceal the identities of the participants during the study process. Participants were not required to state their names if they allowed me to record the interviews.

Ethical clearance.

Before conducting the study, I obtained research permission from the Cape Peninsula University of Technology, Faculty Ethics Committee, as well as the necessary permissions from the principal of the college, the parents of students under 18 years of age, and NCV Level 2 students. The permission letters from Northlink College and CPUT can be found in Appendix A and Appendix B, respectively. Samples of the consent letters provided to the principal and parents of participants under 18 years of age, and NCV Level 2 participants can be found in Appendix C and Appendix D.

Informed consent

Providing participants with information on the purpose of the study and how the data will be treated depends on the way confidentiality is described to the participants (Yin, 2003: 71-72). Harriss and Atkinson (2015) assert that for participants to be interested in and involved in the study, it is crucial to explain the study's purpose to them. I first informed the students, school principal, parents of those learners who are younger than 18 years old about the details of the research before and after data collection. Second, I explained to students the option for voluntary participation and their right to withdraw from the study at any time. Green and Condy (2016) emphasise that students should feel comfortable and aware of their right to withdraw if they feel uncomfortable during the study. Only those students who agreed to participate and had permission from their parents were tested and interviewed. Third, I explained to students my approach to addressing the concerns of confidentiality and anonymity. And finally, to ensure security, I assured the participants about the safety and secure storage of the collected data and that all data would be unloaded to the CPUT eSango secure repository. I successfully obtained consent from participants and parents of participants younger than 18 years in written forms.

3.6 Conclusion

This chapter presents the overview and justification of the methodology and approach used in this study. It starts with an outline of the paradigm, approach and design of the research and provides detailed descriptions of the research methods used, including site selection, participant selection and sampling, data collection, data analysis, trustworthiness considerations, the position of researcher, and ethical issues. I also explained the criteria used to ensure the reliability of the qualitative approach and the steps taken to enhance trustworthiness. In addition, I outline the procedures for addressing ethical considerations. Moreover, I conclude with the procedures for obtaining informed consent and data storage.

4 DATA ANALYSIS AND FINDINGS

4.1 Introduction

In the previous chapter, I presented the research design, data collection methods and the analytical approaches – the analysed content and discourse – that were used in this study. In this chapter, I present the results of that analysis and the findings that emerged from the data.

The analysis and findings addressing the research questions are organised into four sections. First, I provide an overview of the performance of the students on the written tests by analysing the test responses for correct and incorrect answers, and responses that were not completed. Second, I classified the types of errors found in the student test answers. Third, I categorised these different types of errors across the entire dataset. Finally, I show the examined student interview responses and explore the possible reasons behind the errors they exhibited when solving linear equations by using Sfard’s (2012) commognition framework that was discussed in Chapter 2.

Therefore, this chapter addresses the aims of the research, namely, to investigate the kinds of errors produced by NCV Level 2 students when solving linear equations with fractional coefficients, and to understand the underlying reasons for these errors. The findings are discussed in relation to the following research questions:

1. What kinds of errors are demonstrated by students when solving linear equations with fractional coefficients?
2. What are the possible causes of the errors demonstrated by students when solving linear equations with fractional coefficients?

4.2 Test analysis

Students were given a test to assess the level of their understanding of linear equations, and linear equations with fractional coefficients. Once the test was completed, I started the test analysis by marking the learner answers for correct and incorrect responses, and those that were not attempted. I then tabulated the results for each of the questions in terms of correct and incorrect answers, and those that were not attemped. This helped to provide an overall summary of the class performance per question. A percentage column was included in the table alongside the raw numbers of correct and incorrect answers for several different reasons. Percentages make it easier to make comparisons across different questions, and they provide a more intuitive sense of how well students performed relative to the total possible answers.

They also help to highlight the significance of the results and to identify trends in performance across different questions.

TABLE 4.1: TEST RESULTS PER QUESTION

Question	Correct	%	Incorrect	%	Not attempted	%
1.1.1	23	66	12	34	0	0
1.1.2	13	37	22	63	0	0
1.1.3	8	23	26	74	1	3
1.1.4	8	23	23	66	4	11
1.1.5	15	43	16	46	4	11
1.1.6	9	26	20	57	6	17
1.1.7	10	29	21	60	4	11
1.1.8	0	0	34	97	1	3
1.1.9	1	3	28	80	6	17
1.1.10	0	0	26	74	9	26
1.1.11	4	11	23	66	8	23
1.1.12	7	20	18	54	10	29

The table above illustrates the number of students who answered each question correctly, along with corresponding percentages, and the number of students who answered incorrectly, or did not attempt the questions. The analysis of the responses indicated that all 35 students faced difficulties in solving linear equations that involve fractions and variables on both sides of the equation. This is particularly concerning because mastering this skill is crucial for tackling more advanced topics in mathematics like applications of differential calculus and various real-life problems at higher levels (Kunwar & Laxmi, 2023). Most questions have a higher number of incorrect answers compared to correct ones, indicating a general difficulty or lack of understanding in solving linear equations. The later questions showed a higher number of non-attempts, especially those with fractional coefficients, which highlights difficulties with conceptual understanding and problem solving.

Although the table shows the number of correct, incorrect, and unattempted responses for each question, this study focuses on NCV Level 2 students' errors when solving linear equations with fractional coefficients. Rather than emphasizing scores or percentages, the research seeks to understand how students approach these problems, the strategies they use, and the challenges they encounter. This qualitative approach prioritizes meaning and understanding over numerical performance, highlighting the experiences behind the responses rather than the responses themselves.

4.3 Classification of student errors per question

The second part of the analysis involved the selection of the incorrect test answers, and analysing the error and nature of the errors into different categories. Based on the literature review, I created a coding framework to analyse and classify the list of incorrect answers by compiling a list of typical errors in solving equations. The categories of errors identified from the literature include procedural, conceptual, computation, conjoining, inverse, distributive, **minus sign error,** minus ignored, equal sign, **over-generalisation** and carelessness.

These are defined as follows:

- A *procedural error* is a mistake that arises from not correctly following the established steps or methods for solving a problem (Delastri & Lolang, 2023). For example, a student may make an error by not correctly following the steps $3x - 6 = 9$, where the solution is $x = (9 - 3)(6 - 3)$.
- A *conceptual error* is a specific type of mistake that occurs when a student incorrectly performs calculations (Qetrani et al., 2021). For example, the student ignored the constants (-5 and -8) in the numerators.

$$\frac{3x - 5}{5} = \frac{2x - 8}{4}$$

$$\frac{3x}{5} = \frac{2x}{2}$$

- A *computation error* is a mistake a student makes during calculations, often due to carelessness or a lack of clear understanding of basic math rules like addition or multiplication (Lestiana, Herani, Rejeki, & Setyawan, 2016). For example, a student may make an error by accidentally adding and subtracting numbers incorrectly like $(9 + 6 = 14)$ or writing down numbers incorrectly (18 instead of 180), and by misusing the rules of multiplication and division.
- A *conjoin* error is a specific type of mistake a student makes during calculations, often because of problems like misusing inverse operations; $3x - 12 = 4x \text{ becomes } 4x - 3x = 12$: ignoring negative signs $(d - 2d = 3d)$; incorrectly applying the distributive property $5(k - 4) + 2 = 13$, where this becomes $5 + 2 = k - 4 + 13$; and combining unlike terms $3x + 6 = 9x$ (Abay & Clores, 2022).
- An *inverse error* is a specific mistake a student makes during calculations, usually when they do not correctly apply the inverse of basic operations like addition or multiplication (Jupri et al., 2014). For example, $13 = 5 - 2k + 8$, solution $13 + 5 + 8 = -2k$.

- A *distributive error* is a specific mistake that occurs when a student incorrectly handles calculations involving the distributive property rule (Gumpo, 2014), for example,

$$13 = 5 + 2(k - 4);$$

$$13 = 7(-4k)$$

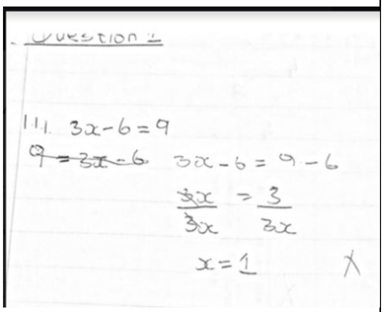
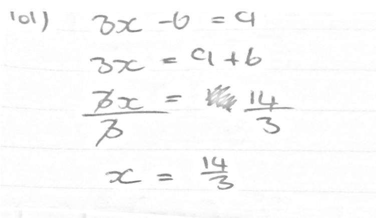
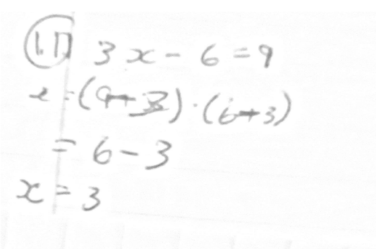
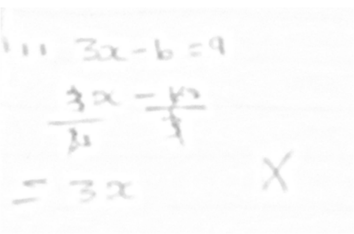
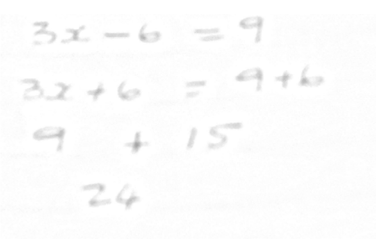
- A *minus sign error* occurs when a student overlooks or disregards a negative sign during calculations or problem-solving (Aydin-Güç & Aygün, 2021), for example,

$$13 = 2k - 8 + 5$$
$$2k = -8 - 13 + 5$$

- A ***minus sign ignored error*** occurs when the negative sign (−) is misinterpreted, misplaced, or incorrectly handled (Pournara, Sanders, & Takker, 2022). This can involve forgetting to apply the negative sign, flipping the sign accidentally, or incorrectly distributing it across terms. An example is the following: $-2(k-4) = 2k - 4$.
- An *equal sign* error is a mistake students make during calculations when they misunderstand the equal sign, treating it not as a symbol that shows two sides are equal, but as a signal to perform more calculations (Machaba, 2017). For example, students might think of it as just a step to calculate the next part, leading them to make errors: $3x + 6 = 9 + 6$, solution $9 + 15$.
- An ***over-generalisation*** **error** happens when a student applies a rule or concept too broadly or inappropriately, often in situations where it does not apply (Aydin-Güc & Aygün, 2021). For example $\frac{y+4}{3} - \frac{y+2}{5} = \frac{y}{15}$ becomes $\mathrm{y} + 4 - \mathrm{y} + 2 = y$.
- *Careless errors* are mistakes made not because a person lacks understanding or knowledge, but due to inattention, haste, or oversight during the process of solving a problem (Marpa, 2019). Example 1: $14 + 6 = 15$ *or* $2d - 20$ is written as $2d - 2$.

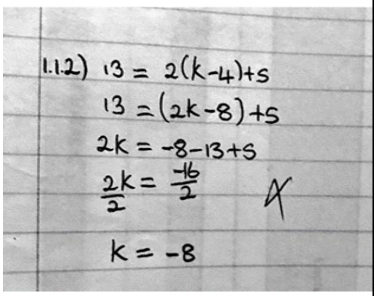
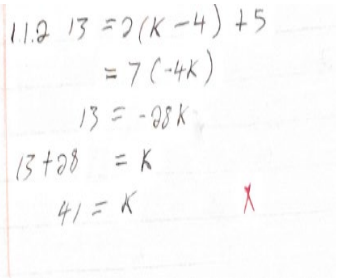
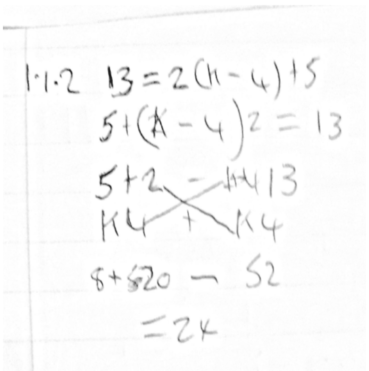
As there were many number of incorrect responses, I had to select the typical incorrect answers given by learners for each question. I then analysed the response and allocated a category of error to each response. These are represented in the section below and helped to address the first research question: identifying the types of errors students make when solving linear equations, especially those involving fractional coefficients. Each of the questions are analysed based on the student errors given in the test answers.

TABLE 4.2: QUESTION 1 [12 INCORRECT]

Question and learner response	Analysis of response	The category of error
	In line 2, the S1 incorrectly places -6 on the right-hand side and ignores transposing it to be positive on the right-hand side. The student also divides by 3x instead of the coefficient, which is 3. I classified this as procedural error.	Procedural error
	In line 3, the S2 incorrectly adds (9 + 6) = 14 instead of 15. I classified this as computation error.	Computation error
	In line 2, the S3 opens two pairs of brackets instead of transposing -6 to the right-hand side. Firstly, the student fails to change the sign (9+6), then fails to divide not subtract (line 3) and gets an answer of x = 3. I classified this as procedural error.	Procedural error
	In line 2, the S4 ignores the equal sign, which I classify as an equal sign error. As a result, the student simplifies the expression instead of solving the linear equation. In line 3, the student say $\frac{6}{3} = 3x$. I classified this as computation error.	Equal sign error Computation error
	In line 2, the S5 makes an error on the left-hand side by combining 3x + 6 and getting 9. I classified this as conjoining error. In line 3, the student ignores the equal sign. I classified this as equal sign error.	Conjoining error Equal sign error

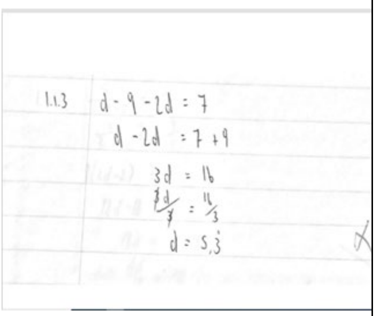
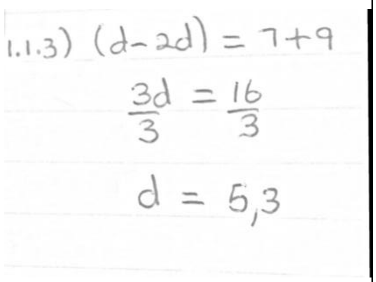
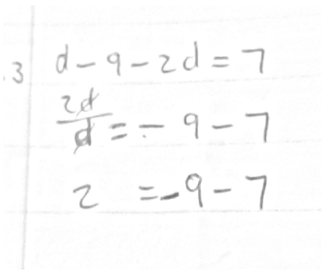
The overall results for Question 1 show that 34% of the students answered the question incorrectly. The item analysis shows that the errors include procedural, computation, conjoining, and equal sign..

TABLE 4.3: QUESTION 2 [22 INCORRECT]

	S6 starts by using the distributive property of multiplication to expand the brackets in line 2. The difficulty arises in line 3, where the student does not change the 2k to -2k when transposing the term. I classify this as a minus sign error.	Minus sign error
	In line 2, S7 combines the numbers 2 + 5 to get 7 and operates incorrectly inside the brackets by conjoining terms k – 4 to make -4k. I classify this as a conjoining error. In line 4, the student uses an additive inverse of -28 instead of the multiplicative inverse. I classify this as a conceptual error.	Conjoining error Conceptual error
	In line 3, S8 does not remove the brackets and use the distributive law. S8 combines the numbers on the LHS and makes this equal to the number on the RHS. I classify this as a distributive error. In line 4, the student adds unlike terms k- 4 to be k4. I classify this as a conjoin error. And also, in line 5 the student ignores the equal sign to make the equation into an expression. I classify this as an equal sign error. The student then ignores the k in line 5 and multiplies all the numbers by 4. I classify this as a computation error.	Distributive error Conjoin error Equal sign Computation error

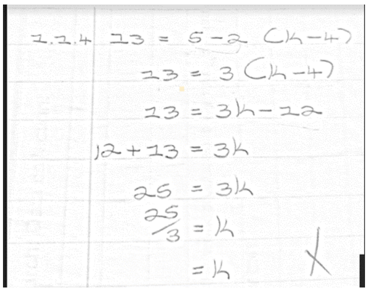
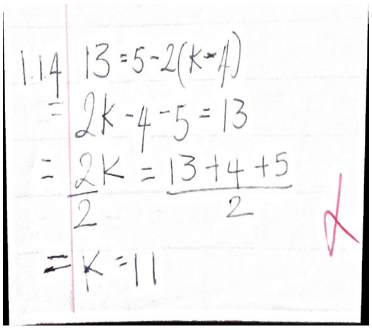
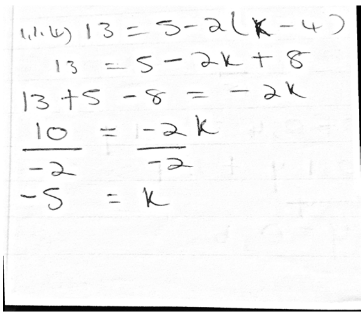
The overall results for Question 2 show that 22% of the students answered the question incorrectly. The item analysis shows that the errors include conceptual, compututation, conjoining, distributive, minus ignored, minus sign, and equal sign.

TABLE 4.4: QUESTION 3 [26 INCORRECT]

	In line 3, S9 ignores the minus signs in front of 2d and adds the letters on the LHS instead of subtracting. I classify this as a minus ignored error.	Minus ignored
	In line 1, S10 transposes the 9 correctly from the original question. I classify this as a conjoining error. However, in line 2, the (d - 2d) becomes 3d and the student ignores the negative sign. I classify this as a minus ignored error.	Conjoining Minus ignored
	In line 2, the S11 combines the ds and divides $\frac{2d}{d}$ = 2, which I classify as a conceptual error. The student also does not change the sign of -9, when transposing it to the right-hand side of the equals, which I classify as an inverse error.	Conceptual error Inverse error

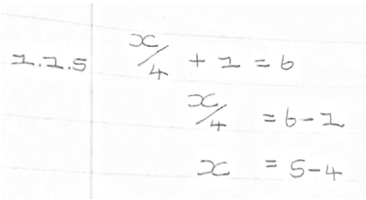
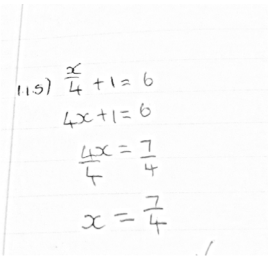
The overall results for Question 3 show that 26% of the students answered the question incorrectly. The item analysis shows that the errors include conceptual, conjoining, inverse and minus sign.

TABLE 4.5: QUESTION 4 [23 INCORRECT]

	In line 2, S12 makes an error on the RHS by not noticing there are two terms, only seeing two numbers and subtracting them. Although S12 solves the problem correctly thereafter, I classify this as a conceptual error.	Conceptual error
	In line 2, S13 did not distribute 2 properly to both terms inside the parentheses. -2(k-4) to get 2k-4. I classify this as a distributive law error.	Distributive law error
	Student S14 calculates correctly from line 1 to line 2. However, in line 3, the student does not change sign of 5 to be -5 when transposing to the left. I classified this as a inverse error.	Inverse error

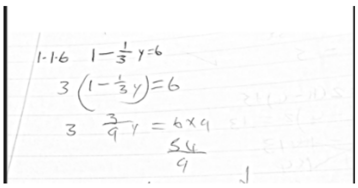
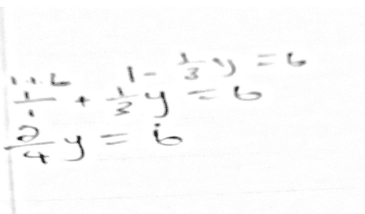
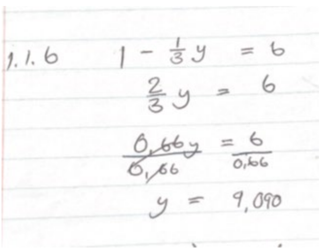
The overall results for Question 4 show that 23% of the students answered the question incorrectly. The item analysis shows examples the errors including conceptual, distributive law, and inverse.

TABLE 4.6: QUESTION 5 [16 INCORRECT]

	In line 2, S15 correctly transposes the +2 from the LHS to -2 on the RHS. In line 3, the student does not multiply by 4 on the right-hand side but subtracts instead. I classified this as a inverse error.	Inverse error
	In line 2, S16 converts x/4 into 4x, which I classify as a computation error. Next, the student does not change the sign when transposing positive 1, which I classify as a inverse error. Thereafter the student correctly divides both sides by 4.	Computation error Inverse error

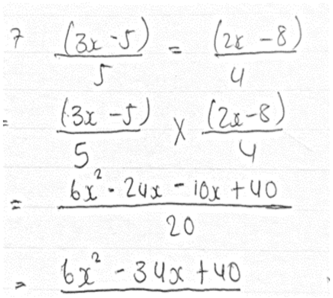
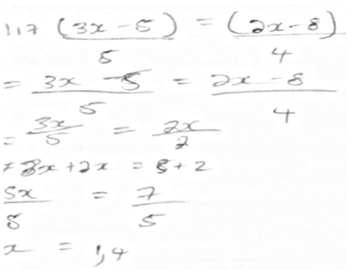
The overall results for Question 5 show that 16% of the students answered the question incorrectly. The item analysis shows examples of errors such as computation and inverse.

TABLE 4.7: QUESTION 6 [20 INCORRECT]

	In line 2, S17 multiplies $3(\frac{1y}{3}) = \frac{3y}{9}$. In line 4, the student leaves the equation unsolved. I classify this as a computation error.	Computation error
	In line 2, S18 changes (negative sign (-) to be positive (+). I classify this as a conjoining error. In line 3, the student adds unlike terms $\frac{1}{1} + \frac{1y}{3}$ to $\frac{2y}{4}$. I classify this as a conjoining error.	Conjoining error Conjoining error
	In line 2, S19 adds unlike terms $3 - 1y$ to get $2y$. I classify this as a conjoining error.	Conjoining error

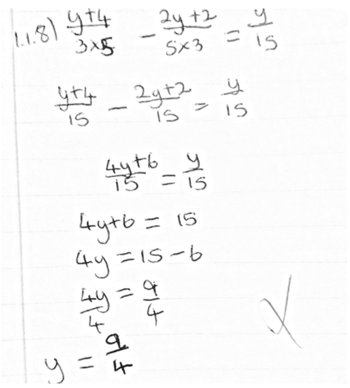
The overall results for Question 6 show that 20% of the students answered the question incorrectly. The item analysis shows examples of errors such as conjoining and computation error.

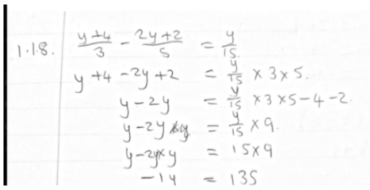
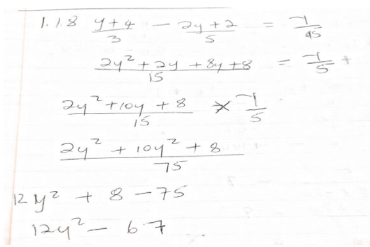
TABLE 4.8: QUESTION 7 [21 INCORRECT]

	In line 2, S20 ignores the equal sign and turns the equation into an expression. The student multiplies the two brackets, effectively altering the problem's purpose. I classify this as a **conceptual error.**	Conceptual error
	In line 3, S21 simplifies the fractions incorrectly. The student ignored the constants (-5 and -8) in the numerators. I classify this as a conceptual error The student combines the numerators and denominators from both sides as if they are like terms, ignoring that they are fractions. I classify this as a conceptual error.	Conceptual error Conceptual Error

The overall results for Question 7 show that 21% of the students answered the question incorrectly. The item analysis shows examples of conceptual errors.

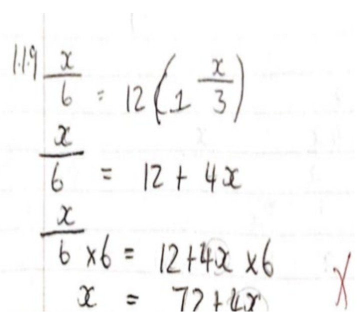
TABLE 4.9: QUESTION 8 [34 INCORRECT]

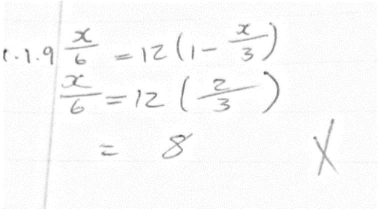
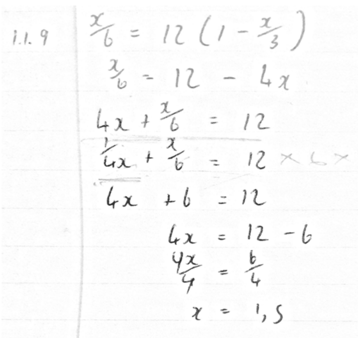
	In line 2, S22 correctly identifies the demoninator but does not make the fractions equivalent on the LHS. The student continues to work on the LHS and combines the numerators while ignoring the minus sign in the second term. I classify this as a minus sign ignored error. In line 4, the student removes the denominator and incorrectly makes the RHS = 15. I classify this as a minus sign ignored error. Thereafter the student correctly calculates for y (carrying the previous error)	Conceptual error Minus sign ignored error Computation error

	In line 2, S23 the student removes the denominators on the LHS incorrectly and multiplies by denominators (3 and 5), but does not apply this consistently across the entire equation. In line 3, the student subtracts the constants (– 4, –2) on the RHS, working with the incorrect error on the previous line. In line 4, the student shows confusion in working with numbers and operations on the RHS ignoring the algebraic fraction. In line 5, the y on the RHS is ignored the student incorrectly solves for x. I classify it as a conceptual error.	Conceptual error, equal sign and minus sign ignored error Procedural error Conceptual error
	In line 2, S24 treats the operation on the LHS as a multiplication instead of subtraction. I classify this as a conceptual error. In line 3, the student ignores changes the equal sign to a multiplication. I classify it as equal sign error. In line 4, the student ignores the y-numerator in the second fraction and multiples the denominators. I classify this as computational error. In line 5, the student changes the dominator into a negative whole number and goes on to simplify the expression.	Conceptual error Equal sign error Computation error Conceptual error

The overall results for Question 8 show that 34% of the students answered the question incorrectly. The item analysis shows examples of errors that include conceptual, equal sign, and computation errors.

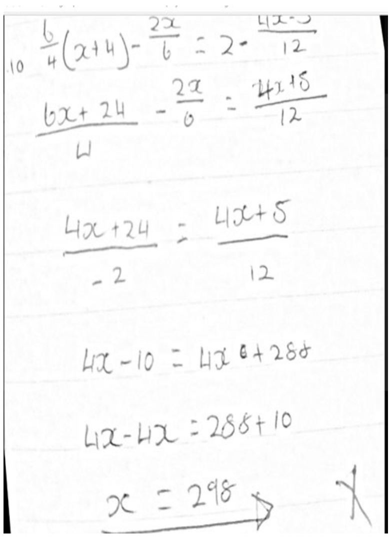
TABLE 4.10: QUESTION 9 [28 INCORRECT]

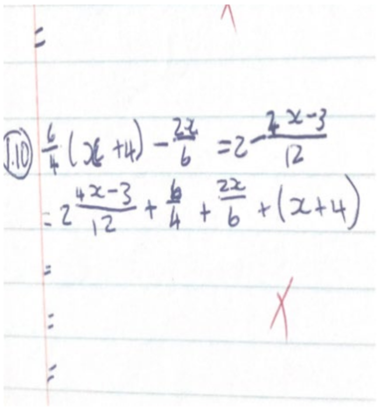
	In line 3, S25 does not multiply both terms by $6\,(12 + 4x)6$. I classify it as a computation error. In line 4, the student does not collect like terms $(x - 4x)$ in the equation, which I classify this as a computation error.	Computation error Computation error

	In line 1, S26 inside the brackets adds the like terms $1-\frac{x}{3}=\frac{2}{3}$ and throw away x. I classify it as a conjoin error. In line 3, the unknown (x) is missing. I classify this as computation error.	Conjoin error Computation error
	In line 3, S27 shows the unclear understanding of implementing $4x$ as a fraction. S27 flip $4x$ to be $\frac{1}{4x}$. I classify it as a computation error.	Computation error

The overall results for Question 8 show that 34% of the students answered the question incorrectly. The item analysis shows examples of errors that include computation and conjoining.

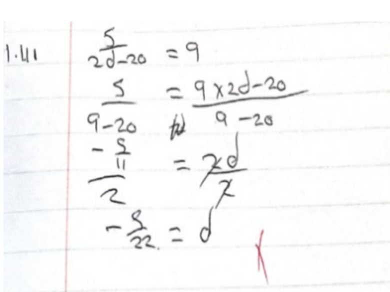
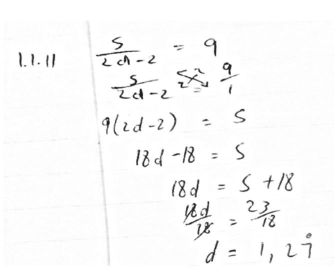
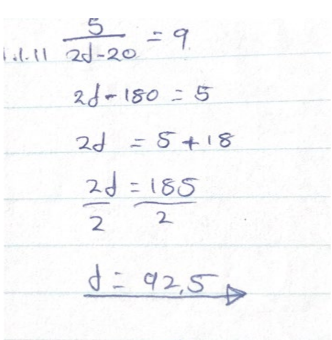
TABLE 4.11: QUESTION 10 [26 INCORRECT]

	In line 2, S28 subtracts the fractions by subtracting both the numerators and denominators, which I identify as a conceptual error. In line 3, the student divides the only constant numbers on numerator by the denominators, and I classify this as a procedural error. Then in line 6, the student subtracts $4x$ from $4\,x$. and gets x, which I classify as a computation error due to incorrect calculation.	Conceptual error Procedural error Computation error

	In line 2, S29 ignores the equal sign, turning the equation into an expression, which I classify as an "equal sign ignored" error. The student separates the first term by taking 6/4 from it and creates another term with $(x + 4)$, showing a lack of conceptual understanding. Additionally, in line 2, the student overlooks the minus sign after 2 that separates the terms and combines 2 with the fraction $\frac{(4x-3)}{12}$.	Equal sign ignored Conceptual error Minus sign ignored

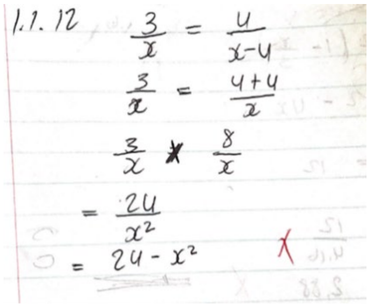
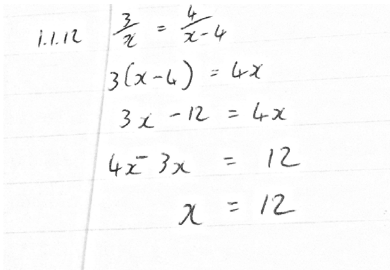
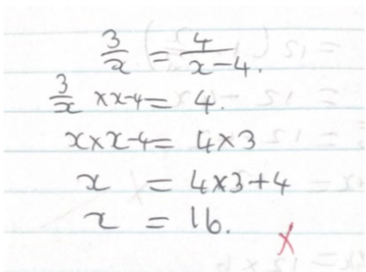
The overall results for Question 10 show that 26% of the students answered the question incorrectly. The item analysis shows examples of errors including conceptual, procedural, computation, equal sign ignored, and minus sign ignored.

TABLE 4.12: QUESTION 11 [23 INCORRECT]

	In line 2, S30 multiplies by the denominator only on the right-hand side, which I classify as a distributive property error of division. In line 3, the student divides 9 by 9 and then -20 by -20, leaving 2d on the right-hand side. I also classify this as a procedural error.	Distributive property error of division Procedural error
	In line 1, S31 writes 20 as 2, which I classify as a careless error. The rest of the solution is correct.	Careless error
	In line 2, S32 does not multiply 2d by 9. I classify it as an operation error. In line 3, the student writes 18 instead of 180, leaving out zero. I classify it as a careless error.	Operation error Careless error

The overall results for Question 11 show that 23% of the students answered the question incorrectly. The item analysis shows examples of errors that include distributive property, procedural, operation, and carelessness.

TABLE 4.13: QUESTION 12 [18 INCORRECT]

	In line 1, S33 removes -4 from the denominator and places it in the numerator. I classify this as an operational error. In line 4, the student moves x^2 from the denominator to the numerator. I classify this as a conceptual error.	Operation error Conceptual error
	In line 2, S34 multiplies 6 by 3 and gives 12 instead of 18, which I classify as a distributive property of multiplication error. In line 4, the student does not change the sign of $4x$ to $-4x$ when transpose to the left. I classify this as an inverse error.	Distributive Law Error Inverse error
	In line 3, S35 cross multiplies by multiplying the numerators together and the denominators together. I classify it as operational error In line 4, the student does not apply the first law of exponents. I classify this as a conceptual error.	Operation error Conceptual error

The overall results for Question 12 show that 18% of the students answered the question incorrectly. The item analysis shows examples of errors that include distributive property of multiplication, operation, inverse, and conceptual.

4.4 Types of Errors

In this section, I focus on the first research sub-question which concerns the kinds of errors made by NCV Level 2 students when they solve linear equations as well as linear equations with fractional coefficients. To categorise the errors, I applied a coding framework taken from the existing literature (Delastri & Lolang, 2023). The categories in this framework consisted of conceptual errors, computational errors, conjoining errors, procedural errors, careless errors, multiplicative errors, balancing errors, and operational errors. A summary table was then developed to indicate the frequency of these errors through the test questions and the errors were classified according to their characteristics and cognitive demands. The analysis showed that the students' answers were very much aligned with the various types of errors that are already mentioned in the literature, thereby supporting the applicability of these classifications to the NCV context.

TABLE 4.1 TABLE OF ERROR ANALYSIS SUMMARY

ERRORS	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
Procedural	2						1	3		1	1		**8**
Conceptual		1	1	1		2	2	1		2		1	**11**
Computation	2	1			1	1		2	4	1			**12**
Conjoining	1	2	1			3			1				**8**
Inverse			1	4	2						1	1	**9**
Distributive		1		1				1			1	1	**5**
Minus ignored		1	2					1		1			**5**
Minus		1						1					**2**
Equal sign	2	1						2		1			**6**
Over-generalise								1					**1**
Careless											2		**2**
	7	8	5	6	3	6	3	12	5	6	5	3	**69**

Table 4.14 provides a detailed summary of the type of errors made by the students when solving linear equations with non-fraction and fractional coeffcients. A total of 69 errors were identified across 12 test questions. An item-by-item analysis showed that Q2 (8 errors), Q8 (12 errors), and Q9 (5 errors) recorded the highest frequencies of errors compared to the other questions. Together, these three items accounted for 26% of the total errors. The most frequent error type was computation errors (12 out of 69), followed closely by conceptual errors (11 out of 69). This highlights a double challenge in which students struggle with basic arithmetic accuracy as well as a fundamental understanding of equation properties. In addition, there were several inverse errors (9 out of 69) and procedural errors (8 out of 69) and indicate difficulties in correctly applying inverse operations and sequencing steps. There were some of the equal sign errors (6 out of 69) identified in the current study which aligned with broader research indicating students often misinterpret the equal sign as an operator rather than a symbol of equivalence. The overall pattern of these errors suggests that student difficulties are not just a result of

carelessness but come from deeper conceptual and procedural weaknesses, underscoring the need for targeted teaching strategies that address these foundational issues.

The next section of the chapter will look at the nature of the type of errors students made and connect these findings to existing literature. This is an important part of the error analysis process, as it provides insights into *why* students make mistakes in solving equations.

Procedural errors

Procedural errors were identified as a common error type in the study. The students made several types of procedural errors in four of the questions. I discovered that many students did not separate the coefficient and a variable, for example S1: $\frac{3x}{3x} = \frac{3}{3x}$.

S3 thought that to solve the equation $3x - 6 = 9$, the solution should be as

$$x = (9 - 3)(6 - 3)$$

Additionally, S30 thought to work with an equation equally

$$\frac{5}{9-20} = \frac{9 \times 2d - 20}{9-20},$$

$$\frac{5}{-11} = 2d$$

The student mishandles the algebraic process of simplifying fractions and ignores the need for equality. These results are consistent with the observations of Qetrani et al. (2021) who found that students frequently commit procedural errors. These errors notably hinder their ability to achieve accurate solutions, despite having a solid grasp of the underlying concepts.

Conceptual Error

This study shows there are errors that stem from a lack of conceptual understanding about how operations affect both sides of the equation.

Example 1: $\frac{6x+24}{4} - \frac{2x}{6}$ = $\frac{4x+24}{-2}$. S28 forgets to balance the denominators or to establish the lowest common denominator.

Example 2: $d - 9 - 2d = -7,\ becomes\ 2d = -9 - 7$. S11 subtracted 2d from d incorrectly ($d - 2d = 2d$). S11 overlooks the need to simplify coefficient properly when solving an equation for d and is unable to find the correct solution. These responses correspond with the findings of Qetrani et al. (2021) who state that students make conceptual errors due to a lack of understanding of equivalent strategies.

Computation Error

When the the students' work was reviewed, the analysis identified computation errors in seven questions. Overall, there were 12 different types of computation errors made. In these questions, computation errors originated from several sources. The following are some examples of these errors:

Example 1: S16 changes a division into a multiplication problem $\frac{x}{4} = 4x$.

Example 2: Another, S17, simplified $3\left(-\frac{1}{3}y\right) = 3\ \frac{3}{9}y$.

Example 3: S24 changed 10y = $10y^2$ to simplify the expression.

$$\frac{2y^2 + 10y + 8}{15} = \frac{2y^2 + 1oy^2 + 8}{15} = \frac{12y^2 + 8}{15}.$$

Example 4: S25 thought that to eliminate the denominator is to multiply only one term by the denominator $\frac{x}{6} = 12 + 4x, then\ \frac{x}{6} \times 6 = 12 + 4x \times 6$ and the solution is $x = 72 + 4x$.

The results indicate that computational errors are the most common mistakes made by students when solving linear equations. The findings of this study corresponds with those of Lestiana et al. (2017) who found that students face difficulties with the four basic operations – addition, subtraction, multiplication, and division – when solving linear equations and working with fractions.

Conjoining error

The conjoining error was also common among the students. Some of the students struggled to solve the five linear equations questions in the study due to the conjoin error. Overall, there were eight distinct types of conjoin errors. These errors arose from various sources, including the incorrect combination of terms or linking of different mathematical concepts or procedures.

For example, some students incorrectly combined unlike terms, S1 as in $3x - 6 = 9$, S7 k - 4 = -4 k , and S10 2d - d = 3d. S18 tried to solve fractional equations by adding numerator and denominator separately, such as $\frac{1}{1} + \frac{1y}{3} = \frac{2y}{4}$. These examples show that students faced difficulties in combining terms and applying procedures because they did not fully understand their proper contexts or how they interact. These findings are similar to Abay and Clores (2022), who state that conjoining errors arose when the student follows the incorrect procedures or operations when solving a problem. Similarly, Pournara et al. (2020) note that conjoining errors occur when students combine unlike terms. In algebra, terms are parts of an expression separated by addition or subtraction. Solving equations by combining like terms involves grouping similar terms together and the accurate implementation of this procedure. The findings of this study suggest that many students continue to struggle to recognise and combine like terms successfully.

Inverse Error

Many students struggle to apply inverse operation when solving equations and incorrectly apply operations.

S15, example: $\frac{x}{4} + 1 = 6 \;\Rightarrow\; \frac{x}{4} = 6 - 1 \;\Rightarrow\; \frac{x}{4} = 5 \;\Rightarrow\; x = 5 - 4$.

S14, example: $13 = 5 - 2k + 8$, the student then writes $13 + 5 - 8 = -2k$.

These errors indicate a fundamental misconception about equations. Instead of viewing them as balanced structures, the students misapply the inverse operation, incorrect solutions. These results align with the findings of Jupri et al. (2014), which indicate that the inverse error occurred due to students' lack of a structural understanding of equations.

Distributive property

The results indicate that S34 solved the equation as $3(x - 6) = 3x - 12$. When the distributive property is not implemented correctly, errors can occur in mathematical calculations. Errors that arise when applying the distributive property, lead to incorrect solutions in equations, such as the S8 solution being $-2(k - 4) = 2 - k4$. Ncube (2016) found that many students encountered difficulties with bracket expansion as a result of applying the distributive property incorrectly in multiple contexts. The distributive property affects the solution of an equation when students cannot correctly remove parentheses (Gumpo, 2014).

Minus Ignored Error, and Minus Error

The minus ignored error is a common error which leads to mistakes in solving algebraic linear equation. This occurred when students overlooked the negative sign, when working with whole numbers and negative fractions. For example, instead of writing $2 - \frac{4x-3}{12}$, S29 wrote $2\,\frac{4x-3}{12}$. And

S22 changed from $-2y\ to\ 2y$. The findings suggest that students frequently modify the problem to suit their needs or to be similar to something they have encountered, such as substituting a minus sign with a multiplication sign. The results demonstrate that a considerable number of students do not comprehend the meaning of the minus sign very well, at times even regarding it as trivial or completely ignoring it. These conclusions are in line with findings by Aydin-Güç and Aygün (2021). They argue that students commit mistakes whenever the negation is overlooked, which is often the result of lack of practice in grasping operations other than addition. The current findings also reveal that students encountered difficulties in solving linear equations with negative coefficients or terms.

For example, S10 gave the solution for $d - 2d$ as 3d. These results are similar to those of Fuadiah and Suryadi (2017), who argue that students face epistemological obstacles with negative numbers and add numbers without considering the sign. Bofferding, Aqazade and Farmer (2017) found that negative signs are easily ignored by students who are unfamiliar with negative numbers in integers and who are used to working with whole numbers, for example, calculating $-4 + 5 = 9$.

Equal Sign Error

In analysing the errors made by S24, several mistakes involved the misuse of the equal sign. For example,

$$\frac{y+4}{3} - \frac{2y+2}{5} = \frac{y}{15}$$

Incorrectly rewritten as

$$\frac{2y^2 \ \ + 10y^2 + 8}{75}$$

$$12y^2 + \ 8 \quad - 75.$$

In this case, S24 dropped the denominators on the left-hand side entirely, forgetting to apply equivalent operations to both sides of the equation while maintaining the structure of equality. This suggests that some students do not recognise that solving an equation involves maintaining the balance between equal sides, and instead their solution collapses into an expression rather than an equation. An equal sign implies that any operation applied to one side must be applied equivalently to the other (Powell, 2012). The results of this study show that many students struggle with the concept of the equal signs. These findings are consistent with the findings of Mc Auliffe, Tambara, and Simsek (2020) who argue that students tended to perceive the equals sign as an operation instruction rather than a symbol of equivalence.

Overgeneralisation Error

The results showed that S22 struggled to solve equations correctly due to the overgeneralisation of the equations. Overgeneralisations examples include,

Example 1:

$$y - 2y \times y = 9 \times 5$$

$$-y = 35$$

Example 2:

$$2 - \frac{4x-3}{12} = \frac{4x+5}{12}$$

The results indicate errors in solving linear equations often occur due to overgeneralisation of arithmetic operations. The overgeneralisation can be the result of procedural or methodological incompetence arising from a lack of understanding of fundamental arithmetic operations. These findings are consistent with the research of Aydin-Güç and Aygün (2021) that also concluded that students' misconceptions frequently happen for these reasons.

Careless Error

Careless errors were identified in this study and show that when working with equations, students exhibit a lack of attention, leading to carelessness. Additionally, rushing through calculations or not paying attention to details can cause errors. S31 misread numbers and wrote them incorrectly, such as $2d - 20$ being written as $2d - 2$. S32 wrote 180 as 18 that also resulted in the incorrect answers. Marpa's (2019) study indicates that students made errors possibly due to their carelessness, as they were confused about whether to add or multiply the terms and coefficients. This was also evidence of such in this study.

Summary

These findings are consistent with existing literature and demonstrate that student difficulties are not just a result to carelessness but are evidence of systematic misconceptions and procedural weaknesses. The results highlight the need for specific teaching interventions that focus on building conceptual understanding of equality, negative numbers and algebraic structure together with procedural fluency.

4.5 Interviews analysed using mathematical discourse analysis

Interviews provide essential data for analysing mathematical discourse, helping to identify the causes of errors students make when solving linear equations. The final part of the data analysis and findings involved one-on-one interviews with a selection of students based on their test answers. This section presents interview data on how students think in their own words about their experiences, opinions, and attitudes when they were solving the test items. Students were interviewed on a selection of items from the written test that they had completed.

I employed a structured approach to investigate student errors in mathematics by categorising the errors and conducting interviews to explore the reasoning or misconceptions behind them. To begin, I examined student work (such as tests) to identify recurring patterns of errors. These errors were then organised into distinct categories, enabling me to pinpoint common themes and patterns in students' misunderstandings. I selected students representing each error category to participate in interviews, aiming to gain deeper insights into the causes of their errors. The interviews were recorded to capture students' explanations of how they solved the equation problems. The class was multilingual, giving me an opportunity to switch codes when a student was struggling to respond in a common language (Gcasamba, 2014). What follows are the selected five transcripts of the conversation between interviewer and interviewee. The interview aimed to discover the thought process of learners behind the errors they made. I aimed to understand how students engage with mathematical concepts, identify where their reasoning deviates, and use this as a foundation to guide them toward accurate and meaningful understanding. This aligns with the theoretical framework of Sfard (2008) who views learning as a transformation of discourse and that errors play a pivotal role in facilitating this transformation.

To give a brief example of how I applied the theoretical framework of Sfard (2008), I analysed how a student communicates while solving linear equations. Each participant was given feedback on their original solution, and I asked three questions aimed at examining their use of language when talking about equations (words in use), to explain the typical problem-solving methods they use step by step (routines), and to unpack their visual mediators, such as symbols or equations. The analysis was based on individual interviews conducted one-on-one. For this study, I focused on three core elements from the theoretical framework of Sfard (2008). The fourth element – endorsed narratives – was not included, as the research was not centered on any mathematical statements or explanations for solutions. The following are the transcripts of the conversations between interviewer and interviewees.

1. How did you solve this question?
2. What did you do in the following step?
3. How did you get your answer?

4.5.1 Student 1 interview

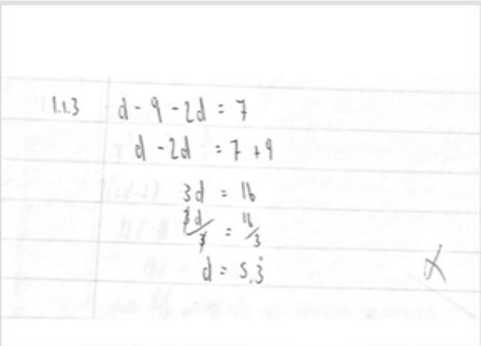


FIGURE 4.1: STUDENT 1 INTERVIEW

The analysis refers to the one-on-one conversation about a response in Figure 4.1 with Student 1, as detailed below:

Table 4.14: Analysis of Interview Student 1

Question 1.1.3	**Interviewer**	**Student 1**	**Data analysis**
	How did you solve this question?	I took -9 to join to 7 on the opposite side (pointing using finger and wavering arm)	*Word use***:** The student used the colloquial words 'join and 'opposite side' Instead of using mathematical terminology 'transpose'. The student was pointing, using her finger and waving her arm to indicate the opposite side. These are also classified under 'word use'. This statement informs the *routine* used to solve this problem: it is a metarule, namely: switch sides, switch signs
	How did you get this answer $3d$?	I added $d - 2d$	Routine (ii) using the metarule: add like terms
	What did you do in the following step?	I am divided by 3 both sides	Routine (iii) using the metarule**:** divide equally on both sides of an equation

Findings

This student demonstrates an understanding of the concept of transposing terms to the other side of an equation and changing signs accordingly. However, the student struggles with using correct mathematical terminology. The visual mediators are the signs during the addition or subtraction of like terms. The student shows a good grasp of implementing the correct procedural steps in problem-solving.

4.5.2 Student 2 interview

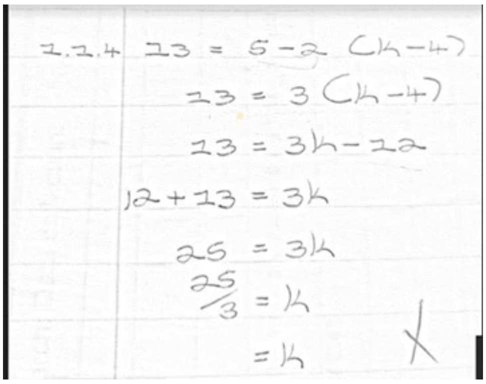


FIGURE 4.2: STUDENT 2 INTERVIEW

The analysis refers to the one-on-one conversation about a response in Figure 4.2 with Student 2, as detailed below:

TABLE 4.15: ANALYSIS OF INTERVIEW STUDENT 2

Question 1.1.4	Interviewer	Student 2	Data analysis
	Explain, how did you get 3?	I took 2 from 5	Word use: Student used colloquial word *took* instead of *subtracting.* Routine and metarule (i): subtract like terms.
	How do you get 25 in the following step?	I have added 12 to 13 Then I multiplied (k-4) by 3. I have divided $\frac{25}{3}$	Word use: Brackets, multiplication, division, the equal sign, addition and subtraction were proper mathematical discourse. Routine (ii) using the metarule: add like terms**.**
	Why did you have to multiply (k-4) by 3?	Because bracket means multiplication, you multiply.	Routine (iii) using the metarule: bracket.

Findings

In analysing the data, it appears this student understands the concept of the minus sign between two terms but does not consistently use the correct mathematical terminology in his/her explanations. Additionally, the student struggles with some of the procedures.

4.5.3 Student 3 interview

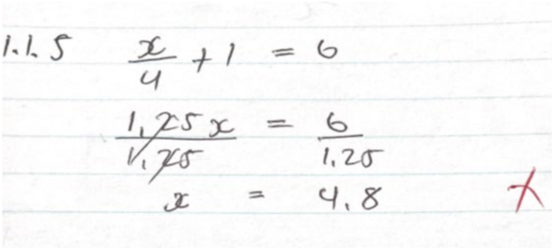


FIGURE 4.3: STUDENT 3 INTERVIEW

Question 1.1.5

The analysis refers to the one-on-one conversation about a response in Figure 4.3 with Student 3, as detailed below:

TABLE 4.16: ANALYSIS OF INTERVIEW STUDENT 3

Question 1.1.5	**Interviewer**	**Student 3**	**Data analysis**
	How did you solve this question, explain step by step?	This x has 1 then .. it becomes as $\frac{1}{4}$ then I say $\frac{1}{4}+1$ the answer is 1,25 and I had to divide by 1.25 both sides to get x value	Word use: Student used *this*, pointing at the equation, and 'becomes as $\frac{1}{4}$'. The student used 'has' instead of 'is equal to'. Routine(i) using metarule: add fractions. Routine(ii) using metarule: divide both sides.

Findings

The student demonstrates an understanding of coefficients and shows familiarity with symbols, integers, and operation signs as visual aids. The student also understands procedural routines and metarules.

4.5.4 Student 4 interview

Question 1.1.7

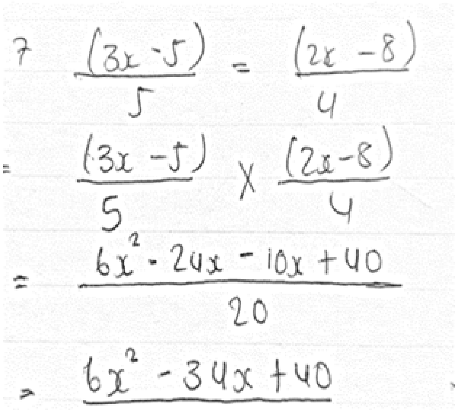


FIGURE 4.4: STUDENT 4 INTERVIEW

The analysis refers to the one-on-one conversation about a response in Figure 4.4 with Student 4, as detailed below.

TABLE 4.17: ANALYSIS OF INTERVIEW STUDENT 4

Question 1.1.7	**Interviewer**	**Student 4**	**Data analysis**
	How did you solve this question, explain it step by step?	I cross multiplied	Word use: The student used the correct mathematical term. Routine (i) using metarule: cross multiplied.
	What did you do in the next step?	I multiplied the brackets on numerator and multiplied 4 by 5 on denominator.	Visual mediators: 4 and 5 Word use**:** Multiplied, brackets, numerator and denominator. Routine (ii) using metarule: brackets.
	How did you get your final answer?	I have collected like terms. Then I was confused to move forward because I have $x^2 and\ x$.	Routine (iii) using metarule: collect like terms. Visual mediators: like terms. Word use: collected.

Findings

The student displayed a strong grasp of the correct mathematical terminology. While understanding the steps needed to solve the problem, there is noticeable confusion during the implementation of procedures (routines). Additionally, the student demonstrated knowledge of collecting like terms (metarule) but struggled with effectively applying operations (metarules).

4.5.5 Student 5 interview

Question 1.1.9

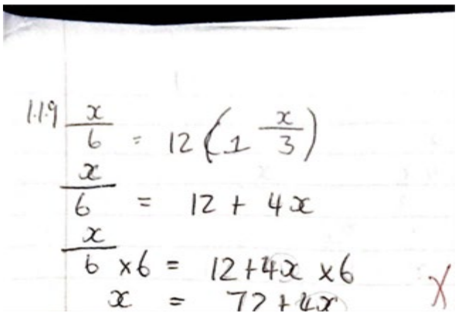


FIGURE 4.5: STUDENT 5 INTERVIEW

The analysis refers to the one-on-one conversation about a response in Figure 4.5 with Student 5, as detailed below.

TABLE 4.18: ANALYSIS OF INTERVIEW STUDENT 5

Question 1.1.9	**Interviewer**	**Student 5**	**Data analysis**
	What did you do in the following step?	I took 6 and multiply both sides	Word use. Student used words instead of *multiplying* by 6 both sides of the equation. Routine (i) using metarule: multiply both sides.
	Explain how you arrived at your answer?	I multiply 12 by 6 to get 72 and just leave 4x.	Visual mediators: multiply was used. Routine(ii) using metarule: multiply.
		ohh I forgot to multiply 4x by 6.	Routine(iii) using metarule**:** forgot Student struggle to solve the equation.

Findings

The student demonstrated procedural understanding (routine and metarule). He shows visual mediator thinking. The student faced challenges to solve the equation.

4.5.6 Student 6 interview

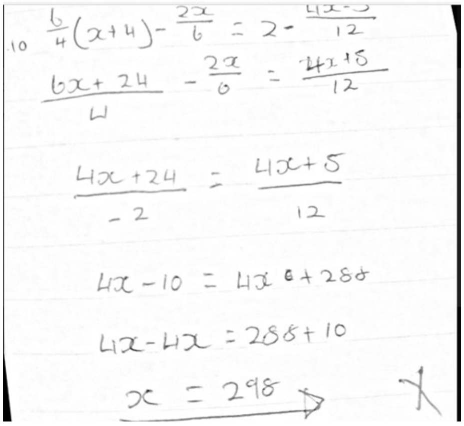


FIGURE 4.6: STUDENT 6 INTERVIEW

The analysis refers to the one-on-one conversation about a response in Figure 4. 6 with Student 6, all detailed below:

TABLE 4.19: ANALYSIS OF INTERVIEW STUDENT 6

Question 1.1.10	**Interviewer**	**Student 6**	**Data analysis**
	Can you explain your solution?	I multiplied $\frac{6}{4}$ in the bracket on this side.	Word use: Student used mathematical terms correctly. Student used on this '*side*' instead of on the left/right side. Routine (i) using metarule: multiplied Routine (ii) using metarule: bracket
		I said 4 x -4 x is equal to x	Visual mediators: Subtract, 4 x was noted and used. The equation 4 x −4 x = x x involves subtracting 4 x from 4 x , which visually cancels out to 0. Routine(iii) using metarule: Subtract Visual mediators: The visual mediators were used discursively in this interview in line 2 to communicate about the operation and relationship of equation.
		and add 2 and 3 like terms	Routines: Student explore adding unlike terms the approach is outcome oriented. Routine (iv) using metarule: add like terms.

Findings

The student showed a clear understanding of correct mathematical terminology and demonstrated a level of visual mediator skills. The student also performed routine tasks, such as adding unlike terms.

4.6 Summary

From the data collected on errors in solving linear equations, several key types of errors emerged, and were classified into different categories. Each error classification shed light on the challenges students face in solving linear equations. The distribution of errors for each question is summarised in Table 4.14. The variety of errors in solving linear equations shows that students face challenges both in understanding fundamental concepts of equations and with accurate application of procedures. According to Qetrani et al. (2021) the use of multiple strategies for solving problems and selecting the more efficient one enhances students' flexibility in solving equations. The student demonstrates substantive gaps in both knowledge and skills for solving linear equations with fractional coefficients.

As the researcher, I identified that students' reliance on informal language, gestures, and procedural metarules in solving equations reflects their current mathematical discourse. These practices demonstrate partial understanding but highlight significant gaps in formal mathematical communication and conceptual articulation. Students often blend colloquial and formal mathematical language, an indication of a transitional stage in their discourse (Sfard, 2008). However, inconsistencies in terminology and reasoning reveal a lack of alignment with established mathematical routines, leading to errors.

The work of Sfard (2008) related to object-level and meta-level thinking can be seen in students' resorting to procedural methods for the operations like cross-multiplication and expanding brackets. These methods reveal the students' knowledge of the meta-level rules, but their lack of understanding at the object level becomes apparent when reasoning such as solving non-linear equations or simplifying accurately is required. Moreover, the students' informal phrases and result-oriented tactics highlight weak mathematical thinking that corresponds to Sfard's classification of ritualised and non-exploratory discourse (Sfard, 2012).

The findings stress that the students are clearly dependent on the use of visual mediators, and at the same time, their use of incomplete routines often leads them to make errors such as forgetting steps, adding unlike terms, or misapplying rules. Following the Sfard (2012) commognitive theory of learning, these challenges indicate that the students should be moved from procedural to exploratory engagement with mathematics. The students' mathematical discourse and reasoning will not improve without the reinforcement of precise terminology, the promotion of conceptual clarity, and the instigation of systematic, rule-based problem-solving.

4.7 Conclusion

The analysis of students' errors in solving linear equations with fractional coefficients shows how their difficulties spread beyond isolated mistakes and reflect deeper conceptual and procedural gaps. The error classification reveals that a considerable number of students have difficulty in keeping equivalence, applying the same operation throughout and handling fractional coefficients. The previously mentioned challenges indicate that students very often are not only lacking the procedural fluency, but also conceptual grounding that would lead them to accurate solutions of equations.

The types of errors that emerge from the data analysis align with broader findings in literature, and suggest that misconceptions about equality, negative numbers, and algebraic structure is systematic, rather than incidental. The need for targeted teaching interventions that create a balanced approach between conceptual development and procedural practice is highlighted to ensure students develop a more integrated understanding of algebra.

Additionally, the discourse analysis emphasises that students' reliance on informal language, gestures, and procedural shortcuts reflect a transitional stage in their mathematical communication. Although students demonstrate some familiarity with mathematical routines, their discourse is mostly ritualised and lacks the conceptual depth necessary for problem-solving. This is consistent with Sfard's commognitive framework, and the findings suggest that students need to be supported in moving from ritualised, outcome-driven practices to exploratory engagement that involves accurate terminology, clarity of conceptualisation, and alignment with formal mathematical routines.

Taken together, these findings emphasise that addressing errors in algebra requires more than corrective feedback – it requires a pedagogical focus on strengthening both the discourse and the conceptual foundations of mathematics. By doing so, educators can help students develop greater flexibility, accuracy, and confidence in algebraic reasoning.

5 DISCUSSION, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter discusses each of the research questions based on the analysis and findings in Chapter 4 and explores more deeply the nature of students' errors and the possible reasons for these errors with a group of students in a TVET college. The research provides insights into how students understand and process mathematical concepts and identifies opportunities for targeted interventions for students. There are also recommendations for curriculum development in the teaching of challenging content topics, for teacher professional development and for future research opportunities.

In Chapter 1, the focus of the study was introduced, aiming to explore how a sample of NCV Level 2 students made errors when solving linear equations with fractional coefficients. This investigation was carried out using the data collection methods discussed in Chapter 3, and the literature review and theoretical framework outlined in Chapter 2 was used to analyse the data, in response to the primary research questions. The next section will discuss the findings in relation to each of the research questions with links to previous research and highlight important patterns and trends that emerged from the empirical data.

5.2 Discussion of research questions

5.2.1 Research sub-question 1

What kinds of errors are demonstrated by students when solving linear equations with fractional coefficients?

According to findings from the test answers (Chapter 4, Table 4.1), the students demonstrated the following types of errors: procedural, conceptual, computation, conjoining, inverse, distributive, minus sign ignored and equal sign, over-generalisation, and carelessness. Given the wide range of errors that address specific student difficulties, it was decided to create four overarching categories for discussion: computational, conceptual, procedural, and operational. These were used to highlight the underlying causes of student challenges, to link existing literature, and to highlight the pedagogical implications for solving linear equations.

First, the test analysis showed that students most frequently made computation errors and were unable to apply appropriate solution strategies when solving linear equations with fractional coefficients. According to Lestiana, Herani, Rejeki, Sri, and Setyawan (2017), computation errors are the mistakes a student makes during calculations, often due to carelessness or a lack of clear understanding of basic math rules like addition or multiplication. The findings of the study by Lestiana et al. (2017) show that most students have problems with computational errors when solving linear equations. This seems to be related to issues with basic operations, recognising errors and organising their written solution. This is similar to the results by Pournara et al. (2022). They highlight basics errors with operations occur even when simplifying simple algebraic expressions. The following shows a student (S2) made a computational error when solving $3x - 6 = 9$.

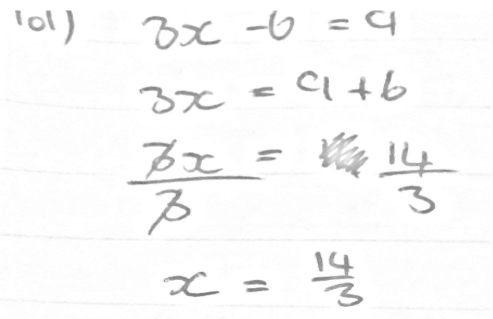


FIGURE 5.1: STUDENT SOLUTION STRATEGY FOR PROBLEM 1.1.1

S2 incorrectly computes $9 + 6$ as 14, instead of 15, and then solving for the value of x. These kinds of errors are commonly made by students who might be rushing to finish the problem, making an error and not verifying the solution. This could be due to several factors, including careless errors and misunderstandings of addition and multiplication rules. Computational errors are well known common student mistakes and need to be addressed by teachers. This may include the use of different pedagogical strategies such as checking answers through substitution and looking for calculation errors regarding number operations.

The second category of errors comprised conceptual errors, especially the equal sign error and the balancing error. Conceptual errors are the specific type of mistakes that happened when a student does not understand the underlying mathematical concept (Qetrani et al., 2021). Below are illustrations of these errors, taken from students' solutions, as examples:

Example 1

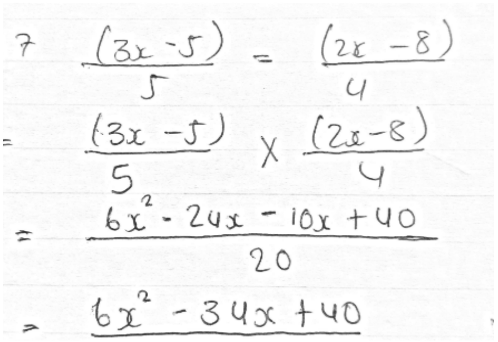


FIGURE 5.2: STUDENT SOLUTION STRATEGY FOR PROBLEM 1.1.7

S20 ignores the equal sign and multiplies across expressions without maintaining equivalence and omitting the denominator in the final answer. This demonstrates a misunderstanding of how equality should be preserved in algebraic equations.

Example 2

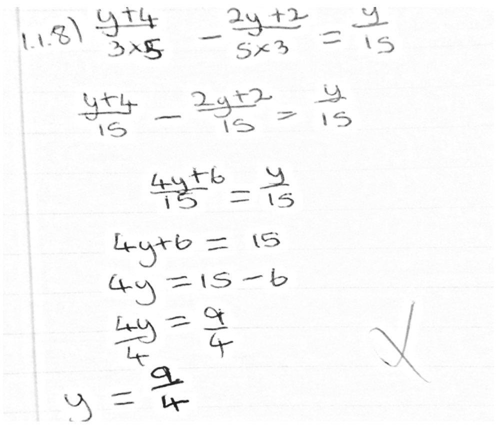


FIGURE 5.3: STUDENT SOLUTION STRATEGY FOR PROBLEM 1.1.8

S22 made a conceptual error by making the denominator the same on the left-hand side of the equation, thus ignoring the numerators and wrongly combining the terms. The student works on both sides of the equation without keeping the balance. This indicates a lack of conceptual understanding for solving equations.

These types of errors are well recognised in literature and research related to algebraic reasoning. This can be related to a poor understanding of the concept of equivalence, which was not well established in the primary school (Qetrani, Ouailal, & Achtaich, 2021). A limited understanding of the equivalence approach, is when leaners did not fully grasp a certain concept or procedure, leading them to struggle to understand other concepts. This difficulty often stems from an inappropriate generalisation of knowledge that comes from a narrow experience with arithmetic. Results from the work of Tastepe and Yanik (2023) highlight that students face difficulties when transitioning from fractions to algebraic fractions. These difficulties emerge when it comes to introducing fractions with variables (algebraic fractions) and making variable the subject of the formula. The findings suggest that teachers need to spend time developing students' conceptual understanding of equivalence when working with equations. When students solve equations, they should not just follow steps without thinking and reasoning. Instead, they need to understand both procedural and conceptual understanding (why, how) and recognise that choosing the right method or approach can make solving equations easier and more accurate. A deep conceptual understanding helps students recognise the value of using effective approaches that make solving linear equations with fractional coefficients easier. This information is useful for the design of teacher professional development workshops.

The third category of error is procedural error, made by students when they apply the solution procedure incorrectly, or apply an incorrect execution of the steps needed to solve a problem. The following student solution illustrates this error:

Procedural Error

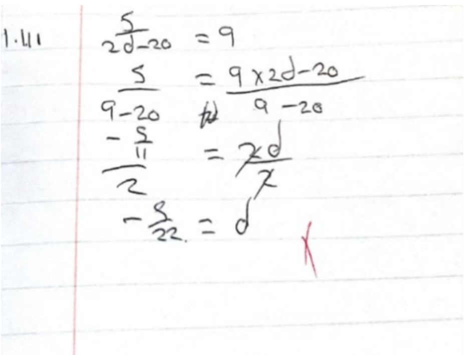


FIGURE 5.4: STUDENT SOLUTION STRATEGY FOR PROBLEM 1.1.11

S30 starts by ignoring the constants and changing the structure of the equation so that it is no longer equivalent, and then combines the variables and the constants to make a new and unrelated equation.

These types of errors were also seen in previous studies such as Delastri and Lolang (2023) who found that students frequently make procedural errors in that they did not apply the correct procedures or algorithms when solving equations. These results also align with the finding of Msomi and Bansilal (2022) which indicates that procedural errors stem from a lack of proficiency in applying mathematical procedures with flexibility, accuracy, and appropriateness. The development of students' understanding when solving linear equation is critical, especially with fractional coefficients. The procedures involved need to be explained so that students are enabled to solve equations. They should understand the reasoning involved in each step of the solution. Teachers can focus on helping students visualise the meaning of fractions in equations, beginning with simplified problems, and using step-by-step modelling such as clearing fractions by eliminating denominators on both sides of the equation.

The last group of errors relates to operations in equations and can happen when students incorrectly use operations that refer to both numerical and algebraic situations, involving addition, subtraction, multiplication, or division.

Operational Error

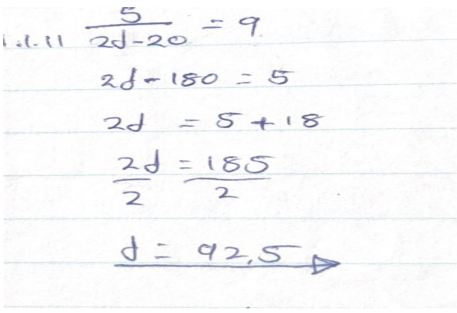


FIGURE 5.5: STUDENT SOLUTION STRATEGY FOR PROBLEM 1.1.11

Here, S32 wrongly utilised the multiplication operation and formed a different equation. This mistake might have been made by the student as reported by Pournara et al. (2022), who suggested that errors were made without due attention to the equation operations. Students sometimes use the cross-multiplication techniques or other methods that they have not yet mastered, resulting in such errors. Jupri et al. (2014) cite similar instances in their research and link this to the poor understanding of operations. Such operation errors can also indicate that students are unable to manipulate equations accurately to arrive at the right solution.

Operational errors happen primarily because students are not well-grounded in basic operations like addition and subtraction, multiplication and division, as well as a good comprehension of the procedures involved and the structure of mathematical expressions. The teaching of linear equations with fractional coefficients can be made more interesting and engaging by using real-life examples and technology, thereby providing an opportunity to improve students' algebraic reasoning and to support the transition from arithmetic to algebra and vice versa, and reinforcing the understanding of operations during the equation solving process. These outcomes confirm suggestions by Takker (2022) that the movement from arithmetic to algebra might be made easier if the arithmetic tasks are changed from simply performing calculations to exploring and identifying relationships between numbers. Yimer and Feza (2019) state that technological tools are indispensable to the teaching and learning processes within mathematics, as they are the ones who not only elevate, but also change and make more stable the level of knowledge that is considered modern. Hence, the combination of technology and real-life applications in algebra instruction is regarded as a necessary measure for the improvement of both students' conceptual and procedural understanding. It is mentioned that students are required to possess wider knowledge and deeper insight in learning equations for both conceptual and procedural knowledge, that is to say, the knowing of how and why.

Summary

The results of this study show that students make different types of errors when solving equations and they can be part of the learning cycle. However, there needs to be a teaching intervention plan to address these challenges, as they have implications for students' understanding and performance. This research, as with other similar research, highlight important factors in the development of students learning algebraic equations, and particularly those with coefficients. These factors include using a variety of teaching strategies such as innovative strategies, the balance, and transposition methods, as these have been shown to be effective for solving linear equations. Using different pedagogical strategies can help students visualise concepts better, and improve mathematical conceptual understanding as well as create a conducive learning environment (Roberts & Le Roux, 2019).

It is also important to consider the integration of real-life contexts when designing problems so that students can understand how equations are useful in everyday life. It helps to provide relatable situations to enhance learning. These strategies can increase students' interest in learning and encourage them to engage more in the mathematics classroom.

In addition, the maintenance of equivalence during the solution process of linear equations is done through using both the principles of equality and inverse operations at the same time. So, in learning, the equality (balance) principle is maintained with the application of both equality and inverse operations. When used correctly, the principle of inverse operations is a good support for the learning of equality. The combining of the two operations (equality and inverse) not only reinforces the development of conceptual knowledge (the 'why') and procedural knowledge (the 'how'), but also ensures the successful application of the equality principle in solving fractional linear equations. The research done by Sanders (2017) highlights that students can tackle more complex equations once they have a good knowledge of these two operations. The utilisation of these operations can provide students with a more profound understanding of solving linear equations with fractional coefficients and thus their success in the long term (Sanders, 2017).

The fundamental principle of equality is the basic rule of manipulation of algebraic equations. Hence, equality in linear equations assist students to achieve an insight into the application of systematic approaches in the hope of reducing typical errors that include inverse errors, operational errors, and errors of signs. Also, logical thinking and problem solving can be acquired by students with the application of the balance approach. They can also identify the changes and flow of processes in equations so that they can correct it in a methodical way (Anthony & Burgess, 2014; Otten et al., 2019). Therefore, it gives students a better understanding of the knowledge of working accurately and confidently when solving linear equations.

The innovative strategy is considered an instructional practice in learning equation, and focus could be put on the implementation of structural sense that builds the ability of the students to discover the interconnections among mathematical concepts and understand the valid transformations that have been tested and proven effective (Roberts & Le Roux, 2019). This approach can allow students to learn algebra with the help of their conceptual knowledge which can be achieved by using this strategy effectively. It also enables students to be capable of minimising the errors that are normally common, like calculation errors and incomplete and unsolved linear equations. It also increases structural sense in problem solving among students, as well as builds confidence in them regarding mathematics because it makes them know the *why* and *how*.

5.2.2 Research sub-question 2

What are the possible causes of the errors made by students when solving linear equations with fractional coefficients?

This discussion explores the causes and reasons behind students' struggles with learning linear equations, to answer the question stated above. The focus on the possible causes and reasons for the errors aim to inform the development of curriculum design and teaching strategies, and to determine students' misconceptions with the aim of conceptual development. The results of the literature review show that poor algebra instruction, lack of prior knowledge, misleading learning materials, and gaps in the curriculum are possible causes of student errors in solving linear equations with fractional coefficients. Students' understanding of problem-solving methods are affected by the above factors, which limit their arithmetic skills and engagement due to the lack of real-life and practical teaching examples. The results of the study show that students' understanding of linear equations with fractional coefficients is limited, and while students demonstrate procedural understanding, their reasoning remains narrow and underdeveloped, as they did not fully grasp the underlying concepts.

Sfard's (2008) commognitive theory was used to address Research question 2, specifically in analysing the causes that led students to make errors. The theory emphasises that learning mathematics is a communicative process. It focuses on the evolution of how students talk about, and think about mathematics – from informal to formal discourse. The findings from the sampled students' responses, reflecting on their experiences, indicate that they struggled with solving linear equations due to difficulties associated with the three fundamental aspects of mathematical discourse: word use, visual mediators, and routines. The main emphasis of the study is the progression of the student's communication and thought processes from informal to formal discourse when doing mathematics. The results obtained from the students' sampled reflections on their experiences show that solving linear equations was a struggle for them due to the three main factors related to mathematical discourse: vocabulary, visual aids, and the use of specific methods. Word use refers to the way students use mathematical terminology and everyday language when explaining their solutions. Visual mediators are symbolic artefacts, such as numbers, tables, algebraic expressions, equations, and graphs. Routines are groups of metarules that describe repeated discursive action. Routines are predetermined patterns for carrying out mathematical tasks.

Example 1

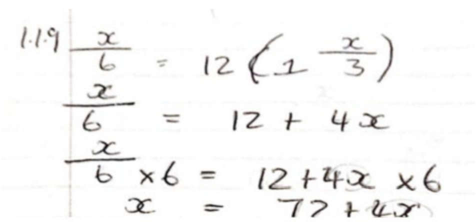


FIGURE 5.6: STUDENT SOLUTION STRATEGY FOR PROBLEM 1.1.9

S3 says,

I took 6 and multiply here this side and this side

I multiply 12 by 6 to get 72 and just leave 4x

Ohh I forgot to multiply 4x by 6.

Word Use

S3 is using informal mathematical terminology/terms; it shows a reliance on memorised procedures rather than conceptual understanding. The student appears to perform steps from habit rather than understanding what operations are being applied mathematically, and why.

Visual Mediators

S3 begins with the equation $\frac{x}{6}$ $=12(1+\frac{x}{3})$ as was evident by using a symbolic mediator, i.e. the standard form of a linear equation. The student observed ritualised $\frac{x}{6}=12+4x$ (distributing 12 to both 1 and $\frac{x}{3}$ correctly). This shows the breakdown in the visual mediator and applying a ritualised step remembered, but without understanding the context. This shows incomplete or incorrect use of visual mediators.

Routine

S3's approach is algorithmic and procedural, following ritualised routines and multiplying both sides. The student misuses the meta-rule procedure when solving equations, which is that one must apply the same operation to both sides in a way that preserves equivalence. The student violates the meta-rule, even though believing that the rule was followed. The routine shows that there is no structural understanding of equations or the logic behind the operations. The final step $x = 72 + 4x$ also shows no strategic awareness of the goal in solving linear equations. The equation was not solved, which shows that the student lacks conceptual understanding of solving linear equations with fractions.

Using Sfard's (2008) theory to understand the student's mathematical discourse of the solution given, helps the teacher to identify what support is needed. This student is beginning to engage in algebraic discourse, but needs help to develop precise mathematical language, to understand the structure of algebraic equations, and to move towards more explorative routines that provide more detailed explanations of the solution.

Example 2

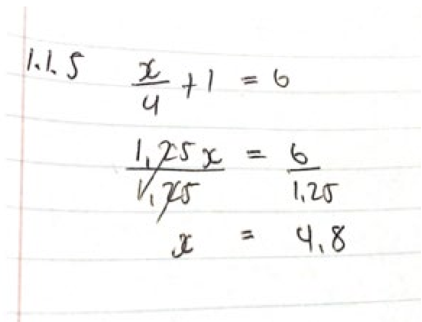


FIGURE 5.7: STUDENT SOLUTION STRATEGY FOR PROBLEM 1.1.5

S5 says,

> *This x has 1 then, it becomes as 1/4*
>
> *then I said 1/4+1 the answer is 1,25*
>
> *and I had to divide by 1.25 both sides to get x value.*

Word Use

Student S5 shows a lack of formal mathematical discourse by using informal or colloquial language such as this, has, here, and this side. The student is indicating a reliance on everyday language rather than mathematical terms.

Visual Mediators

Visual mediators lack support for meta-rule. The structure of the equation when solving it, presented incorrectly. This shows that the interpretation of visual mediators was too literal. S5 interpreted the visual mediators such as the symbols x , $\frac{1}{4}$ and 1.25 numerically, but not structurally. The student shows the lack of understanding of conceptual relationships between terms, and instead, relies on visual appearance.

Routines

S5 demonstrates ritualised and driven routines. The student familiarised procedures without understanding, and shows the use of a memorised routine when applying steps like dividing both sides by, and not as a result of understanding the equation's structure. The student relies is on a rote, procedural approach, without understanding the *what* or *why*.

The student appears to understand the need to isolate x, and that operations need to happen on both sides of the equations showing procedural knowledge. However, there is a lack of understanding of algebraic equations, that is distinguishing between terms and combining terms correctly. The student needs help to develop structural sense through using visual models and strengthen routines with meaning such as error analysis.

5.3 Summary

According to Sfard (1991), students' explanations can be interpreted through a dual framework comprising operational and structural understanding. Using this framework, these students appeared to demonstrate an operational grasp of algebraic procedures, such as solving equations step by step, multiplying brackets, and combining like terms. The students' responses indicated a weak structural comprehension of algebraic equations. They did not appear to have trouble in performing operations; however, they could not detect and decipher the structure and form of the expressions. It means that students have not fully grasped the conceptual foundations, although they can apply the algebraic rules proficiently.

The outcomes support the work of researchers like Pournara (2020) and Msomi and Bansilal (2022), who concluded that students face greater difficulties in the acquisition of structural understanding than in the execution of procedures. The study shows that students can follow the steps (procedures) to solve linear equations, which is an indicator of their partial understanding. However, the students find it difficult to keep both sides of the equation equal throughout the learning process. This implies that they are carrying out the steps without fully comprehending the logic behind them; they are simply memorising and applying the procedures robotically, instead of grasping the reason why the steps work or how they influence the equation.

Sfard's framework (1991) illustrates that students need to demonstrate different types of discourse to be fully competent. The students in this study can display ritual discourse, where they apply the procedural steps without having the smallest idea of the reasons behind such practices. They lack exploratory discourse, when students know the reasoning behind the steps they are following. It is necessary for students to develop a deep and effective understanding of the problems represented by linear equations with fractional coefficients. This implies an exploratory understanding of the reasoning behind the application of correct methods that lead to successful solving of equations. The use of critical thinking can be regarded as a necessary skill among those dealing with solving linear equations with fractional coefficients. Students must acquire this skill, as it allows them to express their thoughts more freely, approach the problem from different angles, and ask context-related questions for better understanding.

Moreover, teachers can help build a productive learning atmosphere where student involvement is high and where they are supported to make the whole process of learning active, positive, and supportive. The Sfard framework indicates that communication and cognitive development are interrelated, with the latter being the ultimate goal of deeper mathematical understanding. Here, the students that are involved in interpersonal communication – talk about the concepts in depth and use mathematical language in spoken and written forms correctly – not only become the ones with the highest thinking skills, but is also influenced the most by their mathematical learning. The framework delineates the path for students to move from the use of the ritual until they reach the exploratory discourse. It thereby supports the understanding of both the procedural and the conceptual sides. This transition promotes students to think deeper, more logically, and more critically, thus, becoming independent, versatile, and conceptually grounded mathematical thinkers (Saunders & Wong, 2020).

5.4 Discussion

Why do NCV Level 2 students perform poorly when solving linear equations with fractional coefficients?

Exploring the errors and their causes when solving linear equations with fractional coefficients provides valuable insight into students' learning challenges, and highlights ways to strengthen their mathematical development. The findings of this study indicate that the majority of errors emerge from basic mistakes in arithmetic operations and the incorrect application of addition and multiplication axioms. Students' difficulties often stem from misunderstandings of the equal sign and the overgeneralisation of algebraic rules. Errors also arise from incorrect problem-solving methods, such as improper arrangement of terms, misuse of the equal sign, and misapplication of cross-multiplication.

The analysis of errors and their reasons in the process of solving linear equations with fractional coefficients offers a useful perspective on students' learning difficulties, and provides suggestions on how to nurture their development in mathematics. This research reports that most errors are due to mistakes in arithmetic operations and misapplications of the rules of addition and multiplication, as well as the incorrect interpretation of the equal sign and the overgeneralisation of the algebraic rules. In addition, errors are caused by incorrect problem-solving methods like incorrect grouping of terms, misuse of the equal sign, and misapplication of cross-multiplication.

These challenges emphasise the necessity for teaching strategies that boost skills to solve problems and grasp the concepts more effectively. The research draws attention to the need of strengthening structure sense, improving students' comprehension of equality, and teaching through problem-solving methods that are systematic. The results also highlight the importance of commognition theory in facilitating students' learning through exploration, engagement, and reasoning. When teaching, commognition is supportive because it helps teachers and students to focus on the concept of understanding and the meaningful use of algebraic language which in turn, helps students to be more reflective and independent mathematical thinkers.

The findings also suggest that students' errors are not only procedural but are also related to gaps in their understanding of the meta-discursive rules that govern how mathematical expressions are structured, interpreted, and communicated. Their difficulties extend beyond computation to include challenges in reasoning, representing, and discussing mathematics. This reveals the importance of helping students to move beyond rote calculation towards a deeper grasp of how mathematics works logically, and how it is expressed.

In conclusion, the research stresses the need for students to be guided to look more closely at their solutions, to give their reasons, and to participate in the learning of mathematics as a conversation. Sfard's commognition theory serves as a solid basis for the teaching and learning of linear equations in a useful way. When learners are talking, reasoning, and reflecting on mathematics, they gradually start to consider errors as chances for acquiring knowledge, instead of being failures. This promotes deeper involvement, clearer comprehension, and the creation of a flexible, conceptually solid approach to algebra learning.

5.5 Conclusion

The results highlight the critical elements needed to solve equations involving word use, visual mediators and routines, which are not being used by students. These are important in the development of mathematical knowledge, as learning happens through communication and thinking (Sfard, 2008). If students are unable to simplify expressions, manipulate equations, and solve problems due to conceptual and procedural errors, this will not be remedied without the direct action of the teacher. This is consistent with Roberts (2016), who found that students face difficulties with specialised vocabulary and interpretation of algebraic notation. Additionally, students face difficulties with visual representations of mathematical concepts that lead to an inability to correctly interpret, represent, and solve linear equations. Sfard's (2008) theory of commognition suggests that teachers should scaffold students' thinking by emphasising key mathematics concepts, with the aim of helping students to improve in the use of mathematical language, visual tools, and logical thinking and reasoning.

The study suggests that teachers should help students develop both a deep understanding of mathematical concepts (conceptual understanding) and the ability to carry out procedures accurately and appropriately. To achieve this, teachers should emphasise the meaning and importance of mathematical language such as terms, symbols, and expressions during instruction. They should guide students step by step through problem-solving processes and help them visualise what each step represents so that they better understand the symbols and terms being used. Encouraging students to participate in discussions about mathematical problems provides opportunities to compare different solution strategies and promotes the use of correct mathematical language. This, in turn, can help reduce the recurrence of common procedural errors.

Finally, it is recommended that a larger group of NCV Level 2 students should be involved in future research to obtain a better understanding of the difficulties experienced by them in the process of solving linear equations. I suggest applying Sfard's commognitive framework which investigates the students' talk, thoughts, and communication during the problem-solving. This method can demonstrate not only the correctness of students' answers, but also the way they reason through the process, and giving insight into the hidden gaps in their conceptual understanding.

5.6 Limitations

This study was conducted with a NCV Level 2 Mathematics class comprising 35 students from a single TVET college. Although this focused sample enabled a thorough investigation of the research problem, the scope of the findings remains limited in terms of their applicability to other institutions, educational levels, or broader student populations. As the researcher, I initially aimed to gather data from a number of different colleges, but was constrained by finance, limited time, and other resources, which led to me conducting the research in only one college. I would also have liked to conduct the research with more than one class in the college, but due to the spread of the COVID-19 pandemic and the resultant restrictions of contact, this was not possible. The restrictions on in-person contact also created a challenge in conducting the study, as it became difficult to determine why students from other classes with different teachers made errors, as well as identifying the specific mistakes they made when solving linear equations. As a result, the study was conducted in only one class, namely my own.

In this study the researcher's dual role as both the teacher and the investigator could also be seen as a limitation. Even though ethical steps were taken such as obtaining informed consent and assuring students that their identities would remain confidential, the existing relationship between teacher and student may have influenced how comfortable students felt about participating. Some may have felt pressure to take part, even if it was unintentionally. This dynamic could also have affected the honesty or openness of their responses. Additionally, being in both roles may have made it more difficult for the researcher to remain fully objective when analysing the data.

The study's findings indicate limitations in transferability of results, since the sampling strategy relied primarily on the availability of participants. Hence, the use of purposive and convenience sampling methods may have introduced selection bias, therefore limiting the generalisability of the results. An additional limitation of the study was the difficulty of generalising the findings, since it is a single case study. The findings of the study are not easily transferable to a larger population because only 35 students were involved in the investigation. This being a small group, the findings cannot easily be applied to all students in general. However, Dehalwar and Sharma (2024) and Sharma (2013) maintain that even small case studies can be valuable. They can provide deep insight into how students think when solving linear equations, and these insights can be used in the transformation of mathematics teaching.

Although qualitative data analysis techniques such as member checking and external review were employed, researcher bias may still have influenced the processes of coding, theme development, and interpretation of participant responses. Furthermore, as the study was confined to a single TVET institution, the transferability of the findings to other educational contexts remains limited.

One potential source of bias originated from the dual role of the researcher as both investigator and lecturer to the participants (students). This dual relationship could have made a difference to participants' responses through tests and interviews, potentially resulting in answers that reflect social desirability rather than their authentic perceptions or experiences. During the semi-structured interviews, participants may have been influenced by the fact that the data was collected on Fridays after school hours, when students are tired and ready to go home. Their responses could also have been shaped by concerns about exceeding the allocated time and potentially encroaching on their personal time. Additionally, some participants were hesitant to talk about and share their solution methods because they were afraid of being judged for making errors. As the researcher, I tried did my best to meet their needs and to reassure them about the purpose of the research.

Given the pressure to adhere to the curriculum schedule and year planner with fixed subjects allotted to certain periods, the collection of data was restricted to a single academic term. As a result, some students may have had limited time to develop a deeper conceptual understanding of the topic.

5.7 Implications of Covid-19

As COVID-19 lockdown restrictions were eased, interviews – both written tasks and face-to-face sessions – were conducted on-site over separate days. This was necessary because the class was divided into two groups. Consequently, students had to skip a day to accommodate the schedule. This adjustment slightly impacted the timeframes, as the number of students attending was affected by the situation, leading to delays in completing all the interviews. This also restricted my ability to collect data from different classes on the same campus.

5.8 Recommendations

This study offers recommendations based on the findings and viewed through Sfard's (2008) theory of commognition, which focuses on student's word use, visual mediators, routines, and communication. The recommendations relate to opportunities for future research, curriculum, teacher practices (improvements to the school curriculum, and strategies that could support and enhance teaching methods), professional development, and textbook development.

Given that this study focused on a single group of 35 students, it is suggested that future research be conducted with a larger sample size with multiple groups of NCV Level 2 mathematics students. This will help ensure a broader and more representative understanding of the challenges and successes within the programme. This is especially important because the NCV Level 2 cohort includes a diverse mix of students, such as those who have dropped out at various grades (8, 9, 10, 11, and 12), as well as students who have failed Grade 12. A larger sample of participants will be examined to understand how their educational backgrounds are connected to their conceptual understanding and how dropout rates vary at primary- and high school. The purpose of expanding the study is to improve students' understanding of what they are learning, and also finding ways to help them deal with the difficulties they experience during learning. Validity of the study's results will be improved by using a larger and more diverse group of participants, as well as making improvements to the NCV mathematics programme so that it becomes more practically useful and relevant to students' real-life needs or future careers.

The study recommends that teacher must help students to improve learning by using the effective strategies, including innovative approaches, the balance method, and the transposition method to make it easier for students to learn and succeed in solving linear equations. Students can develop mastery of linear equations when teachers create a more inclusive, effective, and enriching learning environment by employing diverse teaching strategies.

An expanded study from different backgrounds or levels will help researchers better understand the various difficulties that students experience when learning. A larger and more diverse sample size not only strengthens the statistical validity of the findings, but also gives stronger and more meaningful results, especially for helping to improve the way that mathematics is taught in the NCV (National Certificate Vocational) programme. Students will be more likely to become confident and skilled at solving linear equations if teachers use different teaching methods that are inclusive, effective, and enriching to create a classroom that is conducive for learning (Roberts & Le Roux, 2019). When students are exposed to different strategies, they gain the skills and techniques they need to do well in mathematics. Effective learning can be achieved when teachers help students to think deeply about the steps they take when solving linear equations, exercise how to work together with their classmates (collaborate), and explain their thinking out loud.

Sfard's theory (2008) highlights that communication in learning is very important when students are talking, discussing, and explaining ideas. Sfard's theory also highlights that learning mathematics can be improved by using teaching methods and principles from training that encourage and support communication. The findings of the study agree with Sfard's theory of commognition (2008), which says that learning mathematics well happens when students are encouraged to use and understand mathematical language the words, symbols, and ways of talking used in mathematics. Sfard's theory points out that the way students talk about and engage with mathematics affects how well they understand and solve problems. It shows that both communication and thinking are very important for learning mathematics. It is also suggested that textbook writers should review and improve classroom materials to better support student learning.

Integration of mathematical discourse into the teaching material will enhance students' cognitive growth in mathematical thinking, and strengthen both their procedural accuracy and conceptual understanding. Moreover, the contextual explanations of key mathematical terminology, together with visual representations based on linear equation problems in the textbook, will enhance students' overall understanding.

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7 APPENDICES

Appendix A

Mehlo. Application

Cape Peninsula University of Technology

Dear Principal (Northlink College).

My name is Nikezwa Mehlo, I am currently registered with Cape Peninsula University of Technology (CPUT) where I am doing my Master's degree in Mathematics Education. This research project will be conducted under the supervision of Dr Mc Auliffe and Dr Jacobs at CPUT.

I am hereby seeking your consent to test and interview a class of NCV L2 mathematics students on the topic "Linear equations" at the Northlink College. Bellville campus. My role is to investigate the kinds of errors produced by NCV L2 mathematics students when solving linear equations, including fractional coefficient; to understand the nature of these errors; and to look for possible solutions in minimizing these errors. Students will not be disrupted during the learning processes. Students' participation in this research is voluntary, and if they feel uncomfortable, they have a right to withdraw, at any time, and to review the results of the test and interview. All names will be coded and kept confidential to maintain anonymity and all data records will be secured using password protected files. The data will be destroyed when the research is concluded.

I am attaching a copy of my approved proposal, the consent form which outlines the research process, as well as a copy of approval letter from the Education Faculty Ethics Committee at CPUT.

If you require any further information, please do not hesitate to contact me on 0719873756, nickvkuny@gmail.com. My supervisor: Dr Mc Auliffes (mcauliffes@cput.ac.za) and co-supervisor: Dr Mark Jacobs (jacobsms@cput.ac za)

Thank you for your time and consideration in advance.

Yours sincerely.

Nikezwa Mehlo

SIGNATURE.......

DP : ACADEMIC SERVICES

PRINCIPAL SIGNATURE pp.......

DATE. 5-07-2022 ....

EFEC Form 3 V4_updated 2019

Appendix B

Faculty of Education
Highbury Road
Mowbray
7700
Tel: +27 21 959 6583

FACULTY OF EDUCATION

On the **21 June 2022** the Chairperson of the Faculty Research Ethics Committee of the Cape Peninsula University of Technology granted ethics approval **(EFEC 4-06/2022)** to **N. Mehlo** for an **MEd degree**.

Title:	**Error analysis of linear equations produced by National Certificate Vocational students**

Comments:

The Faculty Research Ethics Committee unconditionally grants ethical clearance for this study. This clearance is valid until **31st December 2025**. Permission is granted to conduct research within the **Faculty of Education only**. Research activities are restricted to those details in the research project as outlined by the Ethics application. Any changes wrought to the described study must be reported to the Ethics committee immediately.

Date: 21 June 2022

Prof. Zayd Waghid

Chair of the Faculty Research Ethics committee

Faculty of Education

Appendix C

Cape Peninsula
University of Technology

Dear Mr/Mrs/Ms……………………………………………………………………………………………

Request permission to observe your child for my CPUT Masters research project

My name is Nikezwa Mehlo, I am currently registered with Cape Peninsula University of Technology (CPUT) for my Masters in Mathematics Education. My research topic is ***Error analysis of linear equations produced by National Certificate Vocational students.***

I am hereby seeking the consent for your child to participate in my master's study on the nature of student errors. I would like to understand what errors students make when solving linear equations and why these occur. I have noticed that students find this topic difficult and I want to understand the nature of these errors; and to look for possible solutions in minimizing these errors.

The student will be required to complete a diagnostic test based on different types of equations and to be available for a follow up one-on-one interview to discuss the test solutions. Your child's participation in this research is voluntary, and if s/he feels uncomfortable, s/he have a right to withdraw, at any time, and to review the results of the test and interview. All names will be coded and kept confidential to maintain anonymity and all data records will be secured using password protected files. The data will be destroyed when the research is concluded.

Please sign the form below if you are willing that your child must be part of this research and to give me permission to continue.

If you require any further information, please do not hesitate to contact me on 0719873756, nickykuny@gmail.com. Supervisor: Dr Mc Auliffes (mcauliffes@cput.ac.za) and Co-supervisor: Dr Mark Jacobs (jacobsms@cput.ac.za)

Yours sincerely

Nikezwa Mehlo

SIGNATURE……………………………

Appendix D

Cape Peninsula University of Technology

Dear Mr/Mrs/Ms……………………………………………………………………………………………

Request permission to observe your child for my CPUT Masters research project

My name is Nikezwa Mehlo, I am currently registered with Cape Peninsula University of Technology (CPUT) for my Masters in Mathematics Education. My research topic is ***Error analysis of linear equations produced by National Certificate Vocational students.***

I am hereby seeking the consent for your child to participate in my master's study on the nature of student errors. I would like to understand what errors students make when solving linear equations and why these occur. I have noticed that students find this topic difficult and I want to understand the nature of these errors; and to look for possible solutions in minimizing these errors.

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Yours sincerely

Nikezwa Mehlo

SIGNATURE……………………………