SECONDARY
AND EARLY TERTIARY STUDENTS' UNDERSTANDING
OF GRAPHS OF MOTION

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DECLARATION

I, the undersigned, hereby declare that the work contained in this dissertation is my own original work and has not previously in its entirety or in part been submitted at any university for a degree.

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SUMMARY

This dissertation deals with typical, widespread student errors with respect to kinematic graphs as revealed by a literature survey, as well as an own empirical investigation into the nature and extent of these misconceptions. The fact that certain misconceptions turned out to be more widespread than initially believed, has serious consequences for educators' assumptions about students' understanding of graphs in general, as well as their ideas on how to minimise some generally occurring "alternative views on graphs".

Students' graphing skills are analysed and described in terms of a number of translations between various representations of physical events involving motion. A special focus is placed on graph transformations, which are translations from one graphical representation to another. It turned out that this provides valuable information about a learner's graphing skills, as well as his understanding of the relevant kinematic quantities and conventions required to make successful transformations.

The main empirical investigation took the form of an open response type questionnaire, which was answered by 278 students from different cultural backgrounds and academic levels. A number of audio-taped clinical interviews provided additional information about students' preconceptions in specific areas.

The study reveals that learners have a poor perception of the meaning of gradient in terms of rate of change. They experience difficulties with graphs of negative kinematic quantities, being generally reluctant to make use of a proper sign convention. Confusion between the meaning of velocity and acceleration often leads to incorrect *v-t* and *a-t* graphs; and when transforming one graph into another related one, they tend to conserve the physical properties of the given graph.

A number of causes of these preconceptions were identified, with the main ones being the fact that graphing skills and graphing terminology are seldom explicitly taught in the science class,

a lack of integration between the science and mathematics curriculum and the fact that students are often not intellectually ready for the abstract approach that is required.

Finally, certain recommendations are proposed in order to improve the prevailing situation with respect to the understanding of kinematic graphs by high school and early tertiary students. The importance of students controlling their own motion experiments as well as the subsequent translations between the various representations of the motion is stressed. The need for instructors to integrate the contents of kinematic graphs with mathematics graphs, as well as the need for instruction in those areas where student preconceptions play a dominant role, is emphasised.

OPSOMMING

Hierdie verhandeling ondersoek tipiese, wydverspreide leerlingfoute met betrekking tot kinematika grafieke. Met die oog hierop is 'n toepaslike literatuurstudie onderneem asook 'n eie empiriese ondersoek na die aard en omvang van hierdie wanbegrippe. Die feit dat sommige wanbegrippe meer wydverspreid voorkom as wat aanvanklik geglo is, het ernstige implikasies vir opvoeders se idees oor leerlinge se begrip van grafieke in die algemeen, sowel as hul idees van hoe om te werk te gaan om hierdie "alternatiewe siening van grafieke" te ontmoedig.

Leerlinge se begrip van grafieke word ontleed en beskryf in terme van 'n aantal translasies tussen verskillende voorstellings van fisiese gebeure wat beweging behels. Grafiek-transformasies as translasies van een grafiek na 'n ander kry spesiale aandag in hierdie ontleding. Hierdie transformasies verskaf waardevolle inligting met betrekking tot leerlinge se begrip van grafieke, sowel as hul begrip van die betrokke kinematiese meetbare groothede en konvensies.

Die empiriese ondersoek het 'n vraelys ingesluit bestaande uit oop-respons-vrae wat deur 278 leerlinge van uiteenlopende kulturele agtergrond en akademiese status beantwoord is. 'n Aantal kliniese onderhoude wat op band opgeneem is, het addisionele inligting verskaf oor leerlinge se begrippe van spesifieke aspekte.

Die studie het aangetoon dat leerlinge 'n swak begrip van die betekenis van gradient in terme van tempo van verandering het. Hulle ondervind probleme met grafieke van negatiewe hoeveelhede omdat hulle traag is om tekenkonvensies na behore te gebruik. Verwarring tussen die betekenis van snelheid en versnelling lei dikwels tot verkeerde *v-t* en *a-t* grafieke en by transformasies van een grafiek na 'n ander is daar 'n neiging om die fisiese eienskappe van die gegewe grafiek te behou.

'n Aantal oorsake vir hierdie wanbegrippe is geïdentifiseer, waarvan 'n hoofoorsaak te vinde is in die feit dat grafiek-begrippe en grafiek-terminologie selde eksplisiet deur leerkragte in die Natuur- en Skeikunde klas behandel word. Ander oorsake is die gebrek aan integrasie tussen

die Natuur- en Skeikunde leerplan en die Wiskunde leerplan, asook die feit dat leerders dikwels nie intellektueel gereed is vir die verlangde abstrakte benadering nie.

Laastens word sekere aanbevelings gemaak om die huidige stand van sake met betrekking tot sekondêre en vroëe tersiêre studente se begrip van kinematiese grafieke te verbeter. Die belangrikheid daarvan dat leerlinge self hul bewegings-eksperimente beheer, sowel as die gepaardgaande tranlasies tussen die verskillende voorstellings van die beweging, word benadruk. Die belangrikheid vir leerkragte om die inhoude van kinematika grafieke te integreer met grafieke in wiskunde, asook die noodsaaklikheid van onderrig in daardie inhoude waar leerlinge se idees bots met dié van vakkundiges, word beklemtoon.

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CHAPTER 1

PROBLEM STATEMENT, MOTIVATION, AIMS AND RESEARCH METHODOLOGY

1.1 PROBLEM STATEMENT AND MOTIVATION FOR THE STUDY

In today's society adults and children are bombarded by a deluge of information, often in the form of large amounts of data. In order to make sense of this wealth of information, graphs are often an useful form of representation. They display the data in such a way that general tendencies and the overall relationship between the variables are usually immediately evident.

Weintraub (1967:345) describes the advantages of graphs as follows:

> *Graphs have assumed an increasingly important role in our society. They present concepts in a precise manner or give at a glance information, which would require a great deal of descriptive writing. They often distill a wealth of information into a very small amount of space.*

Graphs are well adapted to conveying information about experimental measurements, and are often the preferred medium of communication for natural scientists as well as researchers in other fields, such as the social sciences and economics (*cf.* Brasell and Rowe 1993:69).

MacDonald-Ross (1977) as quoted by McKenzie and Padilla (1986:571) stress the importance of a proper interpretation of data in many walks of life:

> *Every discipline in the social, biological and physical sciences, all applied sciences and every instrument of social and governmental policy depend upon the appropriate use of quantitative data.*

It is, however, well documented that students often lack even the most basic graphing skills (see for example McKenzie and Padilla 1986, Brasell 1987, Mokros and Tinker 1987, Wavering 1989 and Clement 1989). They experience problems in making connections between graphical and numerical data and when asked to interpret graphs they often confuse the height and the slope. Students also display a tendency to treat a graph as a picture of the physical problem scene (*cf.* Brasell 1987:77 and Clement 1989:77).

Widespread student difficulty with regard to the interpretation of graphs is reported by Padilla, McKenzie and Shaw (1986:25) who call for ... *more training beyond the plotting of points, especially if we intend to use graphs for interpreting information in high school science courses*. Clement (1989:77) emphasises the importance of developing students' graph interpretation skills:

> *The ability to interpret graphs is important for mathematical literacy and for understanding the concepts of function and variable, as well as for developing basic concepts in calculus.*

An investigation into students' understanding of **kinematic** graphs, which is the main purpose of the present study, should yield valuable information about students' misconceptions about **graphs in general.** The reason for this is the fact that kinematics provides learners with a real-life situation with which they are all familiar. The key concept of rate of change and its relationship with gradients can, for example, be investigated in terms of familiar concepts such as *displacement, velocity* and *acceleration.*

The importance of instruction and training to develop graphing literacy, especially in the field of kinematics is acknowledged by researchers such as Schuster (1983), McKenzie and Padilla (1986), McDermott, Rosenquist and van Zee (1987), Wavering (1989) and others. The main thrust of their research focuses on devising strategies and tasks in order to address specifically identified misconceptions among learners.

The present study will seek confirmation of the typical student errors with regards to kinematic graphs, as reported in the available literature, as well as research **additional** common misconceptions and difficulties not yet reported on. An important question that needs to be answered is: *To what extent are learners' graphing problems in kinematics related to difficulties with graphs in general and to what degree can they be attributed to problems with the ideas and conventions of kinematics?*

The following hypotheses will be investigated more specifically:

- Learners are reluctant to apply knowledge gained about graphing in other subjects, such as mathematics, to kinematics.

- Learners experience particular difficulty with graph → graph transformations.

- The significance of *negative* values for kinematic quantities is poorly understood because learners are reluctant to use a sign convention.

- Confusion between the meaning of certain kinematic quantities and concepts leads to poor graphing ability.

- Learners generally have a poor understanding of the meaning and significance of *gradient* and *area* in the context of graphs.

- Learners display a reluctance to change the attributes of that which is given when making translations between representations.

The causes of these misconceptions can be related to a number of diverse factors, such as inadequate instruction, the power of students' preconceptions, and the absence of a section on "graphing skills" in the curriculum. These causes together with a number of *categories* of graphing misconceptions are reported on in Chapter 5. Certain alterations to the way that kinematic graphs are traditionally taught are proposed in order to attempt to minimise the identified misconceptions.

1.2 AIMS OF THE STUDY

The main aim of the study is to investigate the nature and extent of secondary and early tertiary students' difficulties with and misconceptions about graphing, with special reference to the understanding of kinematic graphs. A number of secondary aims which result from this, include:

- to determine the extent to which poor graphing skills in general contribute to misconceptions about kinematic graphs;
- to attempt a classification of the more commonly occurring misconceptions and mistakes;
- to find out what the possible causes are for the identified misconceptions;
- to investigate the extent of graph transformation difficulties;
- to critically investigate the viability of the present prescribed syllabus with regard to kinematic graphs;
- to propose a number of teaching strategies to make the teaching and learning of kinematic graphs more successful.

1.3 RESEARCH METHODOLOGY

In order to achieve the aims of the study, three different research techniques were employed. These involved a series of one-on-one clinical interviews with selected students, a survey of the available literature on the subject and an open-ended questionnaire which was answered by 278 students from varying academic levels and cultural backgrounds. The clinical interviews had the following main aims:

- To obtain a first idea of some of the most common preconceptions that students have about kinematic graphs.

- To get information about the type of questions which could be included in the main questionnaire.

- To get to understand the reason behind some commonly occurring alternative conceptions, as revealed by the main questionnaire.

The literature study, the results of the personal interviews, as well as an analysis of a pilot study involving 32 Grade 11 and 12 students, provided the material for the questionnaire which was used in the main empirical investigation. In view of the lack of research into learners' understanding of graph-to-graph transformations, the main questionnaire included a number of questions which focuses specifically on these aspects of graphing.

The results of the empirical investigation are analysed on a question for question basis in Chapter 4. This is in order to determine whether the misconceptions identified in the literature study occurred among the test group as well, and if so, to what extent.

Chapter 5 attempts to classify the most commonly occurring misconceptions under various distinctive categories and places a special focus on graph transformation difficulties. This chapter also reviews the didactical implications of students' understanding of kinematic graphs. Own views, as well as those of other researchers into science education form the basis of this discussion, which includes a number of suggestions as to how student difficulties with graphs can be minimised through certain instructional techniques.

1.4 CLARIFICATION OF TERMINOLOGY

In order to facilitate easy reading the following comments with regard to terminology are made:

The words *student* and *learner* are used interchangeably when referring to secondary school pupils and tertiary students alike. Similarly, the words *misconceptions*, *preconceptions* and *alternative conceptions* all refer to those students' views of science which would generally be understood to be in opposition to the accepted scientific view.

The male gender is used throughout this thesis for the sake of convenience. This practice is in no way intended to be discriminatory against the female sex.

CHAPTER 2

CONTENT ANALYSIS OF GRAPHS OF MOTION

2.1 INTRODUCTION

In physics real phenomena can be represented in various ways. These include diagrams, equations, words, graphs, etc. The symbol system which is particularly well-suited to describing kinematic phenomena is graphs, as it is one of the most effective ways to illustrate, describe and predict relationships between variables.

The graphical representation of a motion relates specific kinematic concepts to certain features of a graph. The kinematic concepts might have been studied in the physics class or they could be related to actual observed motions in the real world.

The importance of a proper connection between the various kinematic concepts and their graphical representation should be emphasised in the teaching of kinematics at school as well as tertiary level. Graphs illustrate a specific motion such that key aspects of the motion are immediately noticeable and certain required information about the motion can often readily be extracted from such a representation. They also offer an excellent opportunity to demonstrate the importance and general usefulness of graphs in physics in general, as well as in other subjects. For students not continuing with physics, their experience with graphing could be more important than the actual kinematic principles which they have studied.

A particular physical situation can be modelled into various forms of representation. These usually constitute a verbal description and/or a mathematical description. The mathematical representation of phenomena uses algebraic equations, tabulated data or graphs. The writer takes the position that mastery of the content should include the ability to model and interpret, as well as to translate between various representational forms. These can occur in both directions: from the verbal description of a physical situation to any of the mathematical representations, or from a mathematical representation to a (hopefully) improved verbal description of the physical situation which was originally investigated. Mathematical representations of real phenomena can be said to refine, consolidate and internalise the knowledge that the student has about the situation under investigation (*cf.* Human, personal communication and Janvier, 1978). This chapter deals mainly with a description of the cognitive processes involved in the production and interpretation of kinematic graphs.

The particular ability to transform a given kinematic graph into another related one, will in this study be called a transformation. The possibility of an improved understanding of graphs as representations through the teaching of transformation skills forms an important part of the present study. Other aspects which will receive attention in this chapter are the functionality of kinematic graphs and an analysis of the actual syllabus content, both in this country and in the U.K.

2.2 THE FUNCTIONALITY OF KINEMATIC GRAPHS

2.2.1 General

Graphs as representations of functions describe or illustrate the interdependence of two variables and as such it is a symbol system which is particularly well adapted to conveying information about experimental measurements. In the field of kinematics, graphs serve a useful purpose in the generation, communication and consolidation of information about a specific motion. Important information about a motion can be derived from a graph in the following instances:

- interpreting given quantitative or qualitative graphs of physical situations;

- construction of graphs from data tables followed by analysis and interpretation of the graph;

- construction of graphs from described (or observed) physical situations in order to understand the situation more fully.

In one sense graphs can greatly assist as a facilitator for the improved understanding of a physical situation while in another sense kinematics provides a context for improved understanding of graphs in general. It is even possible for students to exhibit a natural urge to draw graphs of kinematic situations - if only for the sake of appreciating the elegant beauty of this form of representation. The functionality of kinematic graphs will now be discussed under various headings.

2.2.2 The generation of information

Graphs can generate additional information about a specific motion. In the case of accurate graphs which were constructed from data points this additional information can be contained in any of the following features of the relevant motion graph:

2.2.2.1 The height

The height or ordinate gives the value of the kinematic quantity at a specific time. In view of the fact that the graph displays the relationship between continuously varying quantities which appear as discrete quantities in a data table, it is possible to determine the magnitude of the kinematic quantity at any intermediate time value which falls within the given time domain. In the case of linear or curvilinear interpolation, the additional information is derived by assuming that the geometric pattern displayed by discrete points is maintained in reality.

2.2.2.2 The slope

The slope (or gradient) of a graph gives the rate-of-change of the particular kinematic quantity and thus provides valuable information which is not immediately available from the data which was obtained from a specific experiment. Information can be gained from the following aspects of the gradient:

- the level of steepness gives information relating to how quickly the kinematic quantity is changing;
- a positive or negative gradient provides information about the direction of a related kinematic quantity;
- a zero or non-zero gradient indicates whether the kinematic quantity stays at a constant value or not;
- the constancy or variation in gradient could be indicative of a constant or accelerated motion in an *s-t* graph and constant or varying accelerated motion in a *v-t graph.*

The most common information that is gained through gradient is information about the velocity from a displacement-time graph and information about the acceleration from a velocity-time graph

2.2.2.3 The area

Area as a feature of a kinematic graph gives information about the motion under observation much in the same way as the feature of gradient does. The area under the velocity-time graph gives *change in displacement* while the area under the acceleration-time graph gives *change in velocity*. Areas under the time axis are viewed as negative and should be subtracted from the area above the time axis.

2.2.2.4 Position of graph with respect to the time-axis

A graph's position with respect to the time axis provides certain information about the direction of the motion. Negative values are only indicated for the vector quantities of displacement, velocity and acceleration. A crossing of the time-axis in a *v-t* graph has to be associated with a reversal in the direction of the motion, while in an *s-t* graph it has to be viewed as a displacement in the opposite direction to the chosen positive reference direction

2.2.2.5 Coordinates of maximum or minimum values

In the process of plotting a graph, the position of turning points necessarily becomes apparent in reading the data, so that by the time the graph is drawn it has already been observed. The graph does, however, draw immediate visual attention to the maximum and/or minimum points.

2.2.2.6 Intercepts with the coordinate axes

Graphs generate information about the value of kinematic quantities at the time when t is arbitrarily chosen as 0 as it is simply read off from the vertical axis intercept. Similarly the intercept with the horizontal axis yields immediate information about the time values for which the kinematic quantity under discussion becomes zero.

Example of graphical information generation

A single example is cited to demonstrate the usefulness of graphs in information generation. In this example the graph is the sole supplier of information about the motion and it will be shown how it can be used in an analytical-predictive role to yield information about important aspects of the motion.

Example

The graph in figure 2.1 depicts the motion of a car from time t = 0

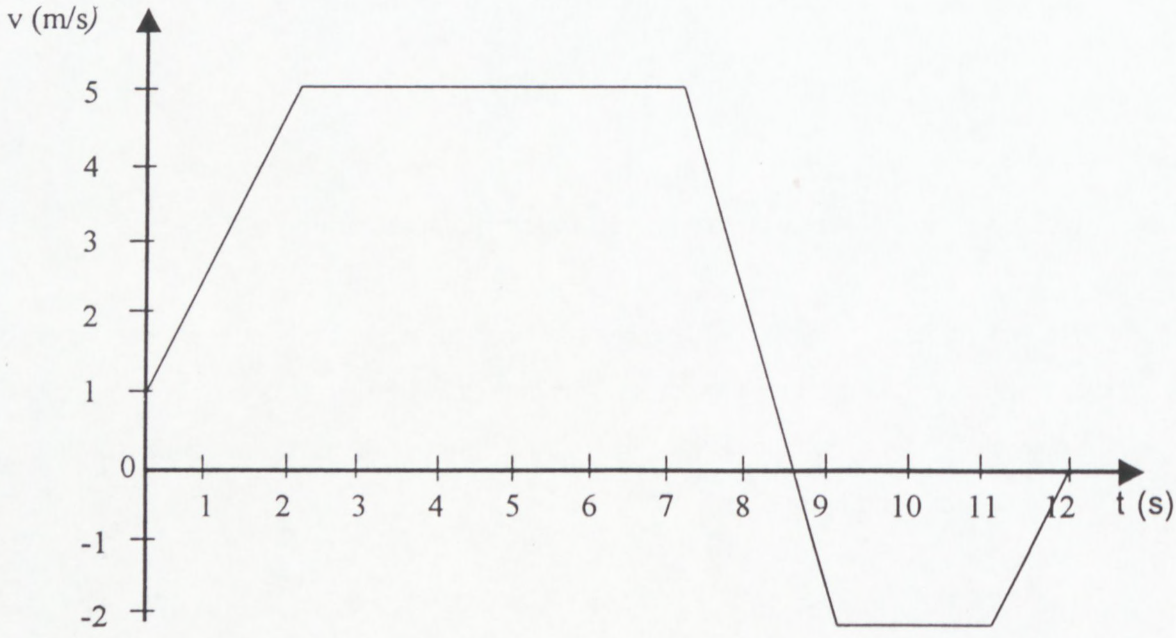


FIGURE 2.1

Using the graph determine the following for the motion of the car.

(i) *the velocity of the car after 1s.*

(ii) *the acceleration of the car between*

a) *0 and 2 seconds*

b) *2 and 8 seconds*

c) *8 and 9 seconds.*

(iii) *Calculate the change in displacement of the car between*

a) *0 and 8 seconds*

b) *0 and 12 seconds.*

(iv) *Explain what happened to the car between 8 and 12 seconds from the start.*

(v) *What is the maximum velocity of the car and when was this maximum achieved?*

Each of these questions refers to the respective five categories of information generation as described in section 2.2.2.

2.2.3 The communication of information

Graphs offer a vehicle for the communication of available information between two persons. Trends and general characteristics of a motion which are often not obvious in the raw data, show up clearly and immediately on the graph. One example from kinematics is the use of *v-t* graphs in motorcar magazines to communicate information about the performance of a new model, so that details such as maximum speed and acceleration are immediately evident from the graph.

2.2.4 The consolidation of existing information

An important function of graphs is to consolidate all the known aspects of a motion. This information can be in the form of data values which were obtained from an experiment, a verbal description of a motion or even another related motion graph.

The new graph (which can be either a sketch graph or an accurate graph) often shows up deficiencies in the available information or displays data points which do not fit in with the general pattern. In the context of consolidation the graph answers questions about a motion such as:

- What information do I have available?
- What information do I need to describe the motion fully?
- Can the missing information be extracted from the physical situation?
- Why were data points generated which do not fit in with the other coordinates?

2.2.5 Improved understanding of the physical situation

The role that graphs play with respect to improved understanding of physical phenomena is highlighted by Leinhardt, Zaslavsky and Stein (1990:20)

> *Graphs serve as representations of real observations and as analytic tools for detecting underlying patterns which in turn inform the observer about the phenomena under investigation.*

McDermott *et a.l.* (1987:513), in a detailed study of students kinematic problems, concluded that many of these misconceptions could be addressed through an increased emphasis on graphing:

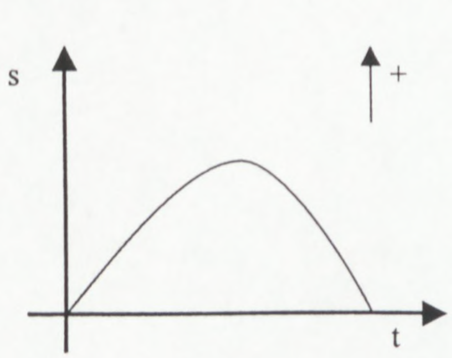
> *...facility with graphing can play a critical role in helping students deepen their understanding of kinematic concepts.*

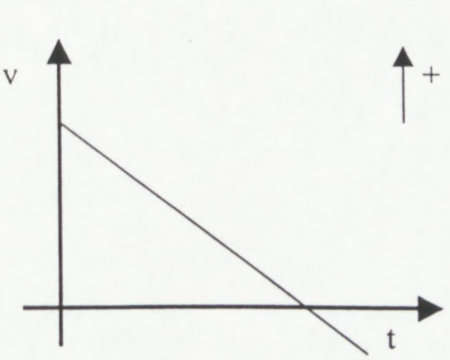
Students must be made aware that the same physical situation can be interpreted in the context of many different representations, such as verbal, tabular and algebraic. This study focuses on graphical representations and the difficulties that students experience, among others, with the way graphs can lead to an improved analytical description of the physical situation that they represent. A single example is cited to show how a graphical representation of a motion underlines the difference between different but related concepts of displacement, velocity and acceleration. The idea is that students gain a better insight into the nature of these concepts by noticing the different shapes of the three graphs representing the same motion.

Example

A cricket ball is tossed vertically upwards and caught again. Draw the *s-t*, *v-t* and *a-t* graphs of the motion of the ball taking up as positive and the throwers hand as reference point.

Solution





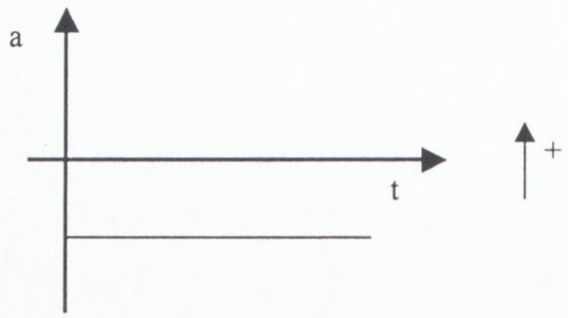


FIGURE 2.2

The fact that the same motion can be represented by three graphs of very different shape not only emphasises the difference between the three kinematic concepts, but also serves to let the student reflect carefully about the described physical situation.

2.2.6 Improved understanding of graphs

Kinematics provides a context for the study of cartesian graphs. It is a context which is familiar to most students and provides a natural source of application for linear and quadratic relationships studied in the maths class. The possibility of an improved understanding of the relationship between an algebraic function such as $y = 4x - 5x^2$ and the kinematic relationship $s = 4t - 5t^2$, when instructors integrate mathematics and science graphs will be explored in Chapter 5.

2.2.7 Graphs for the sake of graphs

Students are often attracted to the natural elegance of graphical representations. They find graphs visually attractive and hence display a tendency to <u>want</u> to draw a graph. This tendency is further amplified by the fact that graphs feature very prominently in the printed media.

It can thus be said that a function of graphs could be merely to impress others with the elegant beauty of the graph itself.

2.2.8 Some disadvantages of graphs

Graphs as representations of real-life physical events have the following disadvantages:

- For those not familiar with the conventions of graphing, it is possible that the advantages are offset by the time wasted in attempting to construct the graph.
- By changing the scale on the axes, it is possible to exaggerate or diminish an observed rate of change if the reader is not "sensitive" to the scale used.
- Graphs are only suitable for representing the relationship between two or three variables.

2.3 KINEMATIC CONCEPTS AND FORMULAS

Before the various graphical representations and translations are described, it is necessary to define a number of important kinematic concepts. This will ensure that there will be no confusion or ambiguity when these concepts are illustrated graphically or when they are used either for the qualitative description of a motion or in quantitative calculations.

The following concepts are fundamental to this study:

2.3.1 *displacement:* the straight line vector from the beginning to the end of a motion during a particular time interval.

2.3.2 *position:* a point in space relative to a specified reference level.

2.3.3 *average velocity:* the time-rate-of-change of displacement.

2.3.4 *distance:* the total length of the path over which a motion takes place during a particular time interval.

2.3.5 *speed:* the time-rate-of-change of distance.

2.3.6 *average acceleration:* the time-rate-of-change of velocity.

2.3.7 *negative acceleration:* acceleration which is in a direction opposite to the reference direction defined as positive.

It must be noted here that a negative acceleration has the effect of either:

- decreasing an object's speed in the case where the object moves in the positive direction; or

- increasing an object's speed in the case where the object is moving in the negative direction.

2.3.8 *instantaneous velocity:* the limit of the ratio of the change in displacement over the time interval as $\Delta t \to 0$

In symbols $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

Note that the magnitude of the instantaneous velocity vector equals the instantaneous speed.

2.3.9 *instantaneous acceleration:* the limit of the ratio of the change in velocity over the time interval as $\Delta t \to 0$

In symbols $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

2.3.10 The basic kinematic equations are stated without proof (rectilinear, uniform accelerated motion is assumed)

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{v}_{av} = \tfrac{1}{2}(\vec{v}_i + \vec{v}_f)$$

$$\vec{s} = \vec{v}_{av} \times t$$

$$\vec{s} = \tfrac{1}{2}(\vec{v}_i + \vec{v}_f)t$$

$$\vec{s} = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$

$$v_f^2 = v_i^2 + 2as$$

with ...

v_f	=	final velocity	in $m.s^{-1}$
v_i	=	initial velocity	in $m.s^{-1}$
v_{av}	=	average velocity	in $m.s^{-1}$
s	=	displacement	in m
a	=	acceleration	in $m.s^{-2}$
t	=	time	in s

2.4 REPRESENTATION OF KINEMATIC PHENOMENA

2.4.1 Introduction

A kinematic situation can be described using a number of different representations which vary in their degree of abstraction. Every representation is an expression of the relationship between two simultaneously changing variables with time usually the independent variable.

The four most common representations used to describe actual kinematic events are verbal descriptions, algebraic equations, ordered data points and graphs. The level of algebraic abstraction is arguably more sophisticated for graphs and algebraic equations than for the other two forms of representation.

The example of the change in velocity of a stone being projected downwards with an initial velocity of 20 $m.s^{-1}$ from a 100 m high tower will be used to illustrate the difference between the four representations.

2.4.2 Verbal description.

This is the earliest form of representation and requires a minimum level of abstraction. Young children at the pre-operational stage are capable of describing the motion of the stone, often in unscientific colloquial language. An 11 year old described the motion as follows: *gravity will cause the stone to hit the ground at a much higher speed than the 20* $m.s^{-1}$.

An older student who has been exposed to some kinematics may employ a more scientific description: *the stone accelerates uniformly at 10* $m.s^{-2}$ *with an initial velocity of 20* $m.s^{-1}$, *which means that its velocity increases by 10* $m.s^{-1}$ *every second.*

Representations focus either on global aspects of the motion or emphasise specific points during the motion. In a verbal representation, a point-wise focus might result in a statement such as that the velocity was 40 $m.s^{-1}$ after two seconds, whereas the global focus describes the motion as a whole.

2.4.3 Algebraic equations

The student has to adapt the general equation for uniform accelerated motion, namely $v = u + at$, to make it applicable to the specific concrete situation. *v* can be expressed as a function of *t*, provided the student:

- recognises the situation as one in which the object is accelerated uniformly in a straight line;
- recognises the need for a sign convention (for example down as positive);
- notes that $u = +20$ $m.s^{-1}$;
- realises that a becomes $+10$ $m.s^{-2}$ (the gravitational acceleration).

Hence the algebraic representation of the verbally described motion is written as $v = 20 + 10t$.

Once this equation has been arrived at, it is important for the student to:

- recognise the linearity of the relationship in terms of the fact that the velocity changes by the same amount between 0 and 1 second as between 1 and 2 seconds;
- associate the equation with the algebraic formula for a straight line graph : $y = c + mx$;

- recognise the continuity of the relationship, i.e. v can easily be determined for any real t value (before hitting the ground);

- realise that if t_1 = time taken for a stone to hit the ground, then $v = 0$ for $t > t_1$, which means the equation is only valid for $0 \leq t \leq t_1$

- be able to determine the value of t_1 by using the equation $s = ut + \frac{1}{2}at^2$ as follows:

$$100 = 20t + 5t^2 \quad \text{(taking down as positive)}$$

$$\therefore \quad t^2 + 4t - 20 = 0$$

$$\therefore \quad t = \frac{-4 \pm \sqrt{96}}{2} \quad \text{(using the quadratic formula)}$$

$$\approx 2{,}9 \text{ s} \quad \text{(ignoring the negative value);}$$

- translate the algebraic equation into a table of ordered pair values;

- translate the algebraic equation into a graphical representation with or without the intermediate step of obtaining data points.

The formula $v = 20 + 10t$ could also be derived by making use of an experimental investigation involving the following actions:

Assume the student knows that the relationship between the velocity of the stone and time in the air is a linear one. The familiar linear function $y = mx + c$ is now written as $v = mt + c$. Two v values with their corresponding t values are obtained experimentally and substituted into this equation which now yields two simultaneous equations in m and c that can easily be solved. It should be noted that neither the initial velocity of the stone, nor the value of g need to be known in this method.

Alternatively, the student could adapt the formula $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ to read $\frac{v - v_1}{t - t_1} = \frac{v_2 - v_1}{t_2 - t_1}$ where (t_1; v_1) and (t_2; v_2) are again two experimentally obtained data pairs. For example, assume it is found that when t = 1, v = 30 and when t = 2, v = 40. It follows that:

$$\frac{v - 30}{t - 1} = \frac{40 - 30}{2 - 1}$$

$$\Rightarrow \quad \frac{v - 30}{t - 1} = \frac{10}{1}$$

$$v - 30 = 10t - 10$$

$$\Rightarrow \quad v = 10t + 20$$

It must be noted here that a student who accepts that the acceleration of the stone is constant can obtain the desired result without focusing explicitly on the linearity of the v vs t relationship. He may argue:

$$a = \frac{v_2 - v_1}{t_2 - t_1} \text{ and } a = \frac{v - v_1}{t - t_1}$$

hence, for constant a :

$\frac{v_2 - v_1}{t_2 - t_1} = \frac{v - v_1}{t - t_1}$ thus implicity suggesting the linearity of v vs t.

A graphical method could also be used to obtain the desired algebraic equation. In this case the two data pairs in the example (say (1;30) and (2;40) as before) are plotted on a system of axes as indicated in figure 2.3.

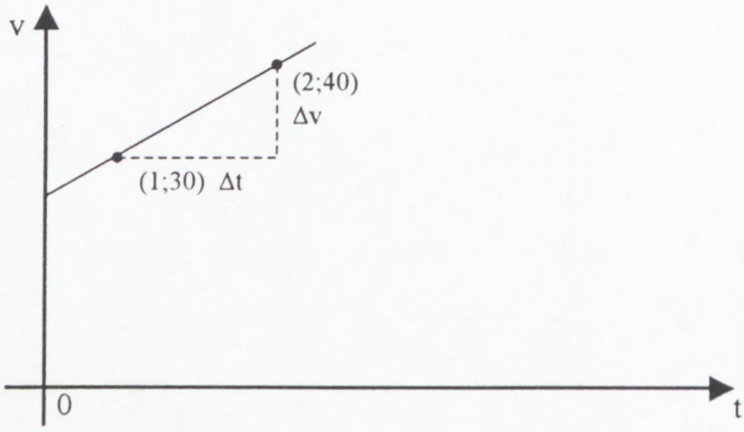


FIGURE 2.3

The gradient is determined from the graph as

$$m = \frac{\Delta v}{\Delta t}$$

$$= \frac{40 - 30}{2 - 1}$$

$$= 20$$

The y intercept is read off as c = 10 and the familiar equation for a straight line y = mx + c is adapted as follows

$$v = mt + c$$

$$\Rightarrow \qquad v = 20t + 10$$

The diagram in figure 2.4 illustrates the various ways in which a specific kinematic formula (such as v = 20t + 100) can be arrived at, starting with the physical situation:

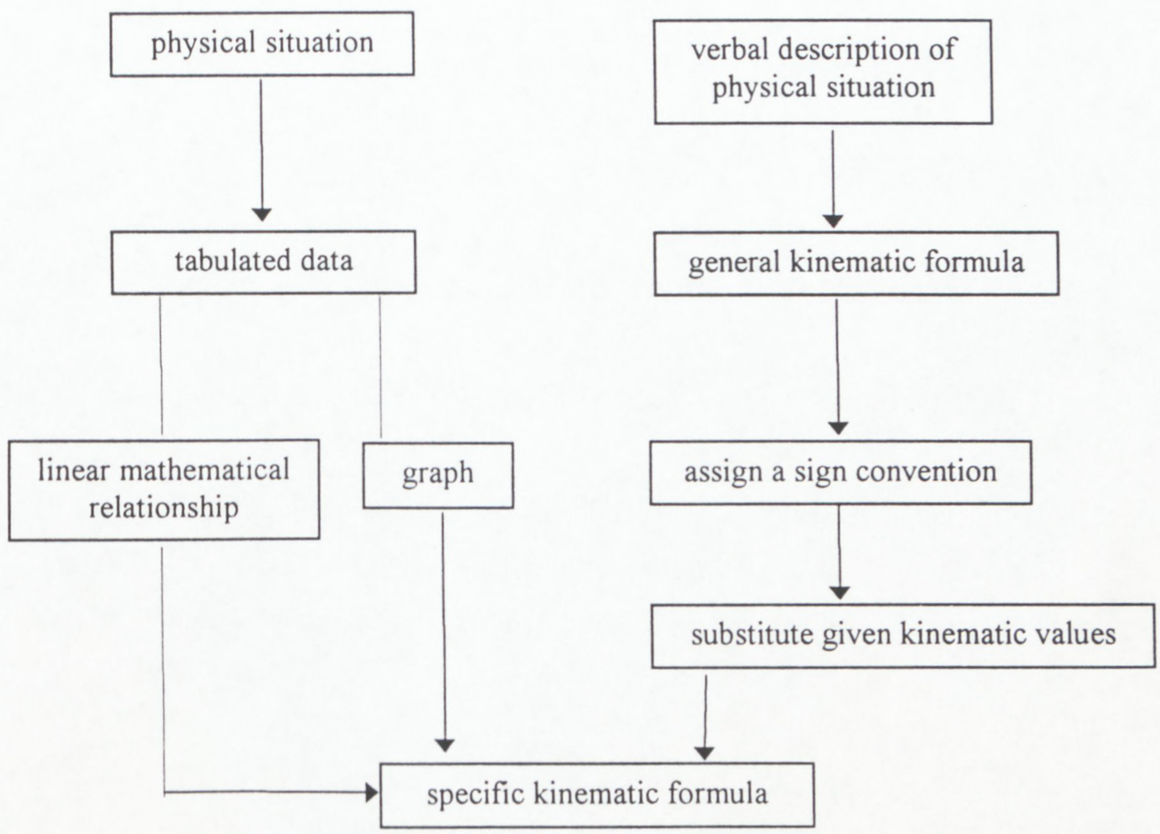


FIGURE 2.4

2.4.4 Ordered pairs

The physical situation can be described in terms of a set of data points. These data points can be in the form of a table of values or as a set of ordered pairs. For the given example this can be written as follows:

t(s)	0	1	2	3
v($m.s^{-1}$)	20	30	40	50

or

$$\{(0;20);(1;30);(2;40);(3;50)\}$$

The source of the data points can vary considerably. Possibilities include the following:

- actual measurements are taken during the experiment.
- an idealised set of measurements are given to the student.
- the data coordinates are extracted from the algebraic equation.
- the data coordinates are extracted from an available accurate graph of the motion.

Tabular values as the result of an imagined or actual kinematic experiment in one sense highlight the functional relationship of the kinematic quantity but at the same time constitute an important intermediate stage between a real phenomenon and its graph.

2.4.5 Graphical representations

A graphical representation of the velocity of the stone as a function of time is illustrated in the figure below. (Down is taken as positive).

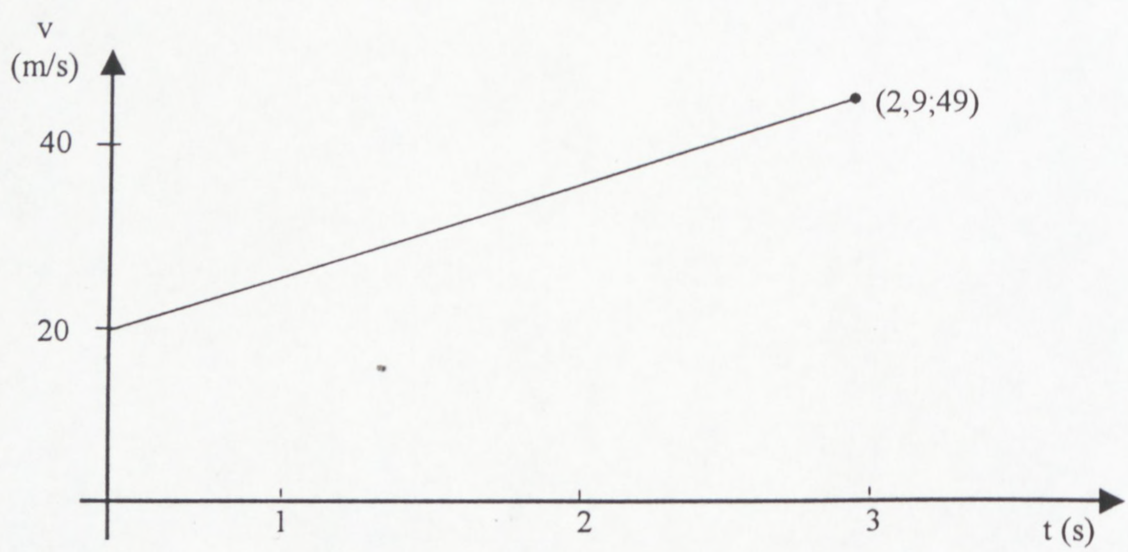


FIGURE 2.5

A graphical representation gives an immediate indication of the nature of the relationship between the two variables - in this case, velocity and time. It also serves as a diagrammatical representation of the *rate of change* concept, with the steepness of the slope giving an indication of rate of change for a specific graphical representation. If the same scale is used on the axes for two different *v vs t* graphs, the one with the steeper slope immediately stands out to be the one which illustrates a larger rate of change of velocity (i.e. a larger acceleration).

The relationship between a kinematic quantity and time is both dynamic and continuous and these characteristics are best illustrated when a motion is represented graphically. Furthermore, it is important that students are capable of going both ways - they should be able to match a narrative passage to relevant features on a graph as well as describe a motion from a given graphical representation.

Many studies have reported that graphs greatly assist towards an improved understanding of kinematic concepts while at the same time providing a meaningful context for the study of graphs in general (Trowbridge and McDermott 1980, 1981; McDermott, Rosenquist, Popp and van Zee 1983; McDermott *et al.* 1987; Clement 1989 and others). Kinematics as a meaningful context for the study of graphs will be further explored in Chapter 3.

2.5 TYPES OF MOTION GRAPHS

2.5.1 Kinematic quantities that can be represented graphically.

The variation in time of the defined kinematic quantities for a particular motion can be represented by one or all of the following graphs (The type of graph is given here together with its symbolic abbreviation):

position- time (or displacement time) .... $\vec{x} - t$ or $\vec{s} - t$

distance-time .... $d - t$

velocity-time .... $\vec{v} - t$

speed-time .... $v - t$

acceleration-time .... $\vec{a} - t$

rate of change of speed-time .... $\frac{\Delta v}{\Delta t} - t$

change in velocity-time .... $\Delta \vec{v} - t$

velocity-distance .... $\vec{v} - d$

It must be noted that in this study for the sake of convenience, arrows will not always be used to denote vector quantities. In the discussion that follows all motion will be assumed to be rectilinear motion with either constant velocity (zero acceleration) or changing velocity at a constant rate (constant non-zero acceleration). Students must be made aware that real observed motion seldom behaves in this idealised manner.

Arons (1984:88) states in this regard:

> *In an effort to make physics 'easier' and to avoid mathematical complexity introductory physics courses tend to ignore situations of non-uniform acceleration.*

Students must also realise that the vector quantities of displacement, velocity and acceleration provide much more information about a motion than their scalar counterparts. This is particularly true for their graphical representations as reversal in direction of a motion becomes immediately obvious on the graph depicting that motion, provided the vector quantity has been graphed.

2.5.2 Sketch graphs

A sketch graph represents an observed or a verbally described motion on a system of axes without indicating any intervals on the axes. It thus illustrates certain key aspects of a motion such as increasing or decreasing velocity or maximum displacement from a certain reference level in a visually easily recognised way. Mokros and Tinkler (1987:320) state that graphing constitutes a key symbol system in science because ...

> *it allows us to use our powerful visual pattern recognition facilities to see trends and spot subtle differences in shape.*

The fact that sketch graphs often provide clearer information about a motion than any other form of representation is also recognised by Wainer (1992:15) who puts it as follows:

> *It is because the ability to understand spatial information is so powerful that humans can do it well, even with flawed graphs*

De Jager (1990:34) suggests that the ability to draw a sketch graph of a given function requires the drawer to make the important connection between the global characteristic of for example, *increasing* with the graphical characteristic of, *rising line*. The graph thus illustrates the characteristics of a given motion qualitatively. These characteristics must be known to the student in order for him to draw the sketch graph.

Leinhardt *et al.* (1990:28) quote Bell and Janvier (1981) who argued that students should be introduced to qualitative graphs of concrete situations and be asked to view them globally instead of point-wise. Students should thus be encouraged to look at the entire graph as an expression of the relationship between two simultaneously changing variables and express the relationship in words.

2.5.3 Accurate Graphs

Accurate or quantitative graphs are usually drawn from a table of values which were obtained from a real or an imagined kinematics experiment. It is also possible to draw the accurate graph from a specific formula which has time as the independent variable. An example would be the accurate drawing of a *v-t* graph from the formula v = 6 + 4t, which is a specific case of the more general formula v = u + at. It has been pointed out earlier (see p. 25) that a formula as a form of representation also generates a number of coordinates which can be tabulated; in fact any number of *v-t* values can be calculated giving a much improved framework for the drawing of the graph.

Accurate graphs require certain skills such as choosing and labelling of axes, marking of intervals and plotting of points which sketch graphs do not require. An important idea that students must understand is the principle of drawing a *best-fit-line* through the plotted data points. It must be pointed out that "best fit lines" are used when the data points are the result of actual physical experimentation where aspects such as friction or inaccuracy of measuring instruments play a role. When a formula (such as v = 20 + 10t) is used, the data points will naturally lie on a straight line or parabola as the mathematical formula describes an idealised situation.

The distinction between a sketch graph and an accurate graph is described by de Jager (1990:39) as follows:

> *The purpose of a sketch graph of a given function is to illustrate the already familiar characteristics (a descriptive role), whilst the purpose of an accurate graph is to obtain additional information (an analytical or predictive role).*

An example of the *analytical or predictive role* in the context of kinematics graphs can be the determination of velocity from the slope of an *s-t* graph or the calculation of change in displacement as the area under a *v-t* graph.

Leinhardt *et al.* (1990:28) distinguish between sketch graphs and accurate graphs, referring to the latter as calling for a *pointwise or quantitative focus* while the former calls for a more *global focus.*

2.5.4 Semi-accurate graphs

It is often important to highlight specific key aspects of a motion. In this case a sketch graph with specific coordinates included might be more appropriate than either a sketch graph or an accurate graph. An example of such a semi-accurate graph could be that of displacement vs time of a ball which is projected upwards with an initial velocity of 80 $m.s^{-1}$:

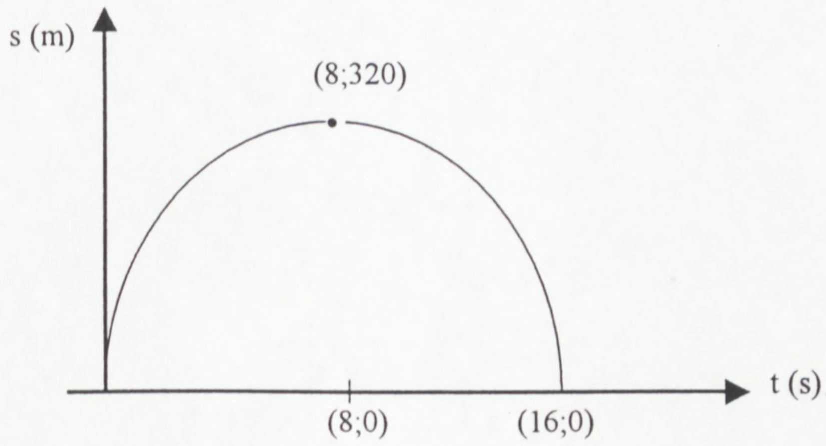


FIGURE 2.6

Students could obtain the displacement and time values at maximum and zero displacement by making use of kinematic equations:

$s = ut + \frac{1}{2}gt^2$

Taking up as positive and the ground as 0 m:

$$0 = 80t - 5t^2$$

$$t(80 - 5t) = 0$$

$$t = 0 \text{ or } 16$$

$\therefore$ Maximum height reached after 8 s:

$$S_{max} = 80 \times 8 - 5x(8)^2$$

$$= 320 \text{ m}$$

An alternative would be to use the formula

$$v^2 = u^2 + 2as$$

at max height : $0 = 80^2 - 2 \times 10 \times s$

$$\therefore \quad 20s = 80^2$$

$$s = \frac{80^2}{20}$$

$$= 320 \text{ m}$$

The advantage of a semi-accurate graph is that it is quick to draw, but still provides important information about a motion. This information, if indicated correctly, can also assist with actions such as:

- interpreting local aspects of the motion;

- constructing graphs of related kinematic quantities;
- communicating information that did not appear explicitly in the narrative passage.

2.6 LEINHARDT *et al.'s* CLASSIFICATION OF GRAPHING TASKS

2.6.1 Introduction

Leinhardt *et al.* (1990) analysed and commented on the research which has been undertaken with respect to students' perception of functional relationships and their graphical representation. Their classification of the results of the research in terms of graphing tasks, student learning and teaching is significant as it approaches graphical representations in terms of content, the action of the student as well as teaching strategies.

Graphing tasks are described in terms of a number of categories which have some implications for a proper development of students' conceptions of kinematic graphs. These categories are *actions of the students*, the *situation*, *variables* and *focus* and they will now be described briefly and illuminated with examples from kinematics.

2.6.2 Actions of the student

The action of the student can be considered to be either focusing on **interpretation** or **construction** of graphs. In the case of interpretation the emphasis could be either on a local or global level. Accurately drawn quantitative graphs where specific data points of kinematic quantities are plotted on a system of axes tend to emphasise local aspects of the motion.

An example would be the accurate determination of the velocity of an object by measuring and interpreting the gradient of a displacement vs time graph.

Qualitative sketch graphs, however, emphasise more global aspects of a motion and they are normally interpreted in order to gain information about aspects such as whether the velocity increases or decreases, whether the acceleration is positive or negative, etc.

The interaction of the student with kinematic graphs is next described and classified in terms of task accomplishments. Leinhardt *et al.* (1990) identify the tasks of prediction, classification, translation and scaling.

2.6.2.1 Prediction tasks

Students construct kinematic graphs in order to predict values of kinematic quantities which were not originally known. This is done through interpolation, extrapolation, drawing best-fit lines through a number of data points, etc. Specific inferences are thus made about the co-variance of two continuous variables such as velocity and time and instructors must ensure that students appreciate the ease with which these predictions are made when graphical techniques are employed.

2.6.2.2 Classification tasks

Classification tasks rely on graph interpretation. Examples from kinematics which require interpretation of motion graphs in order to classify similar events as belonging to a specific verbally described category include the following:

(a) Use the graphs in figure 2.7 to classify the corresponding kinematic events into those involving a constant rate of change of velocity and displacement and those belonging to a non-constant rate of change.

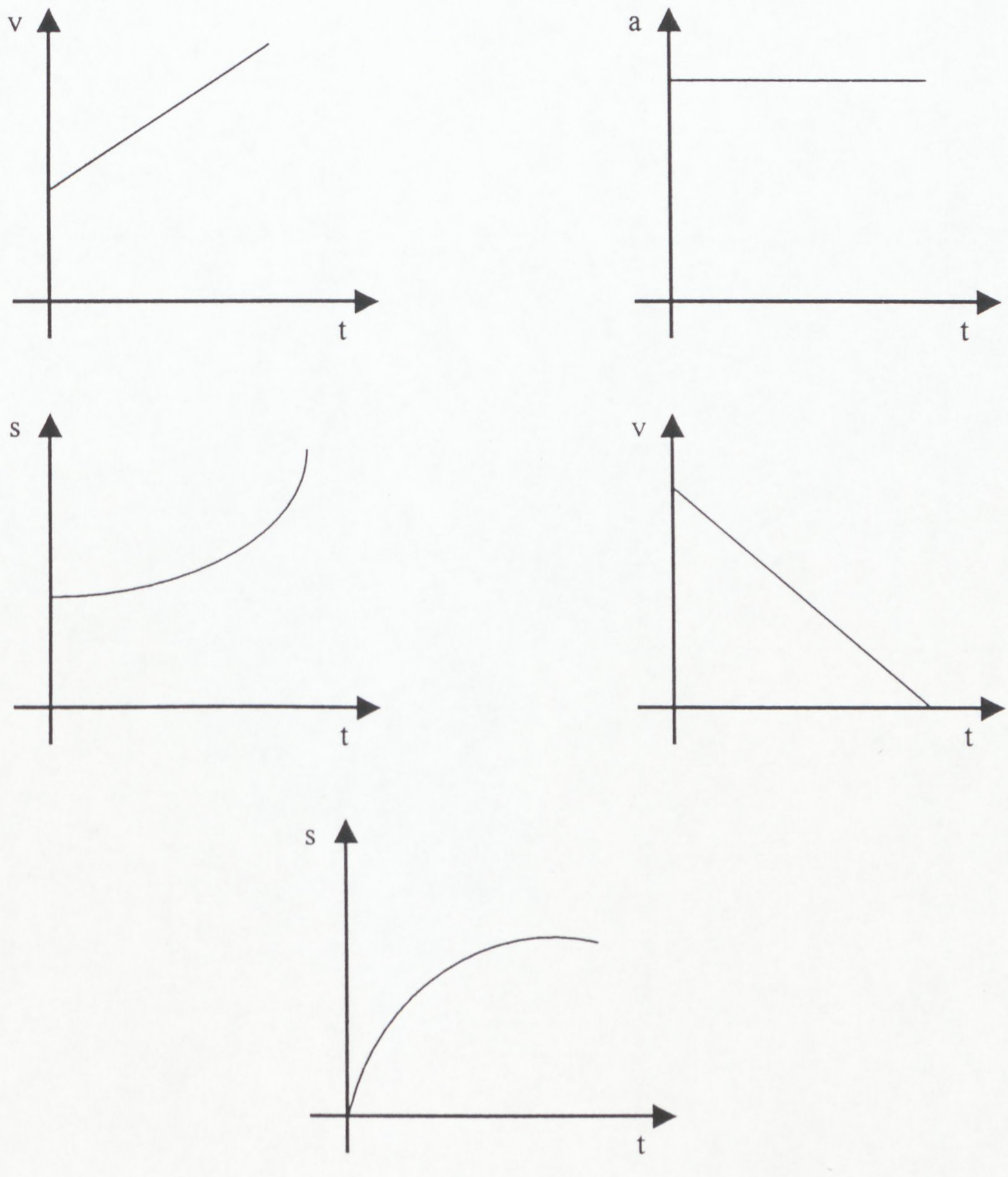


FIGURE 2.7

(b) Which of the graphs in figure 2.8 represent a stone which is thrown vertically upwards? (Considering only the upwards motion and taking up as positive in all cases and ground as reference level.)

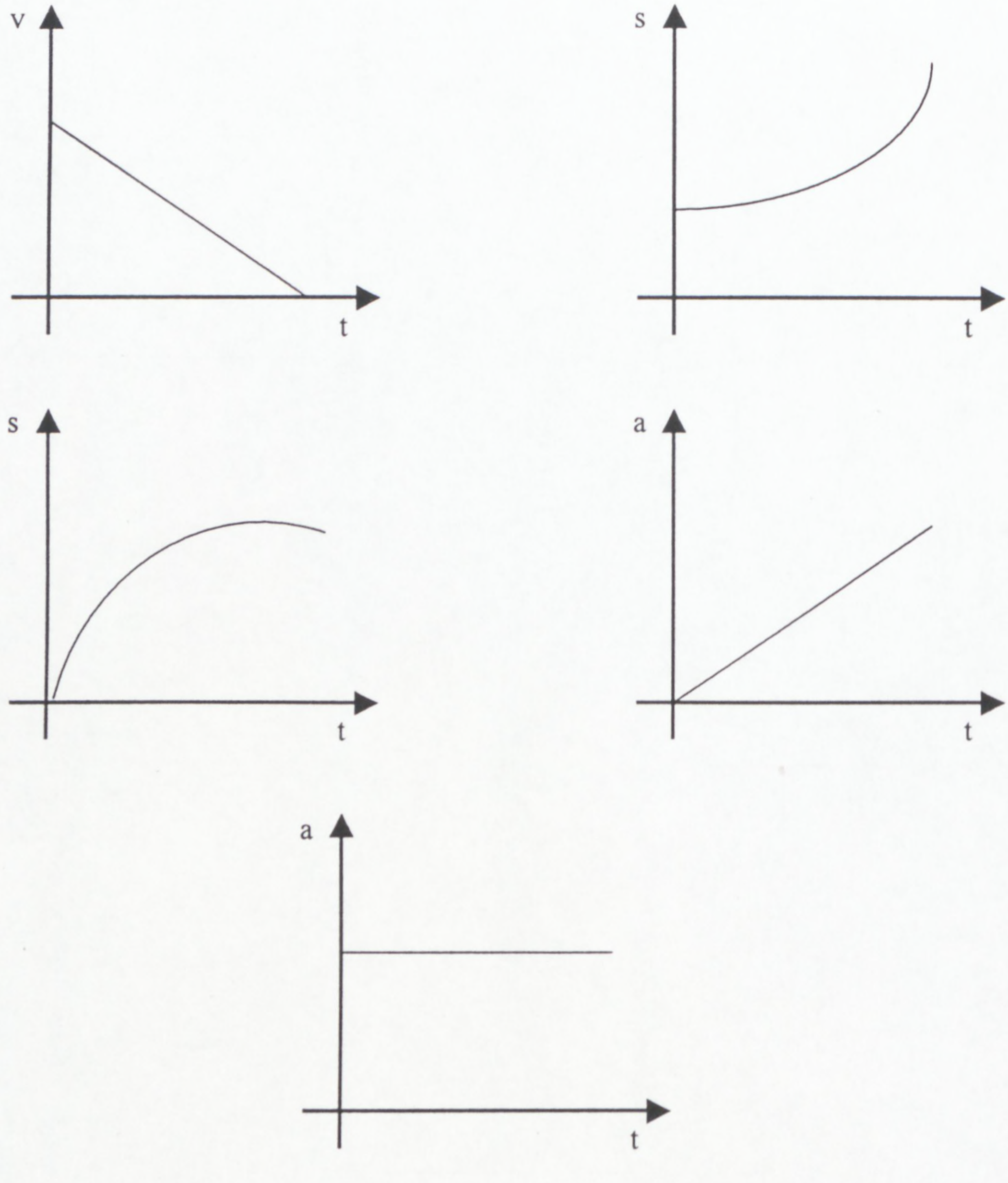


FIGURE 2.8

The idea is that students should be able to identify the correct kinematic graphs (describing different kinematic quantities) of the same event.

(c) A third possibility is to describe the behaviour of a kinematic quantity such a acceleration and ask students to classify a number of different motion graphs into the various behaviour categories. A single example is cited:

Classify the sketch graphs illustrated in figure 2.9 into categories A, B, C or D where

A = motion with zero acceleration

B = motion with constant positive acceleration*

C = motion with constant negative acceleration*

D = motion with increasing acceleration

(*see definition of positive and negative acceleration on page 18)).

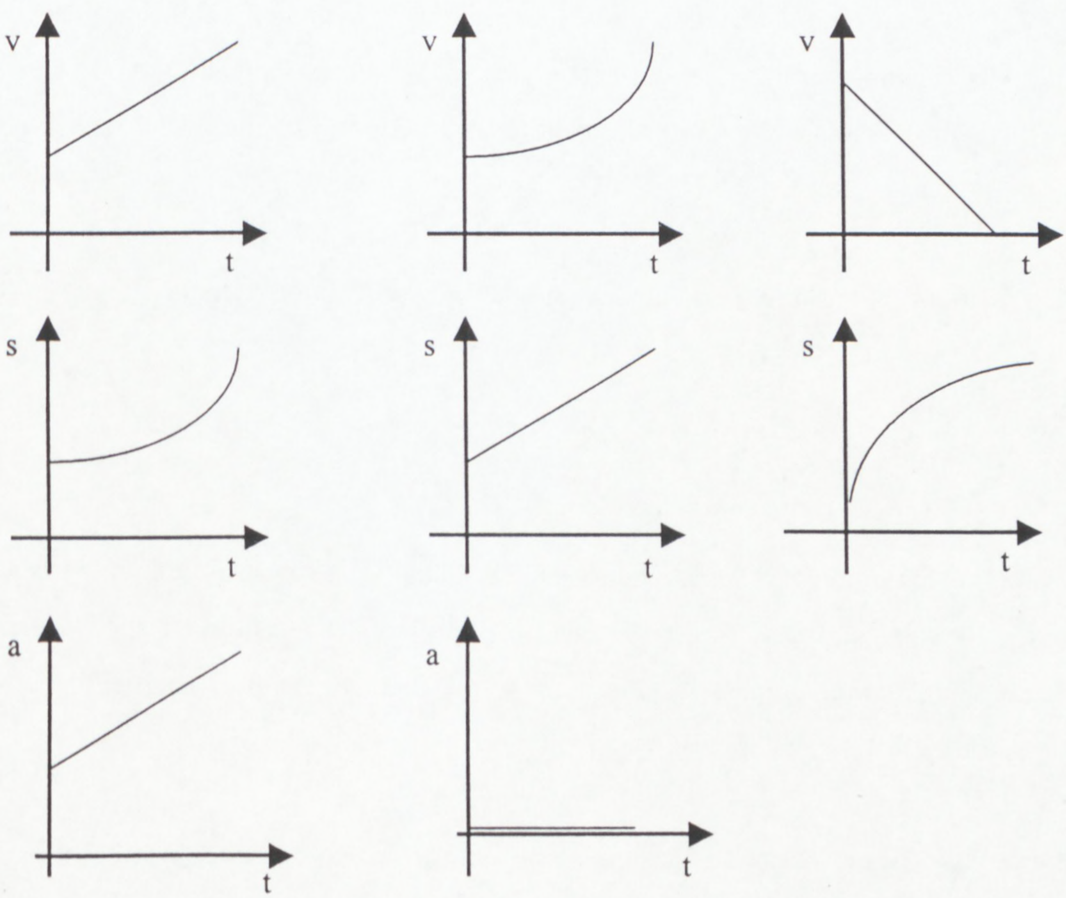


FIGURE 2.9

2.6.2.3 Translation tasks

Leinhardt *et al's.*(1990) translation tasks involve the ability to change from one representation of a motion to another. The representations most frequently used in kinematics include verbal, tabular, algebraic and graphical representations. Verbal representations are encountered by the young child before any formal study of physics has commenced and are obviously closely linked to their intuitions and common sense beliefs.

It must be pointed out that the verbal description of a motion can be stated either in colloquial language or in strict academically correct scientific language. In the case of the former a further translation from the colloquial description of the event to the scientific description is necessary.

The question of translations will be discussed in more detail in a later paragraph (see § 2.7).

2.6.2.4 Scaling

The proper assigning of values to intervals is essential for successful interpretation actions. This is especially true when graphs have to be interpreted locally and applies mainly to accurate quantitative graphs. Scaling is important both for graph construction and graph interpretation.

In the case of construction, scaling ability is an essential prerequisite for a proper transformation from a table of values to the accurate motion graph and *vice versa*. Interpretation of kinematic graphs relies equally heavily on appropriate scaling of both the time axis and the axis representing the specific kinematic quantity. Put differently, the pointwise interpretation of motion graphs cannot be achieved successfully without paying particular attention to scaling skills. Specific aspects which have to be borne in mind when considering scaling include the following:

- Scales need not be the same on the two axes.

- An ordered pair such as (2;10) is always written in the order: (independent variable; dependent variable), which in kinematics implies that time is always the first of the ordered pairs.

- Choose a scale such that an optimum amount of the available area is used.

- The symbol and unit of the relevant kinematic quantity must be indicated on both axes.

- In the case of accurate or semi-accurate graphs each interval must be indicated by the appropriate numerical value.

- Different scales can give graphs representing the same motion very different appearances.

2.6.3 The situation

Leinhardt *et al.* (1990:20) distinguish between the context in which a graphical representation is described and the setting for the instruction. The context of a representation refers to the degree to which familiar real-life aspects are used to make graphs more acceptable to students.

> *The studies that include contextualised situations are often based on the assumption that it is easier for students to deal with problems that build on familiar situations than to deal with abstract situations.* (Leinhardt *et al.* 1990:20)

Two common contextualised situations refer to situations involving *travel* such as a bicycle travelling over a hill, a racing car going around a track and distance-time graphs; or situations involving *growth* such as the growth of bacteria at different temperatures and the average height of boys at different ages. It should be clear that during the construction or interpretation of *kinematic* graphs the context of *travel* is frequently used. Falling objects and objects projected upwards also offer frequently used contexts.

Setting refers to the location where the subject material is presented. It refers to the background which an instructor chooses for the presentation of the subject material and as such is a broader perspective on the subject material than context. The two major settings for graphs would be the purely mathematical approach or a scientific approach. The former has as its aim the understanding of more formal or abstract aspects of function and graphs, while the scientific approach, in the words of Leinhardt *et al (1990:20):*

> *...serve as representations of real observations and analytical tools for detecting underlying patterns, which in turn inform the observer and the learner about the phenomena under investigation.*

It is possible that a mathematical approach to graphs can incorporate real-world applications for the purpose of motivation and towards assisting a deeper understanding of the *mathematical* processes. In the case of kinematic graphs, however, the emphasis appears to be directed towards the interaction between a physical phenomenon and its graphical representation.

2.6.4 Variables

The nature of the data points used to construct accurate graphs plays an important role in any subsequent analysis or interpretation of the graph. The two main categories of data points can be described as either <u>abstract</u> or <u>concrete</u>. It should be self-evident that the variables used in a real-life situation such as kinematics are usually *concrete* as they can be associated with everyday experiences.

Abstract variables can be either *discrete* (eg. the natural numbers) or *continuous* (eg. the real numbers). Contextualised variables on the other hand are usually continuous in nature and often include concepts such as time, distance, speed, temperature, weight and age. Leinhardt *et al* .(1990:23) point out that *speed* is a unique variable, because it can be one of the two explicit variables involved in the *situation*, or it can be inferred indirectly from the slope of a distance-time graph.

Variables that are not time dependent are rare, as most contextualised tasks include a time variable or a time-dependent variable. Leinhardt *et al (1990:23)* make the point that time can be considered a special type of variable as it can be either discrete as in *age of person* or *year* or continuous as in a velocity-time graph of an observed motion.

2.6.5 Focus

The focus of an action involving graphing is described as being either internal to the coordinate system, or on the coordinate system itself. A focus internal to the system of axes focuses on the actual graph and concerns itself with questions of global significance, such as whether an acceleration has a constant positive value for the direction of motion. Sketch graphs often demand an internal focus because the axes are usually left unscaled.

Actions such as accurate graph construction, which usually involve quantitative pointwise interpretation, as well as scaling, demand a focus on the coordinate axes. Clearly, however, the focus during any graphing activity will fluctuate between the actual graph and the axes, depending on the task at hand.

2.6.6 Conclusion

Leinhardt *et al's.*(1990) attempt at classifying graphing tasks in the context of mathematical functions is based on research undertaken by many investigators. This classification was taken as an example to show how an investigation into graphing activities in general can be used to analyse and classify activities which are peculiar to the construction and interpretation of kinematic graphs.

2.7 COGNITIVE PROCESSES INVOLVED IN THE TRANSLATION BETWEEN REPRESENTATIONS

2.7.1 General

Students must be able to translate between the various representations of kinematic situations described in section 2.4. They have to be able to observe and analyse a real physical situation or comprehend a verbally described physical situation so that they are able to describe it in terms of one (or more) of the three more abstract representations, namely algebraic, ordered pairs or graphic. These translations and the accompanying cognitive processes play a crucial role in the proper understanding of kinematics.

This study will focus on translations involving graphs. These translations will be categorised according to the model proposed by de Jager (1990) which was based on the work of Janvier (1978). The classification of translation categories is as follows:

A	:	Physical situation[1]	→	Graph
B	:	Graph	→	Physical Situation
C	:	Algebraic formula	→	Graph
D	:	Graph	→	Algebraic formula
E	:	Graph	→	Related kinematic graph (a transformation).

[1] In what follows, "physical situation" will be understood to mean "verbal description of the physical situation", unless otherwise stated.

This study focuses on translations involving graphs. Translations between a physical situation, algebraic formulas and ordered pairs, although relevant and important, will therefore receive only incidental mention.

2.7.2 Analysis of translations involving graphs

2.7.2.1 Translation A : Physical situation → Graph

One of the main aims of teaching kinematics is to encourage students to make explicit connections between concepts, their graphical representations and the real world. The cognitive processes involved in constructing graphs of actual motions are fundamental to making this connection correctly, with algebraic processes often playing an important part. Brasell and Rowe (1993:66) describe the role of algebra in simplifying the translation physical event → graph in terms of the difference between describing an *event* and a *variable*:

> *When the variable is described, students need only translate the verbal description to a graphic representation (translation only); however, when the event is described, students must first extract relevant information about the variable. The task is more complex, requiring both interpretation of the verbal description and translation from one system of representation to another (interpretation + translation).*

The following example illustrates how, for the same physical situation, the *event → graph* translation often makes greater cognitive demands on a student than the translation: *description of variable → graph*.

Example (a) *(description of variable → graph)*

An object, starting with velocity equal to zero, increases its velocity uniformly for 10 seconds, reaching a final velocity of 20 $m.s^{-1}$. The velocity now stays constant at 20 $m.s^{-1}$ for a further 5 seconds. Draw a neat graph of *v vs t*.

Students who possess a basic understanding of kinematic graphs should have little difficulty in drawing the appropriate (semi-accurate) graph shown in figure 2.10.

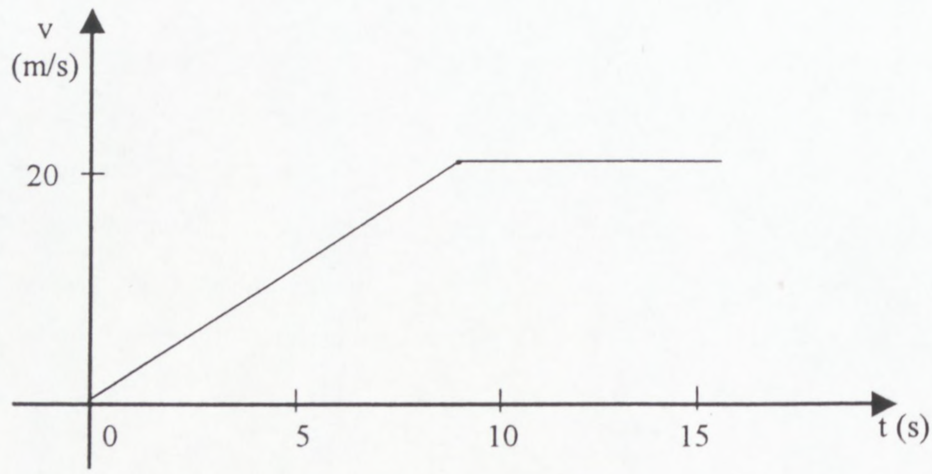


FIGURE 2.10

Example (b) *(event → graph)*

A ball is released from rest in the absence of friction on the ramp shown in figure 2.11 alongside. Draw a *v vs t* graph to illustrate the motion of the ball.

FIGURE 2.11

In this case the *event* (in the form of a verbal description and diagram) must first be interpreted as an example of uniformly accelerated motion from rest, followed by a motion with constant velocity. Students have to do this by interpreting the word *released* as motion from *rest* or motion with initial velocity equal to zero, while the sloping ramp in the absence of friction must be linked with a constant acceleration. Finally, the horizontal section has to be interpreted as causing a constant velocity.

The final sketch graph has the same shape as that of figure 2.10 but, as Brasell and Rowe (1993:66) point out, this task is far more complex, requiring both interpretation and translation.

The path from the physical situation to the successful completion of the graph can follow a number of different routes. Each of these possible routes will now be briefly described and illustrated with an appropriate example.

(a) Observation of physical situation → graph

In this case the student simply draws an appropriate motion graph by carefully observing a specific motion. He obtains sufficient information by using his visual discrimination facility to be able to draw a sketch graph which acts as a global representation of the entire motion.

Example:

A ball is dropped from a certain height in the absence of friction. Draw a displacement-time sketch graph of the motion of the ball.

The student uses his physics knowledge as a basis for the following series of cognitive steps:

The ball experiences a constant gravitational force

⇒ ball experiences uniform acceleration (ignoring friction)

⇒ velocity increases by a fixed amount each second

⇒ displacement increases by progressively larger amounts each second.

A reference direction and reference point are now selected leading to the obvious deduction:

⇒ displacement from reference point in a positive direction is positive.

The final graph is now drawn as in figure 2.12 where the *point of release* was taken as reference point and *down* was taken as the positive direction.

The sequence required here can thus be summarised as: Event → description of variable → graph.

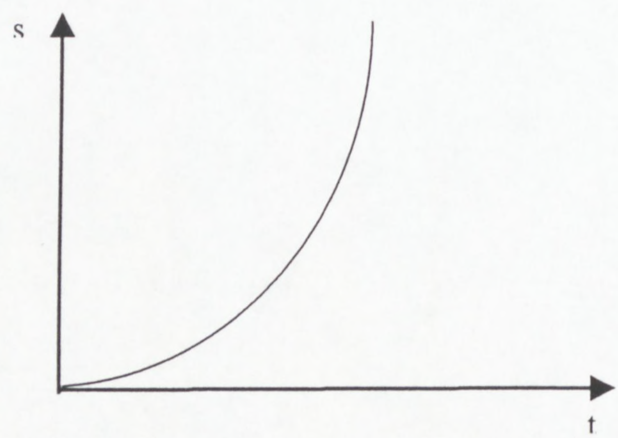
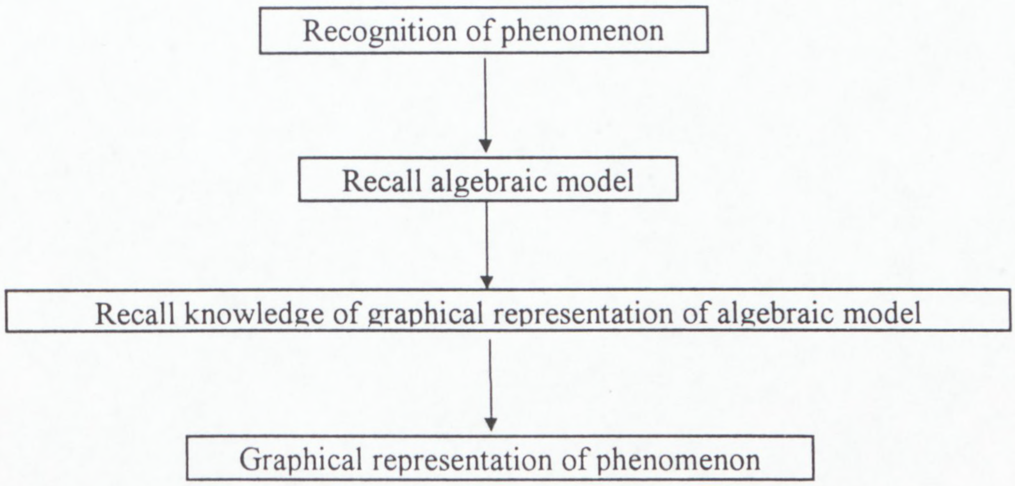


FIGURE 2.12

(b) Recalling the algebraic graph

A student who is familiar with the equations of motion may recall his knowledge about algebra and Cartesian graphs to assist him in arriving at the appropriate kinematic graph for a physical situation. A possible cognitive route could take the following path:



The previous example of a falling ball can be used to explain the various actions described above: The student observes the ball and realises (from his knowledge of kinematics) that the displacement is proportional to the square of the time. He then recalls the algebraic model for proportionality which in this case would state:

$$y \propto x^2$$

$$\Rightarrow \quad y = kx^2$$

He recalls that the graphical representation of this function is represented by a parabola such as shown in figure 2.13.

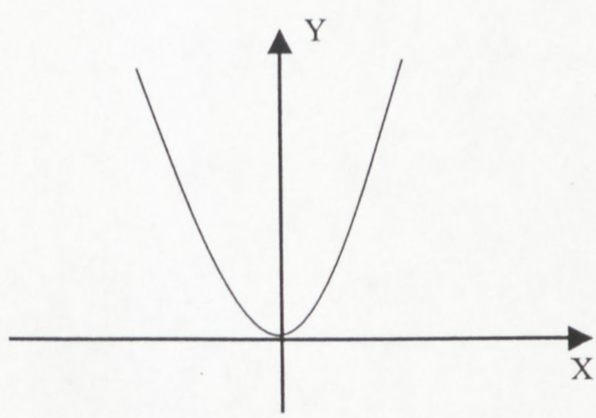


FIGURE 2.13

This finally leads to the correct kinematic graph, provided he realises that negative values on the horizontal axis have no meaning as *t* cannot be negative:

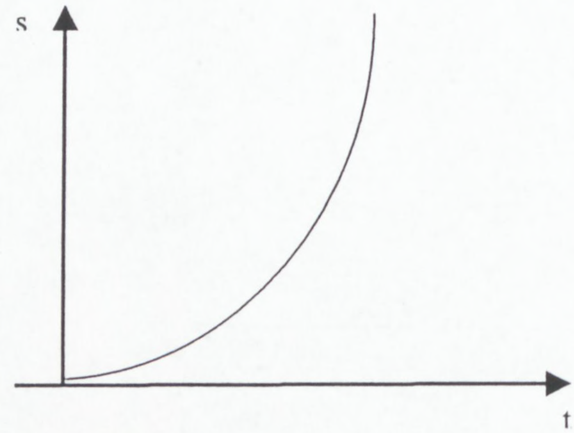


FIGURE 2.14

Example:

The *velocity-time* graph which represents the motion of the falling ball mentioned in (a) can be constructed after a similar cognitive path has been followed. A possible sequence could be:

⇒ student observes motion of the ball;

⇒ notices that velocity increases proportionally to time;

⇒ recalls the mathematical expression for proportionality: $y = kx$;

⇒ recalls the graphical representation of the function $y = kx$ as a graph with gradient k, passing through (0;0);

⇒ chooses the appropriate reference level and sign convention;

⇒ draws the kinematic equivalent of the algebraic graph as indicated in figure 2.15 below.

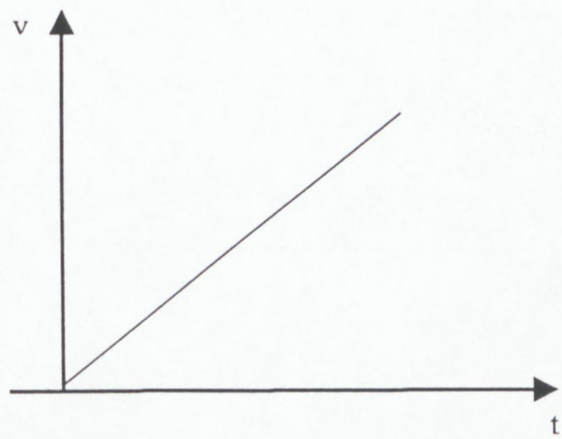
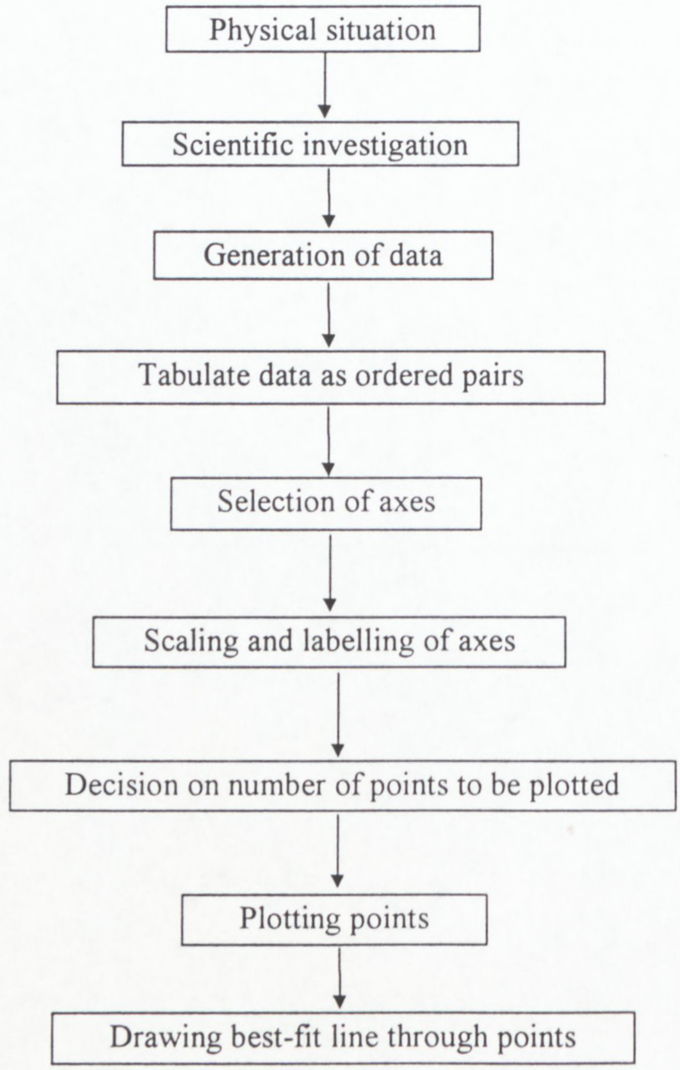


FIGURE 2.15

(c) From physical event to data table to graph

A third possibility is that actual measurements can be made during the motion. This would lead to a data table which in turn provides information for the drawing of an accurate graph. The path or route of the steps can be presented in the form of a diagram:



The actual generation of data can take on many forms, a few of which are listed below:

- stroboscopic multiple exposure photographs of a moving object;
- dynamics trolley and ticker-timer making imprints on a paper tape pulled by the trolley;
- electronic timing devices linked to a microcomputer;

- stopwatch and flags positioned at predetermined positions from the start of a motion.

It is often advisable to present students with data that is not clouded by experimental error or frictional affects. This will enable them to draw an idealised accurate kinematic graph which can in turn be used to generate idealised additional information about the motion (See also Jordaan, 1992:104-108).

The example of a dynamics trolley and ticker-timer generating information for the drawing of a displacement-time graph is cited: Ticker-timer experiments can be performed by students themselves in the laboratory. The generation of dots on a paper tape gives immediate information about the motion of the trolley. Distances travelled can be measured directly on the tape and a specific number of spaces between ticks always represent the same time interval. A single example is given to illustrate the method.

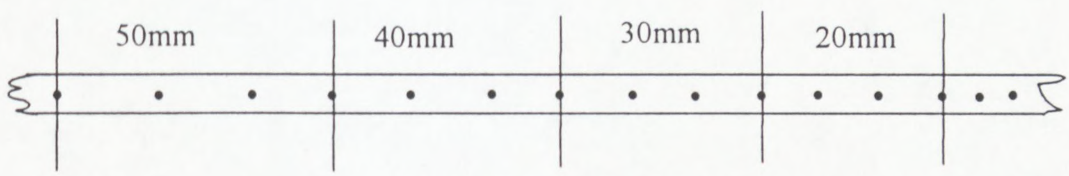


FIGURE 2.16

A ticker-tape which moves from left to right is marked off in strips which are 3 periods apart as indicated in figure 2.16. Students are required to draw a displacement-time graph to illustrate this motion.

The data generated on the paper strip is converted into a table, after which the graph is plotted from these tabulated values. Once again, much can be gained from providing students with data that fits a mathematical description of the motion exactly, thus generating a *perfect* graph such as illustrated in figure 2.17.

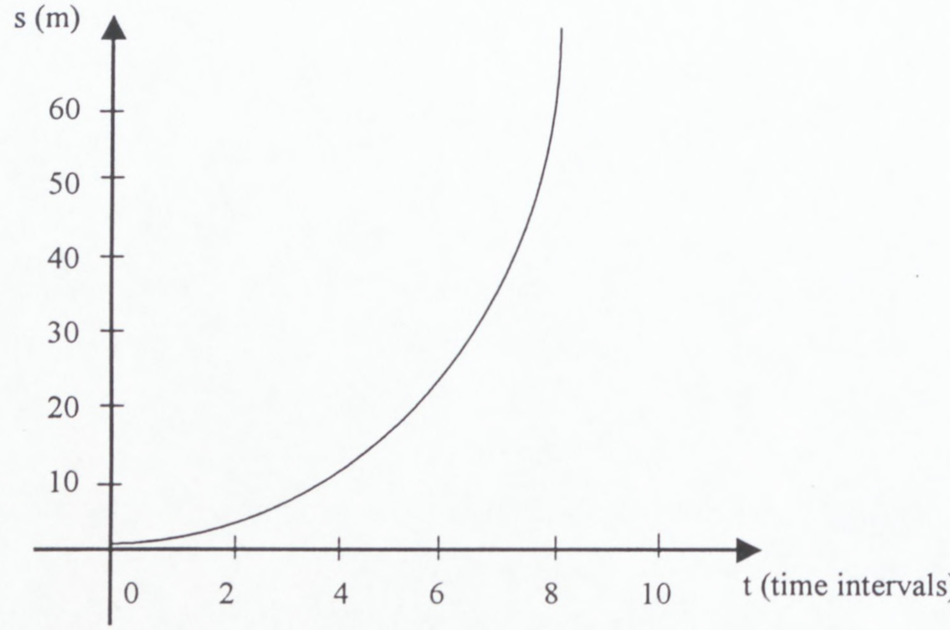


FIGURE 2.17

Students must understand that the Cartesian graph represents the given data as a visual representation of the relationship between two continuous variables. They have to realise that a displacement is associated with any real value of t within the domain prescribed by the given physical situation. Furthermore, students must know how to intrapolate and extrapolate (as processes which yield kinematic values for t-values not explicitly given by the data). The principle of continuity and the ability to intra- and extrapolate are all more clearly illustrated by a graphical representation than a table or a verbal description.
Mokros & Tinkler (1987:369) put it as follows:

> *It (graph) is a symbol system particularly well adapted to conveying information about experimental measurements. Graphing constitutes a key symbol system in science because it summarises the covariance of two or three variables over a large number of measurements.*

Sawyer (1990) has this to say about the transition from a table to a graph:

> *Graphs help us by converting patterns in numbers to patterns in shape.* (1990:22).

Trends which are not always noticeable in the table often leap to the eye when represented graphically. Sawyer states further in this regard (provided the reader is familiar with the complex symbolism used):

> *This is the point of a graph; it delivers its message in matter of seconds* (1990:21).

Experiments with ticker timers recording the motion of a trolley illustrate how a scientific investigation yields data for a data table that is transformed into a graphical representation.

(d) Recognise phenomenon → sketch graph

During this translation the student spontaneously draws sketch graphs of already familiar phenomena. The ability to draw graphs spontaneously is obviously dependent on the state of his pre-knowledge in the particular field (in this case kinematics) and also on his experience with graphing in general.

The cognitive process is quite simple: The learner observes a phenomenon, recognises it as being equivalent to a previously studied situation and spontaneously recalls the sketch graph representing the observed motion.

Example

A student is asked to draw a sketch graph of velocity vs time for a ball which rolls up an incline with a given initial velocity, returning to the original position after rolling back down the incline as indicated in figure 2.18.

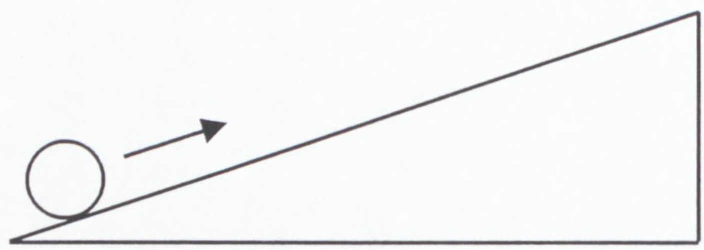


FIGURE 2.18

He recognises the physical situation as being similar to that of a ball being projected vertically and concludes that the required sketch graph must have a similar shape. He proceeds to draw the required graph spontaneously as illustrated below.

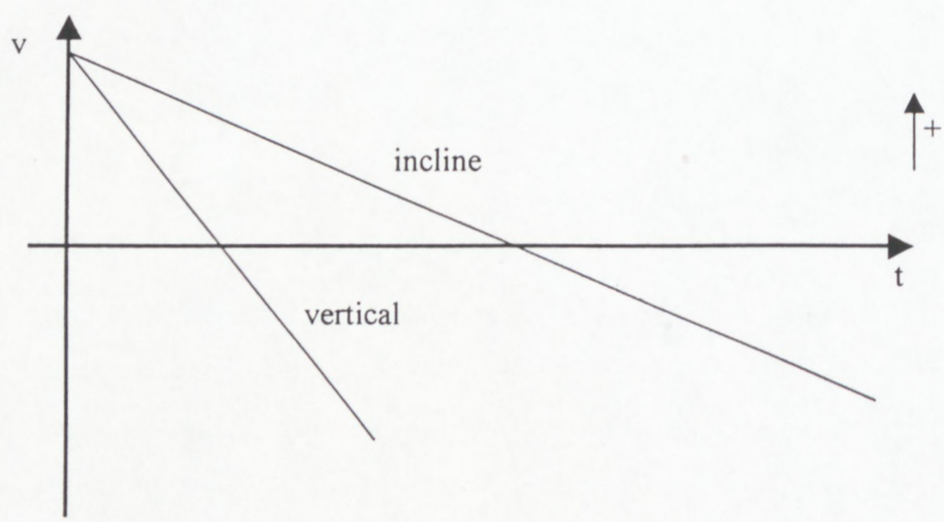


FIGURE 2.19

In this example the stūdents' knowledge of science plays an important role. He must realise that a ball rolling up an incline is equivalent to a vertical motion and then use his maths knowledge by recalling the shape of the *v vs t* graph for vertical motion. Finally, he has to allocate a smaller magnitude for the slope by recognising that in the case of the incline the acceleration has smaller absolute value. The *v vs t* graph for vertical motion and motion up an incline are compared in figure 2.19.

2.7.2.2 Translation B: Graph → Physical situation

The transformation from a graph to a physical situation can take on different forms. Leinhardt *et al.* (1990) differentiate between interpretation as a local process where there is often a point by point attention to the graph and a more global interpretation where the emphasis is on trend detection. In the context of kinematics this classification can be described as a gradual progression from reading of specific coordinates of points to obtaining information about kinematic concepts and finally to a description of the motion as a whole. Wainer (1992) proposed three ways in which an instructor can measure student proficiency in understanding the graphical representation of quantitative data. He measures it in terms of the kinds of questions a student should be able to answer about the graphs, namely:

(i) elementary questions involving data extraction, for example: *determine the velocity after 6 seconds;*

(ii) intermediate level questions involving trends seen in part of the data, for example: *What can you say about the displacement of the car during the first 5 seconds?*

(iii) overall level questions involving the general structure of the data, for example: *use the graph to describe the motion of the car.*

Wainer's classification of questions can, in general terms, be linked to accurate, semi-accurate and sketch graphs respectively, with sketch graphs requiring the answering of questions of a more *qualitative* nature (general trends of the motion). Accurate and semi-accurate graphs, however, focus more on actual data extraction, in other words they require students to give certain *quantitative* answers.

Examples of actual data extraction included aspects such as:

- coordinates (for example intersection of graphs; turning points; intercepts with axes);
- gradients (leading to determination of velocity from *s vs t* graphs and acceleration from *v vs t* graphs);
- areas (leading to determination of change in displacement in the case of *v vs t* graphs and change in velocity in the case of *a vs t* graphs).

The translation from a kinematic graph to the physical situation is next described in terms of the following three possible student activities. These activities often overlap in the context of a specific example:

(a) the demonstration of actual motions;

(b) a verbal description of the motion of an object; and

(c) the determination of kinematic quantities such as displacement, velocity and acceleration.

(a) **Graph →demonstration of actual motions**

In this translation students have to produce motions that are depicted by a given graph. Rosenquist and McDermott (1987:413) proposed a dynamic conceptual approach to teaching kinematics where students are actively engaged in observing and analysing various motions. They state with respect to this translation:

By translating back and forth between a motion and its graphical representations, students can learn to relate the characteristics of an actual motion to the corresponding features of the different types of graphs that describe it.

They suggest that translation B can be illustrated by providing students with a steel ball and a set of aluminium tracks. The tracks must be arranged to produce the motion which is illustrated by a graph such as the one illustrated in figure 2.20:

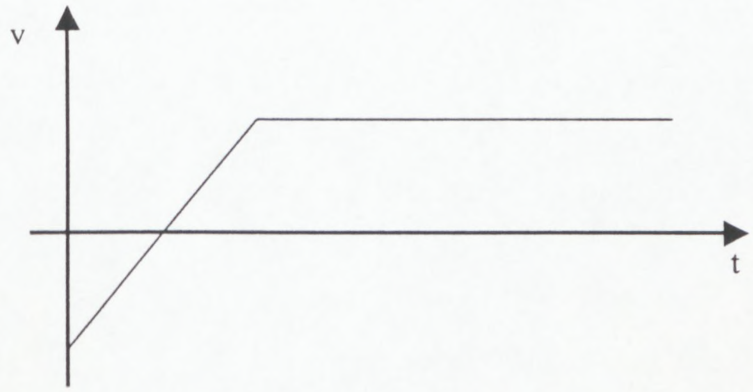


FIGURE 2.20

Before the motion can be produced, students must first extract the following information from the graph:

(i) the ball starts with some initial velocity;
(ii) it then slows down and stops momentarily;
(iii) it turns around and moves in the opposite direction (positive velocity) with increasing speed.
(iv) finally it moves with constant velocity which is larger in magnitude than the initial velocity.

The idea is that students arrange the tracks in such a way that a ball is rolled up an incline from a point part of the way up and then allowed to roll back down to the end of the incline and onto a level section of the track, as illustrated in the following sketch.

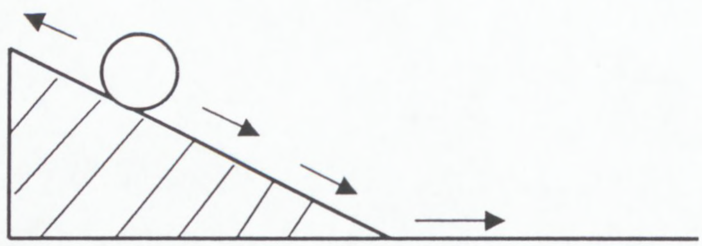


FIGURE 2.21

Students who do not recognise the instant when the velocity is zero or who do not consider the periods of negative and positive velocity will have difficulty with the required task of arranging the tracks in the correct orientation.

(b) **Graph → Verbal description of motion**

This translation involves extracting the relevant information from a graph in order to describe the motion of real objects or objects without context. To be successful at this translation the student must be able to choose the appropriate features of the graph to obtain information about the corresponding kinematic concepts. Woodward and Byrd (1984) report on a lesson plan in which eighth-grade students had to interpret graphs in different and challenging ways. They gave the students a graph (as illustrated in figure 2.22) and then asked them to make up a story to explain the graph.

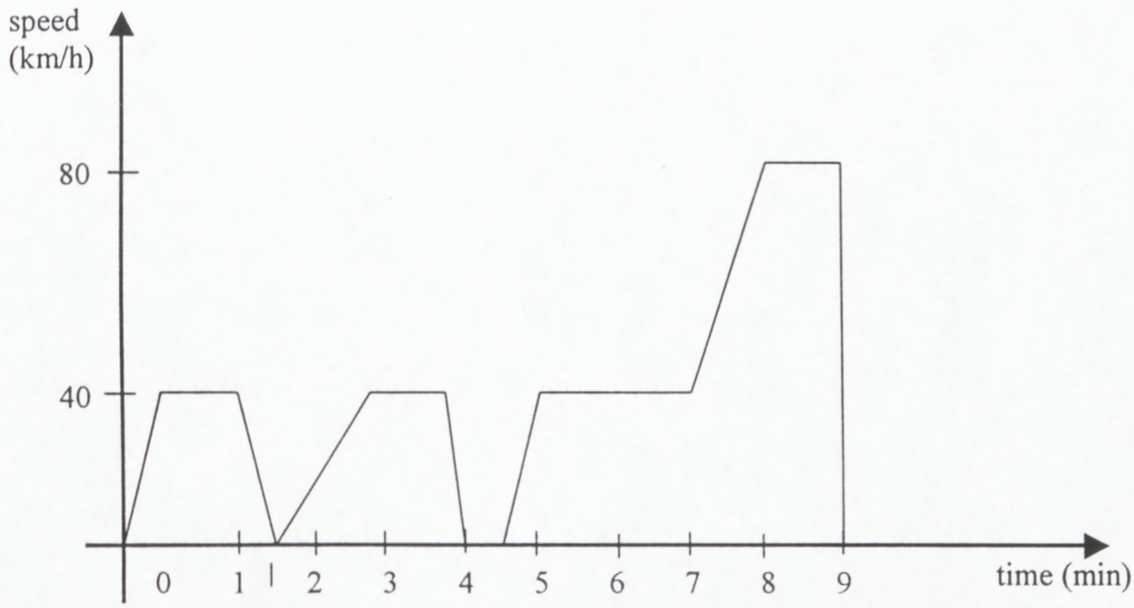


FIGURE 2.22

The students are told that the graph represents the speed at any time during a man's nine minute trip from home to work. They were prompted to explain specifically what could have happened between one and two minutes of his journey as well as between three and four minutes and between six and seven minutes. They also had to determine how far he lives from where he works.

After students had finished writing their stories they discussed what they had written. Woodward and Byrd (1984:34) conclude::

> *After the lesson was over we concluded that the students were much more perceptive about graphs. More important, however, the lesson required them to think and to write what they were thinking - this activity was really problem solving.*

Schuster (1983:331) reports on the kinematics and representations course run by the University of Natal. He includes the following example of how a graph could be used to ask students to answer questions about the motion that it represents:

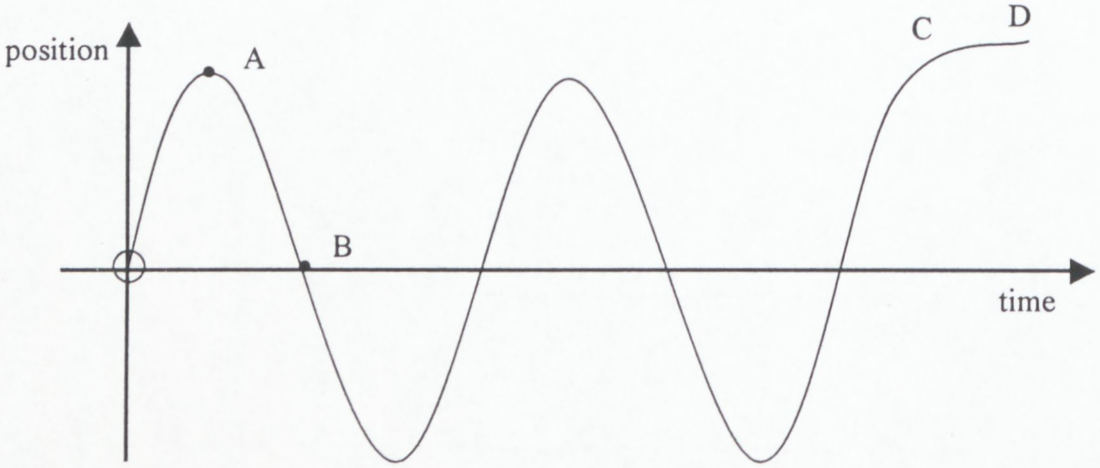


FIGURE 2.23

The motion of an object is represented by the graph in figure 2.23.

(a) Demonstrate the motion.

(b) Describe it in words.

(c) At what points/regions is the object

(i) momentarily at rest?
(ii) moving fastest?
(iii) furthest from the starting point?
(iv) moving backwards?

It has to be noted that this question requires both a pointwise or local interpretation, as well as a qualitative more global description of the motion.

To be able to make the required transformation in this instance it is necessary for the student to:

- recognise the (initially) alternating positive and negative values of the displacement;
- associate this with the periodic change in direction of the motion;
- notice that the change in position per unit time decreases steadily for section OA of the graph;
- associate this with a steadily decreasing velocity;
- recognise that during section AB the rate of change of position is increasing;
- note that this velocity is in the opposite direction because the slope is negative;
- realise that this implies a steadily increasing velocity (in the opposite direction);
- understand that when the graph intersects the time axis the body has arrived back at the reference point;
- understand that section CD is associated with a constant position value of zero slope;
- associate the latter with a zero velocity.

Schuster (1983:331-337) summarises these actions as follows:

We discuss the graphs from both the function and differential point of view, focusing attention on either values or changes. The latter involves ideas of stepwise increments, slope, straightness, curvature, limits and instantaneous and average values.

(c) Graph → Kinematic quantities

The translations graph →kinematic quantities, graph →verbal description of a motion and graph →graph are all related to each other. This is because the isolation and quantitative determination of fundamental concepts such as displacement, velocity and acceleration from the graphical representation of a motion is essential for the verbal description of the motion as well as for the graphical representation of a related kinematic quantity. A student who can determine the velocity at any given time from a position-time graph should have little difficulty describing the motion <u>and</u> have the information available to successfully draw the related velocity-time graph.

The table below summarises the features of various kinematic graphs which can be used to determine the indicated kinematic quantity. The equivalent operation in differential and integral calculus is also indicated.

Given Graph	Feature	Kinematic Quantity	Calculus
$\vec{s}$-t	difference in height	change in displacement	-
$\vec{s}$-t	gradient	velocity	$\frac{ds}{dt}$
$\vec{v}$ - t	difference in height	change in velocity	-
$\vec{v}$ - t	gradient	acceleration	$\frac{dv}{dt}$

Given Graph	Feature	Kinematic Quantity	Calculus
$\vec{v}$ - t	area under graph	change in displacement	$\int_{t_1}^{t_2} v(t)dt$
$\vec{a}$-t	difference in height	change in acceleration	-
$\vec{a}$-t	area under graph	change in velocity	$\int_{t_1}^{t_2} a(t)dt$
$\vec{v}$-t	area under graph	change in distance traversed	-

The following important comments have to be made with regard to the above summary table:

(i) Students must realise that the areas under the *v-t* graph and *a-t* graph give the <u>change</u> in displacement and the <u>change</u> in velocity respectively. Jones (1992:17) cites the following example to illustrate this:

Consider an object moving away from you at constant velocity of 2 $m.s^{-1}$ and that at the point of first observation (t = 0) it is already 4 m from you. The s vs t graph and v vs t graph appear below.

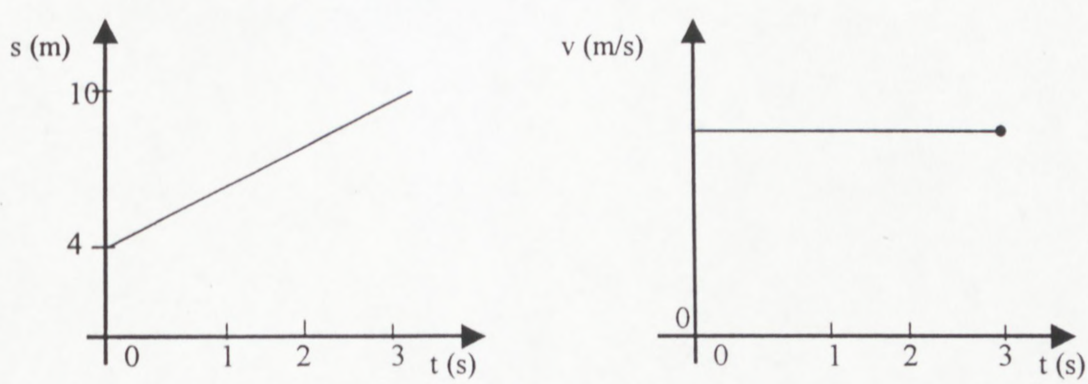


FIGURE 2.24

The area under the v-t graph is 6 m which represents the change in displacement from t = 0 (4 m) till t = 3s (10 m).

(ii) Students must make a clear distinction between a change in velocity and the rate of change of velocity. The crucial role played by time has to be emphasised here. The same argument holds for displacement where *difference in heights* on a *s-t* graph has a different meaning to *difference in height divided by time*.

(iii) The definitions of velocity, displacement and acceleration as described in section 2.3 are closely linked to the corresponding feature of the graph. For example a student must make the following cognitive connections with respect to velocity and the *s-t* graph:

velocity → Rate of change of displacement (Definition)

→ $\frac{\Delta s}{\Delta t}$ (in symbols)

→ vertical change over horizontal change in the *s-t* graph

→ gradient of the *s-t* graph

(iv) The association of a larger gradient with a bigger rate of change, as well as the idea of a variable gradient in the case of a curved graph is fundamental to an effective link between the graph and certain kinematic concepts such as velocity and acceleration.

The graph of *s vs t* for uniform accelerated motion is a parabola as indicated in figure 2.25

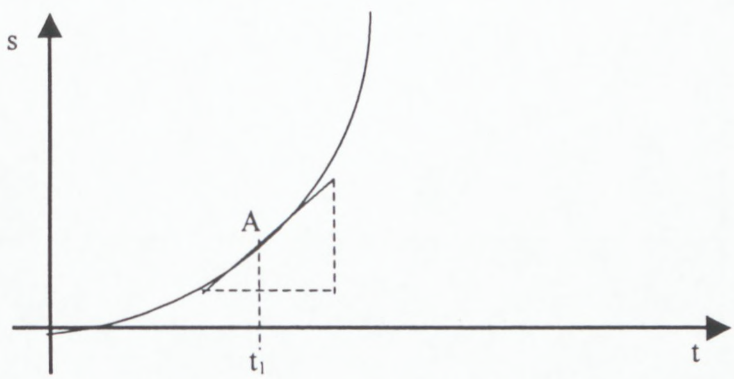


FIGURE 2.25

Students have to associate the changes in slope with changes in velocity. Furthermore the idea of an instantaneous velocity at time $t = t_1$, as being the gradient of the tangent line drawn to the curve at point A is important. The link with the idea of a limit in calculus should be obvious to the alert students.

(v) Students must realise that the unit for the area under the *v-t* graph is $m.s^{-1}$ x s which is m. This indicates the unit for displacement and hence provides an indicator that the area under a *v vs t* graph has something to do with displacement. However, many students might find this confusing as they have come to associate the concept of area with the unit : m^2. Similarly, the unit for the area under an *a-t* graph is $m.s^{-2}$ x s, which is equal to $m.s^{-1}$ which again is the correct unit for change in velocity.

(vi) Many kinematic quantities are vectors and the directional aspect is indicated by a negative gradient or a negative area. The association: decreasing function $\rightarrow$ negative gradient $\rightarrow$ negative velocity has to be made by the student. Similarly an area below the time axis (such as B) should be regarded as negative, so that it has to be associated with a velocity or displacement which is opposite to the direction which was initially chosen as positive. The following example illustrates this:

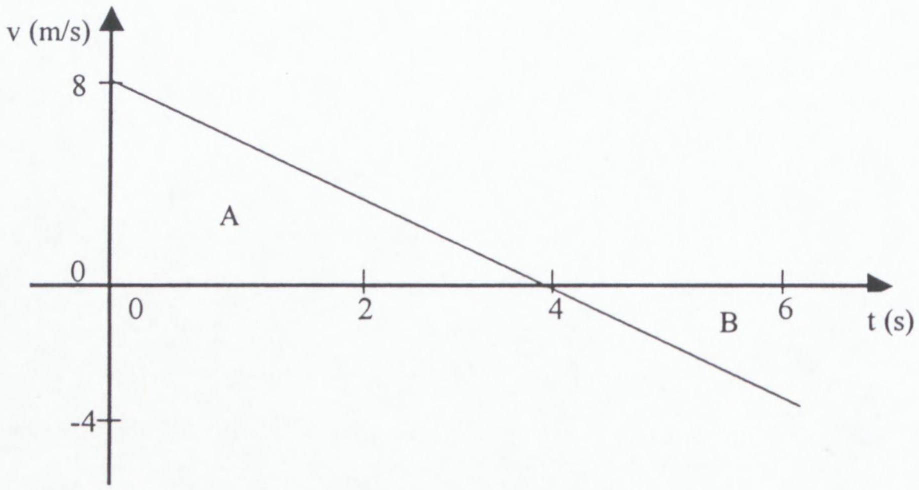


FIGURE 2.26

The graph in figure 2.26 describes the motion of a car travelling for 6 s with an initial velocity of 8 $m.s^{-1}$.

(a) Determine the acceleration of the car.

(b) Determine the total change in displacement after 6 s.

(c) Determine the total change in distance after 6 s.

(d) Describe the motion of the car at the instant when t = 4s. What is the acceleration at this point?

Comment on solutions:

(a) The student should associate the negative slope with a negative acceleration which describes the decreasing velocity, or the negative slope implies that the object experiences an acceleration in a direction opposite to the *initial* direction of motion.

(b) The total displacement must be seen as the sum of a positive area (A) above the time axis and a negative area (B) below the time axis. Students must connect the negative velocity after 4 seconds with motion in the opposite direction and hence allocate a negative sign to the area (or displacement) between seconds 4 and 6. It should be noted that although the displacement is becoming smaller, it is still positive and it (the displacement) would only become negative once area of B > area of A.

(c) The definition of distance as the total length of the path of a motion is the key to the realisation that all areas are added in this instance.

(d) Students must realise that the crossing of the time axis by a *v vs t* graph indicates an instant when the velocity is zero. However, the gradient at t = 4 s is still -2 which implies that the acceleration at t = 4 s is -2 $m.s^{-2}$. The idea of a velocity which is zero at the same instant when the acceleration is non-zero is important. A graphical representation is probably the most effective way to show this.

(vii) Early tertiary students taking a course in physics are usually well acquainted with differential calculus, as they have been exposed to this topic at school. Integral calculus on the other hand is part of most mathematics courses at universities and technikons. It can therefore be reasonably expected of these students to view the gradient of a line or a tangent to a curve as the derivative of the function with respect to time. They should make a similar connection between area under a graph and the definite integral of the given function.

The following is an example to demonstrate the link between integral calculus and the calculation of areas to obtain a change in displacement:

FIGURE 2.27

The graph shows an object with an initial velocity of 4 $m.s^{-1}$ accelerating at 0,5 $m.s^{-2}$ for 6 seconds. Students can be asked to compare the change in displacement obtained by graphical methods with that calculated by means of a definite integral. The logical reasoning that is required to obtain the correct answer is explained below.

Integration Method

We know that $v = u + at$.

Here we have that $u = 4\ m.s^{-1}$ and $a = 0{,}5\ m.s^{-2}$.

Now $s = \int_a^b v(t)dt$

But $v(t) = 4 + 0{,}5t$.

This means that $s = \int_0^6 (4+0{,}5t)dt$ because we are considering the time from 0 to 6 s.

Therefore $s = 4t + 0{,}5\frac{t^2}{2}\Big|_0^6$

$= 4 \times 6 + 0{,}5 \times \frac{(6)^2}{2}$

$= 33$ m

Note that the equation of the graph could also have been derived by considering the general straight line equation ($y = mx + c$).

Area Method

The area under a *v-t* graph gives change in displacement.

The area has the shape of a trapezium and the formula for the area of a trapezium is:

A = ½ (sum of parallel sides) x perp. height

= ½ (7 + 4) x 6

= 33 square units

Therefore the change in displacement is 33 m.

2.7.2.3 Translation C : Algebraic formula → Graph

Algebraic formulas and graphs are both abstract representations of real events. Once the formula for a specific motion is known it is possible to represent it graphically by recalling the equivalent graph that was encountered in the mathematics curriculum. The other alternative is to calculate a set of numerical values which belong to the solution set of the equations and use these coordinates to display the kinematic situation graphically. These two alternatives are illustrated diagramatically below.

i) Physical situation → recall appropriate kinematic formula → recall equivalent mathematical formula → use mathematical techniques to locate key coordinates of kinematic formula → draw sketch graph or semi -accurate graph.

ii) Physical situation → recall appropriate kinematic formula → [substitute numerical values → generate ordered data pairs] → draw graph (sketch, semi-accurate or accurate).

It should be noted that the step indicated by square brackets in the second case involves an intermediate stage of tabulating.

In the examples that follow it will be shown how the study of the straight line graph and the parabola in the mathematics curriculum can be integrated with the study of the graphical representations of kinematic formulas.

Example 1

A cricket ball is projected upward at an initial velocity of 4 $m.s^{-1}$.

(i) Write down an expression for s (the displacement) in terms of t (the time). (Take the upward direction as positive and earth as reference point).

(ii) Hence draw a sketch graph of displacement versus time.

Solution

$$s = ut + \tfrac{1}{2}gt^2$$

$$= 4t + \tfrac{1}{2}x(-10)t^2$$

$$\therefore \quad s = 4t - 5t^2$$

Having made the translation from a verbal description of a real situation to an abstract formula, the student could now follow one of two possible routes to construct the graph. He could substitute a sufficient number of t-values, calculate their corresponding s values, represent these in a table and draw an accurate graph. Alternatively, the cognitive connection between the kinematic formula $s = 4t - 5t^2$ and the formula for a parabola ($y = 4x - 5x^2$) can be made.

The following figures indicate how these two graphs are drawn in the context of (a) the science and (b) the mathematics syllabus respectively.

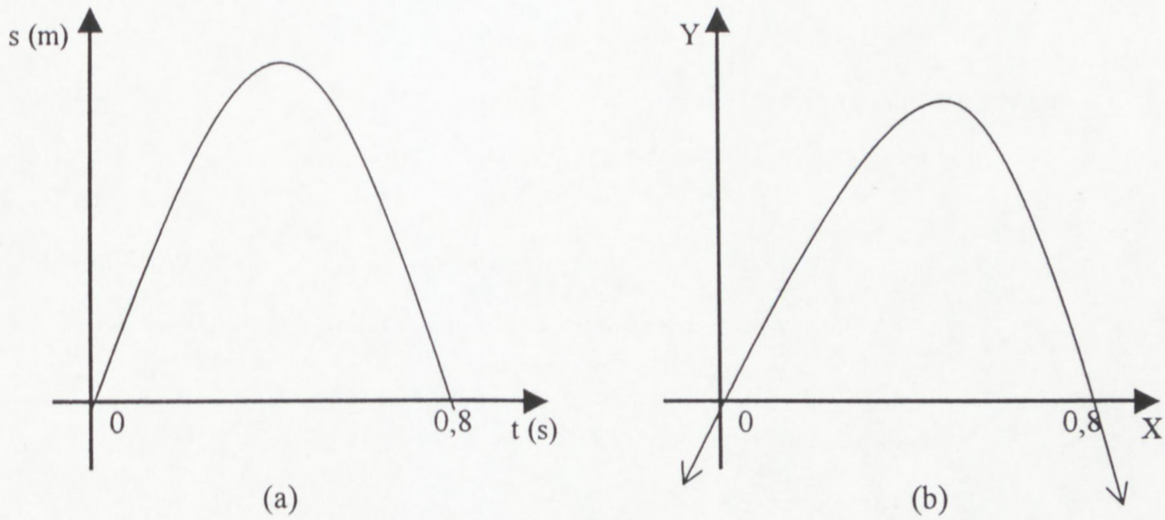


FIGURE 2.28

The following points must be noted with respect to the two graphs:

(i) Students must realise that the symbolic use of "x" and "y" for the independent and dependent variable in mathematics is purely arbitrary and that it is replaced by "t" and "s" in the kinematics graph.

(ii) The graph of *s vs t* does not (unlike the mathematical equivalent) include any points for which $t < 0$.

(iii) It is possible for the *s-t* graph to continue for values for which $s < 0$. Students must understand this as a direct result of the vector nature of displacement and more specifically the choice of reference point.

(iv) The maximum height of the ball can be calculated by substituting t = 0,4 into the expression:

$$s_{max} = 4(0,4) - 5(0,4)^2$$

$$= 0,8\ m$$

This method is in accordance with the standard procedure for obtaining the maximum height of a parabola in mathematics.

(v) A number of other mathematical formulas, relating to maximum points and intercepts with the horizontal axis, could also be employed.

(vi) s can not be negative since the ball is stopped by the earth/thrower.

Example 2

A train with initial velocity of 20 m.s^{-1} east accelerates uniformly for 6 seconds at 4 m.s^{-2} east.

(i) Write down an expression for v in terms of u, a and t.

(ii) Draw a sketch graph of v versus t.

(iii) What does the gradient of your graph represent? Explain your answer.

Solution

Considering east as positive:

(i) $v = u + at$

(ii)

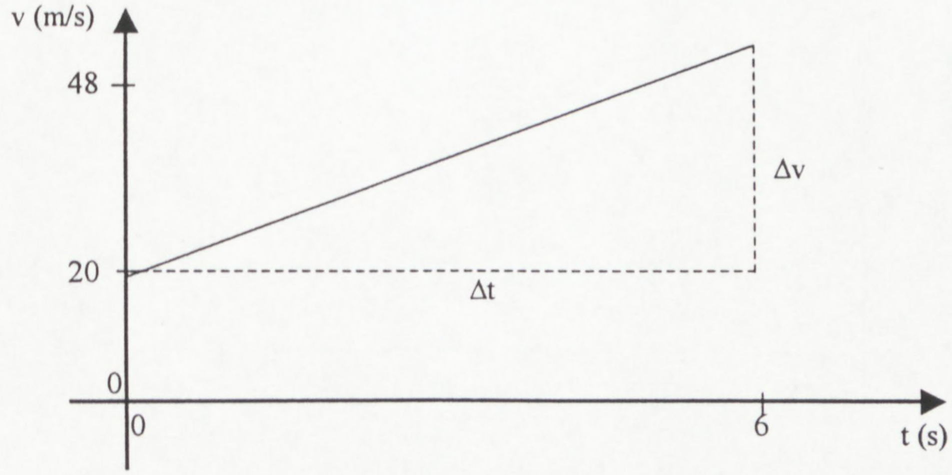


FIGURE 2.29

$v = u + at$

$= 20 + 4t$..........................(1)

$v_6 = 20 + 4(6)$

$= 44\ m.s^{-1}$

Alternatively, the student may make use of the gradient concept to enable him to draw the graph. The following steps are necessary for this:

the given situation is described by v = u + at

⇒ substituting initial conditions: v = 20 + 4t

⇒ this is recognised as being represented by a straight line with gradient 4 and y intercept = 20

⇒ the graph is drawn.

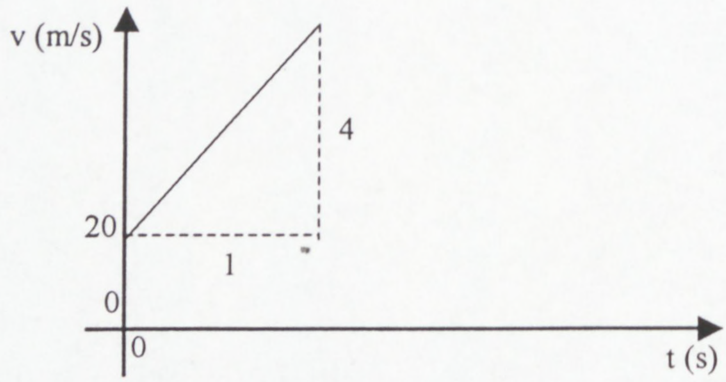


FIGURE 2.30

(iii) The gradient represents acceleration as it gives the change in velocity over a certain time interval.

The corresponding mathematical expression for a straight line is covered as early as Grade 9:

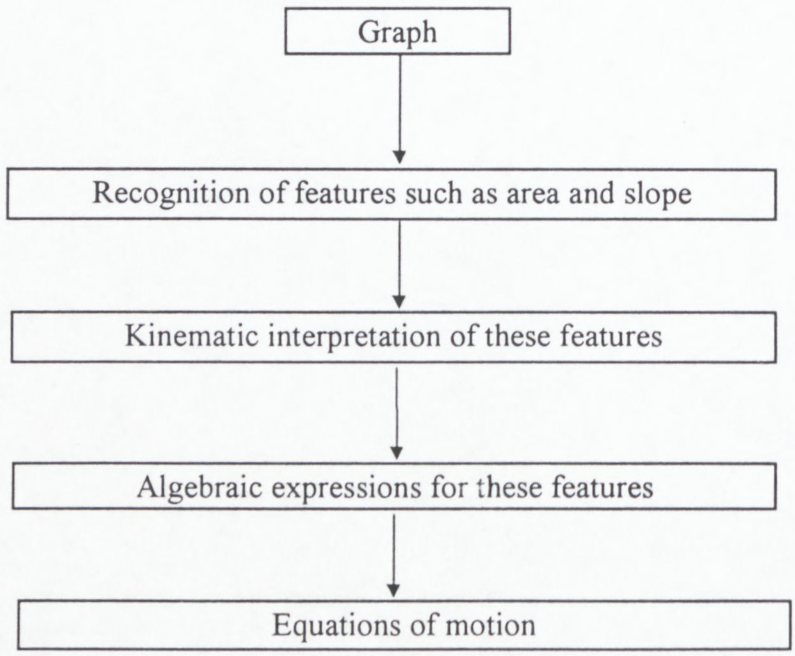
$y = mx + c$

$y = 4x + 20$..........................(2)

This provides an excellent opportunity for integrating the concept of gradient in mathematics with the concept of rate of change of velocity ($\frac{\Delta s}{\Delta t}$) as shown in the semi-accurate sketch graph above. To understand this, students must understand the equivalence of equations (1) and (2).

2.7.2.4 Translation D: Graph → Algebraic formula

In this section it will be shown how the inverse translation of C can be used to yield the algebraic equations which describe various kinematic quantities as a function of time. The cognitive processes involved in this translation are represented schematically below.



As an illustration of the above process the following two kinematic formulas for rectilinear motion will be deduced from a graphical representation:

$\Delta s = ut + ½at^2$ and

$v^2 = u^2 + 2a\Delta s$, where Δs represents a change in displacement.

By definition $a = \dfrac{\Delta v}{\Delta t} = \dfrac{v-u}{t}$

$\therefore\ v = u + at$..............................(1)

Now, using the fact that change in displacement is represented by the area under a velocity-time graph, we draw the graph showing an object with initial velocity *u*, final velocity *v* and added velocity *at*.

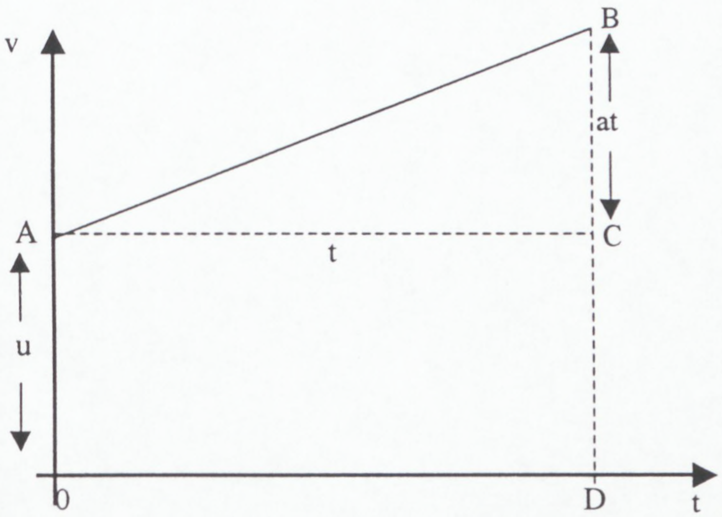


FIGURE 2.31

From the graph: Δs = Area of rectangle OACD + Area of triangle ABC

$$= ut + ½t.at$$

$$= ut + ½at^2$$

Note that $½at^2$ represents the added displacement due to acceleration.

Also, Displacement = Area of trapezium AODB

$$= \frac{1}{2}(\text{sum of parallel sides}) \text{ x height}$$

$\therefore$

$$\Delta s = \frac{1}{2}(u + v) x t$$

$$\Delta s = \frac{u+v}{2} x t \text{..................................(2)}$$

But from 1:

$$t = \frac{v - u}{a}$$

Now substitute into 2:

$$\Delta s = \frac{u + v}{2} \times \frac{v - u}{a}$$

$$= \frac{v^2 - u^2}{2a}$$

$$v^2 = u^2 + 2a(\Delta s)$$

These formulas are widely used in many different examples and it would be unwise to present them to students without proof.

Students must realise that the symbol for change in displacement (Δs) should be used here. The reason is that the object could have a certain displacement at the time that t is arbitrarily taken as 0 seconds.

Nicolas Oresme (1323-1382) is credited with producing the first written record of a graph.

Clement (1989:84) describes his method in some detail. Around 1360, Oresme used a primitive speed-time graph and showed that the area between any two vertical lines represents the distance travelled in that interval (See figure 2.32).

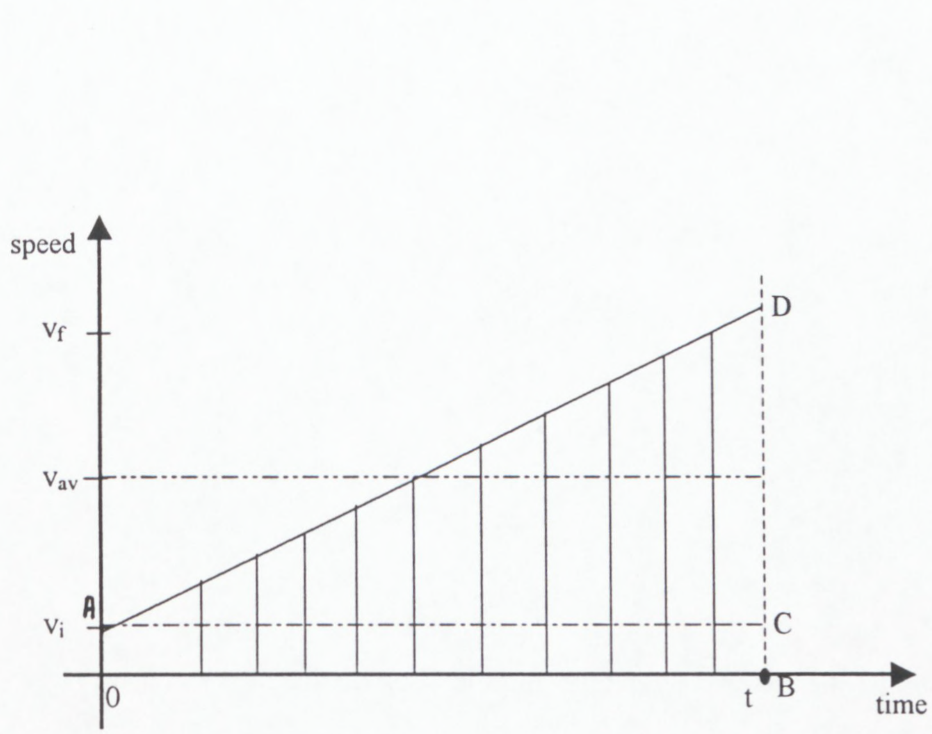


FIGURE 2.32

Oresme was credited with having said: *Everything measurable can be represented by a line* (Clement 1989:84).

Clement goes on to show how the historical development of graphing can be used to teach students about graphing:

> *This suggests that such vertical line graphs are a fairly natural and intuitive symbolisation device and that they would be a useful starting point in training students to draw graphs* (1989:84).

The derivation of an important kinematic equation using Oresme's diagram with modern algebraic notation is another example of how a graph can assist the student to visualise how key aspects of a motion are used to obtain an algebraic formula:

The total area under Oresme's graph can be written as

Area rectangle AOBC + Area triangle DAC

$= \quad v_i t + \frac{1}{2}(v_f - v_i)t$ (where v_i and v_f are the initial and final values for the velocity)

$= \quad (v_i - \frac{1}{2} v_i + \frac{1}{2} v_f)t$

$= \quad \frac{1}{2}(v_i + v_f)t$

But the total area under the graph is the magnitude of the change in displacement experienced during the uniform accelerated motion. This yields the Mean-Speed Theorem:

$$s = \frac{1}{2}(v_i + v_f)t$$

Students must realise that this equation is equivalent to the simpler form:

$s = v_{av}t$ with $v_{av} = \frac{1}{2}(v_i + v_f)$.

Oresme also stressed that it is important to see that the area under the v_{av} horizontal line is equal to the area under the sloping line.

Another route for the translation under discussion in this paragraph which warrants some discussion is the translation: *coordinates on graph → algebraic formula*

On page 23 it was described how an algebraic expression for v in terms of t is derived from two coordinates on the graph of a linear relation. A similar procedure may be followed in the case of a *quadratic* relationship where the graph is a parabola or part of a parabola:

Assume a student is confronted with the following *s vs t* graph with coordinates of the turning point and intercept with the time axis as indicated in figure 2.33.

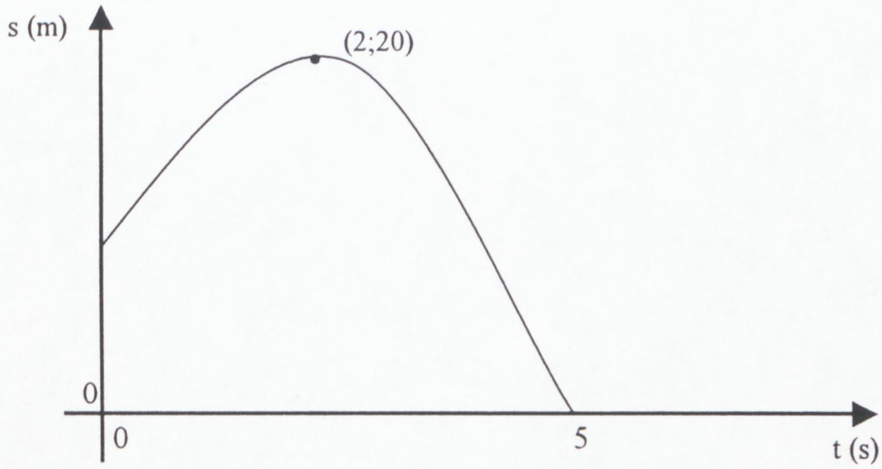


FIGURE 2.33

He is required to derive an algebraic expression for s as a function of t. Now, using a procedure which he has learnt in the mathematics class, he writes:

$s = a(t - p)^2 + q$ which is equivalent to the more familiar mathematical equation:
$y = a(x - p)^2 + q$ where (p;q) are the coordinates of the turning point in both cases.

From the given graph the student substitutes (2;20) for (p;q) and obtains:
$s = a(t - 2)^2 + 20$

To obtain a, he finally substitutes (5;0) for (t;s):

$$0 = a(5 - 2)^2 + 20$$

$$0 = 9a + 20$$

$$9a = -20$$

$$a = -\frac{20}{9}$$

The final expression for s in terms of t now becomes

$s = -\frac{20}{9}(t - 2)^2 + 20$ which after simplification can be written in a more familiar form:

$$s = -\frac{20}{9}t^2 + \frac{80}{9}t + \frac{100}{9}$$

or $$s = \frac{20}{9}(-t^2 + 4t + 5)$$

Personal experience suggests that students usually experience some difficulty with this method for two main reasons:

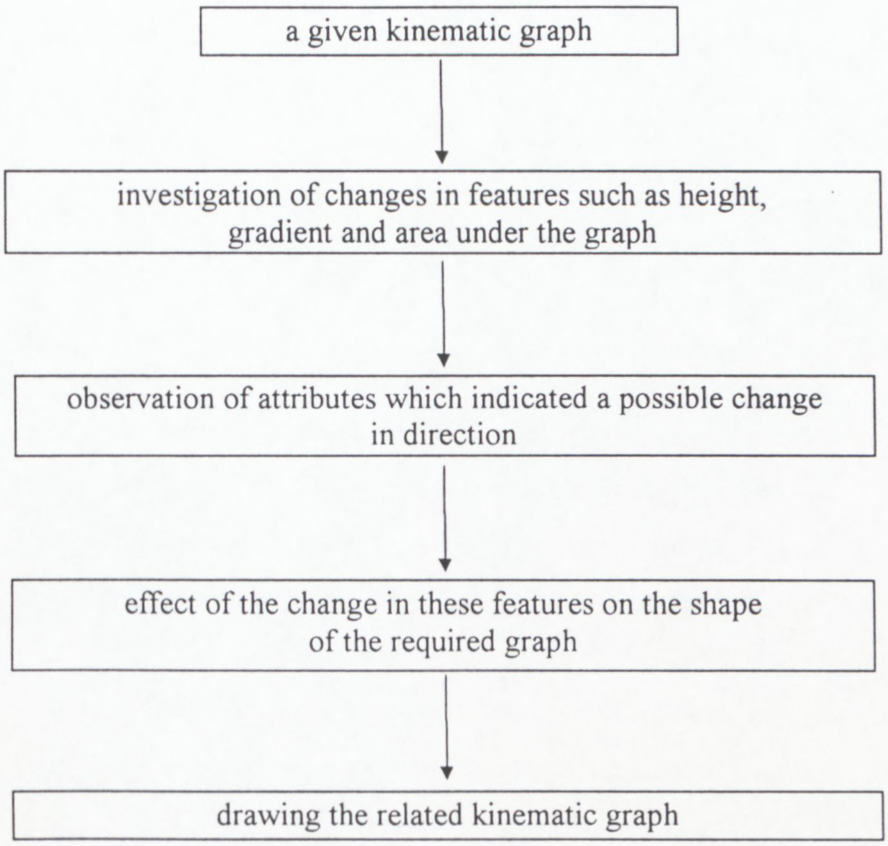
(i) they often fail to apply what they have learned in the mathematics class to other subject areas; and

(ii) an inability to do a mathematical analysis in terms of (say) s and t as variables, when they have been conditioned to use x and y for the independent and dependent variables in the maths class for many years.

2.7.2.5 Translation E : Graph →Related kinematic graph

All kinematic graphs which represent a given motion are related to each other by virtue of the definitions of *displacement, velocity* and *acceleration* and their scalar equivalents. Students must be able to transform a vector graph into its scalar equivalent and a given kinematic graph into another graph representing a related kinematic quantity.

The cognitive actions governing the transformation from a vector graph to a scalar graph depend on the principle that a reversal in direction is ignored in the case of the scalar graph; however, the actions governing transformations between graphs of related quantities are a little more complex. The following diagram attempts to highlight this process.



It can be argued that an understanding of the relationship between an algebraic graph such as $y = ax^2 + bx$ and the corresponding kinematic graph of $s = \frac{1}{2}at^2 + ut$ involves similar mental transformations. However, the nature and basic shape of the graph does not change and hence the transformations between algebraic graphs and kinematic graphs will not be discussed further in this section.

For students to be successful at drawing related kinematic graphs they must have a clear understanding of the following principles:

(i) They must realise that the same motion can be represented by graphs of very different shapes.

(ii) They must identify which features (such as maximum height or time axis intercept) of a given graph can be used to obtain important information about the related graph.

(iii) They should be encouraged to draw related graphs in such a way that corresponding time intervals fall in the same vertical plane.

(iv) More specifically, students should be able to translate the slope of an *s-t* graph into the height of a *v-t* graph as well as an increasing slope of the *s-t* graph into an increasing height on the *v-t* graph. The more problematic transformation of a constantly increasing slope of an *s-t* graph into the constant height of an *a-t* graph is equally important.

(v) In the case of a given *v-t* graph, students must translate the height (or changing height) into the slope (or changing slope) of a required *s-t* graph with due consideration of the sign (positive or negative). If the *v-t* graph has to be transformed into an *a-t* graph the slope (or change in slope) of the *v-t* graph becomes the height (or change in height) of the required *a-t* graph.

Gradient is the most important feature of kinematic graphs because the nature and change of the gradient over a time interval gives direct information about the related graph. The research that de Jager (1990) has undertaken with respect to students' understanding of the gradient concept will be discussed in the next chapter. Here a single quotation will suffice to underline the importance of motion graphs in explaining gradient:

> *Real life physical situations greatly assist in overcoming many problems that pupils have with the concept of gradient* (de Jager 1990:171).

The following example illustrates how a knowledge of gradient assists students in successfully completing graph-to-graph transformations.

Given the sketch graph of $\vec{s}$ *vs* *t* for a stone which is thrown vertically up, draw the corresponding distance-time, velocity-time, speed-time and acceleration-time graphs. (The ground is the reference point and up is taken as positive).

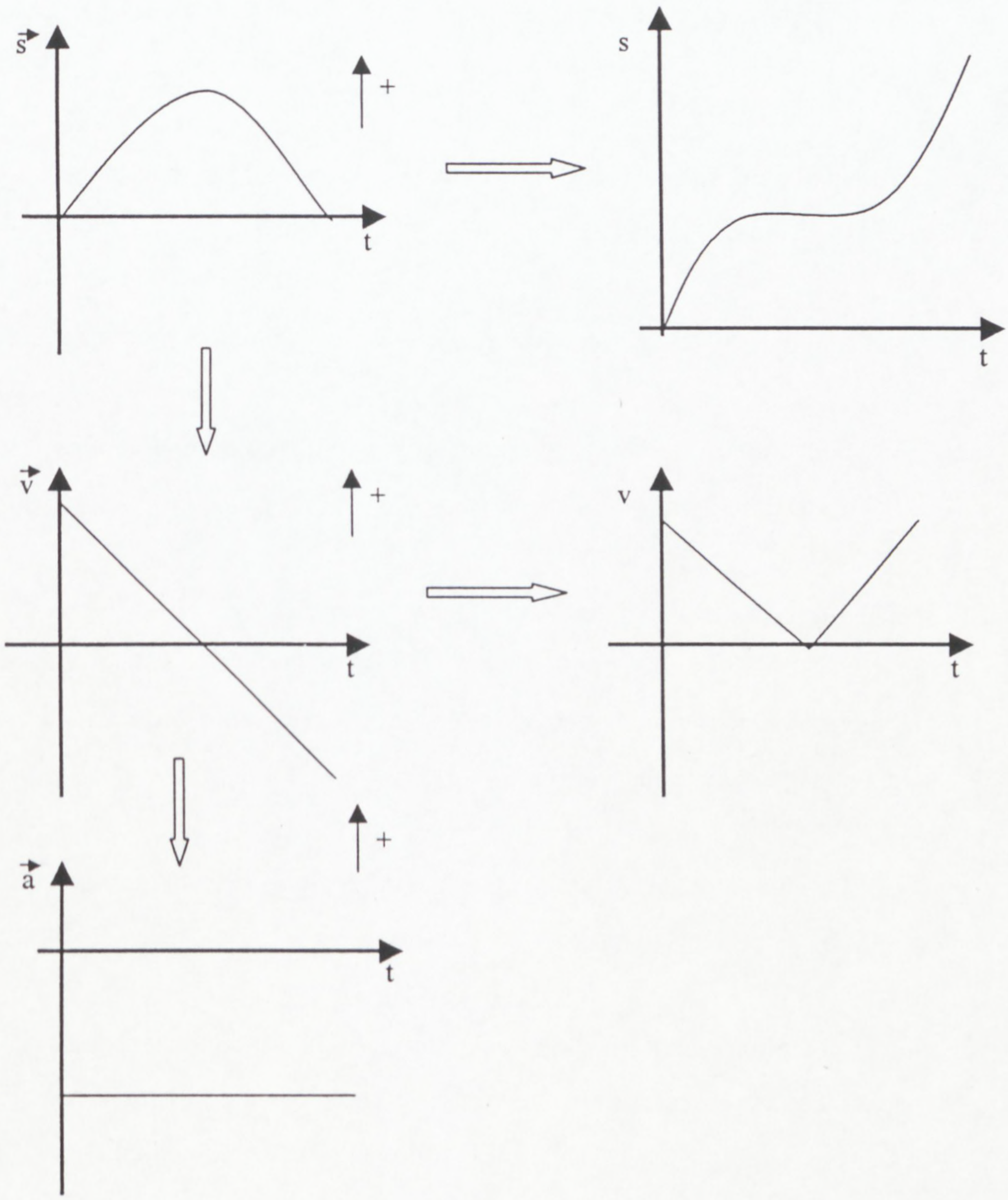


FIGURE 2.34

Successful transformations presuppose that students can make the following mental associations:

- the <u>slope</u> of the *s-t* graph becomes the <u>height</u> of the *v-t* graph;

- a negative/positive slope of the $\vec{s}$-*t* graph is associated with a decreasing/increasing velocity;

- the negative/positive slope of the $\vec{s}$-*t* graph transforms to a negative/positive ordinate on the $\vec{v}$-*t* graph;

- a zero slope of the $\vec{s}$-*t* graph implies zero velocity during this time interval;

- a constant slope of the $\vec{s}$-*t* graph transforms into a straight line parallel to the time axis for the $\vec{v}$-*t* graph;

- a changing slope is associated with a *curved graph;*

- the increasing/decreasing slope of the $\vec{s}$-*t* graph transforms into a straight line with positive/negative gradient for the $\vec{v}$-*t* graph;

- a negative slope of an *s-t* graph which is increasing in absolute value should be viewed as a <u>decreasing slope;</u>

- the distance-time and speed-time graphs are graphs of scalar quantities which means that any negative values for the corresponding displacement-time and velocity-time graphs are taken as positive for their scalar equivalents;

- In the case of a curved graph the slope at any given time must be seen as the gradient of a tangent drawn at the given point.

The transformation velocity-time graph → acceleration-time graph involves similar mental connections which will not be repeated here. Suffice to point out that the same features which assist in the transformation from an $\vec{s}$ -*t* to a $\vec{v}$ -*t* graph are used to transform a $\vec{v}$ -*t* graph into an $\vec{a}$ -*t* graph.

An *a-t* graph will now be considered which is to be transformed into the corresponding velocity-time and finally into a displacement-time graph.

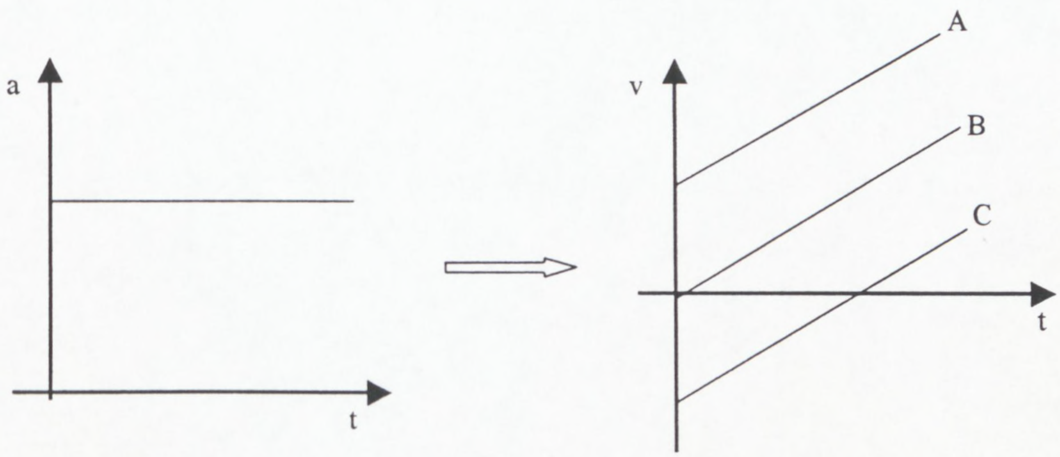


FIGURE 2.35

The area (or change in area) under an *a-t* graph gives information about the shape of the resulting *v-t* graph in the same way that the area under the *v-t* graph gives an indication of the format of the *s-t* graph. Conversely the height of *a-t* and *v-t* graphs gives an indication of the slopes of *v-t* and *s-t* graphs respectively. In the following discussion the latter idea of slopes will be used to summarise the required sequential cognitive thought processes which students have to employ:

The *a-t* to *v-t* transformation of figure 2.35 requires students to notice the following:

(i) the *a-t* graph runs parallel to the time axis;

(ii) the acceleration has a constant positive value, which is obtained from the constant positive values of the graph;

(iii) the gradient of the *v-t* graph gives acceleration;

(iv) the *v-t* graph must have a constant positive gradient;

(v) a sloping straight line running SW-NE has a constant positive gradient;

(vi) hence the required *v-t* graph is any one of the graphs indicated in the figure 2.35 depending on the value of *v* at t = 0.

A similar series of logical mental processes governs the transformation:
v-t graph → *s-t* graph (illustrated in figure 2.36).

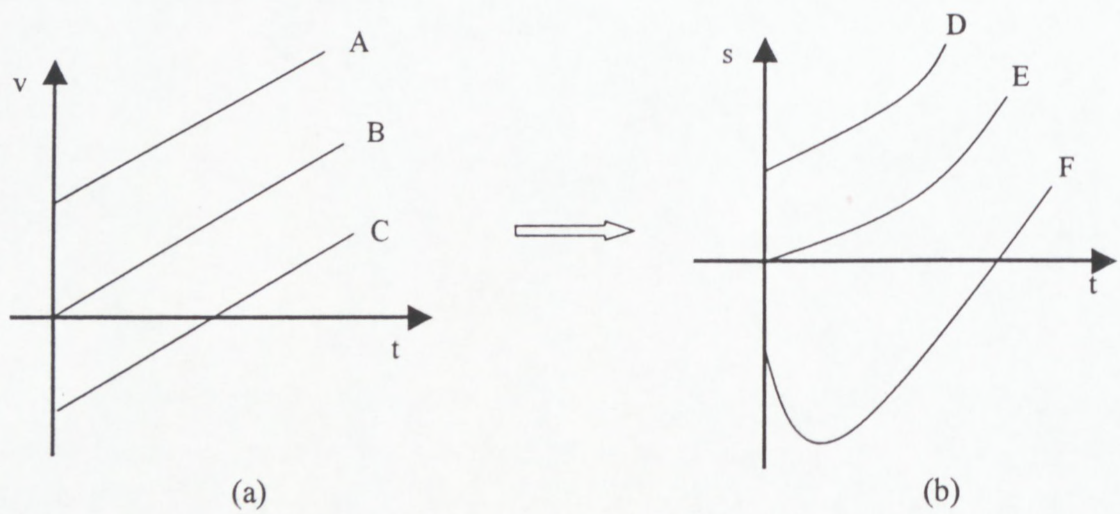


FIGURE 2.36

(i) the $\vec{v}$ -*t* graph is a sloping straight line representing an increasing function;

(ii) the gradient of the $\vec{s}$ -*t* graph gives velocity;

(iii) the $\vec{s}$ -*t* graph must be such that the gradient increases;

(iv) a curve that has the shape of a parabola (concave up) has an increasing gradient;

(v) hence the required $\bar{s}$-t graph is curved parabolically as indicated in figure 2.36.

The fact that the displacement of any body depends on the chosen reference point immediately implies that any one (D, E or F) of the *s-t* graphs in figure 2.36 can be displaced in the vertical plane.

Graph F of figure 2.36 (b) is especially interesting, as it contains a negative as well as a positive gradient. In this case the student must consider three separate parts of the *v vs t* graph (graph C of fig. 2.36 (a)) and transform the algebraic height (or change in height) into a gradient (or change in gradient) on the *s vs t* graph. Figure 2.37 illustrates a number of different graphs, depending on what point was chosen as reference level.

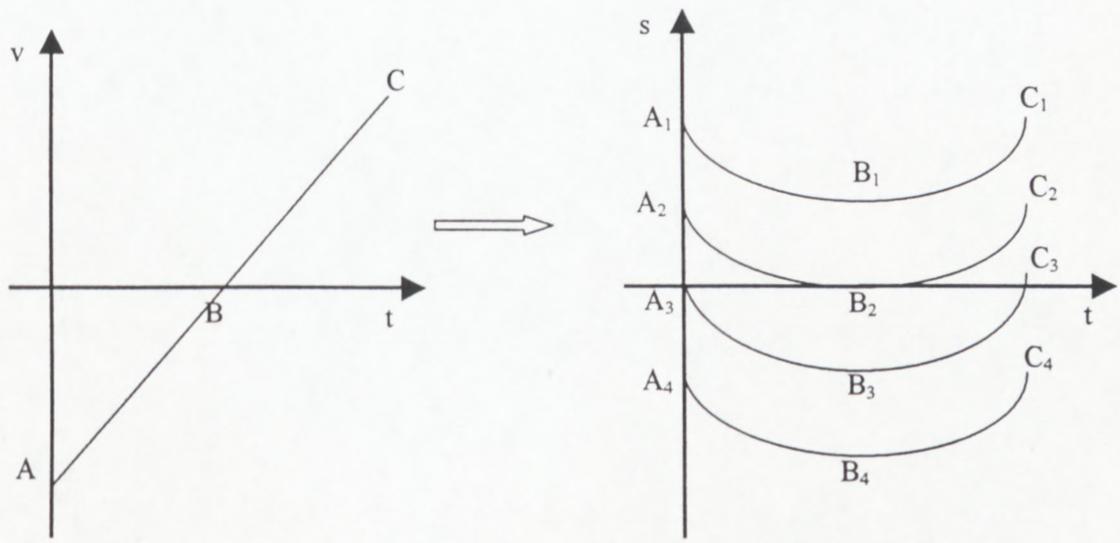


FIGURE 2.37

A final example based on research undertaken by McDermott, Rosenquist and Van Zee (1986) follows. It typifies the level of question that students should be able to answer after some instruction in kinematic graphs.

> *Several interesting times are labelled A, B, C, etc., on the position vs time graph of figure 2.38. Sketch a velocity-time graph for the motion. Label times A-F on your time axis* (1986:505).

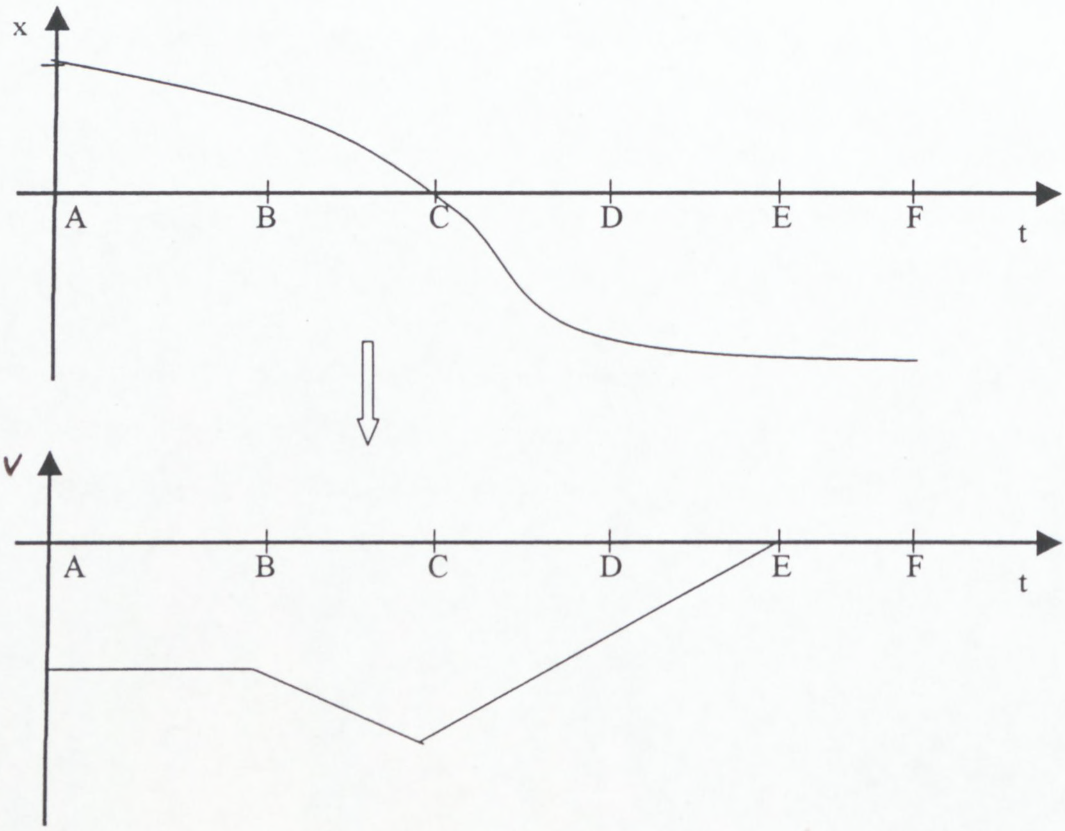


FIGURE 2.38

They stress the important effect of slope of the given graph on the height of the required graph:

> *...to envision an increasing slope on an x vs t graph as an increasing height on a v vs t graph.* (p. 505)

Figure 2.38 illustrates the advantage of drawing corresponding time values vertically below each other for easy comparison. It also shows students how the shape of a *v-t* graph often differs dramatically from the shape of the *x-t* graph.

The slope-height dichotomy will be shown to be a serious obstacle for the proper understanding of kinematic graphs and especially the transformation of kinematic graphs. The next chapter (see especially § 3.7.5) will explore known misconceptions about slopes and heights, whilst Chapters 4 and 5 will include this aspect in an empirical investigation of problems relating to graph → graph transformations.

2.7.3 Scaling

When quantitative graphs are drawn for the purpose of scientific data analysis, issues such as allocation of the independent variable to the horizontal axis, choice of scale and unit become very important.

> *Selecting and constructing suitable scales is an issue particularly in science lab contexts, where a graph describing a situation is plotted based on observations that translate into numerical data* (Leinhardt *et al.* 1990:19).

Scaling tasks require particular attention to the axes and their scales and to the units that are measured. Students must realise that for each axis any two intervals of the same length represent the same number of units and that the scales on the two axes do not have to be identical. The choice of unit depends on the purpose of the scientific investigation and students must learn to transform one unit into another equivalent one which might describe the physical situation more effectively. Leinhardt *et al.* (1990:18) state in this regard:

> *The unit may also need to be transformed. Some units are more sensible and meaningful than others, whereas other units are more efficient and produce "better" graphs.*

An example from kinematics would be the change of the unit for velocity from, for example, $km.h^{-1}$ or mm per time interval of 0,02 s to $m.s^{-1}$.

Learners have to develop an understanding of which features of a kinematic graph will change when the axes are scaled differently and which features are independent of scaling changes. An example of a feature which will change is the visual appearance or shape of the graph in terms of, for instance, the angle that it makes with the axes, while the intercepts with the axes are not dependent on the chosen scale.

2.7.4 Sign convention and choice of reference level

> *Teaching the need for convention is not the same as teaching a specific convention* (Leinhardt *et al.* 1990:52).

The vector nature of many of the kinematic concepts described here makes it important that students understand the importance of the choice of a positive and negative direction as well as choosing an appropriate reference point.

The actual cognitive processes relating to a sign convention are quite significant. The student has to:

- recognise the vector nature of displacement, velocity and acceleration;
- associate a positive or negative sign with opposing directions (usually up-down or left-right);
- allocate the signs and state his chosen frame of reference;
- realise that all vector quantities in a specific situation must follow the same sign convention;
- allocate these signs to the stated vectors;
- associate the appropriate sign with the positive and negative side of the vertical axis;

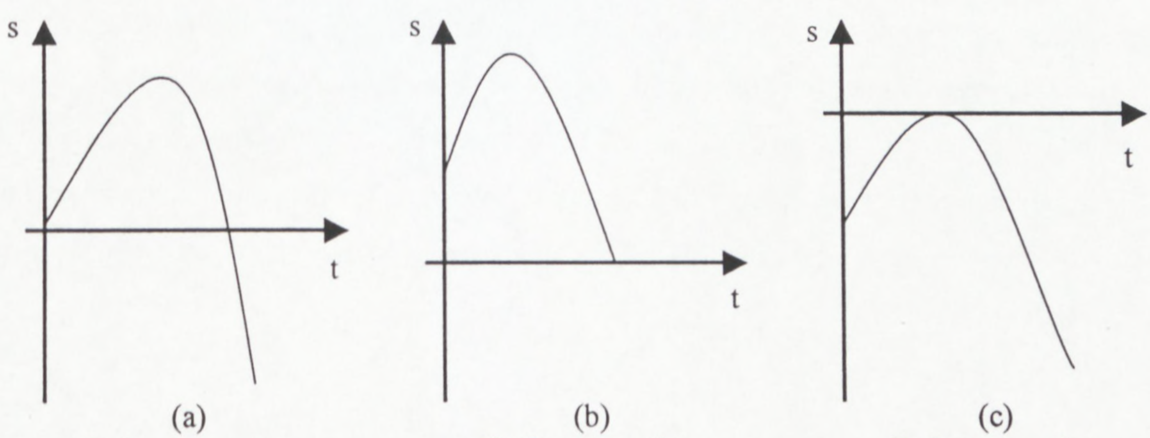
- draw the graph (sketch graph or accurate graph) with due consideration of the sign convention;

- associate certain features of the graph, such as slope, with the appropriate kinematic concept of the correct sign.

The choice of reference level is equally important because effective graphical communication about a motion can only take place if the reference point is clearly stated. This can be illustrated by the following example:

A ball is thrown vertically upwards from the top of a building and is allowed to hit the street below. Draw a sketch graph of position versus time.

Students must understand that the shape of the graph will stay the same, but the positioning in a vertical plane will depend on the chosen reference level. The graphs for three different reference levels are illustrated in figure 2.39 below. (Up has been chosen as positive).



(a) Reference point: top of building. (b) Reference point: ground. (c) Reference point: max height.

FIGURE 2.39

Thijs (1987:45) investigated students' comprehension of reference level and concluded:

Students cannot define motion with respect to a frame of reference other than the ground.

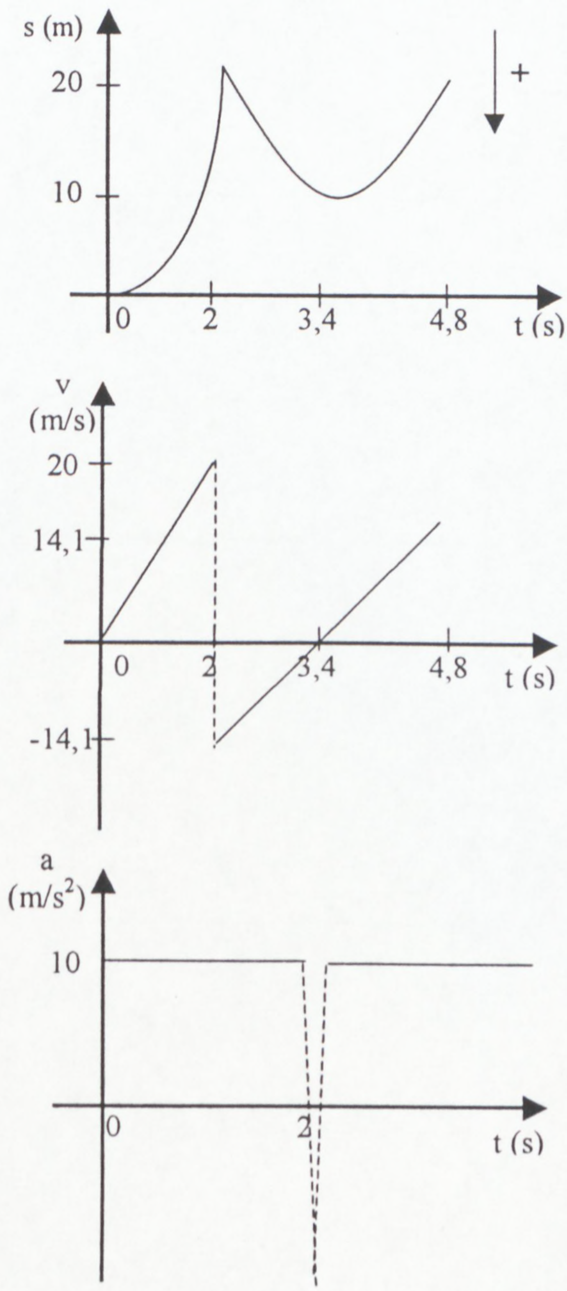
Students must be encouraged to comprehend the relativity of all motion and they must understand that the earth need not necessarily be chosen as a reference frame. Hence graphs (a) and (c) are not less acceptable than (b) in describing the motion in the previous example.

The following example illustrates how graphs with different positions with respect to the axes are generated for different choices of sign convention.

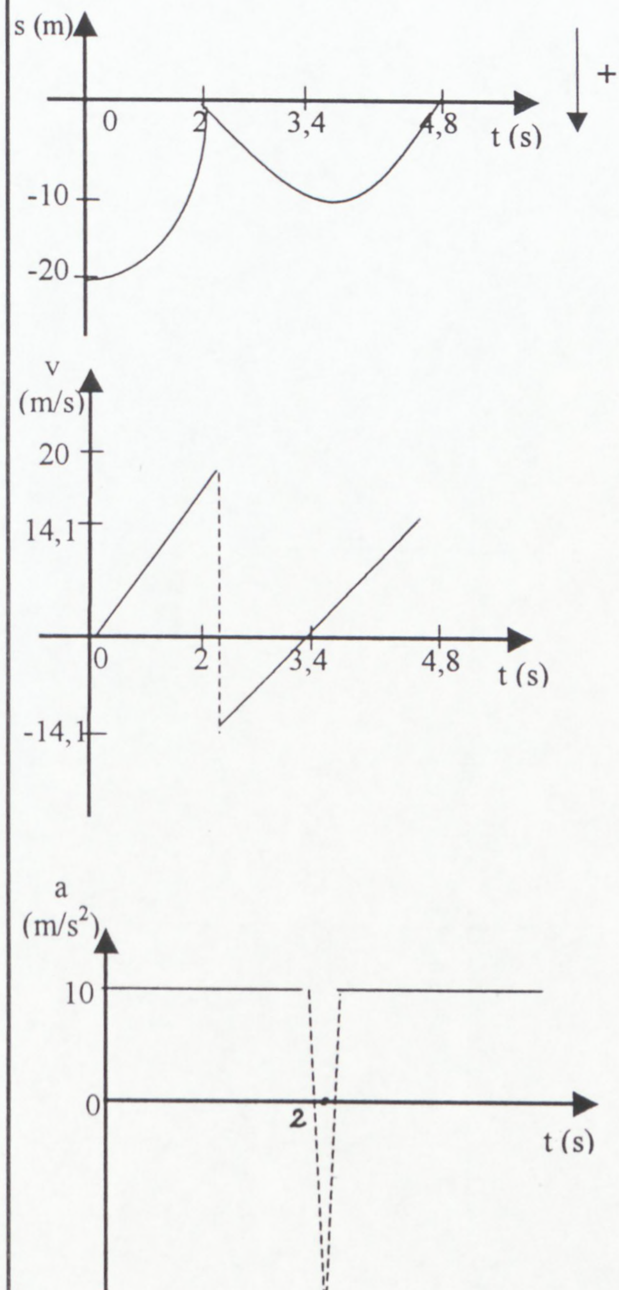
Example: *A girl drops a rubber ball from a bridge 20 m above a concrete surface. It rebounds once, reaching a height of 10 m and bounces onto the surface a second time. Draw neat graphs of* s-t; v-t *and* a-t *and indicate actual values on the axes where possible. (Consider the time from when the ball is dropped until it strikes the surface a second time.)*

The instructor can either give the sign convention and reference point or leave it to the student to define. Either way it is important to insist that it should be stated clearly. It is natural to regard either the bridge or the surface as a reference level together with a stated sign convention for "up" and "down". The four possible combinations of reference level and sign convention yield four different graphs. The four variations are illustrated in figure 2.40, followed by a brief analysis of the cognitive processes involved in making this translation.

reference point: bridge



reference point: surface



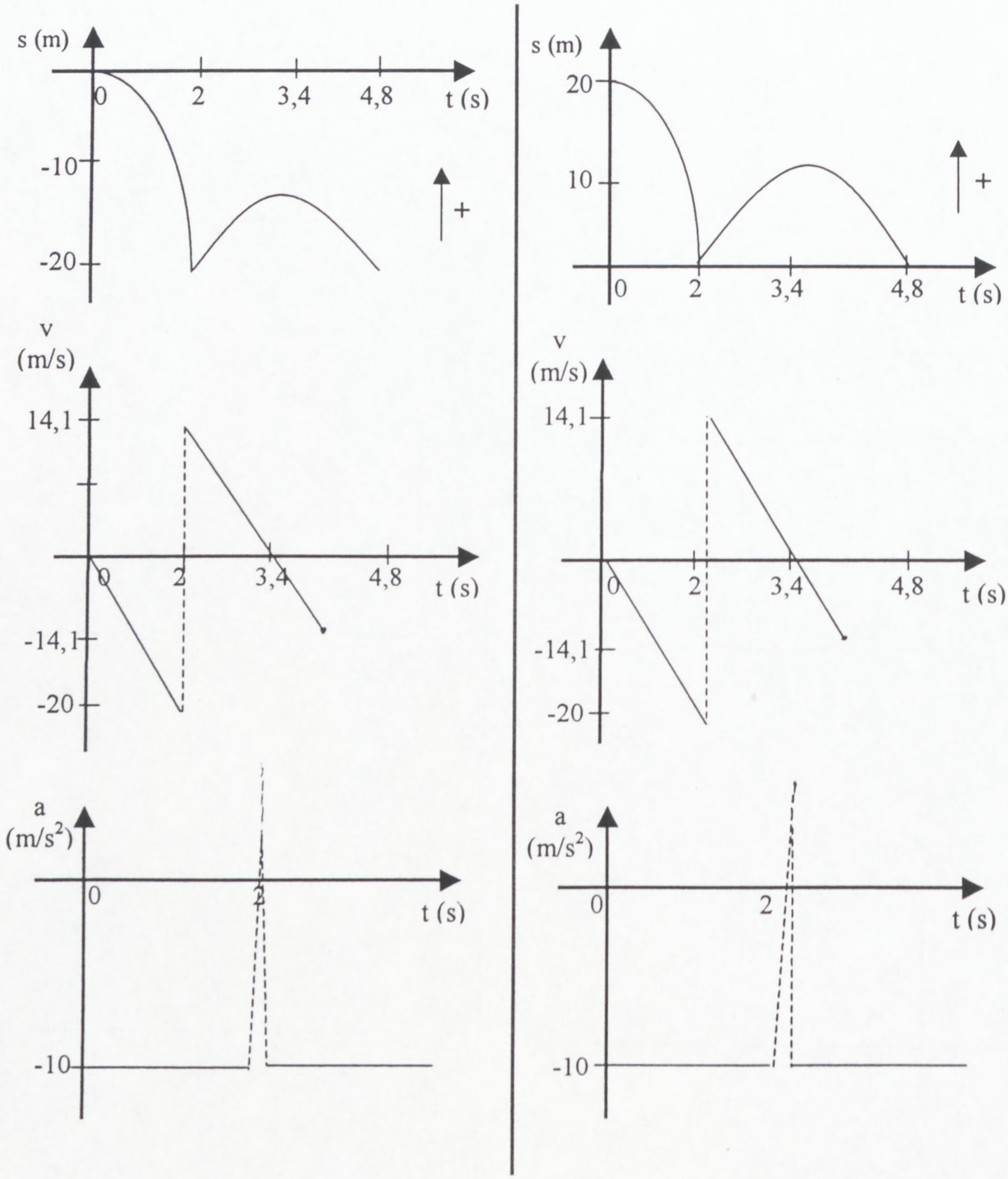


FIGURE 2.40

It has to be pointed out that the choice of reference point is only important in the case of displacement because it is used to determine the initial displacement of a body and/or in which direction (+ or -) a body has been displaced. In the case of velocity, the direction of motion indicates the sign of the velocity vector, while the direction of the resultant force always gives the direction of the acceleration. It follows that the actual direction of motion is irrelevant for the choice of sign for acceleration and displacement.

Calculations [+↑ and bridge is reference point]

$s = ut + ½at^2$

$20 = 0 + ½.10.t^2$

$t^2 = 4$

$t = 2$ s

After first bounce: [+↓ and surface is reference point]

$v^2 = u^2 + 2as$

$0 = u^2 + 2 \times 10 \times (-10)$

$u^2 = 200$

$u = -14,14$ m.s^{-1}

Time (up) after first bounce [+↓ and surface is reference point]

$v = u + at$

$0 = -14,14 + 10t$

$t = 1,414$ s

∴ Time (down) after first bounce = 1,414 s

Alternative method for calculating time (up) after first bounce:

$s = ut + ½at^2$

Taking again +↓ and the surface as reference point:

$$-10 = -14{,}14\,t + \tfrac{1}{2} \times 10 \times t^2$$

$$5t^2 - 14{,}14\,t + 10 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{14{,}14 \pm \sqrt{200 - 200}}{10}$$

$$= 1{,}414 \text{ s}$$

This indication of velocity, position and time values at points on the axes corresponding with sudden changes of direction is important as it gives an indication of scale on the sketch graph.

Students must make the following cognitive connections in order to be successful at this translation:

- understand the need for following a sign convention and stated reference level;

- discriminate between information that leads to changes in height and changes in slope. He must realise for instance that during the downward motion (increasing velocity) the height of the *v-t* graph increases while the slope of the *s-t* graph increases;

- realise that after the first bounce the velocity vector has changed direction and indicate this with a change in sign of the velocity value on the graph;

- visualise both velocity and displacement as a continuously varying quantity, thus indicating their variation with smooth curves or lines;

- realise that a reversal in velocity does not necessarily imply a reversal in acceleration and that in fact the acceleration vector points downwards throughout the motion (except for the very brief moment when the ball strikes the ground during which time the ball experiences a changing acceleration. In that time the acceleration changes from a downward acceleration of 9,8 m/s^2 to an upward acceleration and back to a downward acceleration of 9,8 m/s^2).

Students must also be able to use the appropriate equation of motion for determining velocity and time values. This implies translating the verbal information in the written passage into appropriate values for the algebraic symbols in the equation of motion. It is essential that students use the appropriate sign (negative or positive) to arrive at an answer which is in agreement with the sign convention.

2.8 THE SOUTH AFRICAN SITUATION

2.8.1 General Comments

In this section attention will be given to the prescribed content with respect to kinematic graphs as it is currently taught in this country. The content will be described in terms of its description by (a) the official prescribed syllabus, (b) the interpretation of this syllabus by text book writers and (c) the type of question which students are required to answer in the final Senior Certificate examination.

2.8.2 The prescribed syllabus

The following is an extract from the present Physical Science (Higher Grade) interim syllabus of the Cape Education Department (1995) with respect to graphs of motion:

Content

Displacement-time, velocity-time relationships.

Graphs of displacement/time and velocity/time, limited to linear motion with constant acceleration.

Further Elucidation

Refer to the difference between uniform and accelerated motion.

This section is included to facilitate (a) the interpretation of experimental results (b) the deduction of $v = u + at$, $s = ut + \frac{1}{2}at^2$ *and hence,* $v^2 = u^2 + 2as$. *(Candidates will not be required to derive these formulae in the final examination).*
The drawing and interpretation of sketch graphs.
Simple calculations using the graphs.
Simple calculations using the equations of motion.

Practical Work

(a) *Investigation of the uniform velocity of a moving trolley, leading to the graphs of displacement/time and velocity/time [L].*

(b) *Investigation of the acceleration of a trolley leading to the graph of displacement/time and velocity/time [L].*

General comments

The following comments can be made with respect to the stated syllabus:

(a) The prescribed syllabus appears to be rather incomplete and lacking in detail. This is regrettable in the light of the importance of kinematic graphs in illustrating for a given motion the behaviour of the fundamental kinematic concepts in time.

(b) The specific exclusion of the acceleration-time graph is noted. For uniform accelerated motion this is a straight line with zero slope and as such is directly related to the corresponding velocity-time graph. There seems to be no reason why the graphs of *s vs t, v vs t* and *a vs t* should not be included as a logical group of related graphs.

(c) The drawing and interpretation of sketch graphs is specifically mentioned but it is not clear whether accurate quantitative graphs are included. It is suggested that the phrase "the interpretation of experimental results" could indicate that accurate graphs are included in the syllabus.

2.8.3 The interpretation of the syllabus by textbook writers

The textbook is an important teaching aid used for the instruction of physical science at secondary schools in this country. The interpretation that a particular textbook gives of a specific topic in physics (such as kinematic graphs) is thus of fundamental importance to the teaching and learning of that topic.

In this section a brief comparative description will be given of kinematic graphs as it is presented in four commonly used local textbooks.

The local books which were analysed are:

(a) Physical Science 2000 by Heyns, de Villiers, Gibbon, Naidoo, Fowler and Jordaan (1989);

(b) Senior Basic Physical Science by Pienaar, Walters, Schreuder and de Jager (1987);

(c) Successful Science by Broster and James (1986); and

(d) Physical Science 9 by Brink and Jones (1986).

2.8.3.1 Ticker-tape and timer

All the textbook authors initiate their discussion with a treatment of ticker-tapes and timer for the recording of motion. This practical investigation involving a trolley travelling either at constant velocity or with uniform acceleration (positive or negative) yields data which is used for the drawing of displacement-time and velocity-time graphs.

These graphs are drawn from data points, which are generated by the results of the student's own practical investigation. Heyns *et al.* (1989:43) as well as Broster and James (1986:42) and Brink and Jones (1986:40) ask the students to determine the gradient of their *s-t* graphs and compare it with the magnitude of the *average velocity*. A similar procedure is suggested for the gradient of the *v-t* graph (comparing it to the calculated *acceleration*) and the area under the *v-t* graph (comparing it to the calculated *change of displacement*). The idea is clearly that the student should discover for himself that the mathematical concepts of gradient (or slope) and area give a quantitative description of velocity, acceleration and displacement respectively.

Heyns *et al.* (1989:37) and Broster and James (1986:41) explain to students how someone's walking speed as recorded by a ticker-tape can be represented pictorially by cutting the paper tape into strips of ten intervals each. Each strip represents the distance covered by the person in ten time intervals (say 0,2 s). This represents the average walking speed during the 0,2 s time interval. When these paper strips are pasted next to each other on a piece of graph paper students have in a natural and spontaneous way drawn their first velocity-time "graph"! Figure 2.41 illustrates such a "graph".

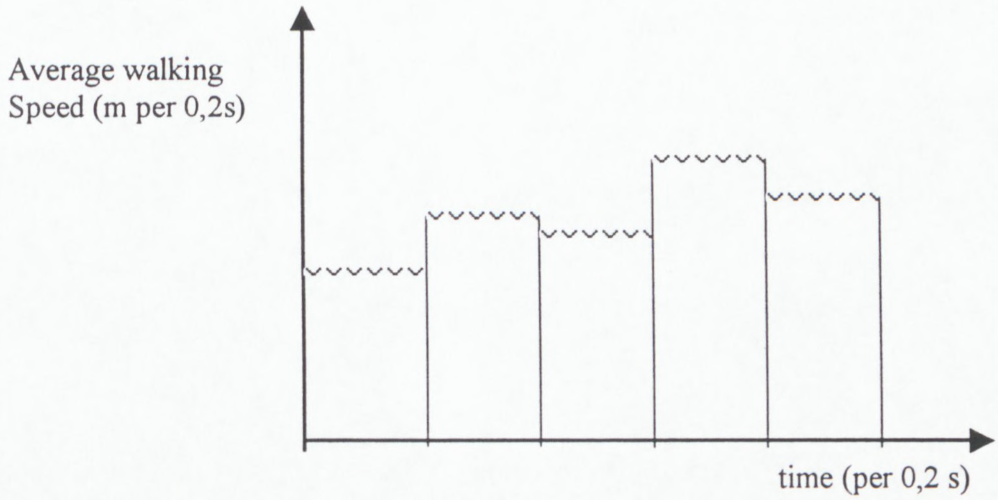


FIGURE 2.41

This approach to graphing (as an actual pasting up of equal-time distance strips) is widely used in the U.K. (see for example Rogers and Wenham (1987:7), Hanson (1989:9) and Pople (1989:17)) and will be referred to again in a later section.

2.8.3.2 Instantaneous velocity

Broster and James (1986) give the following description of the concept of instantaneous velocity for accelerated motion:-

> *Therefore, to find the instantaneous velocity at time = t, from a displacement-time graph, calculate the gradient* $\frac{\Delta s}{\Delta t}$ *of the tangent to the graph at time t* (p. 46).

They motivate this statement by noting that as Δt is made smaller the amount of curvature of the graph "becomes less noticeable".

They then provide actual specimen tabled data values and illustrate how these are used to produce a graphical representation of the motion.

2.8.3.3 Integration with mathematics graphs

Broster and James (1986:42) make two very significant statements which deserve further comment:

(i) *"Draw the best line or curve you can through the points you have plotted. The line shows the trend of the results. Do not draw straight lines between each point."*

and

(ii) *"From mathematics you know that the gradient is equal to* $\frac{\Delta y}{\Delta x}$*."*

The first statement serves as a reminder that when graphs are drawn from real experimental results, we are interested in general trends. Experimental errors caused by friction, inaccuracy of measuring instruments, etc. do not have to be reflected on the graph. A graph is an abstract representation of an event in the real world and as such could be described by a mathematical formula. Students deal only with graphs of exact mathematical functions in the mathematics syllabus and should be well skilled in drawing straight lines and parabolas.

When a student is asked to draw the graph of a function in mathematics he will make sure that the curve passes through each ordered pair. This is because the mathematical formula for the function is given. In science, however, the formula for the graph is not always known, hence students must realise that the curve does not have to pass through all the experimental data coordinates.

The second statement illustrates a rare occurrence in local physical science textbooks where an attempt is made at integrating concepts in mathematics and physics in a direct and explicit way. In mathematics the idea of a gradient is fundamental to the study of graphs and calculus and it is important that students understand that it is this same concept (namely slope or gradient) which is used in an analytical role in graphs of motion.

2.8.3.4 Acceleration opposite to direction of motion

The concept of a "negative acceleration" which is associated with a negative slope of a velocity-time graph is poorly dealt with by all four books which have been investigated.

Broster and James (1986) provide a single example (p. 50) of such a motion in the form of a negatively sloping velocity-time graph, and then explain how to calculate the acceleration as $\frac{\Delta v}{\Delta t}$.

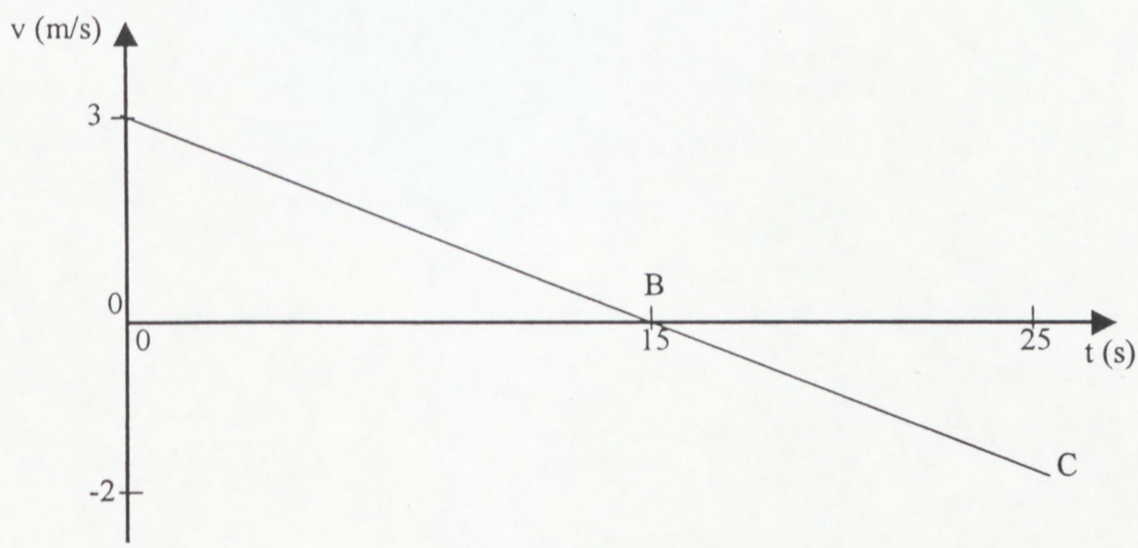


FIGURE 2.42

$$\text{Now } a = \frac{\Delta v}{\Delta t}$$

$$= \frac{-2-3}{25-0}$$

$$= -0{,}2 \text{ m.s}^{-2}$$

$$= +0{,}2 \text{ m.s}^{-2} \quad \text{in opposite direction}$$

Heyns *et al.*(1989) as well as Brink and Jones (1986) give a similar example, stressing that the body moves in the opposite direction with acceleration 0,2 m.s^{-2} during the section marked BC.

Pienaar *et al. (1987:32)* define negative acceleration as:

> *When acceleration occurs in the same straight line as the line of motion, but in the opposite direction, the acceleration is negative since it causes a continual decrease in velocity.*

They proceed to give an example similar to the previous figure, but section BC is omitted. They mention the terms deceleration and retardation as being synonymous with negative acceleration. This could lead to complications as the following example illustrates:-

A car, approaching a stop street, decelerates uniformly from 40 m.s^{-1} to rest in 4 seconds. It then stops for 2 seconds after which it turns around and reaches a velocity of 40 m.s^{-1} in the opposite direction within 2 seconds.

Clearly the acceleration for the first stage is -

$$\frac{-40}{4} = -10 \text{ m.s}^{-2}$$

and for the second stage it is -

$$\frac{-40}{2} = -20 \text{ m.s}^{-2}$$

This example which illustrates that deceleration (slowing down) has the same sign as acceleration (speeding up) in the opposite direction, is best understood by drawing a graph of velocity versus time for the motion (see figure 2.43).

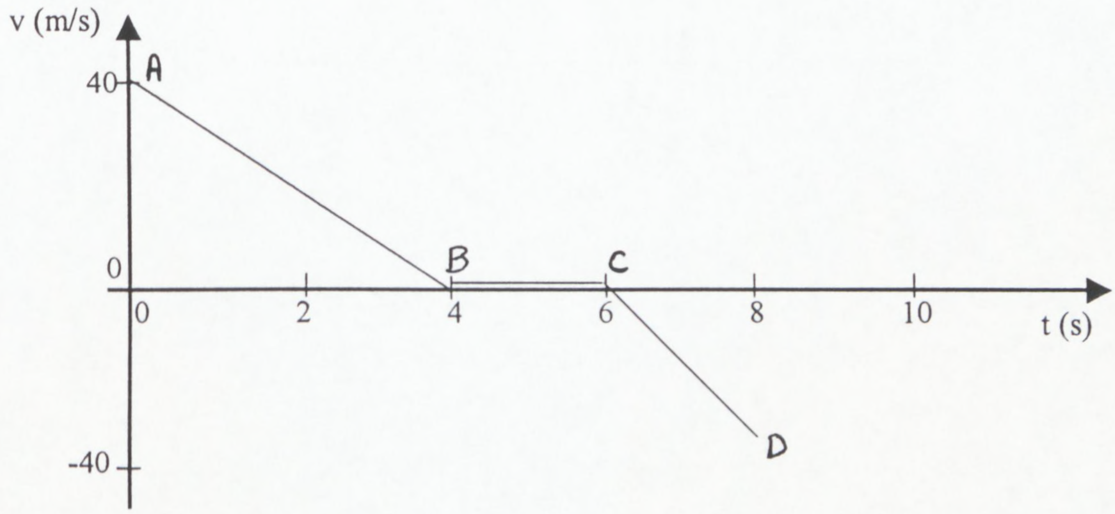


FIGURE 2.43

Students should realise that both sections AB and CD represent negative acceleration, but certainly only section AB can be regarded as representing retardation. It seems that this very important distinction between positive and negative acceleration as well as the distinction between two types of negative acceleration which is fundamental to the study of kinematics,, is largely neglected by local textbooks. The solution appears to lie in the avoidance of terms like "deceleration" and "retardation" and even negative acceleration.

2.8.3.5 Area in terms of narrow rectangular strips

Pienaar *et al.* (1987:36) use the method of narrow rectangular strips to illustrate why the area under any *v vs t* graph gives the (change in) displacement of a body.

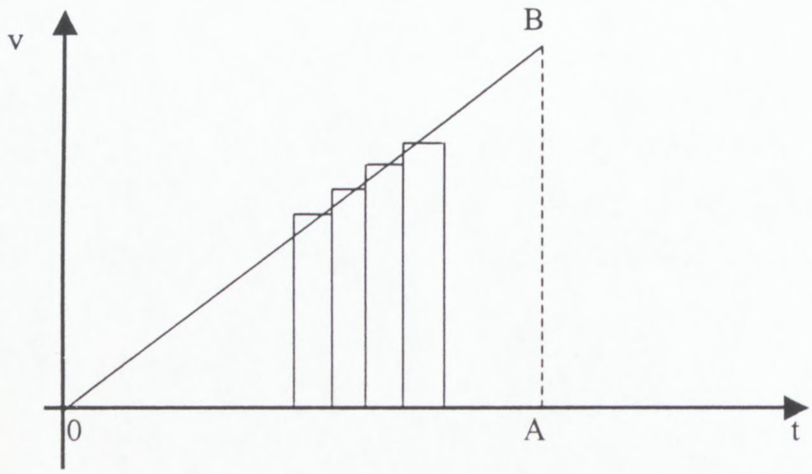


FIGURE. 2.44

In the discussion that follows, it is assumed that students are familiar with the idea that the area under the *v-t* graph represents the change in displacement.

If the area under the *v vs t* graph of figure 2.44 is divided into many narrow rectangular strips, one can view the area of a single strip as the distance traversed in a brief interval of time at a constant velocity. The sum of all the areas of all the strips would then yield an approximation of the area of triangle OAB. With sufficiently narrow strips the total area of the rectangles approaches the area of ΔAOB and thus represents the total change in distance.

It must be pointed out that Pienaar *et al.* (1987:36) err somewhat when they associate the *total distance traversed* with the area under the *v vs t* graph. It is preferable to use the terms *change in displacement* as used by Brink and Jones (1986:38) or even the phrase *the area under the velocity-time graph is numerically equal to the change in displacement* (Broster and James 1986:44). The approach of narrow rectangular strips is essential for two reasons:

(i) It enables students to make a connection between the method of obtaining the displacement under a velocity-time graph for uniform motion and uniform acceleration. The student is familiar with the method of finding the area of a rectangle and this is used in the more general case of determining the area under a sloping straight line.

(ii) This process is fundamental to the study of integral calculus, which means that students are introduced to with a technique which is used at tertiary level.

2.8.3.6 The data source for drawing graphs

Orton (1984:25) stresses the importance of using familiar items to generate data points for drawing graphs when he states:

It may be very important that real-life situations are used to provide data.

This also applies to kinematic graphs where the translation from a table of values to a graphical representation of those values will be better understood by students if it is given in the context of moving cars, falling stones, trolleys, etc.

The following table indicates the nature of the "real life" situation which is used for drawing displacement-time and velocity-time graphs in the four textbooks.

	HEYNS	PIENAAR	BROSTER	BRINK
***s-t* graph (a = 0)**	(i) verbal description of a moving car (with stroboscopic pictures) (ii) trolley	moving car	tabulated results of a moving trolley	an "object"
***v-t* graph (a = 0)**	trolley	moving car	tabulated results of a moving trolley	an "object"
***s-t* graph (a ≠0)**	a moving car	an object in free fall	trolley	an "object"
***v-t* graph (a ≠ 0)**	(i) a moving car (ii) (trolley results: prompt pupils to draw)	a moving "body"	trolley	an "object"

It must be noted that all the textbooks stress that the translation from the observed motion of real objects to an accurate graph depicting this motion, usually presupposes the construction of a data table as an intermediate step.

2.8.3.7 Sketch Graphs

Sketch graphs as qualitative representations of observed or described kinematic situations receive varying degrees of emphasis in the four books that have been studied.

(i) Brink and Jones (1986:46-47) consider a ball rolling up an incline with a certain initial velocity and include the return down the incline. They then proceed to emphasise the important conceptual difference between *distance-time* and *displacement-time* as well as between *speed-time* and *velocity-time* graphs as illustrated in figure 2.45. (Note that the direction up the slope was taken as positive).

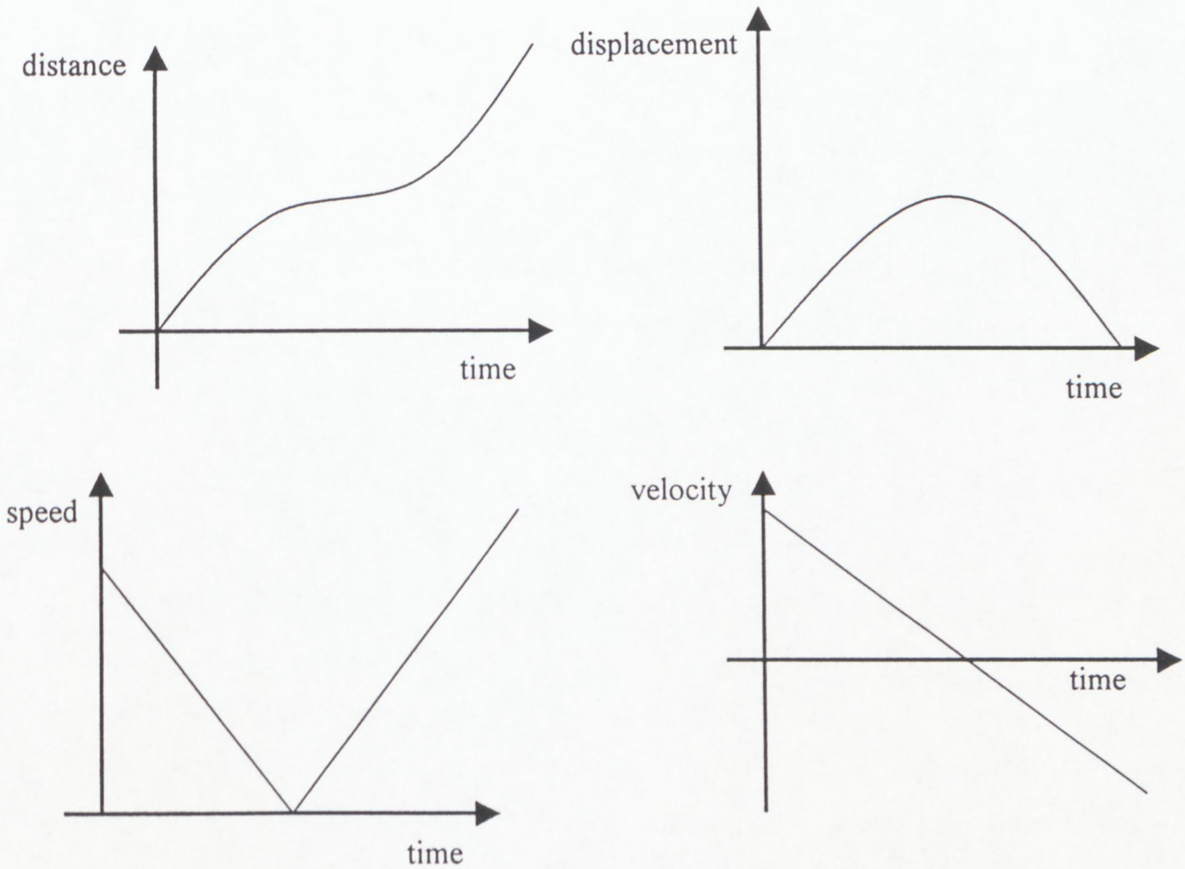


FIGURE 2.45

The acceleration-time graph for the motion is also given:

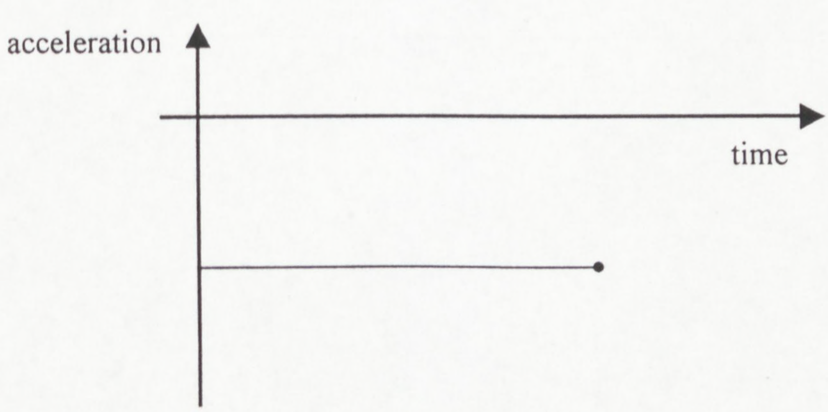


FIGURE 2.46

Brink and Jones (1986:47) state the following as the reason why the graph has zero slope with negative intercept on the acceleration axis:

> *Although the object changes direction the acceleration does not change direction. Since we chose up as positive, the acceleration is negative, indicating that it acts in an opposite direction.*

It should be pointed out that the negative gradient of the *v-t* graph serves as a confirmation that the acceleration of the object is opposite to the direction of motion.

(ii) Broster and James (1986:49) provide *s-t*, *v-t* and *a-t* sketch graphs for motion with constant velocity and uniform acceleration (No mention is made of any real situation and no examples of distance-time or speed-time graphs are provided).

(iii) Heyns *et al.* (1989:50-51) consider the case of a coin tossed vertically upwards and then caught as it falls. The resulting five sketch graphs are similar to those described by Brink and Jones (1986) in (i) above. They provide the following explanation of the conceptually difficult velocity-time graph:

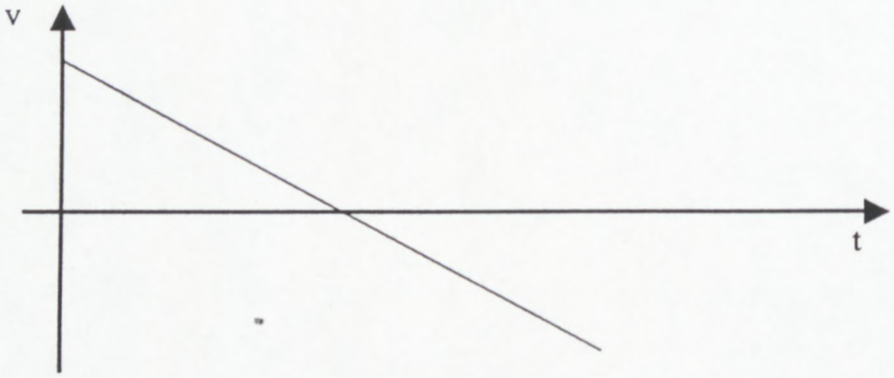


FIGURE 2.47

The graph begins where the velocity has a maximum value. The velocity decreases as the coin moves upwards. The acceleration is *constant, the gradient remains the same and the graph is therefore a straight line. When the coin reaches the highest point (v = 0) the direction changes and the velocity increases to the same magnitude as the initial velocity.* (p. 52)

Mathematically this is not perfectly correct as the graph in figure 2.47 illustrates a decreasing function so that the velocity cannot be increasing at any region of the graph. It would therefore be more correct to say that the magnitude (or absolute value) of the negative velocity increases after the coin has reached its highest point.

Heyns *et al.*(1989) include sketch graphs as a summary of the quantitative trolley (and other) graphs that have been described earlier in the chapter. Unfortunately for the case of a straight line without change in direction, the displacement-time graph is identical to the distance-time graph. Also the shape of the velocity-time graph is identical to speed-time graph. This means that the distinction between the vector vs scalar nature of these graphs is lost.

An obvious error appears on p. 51 when they declare that $v \propto t$ (velocity is proportional to time) for the negative accelerated motion represented by figure 2.48.

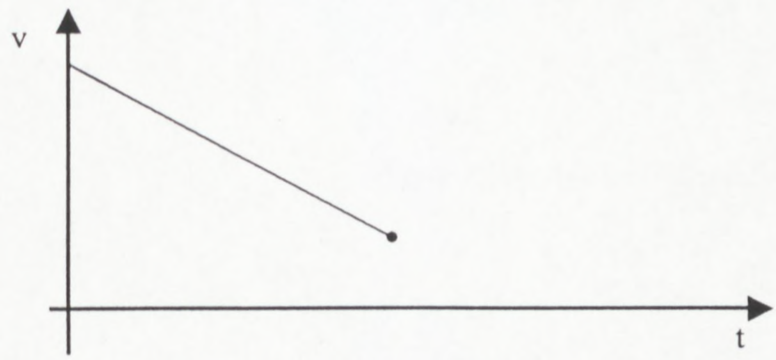


FIGURE 2.48

2.8.3.8 Shortcoming of local textbooks with respect to kinematics

The following aspects of kinematics and kinematic graphs are insufficiently dealt with by local textbook writers: (They are mentioned here without any detailed discussion).

(a) Graphs are usually drawn only for cases where the initial direction of motion was taken as positive.

(b) No mention is made of the importance of frames of reference, especially regarding displacement and velocity.

(c) The concept of a "negative acceleration" is poorly dealt with. Students are not made aware that a negative acceleration can imply speeding up and that a positive acceleration can mean slowing down.

(d) The link between instantaneous rate of change, the gradient of a tangent and basic aspects of calculus is ignored.

(e) Graph transformations, i.e. drawing graphs of related kinematic quantities from a given graph, are not sufficiently discussed.

2.8.4 Senior Certificate Examination Questions

2.8.4.1 General Comments

Examiners of Senior Certificate physics examination question papers play a similar role to textbook writers in determining the interpretation that instructors and learners give to the syllabus. It follows therefore that a content analysis of kinematic graphs will not be complete without a brief discussion of some questions which have been set nationally in the matric examinations over the past few years.

The discussion will take the form of an attempt at classifying a number of sample questions from various examining bodies according to the proposed classification in terms of translations. It must be emphasised that many questions contain sub-sections which can be related to different classification categories. This means that in the context of a particular question it might be possible to distinguish different translations without separating them.

Certain questions are unclear and ambiguous, usually as a result of shortcomings relating to the choice of sign convention and reference point. Others refer to content which is not specifically referred to in the official syllabus. An example of the latter is acceleration-time graphs which are frequently asked in external question papers but get no mention in the syllabus. The fault here seems to be with the syllabus rather than the examiners.

Certain translations involve a visual observation of a specific motion and as such cannot be included in a formal written examination. Consequently type A translations (physical situation → graph) appear only in the context of a verbal description of actual motions.

2.8.4.2 Classification of some questions according to type of translation

Type A : Verbal description of motion of real objects → graph

This translation lends itself to multiple choice questions. A motion of a particular object or particle is described under the influence of the gravitational force (or another type of force). The candidate is then required to choose the correct motion graph from a number of possibilities.

The mental processes which lead the learner to the correct solutions depend on the translation from the description of the object's behaviour into a description of the nature of the changes in the appropriate kinematic quantity. The latter should then be associated with the appropriate sketch graph. This final choice of graph is either based on a direct recall of the appropriately shaped graph or an analytical approach in which the learner matches the nature of the change in a kinematic quantity with the corresponding shape of the graph.

In the case of multiple choice question (a) below, learners could employ the following sequence of cognitive steps:

Ball rolls down incline (physical situation).

⇒ velocity increases (kinematic change) if friction is ignored;

⇒ tangent to *s-t* graph represents velocity;

⇒ *s-t* graph must show increasing tangent;

⇒ choose the correct sketch graph.

Another possibility would be the sequence:

Ball rolls down incline

⇒ velocity increases in absence of friction;

⇒ displacement covered in same time interval increases;

⇒ examine the given *s-t* graphs;

⇒ note that the curved graph gives bigger displacement values for similar time intervals as time increases, by drawing a sketch as in figure 2.49;

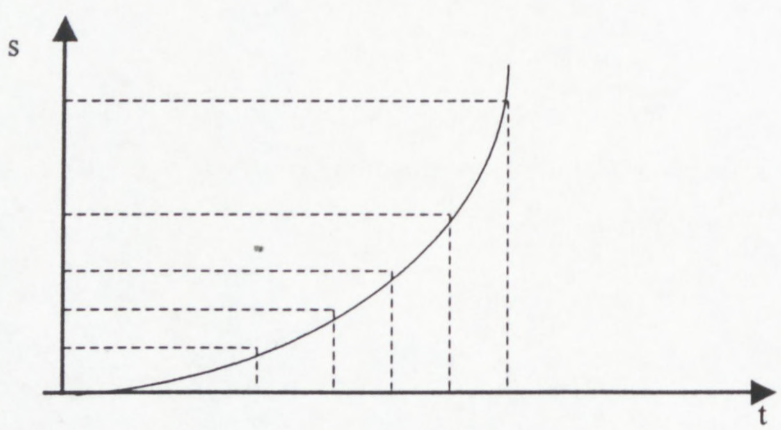


FIGURE 2.49

⇒ choose the appropriate sketch graph.

Examples of Multiple Choice Questions

(a) *A steel ball rolls down an incline of constant slope. Choose the correct displacement-time graph* (Tvl 1993).

answer:

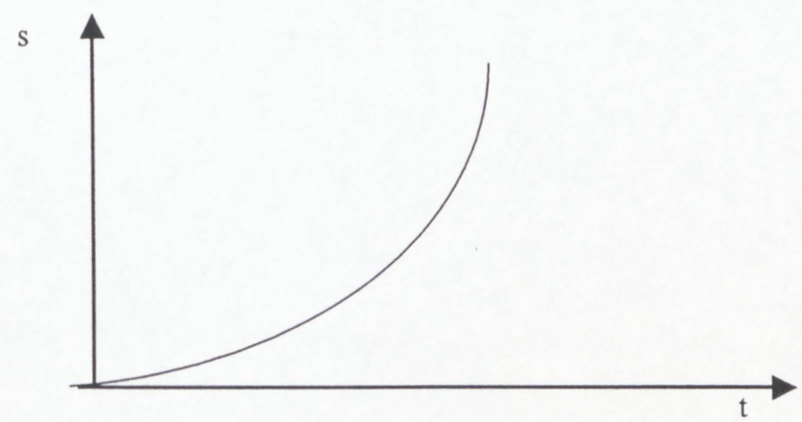


FIGURE 2.50

(b) *A frictionless trolley runs down a sloping runway and then over a frictionless horizontal table. Choose the correct velocity-time graph* (Tvl 1993).

answer:

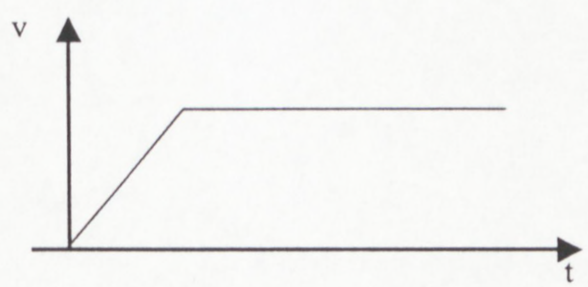


FIGURE 2.51

(c) *An electron moves freely in the electric field of a positive point charge. Choose the correct velocity-time graph* (Cape 1989).

answer:

FIGURE 2.52

[Note that the acceleration is not uniform as the force on the electron is continually increasing. Non-uniform acceleration is not mentioned in the South African syllabus.]

(d) *A ball bounces once after being dropped onto a soft surface. Choose the correct velocity-time graph* (Tvl 1989).

answer:

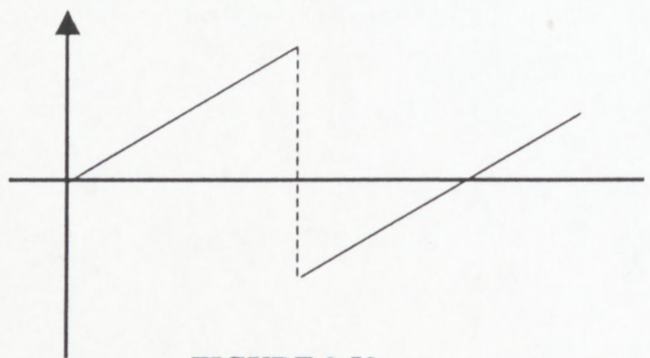


FIGURE 2.53

It must be noted that in none of the questions above was a sign convention suggested. As these were multiple choice type questions, this did not affect the answers; however, the absence of a sign convention can seriously affect pupils' answers in free response type questions.

Examples of "Long" Questions

Example 1

A boy leans over the edge of a 65 m high building and throws a ball vertically upwards. The ball goes up and then falls past the building, striking the ground after 5 s.

(i) *Calculate the initial velocity of the ball;*

(ii) *Show, on a sketch graph, the variation of displacement with time and velocity with time.* (Tvl 1993)

This question is confusing as it does not state clearly whether the 5 seconds refers to the time that it takes for the ball to travel downwards from when it passes the throwers hand until it strikes the ground, or whether it is the total time for the motion. No mention is made of a reference level, neither is a sign convention for up and down suggested. It follows that any of the displacement-time sketch graphs illustrated in the figure 2.54 can be regarded as being correct.

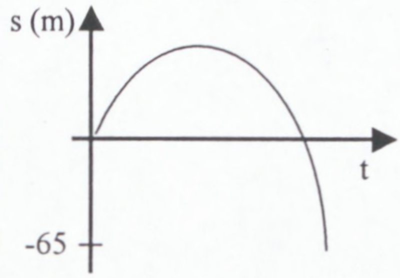
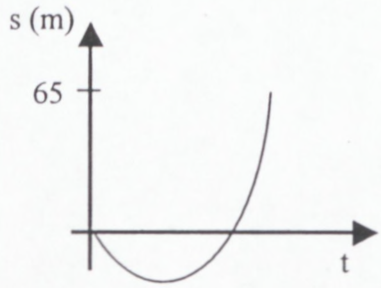
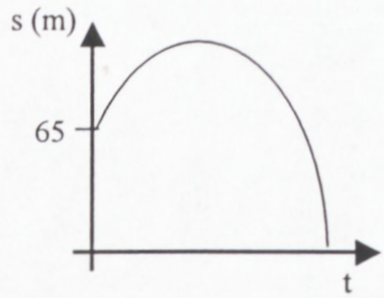
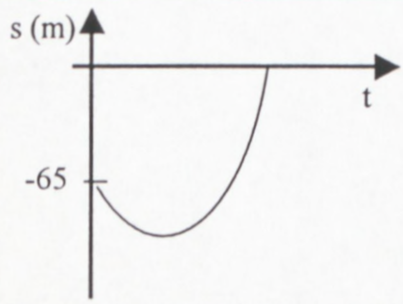
REFERENCE LEVEL	SIGN CONVENTION	GRAPH
Top of building	up is positive	
Top of building	down is positive	
Ground level	up is positive	
Ground level	down is positive	

FIGURE 2.54

The velocity-time sketch-graph can take the shape of any of the two sketches in figure 2.55, depending on the chosen sign convention.

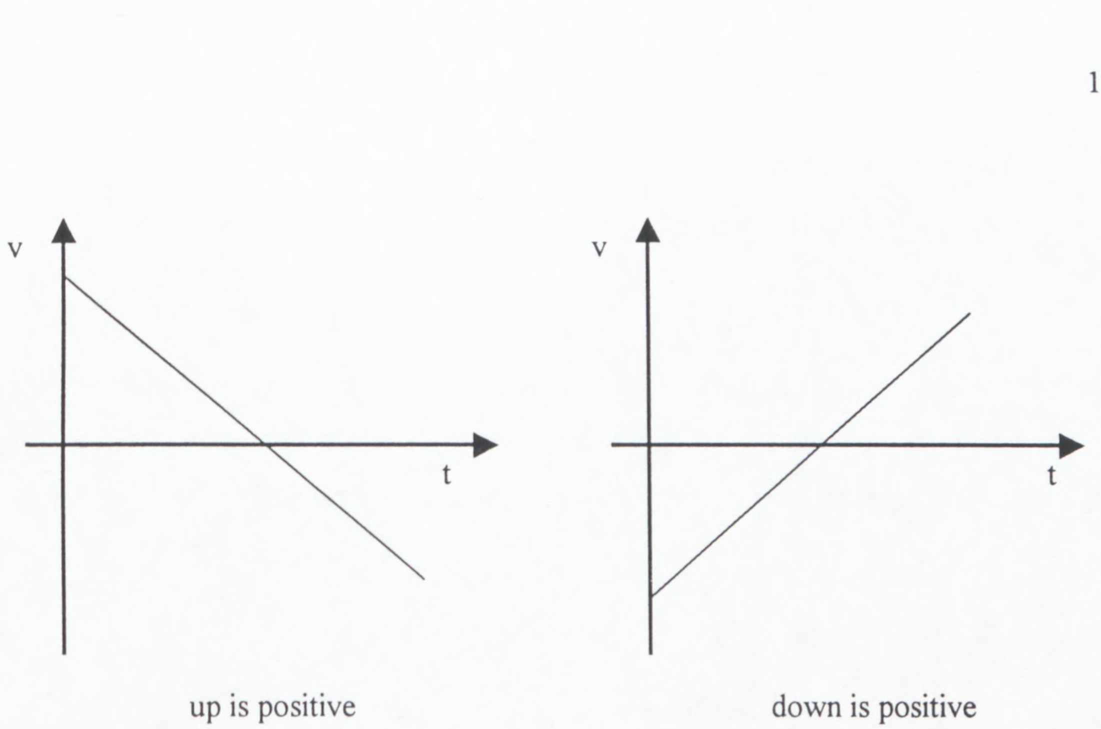


FIGURE 2.55

It is important for both teachers and examiners to emphasise the importance of reference level and the choice of a sign convention so as to avoid the confusion which is illustrated above.

Example 2

A jet aircraft comes in to land and touches down on the runway while travelling at 80 m/s. The aircraft maintains this speed for 6 seconds after which the break system is operated, bringing the aircraft to rest with a uniform deceleration of magnitude 4 m/s^2.

(i) *Illustrate the motion of the aircraft on a labelled velocity-time graph;*

(ii) *Use the graph to determine the length of the runway* (Ind. Exam. Board 1993).

Candidates have to realise that a labelled graph implies that actual values have to be indicated on both axes. This means that they must start off by using a motion equation to find the time during which "deceleration" occurs. They must also view "deceleration" *as negative acceleration which must be indicated by a negative slope on the v-t graph* (if the direction of motion is taken as positive). The association of a horizontal line with the expression "maintains this speed" is required for the initial stage of the motion. It must be noted again that the terms "deceleration" and "retardation" are best avoided as these often leads to confusion .

The required *v-t* graphs are illustrated in figure 2.56. Note that the gradient of the *v-t* graph is positive if the direction opposite to the direction of motion is taken as positive, again illustrating that a positive gradient of a *v-t* graph can in fact indicate the *slowing down* of an object.

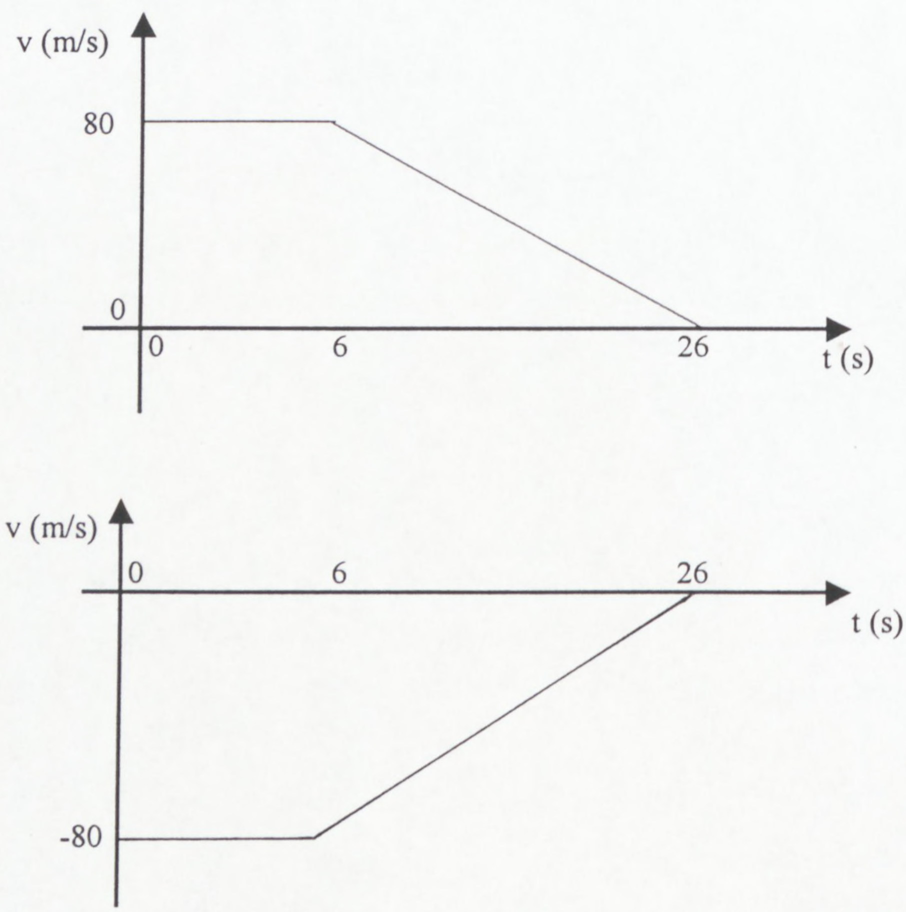


FIGURE 2.56

The determination of the length of the runway is an example of a type B translation where candidates have to associate length of runway with change in displacement which in turn is found from the area under the *v-t* graph.

Type B : Graph → Verbal description of the motion of real objects

A typical example of a question requiring this translation is the following:-

Example 1

The following sketch graph depicts the motion of a motor vehicle. Describe in detail the motion of the vehicle.

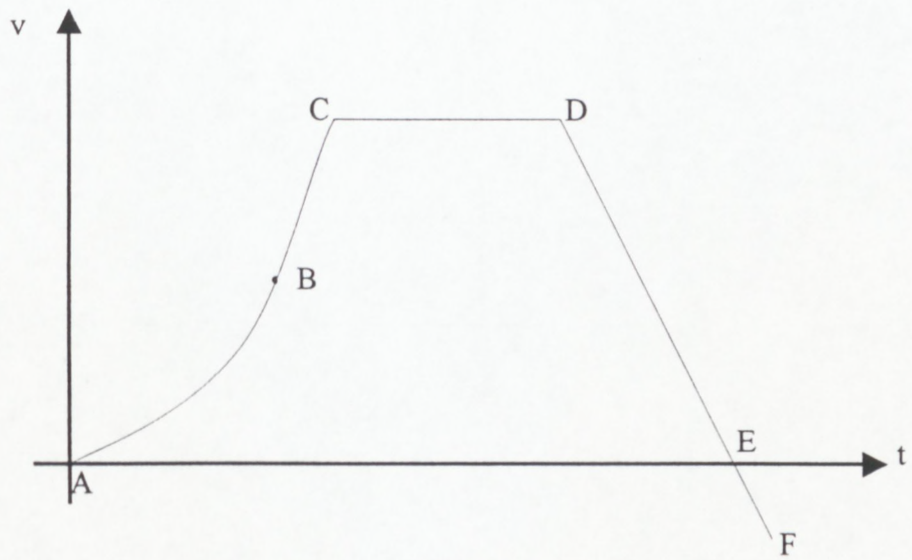


FIGURE 2.57

(N.S.C. 1993)

Solution

The vehicle starts from rest moving with increasing acceleration till B after which it accelerates uniformly up to C. It maintains a constant velocity between C and D and then experiences an acceleration against the direction of motion, coming to rest at E. Between E and F the vehicle moves with uniform accelerated motion in the opposite direction to the initial (positive) direction.

This illustrates how a graph can contain very detailed information about a motion in compact symbolic form. It also requires an examination candidate to understand that a negative slope indicates either a decreasing speed in the original direction or an increasing speed in the opposite direction.

Example 2

Another type B question addressing the concept of negative acceleration, which can cause some confusion for examiner and candidate alike, is the following:

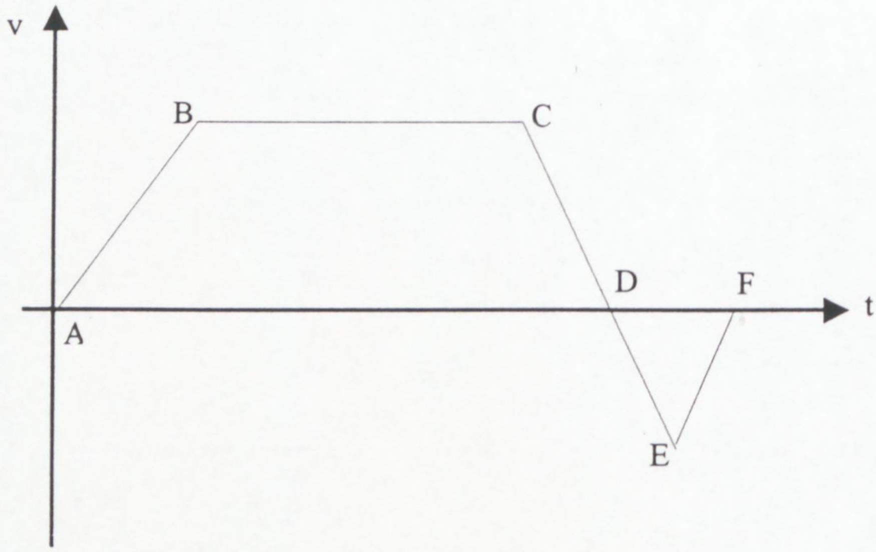


FIGURE 2.58

The graph of figure 2.58 depicts a motor vehicle initially moving in an easterly direction. The motor vehicle accelerates in a westerly direction between

(i) *AB*
(ii) *BC*
(iii) *CD*
(iv) *DE*
(v) *EF* (N.S.C. 1993)

The intention is obviously that (iv) is meant to be correct. It seems more accurate, however, to argue that the line CE (consisting of CD and DE) indicates a negative acceleration which is clearly in a direction opposite to the original positive acceleration as indicated by AB. Hence the car accelerates in a westerly direction between C and E. The *a vs t* graph of figure 2.59 illustrates the reversal in direction of the acceleration vector very effectively:

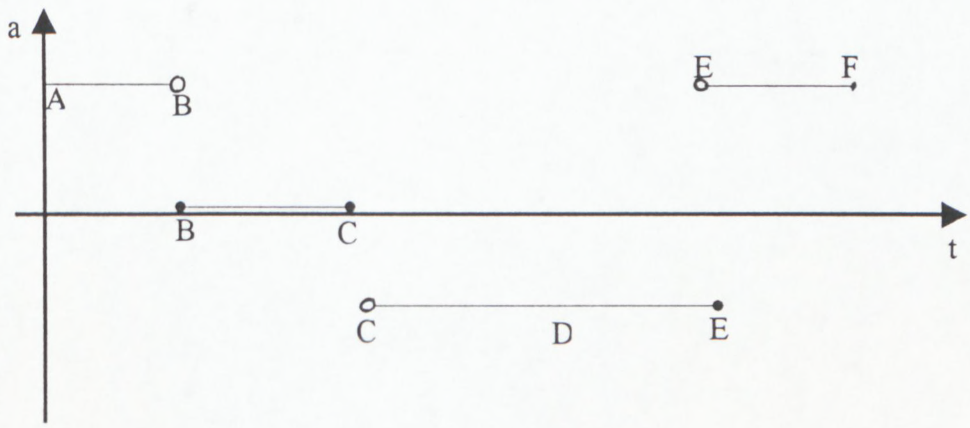


FIGURE 2.59

Example 3

Some questions require students to execute a variety of translations and transformations. The following question contains questions of type B (graph → physical situation) as well as E (graph → related graph) The solution is not included but reference is made to the required translation relevant to each sub-section.

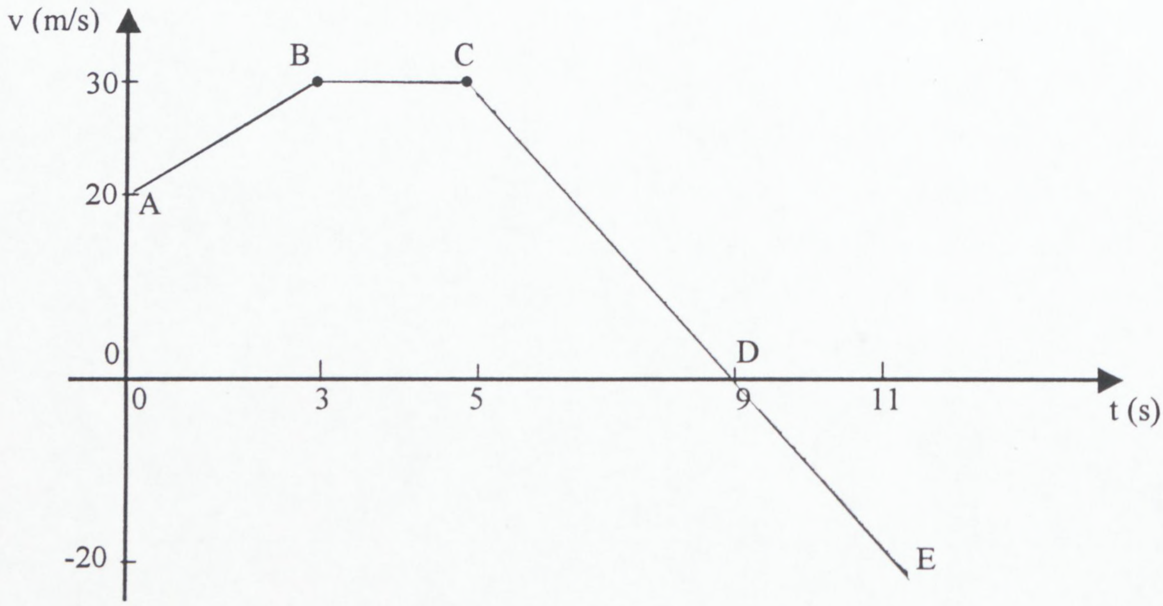


FIGURE 2.60

The motion of an object travelling in a straight line is represented by the velocity-time graph shown in figure 2.60. Answer the following questions, using the graph:-

(i) *At which point is the object furthest from its starting point? (type B)*

(ii) *For what stage of the motion is the object travelling opposite to its initial direction? (type B)*

(iii) *Find the acceleration of the object between A and B. (type B)*

(iv) *Find the displacement of the object for the entire journey. (type B)*

(v) *Draw a rough sketch graph of acceleration against time for the entire journey. (type E)* (Tvl 1989)

Example 4

The following graph shows the displacement of an object to the right of its initial position as a function of time.

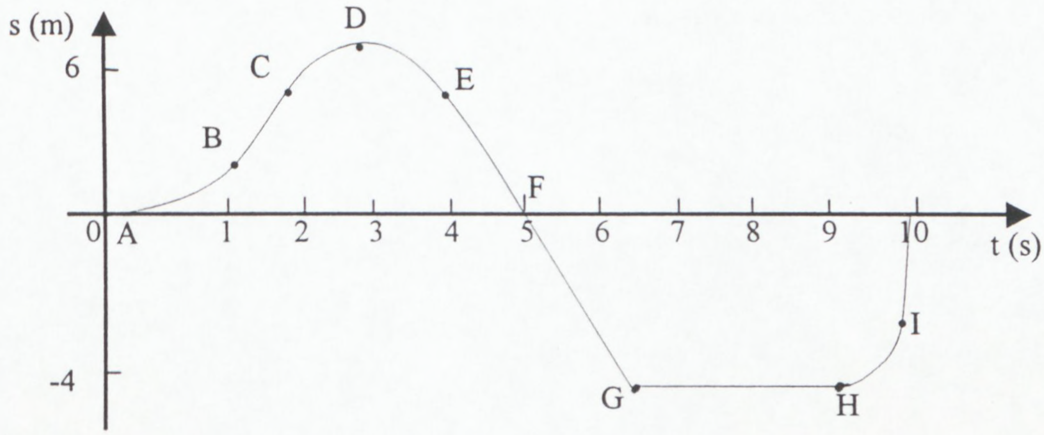


FIGURE 2.61

Make use of the letters on the graph to indicate between which times the object is:-

(i) *stationary for a length of time;*

(ii) *moving with greatest speed;*

Use the graph to find:-

(iii) *the total distance travelled by the object;*

(iv) *the displacement of the object from $t = 3$ s to $t = 10$ s;*

(v) *the instantaneous velocity of the object at position D;*

(vi) *whether the acceleration of the object at position D is zero or non-zero. Give a reason for your answer;*

(vii) the velocity of the object between E and F. (Cape 1989)

In this type of question candidates are expected to associate certain specific features on the graph with the required kinematic concept. The features of the given *displacement-time* graph which have to be used in the cited example are respectively:-

(i) the horizontal section;

(ii) the slope with maximum magnitude;

(iii) the maximum and minimum values;

(iv) the difference between the ordinates of the given co-ordinates;

(v) the gradient of the tangent at a point on the graph;

(vi) whether the gradients of the tangents are changing in the immediate vicinity of D;

(vii) the gradient of the straight line and the recognition of the negative slope indicating a negative velocity.

Type E : Graph → Graph

For the successful transformation from one motion graph to another the candidate has to be able to make the proper correspondence between the features of one graph (eg. slope, turning points or whether it is increasing or decreasing) and the shape of another related graph.

The given graph as well as the required graph can be either a sketch graph, a semi-accurate graph or an accurate quantitative graph indicating actual values on the axes.

The following examples from past senior certificate papers serve as a good illustration of a graph → graph transformation.

Example 1

The following acceleration-time graph is given:

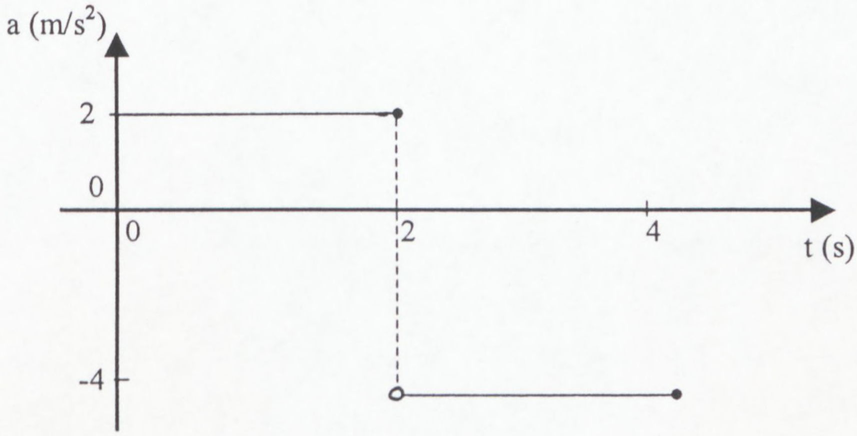


FIGURE 2.62

Draw the corresponding velocity-time graph and show actual values on your graph (OFS 1993).

In this example of semi-accurate graph transformations candidates have to realise that the height of the given graph corresponds with the slope of the required graph. The positive section of the given graph indicates a positive slope while the negative section has to be associated with a negative slope on the required *v-t* graph. An initial velocity of zero is assumed in the following solution:

Solution:

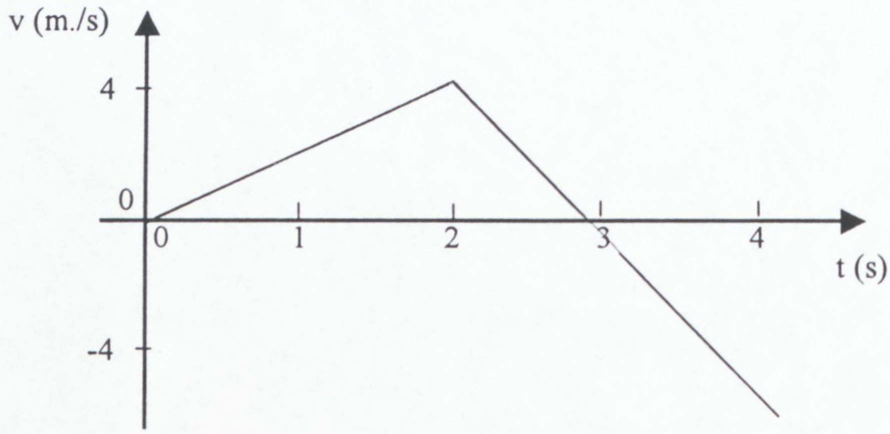


FIGURE 2.63

The following points must be noted with respect to the above question and solution:

- the given *a-t* graph gives information about the <u>change</u> in velocity;

- the area under specific sections of the *a-t* graph may also be used to give the change in velocity during that time interval;

- the original direction of motion was chosen as positive;

- an infinite number of *v-t* graphs with the same shape as that of figure 2.63 but different initial velocities can be drawn.

Finally, three examples of qualitative graph transformations are given (see figure 2.64). In these cases it is the <u>shape</u> of the graphs which is of importance, as no calculations of co-ordinates are required (motion starting from rest is again assumed, in other words $v = 0$ at $t = 0$).

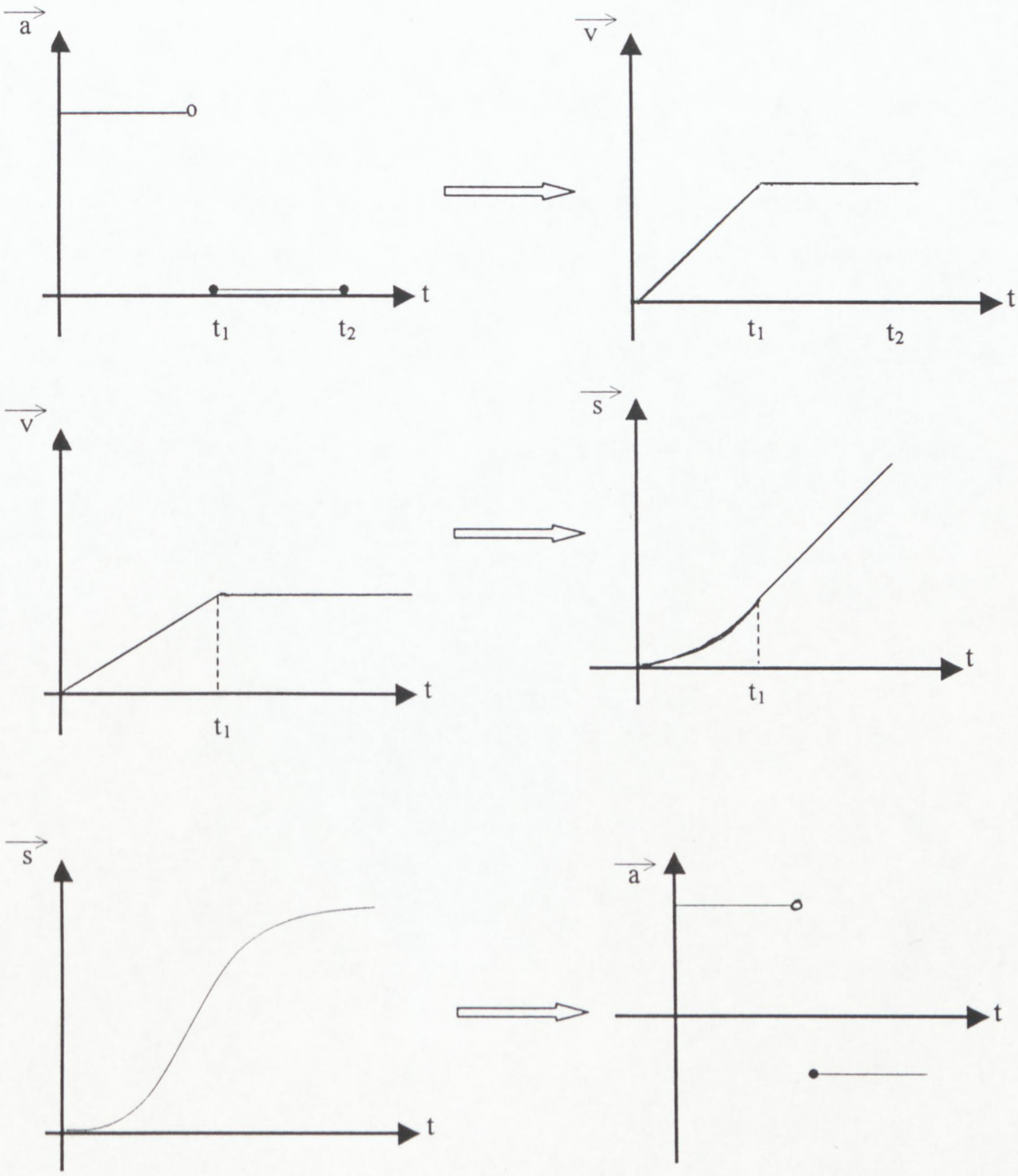


FIGURE 2.64

It is strongly suspected that candidates experience some difficulty with this type of translation as the level of abstraction is fairly sophisticated. The shape of the required graph also usually differs considerably from the given graph. The extent of this suspected difficulty will be investigated in Chapters 4 and 5 of this study.

2.8.4.3 Other Translations

The transformations involving tabulated values and formulas are not frequently asked in senior certificate questions. This fact does not make them less important however. Their value lies in illustrating to students how the same kinematic situation can be described by different symbolic systems and how an investigator can translate between these inter-related representations. Peters (1982:501) emphasises the fact that many concepts in physics can be represented in different ways:

> *Students should be able to approach a physical concept from all directions, from the observable phenomena to the verbal, symbolic and mathematical representations of that concept.*

2.9 THE APPROACH TO KINEMATIC GRAPHS IN A NUMBER OF OVERSEAS TEXTBOOKS

2.9.1 General Comments

The following books which are in use in the senior secondary phase in the U.K. will be commented upon:

(i) Revised Nuffield Physics - Editors E.M. Rogers and E.J. Wenham (1978).

(ii) Physics: A Course for GCSE by G. Rowell and S. Herbert (1987).

(iii) Physics: An Examination Course by E.L. Hanson (1989).

(iv) Coordinated Science - Physics by S. Pople (1989).

A commonly shared feature of all these books is that they are written in uncomplicated easily readable language. Furthermore, a great emphasis is placed on the everyday experiences of students. The authors of these textbooks take advantage of the natural facility that pupils have with concepts such as "speed" and "distance". Such familiar ideas as pupils cycling to school and running rabbits are naturally explored and developed to show how these motions can be represented in a clear and unambiguous manner.

It is significant to note that all the authors, with the exception of Rogers and Wenham, introduce ticker-tape experiments only after motion graphs have been explained from a theoretical perspective. Hanson as well as Rowell and Herbert start off with sketch graphs to indicate general tendencies while Pople uses tables of values of distance and speed against time to draw accurate motion graphs.

Rogers and Wenham follow the local tendency by introducing ticker-timer experiments at the beginning of their section on motion graphs and explain the significance of slope and area by referring to data points generated by motion trolleys undergoing constant accelerated motion.

Rowell and Herbert's presentation of motion graphs (in the form of sketch graphs), describing a ball thrown vertically upwards with a velocity "u" from the top of a cliff of height "h" falling to the beach below, deserves repeating. These graphs are relatively important for the following reasons:

(i) They clearly illustrate the need for a reference level (in this case the beach).

(ii) Their choice of a positive direction (in this case upwards) is important for the drawing of motion graphs.

(iii) They make a clear distinction between vector motion graphs and scalar motion graphs.

(iv) They illustrate the fact that plotting the *vector* quantities (displacement, velocity and acceleration) against time on a graph gives more useful information than using the corresponding *scalar* quantities.

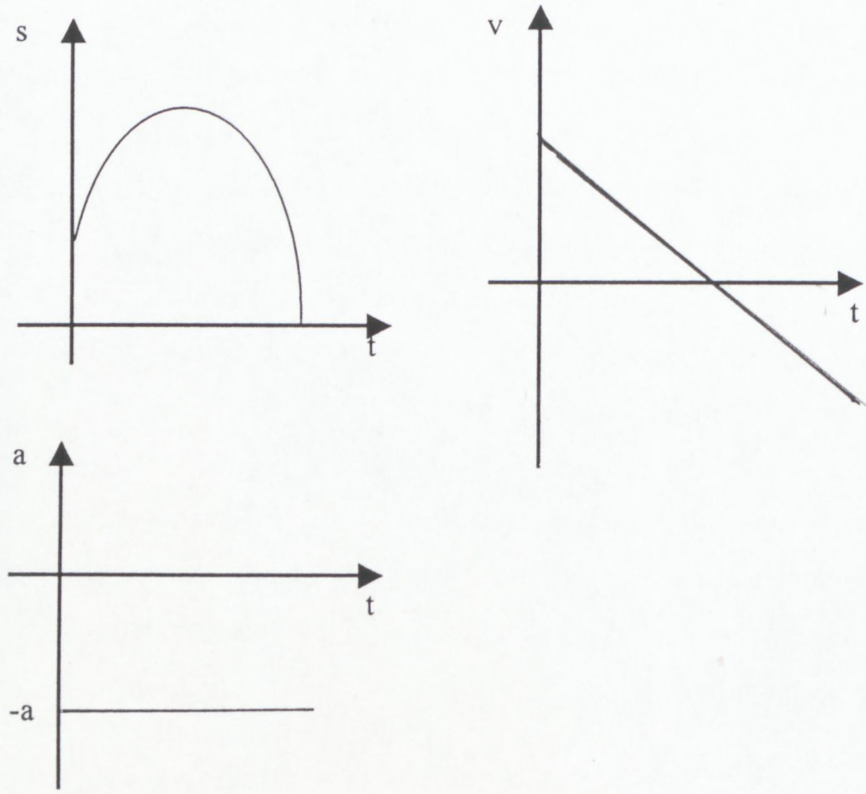
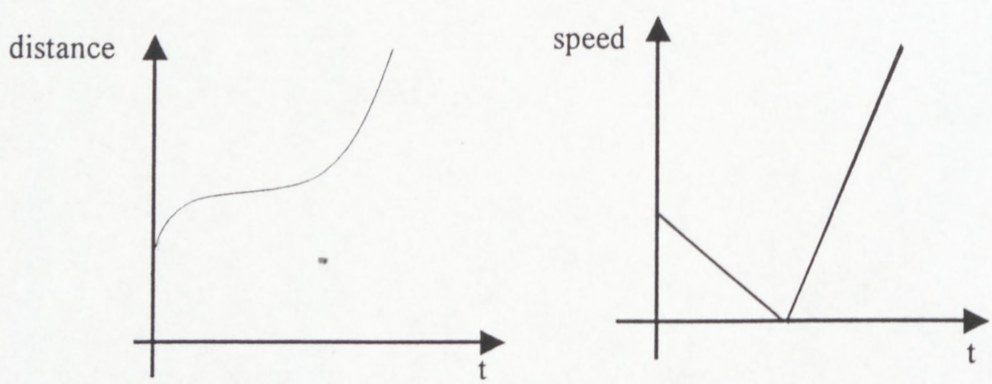


FIGURE 2.65 Graphs of vector quantities



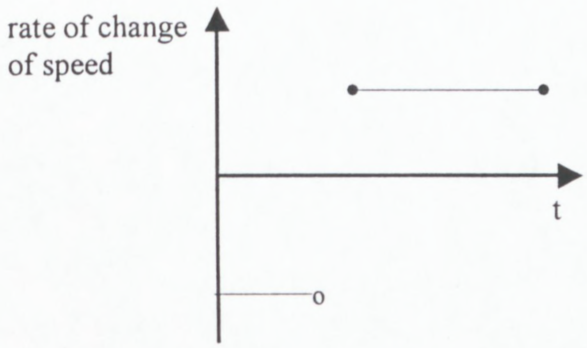


FIGURE 2.66 Graphs of scalar quantities

The graphs of figures 2.65 and 2.66 clearly illustrate how different kinematic graphs represent the same physical situation. Students should (in accordance with translation E) demonstrate their ability to transform any of these graphs into another related one.

2.9.2 Tape Charts

The idea of a tape chart seems to be regarded as fundamental to the practical determination of velocity and acceleration in the U.K. All the books which have been studied place a heavy emphasis on tape charts — an idea which is not regarded as fundamental in local syllabi and textbooks.

The idea is that students generate their own charts by placing side by side the lengths of strips that were generated by a ticker-timer in the same time interval. The following figure illustrates a typical tape chart as explained by Rogers and Wenham (p. 15).

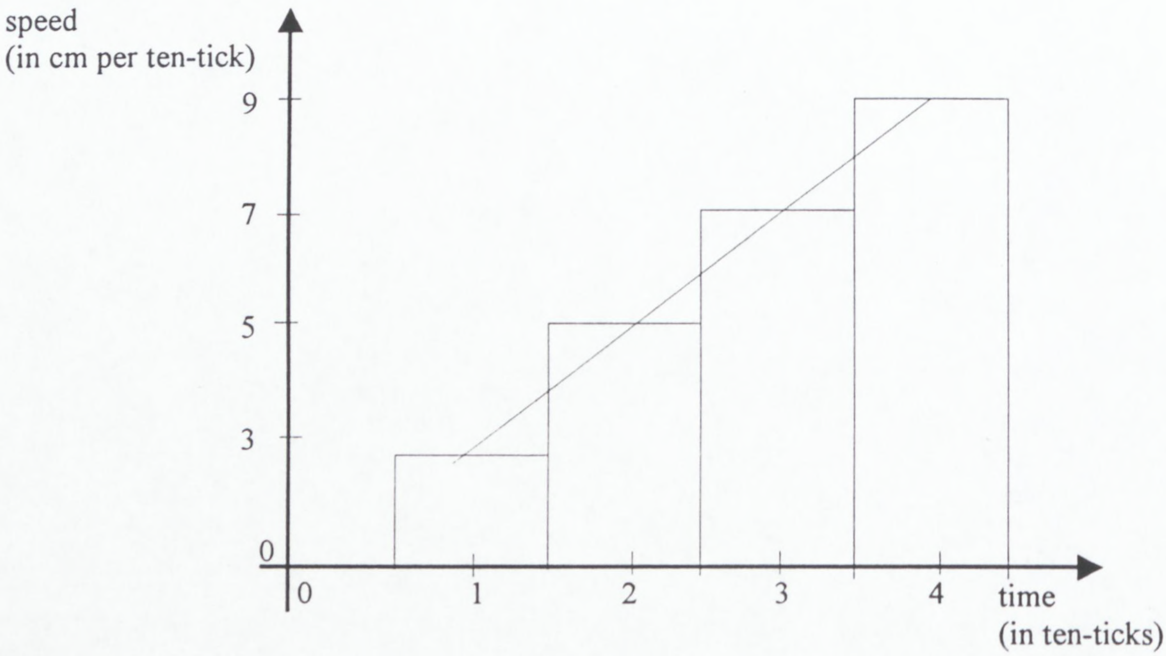


FIGURE 2.67

They proceed to guide the students towards a thorough understanding of these charts and their use as a representation of speed as well as their (indirect) illustration of acceleration. They do this by making use of statement and questioning techniques, for example (with reference to the above figure):

- The chart shows the motion of a trolley moving with constant acceleration.
- What is the increase in speed between (i) tentick 1 and tentick 2; (ii) tentick 2 and tentick 3; (iii) tentick 3 and tentick 4?
- How much is the acceleration of this trolley (in cm per tentick in one tentick)?
- How far (in cm) did the trolley move during (i) the first tentick; (ii) the second tentick; (iii) the first two tenticks?
- How far did the trolley move during the first four tenticks?

- A slanting height is drawn through the tops of the tape lengths. That line may be called a speed-time graph.

2.9.3 Equations of motion

The speed-time graph can be used to derive the equation:

$s = ut + \frac{1}{2}at^2$

Rogers and Wenham (p. 37) give a neat exposition of this familiar proof, when they refer to the concepts of speed and distance rather than the more familiar concepts of velocity and displacement.

Consider the following speed-time graph of an object which is accelerated uniformly from an initial speed of u to a final speed of v in t seconds.

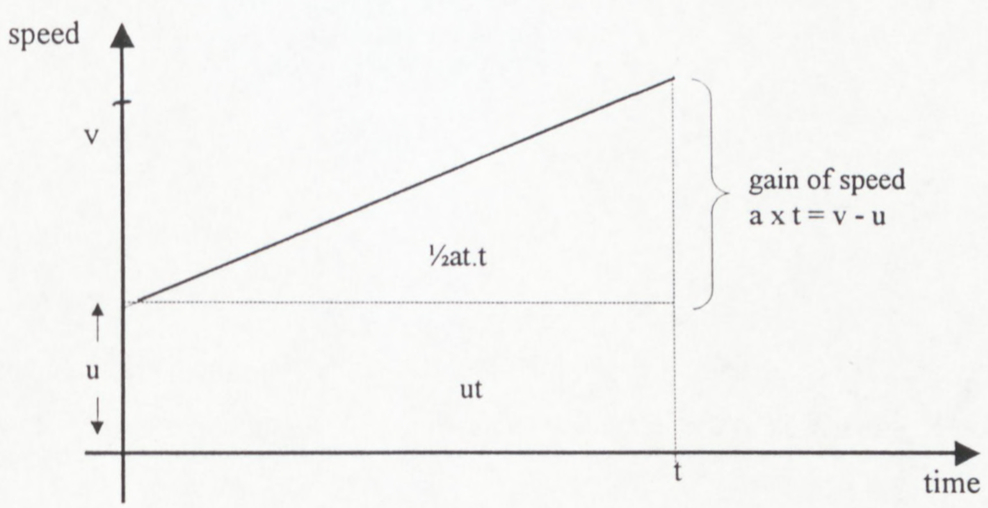


FIGURE 2.68

The rectangle's area is ut and the triangle's area is ½t x (at) or ½(v - u)t.

Now let s be the distance that the object covers in t seconds, then s = area under the speed - time graph

$$\therefore s = ut + \tfrac{1}{2}(v - u)t$$

$$= ut + \tfrac{1}{2}vt - \tfrac{1}{2}ut$$

$$= \tfrac{1}{2}vt + \tfrac{1}{2}ut$$

$$= \frac{v + u}{2}t$$

$$\text{or } s = ut + \tfrac{1}{2}t \times (at)$$

$$= ut + \tfrac{1}{2}at^2$$

It must be pointed out that the students are gradually lead from accepting the idea of distance being equal to the area of the ticker-tape strips placed side by side, to the more abstract deduction of the distance formulas based on the total area under a speed-time graph.

2.9.4 Typical Examples

Two typical examples taken from British text books illustrate the conceptual approach to motion graphs as well as the level of understanding that is required of 16/17 year olds.

Example 1

A man runs a race against a dog. Here is a graph showing how they moved during the race.

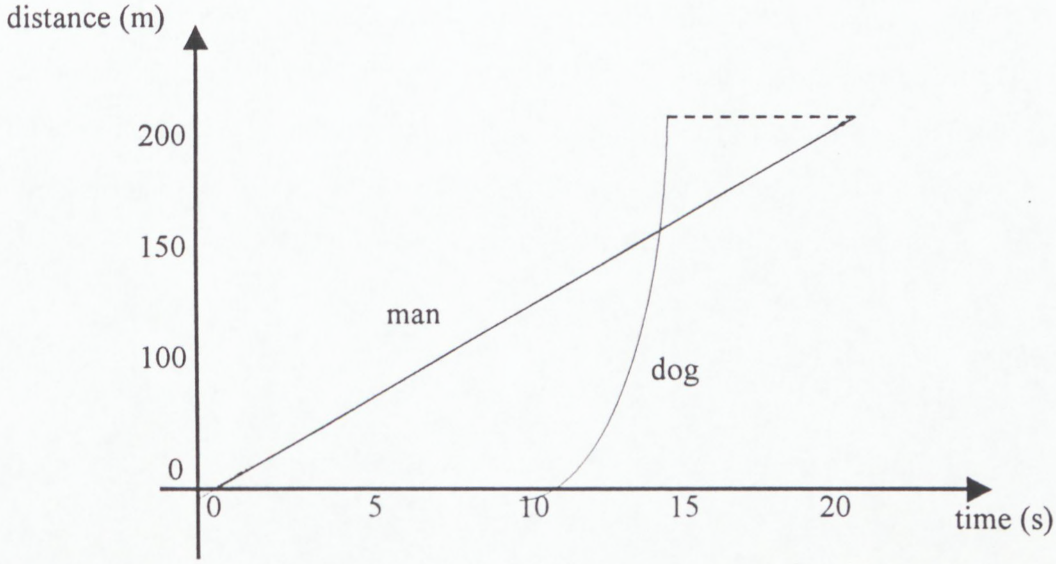


FIGURE 2.69

(a) *What was the distance for the race?*

(b) *After how many seconds did the dog overtake the man?*

(c) *How far from the start did the dog overtake the man?*

(d) *What was the dog's time for the race?*

(e) *What was the average speed of the man?*

(f) *After 8 seconds is the speed of the man increasing, decreasing or staying the same?*

(g) *What is the speed of the man after 18 seconds?*

(Pople 1989:214)

The question is an example of a type B translation (Graph → kinematic quantities). It poses a real-life problem with which pupils can identify themselves easily and is stated in clear uncluttered language. The first four questions involve the reading off of appropriate value on the axes while the last three questions require a slightly deeper understanding of the kinematic principles involved.

Example 2

An object was moved from rest and its speed recorded every second. The following results were obtained:-

Time (s)	*1,0*	*2,0*	*3,0*	*4,0*	*5,0*	*6,0*	*7,0*	*8,0*
Speed (m/s)	*0,28*	*0,53*	*0,78*	*1,06*	*1,33*	*1,35*	*1,34*	*1,31*

(a) *Plot a graph of the speed (y-axis) against time (x-axis).*

(b) *Use the graph to find (i) the time at which the acceleration changed, (ii) the average speed 7,5 s from the start of the motion.*

(c) *State the type of motion before and after 5 seconds have elapsed.*

(d) *Use the graph to calculate the average acceleration 3,5 s from the start of the motion.*

(e) *Outline any one experiment which could be used to find data similar to those shown in the table.*

Hanson (1989:14)

Question 2(a) requires an A(c) translation (see p. 47) and is a good example of how a graphical representation of tabulated values often ensures that trends in the data can be recognised more easily.

The last question (e) deserves a comment. Students are required to make the translation by making use of a quantitative graph that they have drawn themselves from a table of values. The idea is that students describe a configuration similar to that of figure 2.70 below:

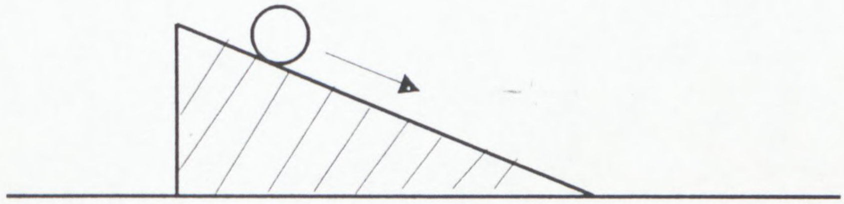


FIGURE 2.70

They have to make the connection between certain features of a graph (for instance positive gradient of a velocity-time graph) with the characteristics of a physical situation (for instance a ball moving with constant positive acceleration down a frictionless slope). When the ball reaches the horizontal section it moves with zero acceleration. The student must associate this with the horizontal section of the velocity-time graph.

2.9.5 An American College Textbook approach

Kane and Sternheim's book: Physics SI Version (1976) is a comprehensive American textbook dealing with introductory pre-calculus algebra-based physics for first year college students. The following points are noted with respect to the section on kinematic graphs:

Kinematic concepts and principles are initially introduced from a purely algebraic perspective. The results are then interpreted graphically and students are encouraged to explore the usefulness of the graphical representations.

Average velocity is defined as

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

and instantaneous velocity as the ratio $\frac{\Delta x}{\Delta t}$ when Δt approaches zero. The graphical interpretation is then stated in terms of the slope of an x-t graph and the slope of the tangent to a curved x-t graph for average and instantaneous velocity respectively.

They also state that acceleration is often a known quantity and from this known value one should be able to determine the corresponding velocity and position changes. The significance of a graphical interpretation of the results is stressed as follows:

By definition $a = \frac{\Delta v}{\Delta t}$

Now
$$\Delta v = a\Delta t \text{ or } v_2 - v_1 = a(t_2 - t_1)\,0$$
$$v = at \text{ if } v_1 = 0 \text{ when } t_1 = 0$$

Also the area under a graph of a vs t in fig (i) is the product of its height a and its width t. This is the velocity change *a.t.*

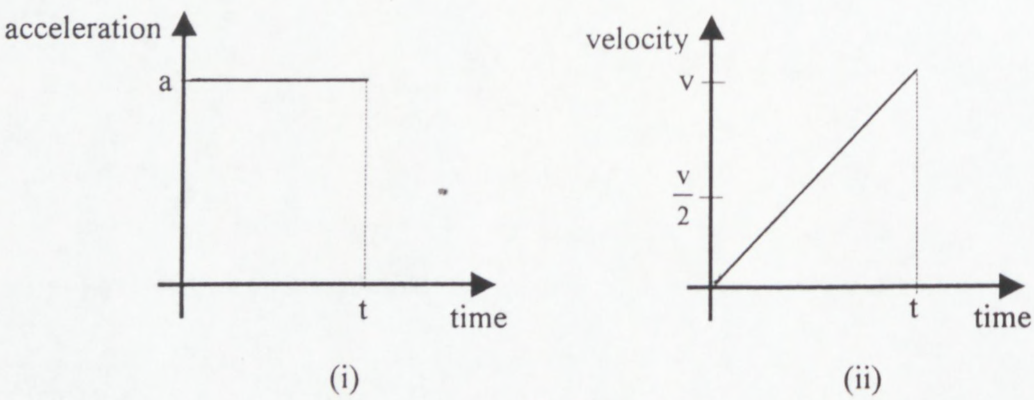


FIGURE 2.71

Thus the change in velocity equals the area under the *a-t* graph over the time interval chosen.

To find the position (x) after a time t a similar route is followed:

By definition $\bar{v} = \dfrac{\Delta x}{\Delta t}$

$$\Delta x = \bar{v}\Delta t$$

But

$\bar{v} = \dfrac{v}{2}$ for the case of uniform acceleration illustrated in graph (ii)

Thus $x = \frac{1}{2}vt$ (assuming $x_1 = 0$ at $t_1 = 0$)

and since $v = at$, this can be written as

$$x = \tfrac{1}{2}(at)t$$

$$= \tfrac{1}{2}at^2$$

Now the area of the shaded triangle under the *v-t* graph of figure (ii) is ½ (base) x (height)

$\therefore$ Area = ½vt

But $x = \frac{1}{2}vt$ (proved above)

Thus the change in position is equal to the area under the *v-t* graph over the corresponding time interval.

It seems clear from this exposition that the graphical interpretation of the relationship between different kinematic concepts is viewed as essential by the author. Students need to handle any subject matter from different perspectives for a clear understanding of the principles involved. The algebraic method together with its graphical interpretation are two such perspectives on kinematics.

2.10 Closing comments

This chapter deals with those aspects of the syllabus contents of kinematics that can be represented graphically. The importance of graphical, algebraic and tabular representations as examples of more abstract, mathematical type of representations is contrasted with the less abstract, verbal description of kinematic events. The importance of the ability to translate between the various forms of translations is stressed.

The remainder of this study will make use of the five categories of translations that were identified in order to attempt to categorise students' misconceptions with regards to translations involving kinematic graphs. Chapter 3 deals with misconceptions identified by other researchers, while Chapters 4 and 5 reports on an own empirical investigation into the problem.

CHAPTER 3

A REVIEW OF LITERATURE CONCERNING THE UNDERSTANDING OF GRAPHS OF MOTIONS

3.1 INTRODUCTION

In this chapter a brief review of the available literature on students' understanding of kinematic graphs will be given. It will be shown that several investigations yielded results which suggest that students often fail to acquire a proper understanding of some of the basic concepts, parameters and relationships relating to kinematics and the graphical representation of kinematic quantities.

Constructivism as a model for learning and the question of what is meant by understanding has to be addressed before research findings with respect to kinematic graphing difficulties are described. Students' kinematic graphing problems could be due to a lack of experience with graphs in general, or an incomplete understanding of the actual kinematic concepts, or an inability to link the two. For this reason research findings on graphing problems will initially be described separately from the results of studies which investigated understanding of kinematic concepts.

A fundamental understanding of *proportional reasoning* and the *ratio concept* is necessary to fully understand all aspects of a graph as a symbolic representation of the relationship between two variables. Another important concept which is crucial to the understanding of kinematic graphs is "rate of change". It will be shown that students generally have a very low level of understanding of this concept and that this is a main contributor to many misconceptions, not only with regard to motion graphs, but also the study of calculus.

3.2 SOME THEORIES OF LEARNING AND THE ROLE OF STUDENTS' PRECONCEPTIONS

3.2.1 Piaget's theory of learning and intellectual development

(a) Intellectual development

Although Piaget's ideas on intellectual development in itself cannot be classified as a *learning theory*, it has to be described briefly in order to understand his learning theory more deeply.

During the greater part of the 1970's Piaget stood out in the eyes of science and mathematics educators as a psychologist whose theory of intellectual development was viewed with a great deal of respect. Attention was mainly focused on his four stages or levels of human intellectual development, namely the *sensory-motor*, the *pre-operational*, the *concrete operational* and finally the *formal operational* level. The first two are usually reached by the time that the child is approximately eight years old, so that these two stages are not usually applicable to post-primary school students.

Researchers have, however, been inspired to use various Piagetian tests to determine whether secondary students in a certain age group (or specific school grades) operate at the *concrete* or *formal operational* stage (see Andersson 1987:13).

Andersson (1987:14) goes on to point out that the teaching material in physics often demands well-developed formal thought from as early as grade 7 (12 - 13 years of age). Aspects such as reasoning with models, systematic logical thinking and proportionality are experienced as very difficult by pupils aged between 12 and 16 because many of them have not yet reached the formal operational stage.

There appears to be a great deal of disagreement about the approximate age that a significant proportion of learners can be expected to have reached the formal reasoning stage. According to Piaget himself (Herron, 1975:146) a learner should have completed his basic intellectual development by the age of 15, which means that most learners should be able to function at the formal operational stage *during the last two years* of secondary school.

Wavering (1989:373) investigated students ability to use the formal operational structures required for *proportional reasoning* and *correlational reasoning* in constructing graphs. He confirmed Piaget's finding, namely that only students who had reached at least grade 11 were able to score consistently high marks in test items which required formal operational reasoning strategies in order to be able to carry out the tasks required by the investigation.

Other studies suggest that many *tertiary* students have not yet reached the stage where they can use formal reasoning ability. According to Herron (1975:146), approximately 50 % of first year tertiary students in the United States do not function effectively at the formal operational stage, while Marais (1997:11) quotes Silberman who puts this figure at 78 %.

Andersson (1987:14) points-out that many modern researchers, whilst acknowledging Piaget's contribution, believe that a person's thinking should be seen as *local* and *content related* rather than as a result of the fact that he finds himself at a certain stage of thinking. According to Andersson (1987:14):

> *The main focus of research shifted from the* ***forms*** *of scientific thinking to its* ***content****.*

(b) Piaget's theory of learning

According to Wadsworth (1979:9-19), Piaget views any cognitive act in terms of an individual's attempt to organise his perceived environment and attempt to adapt to it. To explain this in terms of the individual's mental development, Piaget introduces four basic concepts, namely the concepts of *schema, assimilation, accommodation and equilibrium.*

Schemata are described in terms of cognitive structures or concepts that the individual uses to adapt to and organise the environment. As intellectual development takes place through the various stages, these schemata become increasingly more sophisticated, formal and abstract, i.e. the structures adapt to the particular stage in the cognitive development that an individual finds himself at at that moment. The processes responsible for the change are called assimilation and accommodation.

Assimilation is described in terms of the cognitive process by which new perceptual matter or stimuli are integrated into existing schemata. The process of assimilation allows for the growth of schemata which are already established and does not attempt to change them significantly. Accommodation, however, does allow for the creation of new schemata or the modification of existing ones. If a new stimulus does not fit in with any of the existing schemata, the process of accommodation is used to place it into a modification of existing schemes or into new schemes. The difference between assimilation and accommodation can be described in terms of the fact that in the former, a new conception is forced into existing structures, while in the case of accommodation, the individual is forced to change his structures to fit the new ideas. It follows that assimilation accounts for growth (i.e. a *quantitative* change), while accommodation accounts for development (a *qualitative* change).

Piaget stresses the importance of a good balance between assimilation and accommodation and refers to this balance as *equilibrium*. The individual upon experiencing a new stimulus, tries to assimilate it into an existing schema. If he is successful equilibrium is attained for the moment. If he cannot assimilate the stimulus, he then attempts to accommodate by modifying a schema or creating a new one. When this is done assimilation of the stimulus proceeds and equilibrium is again reached. *Cognitive conflict* refers to an imbalance or disequilibrium between assimilation and accommodation caused by a discrepancy between what the individual expects to happen and what really happens. Such a "cognitive conflict" motivates the person to seek equilibrium once again.

3.2.2 The constructivist theory

Piaget's theory of learning has influenced the development of the constructivist theory of learning and knowing. Andersson (1987:14) summarises the key aspect of this approach in terms of a recognition by the instructor that the learner is active and creative and that ... *he himself constructs his conceptions, and attempts to understand something knew with the knowledge that he already has.*

This means that students should be challenged to change or restructure their preconceptions to make sense of their intellectual environment. The focus of the theory is that people do not learn passively but are always actively engaged in trying to make sense of the world in which they find themselves.

> *Children naturally attempt to make sense of the world in which they live in terms of their experiences, their current knowledge and their use of language.* (Osborne and Freyberg 1985:13).

In trying to make sense out of a situation, learners utilise all their existing knowledge, i.e. actual experiences, book learning, learning from parents and the media as well as through active interaction with the environment. They thus bring all relevant knowledge about a concept to a learning situation, resulting in what Hewson (1987:32-33) refers to as ... *a dynamic interaction between an individual and the facts and events of the world plus the knowledge acquired from cultural, social or philosophical sources. All of these constitute the intellectual environment of the individual.*

Wheatley (1991:10) views the theory of constructivism as based on two principles. The first one states that ideas and thoughts cannot merely be placed by instructors in the heads of learners. Communication between instructor and learner is no guarantee for meaning to be conveyed, but rather *evokes* meaning and this "meaning" is different for different people. Knowledge is thus not passively received, but actively constructed by the learner. Further support for this view of learning comes from Bodner (Marais 1997:18) who said that ... *knowledge is seldom transferred intact from the mind of the teacher to the mind of the* *student .... learners* ***construct*** *understanding. They do not simply mirror and reflect what they are told or what they read.*

The second principle states that the function of cognition (i.e. understanding) can be adapted to serve the way that the learner's experiential world has been organised. This means that knowledge is not true or false (as seen in the traditional view of "ontological reality") but rather that it is good if it works. In the words of Wheatley (1991:10): *In constructivism the concept of truth is replaced by viability.*

According to Hewson (1987:33), the constructivist theory explains why learners bring misconceptions to science classes. In an attempt to explain the physical world around them, learners construct cognitive models that are different from that propounded by orthodox science, and the learner is often committed to that explanation ... *because in some way it is intelligible and meaningful to her, in other words, it works.* This is echoed by Bodner (in Marais 1997:19):

> *In an attempt to create order from the chaos of information with which they are constantly bombarded, they subconsciously construct knowledge that fits their experience, that is, they adapt the information to make sense in terms of their intuitive knowledge of a topic.*

According to Osborne and Wittrock (1983:489-490) young children have definite views about many science topics prior to being taught science at school. According to their *generative learning model* (which has close ties with the basic ideas of constructivism), Osborne and Wittrock argue that children naturally and actively **generate** meaning from sensory input such as sights and sounds while they interact with the natural and technological world around them. Osborne and Wittrock (1985:64) provide the following key postulates to explain their generative learning model:

- a learner's existing ideas influence the way in which his senses react to and select sensory input ;

- a learner's existing ideas also influence which sensory input will receive additional attention and focus;

- the original sensory input has no initial meaning in itself;

- the learner has to construct relationships between selected sensory input and his existing memory store;

- the learner next uses these relationships to actively construct meaning to sensory input;

- the learner may test his constructed meaning against elements of the memory store, as well as against constructed meaning of other sensory input;

- if the constructed meaning makes sense, it is subsumed into memory;

- the necessity for the learner to actively participate in the process of constructing meaning, necessarily implies that he must accept a large part of the responsibility for his own learning process.

3.2.3 Conceptual change

3.2.3.1 The nature of and need for conceptual change

A number of authors emphasise the fact that learners arrive at a formal instructional setting with specific preconceived ideas which often have a profound influence on the quality of the learning that is to take place (see for example Hewson 1980:397 and Driver, Guesne and Tiberghien 1985:4).

West and Pines (1985:2) identify two sources of knowledge in the individual, namely knowledge acquired from *interaction with the environment* and knowledge acquired through *formal instruction*. The first they call *intuitive knowledge* and is influenced by aspects such as language, culture and other individuals. The second is described as a more *disciplined body of knowledge*, which has to be learned through instruction and mastered within a certain time period.

In view of the fact that learners' preknowledge and ideas about the world are often not in agreement with accepted scientific views, these preconceptions stand in the way of whatever the aim of the teaching process might be (see Driver *et al.* 1985:2-4 and Hewson 1987:33). The process of removing or changing these views is, in essence, what these researchers refer to as "conceptual change". Hewson and Hewson (1984:6) emphasise the link with constructivism, when they declare:

The constructivist view of learning, the evolution of concepts in a conceptual ecology, and incommensurability between conceptual frameworks are all ideas which suggest that learning may involve changing a person's conceptions rather than simply adding new knowledge to what is already there.

From this point of view, learning involves an interaction between new and existing conceptions with the outcome being dependent on the nature of the interaction. If the new conceptions cannot be reconciled with the existing ones, then learning requires that existing conceptions be restructured or even changed in order to accommodate the new conceptions. The recognition that changes of this nature might have to occur forms the basis of the conceptual change model of learning.

According to Hewson (1981:385) conceptual change in an individual can happen in a number of different ways. He mentions the following three ways:

- the addition of new conceptions through further experience, personal growth or contact with other people;

- the reorganisation of existing conceptions, triggered either externally by some new ideas or internally as a result of certain thought processes;

- the rejection of an existing conception through conceptual reorganisation or because of the displacement by new ideas.

Hewson (1981:386) explains these ideas in terms of a person whose existing conceptions include a particular conception C and is then confronted with a conception C' which might be an alternative theory about the same set of phenomena. The question can now be asked : *What would happen to the new conception, C' ?* According to Hewson, the new conception could be:

- rejected (either immediately or until further investigation suggests otherwise); or

- included in cognitive structures in one of three possible ways:
  - (a) by rote memorisation;
  - (b) by replacing C with C' and be reconciled with the remaining conceptions;
  - (c) by reconciling C' with C and other existing conceptions.

He (Hewson 1981:386) uses the terms **conceptual exchange** and **conceptual capture** to differentiate between the processes (b) and (c) above. The former is similar to the idea of *accommodation* while the latter has a lot in common with *assimilation.*

Posner, Strike, Hewson and Gertzog (1982) propose the "accommodation" model of conceptual change by stating that learning is more than the acquisition of a set of correct responses, a verbal repertoire or a set of behaviours. The influence of Piaget's model of learning is evident in their use of the terms **assimilation** and **accommodation**. To Posner *et al* (1982:212), *assimilation* refers to the first phase of conceptual change, meaning that students merely use existing concepts to deal with new phenomena. Often, however, these existing concepts are inadequate to grasp a new concept successfully and the student has to replace or reorganise his central concepts. *Accommodation* refers to this re-organisation or re-orientation of the student's existing conceptions in order to "accommodate" the new idea

3.2.3.2 Conditions for conceptual change to occur

Hewson (1981:387) and Posner *et al.* (1982:214) suggest four conditions for conceptual change or exchange to occur:

1. There must be dissatisfaction with existing conceptions.

2. The new conception must be intelligible.

3. The new conception must appear plausible.

4. The new concept should lend itself to further research.

(a) DISSATISFACTION WITH PRESENT IDEAS/CONCEPTIONS

Jordaan (1992:23) quotes Hewson (1982) and Hewson and Thorley (1989) who suggest that the dissatisfaction can be the result of any of three actions. Firstly, it may happen that C (the present ideas) are no longer necessary to explain certain experiences. Secondly, his present conceptions may no longer explain new phenomena with which he is confronted. Finally it may be that dissatisfaction with C occurs if it does not meet up to the expectations that the individual has about his original ideas. An example of the last case would be a growing dissatisfaction caused by an awareness of certain *ad hoc* assumptions that might have been used in C.

One major source of dissatisfaction is the **anomaly.** A person experiences an anomaly if he <u>unsuccessfully</u> attempts to incorporate a new experience into C, his present network of conceptions. In the words of Posner *et al.* (1982:220): *An anomaly exists when one is unable to assimilate something that is presumed assimible.* Posner *et al.* (1982:224) proceed to suggest that the <u>presentation</u> of anomalies will produce dissatisfaction with existing conceptions if:

1. Students understand why an experimental finding represents an anomaly.

2. Students see the need to reconcile their findings with their existing conceptions.

3. Students are prepared to reduce inconsistencies among their beliefs.

4. Attempts to assimilate experimental findings into existing structures are seen not to work.

In the teaching situation instructors have to make the new idea(s), C', as attractive as possible in order to encourage learners to become increasingly dissatisfied with their original conception, C. The following example from kinematics illustrates how this might happen in practice. A student might believe that a negative value for an acceleration implies that a body is slowing down, because his present conception suggests an association of "negative" with " less of something". An instructor may now suggests the existence and use of a sign convention and allows the student to observe an object that accelerates in a negative direction. His present ideas might be inadequate to explain this (to him) new phenomena, resulting in a high probability of a growing dissatisfaction with C (his original idea about negative acceleration).

(b) INTELLIGIBILITY OF NEW IDEAS

According to Hewson (1981:387) a person who is faced with a new conception, will not be able to incorporate it rationally into existing conceptions, unless he can make sense of it. The alternative is to incorporate it through rote memorisation.

Posner *et al.* (1982:216) point out that finding a theory or new concept intelligible requires more than just knowing what the words and symbols mean... *intelligibility also requires constructing or identifying a coherent representation of what a passage or theory is saying.* Instructors have an important role to play in developing real understanding of new ideas. Jordaan (1992:29) summarises the role of teachers of science in this process as the extent to which:

- mutual relationships between concepts contained in the new idea is explicitly explained;
- the new idea is integrated with similar existing ideas;
- the learner can distinguish between examples and non-examples of the new idea;
- the learner is exposed to the necessary experiences which is a prerequisite for learning;
- the teacher (instructor) is focused towards real understanding;
- the teacher's "teaching aim" corresponds with the learner's "learning aim".

Jordaan (1992:29) then goes on to point out that different types of content usually make different demands with regard to the promotion of understanding.

(c) PLAUSIBILITY OF NEW IDEAS

Hewson (1981:388) emphasises that for a new conception to be initially plausible, the student must be able to see that a world in which C' (the new conception) is true is reconcilable with his own conception of the world. This means that its plausibility is strongly dependent on its link to present conceptions, i.e. on the relationship with the existing knowledge of the person considering it. To Hewson a new conception can be plausible if

1. C' can be inferred from existing conceptions, or *vice versa.*

2. C' fits in with personal standards of knowledge, which means for example that C' is experienced as both *elegant* and *economical.*

3. C' fits in with past experience .

Posner *et al.* (1982:218) mentions the following ways by which a conception can become initially plausible.

- It is found to be consistent with current fundamental assumptions.

- The conception is consistent with other theories or knowledge.

- The conception is consistent with past experiences.

- Images can be found for the conception, which match an individual's sense of what the world is like.

- The new conception is found to be capable of solving known problems (i.e. it can resolve anomalies).

A single example is cited from kinematic graphs to illustrate how a new conception can be linked to existing conceptions. A student who has studied the parabolic curve in the mathematics class is asked to draw a sketch graph of *v vs t* for a stone which is dropped from a high building. He is also provided with the equation representing the displacement as a function of time, namely $s = 4{,}9t^2$. The idea is that the student finds $s = 4{,}9t^2$ (The formula describing the displacement of the falling stone as a function of time) "initially plausible" because he can link it to his present conception of a formula for a parabola, namely $y = ax^2$. As this does not always happen spontaneously, learners might have to be encouraged by instructors of science to become aware of the link with existing graphical conceptions from mathematics.

Once this connection is made, the new graph is more likely to become part of his existing conceptions, i.e. C' has been reconciled with C. This, according to Posner *et al.* (1982:218), will occur because the new idea is experienced as consistent with *current assumptions, past experiences and other bodies of knowledge.*

(d) FRUITFULNESS OF THE NEW IDEA

A new idea will not easily replace existing ideas on the grounds of intelligibility and plausibility alone - it has to be found fruitful as well, i.e. it should have the potential to be extended and to open up new ideas of investigation and inquiry. In the words of Posner *et al.* (1982:222):

If the new conception not only resolves its predecessor's anomalies but also leads to new insights and discoveries, then the new conception will appear fruitful and the accommodation of it will seem persuasive.

Hewson (1981:388) identifies a number of ways in which an individual could see a new conception as fruitful:

- If it solves problems experienced by C, i.e. if what was experienced as anomalous with respect to C is no longer anomalous with respect to C'.

- If C' suggests new approaches or new experiments which is not possible with C.

- If C' is generally accepted as part of the discipline and has to be mastered in order to pass conventional tests.

3.2.4 Students' views on science in general and motion in particular

The views of the world and meanings from words that young children have before they are exposed to formal science teaching are sometimes referred to as "children's science", while the meaning or explanation that scientists would give to certain observed phenomena is called "scientists" science (see for example Osborne, Bell and Gilbert 1983:1 and Osborne and Wittrock 1983:490). Hewson (1987:34) describes students' natural ideas about science as ***alternative conceptions*** and stresses that they should be taken seriously because students are committed to those explanations, as they seem intelligible and meaningful to them at the time.

> *If a student's conceptions are taken seriously and addressed by the teacher, there is a chance that real learning might occur and the student could accept and acquire the scientific conception and proceed in her understanding* (of the topic or concept). (Hewson: 1987:34).

Students' own constructed conceptions about the real world are hard to correct or modify through formal instructions. According to Treagust (1988:16):

> *It is, however, well documented that the task of changing misconceptions is extremely difficult, as they have often been incorporated securely into cognitive structures.*

White (1988:ix) also stresses the permanency of views formed from experiences *outside* the classroom and goes on to say that ... *this learning may be more permanent and influential in their lives than anything they are told at school.*

Children's science is tenacious and resistant to change and it is this fact which leads Osborne and Wittrock (1983:491) to support the view held by Hewson above, namely that science teachers must show a respect for "children's science" and be alert to the fact that ... *while children frequently pass tests and other formal assessment hurdles, they often do not really change their ideas of how and why things behave as they do as a consequence of science teaching.* The "generative learning model" is therefore based on the fact that the knowledge and conceptions that students bring to a classroom is recognised as important.

Driver *et al.* (1985) made an extensive study of children's ideas in science and are quoted by Thijs (in Thijs *et al.* 1987:49-50) to have come to the following conclusions about pupils' intuitive ideas in science:

- pupils base their reasoning on observable features in problem situations. An example would be that pupils associate a force which is diminishing with an observed slowing down of an object;

- pupils argue in terms of properties of objects rather than in terms of interaction between elements of a system;

- pupils often focus on change and ignore steady state situations;

- pupils' ideas have properties of several scientific concepts (an example from kinematics is provided in terms of the observation that pupils allocate an “upward” acceleration to an upward moving stone);

- pupils have a tendency to reason sequentially and to have a preferred direction when reasoning about events.

Osborne *et al.* (1983:496) state that there seems to be reason to believe that children and scientists both use their experiences and long-term memory in a similar way in order to give meaning to the world around them. They propose three **reasons** why the views that children generate *differ* from those of scientists:

1 Children tend to view things from a self-centred point of view because they have difficulty with the abstract reasoning ability of scientists.

2 Children use particular explanations for specific events and, unlike scientists, they are not concerned with the need for coherent and non-contradictory explanations for a *variety* of phenomena.

3 The everyday language used by children leads them to have views distinctly different from scientists’ views. Such views may not change as the child gets older, or they may even become increasingly different from scientists’ science.

These views have some significance for this study. The empirical investigation described in Chapter 4 will examine to what extent learners are able to explain and represent abstractly a variety of physical phenomena involving motion, and whether they are able to ignore certain widely held preconceptions in favour of *scientists' science.*

The present study focuses on students' misconceptions with respect to motion graphs and for this reason a few comments will be made about some of the implications of the ideas of constructivism and the nature of learners' preconceptions for the teaching and learning of *kinematics* and *graphs*. A more complete overview of the available literature on the topic will be given in sections 3.5, 3.6 and 3.7.

Students develop their own intuitions about motion and how it can be represented through their day-to-day observations of objects (such as cars, bicycles and other vehicles) which change their position as a function of time. Student intuition about kinematic concepts are not *necessarily* in opposition to the way subject experts view these concepts.

> *Intuitions are features of a student's knowledge that arise largely from everyday experience and sometimes also deeply understood formal knowledge.* (Leinhardt *et al.* 1990:24)

Motion is one of many global observations of physical phenomena that children make from an early age. These observations act as a support basis for the development of related conceptions on a cognitive level, usually under the guidance of an instructor. Leinhardt *et al.* (1990:28) point out that... *we need to understand what students know about graphs and respect the power and utility of their own intuitions at different age levels and after different kinds of instruction.* Students often draw on their everyday experience of "things changing together" as a basis for drawing and interpreting graphs. This is especially true of motion graphs, where certain quantities change as a function of time, resulting in students merely having to understand the nature of the variation in quantities such as *velocity, displacement* or *acceleration.*

Leinhardt *et al.* (1990:28) quote Bell & Janvier (1981) who argued that students should be given qualitative graphs of concrete situations which are viewed **globally** instead of pointwise. Students, in their view, should in this way attempt to reconcile their intuitive experience of two simultaneously changing quantities with both the verbal and the graphical representation of the particular physical phenomenon.

> *Encourage students to look at the entire graph as an expression of the relationship between two simultaneously changing variables and express the relationship in words.* (Leinhardt *et al.* 1990:29)

Halloun and Hestenes (1985) propose the phrase: ***common sense beliefs*** and define these as "*a bundle of loosely related and sometimes inconsistent concepts*" (1985:1058). Common sense beliefs are often at variance with Newtonian kinematics, as they are usually deeply rooted in the perceptual framework of students, and as such should be respected by the instructor. Halloun and Hestenes (1985:1056) state in this regard:

> *Students are not so easily disabused of common sense beliefs, because their own beliefs are grounded in long personal experience ... they should be regarded as serious alternative hypotheses to be evaluated by scientific procedures.*

The point is made by Halloun and Hestenes that most of the misconceptions about motion common among students today were seriously advocated by leading intellectuals in pre-Newtonian times. What is taught in kinematics and how it is taught should be viewed in this light so that students are actively encouraged to modify their beliefs. This modification should take place independent of the authority of the teacher or textbook, but instead should be a natural re-adjustment in the light of prevailing or newly discovered ideas of motion and its graphical representation.

Thijs (in Thijs 1987) reports on an investigation in Zimbabwe where the effect of "amount of instruction received" on "test scores" was investigated in order to measure the resistance of certain types of contents to formal instruction. The test was administered to a group of students training to become teachers and involved everyday problems on simple mechanics. The relationship between test scores and amount of instruction for two groups of questions (groups A and B) appeared to have an approximately linear character as indicated in figure 3.1.

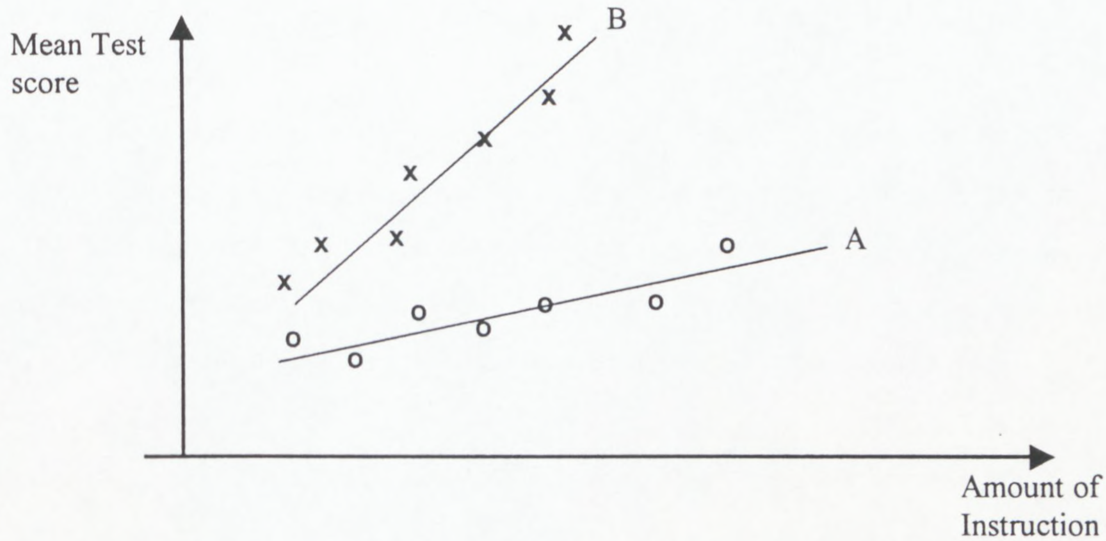


FIGURE 3.1
Effect of amount of instruction on test scores

The slope of a line gives a measure of the resistance of an intuitive idea to instruction. Thijs concluded that the small slope of line A corresponds to topics where deep-rooted preconceptions made these areas of the curriculum resist tutorial correction. Typical examples of common sense beliefs in kinematics which are difficult to correct include the idea of motion being defined with respect to different frames of reference and the fact that a force is not usually part of a moving body (see Thijs 1987:54). An important aspect of this model is that it can be used to identify specific strongly-held intuitive ideas and preconceptions by merely focusing on the slope of the line graph in figure 3.1.

3.2.5 Probing for learners' alternative conceptions

The standard method of investigating the nature and extent of students' preconceptions or alternative frameworks, usually takes the form of a combination of clinical interviews, a forced-option and/or free-response written test followed by further clinical interviews (see for example Freyberg and Osborne 1985, Hewson 1987, Treagust 1988 and Panse 1994).

According to Hewson (1987:37) the open-ended clinical interview is a technique which ... *provides information about student conceptions. It allows for surprises, i.e. the researcher begins the clinical interview with no presuppositions, no agenda, no hypothesis.*

The instructor must encourage students to articulate themselves freely without any attempt at correcting their views or teaching them. Whenever possible students must be encouraged to provide *reasons* for their ideas on a subject. Freyberg and Osborne (1985:15) stress the importance of this approach: *Effective probing of children's real ideas requires a conscious value-free approach which is not at all easy to maintain.*

The alternative conceptions and ideas that were identified during these interviews form the basis of a *free-response test* which is administered to another group to test for the presence of similar alternative conceptions to those identified earlier. Panse (1994:64) suggests that the responses to this test are examined for frequently occurring key notions which could emerge either implicitly or explicitly. Next, he proposes that *a number of interpretative categories of responses* are identified which forms the basis of a forced-option type test. This would give an indication of ... *how widely these conceptions were held, how consistently they were used, and how sensitive they were to the context* (Panse 1994:65).

A variation of the clinical interview is to use the technique developed by Osborne and Gilbert (1980) which makes use of Interviews about Instances or Interviews about Events. This method could be used to obtain greater clarity about the *reasons* why certain misconceptions about a topic are frequently held. In the *interview about instances* the interviewer prepares a representation of an instance such as the *s-t* graph of a falling object and uses this to allow the respondent to reveal her thoughts about falling bodies and their displacement as a function of time. In the case of *interviews about events* the respondent might be asked to give his thoughts on the real life situation of a ball being dropped from a certain height.

Panse (1994:65) sees the final aim of all the above mentioned techniques to...

> *obtain quantitative ideas on the prevalence of alternative conceptions among students, their different aspects and relative strengths in particular situations, and also the consistency with which these notions were used.*

Treagust (1988:161-164) suggests a 10-step research methodology which incorporates the essential features of most of the alternative framework research methodology (for instance interviews and free-response pencil and paper tests). He focuses on a methodology and its subsequent use for developing diagnostic tests to examine and identify students' misconceptions in limited content areas of science. In view of the fact that kinematic graphs can be described as one such "limited content area", Treagust's ten stages are briefly summarised below:

Defining the content

Step 1 *Identifying propostional knowledge statements* which are generally accepted by the scientific community as true statements about the topic.

Step 2 *Developing a concept map.* A map of concepts which relate to the topic is developed.

Step 3 *Relating propositional knowledge statements to the concept map.* This is a reliability *check* that the concepts and propositional statements are indeed examining the same topic.

Step 4 *Validating the content.* The propositional statements and concept map are scrutinised by science subject experts for discrepancies.

Obtaining information about students' misconceptions

Step 5 *Examining the related literature for the occurrence of common misconceptions.*

Step 6 *Conducting unstructured student interviews.*

Step 7 *Developing multiple choice content items with free response.* This brings to light additional misconceptions *as* well as acceptable scientific conceptions.

Developing a diagnostic test

Step 8 *Developing two tier diagnostic tests.* The first part of each item is a multiple choice content question having about three choices. The second part contains a choice of possible **reasons** for the answer given to the first part. These reasons are developed from information gathered from the free-response questionnaire, the interviews as well as the literature.

Step 9 *Designing a specification grid.* This is done to ensure that the diagnostic test fairly covers the propositional knowledge statements and the concepts on the concept map underlying the topic.

Step 10 *Continuing refinements.* Successive refinements of the two tier diagnostic test, to improve its effectiveness and diagnostic potential.

3.2.6 Instruction and students' preconceptions

Thijs (in Thijs *et al.* 1987:44) proposes the following **key tasks for instructors** as a direct response to the above preconditions for conceptual change to occur.

(a) instructors must find out what the nature and extent of the preconceptions are;

(b) instructors should encourage their students to recognise that their existing views require modification; and

(c) students should be challenged to restructure their preconceptions in view of the instructions offered.

In this way teachers acknowledge and respect the (sometimes) strongly held views by students and because they (the teachers) are non-judgmental about these alternative conceptions, students might be more inclined to modify their views, which initially might have been very different from the formally accepted laws of science.

Two models for instruction in science which are both geared towards changing or modifying existing views of learners are those proposed by Barnes (1976) and Rowell and Dawson (1983). The essential features of these two models will be described next:

(i) Barnes' model

Barnes (1976) as quoted by Osborne & Freyberg (1985:104) contends that learners need to take a prominent part in the formulation of their own knowledge. To reduce the teacher's perceived control over knowledge, Barnes believes that students should work primarily in small groups. In practical terms he proposes the following sequence:

- *a focusing stage*, in which the teacher, with the students, prepares the ground by presenting preliminary knowledge (which includes "alternative frameworks" and "children's science"). When the attention of the class is fully focused on the topic, the teacher moves on to an...

- *exploratory stage*, involving much discussion and other activities, including experimentation. Then in the...

- *reorganising stage*, the teacher re-focuses attention and tells the groups how they will be reporting back, and how long they have to prepare for it. Finally, in the…

- *public stage,* the groups of learners present their findings to one another, and this leads to further discussion, presumably resulting in modification of the students' originally constructed ideas.

(ii) Rowell & Dawson's model

Rowell & Dawson (1983) suggest a model which focuses on the confrontation between the student's originally constructed view of science and that proclaimed by scientists. They suggested the following sequence:

- Through questioning, the teacher establishes the ideas which children bring to the problem situation. Conscious awareness of these ideas is of value to both the teacher and the children.

- These ideas are accepted by the teacher as possible solutions.

- Children are asked to retain their ideas, and the teacher states that he or she is going to put forward another possibility which the children will help in evaluating later.

- The "new" idea is taught by linking it to a basic idea already held.

- Once the new idea is available to children the old ideas are recalled for comparison, with each other and with reality.

Rowell and Dawson believe that children are less threatened by this approach than some others, since both "old" and "new" ideas are the pupils' own in the sense that all are pooled knowledge. Assuming that old theories are rarely defeated by contrary evidence but only by better theories, they argue that the children with several ideas available to them are in the best possible situation to accept the scientist's one when it is tested against the others.

3.3 THE MEANING OF UNDERSTANDING

3.3.1 General comments on the meaning of understanding in science

Before considering research that has been undertaken with regard to students' understanding of graphs in general and motion graphs in particular, it is necessary to report briefly on some of the research findings with respect to the nature and meaning of **understanding** in general. Attempts that have been made to propose models for the measuring of the level of students' understanding with respect to specific topics in science, will also be highlighted.

What it means to understand a concept or principle in terms of accepted scientific laws as opposed to a mere intuitive understanding is very important in kinematics, because according to Carramazza (1981) as quoted by Nickerson (1985:204):

> *Real world experience with moving objects does not lead naturally to the extraction of principles that are consistent with the formal laws of motion..*

It is generally agreed by researchers such as Skemp (1975); Ormell (1979); Trowbridge and McDermott (1980) and Nickerson (1985) that it is extremely difficult to describe what is meant by understanding.

Nickerson (1985:221) attempts an explanation by describing understanding in terms of the amount of knowledge one has about a concept. In other words the more knowledge an individual has that is related to a concept or procedure, the greater is his understanding. This, according to Nickerson immediately gives rise to a true paradox:

> *The more one learns about some aspect of the world, the more likely one is to become aware of the depth of one's ignorance. This means one's appreciation of the complexity of that aspect of the world is likely to increase.* (1985:221)

A test for understanding proposed by Nickerson is expressed in terms of a comparison of the individual's view with that of an expert:

> *One understands a concept to the degree that what is in one's head regarding that concept corresponds to what is in the head of an expert in the relevant field.* (1985:222)

Trowbridge and McDermott (1980) also compare an individual's conception of some scientific subject material with that of a recognised expert in the field. They have this to say about what it means for a student to understand concepts in physics:

> *We may consider as an indicator of degree of understanding the extent to which a student's understanding corresponds to that of a physicist i.e. the extent to which a student can define a particular concept in an acceptable operational manner, distinguish it from related but different concepts and apply it successfully* (1980:1020)

The ability to correctly distinguish between similar but different concepts as a test for understanding is advocated by Nickerson as well. He (1985:229) states it in terms of *"the ability to apply principles from one knowledge domain to another"* and to *"recognise similar relationships in different contexts"*. The kinematic concepts of "slope" and "velocity" for example can be viewed differently, depending whether they are used in an everyday context or a scientific context.

Skemp (1976:29) undertook a great deal of research into the question of concept formation and understanding. He refers to man's ability to isolate concepts from any of the examples which give rise to them and stresses that *the construction of a conceptual system is* *something which each individual has to do for himself.* Skemp then goes on to point out that the role of the instructor is to make sure that the material (i.e. examples and non-examples) are to hand as this would enormously speed up the process of concept formation.

Skemp warns against the tendency to communicate complex concepts to students by forcing them to learn a definition as it appears in a book or as it is orally communicated to them by an instructor. He puts it as follows:

> *Concepts of a higher order than those which a person already has cannot be communicated to him by definition, but only by arranging for him to encounter a suitable collection of examples.* (Skemp 1976:32)

Skemp differentiates between three kinds of understanding which can be seen as a hierarchy of quality of understanding, namely *instrumental*, *relational* and *formal understanding*. According to this model these levels of understanding can be described as follows:

(a) instrumental understanding as the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works;

(b) relational understanding as the ability to deduce specific rules or procedures from more general relationships and

(c) formal understanding as the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning.

Skemp's model was developed to describe understanding in mathematics, however it is equally applicable to the understanding of a topic such as kinematics.

Ormell (1979), Trowbridge and McDermott (1980) and Nickerson (1985) all warn against the danger of assuming that a student has understood a concept if he passes conventional tests or if he can solve typical textbook-type problems.

Ormell (1979:35) contends:

> *We must get away from the distorting effect produced by set-piece memorisation, so it is clear that we need to set the candidate some kind of mild "problem" in a fresh situation, which the candidate has never seen before.*

He suggests instructors construct hypothetical situations with sufficient variability and variety. A student who consistently deals with these situations successfully would have demonstrated real understanding of a concept or process.

Nickerson differentiates between prototypical and nonprototypical situations:

> *the ability to deal with nonprototypical situations would seem to require a deeper understanding of a domain than does the ability to deal with prototypical situations.* (1985:224)

He also warns against the fact that many misconceptions that students develop can go undetected because a superficial knowledge of how to manipulate formulas and solve textbook style problems may be enough to carry students through standard course requirements.

Trowbridge and McDermott (1980:1028) state in this regard that ... *The ability to solve conventional problems or to pass usual types of mastery tests does not always indicate conceptual understanding.* Arons (1982:14) discovered that although first year students could solve a specific problem *quantitatively,* they still did not possess the ability to interpret it *qualitatively.* He (Arons 1982:14) comes to the conclusion that ... *the formal solution of a textbook problem gives no assurance of a real grasp of the underlying principles and physical phenomenon.*

What must a student be able to do to demonstrate real understanding of a concept or principle? Nickerson (1985:229-231) proposes four tests which could be used to assess whether an individual really understands certain subject material: The ability to ...

- communicate effectively with people who are knowledgeable about a certain subject domain.
- carry out a procedure and consistently obtain the desired results.
- draw analogies that are considered appropriate by experts in a field.
- demonstrate the understanding of a principle, procedure or concept in a variety of ways.

Ormell (1979:35) sees the test of understanding in terms of an individual demonstrating that he comprehends the meaning of the message or principle because he can make a connection between words and the real world. In kinematics this would mean that a student can be said to understand a principle or definition if he can make a successful connection between actual observed motions and *representations* of this motion (verbal, algebraical or graphical).

Skemp (1976) assesses understanding in terms of an individual's proven ability to abstract the conceptual properties from many examples of varying degrees of difficulty. Furthermore he must demonstrate his ability to ignore data which are irrelevant to a particular communication. Jordaan (1986:93) points out that understanding something is dependant on the nature of the content, so that understanding a law, an algorithm, a model, or a measurable quantity can mean something totally different in each case. Jordaan (1986:93) quotes Davis (1978) who comes to the following conclusion with respect to mathematical contents:

> *Understanding depends on the type of mathematical knowledge. Understanding a concept is different from understanding a procedure.*

In physical science, students can be expected to give examples (and non-examples) of concepts such as metal or acid, but clearly it makes no sense to expect them to give an example of a law because:

> *At most one can give an example of a situation where the law holds or does not hold or where it can be applied or not.* (Jordaan 1986:94)

In the subject area of kinematic graphs, laws do not feature prominently and hence no further exploring of ideas relating to laws will be undertaken at this stage.

3.3.2 Understanding relations and measurable quantities

Reif, Larkin and Brackett (1976:212) contend that *relations* include aspects such as the definition of *acceleration, velocity* and *displacement*. Trowbridge and McDermott (1980) refers to these terms as concepts, while in the present study they will hence forth be referred to as *measurable quantities* or *kinematic quantities*.

For the purpose of the discussion of students' *understanding* of the definition and meaning of the measurable kinematic quantities, these quantities will be viewed in terms of their *relationship* with each other and with other scientific quantities or parameters.

Trowbridge and McDermott (1980:1028) investigated students' understanding of certain kinematic quantities (which they call "concepts") and contend that an indicator of a student's understanding *can be seen as to the extent to which he can define a particular concept, distinguish it from related concepts and apply it.* Reif *et al.* (1976) and Larkin and Reif (1979) proposed similar criteria for assessing students' problem-solving skills as well as their skills in using quantitative relations (such as definitions and laws) effectively. They state in this regard:

> *To use any quantitative relation as a serviceable conceptual tool, a student must possess a number of abilities such as interpreting correctly the*

quantities in the relation, applying the relation appropriately without confusion, and dealing with equivalent forms of this relation. (1976:212)

The complete suggested framework given by Reif *et al.* (1976:213) with respect to what can be expected of learners with regard to *relations* is given below:

A *Statement and example*

State the relation and give an example of its use.

B *Quantities in the relation*

1 Properties (list basic properties: number or vector, possible signs, units, typical magnitudes).

2 Interpretation (interpret various symbolic representations such as words, numbers, formulas, graphs).

3 Conditions specifying meaning (recognise information from which each quantity can or cannot be found).

4 Comparison of quantities (summarise similarities and differences between each quantity and others with which it might be confused).

C *The relation itself*

1 Conditions of applicability (recognise physical situations in which the relation can or cannot be used).

2 Comparison of relations (compare the relation with similar relations, identifying situations where each does or does not apply).

3 Relating quantities (find an expression or value for any quantity in the relation).

4 Dependence (for two different values of one quantity, compare the corresponding values of another quantity).

D Organization of relations

In a physical situation, identify those which are applicable and use them without confusion.

According to the formulation of Larkin and Reif (1979) an instructor can set a number of criteria for assessing whether a student understands the relationship between *s*, *a* and *t* in an algebraic formula in kinematics such as $s = \frac{1}{2}at^2$.

Firstly, he must be able to interpret the symbolic quantities *s*, *a* and *t* in terms of their meaning and whether they are vector quantities or not. The student must realise that the formula describes a motion with uniform acceleration which takes place from rest in the (assumed) absence of air resistance.

Secondly, he must be able to apply the formula in appropriate cases such as objects falling from rest under the influence of the force of gravity or cars accelerating from rest. He must recognise the formula as being applicable in the case where a displacement is required when the acceleration and the time are known and that it is only valid if all units are written in terms of their SI equivalents.

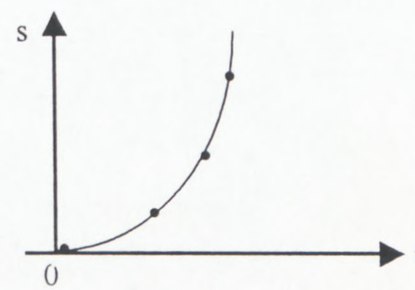
Finally, the student must demonstrate his ability to deal with the following equivalent forms of this relation:

- changing the subject of the formula as in $a = \frac{2s}{t^2}$ leading to $t = \sqrt{\frac{2s}{a}}$.

- ordered pairs which list specific displacement values for given time values.

- a graphical interpretation of the given relationship which may or may not make use of ordered pairs as an intermediate step. This means that students should be able to follow any one of the two routes:

$s = ½at^2 \Rightarrow$ ordered pairs $\Rightarrow$



or

$s = ½at^2 \Rightarrow$ recall shape of graph $\Rightarrow$

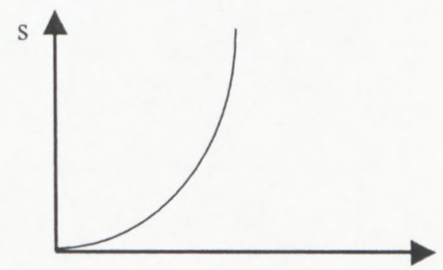


FIGURE 3.2

Reif *et al.* (1976) view a kinematic quantity such as velocity as a *relation* and in view of this fact their "framework of abilities" can be applied to "velocity". The question can now be asked: *What tasks must learners be able to perform with respect to velocity in order to demonstrate their understanding of this "relation"?* By adapting the above framework it can now be contended that learners must be able to perform the following tasks:

A State the definition of average velocity and describe it in terms of other kinematic quantities. It seems reasonable to assume that the learner should (under this category) also be able to perform basic calculations involving velocity as the quotient of a displacement value over a certain time interval.

B Recognise that velocity is a vector quantity which must be given a positive or negative sign depending on the direction convention; represent velocities graphically;

perform calculations involving velocities in algebraic form; calculate velocities from a table of displacement and time values, as well as extract information about velocities from a verbal (or oral) passage.

Compare velocity with other related kinematic quantities and point out the similarities and differences. The proven confusion between velocity and acceleration is a specific example of where a learner must demonstrate his understanding of acceleration as the rate at which the velocity changes. Students should also demonstrate their ability to distinguish between the vector quantity of velocity and the related scalar quantity of speed.

C Recognise a physical situation where velocity should be used instead of speed; be able to determine a velocity as displacement over time (as compared to speed which is distance traversed over time); compare velocities for two different situations with different displacement values (graphically as well, if necessary).

D Translate a given physical situation into an equation, a graphical representation or a table of values, by extracting the appropriate values for velocity and the corresponding times that the velocities were attained.

It should be obvious that different learners will find themselves on different levels of understanding, even when exposed to identical stimuli in terms of formal tuition and/or informal physical observations. It is equally true that the same learner often improves his level of understanding while being exposed to a greater amount of formal teaching. Consequently it can be said that not many students (after an introductory course in kinematics) would be able to respond positively to **all** the categories of abilities as suggested by Reif *et al.* above.

Jordaan (1986) originally classified acceleration, velocity and other kinematic quantities as parameters because they are measurable concepts and therefore unit bound. Instead of using the term "parameter" Jordaan (personal communication) at present also prefers to use the term "measurable quantities". He provides a number of guidelines with respect to aspects which learners must master to demonstrate their understanding of a specific measurable quantity. It is necessary to summarise these actions here, as they relate directly to identified misconceptions about kinematic graphs which are described in §3.7.

The content to which these actions refer were discussed fully in Chapter 2 and will not be repeated again. According to Jordaan the student must be able to:

1 *Know the name and symbol in order to communicate about the parameter.*

2 *Give the definition of the parameter and master the concepts in the definition.*

3 *Recognise examples and non-examples of the parameter.*

4 *Apply the parameter in every-day life.*

5 *Recognise and apply relationships containing the parameter.* (Jordaan cites the example of the understanding of acceleration in terms of the symbolic relationship: ($\vec{a} = \frac{\vec{v} - \vec{u}}{t}$).

6 *Suggest ways of measuring the parameter.*

7 *Determine the parameter from a graph.* An example would be the determination of velocity as the gradient of a displacement-time graph and the recognition of the fact that the units correspond to the units on the two axes.

8 *Give a suitable unit.*

9 *Identify specific attributes of the parameter such as, What is it a measure of? Is it a vector? Which factors influence the parameter? etc.*

3.3.3 Understanding kinematic graphs

The assessment of problem-solving skills is related to the assessment of graphing ability and Larkin and Reif (1979) propose that this be carried out by observing individuals (novices and experts) who were asked to talk out loud about their thought processes while they solved problems. Larkin and Reif (1979) conclude that novices tend to approach a problem sequentially, translating individual pieces of information into an equation. Experts on the other hand apply general principles as part of well-defined coherent methods.

> *experts make use of low detail qualitative description to explore potential difficulties and to plan the execution of a method.* (1979:197)

It will be shown later in this chapter (see §3.7) that the implication that this has for evaluating a learner's understanding of kinematic graphs can be phrased in the form of the question: *Does he make use of verbal or pictorial descriptions of an observed motion so that he initially focuses on the more global aspects and then through a process of "successive refinements" considers the more detailed aspects of the motion?* The global aspects of a motion can include a general description of the motion in a strictly qualitative way while at the same time the able student should be able to demonstrate his understanding of the motion by representing it in the form of a sketch graph. From this general description or representation of the motion certain key aspects can now be investigated further. These include choice of reference level, sign conventions and stated initial conditions. Further refinements such as the determination of specific values for specified kinematics quantities and an attempt to draw a more accurate graph can now be carried out.

In view of Jordaan's categories as well as the meaning which is attached to understanding in general, it is now proposed that students' understanding of kinematic graphs be evaluated according to a number of criteria. These criteria can be separated into two categories, the first referring to broader, more general abilities, whilst the second category describes specific actions or tasks which are carried out by the student when dealing with kinematic graphs. It is important to note that these "abilities" refer to actions which have to be carried out with respect to specific content items as described in Chapter 2.

Examples of **broader abilities** include the following:

- Can he communicate effectively with his instructor on aspects such as how rate of change of position can be found from a position vs time graph?

- Does he consistently obtain correct answers for a wide variety of questions set on the various translations between representations of a motion?

- Can he draw appropriate analogies between aspects such as the area under a *v vs t* graph giving the change in displacement or the area under an *a vs t* graph giving the change in velocity? Furthermore, can he see the relationship between a linear graph in mathematics and the *v vs t* graph for uniform accelerated motion?

- Can the student determine the acceleration of an object from different representations? In other words can he find the acceleration from the graph as well as with a formula or from a table of data points?

- Can he make the proper connection between an observed motion and its graphical representation? i.e. can he successfully make a type A transformation?

- Does the student possess the ability to draw correct motion graphs from many different types of data sources as well as many different types of real life situations?

- Can he interpret a given graph correctly so that every feature is given the correct physical interpretation.

- Does he consistently extract the appropriate information from the gradient or change in gradient of a given graph.

The following are examples of **more specific actions**:

- the ability to chose a realistic system of axes;

- the ability to plot coordinates correctly;

- the ability to read off values correctly from a graph;

- the correct determination of the velocity from a *s-t* graph or the acceleration from a *v-t* graph;

- the ability to correctly translate the height (or slope) of a given graph into the slope (or height) of a required related graph.

The above are examples of actions which students have to be able to perform in order to satisfy an instructor that they have truly understood kinematic graphs. It is suggested that a similar list of required actions is drawn up to evaluate the level of understanding that learners have of any specific science topic.

Understanding of kinematic graphs can now be defined as **the ability to perform any translation between kinematic representations involving graphs correctly so that the resulting answer is totally acceptable to a subject expert in the field**. Furthermore, a student who truly understands kinematic graphs **must be able to construct and/or interpret graphs in the context of original, non-textbook style problems where set-piece memorisation of previously learned "recipes" will not suffice to solve the problem**.

It must be pointed out that understanding of motion graphs presupposes an understanding of both the <u>motion</u> aspect as well as the <u>graph</u> component. The motion aspect involves an *understanding of measurable quantities such as velocity, displacement, acceleration, etc. as well as the relationship between them in terms of definitions and scientific* laws. The graph component refers to *an understanding of a graph as a symbolic representation of two, usually continuous, variables in terms of their distances from a horizontal and vertical axis.*

An understanding of kinematics as it is taught by a subject expert in the classroom as well as an understanding of graphs which are taught in other disciplines such as mathematics, does not however guarantee an understanding of kinematic graphs.

These two aspects become integrated only after a long period of exposure to real-life problems of many different types with different emphasis. As was pointed out in paragraph 3.2, the process of adapting the student's preconceptions about motion and motion graphs is a slow one, and one that has to be approached with great care and circumspection.

Student's understanding of graphs in general as well as their understanding of kinematics as a subject area within physics will be further explored in paragraphs 3.5 and 3.6 respectively.

3.3.4 Understanding mathematical contents and procedures

Learners frequently experience difficulty with specific aspects of Physical Science because they lack certain mathematical skills required for proper grasping of that particular section. Jordaan (1992:102) gives three examples of such contents, namely

- changing the subject of the formula;
- construction and interpretation of graphs;
- ratio and proportion.

All three of the above contents in mathematics play an important part in the understanding of kinematics and kinematic graphs. A single example from kinematics is cited here to illustrate the importance of the first two topics, while ratio and proportion are discussed in the next paragraph.

Pupils who are provided with the familiar kinematic equation v = u + at must be able to change the subject from v to *a* so that it reads: $a = \frac{v-u}{t}$. This, if written in the form $a = \frac{\Delta v}{\Delta t}$, it represents the original definition of acceleration.

The graph of the algebraic relation v = u + at is a straight line with intercept on the ordinate equal to u and (mathematically only!) intercept on the time axis equal to $-\frac{u}{a}$. The confidence with which a pupil can manipulate the familiar straight line equation y = mx + c will have a direct bearing on his ability to draw a graph of *v vs t* for the case v = u + at, as indicated in figure 3.3 below.

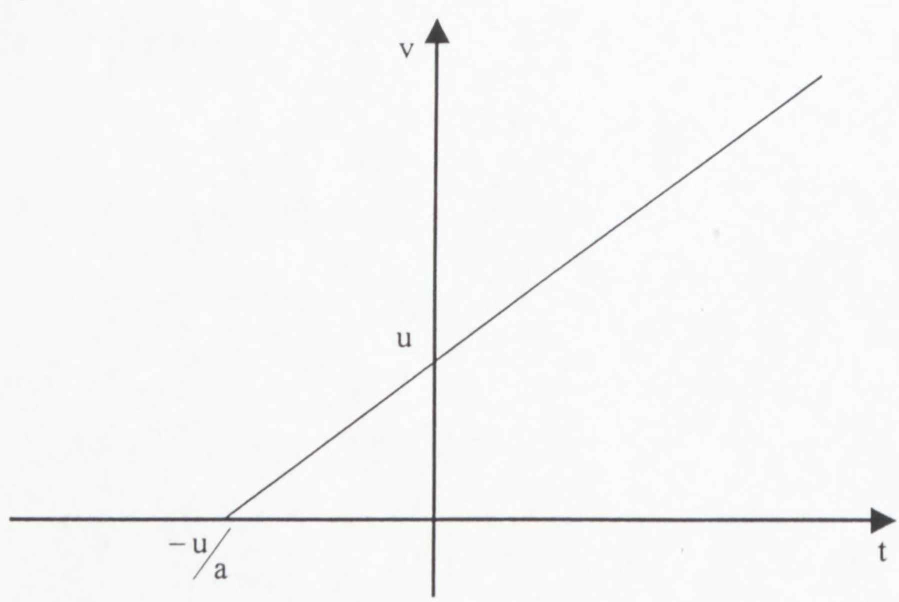


FIGURE 3.3

If u is taken as zero, the equation becomes v = at. Learners must interpret this relation as one which describes *v* as being proportional to *t* with *a* the constant of proportionality.

The answer to these types of questions should not only give an indication of a student's level of understanding, but indeed also bring to light specific misconceptions that he might have. The nature and extent of some typical misconceptions will be discussed in section 3.7 of this chapter.

3.4 RATIO AND PROPORTIONAL REASONING

3.4.1 General

Many authors such as Warren (1979), Orton (1984), Gamble (1986A), Jordaan (1992) and others emphasise the importance of a thorough understanding of proportional reasoning in order to understand certain key aspects of physics and chemistry. Gamble (1986A:355) writes for example:

> *In physics it is impossible for pupils to understand certain laws or relationships unless they share fully with teachers the meaning of the word proportional.*

The concepts of ratio and proportionality is central to the study of kinematics and the graphical representation of kinematic quantities. This raises the following question: *What is the level of understanding of ratio and proportional reasoning by students who enrol for courses where this is vital for the understanding of the course material?*

The following is an illustration of how ratios and proportions can be used to analyse an experimentally obtained set of tabulated values in order to demonstrate to pupils how this approach can be reconciled with a more formal algebraic approach to the same physical situation.

Consider the case of the following table of distance values (in metres) against time (in seconds) for a body travelling in a straight line from a position of rest:

s	0	5	20	45	80
t	0	1	2	3	4

The ratio of $\frac{s}{t}$ and $\frac{s}{t^2}$ yields an interesting pattern if the (0;0) coordinate is ignored.

$$\frac{s}{t} = \frac{5}{1};\frac{20}{2};\frac{45}{3};\frac{80}{4}$$

$$= 5;10;15;20$$

but $$\frac{s}{t^2} = \frac{5}{1^2};\frac{20}{2^2};\frac{45}{3^2};\frac{80}{4^2}$$

$$= 5;5;5;5$$

Students should note that the ratio of $\frac{s}{t^2}$ remains constant, so that it can be written as $\frac{s}{t^2} = k = 5$, where k (or 5) is called the constant of proportionality.

The idea is that students must recognise this equation as a special case of the familiar equation of motion: $s = ut + \frac{1}{2}at^2$ where the initial velocity, u, has been taken as zero and a is taken as 10 $m.s^{-2}$, the gravitational acceleration on earth. Thus, through the use of ratios, the behaviour of a moving body in terms of its distance as a function of time has been analysed and compared to a purely mathematical treatment of the same physical situation.

$$s = \tfrac{1}{2} \times 10 \times t^2$$

$$= 5t^2 \text{ as before.}$$

The fact that children experience great difficulty with proportional reasoning is well documented (see for example Olivier 1984; Gamble 1986 A and B; Jordaan 1992 and Brasell and Rowe 1993). This can seriously affect their understanding of kinematic graphs as they might (for example) not see the link between proportionality and a straight line graph passing through the origin.

Jordaan (1992:102) sites various authors who indicate that many children at age 16 have not yet reached the formal operational stage required to fully understand proportional reasoning. He proceeds to point out that this can cause problems for students with contents that require proportional reasoning. Gamble (1986A:355) puts the problem in perspective:

Given that many children find difficulty with proportionality we must explore the many ways in which this can affect their learning of physics.

He refers to the fact that the word "proportional" as a shorthand notation for a particular and special form of relationship between two variables whose graphical representation is a straight line graph passing through the origin is not understood by many secondary school students.

Fourie (1991) investigated the response of 68 pupils from Grade 11 and 12 to a questionnaire testing for the ability to reason proportionally and reports that <u>no</u> aspect of proportional reasoning was completely mastered by high school pupils. He came to the conclusion that few pupils at high school are at the formal operational stage regarding this aspect. Two significant aspects of this study was his finding that pupils:

i) were not able to give a graphical interpretation to any proportionality involving a square (such as $\Delta s \propto t^2$ for motion with constant acceleration) and;

ii) could not interpret data tables as a direct or inverse proportionality.

Arons (1983) refers to students' failure to master reasoning involving ratios and refers particularly to the difficulty they experience in giving verbal interpretations to abstract algebraical statements involving ratios. Gamble (1986B) investigated pupils competency with equations of the type $a = \frac{b}{c}$ and found that although many pupils could perform routine calculations, although they did not understand the physical quantities involved. He goes on to state the following with respect to the link between mathematical ability and conceptual understanding:

Judgement of what is adequate mathematical ability cannot be made without reference to adequacy of conceptual understanding upon which the mathematical operations are to be based. (1986B:33)

Students who lack a qualitative understanding of proportionality and who are unable to interpret ratios will experience severe difficulty with many concepts and procedures in kinematics. The example cited earlier where a constant ratio of $s:t^2$ should be associated with an equation such as $s = kt^2$, is therefore seldom grasped by students in terms of the proportionality concept. They have particular difficulty with the expression *"constant of proportionality"* because, although they can perform routine calculations involving instrumental understanding, they do not understand the meaning of the constant, k in the equation above. (See for example Gamble (1986B:32 and Fourie 1991:53).

Students' misunderstanding of kinematic graphs can be attributed to many possible factors such as a lack of understanding of graphs or a poor grasp of the kinematic concepts involved. Another important factor can now be added to this list: An inability to understand proportional reasoning and ratios. In the next paragraph it will be shown that learners often do not understand the significance of the proportionality concept as a special relationship between two variables whose characteristic graphical representation is a straight line graph passing through the origin.

3.4.2 Proportional reasoning and graphs

Brasell and Rowe (1993:65) suggest that the formal operational structures required for proportional reasoning are also necessary for correlational reasoning in graphing. They have pointed out, however, that the graphical representation of the relationship between two dynamic variables has the advantage in that it assists to illustrate to students the proportionality concept in a visual manner. This often leads to a better understanding of related concepts such as *rate* and *rate of change*. Jordaan (1992:112) agrees, contending that the graphical method gives an immediate indication of the nature of a relationship because it is more "elegant" than the arithmetical or algebraic method.

Thus the well known kinematic relationship: $v = u + at$, can be represented graphically in terms of a translation, either by employing mathematical techniques (associating it with the straight line equation) or by substitution of values. Either way, the resulting straight line provides an immediate visual representation of aspects such as *rate of change*, *correlations* and *proportionality* (if u = 0 at t = 0, v is seen to be proportional to t).

Arons (1983:578) agrees with this view when he says that one way of helping students master a mode of reasoning, is to allow them to view the same reasoning from another perspective and...a *useful alternative perspective is graphical representation.*

He suggests that instructors should encourage students to view the steepness of a straight line as a property of the system that it describes. For instance the "steepness" of a distance vs time graph should be associated with "speed" and, similarly, the "steepness" of a mass vs volume graph should be associated with "density". Gamble (1986A) distinguishes between proportion and the ratio aspect of proportion and stresses:

> *This (the graphical representation) embodies the ratio aspect of proportion and allows the slope of a graph to be equated to a defined physical variable, such as velocity* (1986A:355)

Gamble goes on to distinguish between proportionality in *physics* as describing the relationship between two variables (such as v = at with a the constant of proportionality) and proportionality in a pure *mathematical* sense, where it is expressed by the four variables: $\frac{a}{b} = \frac{c}{d}$

In the literature this (latter) definition of a proportionality is referred to as a rate. In the case of $\frac{s_1}{t_1} = \frac{s_2}{t_2}$ the rate defines a new variable namely the velocity of a body (Jordaan 1992:106; Olivier 1984:14).

Olivier (1984:15) claims that

> *both uses of ratio are important and must be developed. However, the rate idea, as the relationship between two sets, closely correspond to the function*

concept and is the more important and useful concept. Research indicates that pupils spontaneously use, and are more conversant with, the rate idea.

To illustrate the relationship between the two interpretations of proportionality, consider the following table of values of an independent variable (x) and a dependent variable (y).

x	0	2	4	6	8
y	0	3	6	9	12

It should be clear that the following relationship exists: $\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \frac{2}{3}$

In general $\quad \frac{x}{y} = \frac{2}{3}$

$$\Rightarrow \quad y = \frac{3}{2}x$$

The graphical interpretation of the table of values where actual data points are plotted could now be reconciled with the fact that the constant of proportionality, namely $\frac{3}{2}$, gives the gradient of the line graph so obtained (see figure 3.4)

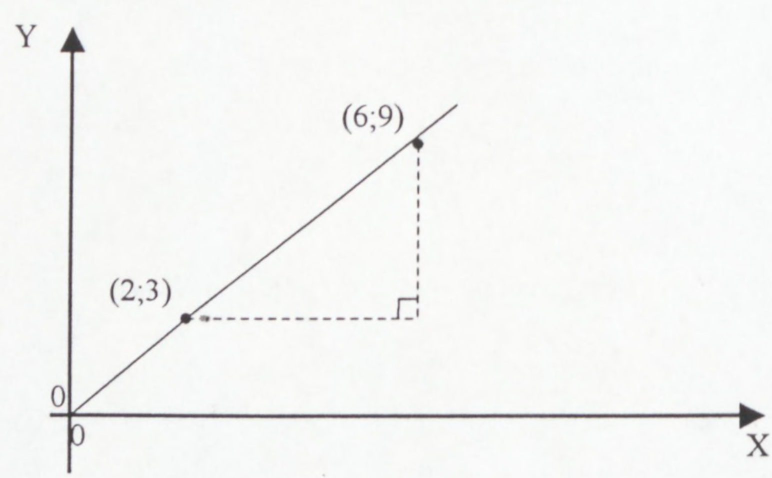


FIGURE 3.4

The following example from kinematics illustrates how a graph can assist in defining a proportionality constant as a physical quantity in terms of a ratio of the original quantities.

Assume that a body has displacement equal to zero at time $t = 0$ and that it moves with constant velocity such that:

$$s \propto t$$

Now, changing the proportionality sign to an equal sign:

$$s = k.t \qquad (1)$$

The graph of s vs t is a straight line through the origin:

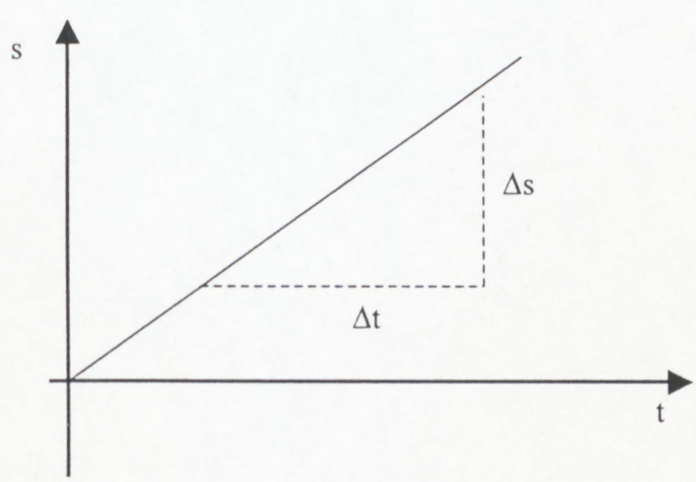


FIGURE 3.5

Now from equation (1) we see that

$$k = \frac{s}{t}$$

and $\frac{s}{t}$ *equals the (constant) slope of the graph, assuming that* $\frac{\Delta s}{\Delta t} = \frac{s_j - s_i}{t_j - t_i} = \frac{s_j}{t_j}$ *(if we take $s_i = t_i = 0$). But the slope* $\left(\frac{\Delta s}{\Delta t}\right)$ *equals the velocity of the body. hence $k = v$, and we can write equation (1) as:*

$$s = \upsilon . t$$

$$\text{or} \quad \upsilon = \frac{s}{t} \qquad (2)$$

Equation (2) now defines velocity in terms of the ratio of the two physical quantities depicted on the graph.

Jordaan (1992:105) follows a similar method by using a table of values of *s vs t* to illustrate what is meant by a "rate". His method is described briefly to illustrate how a table can be used to define a new variable in terms of two known quantities:

Consider the following table of distance-time measurements which were made for a body in motion:

t	0	1	2	3	-----	7	-----
s	0	4	8	12	-----	28	-----

The table depicts uniform motion in a straight line. Now the result $\frac{s_j - s_i}{t_j - t_i}$ = constant where i and j refer to the ith and jth pair of the table respectively.

Now taking $s_i = t_i = 0$, we can write:

$$\frac{s_j}{t_j} = \text{constant}$$

He then encourages students to refer to this constant ratio which exists between two variables as a "rate". This rate represents a new variable namely "speed".

3.4.3 Rate of change

Many relationships in science can be expressed in the form, y = mx or $m = \frac{y}{x}$. In this regard, Jordaan (1992:107) distinguishes between two types of relationships:

i) Those where $m = \frac{y}{x}$ acts as *the defining equation for m*, for example $a = \frac{\Delta v}{\Delta t}$ where the left can "only be fully understood in terms of the right" and;

ii) those relationships where *each variable can be defined independent of the other variables*, such as $a = \frac{F}{m}$.

It is the former type (type (i)) which leads naturally to the concept of rate of change, a concept which many students have difficulty in understanding.

There is little doubt that the idea of "rate of change" is crucial in kinematics and poor mastery of this concept will prevent students from understanding fully the relationship between the various kinematic quantities and their graphical representations.

Nickerson (1985) reports that students frequently confuse "rate" with "amount". This is shown in answers to questions involving such concepts as *velocity* vs *position* and *acceleration* vs *velocity*. He contends that a possible reason for this is that:

> *rate relationships represented by mathematical equations are inherently static and do not facilitate an appreciation of the fact that the relationship is actually dynamic.* (1985:206)

Orton (1984) undertook a detailed study of pupils' and tertiary students' understanding of the concept of rate of change and its implication for the study of calculus. The responses to clinical interviews conducted with students indicated a very low level of understanding of rate of change. He identified confusion between average speed, constant speed and variable speed and further reports that 6th form advanced level mathematics students had serious problems with rate of change in the context of linear graphs. When these students were asked about the rate of change of depth in a tank that receives water at a constant rate, they frequently responded with the height instead of the slope of the given graph. Responses to a similar question based upon a linear graph defined by an equation were worse still, with about one-third of the school pupils giving the y-values instead of the slope.

Another task was based on a curved graph: He gave a number of data points for the distance that a ball has travelled with constant acceleration down a hill as a function of time and also provided a graph of the given co-ordinates. Hardly any pupils were able to describe how to measure graphically the rate of change at which the distance is changing at a specific time, indicating a very limited understanding of instantaneous rate of change.

> *Talking about places on a curve where the function is increasing, where it is decreasing or where it is increasing or decreasing most rapidly may be very important in the context of rate of change.* (Orton 1984:26)

The inability to obtain the correct average rate of change of y with respect to x over a specific interval such as BE in graph (a) of figure 3.6 has direct implications for the understanding of a concept such as average velocity and how this can be determined from the *s vs t* graph in (b).

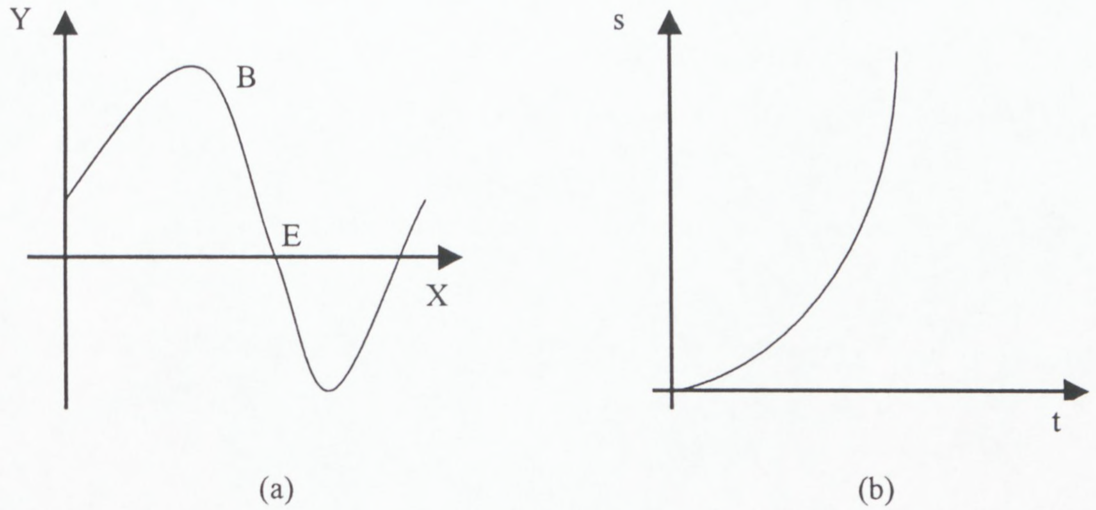


FIGURE 3.6

De Jager (1990) has shown through a series of clinical interviews, as well as pupil responses to a multiple choice questionnaire dealing with the gradient concept, that:

i) pupils generally do not associate larger gradient with bigger rate of change;

ii) pupils have difficulty in describing the behaviour of a function over a specific interval from its graphical representation;

iii) the association between constant/non-constant gradients and straight/non-straight line graphs presents serious difficulties;

iv) real-life physical situations can assist in overcoming conceptual problems with respect to gradient and rate of change.

Confusion with regard to rate of change in kinematics is widely reported by researchers such as Trowbridge and McDermott (1980, 1981), McDermott *et al.* (1987), Thijs (1987), Clement (1989). Even honours students in physics have been shown to display conceptual difficulties similar to those of high school students (see Peters 1982).

Most of the confusion has to do with a lack of distinction between velocity and change in velocity and similarly between position and change in position. When students do include the change in velocity they often ignore the time interval during which the change takes place. (See for example Trowbridge and McDermott 1981).

Clement (1989:80) puts it bluntly when referring to a typical student error:

> *The student has not associated speed with change in position of the object over time.*

Student difficulties with the rate-of-change concept will receive further elucidation in the next paragraph which deals with graphing problems in general.

3.5 THE UNDERSTANDING OF GRAPHS IN GENERAL BY PUPILS AND STUDENTS

3.5.1 Introduction

In order to analyse and discuss the nature of student misconceptions with respect to motion graphs it is first necessary to give a brief review of some of the most significant research findings on the nature of students' graphing difficulties in general. This will be followed in the next section by an overview of research with respect to students' understanding of kinematic concepts. The last section deals with a number of specifically identified misconceptions in kinematic graphs and this will be shown to be often a natural consequence of students' problems with graphs in general and / or problems with specific kinematic concepts.

The necessity to separate problems with graphs from problems with kinematics is stressed by such authors as McDermott *et al.* (1987), Clement (1989) and Brasell and Rowe (1993). Brasell and Rowe state for example:

> *With kinematic graphs, researchers have had difficulty separating students' misunderstandings of kinematic concepts from their problem with graphs as the vehicle for representing the concepts.* (1993:64)

3.5.2 Graphs and abstract reasoning ability

Graphs represent an important symbol system in science and require abstract reasoning ability for proper interpretation. This view is shared by a number of authors (see for example McKenzie & Padilla 1981; Mokros & Tinkler 1987; Wavering 1989 and De Jager 1990). Wavering (1989) conducted a study to determine the logical reasoning necessary to construct line graphs and showed a close fit with the Piagetian model of concrete and formal operational reasoning. He categorised graphs constructed by students in Grades 7 to 12 and showed that many of these students were unable to explain the relationship between the variables. He concluded that the reason for this shortcoming was that line graph construction and interpretation require abstract reasoning ability.

McKenzie and Padilla (1981 32) support this view when they contend that producing graphs and interpreting data are activities which do not come easily for many pupils. They go on to quote a study which reported a 0,77 correlation between formal reasoning ability and graphing skills (Padilla, Okey and Dillashaw 1981). Clement (1989:78) found that students often lacked the cognitive resources to cope with the presentation of variation and covariation in terms of the relationship between a practical situation and its qualitative graph.

Consistent with the findings of the above-mentioned researchers, Brasell and Rowe (1993:65) correlated graphing performance significantly with both verbal ability and reasoning and development scores.

3.5.3 The nature of student difficulties with graphs in general

Many students at school as well as tertiary level take subjects such as physics, chemistry, biology, geography and economics. These and many other subjects make use of graphs to display the relationship between two variables over a large number of measurements. Students often display a mere instrumental understanding of these graphs, a fact that is demonstrated by the frequent inability of students to answer higher order questions about graphs. It appears that students often do not understand that a graph is (merely) a symbolic representation of the dynamic relationship between two or three variables. A number of studies report on important research findings with respect to the nature and extent of student difficulties with graphs.

A Padilla *et al.* (1986) applied a test for assessing the ability of students in Grades 7 to 12 to construct and interpret line graphs. They found that although most students had little difficulty with the simple algorithm for reading data from a given graph or plotting data points on a graph, higher order graphing skills were seriously lacking. Examples of subskills that were most difficult, included the following:

- scaling of axes
- assigning variables to the axes
- using a "best-fit" line
- interpolation and extrapolation
- describing the relationship between the variables.

B Brasell and Rowe (1993) conducted a study among 93 students (average age 17,7) to test students' ideas of usefulness, difficulty and interest in graphs. Furthermore, graphing ability was examined in three ways: First a computer-administered graph interpreting test was given to the group. Secondly the group had to complete a kinematics graph test after a single period discussion involving translation B (graph → verbal description of a motion with the verbal description given in colloquial or technical language). Thirdly they tested for students' ability to use the conventions of Cartesian graphs by asking them to construct a graph of the height that a bouncing ball rebounds to, as a function of the height from which it is dropped. Their findings can be summarised as follows:

a) error rates were much higher for items that described the physical event in colloquial language rather than in terms of an unambiguous scientific description of the event;

b) higher error rates were reported for the A translation (verbal description → graph) than for the B translation (graph → verbal description). A possible reason that they propose is that the latter translation is "easier" because it moves from a less familiar system of representation to a more familiar one;

c) students experienced great difficulty in constructing the graph to represent the bouncing ball data. This indicated a non-existent understanding of how Cartesian graphs represent data as well as an inability to reorganise the data to reveal the relationship among the variables;

d) students tended to select incorrect graphs which were *temptingly similar to a picture of the event;*

e) they failed to link the colloquial description of an event with a scientific description of the relevant variable.

C De Jager (1990) investigated senior pupils' understanding of the gradient concept. He analysed the responses of a number of Grades 10 and 11 students to a series of clinical interviews as well as a multiple choice questionnaire which tested for various aspects of the understanding of gradient.

Among his findings which is significant to the present study is that pupils have great difficulty in making the translation from a table of values to a verbal description of the situation. Two type B translations (graph → verbal description of the physical situation and graph → verbal mathematical description without context) were also poorly handled, with the latter causing particular difficulty.

With respect to the specific concept of negative gradient, De Jager reports the following:

> *Few pupils could identify a graph with negative gradient with the phrase 'as x increases so y decreases'.* (1990:160)

When the same graph was given in the context of speed and time, however, most pupils could associate the negative gradient with the phrase: *as the distance (time) increases so the speed decreases.* The association of a varying gradient with a curved line graph is also reported to present particular difficulties for many pupils, especially if the association has to be made in an abstract mathematical manner (De Jager 1990:162).

The finding that when graphs are presented and interpreted in the context of real-life physical situations they can help to overcome many graphing problems is significant to this study. Motion is a typical and very familiar real-life situation and hence it can be expected that the identification (and attempted correction) of misconceptions with respect to motion graphs should greatly assist the understanding of graphing errors in general.

De Jager (1990:12) refers to Janvier's (1978) research with respect to Cartesian graphs and reports that Janvier found that most pupils have no trouble in mastering skills relating to isolated points such as reading off co-ordinates or plotting of points. Skills relating to interpretation of graphs such as describing the behaviour of graph over an interval, are poorly handled, however. Janvier conducted a teaching experiment involving two groups of pupils who had to interpret some experimental data, the one group using tables and the other graphs. He reports that changes in the function values and rates of change were more readily extracted from the *tables,* while the graphic group found it easier to distinguish between the actual *symbolic meaning* of a graph and the (erroneous) "graph as a picture" interpretation.

Other authors who have undertaken research on student difficulty with graphs include Kerslake (1977), Janvier (1978), McKenzie and Padilla (1981), Linn, Layman and Nachmias (1987) and Leinhardt *et al.* (1990).

D McKenzie and Pandilla (1981) point out that students easily get confused between the responding and the manipulated variable. They often do not differentiate between the two, because they do not understand the cause-and-effect nature of a set of data points. This usually also results in a negation of the convention of plotting the responding variable on the vertical axis and the manipulated variable on the horizontal axis. The use of the phrases "dependent" and "independent" variable seem to do little to overcome this confusion.

E Linn *et al.* (1987) proposed that each individual pupil finds himself at some point on their proposed *ideal chain of cognitive accomplishments* with respect to graphs. They relied on the advice of expert teachers and on observations of students learning about graphs in order to identify a number of links, each consisting of a number of specific graphing skills. They also studied the literature on students' difficulties with graphs and noted recent advances in understanding problem solving and memory.

Linn *et al.* (1987:247) also introduced the idea of a "template" which is described as *stereotypic sequences of activities that are used repetitively in solving problems*. An example of the use of such a template would be a student who uses his first worked out example of a (for him) new physics problem, as a *representation* of how problems falling in this class can be solved. When he encounters another question relating to the same class of problems, he will often attempt to fit it to the template that he has created for that particular class of problem.

They proposed the following four main links in their *chain of cognitive accomplishment*.

(i) Recognition of graph features

This would include aspects such as identifying the title of a graph; location of the axes, labelling the axes with appropriate variables and their units and recognising the meaning of graphically represented data. An example from kinematics would be the construction of a perpendicular pair of axes with time (in seconds) indicated on the horizontal axis and velocity (say) in $m.s^{-1}$ indicated on the vertical axis. It would include the ability to be able to read off a specific velocity value at a given time value. The description of general trends in the variation of the velocity such as a decreasing, increasing or constant velocity would also belong to the category of recognising graph features.

(ii) Using appropriate graph templates

The idea is that the instructor enters into a process of *template building* to modify students templates where necessary. Learners (according to the constructivist model) construct their own understanding of the world, in that they take their first example of a particular phenomenon as a prototype and use their solution as a template to interpret the next example. Modification of this template can occur through (amongst other ways) instruction, selection of appropriate examples and exposure to a particular environment.

Linn *et al.* (1987:251) give the following example (see figure 3.7) to test whether a student has developed an appropriate template for interpreting motion graphs correctly.

Which graph best shows the change in the speed of a ball which started at rest at the top of a slope and was then allowed to roll down the slope?

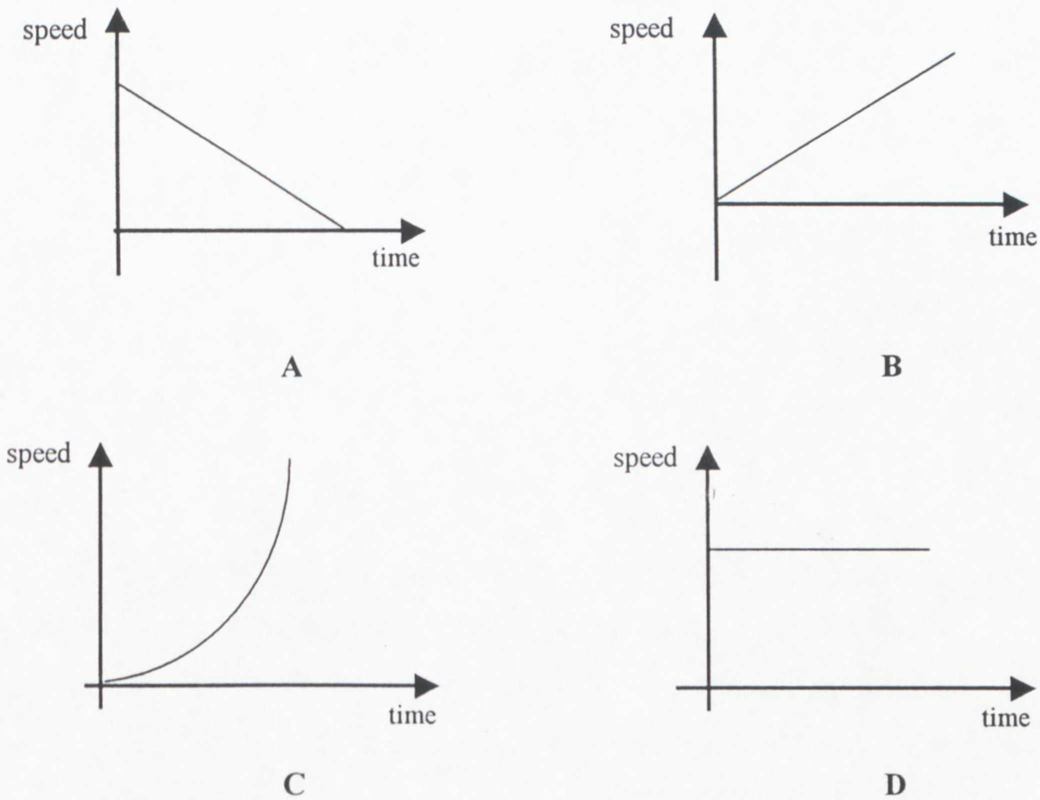


FIGURE 3.7 Test item for motion template

Students who lacked scientifically correct speed templates would opt for A (graph represents situation iconically) or C (speed is confused with displacement). It must be pointed out that the authors obviously assumed that motion takes place in the absence of friction in order for B to represent the correct relation.

(iii) Graph design skills

Graph design skills are described in terms of the ability to apply templates used in a specific subject domain to new problems. More specifically this includes skills such as combining different templates and interpreting a chosen template or combination of templates.

A single example from cartesian graphs will be used to illustrate the category of graph design skills: Assume a student is familiar with a template relating to the drawing of the graph of $y = 3x - 1$. He puts $x = 0$, notes that $y = -1$, then puts $y = 0$, noting that $x = \frac{1}{3}$ and proceeds to draw the graph correctly. He is now confronted with the problem of drawing the graph represented by $y = x^2 + x - 2$ and again puts $x = 0$ (leading to $y = -2$) and then lets $y = 0$ (leading to $x = -2$ or 1). In this way he has obtained key-coordinates for the drawing of the new graph by using a known (and correct) procedure from a similar but different situation.

(iv) Graph problem-solving skills

This, the most advanced link in the chain of cognitive accomplishments, is described in terms of a student's ability to solve graphing problems in a subject domain which is new or unfamiliar to him.

Thus a student who, for example, can determine the displacement of an object from its *v-t* graph as illustrated in figure 3.8 without having received any instruction in kinematic graphs, would have demonstrated his ability to have advanced to the highest level of cognitive accomplishments in graphing.

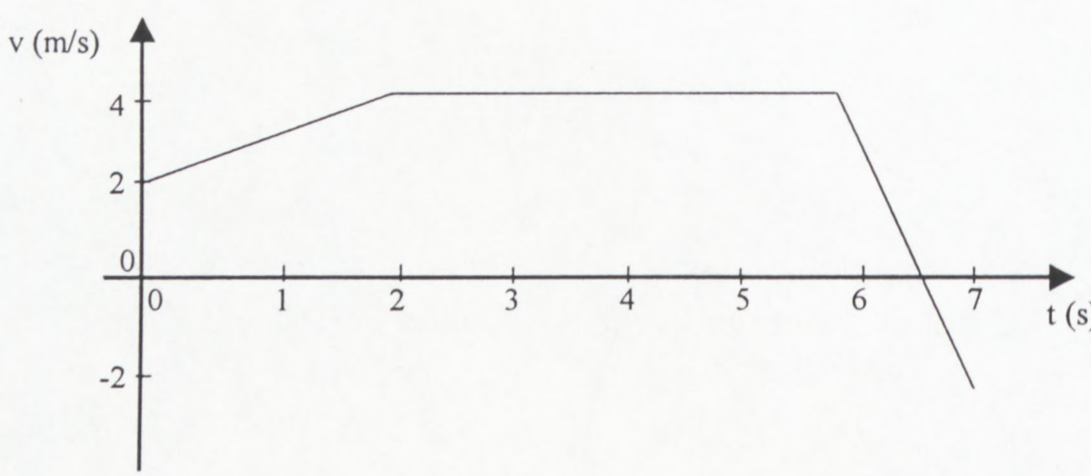


FIGURE 3.8

The identification of the area between the graph and the time axis as representing the change in displacement of the body, appears sufficiently advanced to be classified in the category "problem-solving skills".

Linn *et al.* (1987) tested their classification of cognitive accomplishments by exposing a sample of 8th-grade pupils to a computer program which produces a graph as the experiment progresses. An example of such an experiment is the monitoring of a system that is gaining or losing heat energy. They reported (1987:250) that students made substantial progress in understanding graph features and temperature templates as a result of their exposure to these microcomputer-based laboratories (MBL's). The use of MBL's also greatly increased their ability to identify trends and locate extreme points. MBL's have a great impact on the understanding of motion graphs and will again be referred to briefly in Chapter 5.

F Kerslake 1977 (in de Jager 1980:11) found evidence of the following shortcomings and misinterpretations of pupils with respect to Cartesian graphs:

i) Problems with plotting and reading off points are encountered when decimal numbers are used and interpolation is required.

ii) A very tentative association of equal gradients with parallelism.

iii) Transformations C and D (Graph $\leftrightarrow$ Formula) are poorly handled, especially with respect to specific features of a graph.

iv) Serious difficulty is experienced with translation A. Pupils do not generally know how to go about extracting relevant information from a real-life situation in order to represent it graphically.

G Leinhardt *et al.* (1990:30-44) define misconceptions as *incorrect features of student knowledge that are repeatable and explicit*, and go on to point out that misconceptions with graphs are often the result of deficiencies in previous formal learning. They go on to highlight the following categories of student difficulties:

i) *Functions*

Students possess inaccurate ideas of what graphs of *functions* should look like. They were frequently observed classifying only linear or regular patterned graphs as functions.

ii) *Linearity*

When asked to produce examples of graphs that would pass through two given points, students produced mostly linear graphs. Most of these students indicated with considerable certainty that the straight-line was the *only* graph that could connect the points.

iii) *Continuous versus discrete graphs*

Students were found to err in both directions, that is representing continuous data in a discrete manner and representing (or interpreting) discrete data in a continuous manner. More specifically, students often maintain a strict focus on individual points rather than the lines that join them.

iv) *Representation of functions*

When translations between the various representations of functions were considered, it was found that the particular translation *from a graph to its equation* provides students with particular difficulty as it involves *pattern detection.* One study (Leinhardt *et al.* 1990:35) found that only 5 % of 17-year-olds could generate the equation of a straight line graph with indicated intercepts (-3;0) and (0;5). Particular difficulty with the use and meaning of *slope* is cited as the main cause of this type of difficulty.

(v) *Relative reading and interpretation*

When constructing and interpreting graphs that represent real situations, learners often focus on individual points and ignore the global features of the graph. This difficulty is loosely grouped under the following three categories:

- *interval/point confusion.* When interpreting graphs students often focus on individual points instead of a range of points.

- *slope/height confusion.* Students frequently confuse gradient with maximum (or minimum) values, because they do not attend to the physical quantity described by the slope of a given graph. In kinematics this is illustrated by the fact that students confuse the concepts of speed and distance when a distance-time graph has to be interpreted.

- *iconic interpretation.* Leinhardt *et al.* (1990:39) state that

> *The most frequently cited student error with respect to interpreting and constructing graphs that represent situations is an iconic interpretation.*

In kinematics, students are particularly inclined to draw graphs which are pictures of the problem scene. The extent of this particular student difficulty is further explored on p.229 as well as in Chapter 5.

(vi) *Concept of variable*

Students often do not understand the meaning of a symbol for a variable. For example many students viewed the functional equation, 7 x W + 22 = 109 as different from 7 x N + 22 = 109, suggesting that they missed the idea of a functional relationship between two variables.

(vii) *Notation*

Many students were found to experience difficulty with the idea of an interval scale with many believing that the scales on the two axes *had* to be symmetrical. They also had a poor understanding of the difference between features that are part of the graph itself (like y-intercept) and features that depend on the system on which it is constructed (like the slope of the graph).

SUMMARY OF STUDENT MISCONCEPTIONS ABOUT GRAPHS

An analysis of the preceding research on student difficulties with respect to the construction and interpretation of graphs reveals the following general categories of student misconceptions:

- General difficulty with the construction of graphs to represent an observed physical situation in the case where data points are not explicitly provided.

- The transformation from a graph to a mathematical description in terms of an algebraic formula or equation is problematic for many students. However, they have less difficulty in *describing the graph verbally* when they use scientific terms than when they use colloquial terminology.

- Scaling of axes and interpolation tasks are not handled successfully by (especially) early secondary students.

- Problems with gradient lead to the following difficulties:

  - not comprehending slope as a measure of rate.

  - slope-height confusion when attempting to interpret graphs in terms of real-life situations.

- particular difficulty with the varying gradient of curved graphs.

- not differentiating between a positive and a negative gradient.

- not always accepting that parallel lines have equal gradients.

- A tendency for a pointwise focus on graphs rather than a global view of the graph as a whole. This often results in a failure to recognise trends over a specific interval.

- A confusion between continuous and discreet graphical representation resulting in the following misconceptions:

  - connecting data points on a graph when this is not appropriate

  - a tendency to use straight lines when curves are more appropriate to represent a function

  - not understanding the idea of a best-fit line drawn through experimentally obtained data points.

- Iconic representation where the graph is viewed as a picture of the observed problem scene.

- A confused understanding of variables and parameters as in the formula v = u + at, which leads to the following problems:

  - confusion between dependent and independent variables

  - not understanding that a graph describes the nature of the relationship between an independent and a dependent variable.

- a reluctance to bring to the science class that which has been learned about variables in the mathematics class.

3.5.4 The relationship between mathematics graphs and physics graphs

A graphical representation in mathematics usually requires the student to consider an algebraic formula, determine a number of co-ordinate points which satisfy the given function, plot these points on a Cartesian axes system and join the points with a line (straight or curved). These graphs are often sketch graphs where only certain important co-ordinates, such as intercepts with axes and turning points are indicated. The manipulated (or independent) variable is usually denoted by the symbol x and the responding (or dependent) variable by y. Questions about the behaviour of the given function as depicted by its graphical representation are often included. Alternatively, a sketch graph with certain co-ordinates indicated can be provided with the instruction to determine the defining equation of the function thus depicted.

In the physics course, however, the graph displays the relationship between two physical or real-life variables. The defining equation is not always immediately known, so that the student cannot rely on the method he used in the mathematics class to draw the graph.

McDermott *et al.* 1987 found that when the physical situation is described by a specific formula, students often fail to associate this with the corresponding mathematical formula and hence cannot immediately recognise the shape of the required graph. The problem could well be that they have not received adequate guidance by instructors with regard to the integration of graphs in mathematics with similar graphs in other disciplines.

> *The problem students have with graphing cannot be simply attributed to inadequate preparation in mathematics. Frequently students who have no trouble plotting points and computing slopes cannot apply what they have learned about graphs from their study of mathematics to physics.* (McDermott *et al.* 1987:503)

A simple example relating to ideal gases will illustrate the typical difficulty that a student might encounter with graphs:

Consider the well known ideal gas law

$$pV = nRT$$

Now assume that 4 moles of oxygen at 300 K is allowed to occupy different volumes. Draw a sketch graph to illustrate how the pressure and volume of the oxygen relate to each other.

Students in South African schools taking mathematics in Grade 12 should be very familiar with the mathematical formula for a hyperbola, namely xy = k. The difficulty they have relates to the identification of the term nRT as a constant and thus equivalent to the "mathematical" k. Furthermore, they must associate V with x (as the independent variable) and p with y (as the dependent variable).

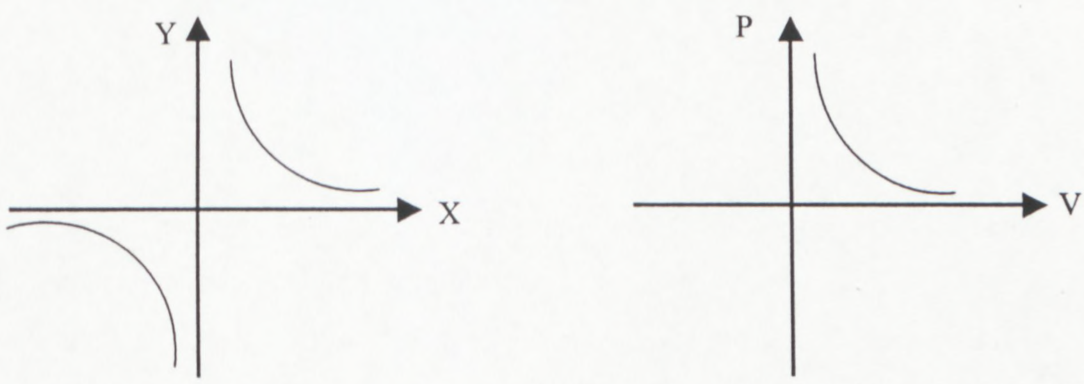


FIGURE 3.9

The fact that the negative values on the horizontal axes are often ignored in the case of the physics graph is also significant. (It must be noted here that negative values cannot <u>always</u> be ignored, as in the case where temperature, in °C, is indicated on the x-axis). Figure 3.9 illustrates the mathematical graph and the *p-V* graph.

3.5.5 Conclusion

Graphs are an efficient and effective way of representing the wealth of information which is accumulated in all walks of life. Data are represented in graphical form in newspapers, magazines, technical books and even on computer screens. It is becoming increasingly important that students understand graphical representations and know how to use them.

It is, however, equally true that most learners experience at least a measure of difficulty with graphing. This difficulty refers to both graph construction as well as graph interpretation and is confirmed by a number of investigations into the level of students' understanding of graphs.

Instructors should be aware of the nature and extent of these difficulties and address them in a sensitive way starting with (and respecting) a student's own personal understanding of graphing.

3.6 STUDENTS' UNDERSTANDING OF KINEMATICS

3.6.1 Students' preconceptions about kinematics

Students who have difficulty in representing a motion graphically could either have problems with graphs in general or they could have difficulty in understanding the meaning of the kinematic quantities involved in the graphical representation. The fact that it is often difficult to separate these two possible underlying causes has been recognised by authors such as Trowbridge and McDermott (1980; 1981), McDermott *et al.* (1983), Clement (1989) and Brasell and Rowe (1993).

Before research findings with respect to the misunderstanding of kinematic concepts are reviewed, it is necessary to refer again briefly to the important role of students' own protoconcepts in this regard.

It seems clear from a number of studies (Trowbridge and McDermott 1980, Andersson 1987, Hewson 1987 and others) that students spontaneously develop their own protoconcepts with respect to an aspect such as motion in the real world. The significance of the students' own constructive ideas has already been explored, so that here a few further comments will be made to accentuate these in the context of kinematics.

> *Our study has shown that prior to instruction the student typically has a repertoire of procedures, vocabulary, associations and analogies for interpreting motion in the real world.* (Trowbridge and McDermott 1980).

Students are generally committed to their own constructed model which gives meaning to the real world and is consequently often resistant to instruction. They find it difficult to reconcile their own protoconcepts with those propounded by orthodox science so that certain beliefs are inclined to remain intact even after intensive instruction. In the field of kinematics the instruction is often quite formal and theoretical so that the student seldom has the opportunity to associate specific concepts with actual observed motions. Rosenquist and McDermott (1987:415) mention for example that *for many students the traditional emphasis on algebraic formalism does not lead to conceptual understanding*.

Trowbridge and McDermott (1980), during their investigation of students' understanding of the concept of velocity, concluded that students held widely different views on motion to those held by their instructors. These views were found to be based on intuition, experience as well as their perception of previous instruction.

3.6.2 Probing for students' understanding of kinematic concepts.

The question of how to go about in order to obtain a better understanding of students' ideas about motion has received attention from a number of authors. Poduska and Phillips (1986) made use of student responses to a series of demonstrations involving such apparatus as toy trains and carts accelerating down an incline in order to investigate the mental structure used by students to think about speed.

The research undertaken by Trowbridge and McDermott (1980) and Schuster (1983) into what students must be able to do to prove their mastery of kinematics, has been reported on in paragraphs 3.3.1 and 3.3.2 and will be briefly stated again. Schuster (1983) tested students' ability to use different representations of a demonstrated motion as a criterion for testing their understanding of kinematic concepts, while Trowbridge and McDermott (1980:1020) propose the following criteria:

- the extent to which a student can define the kinematic concept (or quantity) in an acceptable operational manner,

- the ability to distinguish between different but related concepts such as speed and distance or change of velocity and acceleration,

- the ability to apply concepts successfully.

They make the point that success in conventional test questions does not necessarily imply real understanding. In their investigation of student understanding of the concepts of velocity and acceleration (1980, 1981) they have adopted as a measure of understanding of a kinematic concept ...

> *the degree to which an individual successfully applies that concept to the interpretation of simple motion of real objects.* (1980:1020)

In practice they propose that in an individual demonstration interview the student is confronted with a simple physical situation and asked to respond to a specific sequence of questions. Trowbridge and McDermott have found that a variety of linear motions can be demonstrated by a steel ball or balls rolling along straight aluminium U channels.

3.6.3 Brief description of some specific identified conceptual difficulties

3.6.3.1 Speed confusion

Boulanger (1976) as well as Poduska and Phillips (1986) investigated the mental structures (in terms of the Piagetian model) that children and students use to think about speed. Poduska and Phillips (1986:846) confirmed that although students develop at different rates, they all follow the same sequential steps. Mental operations involving conservation of distance are acquired first, after which follows those involving speed, time and proportional reasoning. They suggest that topics in science should be presented in this order, to be in keeping with the natural order of mental development. If a student lacks the mental structure to accommodate a concept such as speed for example he can do nothing more than memorise what an instructor says in order to pass an examination.

Boulanger (1976) stresses that a child who has not yet reached the formal operational stage can experience serious confusion regarding *speed* because he lacks the ability to use or understand proportional reasoning. This is often manifested by an inability to compare the speeds of two objects which cover different distances in different times. Brasell (1987) as well as Mokros and Tinker (1987) report on studies which indicate a higher proportion of concept errors with *velocity* graphs than with *displacement* graphs and conclude that this indicates that velocity is conceptually more difficult than displacement. Velocity as the rate of change of displacement is more abstract than displacement and hence more difficult to understand.

Orton (1984) contends that a very low level of understanding of the concept of rate of change causes confusion between average speed, constant speed and variable speed. He calls for "real-life" situations to be included in the instruction so as to sort out the difference between these ideas in pupils' minds.

Trowbridge and McDermott (1980) studied students' understanding of velocity in one dimension by interviewing them about their interpretation of the motions of two identical balls rolling on parallel tracks. They report the following student misconceptions with respect to velocity:

- When two objects reach the same position, they have the same speed. This means that students have used a position criteria to determine the time when the speeds of two balls are the same, or put differently: They often identify the instant when one object *passes* the other as a time when the speeds of the two objects are equal.

- The object that is *ahead* travels *faster*. This association of being ahead or being behind as being faster or slower can also be related to a confusion between *displacement* and *rate of change of displacement*. Relative velocities are thus compared by simply comparing positions.

- Students do not understand the idea of *instantaneous velocity* as a value that refers to a specific instant. The idea of $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ as describing an instantaneous velocity seems far removed from any direct observation.

- Students seldom make a connection between *speed* and the ratio of *distance travelled to elapsed* time. This is in spite of the fact that they can often give an acceptable "textbook" definition of speed.

- No discrimination is made between *velocity* and *change in velocity* and no consideration is given to the *time intervals* during which these changes in velocity take place.

3.6.3.2 Frames of reference

Thijs (in Thijs *et al.* 1987:46) quotes Saltiel and Malgrange (1980) who found that students think of velocity as an intrinsic property of an object and are generally not aware of the frame of reference used. For a boat crossing a fast flowing river for example, students do not recognise that the component of the velocity produced by the propulsion system of the boat is independent of the speed of the current.

Aguirre and Erickson (1984:441) found that many students, instead of adopting a common reference point, tended to use several different reference points to describe the position of a body in a number of different locations.

Panse (1994) studied students' understanding of frames of reference and concluded that most students held strong alternative conceptions, often associating *frames of reference* with concrete objects and regarding particular phenomena as belonging to a specific frame of reference.

> *A frame of reference in students' alternative view is a kind of chunk of space tied to a concrete body, bounded (but not closed) by the size and shape parameters of the body, sitting inside which an observer looks at phenomena.* (Panse 1994:75)

The close link between frames of reference and relativity of motion indicates that this problem can effect a student's fundamental understanding of motion. The choice of reference point or reference level when describing a displacement is one example of the importance of a good understanding of frames of reference. Another (less obvious) example is the fact that the earth is usually taken as the frame of reference when dealing with velocities. Personal experience indicates that when a student (sitting stationary at a desk) is asked what his velocity is, the answer is usually *zero*. This student ignores the fact that he is part of the planet earth which possesses a number of rather complicated velocity components.

3.6.3.3 Acceleration confusion

Students seem to have conceptual difficulties with acceleration which are similar to difficulties experienced with velocity, only they are of a more severe nature (Warren 1979, Trowbridge and McDermott 1981, Thijs 1987)

Warren (1979) reports that students have problems with differentiating between concepts of "average velocity", "instantaneous velocity" and "acceleration". This results in students having great difficulty with the idea that a body can be momentarily at rest and yet be accelerated (such as a ball reaching its highest point after having been thrown vertically upwards). A situation where the velocity increases and the acceleration decreases is equally difficult for students to grasp. An example of such a situation is a ball rolling down a hill with a hollow curve in the slope, as shown in figure 3.10.

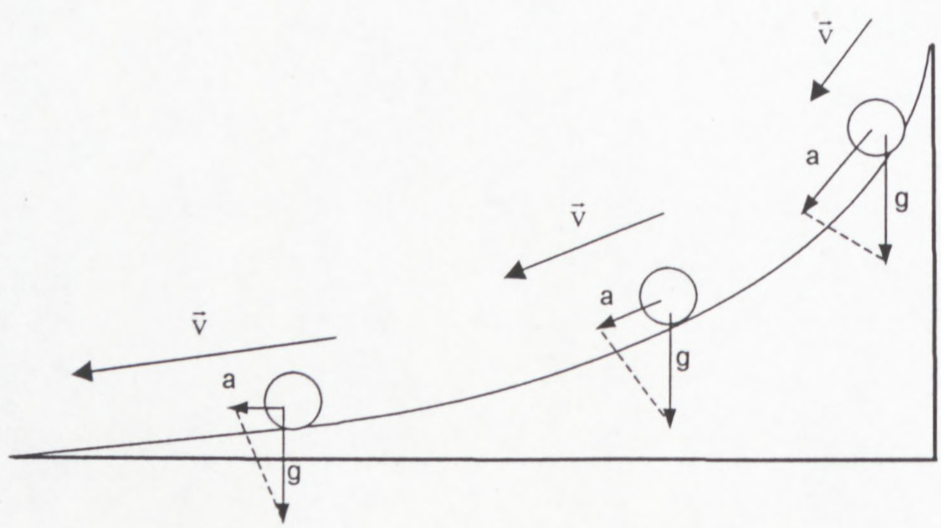


FIGURE 3.10

Trowbridge and McDermott in a follow-up study to their study on the understanding of velocity (1980) investigated students' understanding of acceleration in one dimension (1981). The criterion they used was as before: *Could the student apply kinematic concepts to the interpretation of the motion of real objects?* Their findings can be summarised as follows:

- The identified confusion between velocity and change in velocity of the earlier (1980) study caused problems with a practical interpretation of acceleration.

- Even when students realised that the concept of acceleration includes the idea of change in velocity they ignored the corresponding time interval during which the change takes place.

- Students believed that the acceleration of two objects must be the same if they are on the same incline, even when they observed that (due to different frictional effects) one object takes less time to undergo the same change in velocity.

- Two balls that reach the same position are perceived to have the same acceleration.

- The existence of a constant acceleration while the velocity undergoes constant change was not readily accepted.

- They often lack even a qualitative understanding of the concept of acceleration as the ratio $\frac{\Delta v}{\Delta t}$. The more rigorous definition of acceleration as the ratio of the change of instantaneous velocity to time is often merely memorised without any real understanding.

3.6.4 Sign convention and negative kinematic quantities

Very little research has been done to investigate student difficulties with regard to *negative values* for the main kinematic quantities of *displacement, velocity* and *acceleration*. Personal experience suggests that many students are extremely hesitant to deal with directional information. They often do not state which direction was chosen as positive when dealing with a calculation in kinematics or when they have to describe an observed motion verbally.

Goldberg and Anderson (1989:258) investigated possible reasons for students' difficulty with negative velocity and concluded that ... *in everyday life students are familiar with the magnitude of velocity, namely speed ... everyday usage may cause them to think of the magnitude and direction as completely separate attributes.*

Goldberg and Anderson (1989:258) also discovered that many students believed that a negative quantity somehow implies a "lesser amount" of that quantity. In this sense they did not accept that a body could be moving "negatively" but rather less positively.

The present study will investigate the extent of student difficulty with the following aspects of sign convention and negative values:

- A *reversal in direction* of a motion is associated with a *change in sign* of the velocity vector.

- A *negative displacement* is a displacement from a reference point which is *opposite* to the direction which has been chosen as *positive*

- A *negative acceleration* occurs when there exists a *resultant force* in the *negative direction.*

3.7 IDENTIFIED STUDENT MISCONCEPTIONS WITH RESPECT TO KINEMATIC GRAPHS

3.7.1 General

Researchers are generally in agreement that students often experience considerable difficulty with graphical representations in science as well as in other subjects (see for example Padilla *et al.* 1986, Wavering 1989, Leinhardt *et al.* 1990, and Brasell & Rowe 1993). These students often have little trouble with graphs in the mathematics course, so that the problem is likely to be an inability to apply graphing skills to a particular field of study (see for example McDermott *et al.* 1987:503). A number of key questions arise naturally from this state of affairs:

- What is the nature of the difficulties that students experience when they attempt to apply certain graphical skills to illustrate or describe a kinematic situation?

- How common are these difficulties, in other words what is the extent of the problem?

- How can instructors go about identifying the nature of typical student graphing errors in kinematics?

This section will focus on the nature and extent of specifically identified misconceptions in motion graphs as described by various researchers. Instructional strategies to remedy these problems will not be suggested at this stage.

3.7.2 Translation A difficulties (Physical situation → Graph)

3.7.2.1 General

Students experience great difficulty in making the translation between a physically demonstrated motion and its graphical representation, because they do not know which features of a graph to use to represent specific characteristics of the observed motion. Specific difficulties in this regard have been reported by (among others) Peters (1982), McDermott *et al.* (1983, 1987) and Rosenquist and McDermott (1987). McDermott *et al.* (1983:14), when reporting on the difficulty students have in connecting graphs and scientific concepts to actual physical systems, state the following:

> *Our students have considerable difficulty in relating graphs representing moving objects to the actual motions of real objects in the laboratory. They*

have trouble in going in both directions: in producing a graph from a real motion and in reproducing a motion represented by a given graph.

Peters (1982:503) asked students in an introductory honours physics course to sketch position vs time and velocity vs time graphs for the motion of a glider on an inclined air track. He reports that only 30% of the students represented the motion reasonably accurately and that they had particular difficulty with the representation of features such as speeding up and slowing down as well as being confused about the meaning of position. They also indicated the velocity to be positive despite a reversal in direction. These students were enrolled for an advanced course in physics, so the problem is likely to be more serious among students from the general population.

In an important investigation McDermott *et al.* (1983; 1987) presented students with a laboratory set-up consisting of a steel ball rolling along sections of a straight aluminium track, as shown in figure 3.11. The students were required to use meter sticks and stopclocks to make appropriate measurements for plotting *position-time* and *velocity-time* graphs for the motion of the steel ball.

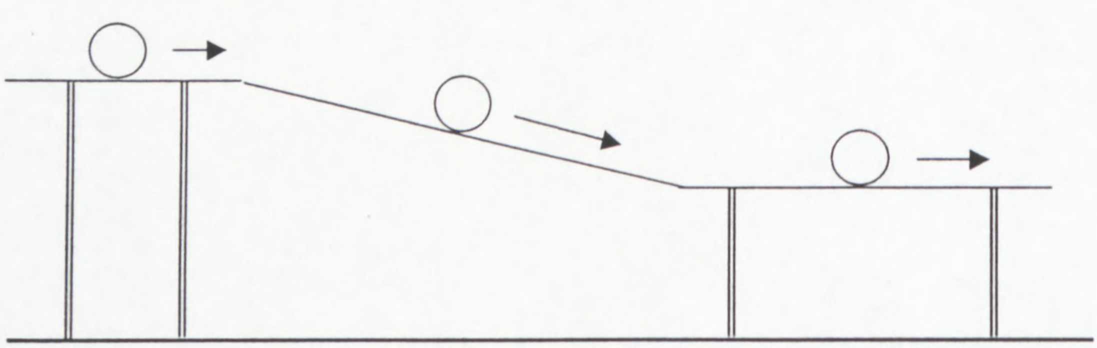


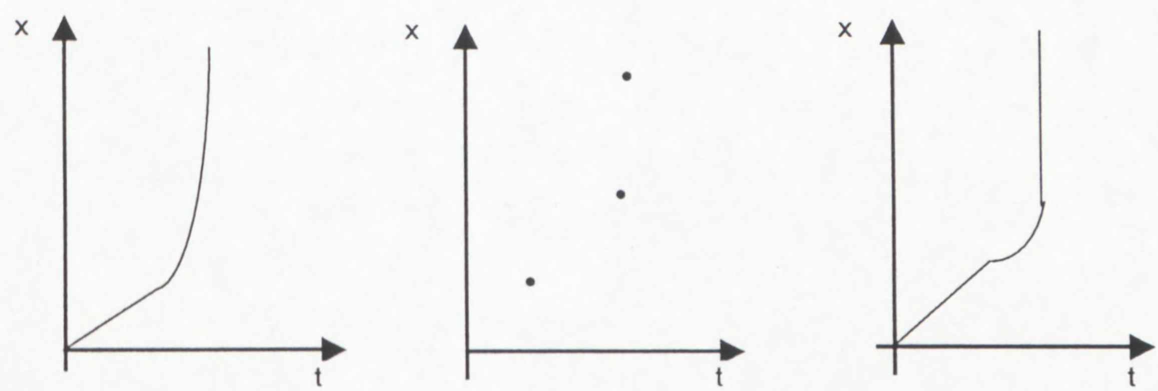
FIGURE 3.11

Although this configuration does not produce rectilinear motion in the strictest sense of the word, it nevertheless provides an experimental set-up that effectively tests students ability to produce graphs from a real motion.

The track above initially produces uniform motion at a low speed, followed by the sloping section producing accelerated motion and finally a second horizontal section producing uniform motion at higher speed. McDermott *et al.* (1983) found that, although the students that were tested studied position vs time graphs in the mathematics classes, they nevertheless experienced great difficulty in representing the motion of figure 3.11 graphically. Reported student difficulty with *position-time* and *velocity-time* graphs of the configuration represented in figure 3.11 is summarised below.

A position-time graph

The correct *x vs t* graph is given in figure 3.12(a). (It must be noted that all frictional effects are ignored).



(a) (b) (c)

FIGURE 3.12

Note that the correct graph should consist of a straight line (of small slope) followed by a curved section representing the acceleration and finally a straight section of larger slope. Student difficulty with *x vs t* graphs for the described physical situation can be summarised as follows:

(i) An inability to make the correct measurements with metre sticks and clocks, resulting in many student plotting the lengths of the segment of the track vs the clock reading when the ball passes the end of the segment. Figure

3.12(b) shows such a graph, illustrating clearly how these students plotted Δx (the displacement) for three consecutive time intervals.

(ii) A reluctance to draw curves on the *x vs t* graph because the graph is perceived to have straight lines like the track.

> *These students seem to think of an* x vs t *graph as a picture of the motion as it appears in the laboratory rather than as representation of how the ball's position changes with time.*
>
> (McDermott *et al.* 1983:15)

(iii) Students are inclined to join successive line segments with a kink or cusp as shown in figure 3.12(c). These students often know that the gradient of the *x vs t* graph yields velocity and yet they show no concern for the abrupt change in slope of the graph.

B velocity-time graph

The correct *v vs t* time graph is drawn in figure 3.13(a).

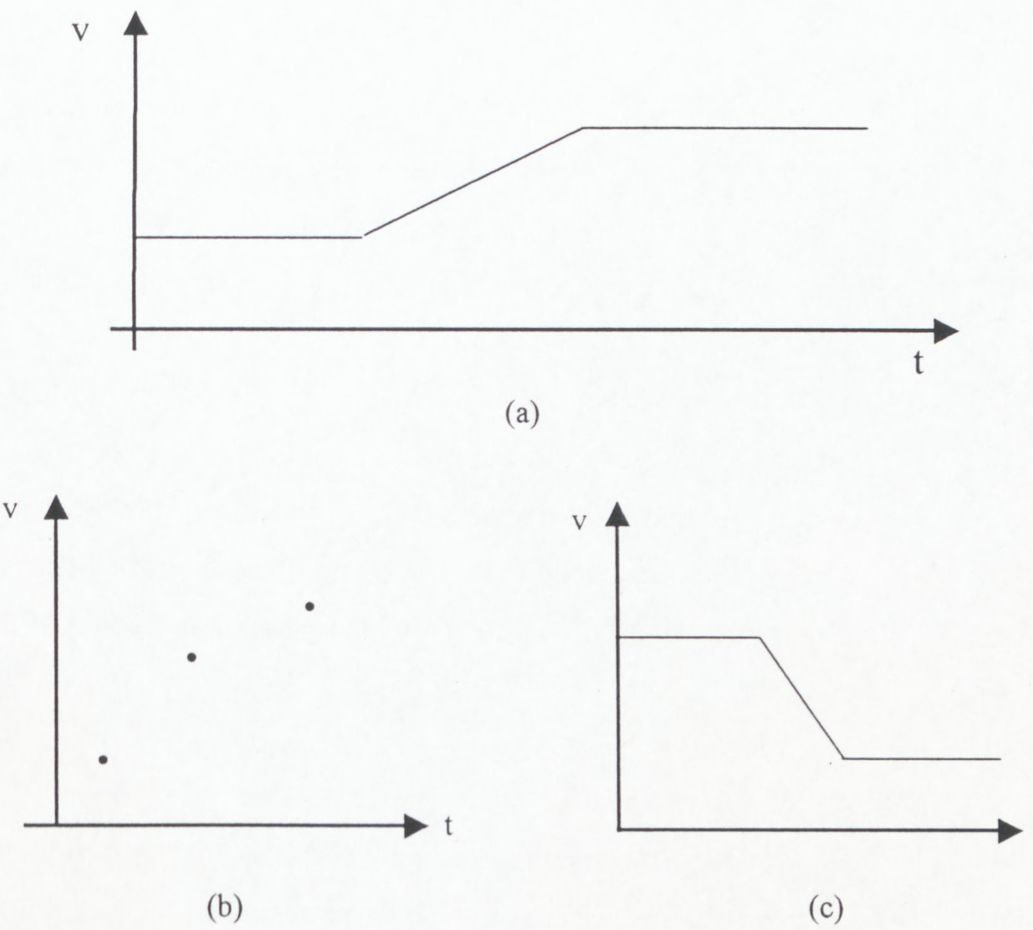


FIGURE 3.13

The correct graph should consist of two horizontal sections (representing sections of uniform velocity) joined by a sloping line (representing the accelerated motion). Commonly occurring errors were:

(i) An inability to associate the motion on the incline (acceleration) with a slanting straight line.

(ii) Not plotting a continuous graph but rather three separate points, as illustrated in figure 3.13(b). These students do not visualise velocity as a continuous varying quantity with a distinct value at each instant. Instead they associate a single velocity with each segment of the track.

(iii) The single velocity associated with each section of the track is often taken as the velocity at the end of the track.

(iv) Even for the level sections, points were plotted rather than lines, showing that these students, even for constant velocity events, did not understand that a graph should show all (i.e. continuous) values of the velocity. It seems that many students cannot correctly interpret the meaning of a graph which runs parallel to the time axis.

(v) Drawing graphs such that the slope of the graph resembles the path of the motion. A typical example of such an error described by McDermott *et al.* (1987) is illustrated in figure 3.13(c). The *v-t* graph drawn by these students is simply an iconic representation of the physical situation that the student has observed.

(vi) Some students confused the concepts of velocity and displacement. They simply plotted a number of position values and drew a best-fit line through these points. The result is a graph which resembles the position-time graph as indicated in figure 3.13(d)

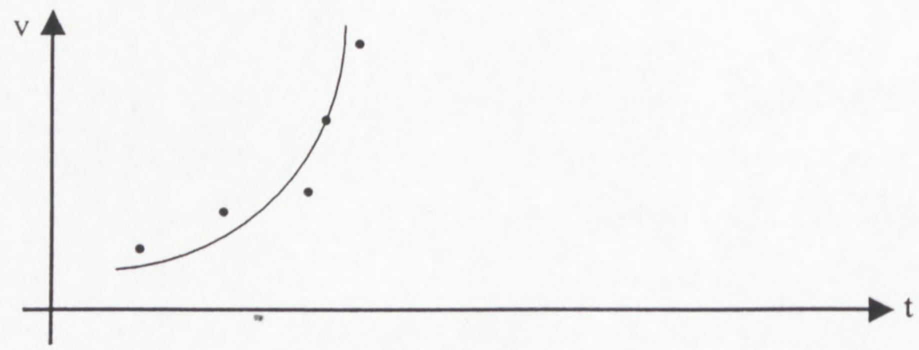


FIGURE 3.13(d)

In another experiment McDermott *et al.* (1987:509) demonstrated a motion which involved a ball moving up an incline with decreasing velocity, turning around and moving down the same incline with increasing velocity in the opposite direction. Many students could not draw the correct *v vs t* graph because they displayed a distinct unwillingness to cross the horizontal axis. Instead, the reversal in direction of the motion was represented by a change in direction of the graph as indicated in figure 3.14.

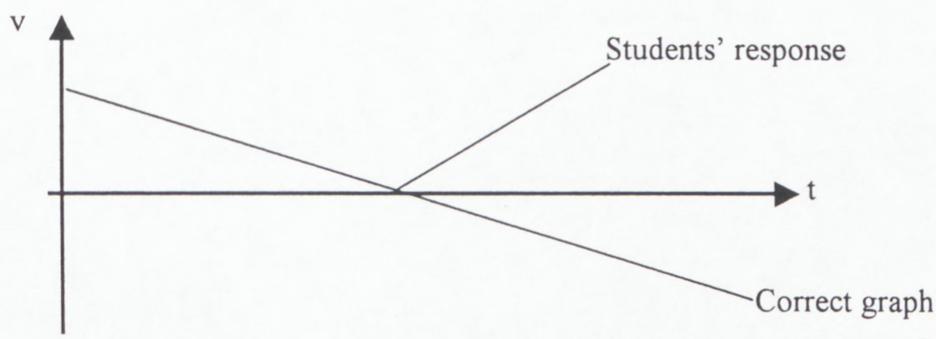


FIGURE 3.14

The corresponding *a vs t* graph was often drawn as follows:

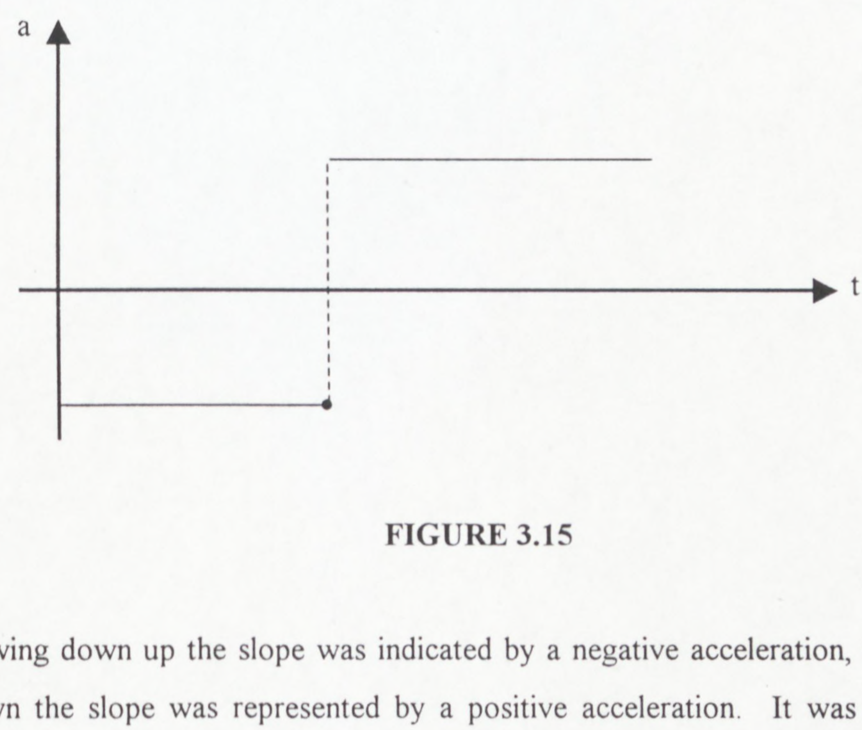
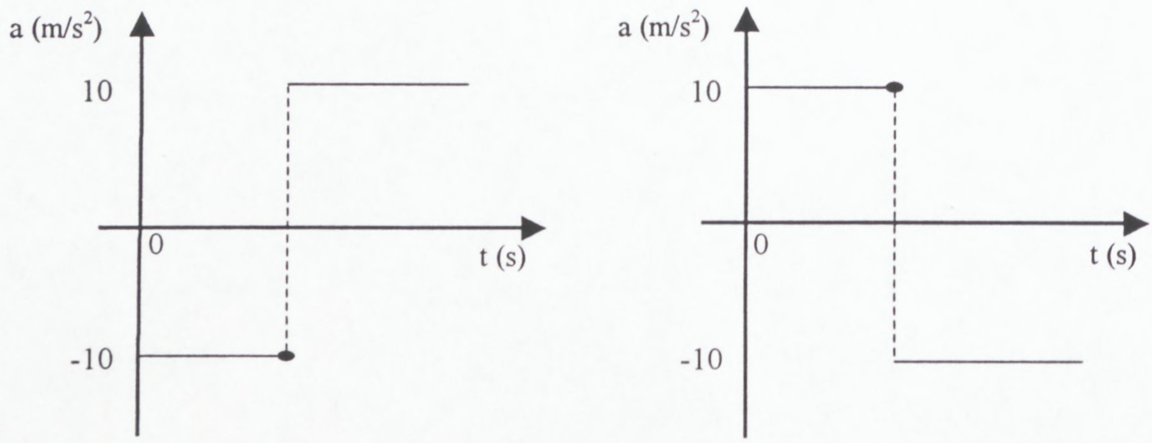


FIGURE 3.15

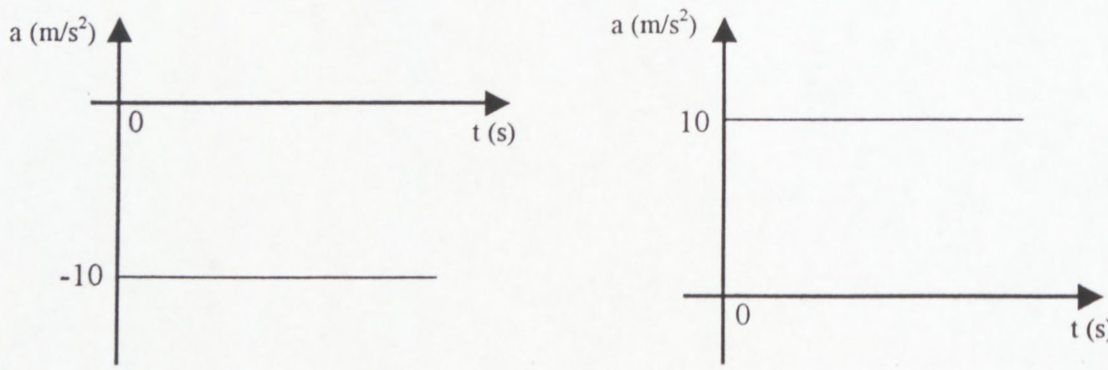
The slowing down up the slope was indicated by a negative acceleration, while "speeding up" down the slope was represented by a positive acceleration. It was not made clear whether the reason for this misconception is a misplaced definition of acceleration or whether it is derived from the incorrectly drawn velocity vs time graph.

If it is a misplaced definition of acceleration which gives rise to the incorrectly drawn graph, the reason is likely to be that students cannot accept that "slowing down" in a given direction is equivalent to "speeding up" in the opposite direction. It must be pointed out that experience suggests that students frequently make this error when asked to draw an acceleration vs time graph of a ball projected vertically upwards, where the downward motion is considered as well. Figures 3.16 (a) and (b) give typical erroneous answers, while figures 3.17 (a) and (b) are examples of correctly drawn graphs.



(a) (b)

FIGURE 3.16



(a) (b)

FIGURE 3.17

The above errors can also be ascribed to a tendency to associate a reversal in the direction of a motion with a change in sign of the acceleration vector.

Jordaan (1992:82) quotes Pienaar *et al.* who, in a physical science textbook widely used in South Africa, suggest that pupils take g as positive for downward motion and negative for upward motion. Jordaan points out that the instrumental learning of this rule can easily cause learners to draw the (incorrect) graph of figure 3.16(a) for a stone which is projected vertically upwards.

McDermott *et al.* (1987:510) also asked students to draw a graph of the position vs time of a steel ball rolling along a track with constant velocity. The correct graph as a sharply inclined line was often not given. Instead horizontal lines or isolated points were frequently indicated as illustrated by the typical responses of figure 3.18.

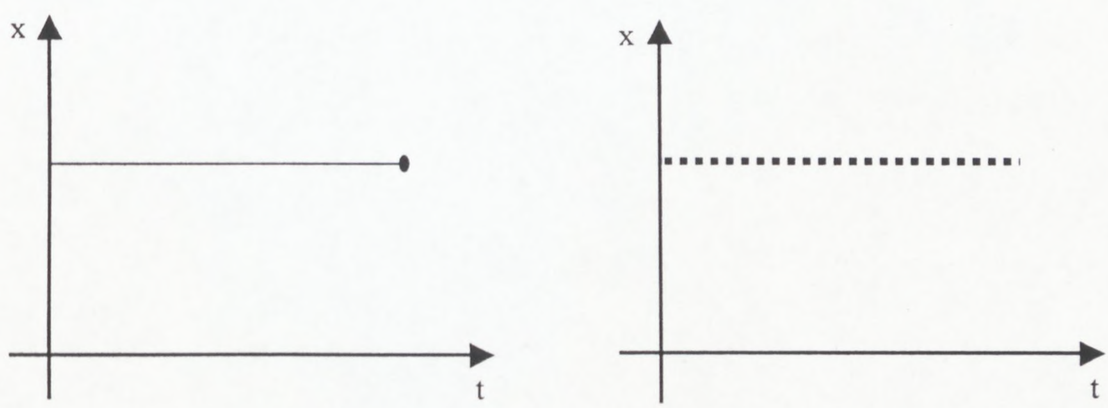


FIGURE 3.18

3.7.2.2 "Graph-as-a picture" confusion

Students frequently fail to take into account the time-rate-of-change aspect of the required kinematic quantity when drawing a graph, focusing instead on the physical orientation of the problem scene. As this representation appeals to the visual senses of the individual it is tempting to include the actual path of the motion in the graph. Brasell and Rowe (1993:65-69) found that students often select graphs that were *temptingly similar to the mental image of the event* and cite the reason for this as *finding the incorrect picture-like graph so visually compelling that they select it without really thinking about the other options.*

Clement (1989:82-83) differentiates between two manifestations of treating the graph as a literal picture of the problem situation, namely a global correspondence and a local correspondence error. In the case of a global correspondence the student makes a figurative correspondence between the shape of the graph and some visual characteristic of the problem scene. When students were asked to draw a speed vs time graph for a bicycle travelling over a hill for example, many students simply drew a picture of the hill.

> *The shape of the entire problem scene is matched to the shape of the entire graph in a global manner.* (1989:82)

An example of a local correspondence error is illustrated by the graph of figure 3.19 depicting the motion of two cars:

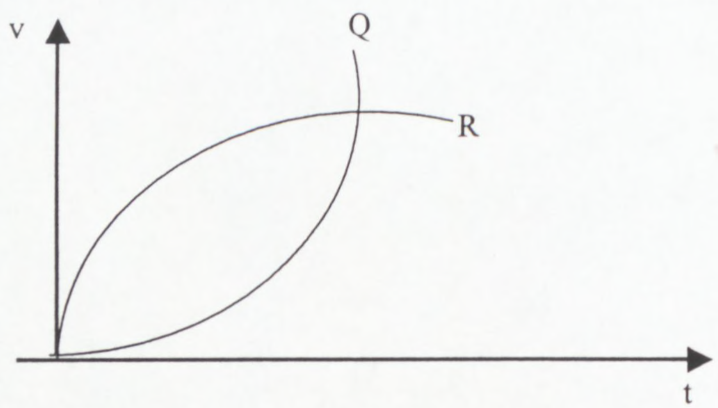


FIGURE 3.19

Students often say (incorrectly) that the two cars represented in the graph were passing each other at the point where the graphs cross. These students seemed to be associating a local feature (same location for both cars) with a similar feature of the two graphs (graphs have same location at point of intersection). In the words of Clement (1989:83):

> *A local visual feature of the problem scene (same location) is matched to a specific feature of the graph (intersection).*

It must be pointed out that another reason for this error could be a failure to distinguish between *v-t* and *x-t* graphs. On an *x vs t* graph of two cars Q and R such as shown in figure 3.20 it <u>does</u> follow that they pass each other at the point of intersection of the two graphs.

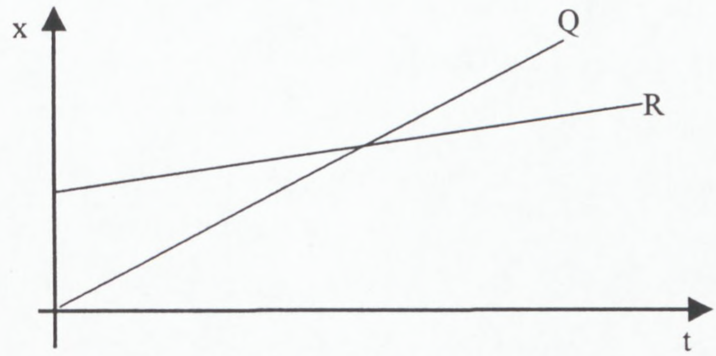


FIGURE 3.20

These students probably failed to take proper notice of the <u>labelling</u> of the axes, resulting in the drawing of inappropriate conclusions from specific features (such as intersection of graphs) of the graphical representation of a motion.

3.7.2.3 Table to graph translations

Real world physical investigations often give rise to tabular sets of data points. These data values are merely an organised way in which the numerical values of certain measured quantities (or variables) are recorded. Students of kinematics are often required to display such a record of data values graphically in order to stress specific aspects of the motion and represent general tendencies in a visual manner.

Difficulties with the graphical representation of data points is not well documented, although one study (Ponte: 1984 as reported by Clement 1989:77) found that many secondary students and even pre-service teachers have difficulty in making connections between the graphical and numerically recorded data of an observed motion.

Consider a table of data values derived from a certain experiment such as the following:

t (s)	0	1	2	3	4	5	6	7	8
v ($m.s^{-1}$)	4,0	5,1	6,0	7,2	8,1	8,0	8,2	8,0	8,1

The usual translation of this representation would be to present the data graphically. This is not difficult, as it merely requires the student to be able to choose and label a set of axes, assign a suitable scale and plot the data values. Most students would even join the points (although not necessarily with a best-fit line) as they have been trained to do so in the mathematics class. However, this does not require any *interpretational* skills, so that even when these points have been plotted, students have not yet demonstrated their understanding of the physical situation which is described by the data values.

Another possible route would be one that includes a *verbal description* of the event and can take the form:

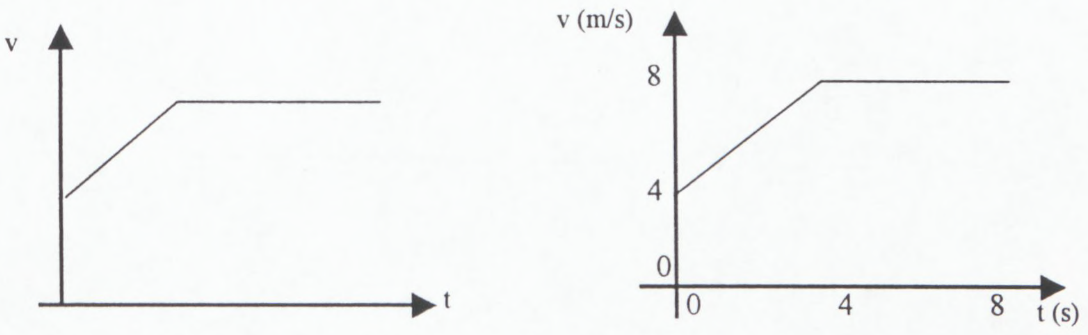
table of values → graph → verbal description

or

table of values → verbal description → sketch graph

No particular difficulty is reported for the first mentioned path, however the second case can be problematic for many students. De Jager (1990:169-170) reports considerable student difficulty in making the translation from a table of values to the verbal description of the situation. This translation involves changing patterns in number into a description of changes in key aspects of a motion and can be especially troublesome if the verbal description has to be represented graphically in the form of a sketch graph.

The table of data values given above would require a student to perform the following task: Describe the motion in terms of a body initially travelling at a velocity of 4 $m.s^{-1}$ accelerating uniformly (ignoring experimental errors) to approximately 8 $m.s^{-1}$ after 4 seconds and then maintaining this velocity for at least another 4 seconds. The graph of this verbally described motion can be a sketch graph as in figure 3.21(a) or a semi-accurate graph as in figure 3.21(b)



(a) (b)

FIGURE 3.21

3.7.3 Translation B difficulties (Graph → Physical situation)

3.7.3.1 General comments

To be able to link an appropriate verbal description of a motion to a given kinematic graph, learners have to do more than merely reproduce a previously learned algorithm or definition of a kinematic quantity. They have to translate between two kinds of representation: a *verbal* representation (usually in written form) and a more abstract *graphical* representation.

McDermott *et al.* (1987:506) analysed students' responses to multiple choice type questions where they had to match up an appropriate verbal description with a given graphical representation of a motion. They found that the lack of attention students paid to the detail of a written description meant that they experienced trouble in matching the passage to relevant features on the graph. Particular difficulty was reported in comparing two motions represented on the same graph (1987:507). Students also displayed an inability to visualise the motion depicted by graphs such as the one illustrated in figure 3.22.

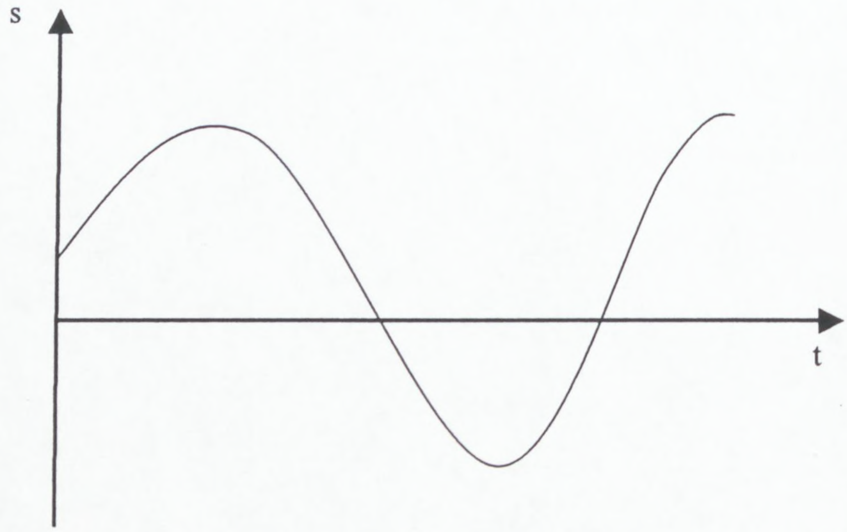


FIGURE 3.22

Problems with the real-life interpretation of the motion represented by figure 3.22 were reported to be due mainly to the following factors:

- an inability to correctly interpret the negative and positive slopes of the graph;
- an inability to describe the kinematic situation represented by the crossing of the t-axis;
- a lack of differentiation between curved and straight sections of the given graph;
- confusion between displacement, velocity and acceleration. (McDermott *et al.* (1987:506)

Schuster (1983:336) points out that it takes time and practice before students are able to "read" graphs fluently and quickly, while Brasell and Rowe (1993:69) stress that the level of scientific sophistication of the language plays an important role in determining the success that students have in translating from a graphical representation to a verbal description of a motion. They discovered that students made errors associated with linking the graph to the verbal description of a given event as well as linking the colloquial description of an event to a scientific description of the relevant variable. (This was also referred to in section 3.5.3 and is a problem which applies to all graphs in science.)

Brasell and Rowe (1993) as well as Poduska and Phillips (1986) found that students had more difficulty in comprehending graphs of derived variables such as velocity and acceleration than with graphs of fundamentally directly measurable variables such as length. A typical misconception with *a vs t* graphs is reported by McDermott *et al.* (1987:510). They found that students would not accept that it is impossible to tell whether an object with a negative acceleration is speeding up or slowing down. Students did not believe that...

> *one cannot tell from an* a vs t *graph whether an object is speeding up or slowing down or in what direction it is travelling.*

Another study (Brasell 1987:394-5) revealed that students were inclined to select a horizontal distance-graph or a sloping velocity graph for a constant velocity event. The same study found evidence of the fact that they also did not know how to present the *direction* of movement of a verbally described motion. Brasell cites a lack of basic graphing skills as the underlying cause of these deficiencies.

> *Improvement in the understanding of the conventions of graphing will cause a gain in the ability to translate between the verbal and graphic description of a motion.* (Brasell 1987:394)

It appears somewhat harsh to suggest that a lack of graphing skills is the only cause of translation difficulties. Even students who are totally conversant with the conventions of graphing often appear to have difficulty in giving an accurate verbal description of a graphically represented motion.

3.7.3.2 Identifying appropriate features of a graph

Students often have difficulty in deciding which feature of a graph should be associated with a required kinematic quantity. Often they do not know whether to extract the required information from the height, change in height, slope, change in slope or area under a graph (see § 2.2.2). This problem is highlighted by Schuster 1983, McDermott *et al.* 1983, 1987 and Orton 1984.

McDermott *et al.* (1983:9)) make particular mention of the fact that students find curved graphs very difficult to interpret when information has to be extracted from changes in height or changes in slope. Students were given an *x vs t* graph such as the one depicted in figure 3.23 and asked questions such as:

i) *Where is the motion slowest?*

ii) *Where is the object speeding up?*

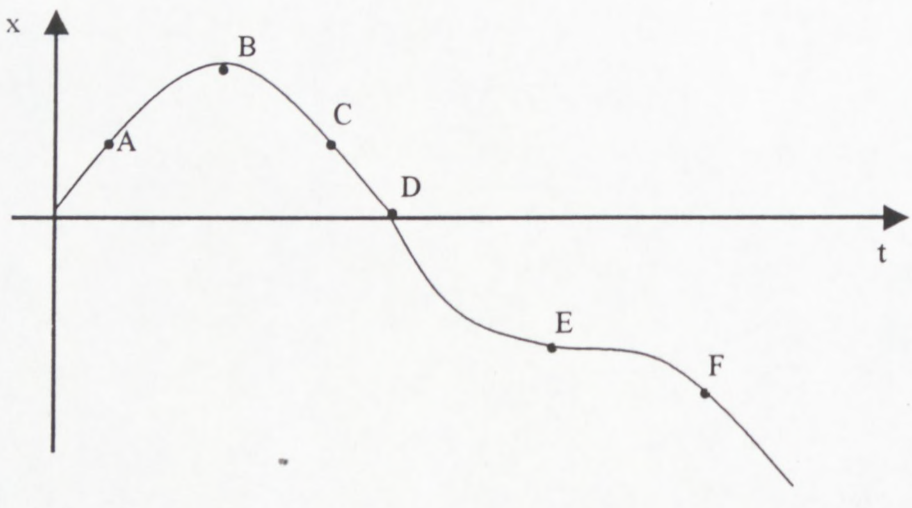


FIGURE 3.23

Surprisingly few students realised that the graphs had to be examined for changes in slope (and not changes in height). The question of slope-height confusion will be examined in more detail in the next section.

Schuster (1983:331) suggests that the reason for these shortcomings is that students do not receive sufficient instruction about aspects of a position-time graph such as finding points on the graph where the object:

- is momentarily at rest,
- moving fastest,
- furthest from its starting point,
- moving backwards.

Orton (1984:25) emphasises that the problem that students have with relating graphs to kinematic concepts can often be ascribed to a low level of understanding of rate of change in a graphical context (see p. 194). Students struggle particularly with the idea that it is possible, in the case of curves, that for every point on the curve the rate of change will have a different value. The fact that students often cannot obtain the instantaneous velocity at a certain time from a curved displacement-time graph can be mainly attributed to this problem.

The association of the area under a velocity-time graph with displacement is problematic, not only for students but even some instructors! (Arons 1983, McDermott *et al.* 1987, and Jones 1992). Jones (1992:14) points out that textbooks often do not make it clear that it is the <u>change</u> in displacement which is represented by the area under the *v vs t* graph. The (mistaken) definition of this area as the actual displacement of the body can be related to the mistaken belief that at time $t = 0$ the displacement of a body must be zero. The reference level for any rectilinear motion is chosen arbitrarily, a fact that has to be stressed by instructors. A similar argument for the area under an *a vs t* graph being equal to the *change* in velocity is put forward by Jones (1992:14-15). (See also pp 77, 130 and 142).

McDermott *et al.* (1987:506) as well as Arons (1983:580) report that students find it difficult to associate a displacement (unit: m) with an area (unit: m^2):

They come to associate numerical values of area as the output of formulas of unknown origin and do not visualise the meaning of the numbers in terms of squares or cubes. (Arons 1983:580)

3.7.3.3 Slope-height confusion

Peters 1982, McDermott *et al.* 1983, Rosenquist and McDermott 1987, Clement 1989 as well as Brasell and Rowe 1993 all report serious student confusion between the slope and the height of kinematic as well as other scientific graphs. The most commonly occurring problem is that students tend to use the height or ordinate values of a graph when they should be using the slope.

Clement (1989:81) quotes Janvier (1978) who reported serious height-for-slope errors made by students in a variety of practical, everyday examples. In one of the problems (which was briefly referred to on p. 194) Janvier asked students to draw graphs of height vs time for the water level in different jars being filled with water. He showed them graph A for a wide jar and asked them to draw the graph for a narrower jar being filled from the same source (see figure 3.24). The idea was that students had to focus on the *slope* of graph A which gives an indication of the *rate* at which the water levels in the jar was increasing.

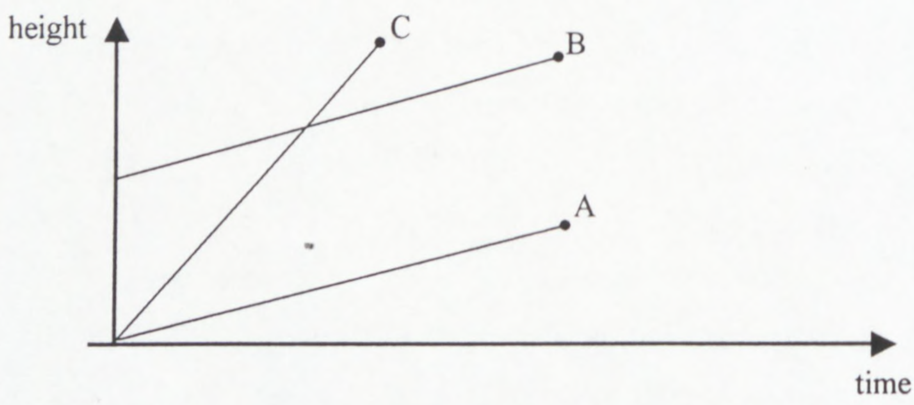


FIGURE 3.24

Many students drew graph B instead of graph C with increased slope. This confusion is also reported by among others Orton (1984) and De Jager (1990:171) who both confirmed that students often do not associate a larger gradient with a higher rate of change. One suspects that an added problem is that the concept of "rate-of-change" is also not fully understood in the context of the specific physical situation which is being investigated.

The confusion between the position and velocity of a graphically represented motion seems to be a direct result of this slope-height confusion. Rosenquist and McDermott (1987:409) report on serious errors committed by students who were given a written task administered as part of a study on graphing skills. They found that the tendency to use position criteria to make judgements about relative velocity was more prevalent among those who had no formal instruction in physics than among those who had. This could account for the fact that when comparing two motions plotted on the same set of axes, many students incorrectly identified the point of intersection of the position vs time graphs with the time when the speeds were the same (see p 230).

Trowbridge and McDermott (1980:1025) investigated the nature of speed-position confusion among students by asking them whether two objects that reached the same position at the same clock reading would necessarily have the same speed at that instant. Many answered in the affirmative, stating that a graph such as the one drawn in figure 3.25 clearly shows when the two speeds are equal.

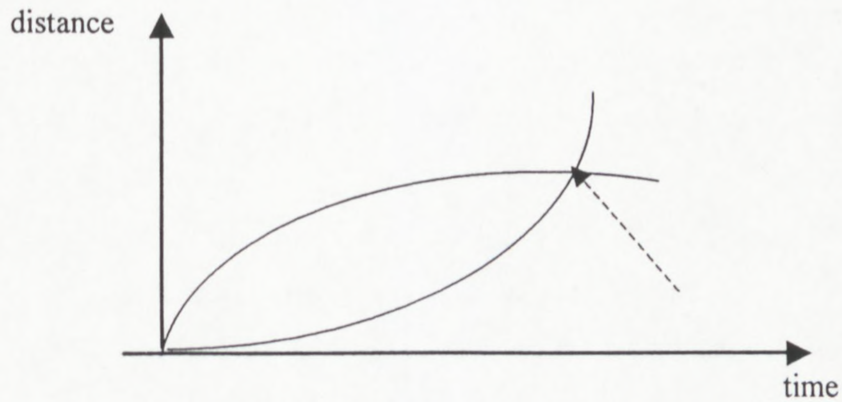


FIGURE 3.25

A typical response was: *True, because the instantaneous velocity at that instant is* $\lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$ *and the Δd value is the same*. These students display a serious lack of understanding of the rate-of-change concept as well as the concept of a limit. It could also be that they confuse "d" and "Δd", because although the value of "d" is the same at the point indicated by their arrow, the change in d over a specific time period is very different for the two graphs.

McDermott *et al.* (1987:504) report on a study of science-oriented college students who were asked the following question referring to the graph in figure 3.26.

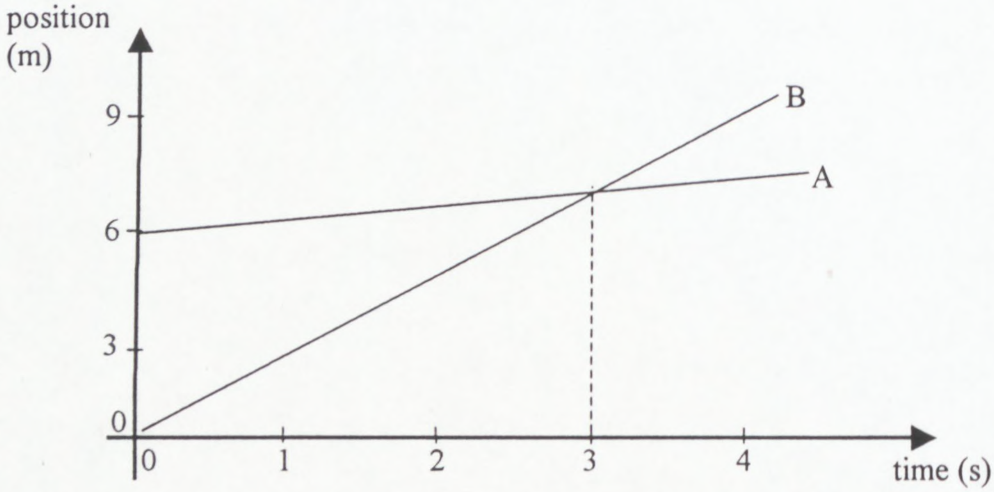


FIGURE 3.26

"At the instant t = 2 s, is the speed of object A greater than, less than or equal to the speed of object B?" As many as half the students incorrectly answered "greater than". When asked whether they will ever have the same speed, many said that A and B will have the same speed at t = 3 s. Clement (1989:80) generalises this mistaken use of the graphical feature of height instead of slope to represent speed as *a misplaced link between a successfully isolated variable and an incorrect feature of a graph.* He attributes it to the fact that students have difficulty with the idea of covariation. They have to associate *speed* with *change in position of the object over time* and decide that, compared to A, there is a greater change in position per time unit in graph B.

McDermott *et al.* (1983), however, point out that it is difficult to assign a single cause to this error since students have been observed confusing the physical concepts of speed and relative position in other tasks which do not involve graphs.

Slope-height confusion is also responsible for the fact that students often do not know how to determine the acceleration of a body from its *v vs t* graph. Rosenquist and McDermott (1987:411) claim that the reason is that many students cannot make the distinction between change of velocity and its rate of change - i.e. acceleration. Students do not understand the significance of the corresponding time interval which is associated with the change in velocity.

There is little doubt that a significant amount of direct instruction is necessary to overcome slope-height confusion, as it is a misconception which is often very well-established.

> *The belief that a position criterion may be used to compare relative velocities seemed to remain intact in some students even after several weeks of instruction.* (Trowbridge and McDermott 1980:1027)

3.7.4 Translation C and D difficulties (Formula ↔ graph)

The nature and extent of student difficulties with respect to formula-graph translations has not been fully researched. Clement (1987:77) contends that a minimal interpretation of the relationship between an algebraic equation and its graph is quite simple; the graph displays the co-ordinates corresponding to the pairs of numbers belonging to the solution set of the equation. However, when it comes to the graphical interpretation of a formula in kinematics, such as $s = ut + \frac{1}{2}at^2$, McDermott *et al.* (1987:513) found that the interpretation of the relation between such an algebraic relation and its graph is difficult for most students.

There is a strong association between graphs such as parabolas and straight lines in the mathematics syllabus and graphs of kinematic quantities. This correspondence is not always recognised by students, because subject matter in different subjects is usually viewed in isolation. (see McDermott *et al.*, 1983:3, Leinhardt *et al.*, 1990:3 and Nickerson, 1985:229). Nickerson states for example that a test for real understanding is whether a student:

> *can apply principles from one knowledge domain to another and recognise similar relationships in different contexts.* (1985:229)

A student who has no difficulty with the drawing of the graph of $y = ax^2$ in the mathematics class, might experience considerable difficulty with the graphical representation of $s = \frac{1}{2}gt^2$ in the science class. The role that "mathematics graphs" play in facilitating the understanding of kinematic graphs will be investigated and reported in Chapter 5.

3.7.5 Transformation E difficulties (Graph → Graph)

Kinematics offers a unique opportunity for students to demonstrate their ability to transform a particular graph into another related one. In order to do this students must be able to focus on appropriate aspects, such as change in height, change in slope, or whether the given graph is above or below the time axis.

Students often find it hard to ignore the shape of the original graph because they find it difficult to accept that the same motion can be represented by graphs of widely different shape. This fact has been confirmed by researchers such as Peters (1982) and McDermott *et al.*(1983, 1987).

Peters (1982:502) in his research on post-graduate students' understanding of kinematics found that these senior students drew velocity vs time graphs that were very similar in shape to the position vs time graph of a specific demonstrated motion. The reason for this seems to be that they did not interpret the *v vs t* graph as a plot of the slope of the *x vs t* graph. McDermott *et al.* (1983, 1987) came to the same conclusion. They report that although students experienced some difficulties with the translation of the slope of an *x vs t* graph into the height of a *v vs t* graph, their real difficulty lay with the translation of an increasing slope into an increasing height. Their findings are illustrated in figure 3.27.

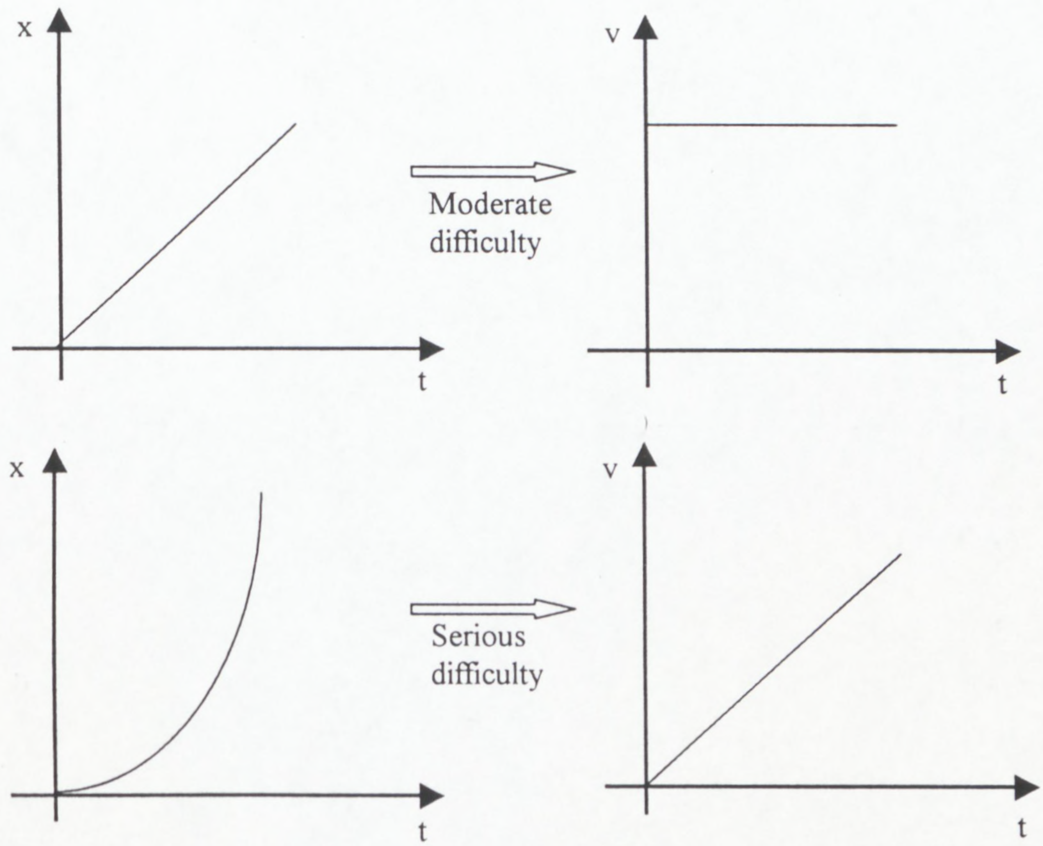


FIGURE 3.27

McDermott *et al.* (1987:510) observed also that most students cannot distinguish between different motion graphs and found that a large proportion of students tended to draw acceleration vs time graphs that contained inclined lines to match a given velocity vs time graph. Instead of using horizontal lines to represent a constant acceleration on a *a vs t* graph, these students drew straight inclined lines running roughly parallel to the accelerated portion of the corresponding *v vs t* graph.

Similar mistakes were made when *v vs t* graphs were drawn for the motion demonstrated by the steel ball rolling on the aluminium track in figure 3.11. Data points were plotted properly, but another point at v = 0 was added. The points were then connected by a curved line so that the resulting *v vs t* graph matched the correct *x vs t* graph as shown in figure 3.28.

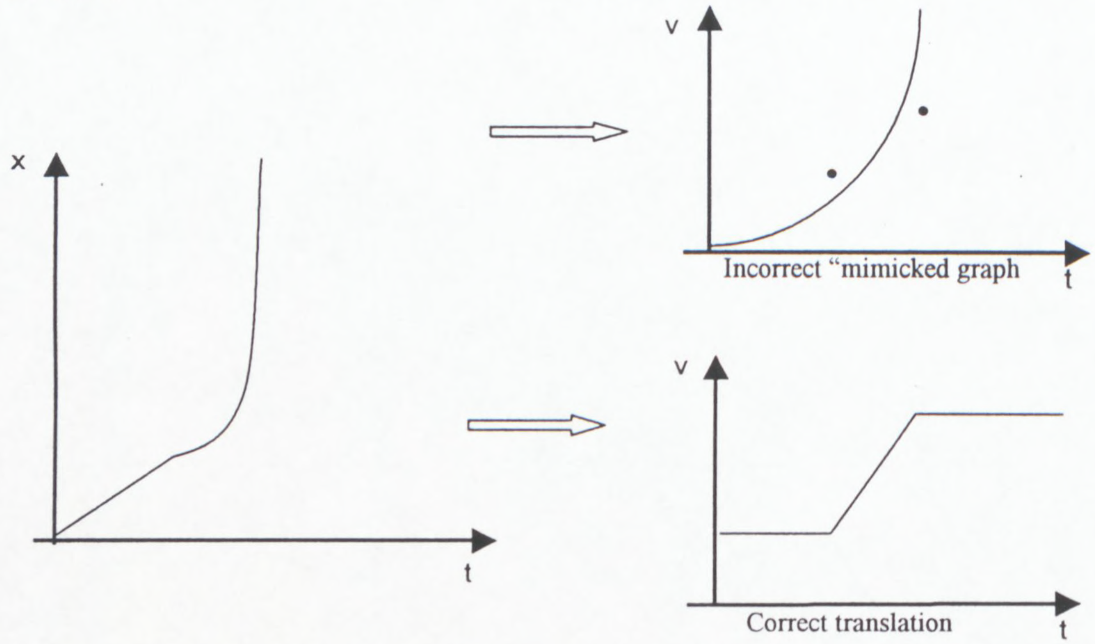


FIGURE 3.28

3.8 CONCLUSION

This chapter described some of the more commonly occurring misconceptions that have been identified by various researchers with regard to graphing in general and kinematics and kinematic graphs in particular. Some of the underlying **reasons** for these misconceptions were described in terms of certain cognitive structures which give rise to identifiable alternative frameworks which students use to make sense of the world around them.

Kinematic graphs are often a very effective way of illustrating the behaviour of certain kinematic quantities as well as the relationship between these quantities. The graphs are unique in that they offer an opportunity for transforming a given graph into that of a related kinematic quantity, often by the exclusive use of height, gradient or area considerations. The nature of student difficulties with these graph transformations has not yet been thoroughly investigated.

This study will next report on the results of an own empirical investigation into the nature and extent of students' misconceptions with respect to motion graphs and, more particularly, graph-graph transformations. Students understanding of specific aspects of kinematics will first be evaluated in order to gain some idea of whether the errors that occurs during graphing is due to a misplaced understanding of certain kinematic quantities and the relationship between them or whether it could be ascribed to other factors.

Finally, certain recommendations will be made which hopefully would lead to the elimination or (at least) the reduction of identified conceptual and relational errors made by high school and early tertiary students when dealing with kinematic graphs.

CHAPTER 4

AN EMPIRICAL INVESTIGATION INTO STUDENTS' DIFFICULTIES WITH KINEMATIC GRAPHS

4.1 MOTIVATION FOR THE INVESTIGATION

The literature survey has shown that students experience great difficulty with the translation between real observations and representations thereof (see, for example, Schuster 1983; Halloun & Hestenes 1985; Nickerson 1985; McDermott *et al.* 1987; Goldberg & Anderson 1989 and Brasell & Rowe 1993). The present investigation attempts to reveal further information about the difficulties experienced by students when attempting to represent unidirectional motion graphically.

The particular problems associated with the transformation of a specific *graphical* representation into another *related* one have not received much attention. This study, by focusing on such graph to graph transformations, will attempt to show how an analysis of student misconceptions can assist educators to structure the syllabus and organise the teaching such that improved understanding of not just motion graphs, but also kinematics and graphs in general, may result.

4.2 THE NATURE OF THE INVESTIGATION

In order to report on students' views of certain aspects of kinematics and its graphical representation, a variety of testing techniques were used. From an analysis of the responses a fairly detailed picture of students' conceptions of those items that were tested was formed. The investigation consisted of four phases, namely an initial series of clinical interviews, followed by a pilot study which lead to a final questionnaire. A final series of personal interviews was also conducted.

4.2.1 Phase 1 : Clinical interviews

In order to gain a first impression of the state of students' understanding of certain aspects of kinematics and kinematic graphs, a series of audio-taped individual interviews were conducted with ten first year technikon students. These students were selected so as to represent a cross-section of varying academic ability and sociological backgrounds.

The students were firstly asked to respond (mostly verbally, but in some cases with pen and paper) to a number of questions on kinematics. The questions focused primarily on a number of key aspects of kinematics such as:

- the nature of and need for *negative* kinematic quantities;
- the meaning of the *gradient* of a graphically represented relationship;
- the role of *reference point* in the case of displacement;
- the importance of a sign convention when dealing with numerical values of displacement, velocity and acceleration;
- the fact that not all graphs of kinematic relationships represent real motions.

The second part of the interview consisted of a paper and pencil activity where the students were asked to perform a number of graph → graph transformations (translation E, as defined in Chapter 2). They were required to perform seven transformations involving the drawing of the appropriate *s-t* graph from a given *v-t* graph. The level of difficulty ranged from very elementary to a more complex semi-accurate transformation and the subjects were asked to "think out loud" while they attempted to draw the graphs. Three examples of a *s-t* $\Rightarrow$ *v-t* transformation were also included.

4.2.2 Phase 2 : The Pilot Study

The responses to the clinical interviews were analysed and categorised and the resulting framework of student misconceptions, together with the information gained from the study of the available literature, formed the basis of a questionnaire which served as a pilot study. The main purpose of the pilot study was to validate and evaluate a proposed final questionnaire for shortcomings, omissions and vagueness in order to use it as the main measuring instrument in this study.

Certain questions were added to those used in the clinical interviews, while others were adapted to make them more suitable for a written response, so as to obtain greater clarity on the state of students' thinking with regard to specific issues.

In general terms, however, the questionnaire investigated the same aspects of kinematics as the clinical interviews, namely the level of comprehension of certain key aspects in kinematics as listed on p. 247. Students were also requested to provide *reasons* for all responses in order to gain a more complete understanding of their comprehension of specific concepts, definitions and representations, as well as their ability to translate between representations. An important addition was a request to *verbally* describe a number of *graphically* represented motions.

4.2.3 Phase 3 : The final questionnaire

The results of the pilot study were analysed so that this questionnaire could be adapted to prepare a final questionnaire which was used to gain a deeper insight into the way students think about motion and its graphical representation. The final questionnaire (which was drawn up in English and Afrikaans) was distributed to 278 students ranging from Grade 11 through to second year technikon students and targeted different cultural and academic-ability groups. The questionnaire was also given to two subject experts for scrutiny to check for accuracy and clarity of questioning. A copy of the final questionnaire has been included as Annexure A.

4.2.3.1 Nature of questions

The original questionnaire used in the pilot study yielded some very promising results, so that it was decided to make only a few minor changes before it was distributed for answering by the larger sample of students. An important question which was included was one which asked the students to explain the basic kinematic vector quantities in the way that they would explain them to a friend with little knowledge of science. This served to test for their conceptual understanding of *displacement, velocity* and *acceleration* and the relationship between these vector quantities. Students could answer the questions in English or Afrikaans.

4.2.3.2 Honest participation

In order to obtain the serious co-operation of the respondents, they were requested to include their names as well as the names of their schools. It was emphasised that their answers would be treated as confidential and that their responses would make a significant contribution to an important research project. The time constraint was played down in order for as many students as possible to complete the questionnaire, thus encouraging them to attempt an answer to *all* questions.

4.2.3.3 Profile of the test group

The questionnaire was administered to a heterogeneous group of students of varying intellectual ability (as measured by academic results), cultural background and years of exposure to physics. Only students who offer or have offered physical science on the higher grade were involved in the investigation, as kinematic graphs are not examined in the standard grade syllabus in South African schools. Table 4.1 summarises the personal information of the test group. (Respondents who have not indicated their status in a particular category were not included in a distribution.)

TOTAL	Grade 11	Grade 12	TECH 1	TECH 2
276	145	66	33	32
100 %	53 %	24 %	12 %	11 %

(a) distribution by academic level

TOTAL	MALE	FEMALE
273	179	93
100 %	66 %	34 %

(b) distribution by sex

TOTAL	ENG	AFR	ENG/AFR	BLACK	OTHER
276	170	26	8	61	11
100 %	62 %	9 %	3 %	22 %	4 %

(c) distribution by language group

TOTAL	< 40 %	40 - 59 %	60 - 79 %	> 80 %
227	8	140	70	9
100 %	3 %	62 %	31 %	4 %

(d) distribution by most recent physical science HG result

TOTAL	< 40 %		40 - 59 %		60 - 79 %		> 80 %	
	HG	SG	HG	SG	HG	SG	HG	SG
219	14	4	73	18	64	24	15	8
100 %	6 %	2 %	33 %	8 %	30 %	11 %	5 %	5 %

(e) distribution by most recent maths result

TABLE 4.1
DISTRIBUTION OF PERSONAL INFORMATION OF TEST GROUP

4.2.3.4 Analysis of data

The individual responses to the questionnaire were marked according to a scheme of codes which were fed into the SASPAC-programme at the University of Stellenbosch in order to analyse them statistically. A number of cross-tabulations were also made in order to:

(a) obtain an idea of the difference in understanding between the various groups, and

(b) draw comparisons between responses to different questions testing the same or similar principles.

An analysis of the computer printout revealed that certain questions were left unanswered by a relatively large percentage of the respondents. The reason for this could be attributed to any of the following:

- an eagerness to complete the questionnaire within the allocated time;
- spending too much time on certain questions;
- a lack of motivation to answer all questions;
- a supervisor who underplayed the importance of the questionnaire.

For the purposes of the statistical analysis of the questionnaire it was thus assumed that an unanswered test item did not necessarily indicate a lack of understanding of that question. Unanswered questions were consequently *not* included in the calculation of percentages of correct and incorrect responses.

4.2.4 Phase 4 : Final personal interviews

18 additional clinical interviews were conducted with students from the sample group as well as with students who did not participate in any of the previous phases. The responses of the first group assisted in classifying certain responses under the appropriate answer codes, in cases where this was not initially clear from a particular written response. The second group of students was interviewed in order to

- confirm the presence of certain generally held misconceptions;

- probe for the origins of these misconceptions;

- gain additional information about students' views of certain test items.

4.3 METHODOLOGY OF REPORT-BACK ON EMPIRICAL INVESTIGATION

The results of the empirical investigation is reported in the form of a question-by-question analysis of the various responses to the main questionnaire.

The question-wise discussion commences with a table of the more commonly occurring responses, each of which has been allocated a numerical code and includes the popularity of a particular response, expressed as a percentage of all respondents who attempted to answer the question. Codes which correspond to responses which can be classified as correct have been marked with an asterisk (*). Correct responses are separated from incorrect ones with a horizontal line.

4.4 QUESTION-BY-QUESTION ANALYSIS OF THE MAIN QUESTIONNAIRE

QUESTION 1

Explain briefly how you would explain the concepts of displacement, velocity and acceleration to a friend who has very little knowledge of science

In this question the respondents had to illustrate how they would explain the basic kinematic vector quantities in ordinary, uncluttered language and so demonstrate their real understanding of these quantities. Not all students managed to stay clear of the traditional textbook definition of *rate of change of displacement* and *rate of change of velocity* and it was decided to accept these explanations as “correct”.

RESULTS

Displacement (success rate 73 %)

RESPONSE CATEGORY	ANSWER CODE	%
Straight line distance from start to end	1*	39
Total distance from start to finish	2*	14
Straight line distance from start to end with direction	3*	10
Distance covered in a certain direction	4*	7
Uses a sketch - correctly	5*	3
How far it goes / how much it moves	6	8
Change from a point of rest	7	4
Other incoherent replies	8	15

TABLE 4.2
RESPONSES TO QUESTION 1 (DISPLACEMENT)
(expressed as a percentage of the number of students who attempted the question; n = 273)

An encouraging 73 % of the students managed to give an acceptable explanation of displacement. The directional aspect was, however, often left out (see codes 1 & 2), or when this was included the idea of a straight line was not mentioned (code 4). Code 3 is totally correct, but as the question required students to state the explanation in their own words, it can be accepted that codes 1, 2, 3, 4 & 5 all indicate a sufficient understanding of displacement in order to differentiate it from the related quantities of distance, velocity and acceleration.

Confusion between displacement and distance probably accounts for the fact that as many as 14 % of the respondents simply thought of displacement as the total distance from start to finish. After interviewing a view students who gave this response (see code 2), it transpired that they purposely avoided the term *straight line* and *direction* because of the requirement to state the concept of displacement in their own words.

Velocity (success rate 30 %)

RESPONSE CATEGORY	ANSWER CODE	%
Rate of change of displacement	1*	17
Displacement per unit time (detailed explanation)	2*	13
Speed with direction	3	16
Similar to speed	4	11
Distance per unit time	5	11
How fast something is moving in a direction	6	5
Increase in speed in direction / acceleration	7	3
Rate of movement	8	3
Rate of displacement	9	2
Incoherent - other	10	19

TABLE 4.3
RESPONSES TO QUESTION 1 (VELOCITY)
(expressed as a percentage of the number of students who attempted the question; n = 265)

The concept of velocity can only be explained adequately in terms of displacement - a fact which was recognised by only 30 % of the respondents with many simply giving the "official" textbook definition of rate of change of displacement (code 1).

A further 30 % (codes 3, 4 & 7) attempted to explain velocity in terms of speed, often believing (incorrectly) that if a direction is added to speed, velocity is always obtained. These students did not realise that *speed* is related to *distance* and *velocity* is related to *displacement* - a mistake also made by an additional 11 % of the respondents who stated, according to code 5, that velocity is *distance per unit time*.

Acceleration (success rate 41 %)

RESPONSE CATEGORY	ANSWER CODE	%
Rate of change of velocity	1*	29
Change in velocity over time	2*	12
Change / increase of speed per second	3	10
Increase in speed	4	9
Increase of velocity	5	5
Change in velocity	6	4
Change in speed	7	3
Increase or decrease of velocity	8	3
Increase in velocity over time	9	3
Going faster & faster	10	3
Other incoherent	11	19

TABLE 4.4
RESPONSES TO QUESTION 1 (ACCELERATION)
(expressed as a percentage of the number of students who attempted the question n; = 261)

Once again a significant proportion (29 %) of the respondents simply gave the "official" textbook definition, while only 12 % (code 2) were able to explain the meaning of acceleration in terms of a change in velocity divided by the time for which the change occurs.

Most of the other students (25 % of all the respondents) attempted an explanation which associated the word "acceleration" with an *increase* in either the velocity or the speed - usually negating the role of time. To these students acceleration is not something one would associate with an object which is moving with *decreasing* velocity. It is also evident that the terms *speed* and *velocity* are loosely used when trying to explain the meaning of acceleration, without considering the definitions of these two quantities.

The *rate of change* concept was poorly understood by many respondents. In trying to give their own versions of the meaning of acceleration as the time-rate of change of velocity, altogether 27% of students neglected to refer to time (see codes 4, 5, 6, 7, 8 & 10). When responses *did* include (sometimes by implication) a reference to time, they were often confusing or even totally incoherent. Examples of the latter include:

> *acceleration is the rate of change of velocity divided by the rate of change of time* or *acceleration is the rate at which speed is measured.*

A cause for concern is the fact that approximately 19 % of the respondents had no idea whatsoever about the meaning of acceleration as illustrated by responses such as: *it is the time for a body to move from point A to point B.*

The fact that a much larger proportion of the respondents merely gave the textbook definition for *acceleration* than was the case for *velocity*, might also be attributed to a poorer understanding of the concept of acceleration. Students could have opted for the learned phrase "rate of change of velocity" because they do not understand the real meaning of acceleration. This hypothesis will be investigated further during the follow-up clinical interviews.

QUESTION 2

A pupil makes the following statement: ***A car <u>can</u> travel at a velocity of –20 m.s^{-1}.***

Cross the appropriate block to indicate your view on this statement.

The purpose of this question was to find out whether students accepted that real objects can travel with negative velocities and what physical meaning they attached to the negative sign.

RESPONSE CATEGORY	ANSWER CODE	%
Agree	1*	65
Disagree	2	29
Unsure	3	6

TABLE 4.5
VIEW ON NEGATIVE VELOCITIES
(expressed as a percentage of the number of students who attempted the question; n = 272)

Reason:

RESPONSE CATEGORY	ANSWER CODE	%
Moving backwards / reversing	1*	26
Moving in opposite direction to the positive	2*	23
Travelling in a negative direction	3*	5
Speed cannot be negative	4	7
Velocity cannot be negative	5	6
It is decelerating / slowing down	6	4
Incoherent	7	10

TABLE 4.6
REASONS FOR NEGATIVE VELOCITIES
(expressed as a percentage of the number of students who attempted the question; n = 240)

It is encouraging to note that 65 % of the respondents are comfortable with the fact that objects *can* travel at negative velocities, and that 54 % of all the respondents understand the negative sign to indicate the *directional* aspects of the motion of the car (codes 1, 2 and 5). It has to be pointed out that response code 1 (dealing with reverse motion) does not necessarily have to be associated with a *negative* velocity. A car moving backwards can have a *positive* velocity if the backwards direction is taken as positive. Students usually seem to assume, however, that a "forward" moving car travels in a *positive* direction.

A number of students recalled the fact that *v-t* graphs often intersect the time axis. When interviewed, it transpired that they were fully aware that this is associated with a change in direction and hence these responses were included under answer code 2. The following written response serves as an illustration:

If drawn on a velocity vs time graph, the velocity can be represented by a negative number to show a change in direction in which the person/object is moving.

Some students, whilst acknowledging that a velocity can have a negative *direction*, did not accept that a velocity can be given a negative *value*. This viewpoint was expressed in the clinical interviews and pilot study as well. A typical response was: *A car can travel with a velocity in a negative direction, but not with a negative velocity.*

Others believed that a negative velocity had to be associated with a *decrease* in velocity, resulting in written responses such as: *the car is decelerating; the object is slowing down* and even a combination of directional change and decreasing velocity as in: *negative velocity occurs like in slowing down in the opposite direction.*

The association : positive value → increasing kinematic quantity and
: negative value → decreasing kinematic quantity

which seems to be responsible for this misconception, occurs frequently in the case of graphical transformations and will be discussed in greater detail in Chapter 5

The following interesting comments regarding negative velocities were also received:

- *even when a car is reversing its velocity is still positive*
  (negating the possibility of a negative velocity).

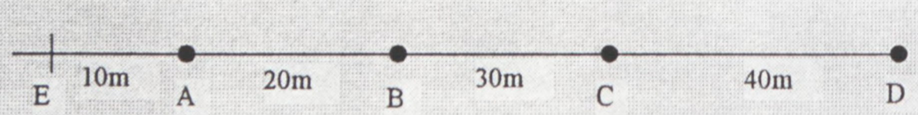
- *there is no -20* $m.s^{-1}$ *on the speedometer of a car*
  (confusing speed and velocity).

- *velocity is distance over time and distance and time are both positive - therefore velocity is positive*
  (using an incorrect definition of velocity).

- *a car can travel with a speed of -20 m/s, but velocity of -20 m/s + direction*
  (does not understand the directional significance of the negative sign).

QUESTION 3

Consider the following figure representing the positions of four cars A, B, C and D.



A student states that the displacement of car D is 100 m.

Comment on this statement.

This question examines the student's ability to recognise the *directional* aspect, as well as the importance of a *reference point*, when dealing with a *displacement*.

RESULTS:

RESPONSE CATEGORY	ANSWER CODE	%
False, no reference point was given	1*	20
True, if started from point E	2*	15
False, value of displacement depends on reference point <u>and</u> no direction was stated	3*	3
Statement is correct	4	7
Displacement should be 90 m	5	7
False - *distances* all add up to 100 m	6	7
Correct - all lie in straight line	7	5
False - the distance is 100 m	8	5
Displacement should be 50 m (increases by 10 m each time)	9	5
False - no displacement, cars are not moving	10	4
False, no direction was given	11	4
Incoherent	12	18

TABLE 4.7
COMMENT ON NATURE OF DISPLACEMENT
(expressed as a percentage of the number of students who attempted the question; n = 253)

Very few (3 %) of the respondents realised that for a displacement to have a valid meaning, the *direction* as well as the *reference point* have to be considered. The real purpose of the question was to test for the importance of reference level or reference point, which means that the first three response categories can be considered to constitute a valid response. This means that only 38 % of the student stated explicitly or by implication that a displacement is only meaningful if it is given relative to a specific point or line.

An interesting observation is the fact that there appears to be a significant level of confusion between the terms *displacement* and *distance*. Responses no. 6 and 8 both include a reference to *distance* in order to make the given statement true. These students (totalling 12 % of the sample) seem to believe that cars travel "distance" (where starting points are irrelevant) but cannot be "displaced".

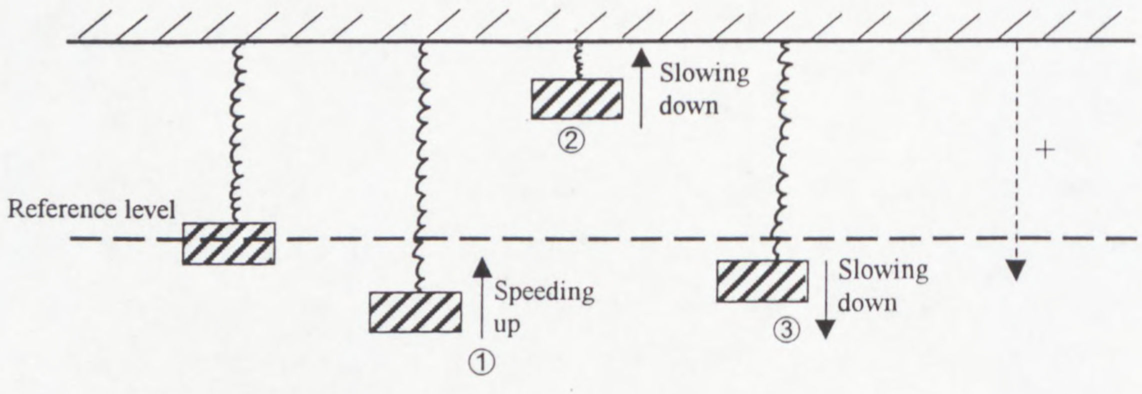
Another group believed that, unless the nature of the actual motion of an object is clearly described, there can be no displacement (see code 10). For these students the displacement of car D is zero because this car has not *moved* from a certain position to its present position as indicated on the given sketch. A number of these students, who were adamant that no displacement can occur without an observed motion, explained that this, to them, was far more significant than the absence of a reference level. These responses were therefore all grouped together under code 10.

Some students thought that the condition of linearity was sufficient to define a displacement (code 7), while others assumed that the displacement of car D *had* to be given relative to car A prompting them to arrive at an answer of 90 m (the sum of 20, 30 and 40 - see code 5).

A disturbing 18 % of all students made comments that can best be described as irrelevant or idiosyncratic, indicating a total lack of understanding of the question. Response code 9 is a typical example of such a response where students simply observed the given arithmetical number sequence 10; 20; 30; 40 and concluded that the required value has to be 50 m to fit in with the sequence. Another student, misreading the question, stated that the statement is true *because the velocity increases as you move away from E*.

QUESTION 4

The sketch shows a wooden block suspended from the ceiling by means of a coiled spring. It is stretched from its position of rest to position (1) after which it is released, moving upwards through position (2) and eventually downwards through position (3).



Taking down as a positive direction and noting the reference-level as indicated, fill in the signs (i.e. + or -) of the displacement, velocity and acceleration at positions (1), (2) and (3) in the following table:

	1	2	3
displacement			
velocity			
acceleration			

The purpose of this question was to establish whether students could identify positive and negative values of the main kinematic quantities in the context of a verbally described motion illustrated by a diagram, in a case where the reference level and sign convention are explicitly described and indicated on a sketch.

RESULTS

	Position 1		Position 2		Position 3		Average
	Correct sign	%	Correct sign	%	Correct sign	%	%
displacement	+	75	-	65	+	81	74
velocity	-	27	-	67	+	57	50
acceleration	-	29	+	32	-	58	40

TABLE 4.8
SIGNS OF KINEMATIC QUANTITIES
(expressed as a percentage of the number of students who obtained the indicated correct respose (n = 261).

It is evident from the results that the level of the respondents' understanding of sign convention shows a noticeable decrease from displacement to velocity to acceleration.

The average success rate for allocating the appropriate sign for the *displacement* in the three indicated positions was 74 % indicating that students do take note of a reference level and sign convention when considering displacement *when these are explicitly stated in the question.*

In the case of velocity, a large majority (73 %) of the students believed that the velocity in position no. 1 had to be positive, in spite of the fact that the motion occurred in a direction *opposite* to that chosen as positive. This could be attributed to the fact that the block was speeding up in this position, resulting in the association:

increase in speed → positive velocity.

Further confirmation of this type of thinking is provided by the fact that a relatively large proportion of the respondents (67 %) correctly said that the velocity in position number 2 is *negative*. Instead of noting the negative direction of the motion, these students could have made the association

decrease in speed → negative velocity.

Another possible reason for the poor response to the sign of the velocity in position 1, is that some students might have allocated the same sign as for the displacement in that position. These students could have argued that since velocity is related to displacement and displacement was (correctly) identified as positive, it means that velocity is also positive. A similar (erroneous) argument might have been responsible for the *correct* reply to the sign of the velocity in position no. 2.

The way students think about negative and positive velocities can thus be influenced by different reasons. The correct allocation of the sign to the velocity in positions no. 1 and 2 could be due to any of the following reasons:

- the velocity vector has the same sign as the displacement vector in that position.

- a body that slows down has a negative velocity whilst a body that speeds up has a positive velocity.

- the sign of the velocity vector is determined by the direction of motion and the given (or chosen) sign convention - the correct method.

- guessing.

The uncertainty of the correct sign allocation for velocity in position number 3 (a 57 % success rate) further reinforces the belief that students do not know which factors to consider in this case. "Slowing down" might have prompted a reply of *negative;* however, the other two factors (except guessing!) yield a positive answer for the velocity vector. This question will be addressed again during the individual interviews.

The allocation of a sign to the *acceleration* vector was very poorly handled. Students frequently confused acceleration with velocity and also had a very poor understanding of the interrelationship between increasing / decreasing speed, the chosen sign convention and the resulting allocation of sign to the acceleration vector.

58 % of the students readily accepted that "slowing down" in a positive direction implies a negative acceleration, probably because of the fact that this motion is often (somewhat erroneously) referred to as "deceleration", which in turn is often associated with "negative" acceleration. Very few (29 %) of the students accepted, however, that *speeding up in a negative direction* also implies a negative acceleration and, similarly, that *slowing down in a negative direction* gives a positive acceleration (success rate only 32 %).

The confusion between velocity and acceleration is further illustrated by the fact that about 75 % of all respondents who allocated the correct negative sign to the velocity vector in position 2 (for whatever reason), proceeded to allocate (incorrectly) the same sign to the acceleration vector in this position. Another reason for the popularity of the negative sign for acceleration in position 2 is the fact that *slowing down* is immediately associated with a negative value, just like speeding up (in position 1) is associated with a positive acceleration. **The direction which was chosen as positive is often simply ignored when the sign of the acceleration at a given point has be considered.**

A good illustration of the extent of the confusion among students with regard to reference level (for displacement) and sign convention is provided by the fact that only **one** student managed to allocate all nine signs correctly for the kinematic quantities in the various positions.

QUESTION 5
State the importance and mention all the information that can be obtained from gradients in displacement-time, velocity-time and acceleration-time graphs.

Gradients are frequently used when learners have to extract information from kinematic graphs and they are particularly important tools which assist greatly in graph-graph transformations. It is therefore important to investigate whether students have a good understanding of the meaning and use of gradients in the context of the three basic kinematic graphs.

RESULTS:

RESPONSE CATEGORY	ANSWER CODE	%
Gradient of *s-t* graphs		
Gradient of graph gives velocity	1*	45
Displacement over time	2*	5
Positive/negative gradient gives direction of velocity	3*	5
Includes reference to gradient of tangent to curved graph	4*	3
Mentions instantaneous velocity	5*	1
Gradient gives speed	6	2
Gradient gives acceleration	7	2
Can calculate velocity and acceleration	8	2
Incoherent	9	35
Gradient of *v-t* graphs		
Gradient gives acceleration	1*	49

RESPONSE CATEGORY	ANSWER CODE	%
Velocity over time	2*	3
Gradient gives displacement	3	5
Gives information about increasing/decreasing/const. Velocity	4	4
Other incoherent response	5	39
Gradient of *a-t* graphs		
Gradient is always zero	1*	7
Acceleration over time	2*	7
Rate of change of acceleration	3*	3
Tells whether acceleration is constant or increasing	4*	3
Gives velocity	5	13
Gives displacement	6	10
Cannot find the gradient	7	6
Gives acceleration	8	5
Other incoherent response	9	46

TABLE 4.9
MEANING OF GRADIENTS IN CERTAIN KINEMATIC GRAPHS
(expressed as a percentage of the number of students who attempted the question; n = 150)

The results seem to point to the fact that students are seldom able to give more than a very simplistic explanation of the gradient concept in the context of kinematic graphs. Very few respondents linked the negative gradient of an *s-t* graph to a negative velocity. The questions did not require an explicit comment on the gradients of tangents to curved *s-t* graphs, so that the poor response rate in this regard has to be ignored.

Secondary students receive direct instruction about the meaning of gradients, both in the science and mathematics class, and it is therefore surprising to discover that 27 % of the respondents gave an incoherent response to the question on gradients of *s-t* graphs.

49 % of the students correctly stated that the gradient of a *v-t* graph gives acceleration, with a further 3 % associating it with *velocity over time*. Curved *v-t* graphs are not included in

the high school syllabus, so it was not expected that students would mention the "gradient of tangent" idea with respect to *v-t* graphs. It was, however, surprising that (as with *s-t* graphs) no student chose to explain the gradient of a *v-t* graph as giving *the rate of change of velocity.*

Some students confused the *graph* of *v-t* with the *gradient* of the graph leading to the following typical idiosyncratic statement: *if the gradient is curved then the velocity is increasing, but if this is straight it is travelling at constant velocity.*

The response to the question on gradients of *a-t* graphs was extremely confusing. The students seem to have been surprised at the question, because they were unable to give a previously learned definition or explanation. Although motion studied at school only deals with that which is characterised by a *constant* acceleration, it was nevertheless felt that they should be able to associate the gradient of the *a-t* graphs with *the rate of change of acceleration* or at least with an expression such as *change in acceleration divided by time.*

"Gradient of *a-t*" codes 1, 2 and 3 (the latter dealing with uniform acceleration) all contain elements of truth, although students did <u>not</u> use the scientifically correct expression of ***change** in acceleration over time* in code 2. This means that a mere 18 % of the respondents were able to attach an acceptable meaning to the gradient of an *a-t* graph.

Some students believed that the gradient of the graph of a kinematic quantity must necessarily yield another related kinematic quantity. In the case of a-t graphs a total of 28 % of the respondents thought that the gradient represents either the *velocity,* the *acceleration* or the *displacement* (codes 5, 6 and 8).

Students' understanding of gradients of motion graphs can be summarised as follows:

- a reluctance to associate it with the "rate of change" concept.

- describing it as the specific kinematic quantity over time, instead of the *change* in the quantity over time.

- not comprehending the meaning of the gradient of an *a-t* graph.

QUESTION 6

An object moving with constant negative acceleration can be "speeding up". Agree, disagree or unsure?

The purpose of this question was to find out whether students realised that a negative value for an acceleration merely provides information about the *direction* of the acceleration (or more specifically the fact that the resultant force acts in a direction opposite to that which was chosen as positive). An object with a negative acceleration can thus be "speeding up" in the *negative* direction or "slowing down" in a *positive* direction.

RESULTS

RESPONSE CATEGORY	ANSWER CODE	%
Agree	1*	48
Disagree	2	48
Unsure	3	4

TABLE 4.10
NEGATIVE ACCELERATION AND SPEEDING UP
(expressed as a percentage of the number of students who attempted the question; n = 266)

RESPONSE CATEGORY	ANSWER CODE	%
Speeding up in a negative direction	1*	4
g can be -10 m/s^2 so object is speeding up	2*	3

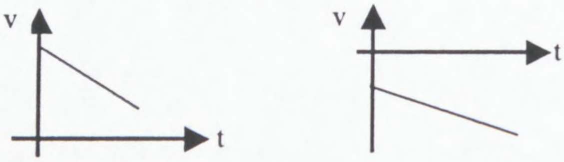
RESPONSE CATEGORY	ANSWER CODE	%
v-t graph "slows down" (fig (a)), or "speeds up" (fig (b))  Figure (a) Figure (b)	3*	2
Object is slowing down (velocity is decreasing)	4	18
Moving in opposite direction	5	17
Moving backwards (reversing)	6	12
Object is decelerating	7	10
Constant acceleration means **no** speeding up or slowing down	8	10
Acceleration is negative, but it is still acceleration	9	2
Incoherent	10	22

TABLE 4.11
REASON FOR NEGATIVE ACCELERATION AND SPEEDING UP
(expressed as a percentage of the number of students who attempted the question; n = 249)

Approximately half of the respondents accepted the fact that a negative acceleration may be associated with an object which is speeding up. However, when asked for an explanation a wide variety of (often confusing) reasons were given.

Reason codes 5 and 6 (given by 29 % of the respondents) deal with some form of reversal of direction. For these students a negative acceleration means "speeding up" if the direction of motion is simply reversed (presumably from positive to negative). They confused acceleration with velocity, not realising that it is necessary to add that the object has to speed up in this negative direction. The information that the movement is in a negative direction is thus *necessary but not sufficient.*

28 % of the students associate a negative acceleration with a decrease in the velocity of the object (codes 4 & 7). These students (who thus did not accept the given statement)

indicated explicitly or by implication that they regarded the expressions *negative acceleration*, *deceleration*, *slowing down* and *decreasing velocity* as synonymous.

A small minority were able to give an explanation which contains sufficient elements of truth to be classified as acceptable by most subject experts. Codes 1, 2 and 3, which make up a mere 9 % of the students, represent three different approaches to the interpretation of negative acceleration. Code 1 describes the motion in general terms, as in: *the object is slowing down in the positive direction and therefore speeding up in the negative direction.*

Code 3 illustrates the motion graphically in terms of a negative velocity of increasing magnitude and accompanying negative gradient, while code 2 makes use of the well-known example of the gravitational acceleration, where "up" is taken as positive.

Two students recalled their knowledge of number lines, learned in mathematics, stating the reversal in direction in terms of a *negative velocity* of *increasing magnitude*. A sketch such as figure 4.1 was included with the explanation: *the body can increase its speed by moving backwards like on a number line.*

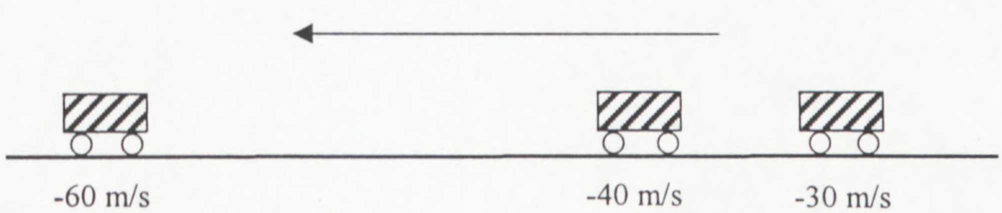


FIGURE 4.1

The following misconceptions were displayed by a limited number of respondents:

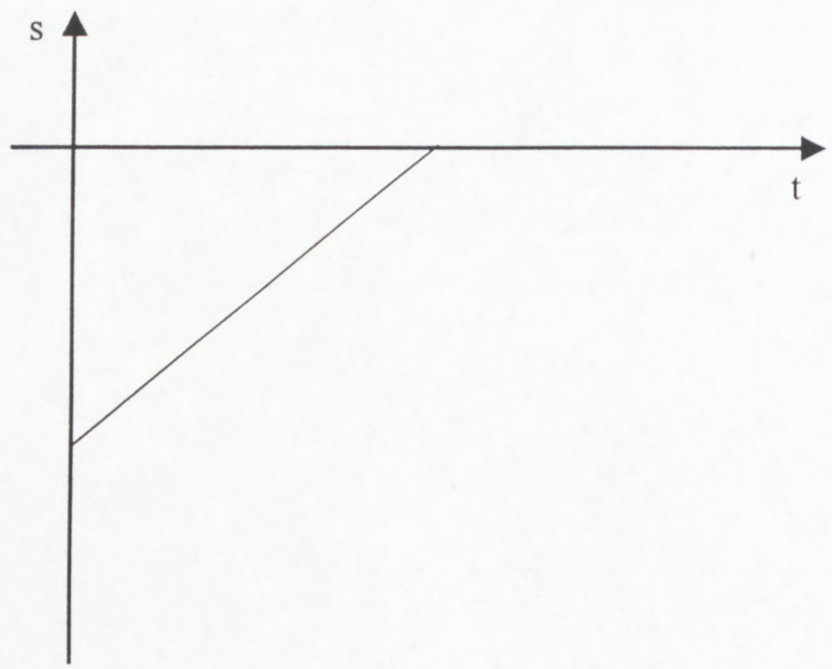
- A negative value for acceleration is seen as a **decreasing** positive acceleration. These students agreed with the statement, but believed that the *rate of speeding up is decreasing.*

- 10 % of the students focused on the word *constant* and concluded that while the acceleration does not change, the velocity also does not change. To these students constant acceleration therefore means constant velocity (see code 8), thus demonstrating a basic failure to differentiate between these two quantities.

- A 2nd year tertiary student argued that the change in speed of an object is independent of its acceleration, probably recalling the definition of acceleration in terms of *velocity* (and not *speed*). He states: *speed can increase without influencing acceleration, because acceleration is related to velocity.*

QUESTION 7

Briefly describe the motion represented by the given *displacement-time* graph.



The purpose of this question was to find out whether the respondents could interpret the graphical representation of a motion which is represented by linearly increasing, but **negative**, displacement values. The motion starts at a negative displacement in terms of the chosen reference point and sign convention, and proceeds with constant positive velocity (indicated by the constant positive gradient) to stop at the chosen reference level.

The first five responses in the table (representing 41 % of the students) describe various aspects of the motion correctly. It is interesting to note, however, that only 14 % stated that the motion that was represented by the graph was that of a body moving at *constant* *velocity* (codes 2 & 5). Approximately half of these students noted that this constant velocity was **positive**.

RESULTS

RESPONSE CATEGORY	ANSWER CODE	%
Moving towards a starting point	1*	13
Moving at constant velocity / speed	2*	11
Moving from a negative displacement to a positive displacement (or with decreasing negative displacement)	3*	7
The motion causes displacement to increase	4*	7
Moving at constant velocity towards reference point	5*	3
Moving with increasing velocity / acceleration	6	14
The velocity is decreasing / deceleration	7	10
Moving backwards	8	8
The body travels with constant negative velocity	9	6
Moving forward	10	2
Other	11	19

TABLE 4.12
DESCRIPTION OF *s-t* GRAPH
(expressed as a percentage of the number of students who attempted the question; n = 227)

The graph was drawn for negative displacement values only and although only one student stated (erroneously) that no such motion exists, many had difficulty with recognising the constant positive gradient and associating this with a constant positive velocity.

Many respondents clearly felt a need to make mention of the behaviour of the velocity. Those who did not accept a constant positive velocity, were divided about whether the velocity was increasing in line with the increasing s-values (code 6), or decreasing as a result of the decreasing <u>magnitude</u> of the s-values (code 7). These students incorrectly focussed on the ordinates of the given *s-t* graph in order to gain information about the velocity.

Those who did recognise that the velocity was constant but incorrectly viewed it as <u>negative</u>, probably argued that the observed negative displacement values resulted in a negative velocity as well. One student answered as follows:

$$v = \frac{s}{t}$$

$$= \frac{\text{neg.value}}{\text{pos.value}}$$

$$= \text{neg. value}$$

The error of defining velocity as *displacement over time* instead of ***change*** *in displacement over time*, thus leads this student to a serious incorrect conclusion about the direction of the velocity vector.

It is encouraging to note that a number of the respondents chose to describe the motion in terms of its position relative to a reference point. 16 % (by codes 1 & 5) stated this explicitly, while a further 7 % (code 3) implied the existence of a reference point by referring to the fact that the displacement would have become positive after a further elapse of time. The expression: *moving backwards* (code 8) also seems to imply a vague attempt at describing an object which is moving back to a reference point, after having moved forward. One student attempted to give an explanation of the motion in terms of a real-life situation, choosing (by implication) sea level as his reference level: "*an object released from the sea bed, floating to the surface after being dropped into the ocean (graph drawn from sea bed)*".

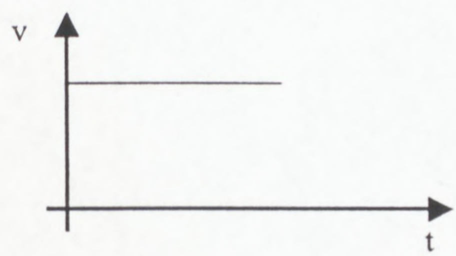
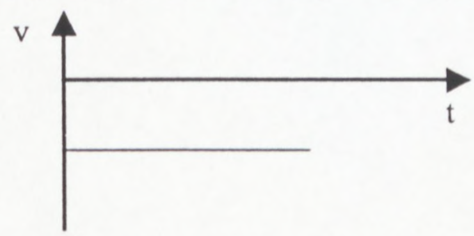
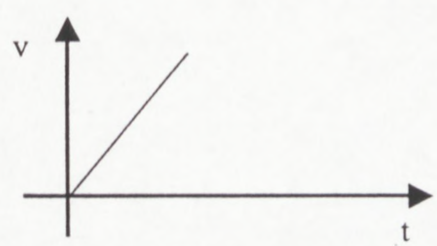
Some students confused the terms *increasing, decreasing* and *decreasing magnitude.* 7 % stated, correctly, that the displacement was increasing; however, about half the respondents, who according to code 3 mentioned the change in displacement from negative to positive, associated this with a *decreasing* displacement!

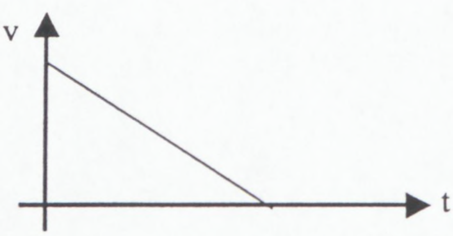
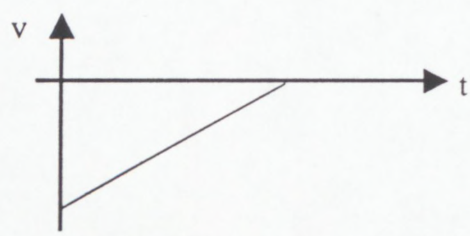
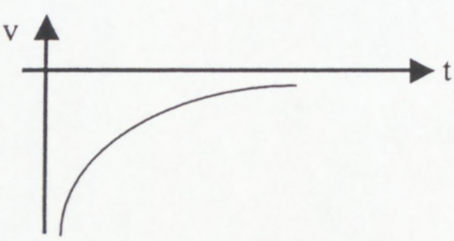
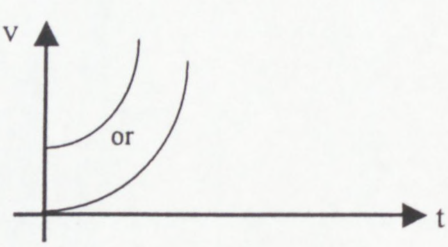
QUESTION 8

Draw the *velocity-time* graph which describes the same motion depicted by the graph in question 7. Give a reason for your answer.

The students were required to deduce from the constant positive gradient of the given graph (or otherwise) that the velocity was constant and positive and hence draw the appropriate *v-t* graph for the motion.

RESULTS

GRAPH	ANSWER CODE	%
	1*	26
	2	20
	3	13

GRAPH	ANSWER CODE	%
	4	12
	5	7
	6	3
	7	3
OTHER	8	16

REASON	ANSWER CODE	%
Constant positive gradient gives constant positive velocity	1*	7
Constant velocity / zero acceleration / uniform motion	2	25
v is constant and negative because *s-t* graph is "negative"	3	15
Velocity decreases because *s-t* graph indicates neg. acceleration	4	12

REASON	ANSWER CODE	%
Velocity / speed increases at constant rate / constant acceleration	5	11
$v \propto t$	6	3
Moving in opposite direction (backwards)	7	3
Velocity is zero because s_{res} is zero	8	3
Incoherent	9	24

TABLE 4.13
GRAPHS & REASONS FOR *s-t* → *v-t* TRANSFORMATION
(expressed as a percentage of the number of students who attempted the question; n = 174)

Although 26 % of the students drew the correct graph (code 1), only 7 % of the total number of respondents (or 27 % of those who drew correct graphs) stated that the observed constant **positive** gradient of the *s-t* graph yields a constant **positive** *v-t* graph (reason code 1). The rest of the respondents made no mention of *gradient* in their comments - frequently simply stating that the velocity is constant or that the motion is uniform (reason code 2). (It must be pointed out, however, that it is possible that some students who did make use of gradient considerations might not have included this fact in their *written* response.)

20 % of the respondents drew a *v-t* graph that represented a motion with a constant *negative* velocity (graph code 2). These students recognised the fact that the velocity stayed constant but thought that it had to be negative, like the given *s-t* graph (see reason code 3). The following comment made by a technikon student illustrates the sign-confusion that this question generated: *the v-t graph runs parallel to and below the t-axis because it must be a negative velocity in a positive direction.*

Graph codes 3 and 5 both show *v-t* graphs that increase linearly with time. The fact that a total of 20 % of the students drew this type of graph is significant, because they seem to believe that an increasing *s-t* graph should necessarily yield an increasing *v-t* graph. The explanation of the linearly increasing *v-t* graph often included phrases like: *the speed is* *increasing at a constant rate*, or *v increases steadily because s increases steadily*, or simply *v is proportional to t*.

Another common mistaken graph transformation involved a steadily decreasing (but positive) *v-t* graph (see graph code 4). The reasoning was usually given in the form of the following argument:

given *s-t* graph shows decreasing magnitude of displacement,

also given *s-t* graph has positive gradient,

$\Rightarrow$ *v-t* graph must also be decreasing and positive.

Some students even associate the given *s-t* graph with a negative *acceleration*, giving this negative acceleration as their reason for the negative slope of the *v-t* graph (see reason code 4 and graph code 4). Another group who attempted a motivation for graph code 4, argued that the velocity must have a decreasing magnitude (to match the observed decreasing magnitude of the displacement). However, they kept the v-values positive to match the positive gradient of the *s-t* graph.

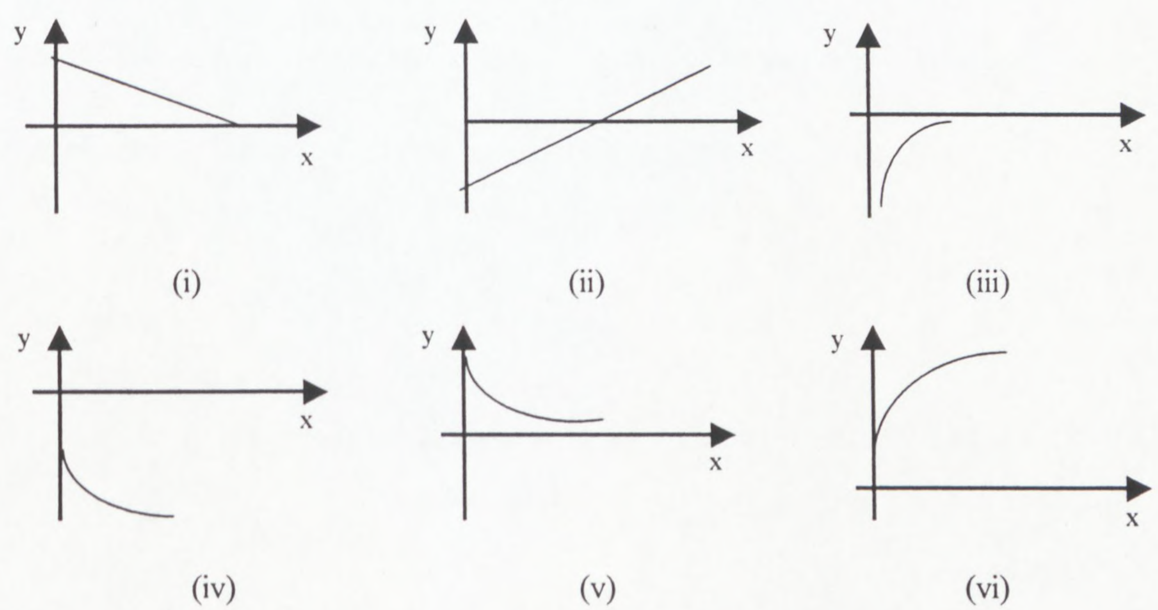
Five students stated that the velocity is zero at all times, because they interpreted the given graph as a *vector diagram* which shows a resultant displacement of zero (reason code 3). These students used the textbook definition of velocity as $\frac{\Delta s}{\Delta t}$ and concluded that v = 0 because $\Delta s = 0$. Another possibility is that they took $s_{final} = 0$ m and then conclude that $v = 0$ because $v = \frac{s}{t}$.

Students who drew curved graphs (similar to graph codes 6 and 7) confused the *s-t* $\Rightarrow$ *v-t* transformation with the opposite *v-t* $\Rightarrow$ *s-t* transformation. They simply drew a curved *v-t* graph of *increasing* gradient (code 7) to show the given *increasing* displacement values. Alternatively they drew a curved *v-t* graph with decreasing positive gradient to illustrate the *decreasing magnitude* of the given displacement values. In the latter case the *v-t* graph was drawn for *negative* v-values, similarly to the given graph.

The perceived "movement in an opposite direction" reported by some students (reason code 7) can possibly be attributed to a belief that the negative displacement values imply a direction which is opposite to a specified positive direction. These students usually gave either graphs 2 or 5 as their corresponding *v-t* graph.

QUESTION 9

Consider the following graphs, and write down the numbers of those with a *negative* gradient:



The ability to distinguish between a positive and a negative gradient is of great importance in all translations involving real motions and their graphical representation, as well as the transformation from one kinematic graph to another related one. This question attempts to test students' ability to identify graphs with a negative gradient when they are confronted with a number of graphs of different shapes and varying orientations with respect to the co-ordinate axes.

RESULTS

THOSE WITH NEGATIVE GRADIENT ARE:	ANSWER CODE	%
(i); (iv) & (v)	1*	35
(ii); (iii) & (iv)	2	20
(i); (iii) & (vi)	3	5
(i); (iii); (iv) & (v)	4	4
(iii) & (iv)	5	4
other combinations	6	32

TABLE 4.14
RESPONSES TO NEGATIVE GRADIENT
(expressed as a percentage of the number of students who attempted the question; n = 270)

In view of the fact that many different combination with limited support were reported, only the five most popular combinations are indicated. Students were not required to give a reason for their choice.

Only 35 % of the students opted for the correct combination (code 1), indicating a limited understanding of negative gradient among the test group. The reason for the poor response could be due to either or both of the following two factors:

- some graphs were curved, which meant that the gradient was changing. This might have made the identification of a *negative* (but *increasing* or *decreasing*) gradient more problematic than would have been the case for normal straight line graphs;

- a number of graphs were drawn at least partially below the time axis, which meant that students had to focus purely on the *direction* of the graph and ignore the fact that it described negative values.

Graph (iii) has a decreasing positive gradient although it is drawn for negative y-values. It is significant that this graph is included in all the incorrect combinations represented by codes 2, 3, 4 & 5. This seems to indicate that these students (at least 33 % of the total) confuse the sign of the *gradient* with the sign of the *ordinate* values. A further confirmation of this hypothesis is that a further 20 % opted for the combination (ii), (iii) & (iv), which are the only graphs with negative values for the dependent variable (y).

Graphs (iii) and (vi) are the only ones with a *decreasing positive* gradient. Students who chose option code 3 combined these two graphs with (i) (which they accepted must have a negative gradient). This seems to indicate that these students (5 % of the respondents) view a function for which the rate of increase *decreases* also to have a negative gradient. In other words they seem to have made the association: decreasing gradient → negative gradient.

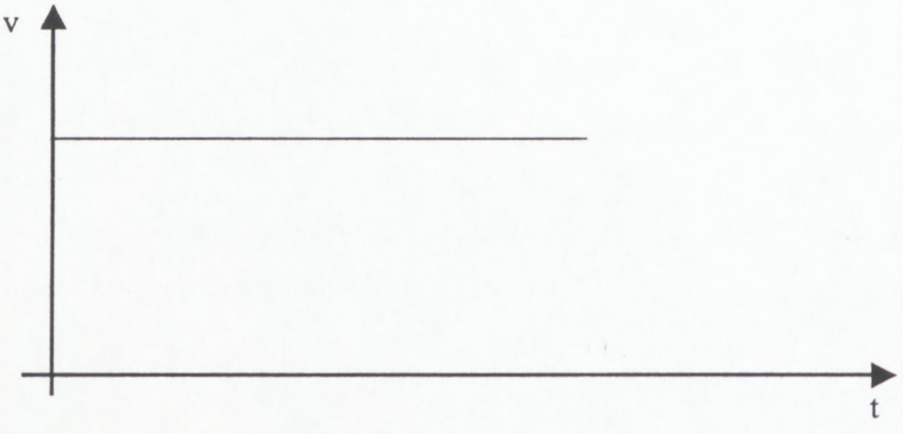
A large number of <u>other</u> combinations were given of which two are recorded in the table. Code 4 represents the correct combination with the addition of graph no. (iii) which lies wholly below the x-axis. Code 5 refers to the two graphs which are curved and lie below the x-axis. Students opting for code 5 (4 % of the respondents) probably argued that a graph has to be negative <u>and</u> curved in order to be classified as a negative gradient.

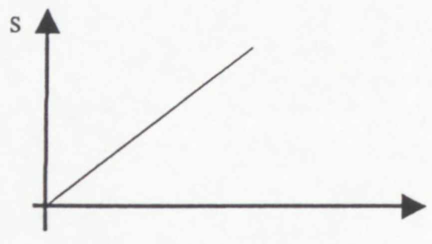
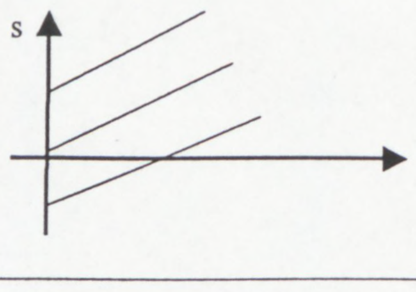
Questions 10, 11, 12 and 13

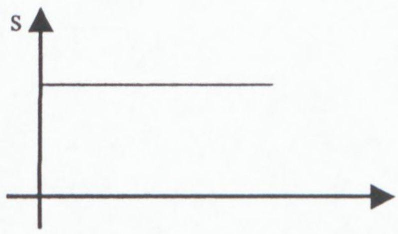
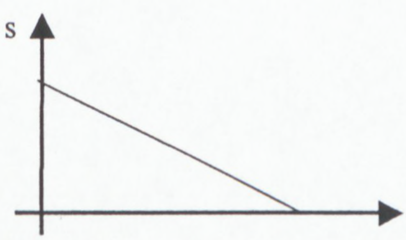
Questions 10, 11, 12 and 13 required the respondents to draw *displacement-time* graphs to match the given *velocity-time* graphs. **Reasons** for their answers were also required.

QUESTION 10

Given:



s-t GRAPH RESPONSES	ANSWER CODE	%
	1*	74
	2*	3
 (mentions various alternatives)	3*	2

s-*t* GRAPH RESPONSES	ANSWER CODE	%
	4	7
	5	3
	6	2
	7	2
OTHER	8	7

REASON CATEGORY	ANSWER CODE	%
Constant velocity (zero acceleration)	1*	25
Displacement increases due to constant velocity	2*	20
Gradient of *s*-*t* graph gives constant velocity	3*	7
$s \propto t$	4*	5

REASON CATEGORY	ANSWER CODE	%
Velocity is constant, therefore displacement is uniform	5	14
Velocity is constant, so displacement is constant	6	12
Acceleration is constant	7	4
Incoherent / other	8	13

TABLE 4.15
GRAPHS & REASONS FOR *v-t* → *s-t* TRANSFORMATION (Q10)
(expressed as a percentage of the number of students who attempted the question: n (graphs) = 246, n (reasons) = 183)

It was decided to start off the series of *v-t* graph to *s-t* graph transformations with the simple case of a constant positive velocity transforming into an *s-t* graph which takes the form of *any* straight line with *positive* slope. The relatively simplistic nature of the question resulted in 79 % of all respondents drawing a correct *s-t* graph (see graphs 1, 2 & 3). Reason codes 1, 2, 3 and 4 all contain "some elements of truth", so that in broad terms it can be stated that 62 % of the respondents gave an acceptable reason for the shape of their (correct) sketch graph.

It is interesting to note that almost all students who drew a correct *s-t* graph started their graphs at the origin (another 3 % started at a positive s-value), while only 2 % indicated that a number of different parallel lines are possible. This implies that most students believed that *the displacement at the beginning of a motion had to be zero*, thus ignoring the fact that the choice of reference level is purely arbitrary.

Students had difficulty in explaining why they drew a linearly increasing *s-t* graph, often using expressions such as: *the displacement is uniform; the displacement increases equally; the same distance is covered each second* or *a constant increase in displacement*. The more acceptable explanation in terms of *the positive gradient of the s-t graph stays constant* was only used by 7 % of the respondents (see reason code 3).

Graph no 4 (given by 7 % of the students) was motivated by reason code 6 (12 %). Other students, however, also used the incorrect reason code 6 (velocity is given as constant therefore displacement must be constant) to motivate a **correct** *s-t* graph (code 1). This means that these students interpreted the constant *gradient* of the (correct) *s-t* graph incorrectly as *constant displacement values*.

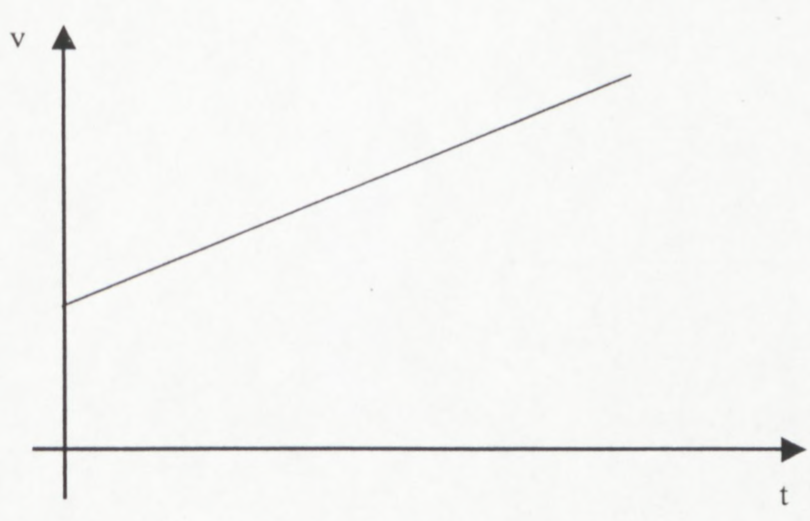
Students who responded by drawing graphs similar to graph code 5 interpreted the given motion in terms of an object approaching an observer and stated typically: *s gets smaller as the object nears its destination.* These students were obviously not perturbed by the fact that their *decreasing s-t* graph represented a *negative* velocity.

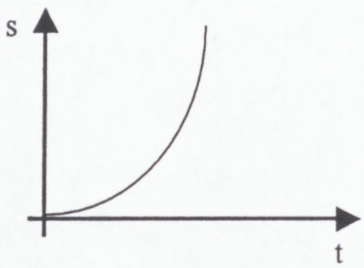
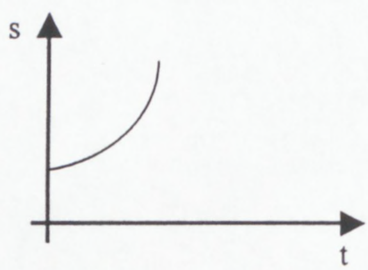
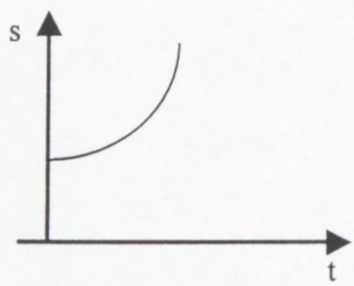
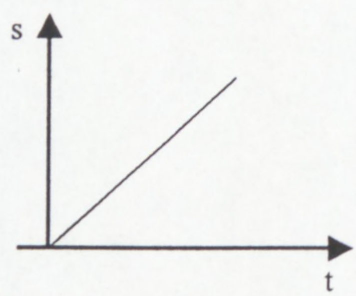
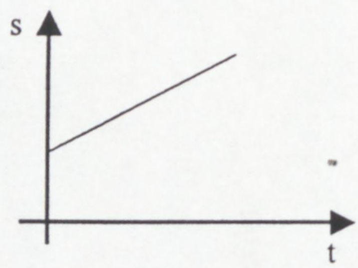
The curved *s-t* graph of code 6 was usually accompanied by an explanation typified by code 7, involving the idea of a constant non-negative acceleration. These students seemed to confuse velocity with acceleration and argued that a constant velocity implies a constant acceleration which in turn is represented by a *curved s-t* graph.

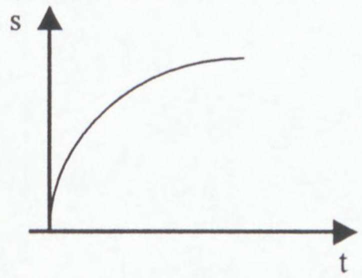
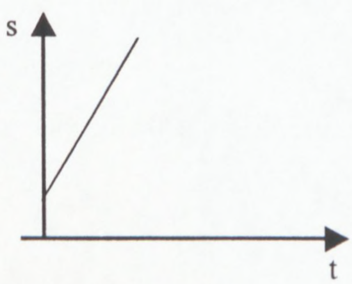
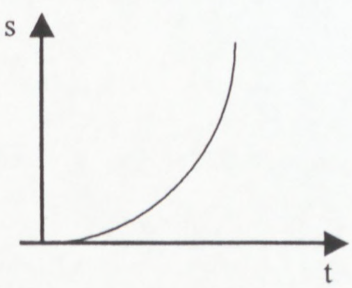
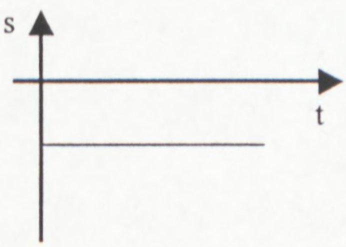
In a few cases velocity was even confused with *displacement*. Graph code 7 (horizontal line on t-axis) was usually motivated in terms of the fact that if the velocity does not change, then the displacement does not change, resulting in a value of zero for the displacement throughout the motion.

QUESTION 11

Given:



s-t GRAPH RESPONSES	ANSWER CODE	%
	1*	36
	2*	7
	3	10
	4	9
	5	6

s-t **GRAPH RESPONSES**	**ANSWER CODE**	**%**
	6	4
	7	4
 (indicating an initial velocity of zero)	8	4
	9	3
OTHER	10	13

REASON	**ANSWER CODE**	**%**
Velocity increase ⇒rate of change of displacement increase	1*	30
Ref. pt determines starting point of graph	2*	1
Velocity increase ⇒displacement increase	3	19
Velocity is increasing (constant acceleration)	4	18
Velocity starts above rest, so displacement increases from above rest	5	7

REASON	ANSWER CODE	%
Displacement-time graph has increasing gradient	6	6
Uniform velocity	7	2
Other / incoherent	8	17

TABLE 4.16
GRAPHS & REASONS FOR *v-t* → *s-t* TRANSFORMATION (Q11)
(expressed as a percentage of the number of students who attempted the question: n (graphs) = 244, n (reasons) = 162)

Students had to consider the given *v-t* graph and deduce the shape of the accompanying *s-t* graph by employing any (or a combination) of the following methods:

- Draw a curved *s-t* graph with gradually increasing positive slope to represent the linearly positive increasing v-values.
- Consider the increasing area per unit time interval under the *v-t* graph, and associate this with a progressively *bigger increase in displacement per time unit*, as time increases.
- **Recall** the shape of the *s-t* graph from the earlier study of kinematic graphs.
- Use the definition of velocity as *change in displacement per unit time* to draw a parabolic *s-t* graph - illustrating a rapidly increasing change of displacement per unit time.
- Visualise a real motion for which the velocity increases, and *spontaneously* draws the required *s-t* graph.

Students also had to note that the tangent to the *s-t* graph at t = 0 had to be positive to represent the positive velocity at t = 0.

An examination of the graphical responses reveals that the respondents usually realised that the required graph had to be curved with increasing positive gradient. Graph codes 1, 2, 3 & 8 (representing 57 % of all responses) meet this criterion. Codes 3 and 8, however, incorrectly show a gradient of zero at t = 0 indicating that these students (11 % of the total) did not note the *non-zero* velocity at t = 0.

As before, a large majority of the students who drew the correctly shaped graph insisted that it had to start at the origin. Only 7 % drew their *s-t* graphs starting at a random non-zero s-value illustrated by graph code 2. It is possible that these students imitated the positive *v-value* at t = 0 of the given graph, *without* actually considering the arbitrary nature of the s-value at t = 0, a suspicion confirmed by statements such as: *v starts above the rest, so s increases from above rest.* However, one student argued (correctly): *the chosen reference point determines the starting point of my graph*, hence displaying a rare insight into the importance of a reference point to indicate a displacement in a physical situation.

Generally, the attempted explanations for the curved shape of the resulting *s-t* graph varied in the amount of detail given and also in the accuracy of the exposition. 18 % simply mentioned the fact that the given graph indicated an increasing velocity (code 4). A further 19 % (code 3) added that this resulted in an increasing displacement as well, without explaining the *nature* of this increase. It must be pointed out that during the interviews with some respodents who opted for reason code 3 (displacement increases), it appeared evident that they did not view this as a change of rate of displacement. For this reason answer codes 1 and 3 were seperated. An encouraging 30 % of <u>all</u> students participating, <u>did</u> attempt to describe *how* the displacement increases. They used some rather unique expressions and phrases to describe the shape of the parabolic *s-t* graph, with some being more acceptable than others. These have all been grouped together under reason code 1. The following are examples of student reasons with regard to reason code 1, all attempting to refer to the increasing rate of change of the displacement.

- *a large displacement has been made within a shorter period of time;*

- *the increase in distance every second is greater and greater;*

- *the number of metres covered per second will increase;*
- *the rate of increase in displacement is increasing;*
- *displacement increases with a curve;*
- *displacement increases exponentially;*
- *displacement increases parabolically;*
- *displacement varies not proportionally with time;*
- *displacement increases rapidly.*

The willingness amongst the respondents to attempt a verbal explanation for their *s-t* graph was most encouraging and usually revealed a remarkably sound insight into the behaviour of this particular graph.

Code 6 specifically describes a correct explanation of the *s-t* curve in terms of gradient considerations. The unwillingness on the part of respondents to make use of gradients which was reported in question 10 was again evident here, as a disappointingly low percentage (6 % of all the students questioned) stated that they noted the increasing gradient of their own *s-t* graph. Even among these few students a lack of complete insight into the role of gradient still appears to exist. Once again height was often confused with gradient and this caused these students to give an incorrect explanation for a correct graph, as illustrated by the following typical response: *the v-t graph has a steady increase in gradient, therefore the s-t graph must also have a steady increase in gradient.*

Graph code 7 (response factor 4 %) deserves special mention because these students viewed the observed increasing rate of change of displacement as resulting in an *s-t* graph with a much *steeper* gradient than the one that they drew in question 10. Their reasoning is illustrated in figure 4.2.

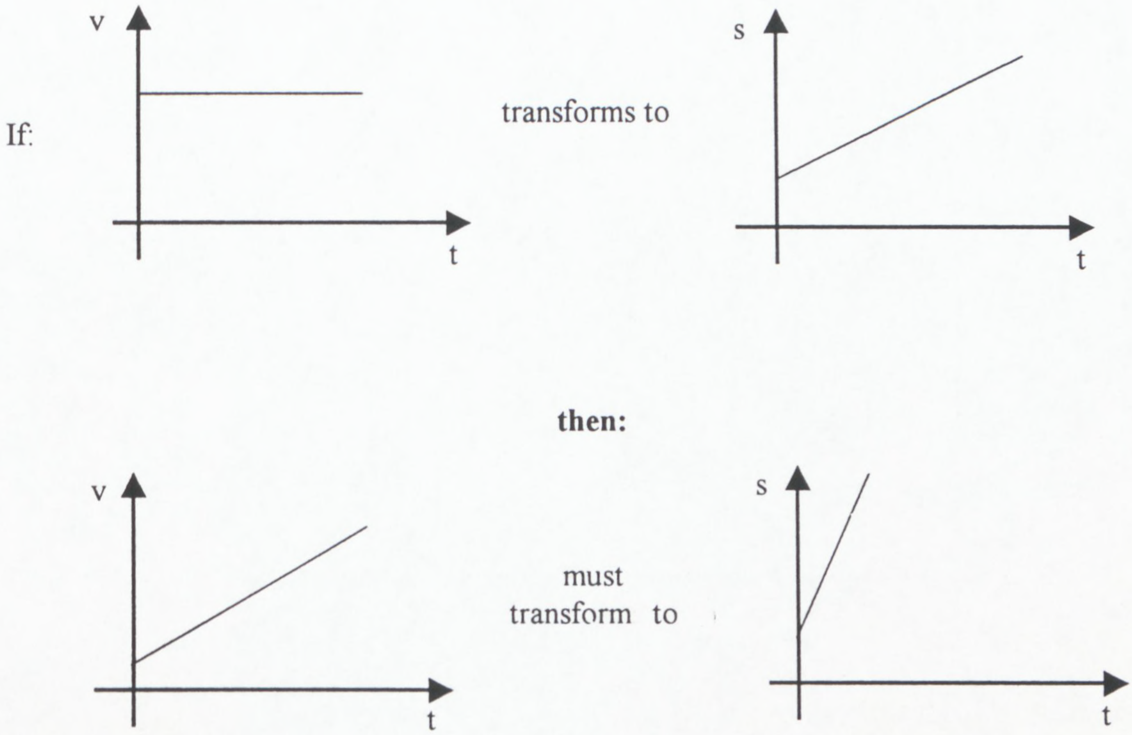


FIGURE 4.2

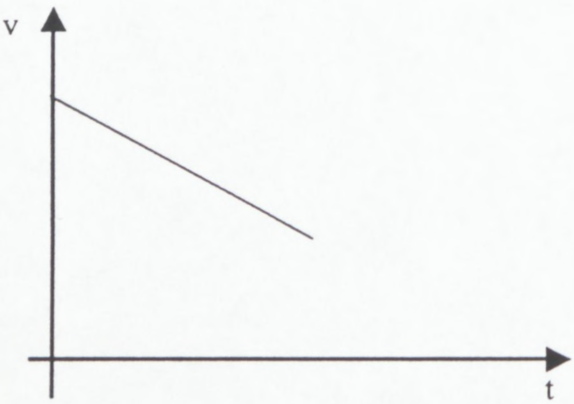
This refusal to change the linear character of the *s-t* graph was also evident in graph code 5 which is simply a copy of the shape of the given *v-t* graph. These students seem to believe that velocity is so closely related to and dependent on displacement that the graphs of *v-t* and *s-t* describing a particular motion should have *at least* similar appearances.

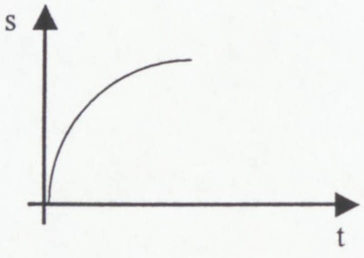
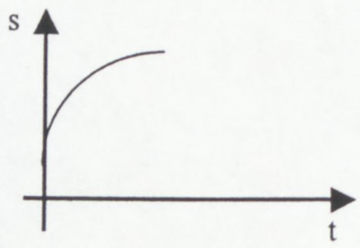
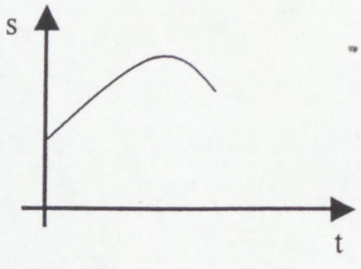
The "concave down" curve of graph code 6 **did** indicate an acceptance of the non-linearity of the *s-t* graph, but assumed that the displacement values converged to some final fixed value.

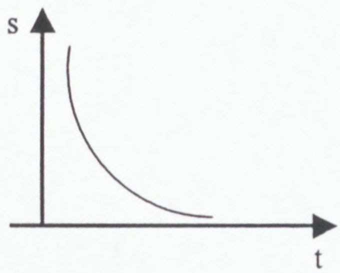
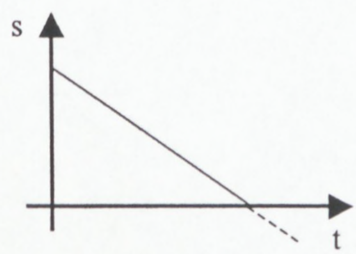
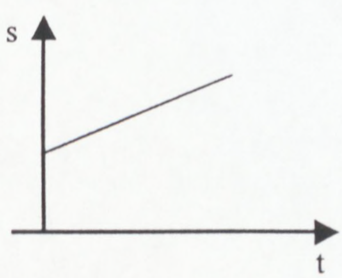
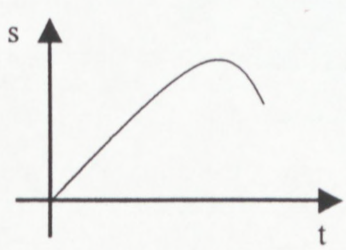
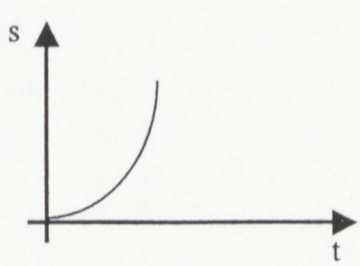
One student illustrated a total lack of understanding of the meaning of graphs by declaring - after studying the given *v-t* graph: *the body is moving in a NE direction*, thus interpreting the line as the actual velocity vector, rather than viewing it as an illustration of how velocity varies with time.

QUESTION 12

Given:



s-t **GRAPH RESPONSES**	**ANSWER CODE**	**%**
	1*	28
	2*	5
	3	14

s-t GRAPH RESPONSES	ANSWER CODE	%
	4	11
	5	9
	6	9
	7	8
	8	3
OTHER	9	16

REASON	ANSWER CODE	%
Decreasing v, i.e. negative gradient implies decreasing rate of change of displacement (or smaller change in s in same time, etc.)	1*	19
Decreasing velocity implies decreasing positive gradient of *s-t* graph	2*	6
Velocity is slowing down but distance is still covered so s increases <u>more slowly</u>	3*	5
Area under *v-t* graph gives change in displacement	4*	1 student
Velocity decreases / deceleration / negative acceleration	5	25
Velocity decreases, so displacement decreases	6	19
v-t graph has negative gradient so displacement is negative	7	3
Incoherent	8	23

TABLE 4.17
GRAPHS & REASONS FOR *v-t* → *s-t* TRANSFORMATION (Q12)
(expressed as a percentage of the number of students who attempted the question: n (graphs) = 229; n (reasons) = 150)

The purpose of the question was to find out whether students were able to interpret the given decreasing but positive velocity in terms of progressively smaller displacements per time unit, thus leading to *s-t* graphs such as illustrated by codes 1 and 2. As before, it was anticipated that an analysis of students' reasons would give some idea of the method by which they arrived at their *s-t* graphs.

The fact that only 33 % of the respondents were able to draw a correct *s-t* graph, with nearly all of them starting their graphs at the origin, has two important implications:

- the continuing pattern of a drop in the number of correct responses (compare 79 % for question 10 and 43 % for question 11);

- another illustration of the tendency of students to begin their graphs at the origin.

Reasons for a correct graph usually included a reference to the fact that the velocity was decreasing; however, relatively few candidates made an acceptable inference about the effect of this on the *shape* of the *s-t* graph. Explanation codes 1 & 2 (totalling 25 % of the students) represent the *only* instances where students who produced the correct graph demonstrated their ability to communicate their understanding of the appropriate shape of the *s-t* graph. Of these 25 %, only 6 % referred (correctly) to the *change* of the gradient in the *s-t* graph, while the other 19 % (code 1) attempted to explain the curved *s-t* graph in terms of expressions such as: *less displacement in the same time* or *same displacement in more time*. These students failed to note that they should have referred to **change in displacement** instead of displacement.

Reason code 3 (representing 25 % of the respondents) does not sufficiently explain this graph transformation. None of these students, when interviewed could improve on their explanations. All responses which merely referred to the observed decreasing velocity, were thus grouped together under code 3 and classified as "incorrect".

Some made use of the scientifically acceptable phrase: *rate of change of displacement decreases*, thus linking the observed decreasing velocity to the definition of velocity, while others used the more colloquial expression: *a slower change in displacement*.

Students were extremely reluctant to use the area under the *v-t* graph to gain information about the displacement. Only one student used this method, stating: *the decreasing area per second gives a decreasing displacement covered every second.*

The inability or unwillingness of learners to make use of *area* considerations when making graph → graph transformations seems to be even more common than the absence of *gradient* arguments.

An interesting feature of the incorrect *s-t* graphs drawn by the candidates is the fact that they are very diverse in shape, with many different graphs gaining a significant proportion of the support (see graph codes 3, 4, 5, 6 & 7). Shape no. 3 resembles the correct graph but has a decreasing portion (i.e. with negative slope) to resemble the given graph. These students (14 % of the total) understood the essential features of the required graph, but could not resist the temptation to draw at least part of the *s-t* graph similar in shape to the given *v-t* graph.

A further 11 % (graph code 4) drew a hyperbolic *s-t* curve. Few attempted an explanation, but it is possible that they reasoned that the displacement had to decrease with time in accordance with the decreasing velocity values, except that they recalled that *s-t* graphs are often curved. One student confused *decreasing* with *inversely proportional* when stating: *we know that velocity decreases, this means that displacement is also inversely proportional to time.*

Many other students also displayed a lack of understanding of the idea of a *decreasing* graph. Half of the 19 % of the respondents who opted for reason code 6 (the incorrect referral to a *decreasing displacement*) still managed to draw a correct *s-t* graph. They interpreted their decreasing but positive *gradient* as a decreasing *function*. Candidates even called curves 1, 2 and 3 "*negative*" curves, contrasting them to what they believe are "*positive*" curves as illustrated in figure 4.3:

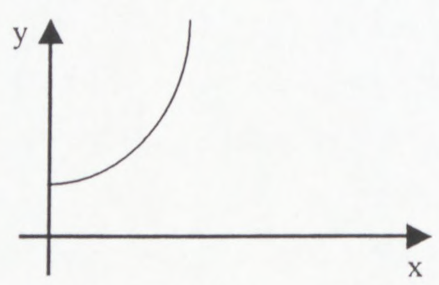


FIGURE 4.3

This suspicion would be investigated further by means of personal interviews.

Graph codes 5 & 6 (gaining equal support of 9 % each) show linearly increasing and decreasing displacement values respectively. Graph 6 was usually accompanied by reason code 3, arguing that while there exists a velocity, a distance has to be covered (even if the velocity is decreasing). For these students this "increased distance" is represented by a *positively* sloping *s-t* graph.

Graph 5 is accompanied by the familiar *like v-t like s-t* type argument. These students, when attempting to explain their graph, used expressions such as *decreasing graph, negative gradient, negative graph* and even *decreasing gradient* in a haphazard and confusing manner. The following are examples of students' failed attempts at stating the more acceptable argument that *a decreasing* positive velocity gives an *s-t* graph with *decreasing positive gradient,* which is based on reason code 1:

- *velocity decreases, so the gradient of the s-t graph is negative;*

- *velocity decreases, so the s-t graph is negative;*

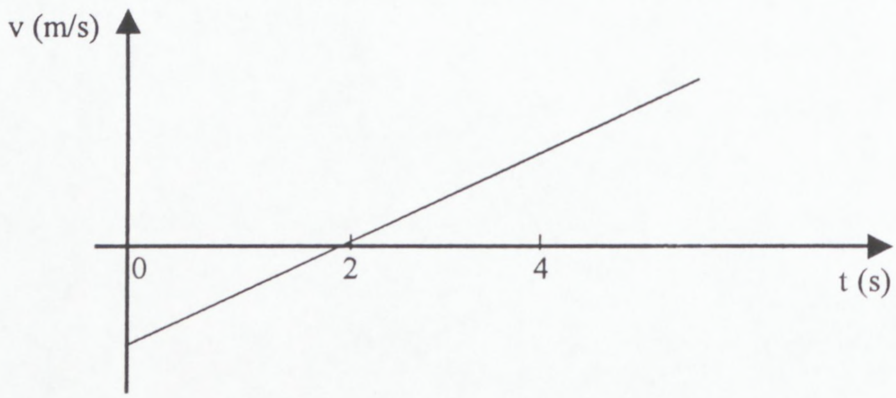
- *velocity decreases, so displacement decreases;*

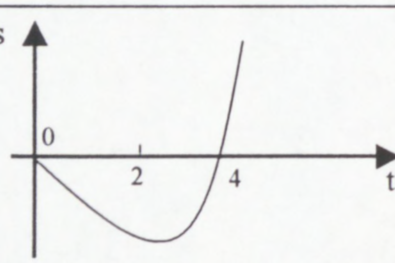
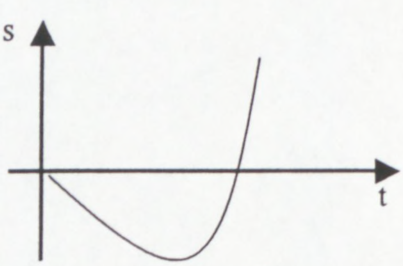
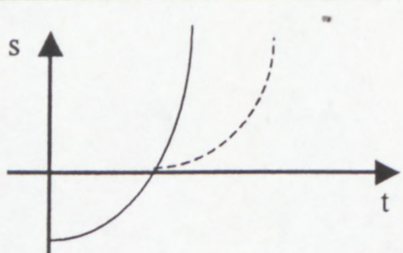
- *v-t graph shows decrease in gradient, so s-t graph also shows decrease in gradient.*

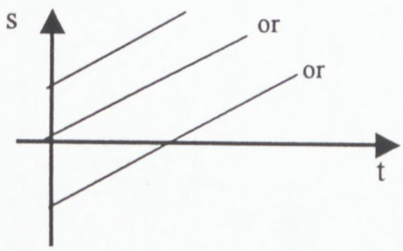
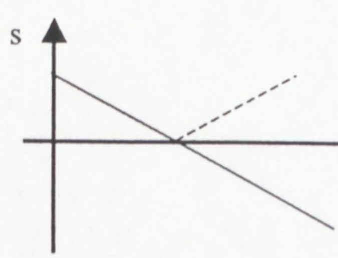
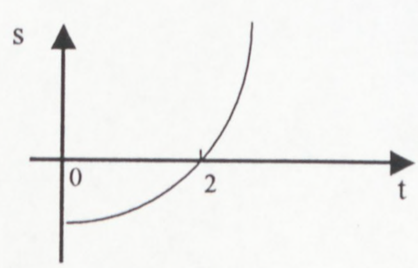
One other graph which gained significant support was no. 7, which was given by 8 % of the respondents. This *s-t* graph is similar in appearance to no. 1, except that, during the final time interval, it indicates a decrease in s-values. These students probably spontaneously drew part of a concave-down parabola, imitating the way that they drew it in the mathematics class simply ignoring the significance of the physical meaning of the downward sloping section.

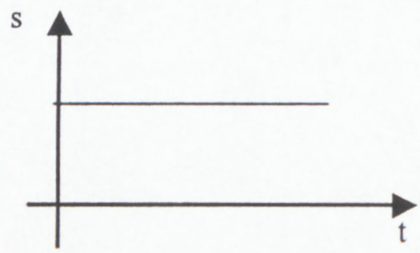
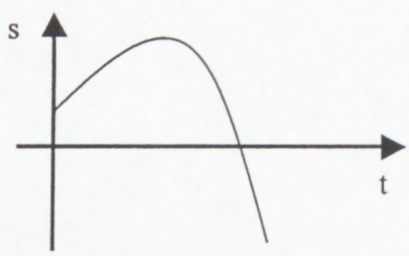
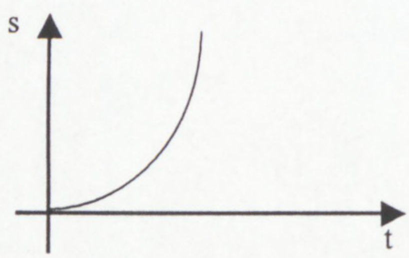
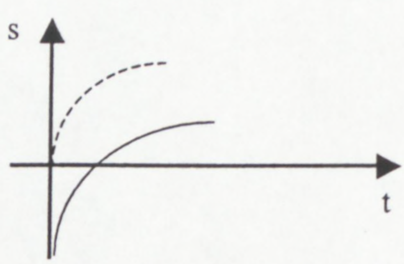
QUESTION 13

Given:



s-t **GRAPH RESPONSES**	**ANSWER CODE**	**%**
 (or any vertical shift)	1*	9
 (or any vertical shift)	2*	2
	3	18

s-t GRAPH RESPONSES	ANSWER CODE	%
	4	14
	5	8
	6	7
	7	6
	8	5

s-t GRAPH RESPONSES	ANSWER CODE	%
	9	5
	10	4
	11	3
	12	3
OTHER	13	16

REASON	ANSWER CODE	%
Object reversed, then went forward (or negative direction followed by positive direction)	1*	17
Describes the motion of a real object (e.g. car slows down in negative direction, turns around and accelerates in opposite direction)	2*	3
Gradient of *s-t* graph changes from negative to positive	3*	2
v increases uniformly, so s increases with a curve	4	14
Acceleration in first 2 seconds is negative	5	10
Object is moving with positive acceleration	6	9
Negative velocity gives negative displacement, then increasing velocity gives curved increasing displacement.	7	7
Constant velocity gives constant displacement / velocity slows down, then speeds up	8	6
v increases uniformly, so s increases with a straight line	9	3
Other wrong or incoherent response	10	26

TABLE 4.18
GRAPHS & REASONS FOR *v-t* → *s-t* TRANSFORMATION (Q13)
(expressed as a percentage of the number of students who attempted the question: n (graphs) = 211, n (reasons) = 110)

Candidates were required to study the given *v-t* graph and notice that the object starts off by moving in a negative direction with decreasing speed until it comes to rest after 2 s. It then turns around, moving with increasing speed in the opposite (positive) direction. The *s-t* graph could now be deduced from gradient considerations as follows: The initial gradient of the *s-t* graph is large and negative (corresponding to the initial negative velocity). It then decreases in magnitude until it becomes zero (corresponding to the zero velocity at t = 2 s). The gradient then becomes positive and increasing, resulting in a curved *s-t* graph similar to graph code 1.

Alternatively, the *s-t* graph could have been deduced from area considerations. The area between the *v-t* graph and the time axis (negative initially and positive after 2 seconds) gives successive changes in displacement values, which can be plotted to yield the appropriate *s-t* graph, showing a negative change in displacement for t < 2 s. A number of other correct methods, discussed under question 11, are equally appropriate here and will not be repeated again.

It was not surprising that the lowest success rate of 11 % was recorded for this *v-t* → *s-t* transformation, as it involves a motion with both negative and positive values for the velocity. 9 % managed to draw the appropriate graph *and* label the appropriate values on the time axis, while 2 % drew the correctly shaped *s-t* graph without noting any numerical values (see codes 1 & 2 respectively).

The reason table indicates a variety of reasons, and it was expected that the alert students would mention the crucial reversal in direction after 2 seconds. Reason codes 1, 2 and 3 (totalling 50 % of those who drew the correct graph and 22 % of all the recorded responses) did indicate some reference to a changing direction. Some stated this in terms of a motion that changes from reverse to forward, while others explained it in terms of a negative direction of motion gradually changing into a positive direction.

Reason code 2 represents a more detailed verbal explanation of the observed motion. Not surprisingly, these students (3 % of the total) invariably managed to draw a correct *s-t* graph. A typical example of such an explanation was given by a first year technikon student. It is stated here to illustrate how a real understanding of a graph in terms of a physical situation **can** assist in the drawing of the correct related graph:

A car moving in a negative direction slows down gradually until it stops after 2 seconds. It then turns around and accelerates in the opposite direction. This means that its displacement first decreases and then increases the shape of a parabola.

The most popular incorrect *s-t* graph is represented by graph code 3, representing 18 % of the respondents. Explanations for this graph were usually of the form *like v, like s*. This could also be classified as a *corresponding characteristics* type of error, as illustrated by comments such as:... *the observed velocity increases uniformly, therefore the displacement increases in a curved graph,* or ...*the initial negative velocity gives a negative* *displacement followed by an increasing velocity giving an increasing displacement-time graph* (reason code 7). The first comment suggests that these students treat the given *v-t* graph similarly to the one in question 11, i.e. ignoring the significance of the negative part of the velocity. The second comment addresses the negative velocity, but simply allocates a negative displacement to the observed negative velocity. These students believe that the *s-t* graph has to be drawn negatively up to 2 seconds and positively thereafter (see graph codes 3 and 7). Graph codes 4, 8 and 12 also contain elements of this type of interpretation and total 30 % of all respondents.

A surprisingly large number of students (10 % of the respondents) thought that the acceleration during the first 2 seconds was negative. They interpreted the sloping straight line *v-t* graph as representing a constant acceleration, but clearly believed it to be negative - just like the velocity. Few managed to translate this into an appropriate *s-t* graph, but graph codes 10 & 12 were given by a few students as graphs which fulfilled their criteria of an initial negative acceleration. These students usually had no trouble in accepting the positive acceleration *after* 2 seconds, indicating a total lack of focus on the *gradient* of the *v-t* graph to yield information about the sign and magnitude of the acceleration.

The view that the velocity "slows down" and then "speeds up" (see code 8) indicates a confused interpretation of the observed decreasing magnitude of the negative velocity followed by an increased magnitude of the positive velocity. Graphs 5 & 8 all contain points of inflection in an attempt to represent the "*slowing down - speeding up*" interpretation as a sudden change in the way that the gradient (of the *s-t* graph) changes. It appears that these students preferred to use an inflection point instead of a turning point, because they failed to notice the reversal in direction of the motion. The fact that 13 % of the respondents drew either graph 5 or 8 suggests that the view described here is more widespread among students than might have been believed before. One student even included a type of asymptote in his *s-t* graph, arguing that because *velocity* changes from negative to positive, displacement also has to change from negative to positive. He simply changed a *negative* displacement to its corresponding positive displacement during the 2nd second of the motion, whilst maintaining a parabolic shape for the graph. (See figure 4.4).

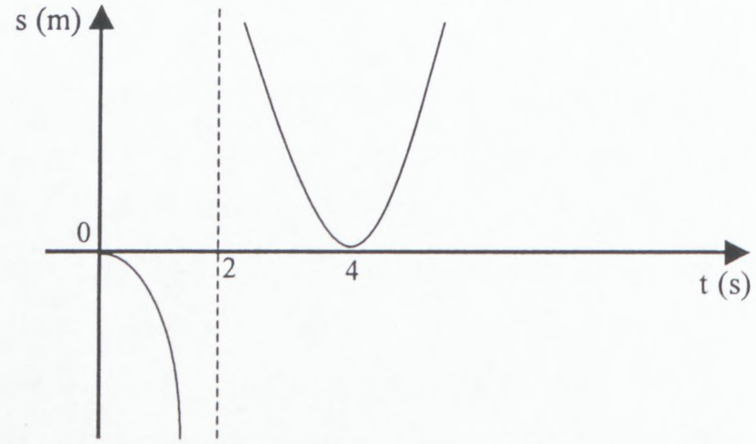


FIGURE 4.4

9 % of all candidates stated (correctly) that the object which is represented by the given *v-t* graph moves with positive acceleration. What is significant is that not a single one of these students was able to draw an acceptable *s-t* graph, because they did not interpret the transition from a negative velocity to a positive velocity correctly

Graph code 9 accounting for 5 % of the respondents, represents those that thought that the gradient of the *v-t* graph represents displacement, instead of the other way around. These students usually reasoned as follows: *gradient of v-t is constant and positive → displacement is constant and positive → graph of s-t is a positive straight line parallel to the t-axis.*

Very few students attempted a description of the gradient characteristics of the *s-t* graph in accordance with the height (negative or positive) of the given *v-t* graph. The closest any student came to such an interpretation was the two students who simply stated that the gradient of the *s-t* graph must change from negative to positive, because the height of the *v-t* graph changes from negative to positive.

No student mentioned that the area between the *v-t* graph and the t-axis can provide valuable information about the shape of the *s-t* graph. Students usually know how to obtain the displacement from a *v-t* graph in terms of areas when explicitly asked to do so.

However, when a graph transformation can be successfully completed by using the same area techniques, they do not seem to spontaneously implement this method.

The question of reference level was seldom explicitly mentioned; however, those who drew correctly shaped *s-t* graphs positioned them at different vertical orientations to that indicated by graph codes 1 & 2. Figure 4.5 illustrates the various correctly shaped *s-t* graphs that were recorded (including the percentage popularity of each graph).

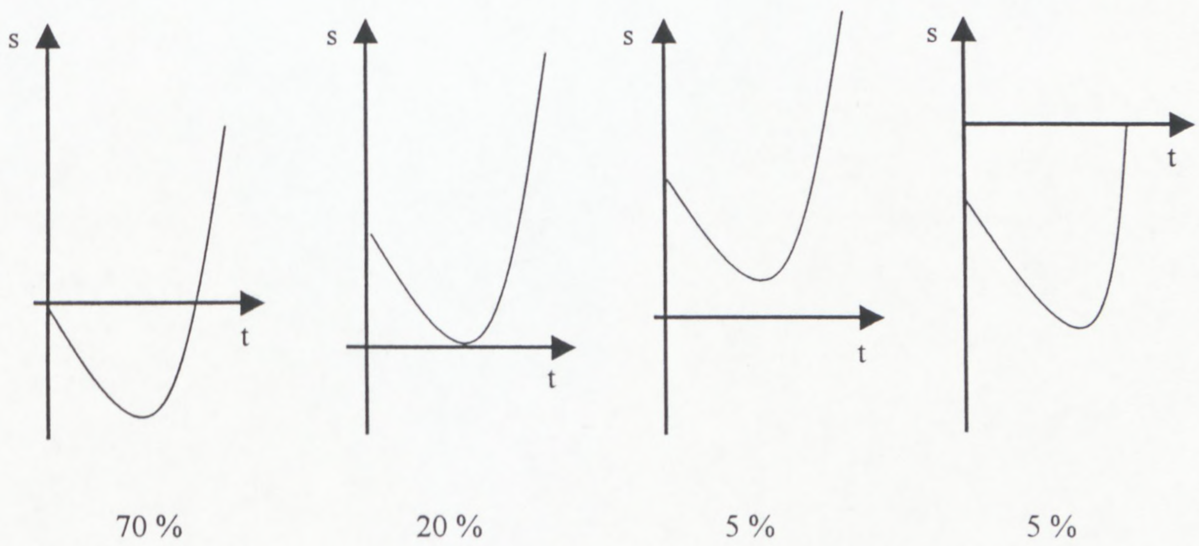


FIGURE 4.5

It is not surprising that a large majority (70 %) drew an *s-t* graph with a negative part corresponding to the negative section of the given *v-t* graph, while 25 % of the students who recorded correct graphs did not believe that the displacement should *ever* be negative. It must be mentioned here that three students believed that the given negative velocity cannot yield a displacement at all, resulting in a graph such as illustrated in figure 4.6 which shows s values for $t \geq 2$ only.

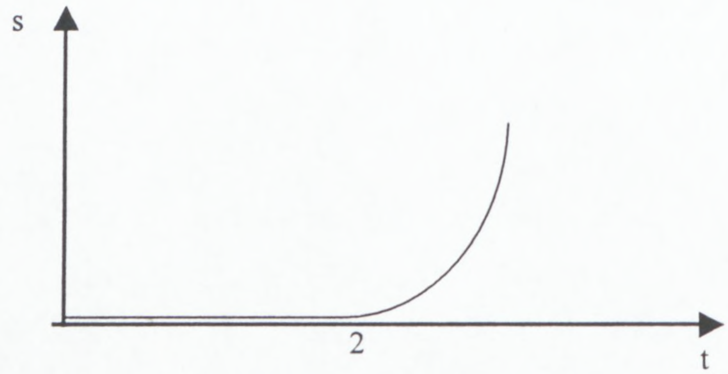
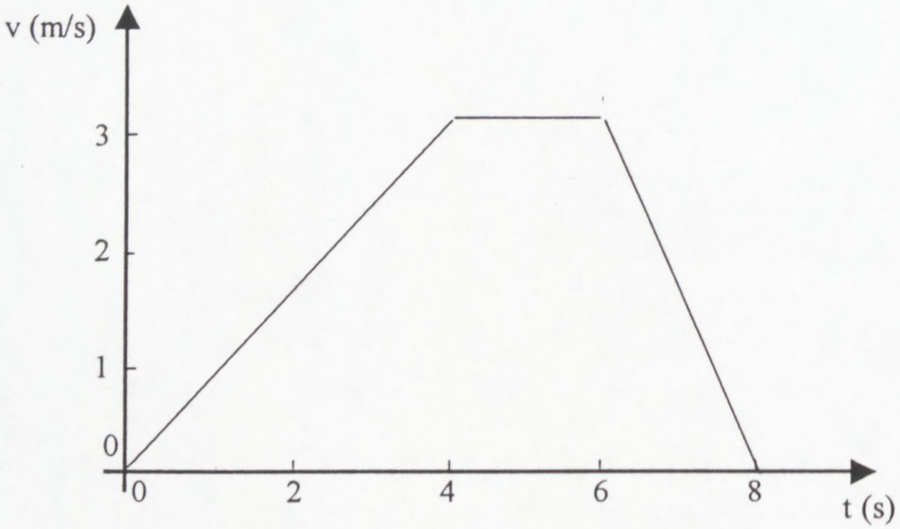


FIGURE 4.6

QUESTION 14

You are given the following *velocity-time* graph of a moving car. Draw the corresponding *displacement-time* graph on the given system of axes, taking s = 0 at t = 0. Indicate actual numerical values on the vertical axes corresponding to specific time values.



Describe briefly how you arrived at your answer.

(i) The purpose of the question

The purpose of this question was to investigate students' ability to transform a given *v-t* graph, representing a physical situation (the motion of a car), into the correct *s-t* graph.

This involved an examination of the given graph, which contained a *combination* of the types of motion described in questions 10, 11 and 12, and from this make appropriate deductions about the appearance of the require *s-t* graph.

Both the given as well as the required graph were of the *semi-accurate* variety, which meant that actual displacement values had to be calculated and indicated on the graph.

The *v-t* graph represents a car accelerating from rest for 4 seconds, after which it maintained a constant velocity of 3 m/s for 2 seconds and finally decreases its velocity uniformly, reaching a velocity of zero after a further 2 seconds. The various displacement values could be calculated either by using areas or equations of motion. In the case of areas the students had to calculate the area between the graph and the time axis separately for the three sections, by making use of the well-known formulas for the area of a triangle and a rectangle. Alternatively, they might have opted for a method involving kinematic equations using either of the equations $\Delta s = ut + ½at^2$ or $v^2 = u^2 + 2a\Delta s$ in order to calculate the change in displacement[1] of the car. In both instances, however, the acceleration had to be calculated first, either from the gradient of the graph or from a kinematic equation such as $v = u + at$. Candidates were also required to note that the acceleration was positive at first, then zero and finally negative during the last 2 seconds.

The graph responses for the three sections are reported separately in tables 4.19, 4.20 and 4.21. Table 4.22 summarises a number of <u>global</u> responses in terms of the general shape of students' *s-t* graphs, as well as the extent to which numerical <u>values</u> were included.

[1] In the discussion that follows the *change in displacement* will simply be called *displacement*

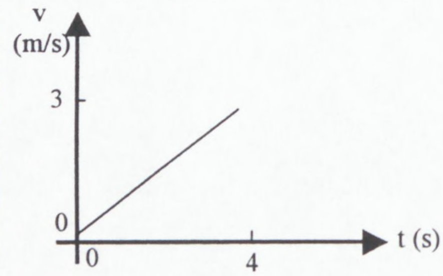
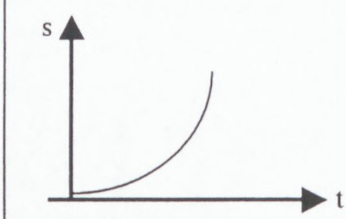
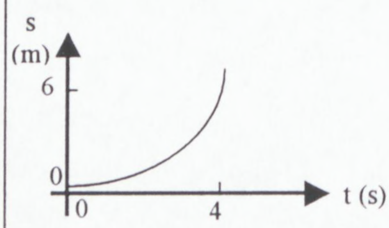
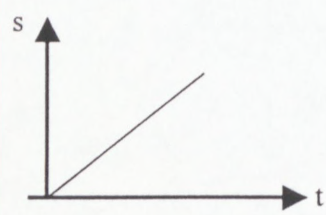
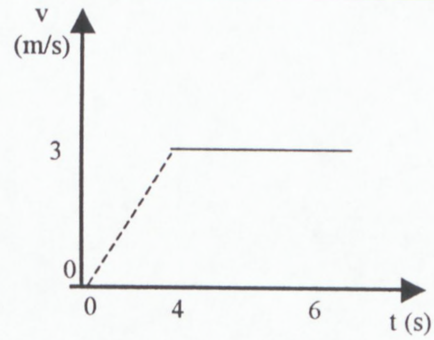
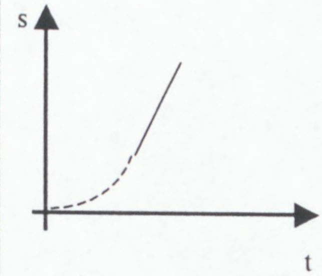
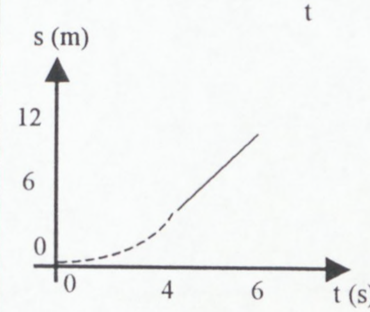
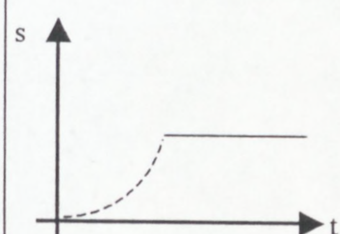
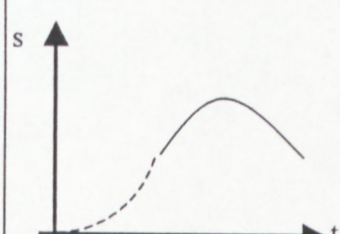
GIVEN *v-t* GRAPH SECTION	*s-t* GRAPH RESPONSES	ANSWER CODE	%
	 no values or wrong values	1*	42
	 correct values	2*	15
		3	27
		4	5
	OTHER	5	12

TABLE 4.19

***s-t* GRAPH RESPONSES FOR FIRST SECTION OF *v-t* GRAPH**

(expressed as a percentage of the number of students who attempted the question; n = 204)

GIVEN *v-t* GRAPH SECTION	*s-t* GRAPH RESPONSES	ANSWER CODE	%
		1*	40
		2*	18
		3	19
		4	6
		5	4

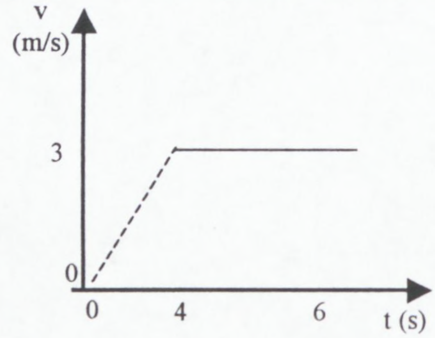
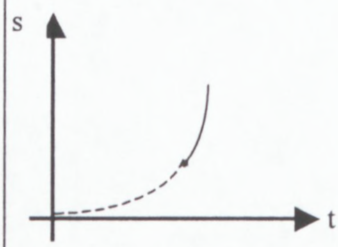
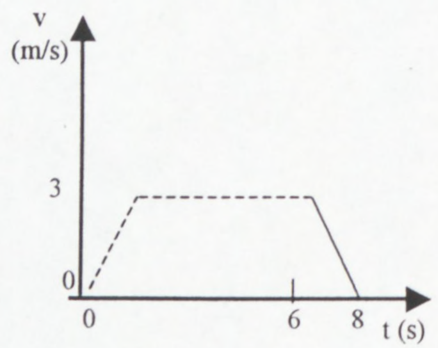
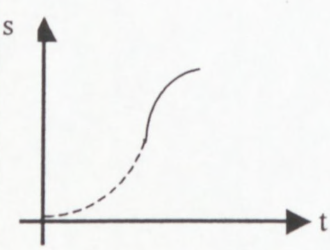
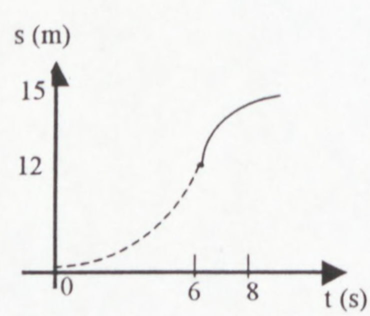
GIVEN *v-t* GRAPH SECTION	*s-t* GRAPH RESPONSES	ANSWER CODE	%
		6	3
	OTHER	7	10

TABLE 4.20

***s-t* GRAPH RESPONSES FOR MIDDLE SECTION OF *v-t* GRAPH**

(expressed as a percentage of the number of students who attempted the question; n = 202)

GIVEN *v-t* GRAPH SECTION	*s-t* GRAPH RESPONSES	ANSWER CODE	%
		1*	14
		2*	6
	complete *s-t* graph resembles given *v-t* graph (see also table 4.22)	3	38

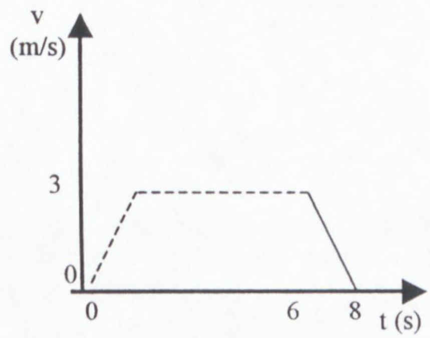
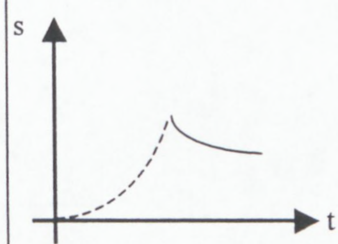
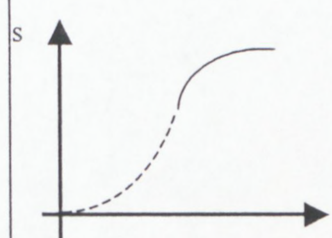
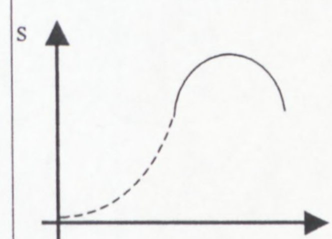
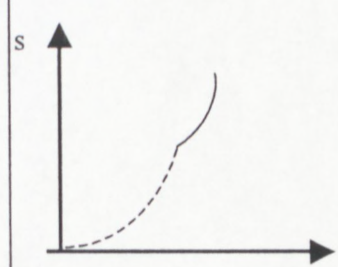
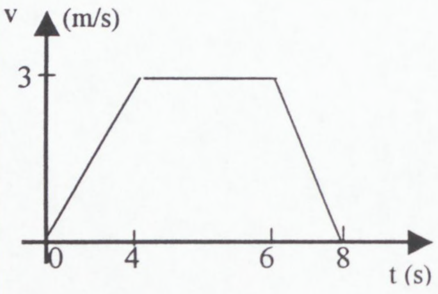
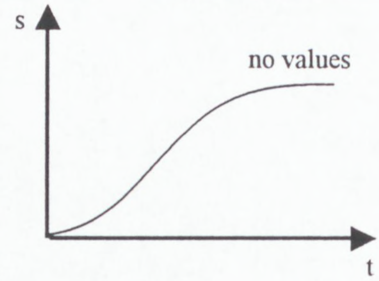
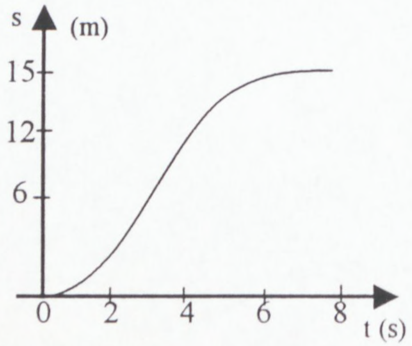
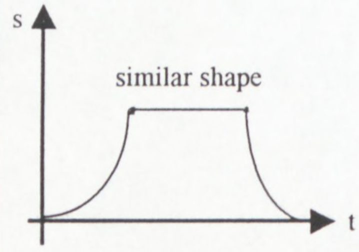
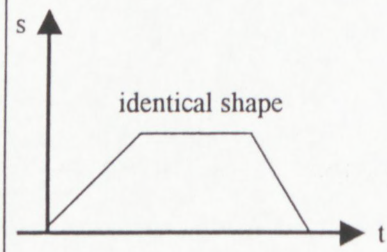
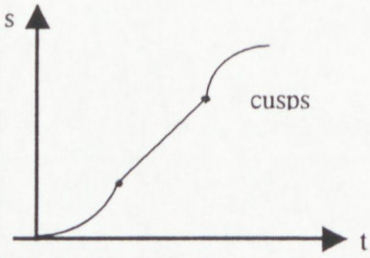
GIVEN *v-t* GRAPH SECTION	*s-t* GRAPH RESPONSES	ANSWER CODE	%
		4	9
		5	5
		6	5
		7	2
	OTHER	8	21

TABLE 4.21

***s-t* GRAPH RESPONSES FOR LAST SECTION OF *v-t* GRAPH**

(expressed as a percentage of the number of students who attempted the question; n = 199)

GIVEN *v-t* GRAPH SECTION	COMPLETE *s-t* GRAPH RESPONSES	CODE	%
		(a)*	14
		(b)*	6
		(c)	26
		(d)	12
		(e)	9

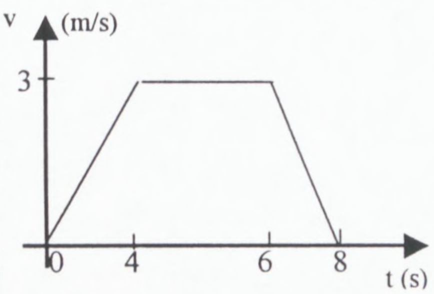
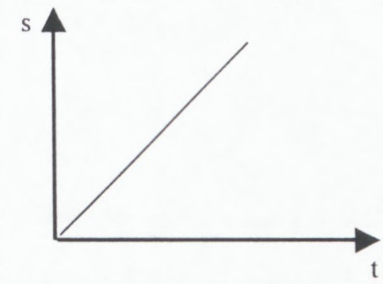
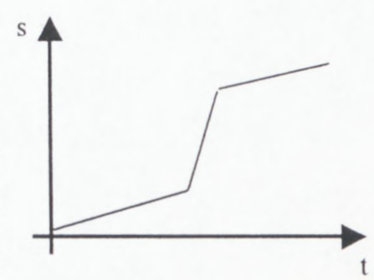
GIVEN *v-t* GRAPH SECTION	COMPLETE *s-t* GRAPH RESPONSES	CODE	%
		(f)	6
		(g)	4
	OTHER	(h)	23

TABLE 4.22 (a)
GLOBAL *s-t* GRAPH RESPONSES
(expressed as a percentage of the number of students who attempted the question; n = 199)

REASON / METHOD	ANSWER CODE	%
s-values correct according to area under *v-t* graph	1*	16
Explanation correct, but did not calculate s-values	2*	10
Explanation correct, but *incorrect* s-values	3*	6
Explains effect of changing velocity values on displacement values, with displacement values correctly calculated.	4*	6
s-values correct according to equations of motion	5*	5
Car accelerates, then moves with constant velocity, then decelerates	6	13
s-values *incorrectly* calculated with areas	7	9
s-values *incorrectly* calculated with equations of motion	8	7
Incorrect description of the effects of the *v-t* graph on the *s-t* graph	9	2
Incoherent	10	26

TABLE 4.22 (b)
POPULAR METHODS AND REASONS FOR THE GRAPH TRANSFORMATION OF QUESTION 14
(expressed as a percentage of the number of students who attempted the question; n = 172)

(ii) Discussion of results

An analysis of tables 4.19, 4.20 and 4.21 reveals that the two major areas of confusion were an inability to represent the last section of the motion on the *s-t* graph, together with a reluctance to calculate appropriate displacement values at specific time values.

The basic *shape* of the *s-t* graph for the first two sections was correctly drawn by 57 % of the respondents, while the same proportion succeeded with the transformation of the middle section with constant velocity. However, when *actual values* are required as well, these percentages drop to a disturbing 15 % and 18%, indicating a lack in ability of the sample group to successfully transform semi-accurate graphs.

Table 4.19 indicates that the two most commonly occurring incorrect *s-t* graphs for the first section involved either a straight line (imitating the given graph), suggested by 27 % of the respondents, or a curved graph with decreasing positive gradient (5 % of the students). This latter group seems to have remembered that the *s-t* graph had to be curved, but had no idea about the *way* it had to curve.

The middle section consisted of a motion at constant positive velocity. Of the 42 % who drew inappropriate *s-t* graphs the most popular shape was a straight horizontal section (table 4.20, code 3), again copying the middle point of the given *v-t* graph. Typical motivations for this were: *if velocity is constant then displacement cannot change* or ... *a constant velocity implies a zero displacement*. A number of other *s-t* graph responses were recorded in table 4.20, but very little was revealed by way of explanations. It is likely that many responses in this case were the result of pure guessing.

The last section of steadily decreasing velocity proved particularly troublesome to most students. Only 6 % managed to draw the correct *s-t* graph with appropriate values indicated on their graphs. This percentage rose to 20 % when only the *shape* was considered (see table 4.21, codes 1 and 2). Students were extremely reluctant to draw an **increasing** graph of *any* form to represent the **decreasing** portion of the *v-t* graph. This is in accordance with the observation reported earlier under question 12.

The erroneous decreasing *s-t* graphs suggested by certain students were sometimes curved (concave down or concave up) and sometimes straight. The result was that the students' complete graphs often resembled the given *v-t* graph. When this occurred, a code 3 was allocated and then included in table 4.22(a) (global responses).

For the global responses, illustrated in table 4.22(a), a graph which at least *resembled* the given graph globally was allocated code (c), whereas a graph which had the **exact** same shape as the *v-t* graph was given code (d). From the table, it immediately transpires that 38 % of all respondents *at least* imitated the given graph, while 12 % simply copied the given *v-t* graph.

Code (e) represents a correct graph, but includes *cusps* which indicated (incorrectly) sudden abrupt changes in the displacement, while code (f) indicates a straight line representing the displacement *vector* for the resultant displacement of the graphically represented motion.

(iii) Students' reasons and methods

An attempt at the use of *areas* proved to be the most popular method to gain information about the actual displacement of the car after 4, 6 and 8 seconds. Of the 25 % of the students who opted for this method, approximately two-thirds were successful (see reason codes 1 & 7). The given *v-t* graph described a motion with positive velocity throughout and started and ended on the time axis, so that the use of areas is in fact the obvious method for calculating displacements.

Most respondents, however, merely explained the motion which was represented by the *v-t* graph, with little or no reference to the effect of particular sections of the given graph on the shape of the required *s-t* graph (code 6). About half of these students managed to draw correctly shaped *s-t* graphs for at least the first two sections of the motion, which suggests that they understood the essential principle of the transformation without stating this explicitly as a "reason".

Reason code 1 required students to explain the correct shape of the *s-t* graph as well as give an indication of how specific changes in displacement values were calculated for the various time intervals. The fact that only 6 % of the candidates did this can possibly be ascribed to the open-endedness of the question, or alternatively some students might have actually believed that an indication of the calculations of s-values is **all** that was required.

12 % of the respondents attempted to use kinematic equations to calculate s-values, of which about half were successful. Not unexpectedly, the most common error made by these students was a failure to use the appropriate value and sign of the *acceleration* for the three separate stages of the motion.

The following common errors made by specific students, typifies certain specific misconceptions.

- During the last section of the *v-t* graph *the car decreases its speed, returning to its original position.* These students associated a decreasing positive velocity with a decreasing displacement from a chosen reference point and believed that the car ends where it started off, because the graph returned to the time axis.

- *If the velocity is constant from seconds 4 to 6, then the displacement has to be zero.* In this case, the respondents confused the effect of a constant velocity on displacement with its inverse: the effect of a constant displacement on velocity. These students usually drew an *s-t* graph which ran parallel to the t-axis from seconds 4 to 6, which seems to indicate that the displacement of zero is actually interpreted by them as a *change* in displacement of zero.

- A combination of the above is illustrated by students who correctly used the equation $s = \left(\frac{v+u}{2}\right) \cdot t$ to obtain the change in displacement values of 6 m, 6 m and 3 m for the three sections. They usually interpreted the middle value of 6 m to represent a constant displacement between 4 s and 6 s, because as one student put it: *it still equals the* original 6 m. The confusion is again between *displacement* and *change in displacement.*

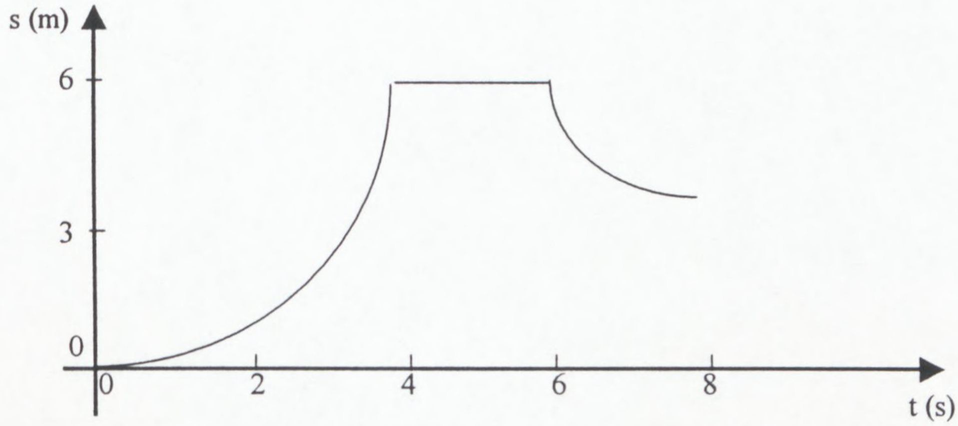


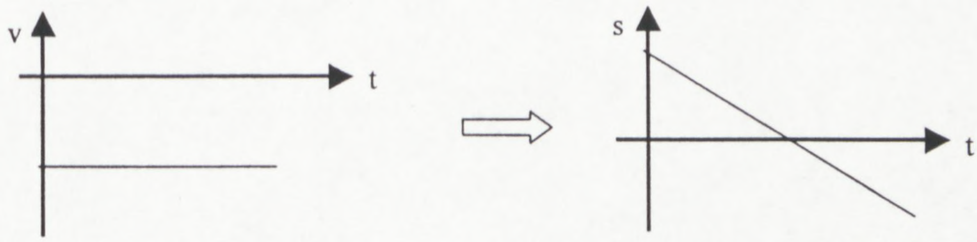
FIGURE 4.7

The last section is usually curved (but decreasing) in an attempt to represent the constantly decreasing velocity over the last 2 seconds (see figure 4.7).

- 6 candidates believed that the symbol *s* on the vertical axis of the required graph represents *speed.* These students proceeded to draw an identical graph to the given one, with one student motivating his answer in terms of the fact that in the absence of a reversal in direction, the speed of an object will always be equal to its velocity.

QUESTION 15

A pupil was asked to draw a *displacement-time* graph corresponding to a given *velocity-time* graph as in figure (a) below. His answer is illustrated in figure (b).



(a) (b)

The student's answer is:

Completely correct ☐

Partly correct ☐

Completely wrong ☐

STUDENTS' RESPONSE

STATEMENT IS:	ANSWER CODE	%
Correct	1*	31
Partly correct	2	41
Wrong	3	28

TABLE 4.23
(expressed as a percentage of the number of students who attempted the question n = 239)

VALIDITY OF STATEMENT	REASON	ANSWER CODE	%
Correct	Constant negative velocity gives constant negative gradient of *s-t* graph	1*	5
	Negative velocity gives constant decreasing displacement	2*	4
	Mentions arbitrary nature of initial displacement	3*	3
	Negative velocity gives negative displacement	4	8
	Velocity is constant and negative	5	3
	Object is moving backwards/reversing/ opposite direction	6	3

VALIDITY OF STATEMENT	REASON	ANSWER CODE	%
	Velocity is negative, therefore displacement should increase in a negative direction - draws graph as follows: 	7	23
Partly correct or wrong	Displacement should *increase* with constant velocity	8	8
	s-t graph shows *decreasing speed* which is not in agreement with the given *constant* velocity	9	7
	Velocity is *constant* and negative, so displacement should increase from a *negative* starting point	10	5
	s-t graph *also* starts negative, draws *s-t* graph as follows 	11	5
	Displacement is never negative (gives definition of displacement)	12	4
Partly correct or wrong	Pupils graph shows object moving towards observer, then turns around, but given *v-t* graph shows object moving *away* from reference point.	13	3
	The given graph illustrates a constant negative **acceleration**	14	3
	Other/incoherent	15	16

TABLE 4.24

(expressed as a percentage of the number of students who attempted the question n = 160)

The purpose of this question was to find out whether the test group could associate the given **constant** velocity with a *slanting* straight line *s-t* graph and the fact that the velocity is **negative** with a *decreasing s-t* graph. The initial s-value (at t = 0) could be chosen anywhere on the s-axis depending on the reference point. This implies that the fictitious pupil's response in the question had to be classified as completely correct.

The fact that only 31 % of the respondents stated that the pupil's graph is totally correct, has to be seen in the light of the results of question 10, where it was reported that 79 % of the respondents correctly transformed a *v-t* graph with constant **positive** velocity into the appropriate *s-t* graph. The question can therefore be asked: *What is it about this negative v-t graph which resulted in such a poor response success rate, compared with the positive v-t graph transformation*? An analysis of respondents' reasons reveals that although most students accepted the negative gradient of the *s-t* graph, many disagreed with an *s-t* graph that represents both positive and negative displacement values. Of those that agreed with the *s-t* graph, only 12 % (reason codes 1, 2 and 3) could offer an acceptable explanation for this graph. Reason code 4 usually accompanied a "correct" response to the pupil's graph, which indicates that these students might have meant *negative gradient* (of *s-t* graph) when they stated that a *negative displacement* results from a *negative* velocity.

The most popular alternative that was offered to the "pupil's graph" was one which displays the same negative gradient, but starts at the origin, thus covering negative *s*-values only (reason code 7), or one that starts at a negative s-value and continues with negative gradient. These two alternatives were stated by a total of 28 % of the respondents. Essentially these students seem to saying: ***If the velocity is negative then the displacement may not be positive.***

Reason codes 8 & 9 both emphasise the fact that the velocity is **constant** and as such underplay the importance of the negative sign. Code 8 represents the view that a constant velocity results in an *increasing* displacement, while code 9 attempts to interpret the *s-t* graph as a *speed-time* graph, which according to these students may *not* be decreasing. Code 10 is similar to code 8, except that it indicates a displacement which *increases* from a negative value at t = 0 (again in keeping with the observed negative velocity).

A limited number of respondents, represented by code 6, associated the negative *v-t* graph with a movement *in an opposite direction* which means that, to these students, a reversal in direction of a motion **must** be accompanied by a change in sign *from positive to negative*. This view also came out clearly in the responses to question 2 and can be summarised as follows: *students often believe that a **"forward"** moving body may not have a negative velocity.*

Other student conceptions that came to light included a belief that according to the *definition* of displacement it may not be negative (code 12) and the view that the pupil's *s-t* graph represents a reversal in direction which is not illustrated by the given *v-t* graph (see code 13).

Interviews with certain students revealed that they did not differentiate between the terms "partly correct" and "wrong". It appears that most students who replied "partly correct", implied that there where some fundamental mistakes in the fictitious pupil's response. This is confirmed by the fact that no noticeable differences in students' explanations for "partly correct" and "wrong" were noted in the written responses.

4.5 EFFECT OF RACE, SCIENCE MARKS AND ACADEMIC LEVEL

4.5.1 The nature of the sample and test items

In this paragraph a brief analysis of possible differences between specific groups in the success-rate for answering certain questions in the main questionnaire will be given. The three categories for which possible difference in responses will be investigated are:

- **race:** students whose home language is a black language vs those who speak a non-black language.

- **science marks:** students who in their most recent exams achieved above 80 % for science, between 50 % and 80 %, and less than 50 %.

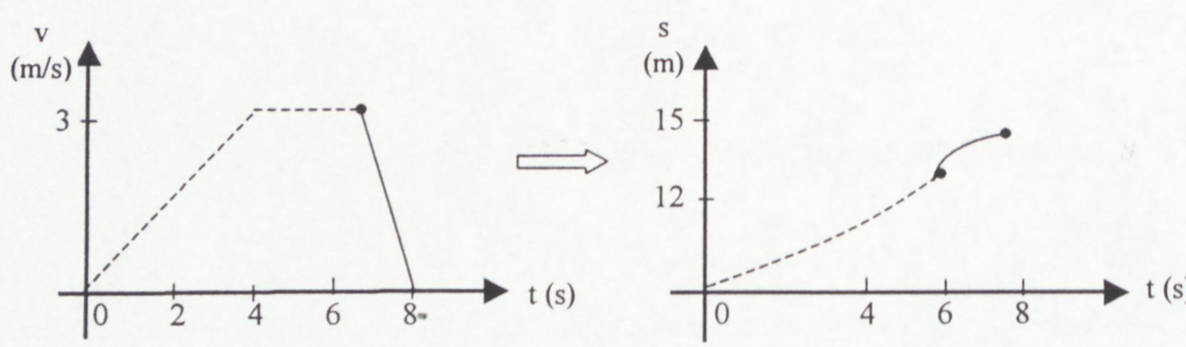
- **academic level:** the four academic levels are: Grade 11; Grade 12; first year technikon and second year technikon

A sample of three questions was chosen for which cross-tabulations were made to obtain a breakdown of the responses for the various groups described above. The questions were:

Question 1 Explain the concepts of *displacement*, *velocity* and *acceleration* to a friend with little knowledge of science.

Question 11 Make the following appropriate graph transformation:

Question 14 Transform the *last* section of the semi-accurate *v-t* graph into the correct *s-t* graph.



4.5.2 Black students vs other students

HOME LANGUAGE	QUESTION 1			QUESTION 11	QUESTION 14
	s (%)	v (%)	a (%)	%	%
Black	50	31	36	9	3
Non-black	80	30	44	60	25
All students	77	30	41	43	22

TABLE 4.25
PERFORMANCE OF BLACK STUDENTS vs OTHER STUDENTS IN QUESTIONS 1, 11 AND 14
(expressed as percentages)

Table 4.25 reveals that students whose home language is black fared significantly worse at graph-graph transformations than their "white" counterparts. However, when it comes to a verbal description of certain kinematic quantities this difference was not nearly of the same magnitude.

Question 11 and 14 require a certain level of abstract thinking and spatial orientation in so far as the required *s-t* graphs are alternative versions of a given symbolic representation of certain real motions. In order to succeed with this, many examples of such transformations have to be practised and studied. Question 1 does not require as much practice because it merely requires of the student to explain certain basic, everyday concepts in kinematics that are regularly used in the study of this branch of physics.

Black students therefore appear to do far worse than their white counterparts at those questions in kinematics that require *much practice* and *good teaching* in order to be successful at answering them. The present crisis in black education is well-known and the absence of effective and regular instruction by teachers and the resulting lack of motivation of students to study and practise their physics, could be some of the reasons for this discrepancy. Other possible reasons include problems with language, a poor grounding in science and a poor perception of representations.

4.5.3 Science marks

SCIENCE MARK (%)	QUESTION 1						QUESTION 11		QUESTION 14	
	s %		v %		a %		%	%	%	%
0 - 29	50		0		0		50		0	
30 - 39	60	63	8	24	33	37	30	40	25	20
40 - 49	64		27		41		41		19	
50 - 59	69		31		40		39		12	
60 - 69	88	76	22	28	39	41	63	46	28	22
70 - 79	84		29		48		64		35	
80 - 89	80		30		65		53		23	
		82		36		67		57		24
90 - 100	100		100		100		100		50	
all students	77		30		41		43		22	

TABLE 4.26
STUDENT PERFORMANCE IN QUESTIONS 1, 11 AND 14 IN CONTEXT OF PROVEN SCIENCE ABILITY
(expressed as percentages)

Table 4.26 summarises the relative performance of so-called "weak students, average students and scientifically gifted students in the descriptive type question (Q1), as well as the more abstract type (Q11 and Q14). For easier comparison, the performances have been grouped into three categories of proven science achievement, namely a "bottom" group that obtained below 50 % for science, the "middle" group (achieving 50 - 79 %) and finally the "gifted" group (achieving 80 % and above).

The general trend is not unexpected. Students who do well in science examinations were more successful in answering question 1, 11 & 14 than their counterparts who performed poorly in examinations. This pattern holds for both the descriptive type of explanation of question 1 as well as the more abstract graph transformations of questions 11 and 14.

An interesting observation is that the difference in success rate between the "bottom", "middle" and "gifted" group is not as large as might be expected. For example 24 % of students in the "bottom" group could give a reasonable explanation about the meaning of velocity - a proportion which only improved to 36 % for the "gifted" group. In the case of the semi-accurate graph transformation required in question 14 the differences are even less significant, with an improvement of only 4 % from the "bottom" to the "gifted" group. This indicates that the following hypothesis might be valid:

Compared to poor achievers, top achievers have a superior mastery of kinematics; however, this superiority is not in proportion to students' proven overall achievement in science.

4.5.4 Educational level

EDUCATIONAL LEVEL	QUESTION 1			QUESTION 11	QUESTION 14
	s (%)	v (%)	a (%)		
Grade 11	70	17	37	50	25
Grade 12	64	38	40	18	11
Tech 1	94	30	48	63	25
Tech 2	67	40	29	42	8
All students	77	30	41	43	22

TABLE 4.27
STUDENT PERFORMANCE IN QUESTION 1, 11 & 14 IN CONTEXT OF EDUCATIONAL LEVEL (expressed as percentages)

Table 4.27 indicates that, generally speaking, there is no correlation between a student's academic level and his ability to explain the kinematic concepts of *displacement*, *velocity* and *acceleration*. Kinematics is not taught explicitly in Grade 12, nor is it included in the syllabuses studied by the sample of "Tech 2" students which participated in the survey and it appears therefore that, once understood, learners do not easily forget the meaning of the basic kinematic quantities.

Two exceptions to this observations appear to be the following: (i) the relatively small proportion of Grade 11's who managed to explain what is meant by velocity, and (ii) the large number of first year technikon students who *could* explain the term *displacement*. The first is difficult to explain and could be the result of random variation of the test results, while the second observation might be due to the fact that the idea of a *displacement* was explicitly covered in the technikon physics 1 course. It often receives more attention than other kinematic quantities, because of its importance for the study of *work, energy, levers* and other related topics in physics.

Of special interest is the observation that, in the case of v-$t \rightarrow s$-t graph transformations, the Grade 11 and Technikon 1 group fared considerably better than the Grade 12 and Technikon 2 groups. The most likely explanation for this is that the first mentioned groups have covered the work on kinematic graphs at school and at technikon before and very close to the time of this investigation. The Grade 12's and Technikon 2 groups, however, have not received direct instruction in kinematics for at least one year before the questionnaire was answered.

This seems to indicate that students tend to easily "forget" the basic principles of graph transformations over the course of 12 months. Conversely: Students who have been exposed to kinematics just prior to this investigation, achieved a higher proportion of correct answers because the material apparently was familiar and fresh to them.

4.6 CONCLUSION

This chapter describes the results of an empirical investigation into typical student errors with regard to the graphical representation of motion. The literature contains very little reference to student misconceptions with regard to the transformation of one kinematic graph into another related one. For this reason the present investigation focuses on this particular aspect and identifies some serious commonly occurring errors.

These errors and commonly occurring misconceptions can be described in terms of a number of *categories*. The next chapter will attempt to classify the identified student views on kinematic graphs and their transformations and relate these to a number of possible root causes. A teaching strategy, that will have as its aim at least the *partial* elimination of some of these widespread student misconceptions, is also suggested.

CHAPTER 5

CATEGORIES OF COMMON MISCONCEPTIONS ABOUT KINEMATIC GRAPHS, THEIR CAUSES AND HOW THEY CAN BE ADDRESSED

5.1 INTRODUCTION

This chapter provides a more structured account of the most commonly occurring misconceptions, non-conceptions and mistakes made by learners. The literature study reported on in Chapter 3, the results of the own empirical investigation reported on in Chapter 4, as well as the outcome of a number of clinical interviews will be used as a basis for this classification.

Where possible the origins or causes of certain fundamental misconceptions will also be investigated and described. It has to be borne in mind that it is often difficult to distinguish between the actual error and its cause. For example, a student might ignore the reference point in a problem where this point makes up an integral part of the question, or, alternatively, he may not manage a particular graph transformation because he ignores the crucial role of the reference point.

In the first instance the non-conception of a reference point is the actual error, while in the second case, it is the cause of the inability to make the appropriate transformation.

It will be shown that many of the difficulties that students experience with kinematic graphs can be traced back to the syllabus, and more particularly, to how it is taught. Aspects such as the role of language, the current state of the intellectual development of the students and the extent to which they are encouraged to integrate mathematics graphs with science graphs, are all important contributing factors to their understanding of motion graphs.

Finally, in an attempt to address these misconceptions and their causes, a number of untested instructional strategies will be proposed.

5.2 CLASSIFICATION OF COMMON ERRORS AND THEIR CAUSES (OTHER THAN TRANSFORMATION DIFFICULTIES)

An attempt will now be made at classifying the most frequently occurring misconceptions with respect to kinematics and the graphical representation of kinematic events. The information gained from the empirical investigation described in Chapter 4 and the results of the clinical interviews, together with the literature study reported on in Chapter 3, can be used to identify five broad categories of potential problem areas. These are...

- A poor perception of reference point and sign convention.

- Not differentiating adequately between certain kinematic quantities.

- An unwillingness to use gradients and areas to interpret graphs.

- A tendency to conserve the features of a given graph when having to construct a related graph.

- Viewing graphs as pictures of the physical problem scene.

These categories should not be viewed in isolation; rather it must be remembered, for example, that a poor grasp of the meaning of sign convention could encourage students to view graphs as pictures. The present investigation reveals serious shortcomings in students' understanding of *negative* kinematic quantities and their graphical interpretation and it will be shown that these problems are likewise attributable to problems with the understanding of sign convention. Another example of a link between categories is found in the fact that confusion between *velocity* and *acceleration* can often be related to a poor understanding of gradient as "rate-of-change".

Table 5.1 indicates which misconceptions were tested for by the various questions of the main questionnaire. Graph-as-picture errors were not tested by the main questionnaire; instead these were investigated by clinical interviews conducted with individual students. The remaining categories of student errors, while being dealt with explicitly by the questionnaire, were also further investigated by interviews with students

Question	Conservation	Conventions	Gradient/Area	Kin. quantities
1				*
2		*		
3		*		*
4		*		*
5			*	
6		*		*
7			*	
8	*		*	
9			*	
10	*		*	
11	*		*	
12	*			
13	*	*	*	
14	*			
15	*	*	*	

TABLE 5.1
Misconceptions tested for by the various questions of the main questionnaire

In view of the fact that graph-graph transformations formed a significant part of the main investigation, misconceptions relating to these transformations are described separately from other graphing problems.

Table 5.2 provides some idea of the extent to which the main questionnaire tested the various main classes of misconceptions. All questions probing for a specific error or alternative conception were multiplied by the number of respondents who attempted to answer it. The total number of opportunities taken (i.e. to hold the particular misconception) is compared to the potential number of opportunities offered and this ratio is expressed as a percentage. It should be emphasised that most of the graph transformations did not *explicitly* test the respondent's comprehension of gradient and area as most of the transformations could have been successfully achieved by carefully analysing the relevant problem scene. In the case of gradient and area, table 5.2 therefore refers to the proportion of respondents who either *chose* not to use gradients or areas or to those who used it *unsuccessfully*.

	conservation of features	**conventions**	**gradient**	**area**	**kinematic quantities**		
					s	**v**	**a**
Opportunities for misunderstanding	1463	1742	1576	172	526	265	1037
opportunities taken	380	952	1155	134	230	185	675
% opportunities taken	26,0 %	54,6 %	73,2 %	77,9 %	43,7 %	69,8 %	65,1 %

TABLE 5.2
Proportion of respondents who held misconceptions offered by the main questionnaire

Table 5.2 underlines the reluctance shown by learners to use gradient and area to solve kinematic graphing problems, especially when they have to do this spontaneously, i.e. in cases where instructors give no hint about the method that might be used. The high incidence of the lack of understanding of the real meaning of velocity and acceleration can be ascribed to the fact that students tended to confuse these quantities with each other, as well as to a serious misunderstanding of the meaning and significance of *negative* velocity and acceleration.

The various categories of student misconceptions with respect to kinematic graphs are now described in more detail.

5.2.1 Problems with reference point and sign convention

5.2.1.1 General misconceptions

Table 5.2 shows that just more than half the students responding to the main questionnaire demonstrated a lack of understanding of the role of reference point and/or sign convention in problems where it is vital to consider these conventions for a proper comprehension of the problem scene that is being represented. When students are expected to define this for themselves, the problem is particularly severe because they frequently describe and represent a physical situation graphically without *any* reference to sign conventions or reference point. One study (Aguirre & Erickson 1984:440) reveals that students tended to use several reference points for different locations of a moving body. In other words, they would not easily accept a *common* reference point to describe several locations in the same setting.

In the case of velocity and acceleration the earth is usually naturally accepted as the frame of reference, although this is seldom stated by textbook writers. However, in the case of displacement the reference point is not always self-evident — hence the need for it to be stated clearly and unambiguously.

During the clinical interviews students were asked to comment on the displacement of boy B_1 at the particular moment during a 100 m race as illustrated in figure 5.1.

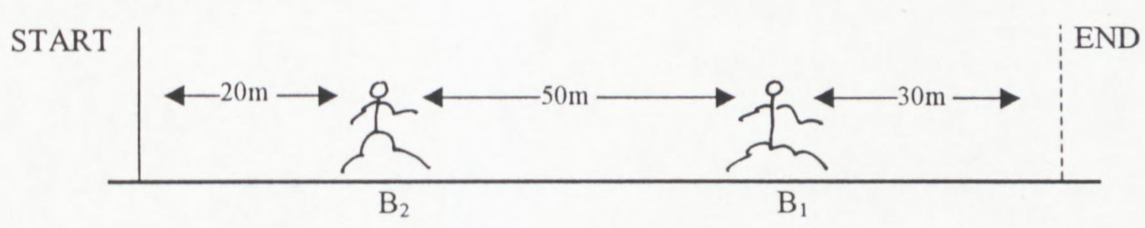


FIGURE 5.1

Most students gave an answer of 70 m, simply assuming, but not *stating* that the start of the race has been taken as reference point. When asked about the reference point they were usually surprised, stating that it was *obviously from where they started to run*. When asked about B_1's displacement with respect to the finishing post, most students ignored the need for using a sign convention, stating that it was 30 m and added the negative sign only *after* they were prompted to do so by the interviewer in terms of a question such as: *Are you **sure** that it must still be **positive** 30 m, just like the other displacements?*

Some students believed that *moving* bodies cannot possess displacement as illustrated by the following extract of an interview:

I: *So what is the displacement of B_1?*

S: *But he is running, so how can he have a displacement?*

I: *Does this mean that moving bodies cannot ever be displaced?*

S: *Sure they can, but it (the displacement) changes all the time, so you can't get a value for it.*

The difficulty that learners experience with the recognition and interpretation of negative kinematic quantities can, at least partially, be attributed to a poor understanding of sign convention. They seldom verbalise their interpretation of this important aspect of kinematics in the way that scientists would, namely that for rectilinear motion...

***Velocity** will have a negative value if the direction of motion is opposite to that direction which was arbitrarily chosen as positive.*

***Acceleration** will be indicated by a negative quantity if an object speeds up while moving in a negative direction or slows down while moving in a positive direction.*

***Displacement** is regarded as negative if the direction from an arbitrary chosen reference point to the present position of the object is opposite to the direction, which was originally chosen as positive.*

The fact that learners seldom view the kinematic vector quantities in this light is illustrated by the following typical comments made by different students:

- *If something slows down, its velocity is negative.*

- *Velocity can be negative, but displacement cannot actually be negative.*

- *I have not encountered a problem involving negative displacements.*

The idea of a negative velocity being associated with the slowing down of a body, as implied by the first student above, is not uncommon and is also reported by Goldberg & Anderson: (1989:258).

> *Another possible source of difficulty may be the belief that a negative quantity somehow implies a "lesser amount" of that quantity.*

The existence of negative velocities is thus readily acknowledged by the majority of students (for example 65 % of the respondents agreed in question 2 that cars *can* travel with negative velocities). However, the *reason* for this is seldom associated with a sign convention, but rather with factors such as "a decreasing speed" or a "velocity in the opposite direction". In fact 26 % of the students participating in the main investigation believed that an object travelling "backwards" or "reversing" **had** to have a negative velocity.

Another commonly occurring misconception is contained in the argument that if a *displacement* is negative, the *velocity* of the body in that position must **also** be negative. These students believe that the sign of the displacement vector determines the sign of the velocity vector, because of the definition of velocity as displacement over time (instead of change in displacement over time). In the words of a Grade 11 student: *Because velocity is displacement over time and displacement can't be negative, it means that velocity can't be negative.* A refusal to accept the vector nature of displacement thus lead this student to a non-acceptance of the existence of negative velocity values as well.

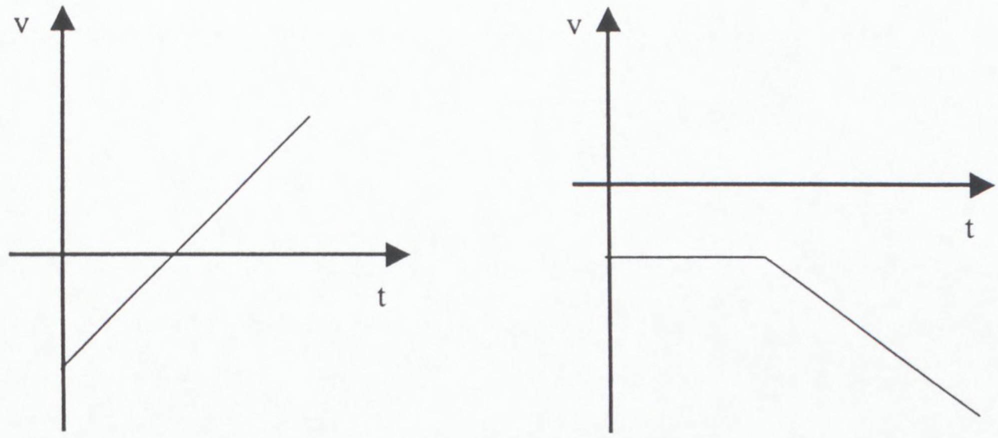
Goldberg and Anderson point out that the day-to-day usage of the term speed further confuses the issue for many students:

> *In everyday life students are familiar with the magnitude of velocity, namely speed. Although they may recognise, through their course work in physics, the directional aspects of velocity, everyday usage may cause them to think of the magnitude and direction as completely separate attributes.* (1989:258)

This view is confirmed in this study when it was found that students frequently stated that a car cannot travel at a negative velocity, but acknowledged that its direction (as a separate entity) can be negative. In this way the *magnitude* of the velocity vector was completely separated from its directional attributes. To these students an answer such as -60 $m.s^{-1}$ is not acceptable. Instead they would insist that it is expressed as *60* $m.s^{-1}$ *in a negative direction.*

5.2.1.2 Graphs involving negative kinematic quantities

Students' unwillingness to define and/or use an appropriate sign convention for rectilinear motion is often the cause of serious misunderstanding of the directional information that is contained in a motion graph. When confronted with the *v-t* graphs of figure 5.2, they had great difficulty in combining the direction of motion with statements about whether the velocity was increasing or decreasing. Further confusion between changes in *velocity* and changes in *speed* were also very evident.



(a) (b)

FIGURE 5.2

A matric student of above average proven academic ability in science was asked to describe the motion represented by figure 5.2 (a). The following is an extract of the transcription of the interview:

I: *Can you describe the motion of the object represented by this graph?*

S: (after a long pause) *The velocity is definitely increasing because of the positive slope... but, wait a minute, it gets closer to zero which means that it is decreasing.*

I: *So, what is your final conclusion?*

S: *it is decreasing to zero and then increasing*

I: *What is "it"?*

S: *Well, the velocity because that is what is being represented on the axis* (points to the v-axis)

When students were asked whether the graph of figure 5.2 (b) represented a real motion, they usually had no hesitation in agreeing that such a motion is indeed possible, but were very unsure about the behaviour of the velocity during the latter part of the motion. Comments that occurred fairly frequently included: *it is moving with decreasing speed in a negative direction* and ... *the velocity is becoming more and more negative as time increases.*

These students, by failing to interpret the negative velocity values correctly, were thus not able to connect the given graph to the appropriate physical motion of an object.

The results of the questionnaire (typically questions 13 & 15) confirm the observation that students refuse to differentiate between *velocity* and *magnitude of velocity* when dealing with graphs of negative velocities. **No** student that participated in the main investigation managed to explain a positively sloped line representing negative velocity values as an *increasing velocity* but *decreasing magnitude* of velocity.

Goldberg & Anderson (1989) reported similar problems with students tending to interpret the positively-sloped portions of graphs such as those depicted in figure 5.3 (a) & (b) as representing an object "speeding up".

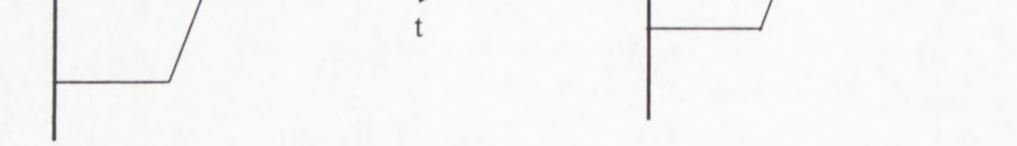


FIGURE 5.3

They classified this as a "displaced-curve" error because the students were assumed to interpret the graph as if it were entirely above the time axis and were thinking only of changes in *speed*. Graphs of displacement vs time, where at least part of the graph is drawn for negative displacement values, caused an equal amount of confusion.

Students who were asked to describe the motion depicted by the *s-t* graph of figure 5.4 often could not give an explanation that adequately accounts for the linearly increasing displacement values.

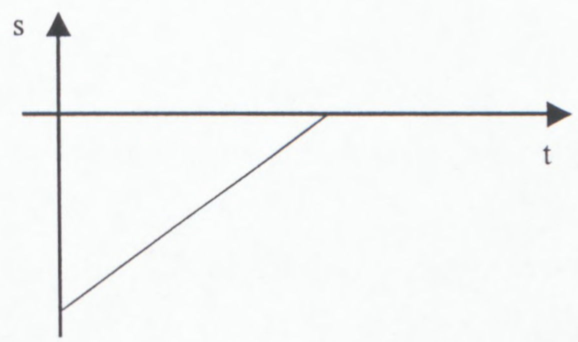


FIGURE 5.4

Only 33 % of the respondents in the case of the pilot study recognised this as a motion with constant velocity, whilst a further 33 % had no idea what this graph represented, often explaining it as *a motion with increasing speed*. The respondents in the main questionnaire tended to describe the motion in terms of a *decreasing displacement* or an object *returning to its reference point*, with a mere 14 % referring to its constant velocity (half of whom correctly mentioned that this constant velocity was *positive*).

5.2.1.3 Reversal in direction

Problems that students experience with graphs depicting a reversal in direction can also be related to their problems with graphs of *negative* kinematic quantities. Students that participated in the main questionnaire, as well as those interviewed, seldom associated a *v-t* graph that crosses the time axis with a reversal in direction.

The fact that a crossing of the time axis in the case of an *s-t* graph almost **never** implies a change in direction seems to complicate the situation further for most students. Not a single student interviewed could give the following acceptable summary of how the various graphs illustrate a change in direction of a motion.

i) ***s-t* graph** : the change in direction occurs at the maximum or minimum point on a parabolic curve

ii) ***v-t* graph** : the change in direction occurs at the point where the graph crosses the time axis

iii) ***a-t* graph** : a change in direction of a motion *cannot* be detected on an *a-t* graph.

It was reported earlier (see p. 336) that the reason for students' inability to associate the appropriate graphical features with "change in direction" can be largely attributed to their poor perception of sign convention.

A number of students were asked to draw a *v-t* graph of a ball which is projected vertically and then caught as it comes down. The most popular graph, depicted in figure 5.5, was that which represented the reversal in direction by a **sudden change in the sign of the gradient** of the graph.

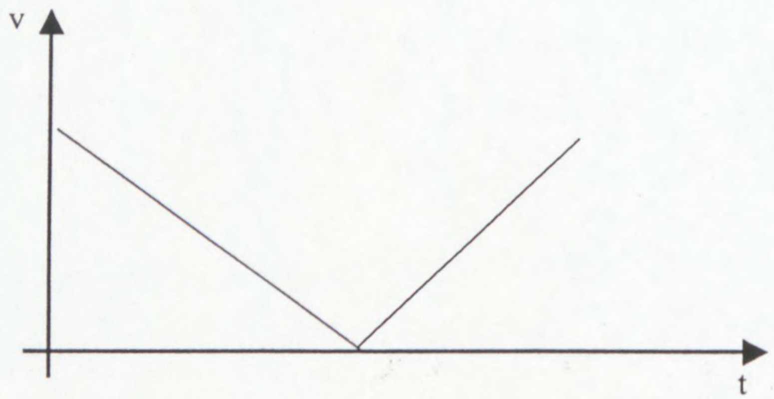


FIGURE 5.5

These students simply ignored the fact that the directional aspect of velocity is strongly linked to a sign convention and simply drew a graph of *speed vs time*, thus avoiding negative values of the velocity altogether. The research undertaken by McDermott *et al.* (1983, 1987) into similar student difficulties yielded similar results to those reported here and were fully described earlier (see p. 227)

Students' often held views about the directional characteristics of displacement and velocity can now be summarised as follows:

(1) ***displacement***

- reference points are irrelevant when determining the sign associated with a displacement.

- a non-acceptance of the idea of an *instantaneous* displacement for moving bodies.

(2) ***velocity***

- *negative* velocities are associated with objects that are *slowing down.*

- a *reversal in direction* is a necessary and sufficient condition for a velocity to be *negative.*

- a *negative displacement* leads to a *negative velocity*, because velocity is defined in terms of displacement.

- the *directional* characteristics of a velocity are often seen as a *separate* aspect to its *magnitude.*

- confusion between *speed* and *velocity* when considering graphs representing negative velocities.

- graphically, a reversal in direction of a motion is frequently represented by a change in the sign of the gradient of a *v-t* graph.

5.2.1.4 Starting graphs at the origin

Students typically tend to start *s-t* and *v-t* graphs at the origin, irrespective of the described or observed physical situation. In the case of an *s-t* graph the reason is often due to a lack of understanding of the role of reference point, as illustrated by the student who bluntly observed that: *the displacement at the beginning of a motion has to be zero.*

A student achieving below average marks in science was asked to draw an *s-t* graph of an object projected vertically upwards by a boy standing on a bridge. The student spontaneously drew the graph depicted in figure 5.6.

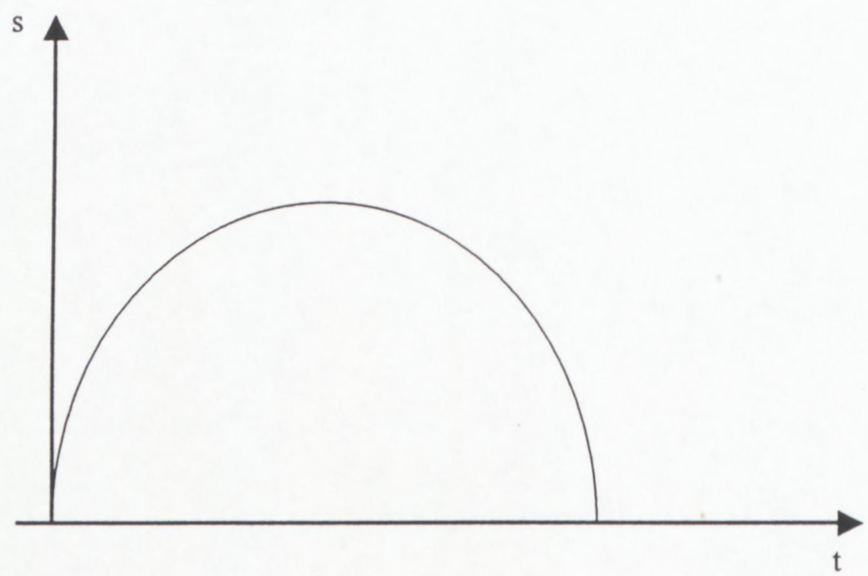


FIGURE 5.6

The interview proceeded as follows:

I: *Why did you draw a curved graph?*

S: *I actually don't know - I just remember that it has to be like this.*

I: *I noticed you started your graph at s = 0. Why?*

S: *The ball starts on the bridge and has not gone anywhere before it started, so it must start at 0. In maths it is called the origin.*

I *So these graphs are all wrong?*

(interviewer draws graphs like those in figure 5.7)

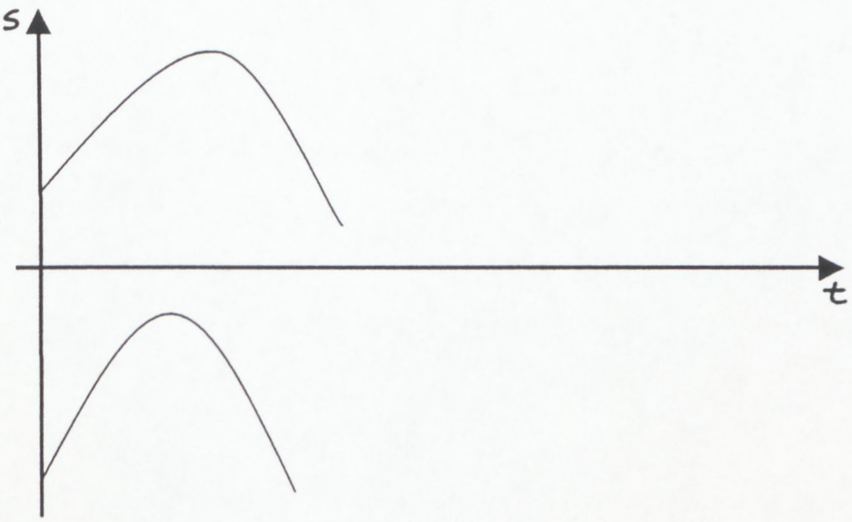


FIGURE 5.7

S: *Yes they are both wrong.*

The tendency to accept that s = 0 at t = 0 is further illustrated by the responses to question 15 of the main questionnaire which demonstrated that learners generally did *not* believe that a graph of *s vs t,* such as the one shown in figure 5.8 (a), illustrates the motion of a body moving with constant negative velocity.

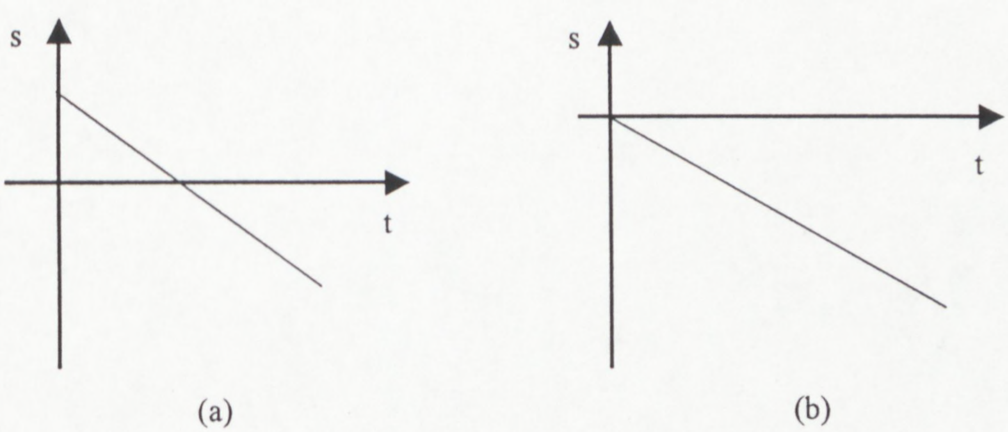


FIGURE 5.8

A large majority of the students were, however, quite happy to accept the graph of figure 5.8 (b) *which starts at the origin* and thus simply assumes a different reference point.

The notion that the displacement at the beginning of a motion must be zero and the belief that this *has* to be reflected on an *s-t* graph appears very strongly developed among high school and early tertiary students.

The tendency to start *v-t* graphs at the origin, however, is not quite as strongly developed. The only incidence of a motivation for a *v-t* graph incorrectly starting at (0;0) was the observation made by one student that... *if a displacement is zero at the beginning of a motion then the velocity at the start must also be zero.*

5.2.2 Confusion between the meaning of kinematic quantities

Table 5.2 indicates that students experience a great deal of confusion about the meaning of *velocity* and *acceleration*, while their understanding of *displacement* has been found to be considerably more acceptable.

Only 12 % of students sampled could give a satisfactory explanation of the meaning of acceleration without resorting to the traditional textbook definition of *rate of change of velocity*. It is often confused with velocity because it is viewed by some students as a *velocity that gets bigger and bigger* (without considering the time during which this takes place). This is also reported by, amongst others, Trowbridge & McDermott (1981:245), Peters (1982:502) and Halloun & Hestenes (1985:1059). Trowbridge & McDermott discovered particular confusion among students between the concepts of velocity and acceleration when they had to perform tasks on simple motion of *real* objects:

> *A significant number of students from a wide variety of courses confused the concepts of velocity and acceleration ... fewer than half of the students demonstrated sufficient qualitative understanding of acceleration as a ratio to be able to apply this concept in a real situation* (1981:253)

Learners often interpret acceleration in terms of increasing speed only and view *deceleration* as the opposite "phenomenon" to acceleration, namely slowing down. A third-year technikon engineering student was asked what she thought acceleration meant:

I: *What do you think is meant by acceleration?*

S: *Isn't it an increase in speed per second?*

I: *How do you mean?*

S: *Well, it is when an object moves faster and faster.*

I: *What about if it moves slower and slower?*

S: *In that case it is not called acceleration.*

I: *What is it called then?*

S: *That's deceleration.*

The same student also agreed that a *negative* acceleration implies that a body is decreasing its speed and that *deceleration* therefore equals *negative acceleration*. This confusion about the vector nature of acceleration is common amongst students at all levels and is further illustrated by the replies to questions 4 & 6 of the main questionnaire. In spite of the fact that about 50 % of the respondents agreed that negative acceleration could mean speeding up, only 28 % could recognise a negative acceleration in the context of a *physical* *problem* with reference direction specified where an object is *seen* to speed up in a negative direction.

Students do not usually accept that the chosen sign convention is as critical for *acceleration* as it is for *velocity* and that slowing down in a *positive* direction is equivalent to speeding up in a *negative* direction in terms of the chosen sign for the acceleration vector. The converse is equally valid i.e. speeding up in a positive direction is seldom associated with the same sign for acceleration as slowing down in a negative direction.

One possible reason for the relative difficulty which learners experience with the concept of acceleration can be linked to the problems they have with the comprehension of the "rate of change" concept. Orton (1984) and Clement (1989) both found that this could seriously impair a student's progression in a subject such as kinematics, because the slopes of graphs are not associated with **rate**. In the present study this is confirmed by the relatively large number of respondents who could correctly state that acceleration is the rate of change of velocity, while only 5 % of *these* students could explain the gradient to an *a-t* graph in terms of the *rate of change of acceleration*.

Two further misconceptions about acceleration which were revealed by the questionnaire and confirmed by personal interviews were:

- *an object travelling at **constant** acceleration cannot be "changing" its speed.* These students focus on the word *constant* and immediately seem to be thinking in terms of the velocity being constant. Objects, according to these students, can only speed up or slow down if their acceleration *changes*.

- *a negative acceleration is a decreasing positive acceleration.* The South African syllabus for Physical Science does not provide for non-constant acceleration. This could be the reason why students tend to give their own interpretation to the idea of a decreasing positive acceleration. They simply equate this to negative acceleration. The association of *negative* with *decreasing* (and *positive* with *increasing*) seems to occur quite commonly amongst learners (see pp. 263 - 264 where it was shown that *negative velocity* was often associated with the slowing down of an object). Negative acceleration, to these students, is characteristic of an object which is speeding up, with the amount by which it speeds up getting "less and less". A graph such as the one in figure 5.9 often accompanies the explanation and is occasionally even called *a negative graph*!

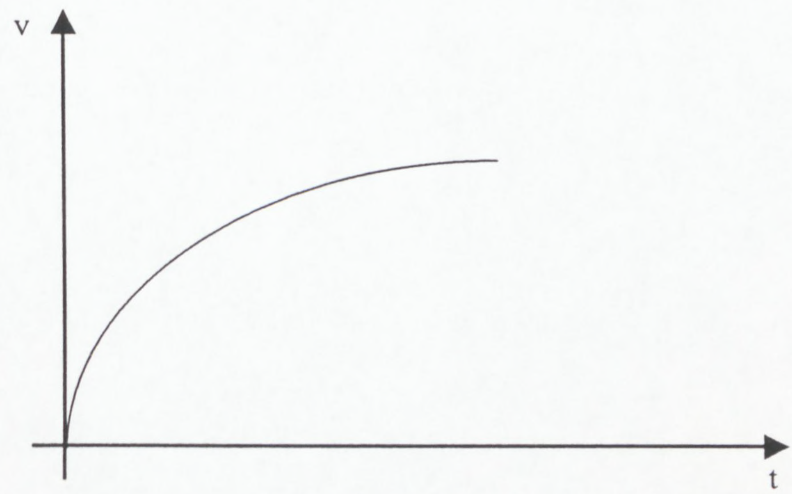


FIGURE 5.9

Students sometimes seem to be inclined to draw *v-t* graphs when they are asked to draw the *a-t* graph of a particular motion. Once again this appeared particular prevalent amongst students who achieve relatively low marks in physical science.

One student repeated the *speed-time* graph of figure 5.5 when he was asked to draw an *a-t* graph for the vertically projected ball, thus obtaining the graph of figure 5.10 below.

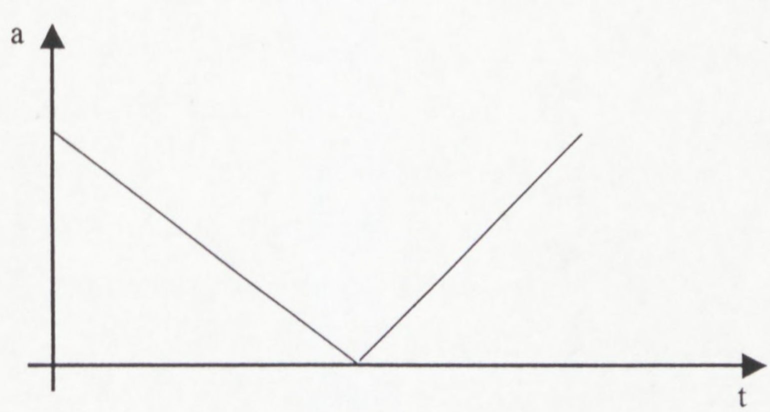


FIGURE 5.10

When asked what the *v-t* graph would look like, this student replied that it would have the same shape as his *a-t* graph. He appeared to have **two** misconceptions:

- he believes that *velocity* and *rate of change of velocity* will have similar appearances when represented graphically as a function of time;
- he confuses *velocity* with *speed* by ignoring the negative values on the graph of the kinematic quantity.

5.2.3 Reluctance to use gradient and area

Learners generally show a reluctance to use gradients to gain information about a graphically represented motion. This is particularly true when they have to differentiate between negative and positive gradients in the case of graphs depicting *negative* kinematic quantities.

According to table 5.2 about 73 % of the respondents in the main questionnaire chose not to use gradients in cases where this might have considerably simplified the solution of the graphing problem. This is illustrated by the general tendency not to use gradients in order to find the *velocity* from an *s-t* graph and the *acceleration* from a *v-t* graph. Even when the gradient of a *v-t* graph is given (by implication) and this has to be used to construct the semi-accurate *a-t* graph, students still tend to prefer to use equations of motion to obtain the required co-ordinate values. This is demonstrated by the fact that a majority of students interviewed, when asked to draw a *v-t* graph representing a motion with initial velocity of 2 $m.s^{-1}$ and acceleration equal to 4 $m.s^{-2}$, calculated a number of v-values by using the equation $v = u + at$. The simpler method of drawing a graph with v-intercept = 2 and a gradient of 4, as illustrated in figure 5.11, was seldom used.

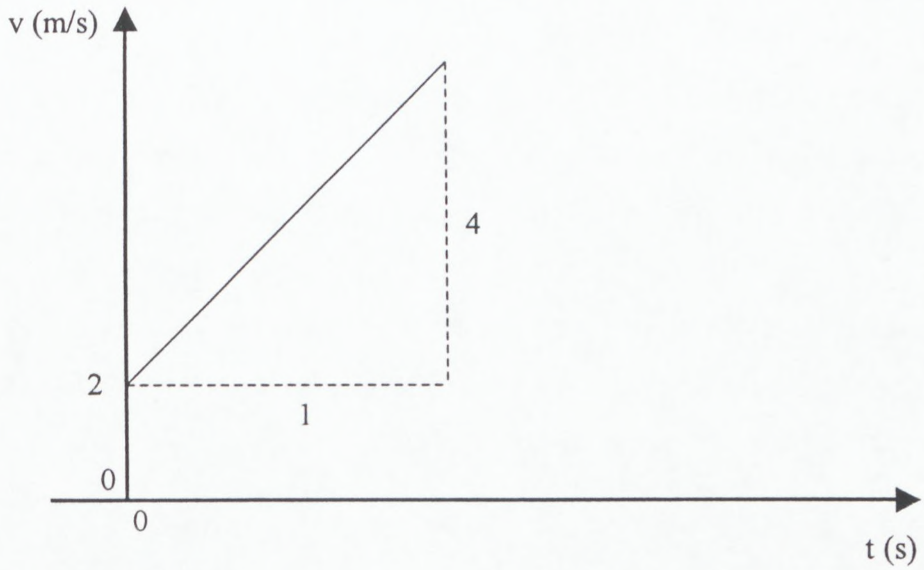


FIGURE 5.11

The use of gradients of tangents to **curved** *s-t* graphs is a serious problem, even among top achievers. Students normally study calculus in the maths curriculum *after* they have completed the section on kinematics, so that they are often not familiar with the concept of an *instantaneous rate of change*, causing them to avoid using gradients in the case of curved graphs

An analysis of the interviews reveals that the students questioned had little or no problem with the idea of a gradient. Most of them gave a satisfactory explanation of gradient in terms of $\frac{\Delta y}{\Delta x}$ and, when prompted, usually mentioned the importance of the slope of a tangent in the case of a curved graph. Question 5 of the main questionnaire indicates, however, that students were generally reluctant to **apply** the idea of a gradient to a tangent in graphs that represent the variation of a kinematic vector quantity with time in cases where the graph is *not straight*.

The response to question 9 of the main questionnaire gives a good indication of the state of the test group's understanding of *negative* gradient. It was reported that the two main areas of confusion are *curved* graphs and graphs that describe *negative* values of the dependent variable because in these cases students exhibited particular difficulty in distinguishing between negative and positive gradients (see p. 279).

Table 5.2 shows that students are even more reluctant to use **areas** under graphs than gradients in order to solve problems involving graphs. Questions 13 & 14 of the main questionnaire required students to calculate displacement values for various time intervals by making use of a given semi-accurate *v-t* graph. Very few students used areas, opting rather to use equations of motion in order to obtain the required displacement values.

Gradients and area play an important part in *graph → graph* transformations and the extent to which learners make use of these mathematical techniques in performing type E translations will be explored further in paragraph 5.3.

5.2.4 Graphs as iconic representations of physical events

The investigation reported on in Chapter 4 reveals that learners display a noticeable reluctance to change the physical appearance of that which is given (for example an observed motion, a described motion or a mathematical representation of a motion). This leads to a number of serious misconceptions with regard to graphical representations and their transformations. Two manifestations of this tendency are *graphs-as-pictures* and, in the case of graph transformations, *conservation of properties*. (Conservation of properties is described in paragraph 5.3.3).

Personal interviews with students confirmed the tendency to conserve the physical characteristics of a kinematic event. Students were asked to comment on the *v-t* graph illustrated in figure 5.12 of a ball projected vertically upwards and allowed to return to its original position.

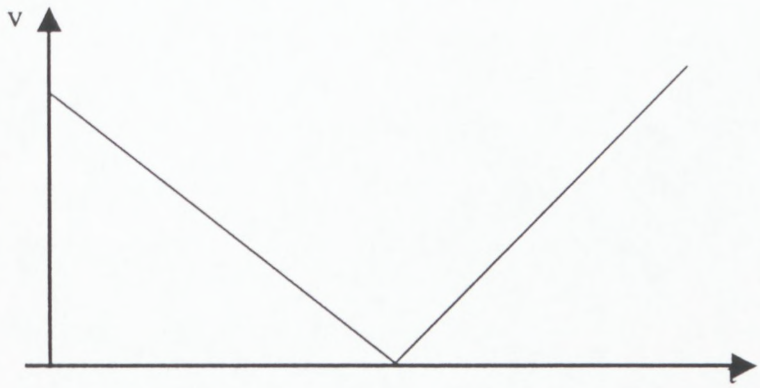


FIGURE 5.12

I: *Many students say that this graph (points to fig. 5.12) describes the motion of a ball projected vertically upwards and allowed to come down again.*

S: *The graph is correct.*

I: *Why?*

S: *I have done many examples like that and know that the line changes direction because the ball changes direction.*

The fact that a number of other students interviewed gave similar explanations, confirms that **students often have an urge to conserve an observed directional switch in a real-life situation by a sudden change in direction of the line graph representing that situation.** This confirms that the misconception is not about graphs, but about the conventions used to describe vectors. It was suggested earlier that the reason for the popularity of the *v-t* graph of figure 5.12 is due to a belief that the velocity of the ball can never be negative. The interviews suggest that problems with the interpretation of a change in direction could be another cause of this common error. Students do not know how to represent the observed change in direction on a *v-t* graph and mimicking the physical problem scene seems to them an easy way out of the dilemma.

Mokros and Tinker (1987:381) provide one possible explanation for the relatively common occurrence of graphs as iconic representations of a real-life situation:

> *The earliest graphical representations students encounter are picture graphs for which the graph* ***is*** *a picture* (of the real situation). *Thus graphs-as-picture represents a good strategy and can lead to understandable confusion.*

Students' who hold this view of a graph are often observed to make the error of drawing a *v-t* graph such as that of figure 5.13 for a bicycle travelling down a steep hill (Clement 1989:82).

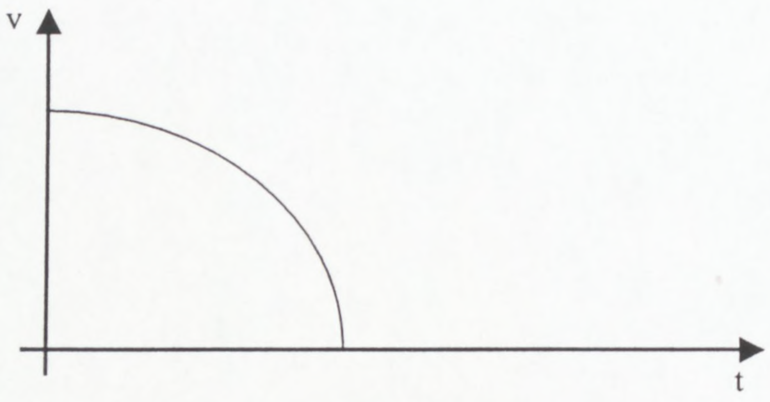


FIGURE 5.13

5.3 GRAPH TRANSFORMATION DIFFICULTIES

5.3.1 Introduction

The present study has shown that the literature contains a host of studies which investigate students' difficulties related to the construction and interpretation of graphs representing real kinematic situations. These difficulties were shown to be primarily the result of interval/point confusion, slope/height confusion, iconic interpretations and a failure to understand the significance of *negative* kinematic vector quantities.

Very little research has, however, been undertaken to investigate the nature and extent of student errors with regard to *graph → graph transformations* (translation E). McDermott *et al.* (1983, 1987) report serious difficulty in associating a slope or increasing slope of an *x-t* graph with the height or increasing height of a *v-t* graph. They also found that when students tried to construct one graph from another, they often seemed unable to ignore the *shape* of the original graph.

Schuster (1983) reports that *kinematics* and *representations* make up an important component of the first year physics course at the University of Natal. He suggests that it takes *time* and *practice* before students can be said to have mastered the techniques involved in the transformation between different graphical representations of the same motion. One of the reasons for this, he claims, is that a particular feature of one graph points to a totally different feature of the related graph:

> *Transformations between position and velocity graphs may involve a quantity (e.g. displacement, velocity) being represented as a value, as a slope, or even as an area.* (1983:336)

Apart from these investigations, however, very little else is reported in the literature dealing with graphical representations of motion about problems experienced with regard to the transformation of one kinematic graph into another related one. The present study will endeavour to show that it is possible to categorise the various typical student errors which are made in this regard.

The investigation concentrated on students understanding of the transformation from a given *v-t* graph to the *s-t* graph representing the same motion. The degree of "difficulty" varied from an elementary *v-t* graph representing constant positive velocity, to a more complex graph involving both negative and positive velocity-values. In all cases students were required to provide reasons for their *s-t* graph response.

Most of the graph transformation questions used in the pilot study were kept unchanged in the main investigation. For this reason certain common responses which came to light in the first study will be included in the discussion of typical student misconceptions.

Some of the student difficulties with drawing matching graphs representing different kinematic quantities can be attributed to their unwillingness or incapability to make proper use of *gradient* and *areas*. This factor , together with a noticeable inability to visualise the represented physical situation, results in students producing transformed graphs which simply copies the general shape of the given graph. The difficulty that students have with *negative* kinematic values in general is also manifested in graph transformations, especially when negative values and gradients have to be combined in order to obtain the related graph.

The tendency to start *s-t* graphs at the origin, regardless of any reference to reference points, was also noticed in the case of graph transformations when a *v-t* graph had to be transformed into an *s-t* graph.

5.3.2 Problems relating to gradient and area

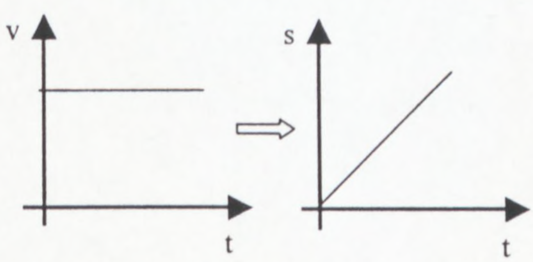
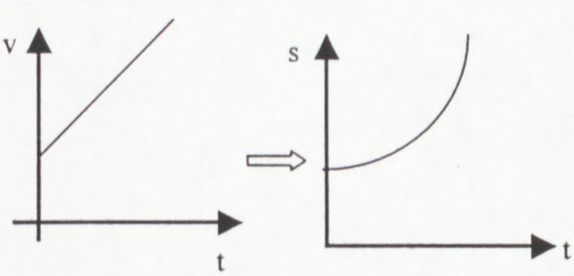
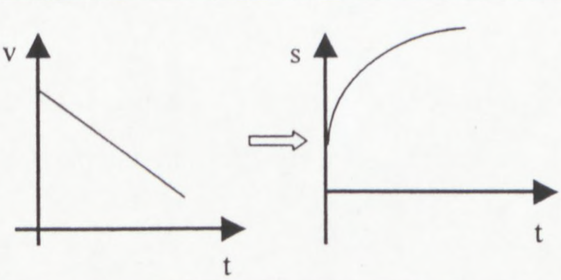
5.3.2.1 General comments

(a) *Gradient*

The present study confirms that the poor understanding of gradient can be regarded as the origin of many student problems with kinematic graphs. The reason for this is that they *do not comprehend the significance of the slope of a graph as a measure of rate of change* (see for example Orton (1984), Clement (1989), Leinhardt *et al.* (1990) and others). Students seldom verbalise their thoughts about gradients when they have to make a graph transformation and even when they do use it, it is explained in terms of expressions such as: *one quantity increasing at a steady rate* or *the quantity increases by bigger amounts in the same time*.

The analysis of the main questionnaire shows that, although not explicitly asked, on average fewer than 8 % of the respondents spontaneously explained a correct graph → graph transformation in terms of gradients or change in gradients. Table 5.3 shows how the proportion of students who successfully used gradients to explain certain *v-t* → *s-t* transformations decreases as the given *v-t* graph becomes increasingly complex, starting with a constant velocity, then an increasing positive velocity, a decreasing positive velocity and finally an increasing velocity starting with a negative value.

Transformation no. 4 of table 5.3 is relatively easy to achieve by observing that the required graph has to start with a negative gradient which decreases in magnitude to zero, whereafter it increases positively in accordance with the ordinates of the given *v-t* graph. The fact that almost nobody used this type of argument, probably explains the very poor success rate with this transformation, as well as the disappointing proportion who obtained the correct answer to graphs 2 and 3.

NO	*v-t* → *s-t* TRANSFORMATION	% CORRECT GRAPHS	% OF CANDIDATES SUCCESS-FULLY USING GRADIENT
1		79 %	7 %
2		43 %	6 %
3		33 %	5 %

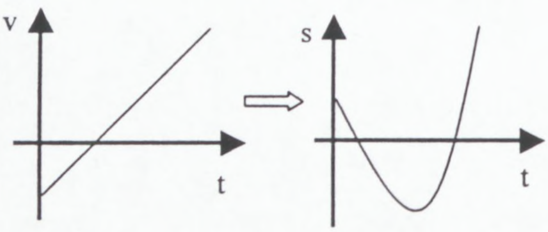
NO	*v-t* → *s-t* TRANSFORMATION	% CORRECT GRAPHS	% OF CANDIDATES SUCCESS-FULLY USING GRADIENT
4		11%	2%

TABLE 5.3
PROPORTION OF STUDENTS USING GRADIENTS TO EXPLAIN GRAPH TRANSFORMATIONS

Further evidence for the fact that the failure to use gradients can seriously impede the graph transformation ability of students is found in the response to question 15. Of the 31 % who agreed that the *s-t* graph does indeed represent the same motion as the *v-t* graph (see figure 5.18) only 5 % attributed this to the fact that the *s-t* graph had a *constant negative gradient* to accompany the *constant negative ordinates* of the given *v-t* graph. In fact, 8 % of the candidates simply stated that the *s-t* graph is *also a negative graph*!

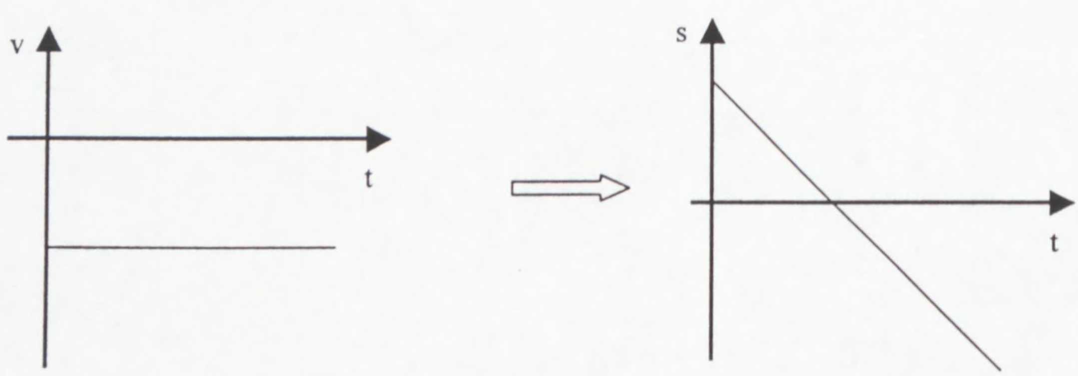


FIGURE 5.18

The relatively simpler *s-t* → *v-t* transformation of figure 5.19 requires students to notice the constant positive gradient and translate this into a positive straight line parallel to the t-axis.

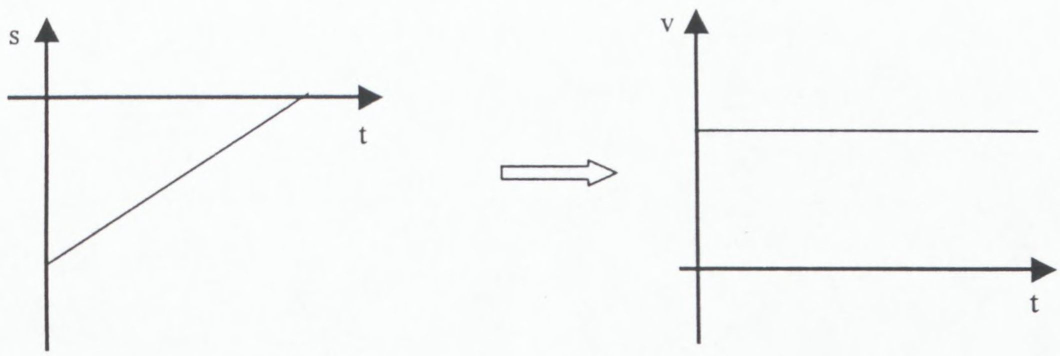


FIGURE 5.19

Of the 26 % who achieved this, a mere 7 % stated that they used the method involving gradients described above.

An analysis of the results of the empirical study reveals that, with respect to transformation no 4 of table 5.3, 17 % of all respondents who proved their ability to differentiate between negative and positive gradients were successful at drawing the correct *s-t* graph, a moderate improvement on the general success rate of 11 %. However, when ability to recognise negative gradient is correlated with the recognition of the validity of the transformation illustrated by figure 5.18, no improvement is found.

This seems to indicate that the poor transformation skills exhibited by students with regard to figure 5.18 and figure 4 of table 5.3 can *not* be attributed to a failure to recognise the difference between positive and negative gradients.

When knowledge and understanding of the role of gradients of tangents to *curved* graphs is cross-tabulated with success rate at achieving the transformations of table 5.3 resulting in *curved graphs*, a significant improvement is noted. This is shown in table 5.4

This seems to indicate that learners struggle with *v-t* → *s-t* conversions <u>because</u> they do not think of their *s-t* graphs in terms of *continuously changing gradients, to match the continuously changing height of the given v-t graph.*

The success rate for no. 4 remains low however, possibly because many students when thinking in terms of changing gradients still experience difficulty with aspects such as *a decreasing magnitude of a negative gradient*.

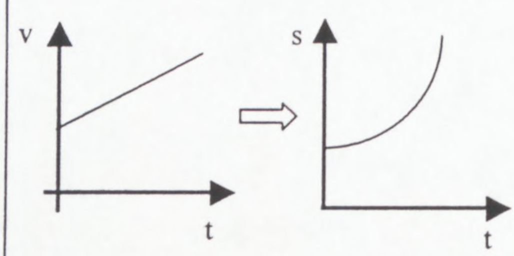
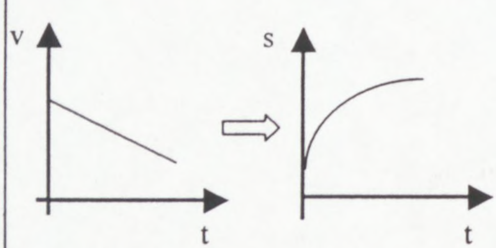
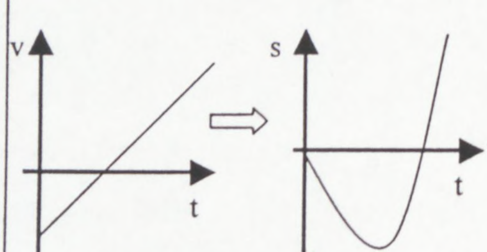
NO	*v-t* → *s-t* TRANSFORMATION	OVERALL SUCCESS RATE	SUCCESS RATE FOR CANDIDATES WITH PROVEN UNDERSTANDING OF GRAD OF TANGENTS
2		43 %	83 %
3		33 %	67 %
4		11%	18%

TABLE 5.4
COMPARISON OF SUCCESSFUL GRAPH TRANSFORMATIONS : ALL RESPONDENTS VS THOSE WITH UNDERSTANDING OF TANGENTS

Changing gradients did, however, assist some students in noting the fact that the turning point of the *s-t* graph occurs at the intercept of the *v-t* graph with the time axis. This is best illustrated by the student who declared: *the gradient at the turning point must be nought, so the speed must be nought here as well, so this is where we are on the v-t graph* (points to intercept of *v-t* graph with time axis).

(b) *Areas*

Students do *not* use areas during graph → graph transformations involving *sketch* graphs. This was confirmed by both sets of questionnaires as well as the interviews conducted with individual students. Even when prompted, students who were interviewed did not know how to relate the changing area under the *v-t* graph to the height (or changing height) of the required *s-t* graph.

Only one student verbalised his knowledge of the use of areas when attempting transformation no 3 of table 5.4: *After each second the added area gets less and less, but is still increasing, so the s-t graph must have a curved shape* (proceeds to draw appropriate *s-t* graph).

In the case of semi-accurate graph transformations, however, areas were used more readily. 25 % of the respondents to the main questionnaire *attempted* the use of gradients to make the successful transformation indicated by figure 5.20.

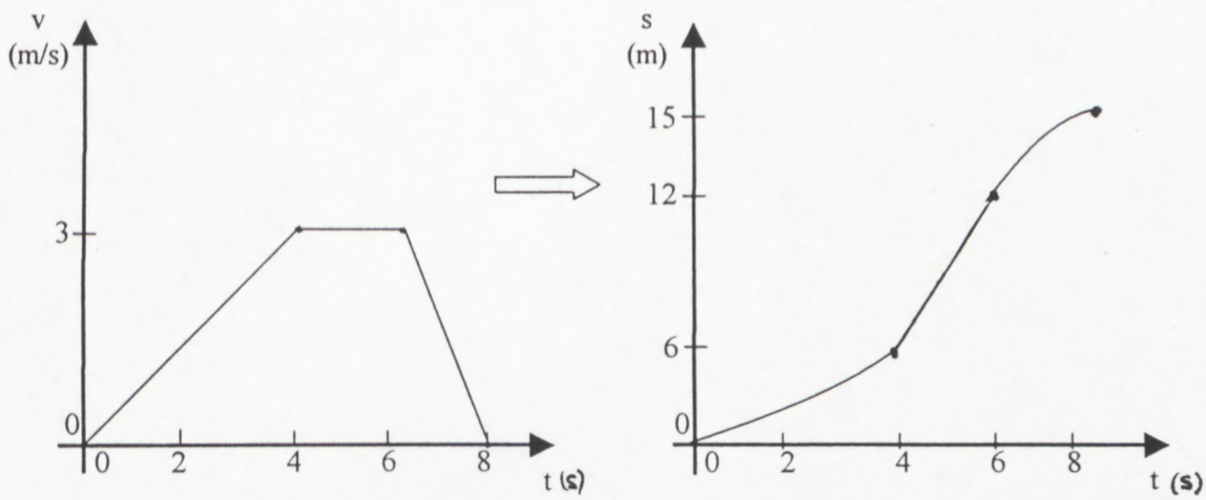


FIGURE 5.20

16 % of all the respondents were totally successful, while 9 % made either a careless error or erred in using their results to plot their displacement values on their *s-t* graphs.

It appears therefore that although some students are comfortable with the idea of using areas to obtain *actual values* in the case of accurate or semi-accurate graphs, they do not easily employ areas to analyse *tendencies* in the shapes of *s-t* graphs when only a sketch graph of *v vs t* is given.[1]

5.3.2.2 Transforming graphs of negative values

The result of the main investigation show that a dramatic drop in the success rate of graph → graph transformations occurs as soon as negative kinematic values are involved. The students did not seem to accept that an *s-t* graph that represents negative values of a displacement could be related to the same *v-t* graph as an *s-t* graph that represents positive displacement values.

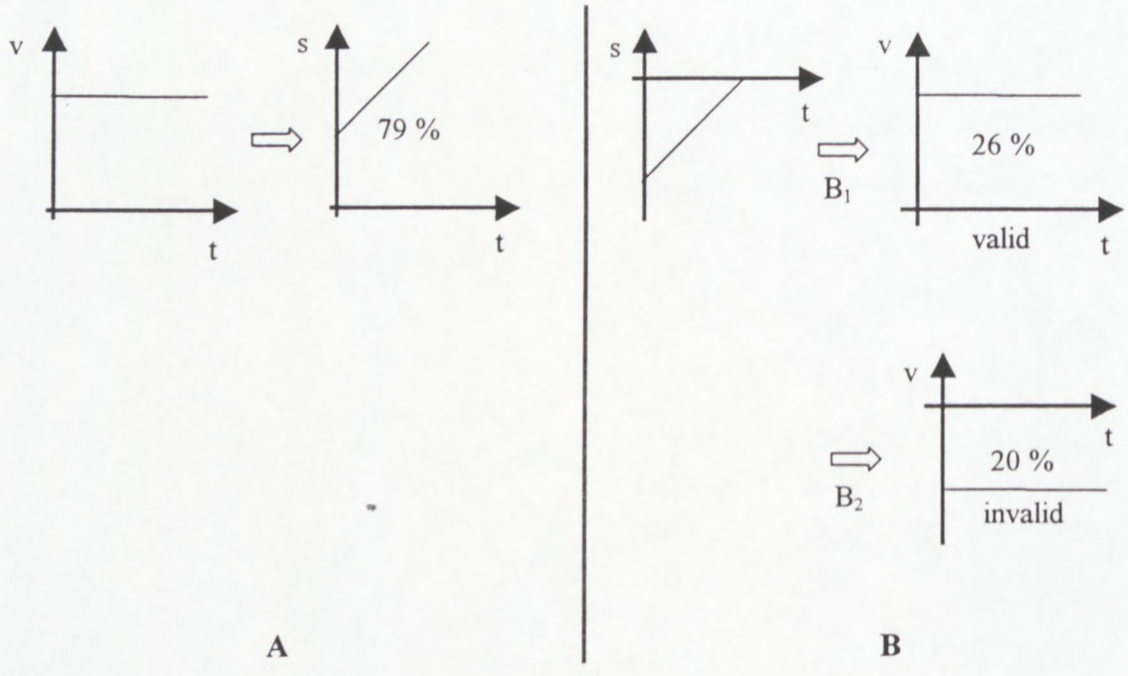


FIGURE 5.21

[1] In the case of *a-t* graphs, only flat horizontal graphs are included in the syllabus so that the change in area stays constant with time.

Translations A and B of figure 5.21 above differ only in the *direction of the translation* and the *sign* of the given graph. Very few (26 %) of the candidates made the correct transformation B_1; however, when only these students are considered, almost all (90 %) managed the "positive" translation A. 20 % believed that the negative *v-t* graph of B_2 is correct; however 90 % of these students *also* managed the correct answer to A.

When the transformation of figure 5.22 below is added to table 5.4 as the fifth type, the low success rate of transformations involving negative values becomes immediately evident.

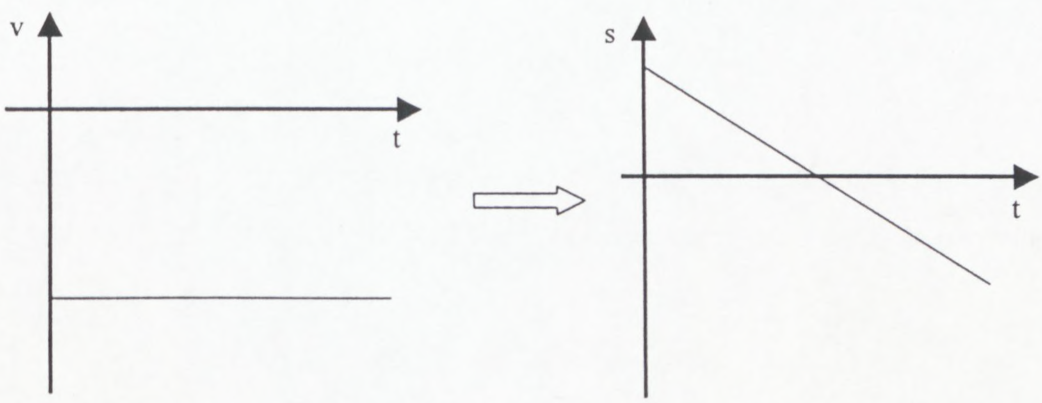


FIGURE 5.22

Only 31 % of the respondents in the main questionnaire accepted this transformation as valid. This means that the *average* success rate of translation types 4 & 5 (involving negative velocities) is 21 % compared to the average success rate of 1, 2 & 3 (involving positive *v-t* graphs) of 52 %.

Students' failure to distinguish between valid and invalid transformations is also evident when the required *s-t* graphs are *curved,* i.e. representing motion with uniform acceleration. In the pilot study, students were asked whether graphs (a) & (b) of figure 5.23 were both valid transformations of the same *v-t* graph. Only 17 % of the respondents agreed that this was indeed the case.

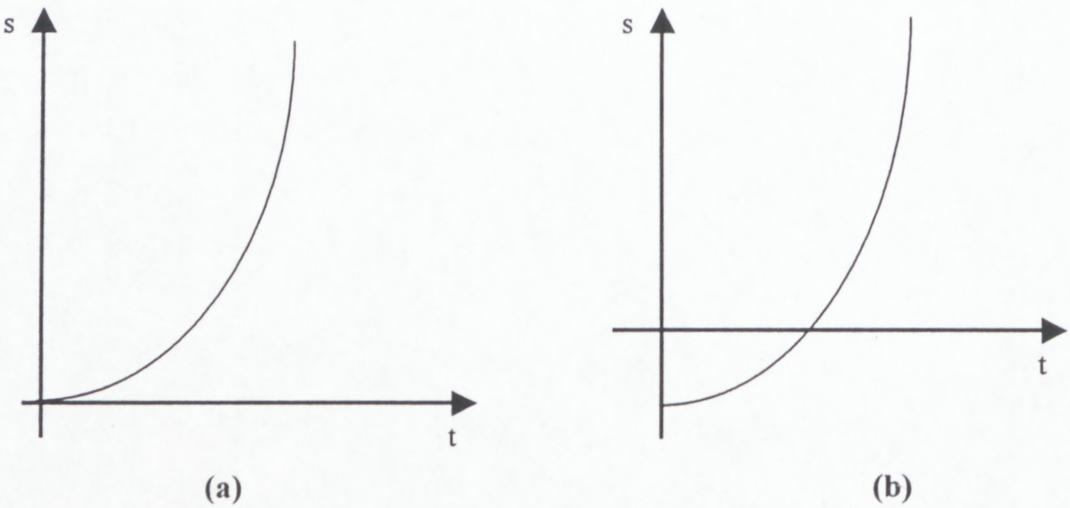


FIGURE 5.23

The problem that students have with negative transformations is further illustrated by the student who was asked to explain how she would draw appropriate *v-t* graphs for the two cases of figure 5.24.

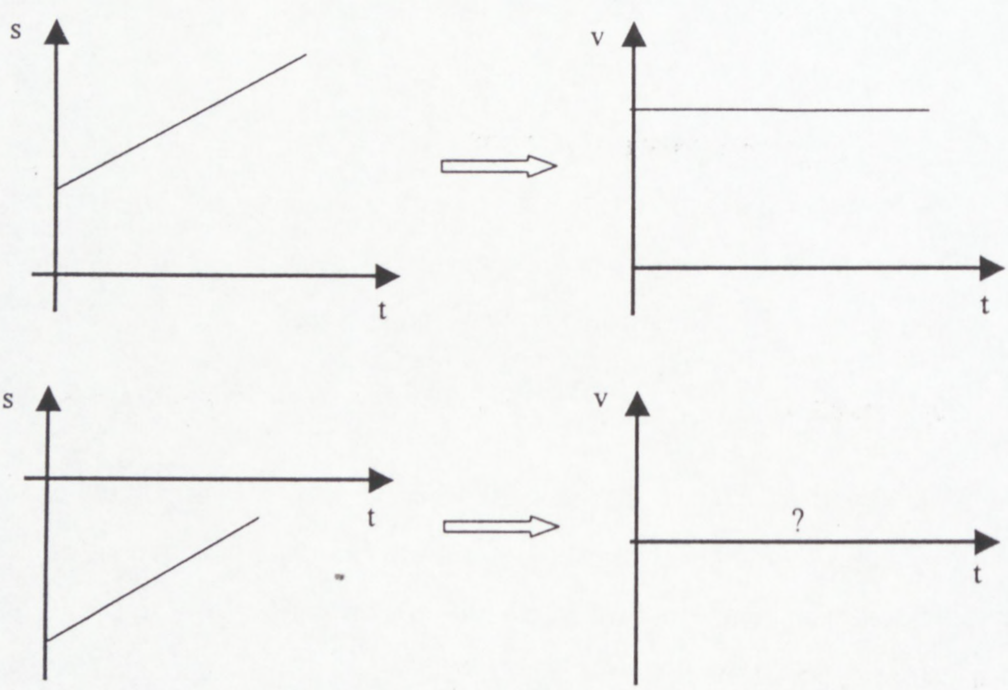


FIGURE 5.24

She had no problem with the positive case, noting that because the displacement increases such that equal distances are covered in equal times, the velocity had to be constant, adding (after being prompted by the interviewer) that it (the velocity) *also* had to be positive. The negative case, however, caused serious doubts in the mind of this student. After a long pause she admitted that she was not sure about this transformation, because she was unable to visualise the motion that was represented.

This student was thus unable to draw a *v-t* graph from a negative *s-t* graph, although she had no hesitation in drawing the same graph for positive displacement values.

Students' problems with kinematic graph transformations involving negative values thus seem to be the result of two fundamental shortcomings, namely:

- not visualising the motion which is represented by the negative kinematic graph;
- the failure to make use of gradient and area techniques to gain information about the *related* kinematic graph.

5.3.2.3 Inverse use of gradients

Instead of translating the constant gradient of a sloping *s-t* graph into a constant velocity, students sometimes invert this reasoning by believing that the gradient of a given *v-t* graph gives the *displacement* of the motion. This results in transformations such as illustrated in figure 5.25.

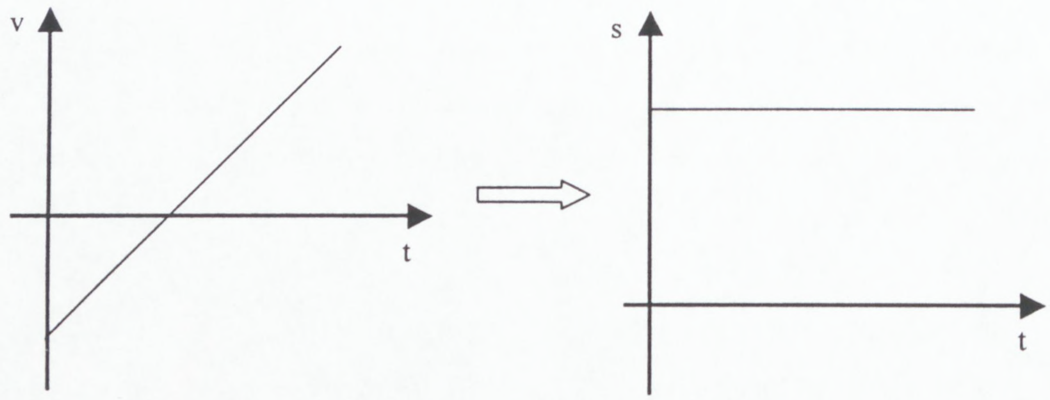


FIGURE 5.25

The observed incidence of this type of error in the present study was only 5 % for the main questionnaire and 7 % in the pilot study. For this reason only one other observed occurrence of this error will be briefly referred to:

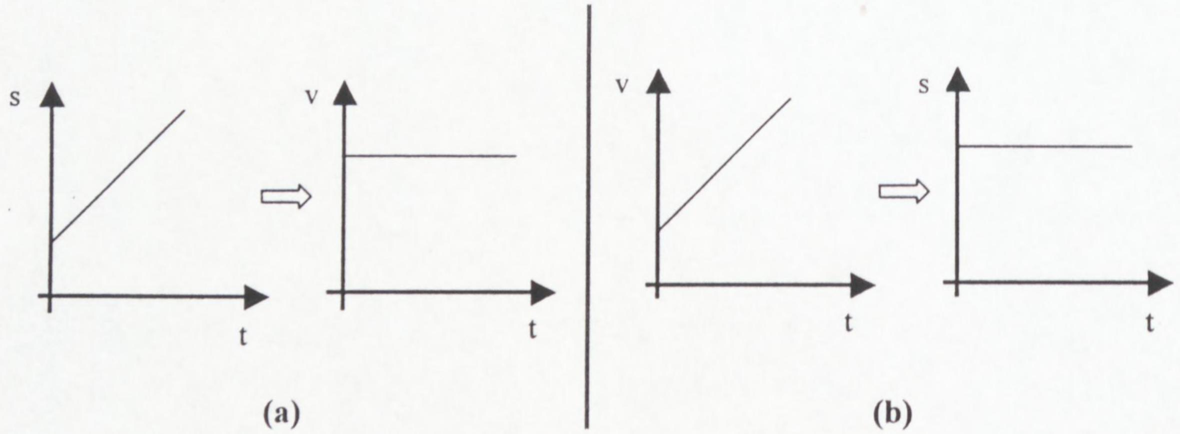


FIGURE 5.26

A Grade 11 student made the transformation illustrated in figure 5.26 (b) and explained that he remembered the validity of the transformation (a) and stated that he believed that: *if a straight s-t graph gives a constant velocity, then a straight v-t graph must give a constant displacement.*

5.3.3 Conservation of properties and inferences

When an attempt is made at classifying common student errors with regard to graph transformations it becomes immediately evident that one of the main underlying **causes** for these errors or "preconceptions" can be expressed in terms of a tendency towards *conservation of the given representation*. Students tend to keep the given characteristic of a graphical representation unchanged when attempting to make a related graphical representation. The same holds for specific *inferences* that might have to be made, in so far as that they (the students) tend to copy a known or given conclusion, when having to draw conclusions about a *related* graph.

Conservation of properties and inferences are encountered in three different *classes* or *categories* of kinematic translations. In this study they will be called γ, α and β errors.

- **a γ-error:** conserving properties or inferences when translating a graphical representation of a certain kinematic situation to the **same** graphical representation of a **different but related situation**.

- **an α-error:** conserving properties or inferences when transforming a given graphical representation of a kinematic situation to **another** graphical representation of the **same situation**

- **a β-error:** accepting a specific graphical transformation as valid and then applying the same or similar properties to the graphical transformation of a *new but related kinematic situation*.

α- and β- errors involve graph → graph transformations whilst a γ-error relates to problems with a different graph for a different situation. An example of each of these types of error will be given next, after which α- and β- errors and their practical manifestations in student responses will be discussed in more detail.

Example of a γ-error:

The given a-t sketch graph (drawn in figure 5.27) represents the motion of a car moving with diminishing speed in an easterly direction.

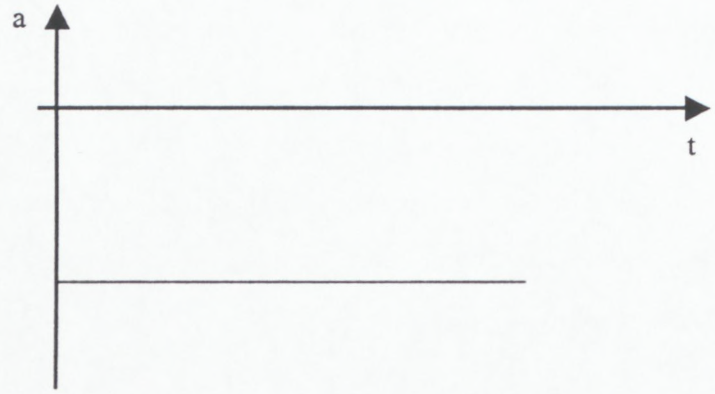


FIGURE 5.27

Now (using the same sign convention) draw the a-t graph of the same car moving with diminishing speed in a westerly direction.

Most students who were interviewed drew an *a-t* graph similar to the given graph. They seemed to focus on the word "diminishing" which appeared in both descriptions and generally tended to ignore the directional switch, which meant that they did not realise that the sign of the acceleration vector changed from negative to positive. They thus can be said to have conserved the shape of the given graph when asked to make the same graphical representation (namely *a vs t*) of a related physical situation.

Example of an α-error:

The v-t graph in figure 5.28 represents the motion of an aeroplane on the runway of an airport. Make use of this graph to draw an s-t graph of the motion of the plane:

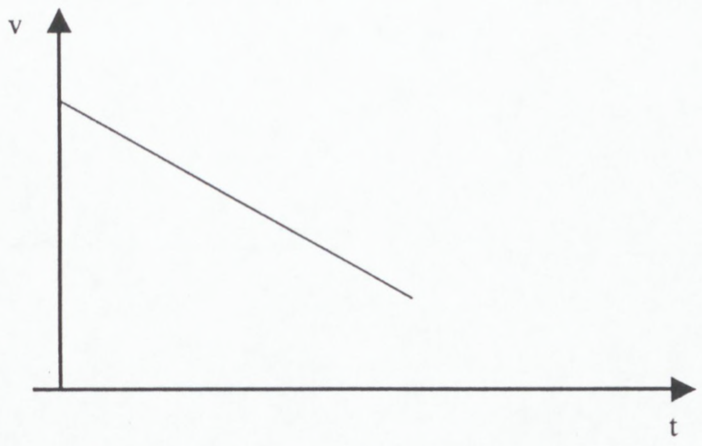


FIGURE 5.28

As many as 20 % of the respondents in the main questionnaire incorrectly drew an *s-t* graph which also had a negative gradient and was either curved or straight, thus conserving the given graphical properties of *slope* and *sign*. Most of the errors which occurred in responses to the main questionnaire were the result of α-errors, because students were often seen to draw *s-t* graphs which resembled the given *v-t* graph in at least certain respects.

Example of a β-error:

Students might be familiar with the *v-t* $\rightarrow$ *s-t* transformation illustrated in figure 5.29, either because it was given to them or they have been successful at achieving it for themselves:

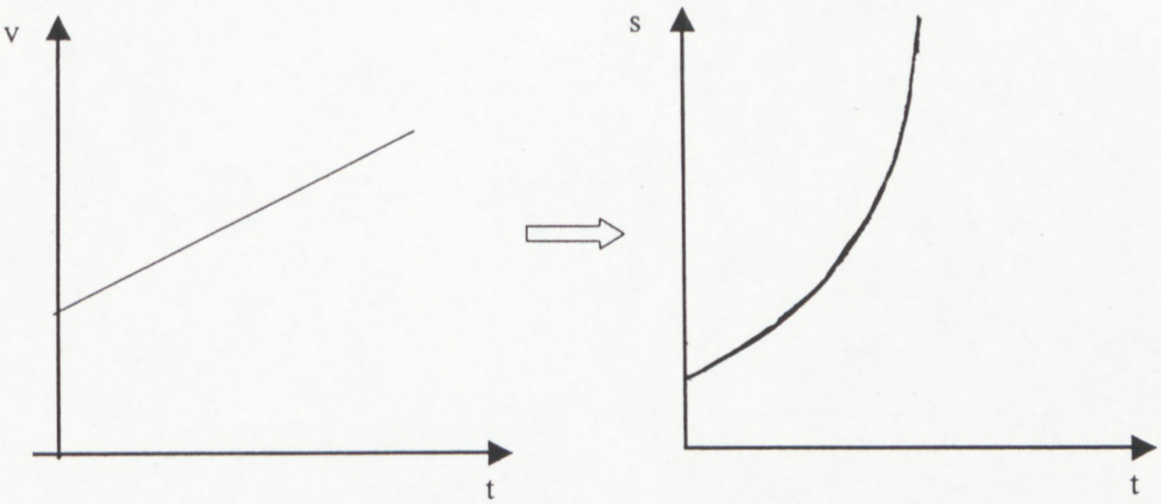


FIGURE 5.29

This often leads to an incorrect transformation such as the one illustrated in figure 5.30.

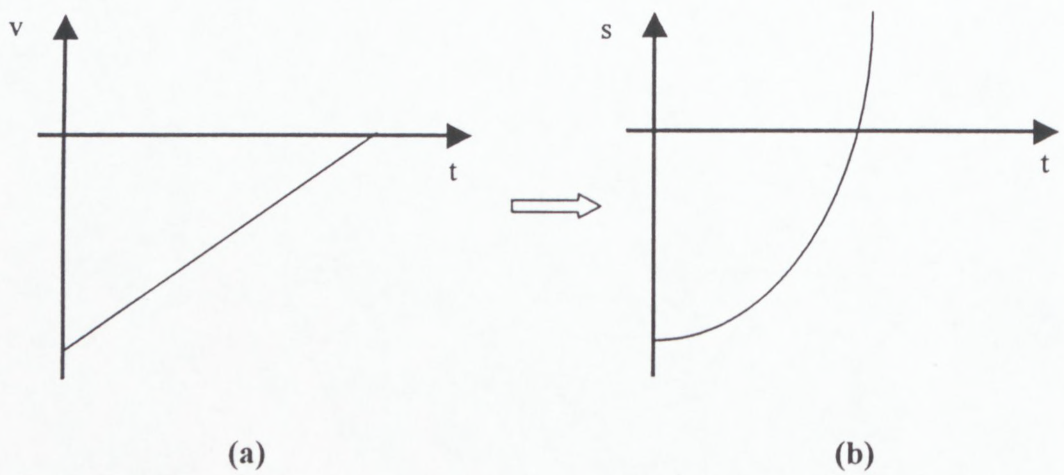


FIGURE 5.30

Instead of focusing on the increasing *magnitude* of the negative velocities, these students concentrated on the positive slope of the *v-t* graph and made similar inferences to those of figure 5.29 about the shape of the resulting *s-t* graph. The following extract from a recorded interview, where a student was asked to think out loud while attempting the *v-t* → *s-t* transformation illustrated in figure 5.30, clearly shows how the student copies the more familiar transformation of figure 5.29:

I: *...so how would you go about drawing the appropriate s-t graph for* *<u>this</u>* *motion?* (points to given *v-t* graph of figure 5.30(a))

S: *I would say the body is accelerating*

I: *How can you tell?*

S: *Because of the positive gradient*

I: *Does this help you to draw the s-t graph?*

S: *Well, I remember the easy case* (draws transformation of figure 5.29) *and this problem is almost the same as that, so it is probably the same shape.*

I: *But what about the fact that the velocity is negative?*

S: *Oh, so the s-t graph must start negatively as well* (Proceeds to draw graph of figure 5.30(b)).

The properties that students tend to conserve during graph → graph transformations are the following:

- conservation of sign;
- conservation of slope;
- conservation of shape (both locally and globally);
- conservation of intercept with the axes.

The extent to which each of these properties is conserved, as well as the incidence of the copying of *familiar or given* **inferences** to a *new* situation is discussed next. The observed tendency of students to make α-errors (conserving properties from one graphical representation to a related one) and β-errors (conserving deductions made from one graphical transformation to the same graphical transformation of a related situation) will be emphasised. It will be shown that students tend to keep the gradient the same, transform positive values into positive values and negative into negative and maintain the same basic shape of the given graph.

(a) *conservation of slope*

Students often believe that if a specific kinematic quantity is seen to be increasing (or decreasing), then a related kinematic quantity must also be increasing (or decreasing) and this idea is extended to the *drawing* of the related graph. Even when the given kinematic quantity stays *constant*, there seems to be a tendency to draw the related graph parallel to the time axis as well, thus conserving the zero slope.

When students' responses to the required graph transformation of table 5.5 are considered it transpires that a significant proportion of them drew *v-t* graphs that displayed the same positive gradient as the given *s-t* graph.

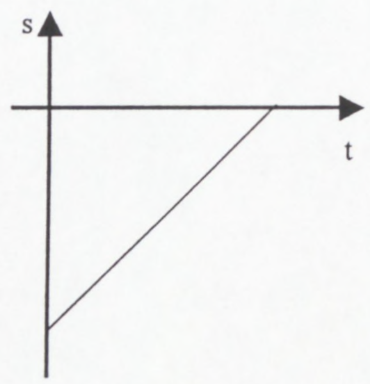
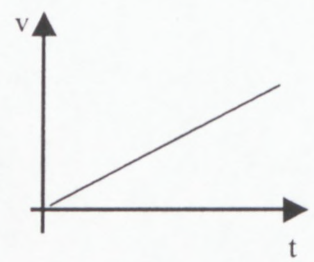
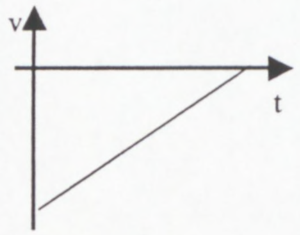
GIVEN GRAPH	STUDENT'S RESPONSE	RESPONSE %	
		MAIN	PILOT
		13 %	17 %
		7 %	4 %

TABLE 5.5
CONSERVATION OF GRADIENT DURING *s-t* → *v-t* GRAPH TRANSFORMATIONS

When the approximately 20 % of all students who made this error is considered and *their* reply to the graph → graph transformation illustrated in figure 5.31 is compared to the average success rate it transpires that only 58 % of <u>these</u> students managed this transformation (compared with the overall success rate of 78 %)

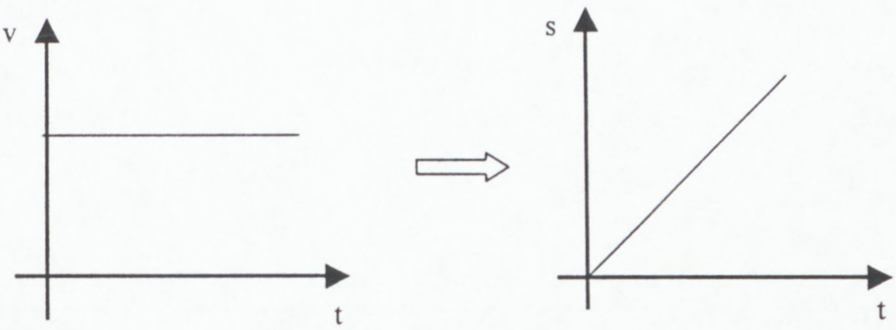


FIGURE 5.31

The reason seems to be that they could not easily accept the different shape of the *s-t* graph, with 13 % of them maintaining that the required graph also consisted of a positive horizontal line, thus conserving a slope of zero.

Another popular misconception relating to the conservation of slope is the *v-t* → *s-t* transformation of figure 5.32.

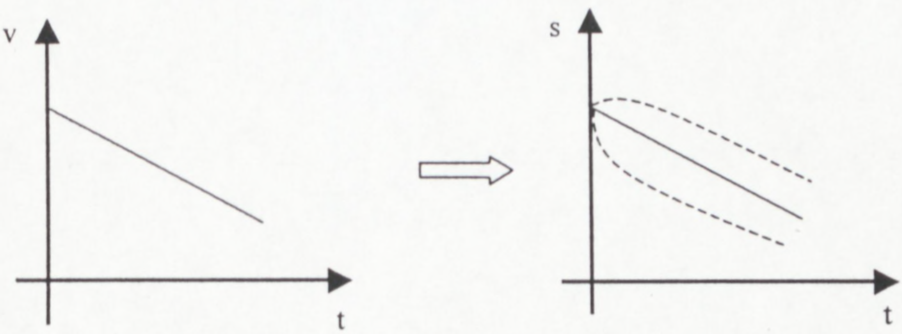


FIGURE 5.32

20 % of the respondents to the main questionnaire and 30 % of the pilot study drew *s-t* graphs which mimicked the observed negative gradient. These students simply ignored the fact that as long as the velocity is <u>positive</u>, the displacement had to <u>increase</u>.

A few popular responses to the conceptually more difficult transformation illustrated in table 5.6 are illustrated below. This once again shows that these students, instead of considering the effect of the increasing *magnitude* of the negative velocity on the displacement, simply ensure that their *s-t* graphs has a positive slope, in accordance with the positive slope of the given graph.

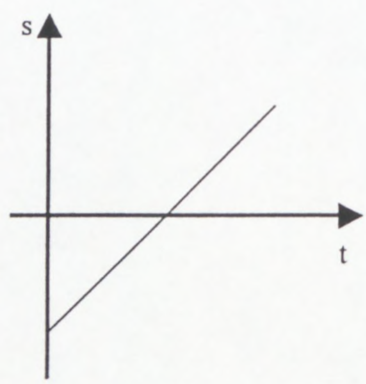
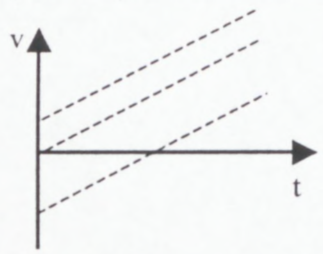
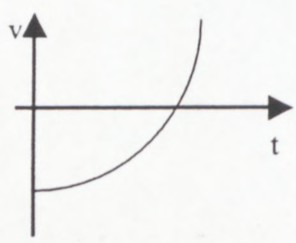
GIVEN GRAPH	STUDENT'S RESPONSE	RESPONSE % MAIN	PILOT
		14 %	29 %
		24 %	18 %

TABLE 5.6

GRADIENT CONSERVATION IN THE CASE OF MORE DIFFICULT TRANSFORMATIONS

In some cases the slope of the required graph *was* the same as the given graph, except that it was curved when the original graph was straight and vice versa. To find out why as many as 29 % of all respondents drew *s-t* graphs (to match the given *v-t* graph) like those illustrated in table 5.6, a student was asked to explain his reasoning for drawing such a graph.

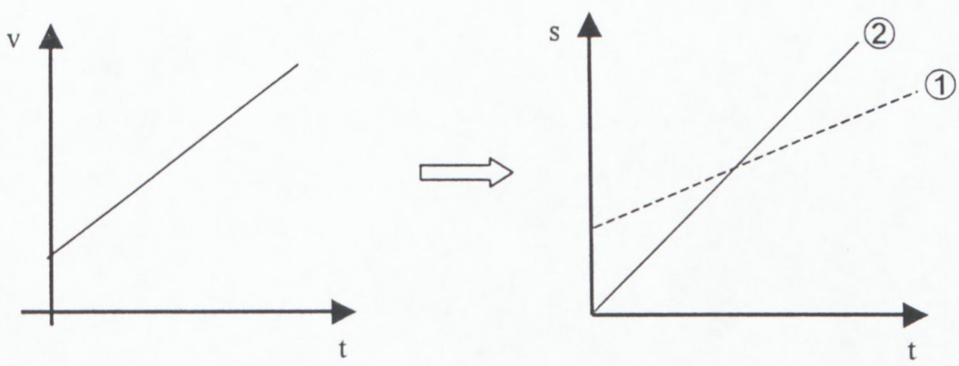


FIGURE 5.33

A transcript of the interview shows clearly how he copied the positive slope of the *v-t* graph:

I: *Explain how you would change this graph into an s-t graph.*

S: *Let me see: more velocity more time, so the object is moving faster.*

I: *So what happens to the displacement?*

S: *It also gets more just like the velocity* (Draws graph no 1 of figure 5.33).

I: *Are you sure about the shape?*

S: *It is straight, like the velocity.*

I: *And the start of your graph?*

S: *Oh, sorry, it must start at zero* (Draws graph no 2 above).

Many students are thus inclined to ignore the effects of gradient on graph-graph transformations. Instead they rather *maintain* an observed decreasing or increasing function, with the result that their transformed graphs have the same slope as the given graph.

(b) *conservation of sign and intercepts*

Students' graphs frequently match given graphs as far as the sign (positive and negative) and the intercepts with the axes are concerned. The curved *s-t* response of table 5.6 is a good example of sign and intercept conservation and is repeated in figure 5.34 below.

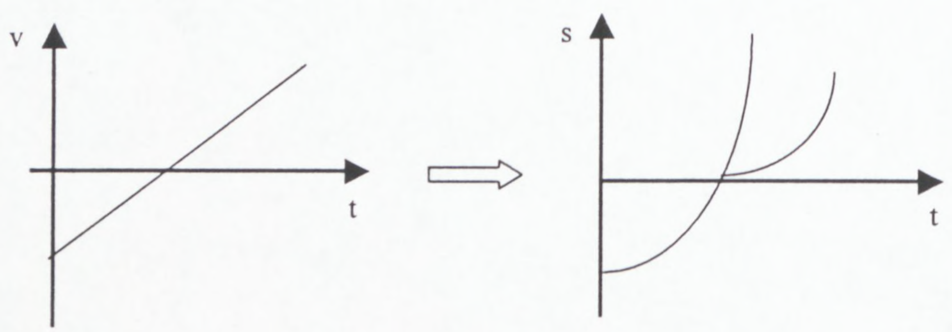


FIGURE 5.34

24 % and 18 % of the respondents to the main questionnaire and pilot study respectively drew *s-t* graphs such as illustrated above. They evidently believed in conserving the following properties:

1. the vertical axis intercept (not realising that the s-intercept in purely arbitrary);

2. the negative values of the kinematic quantities between t = 0 s and t = 2 s;

3. the t-axis intercept at t = 2 s (not interpreting a zero value of the velocity as the turning point of the *s-t* graph);

4. the positive-values after t = 2 s.

The extent of the tendency by students to copy a given time axis intercept is further illustrated by the fact that an *additional* 16 % of the respondents drew differently shaped *s-t* graphs to the one illustrated in figure 5.34 *that also intercepted the t axis at t = 2 s.*

Another illustration of the conservation of the sign of a given graph is provided by the statistics of student responses to the transformation illustrated in table 5.7.

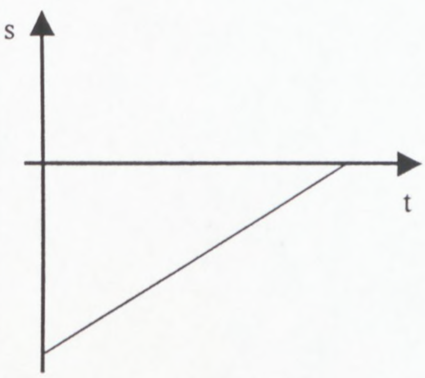
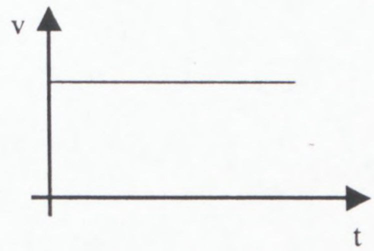
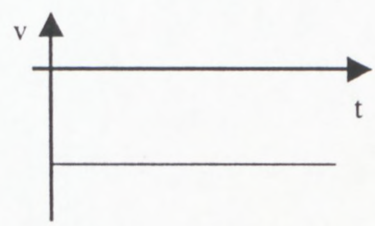
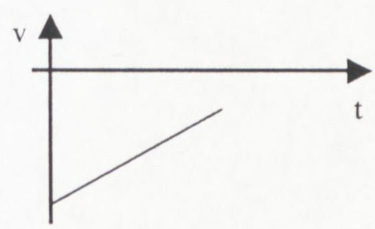
GIVEN GRAPH	STUDENT'S RESPONSE	RESPONSE %
		26 %
		20 %
		7 %

TABLE 5.7
INCIDENCE OF CONSERVATION OF SIGN IN GRAPH TRANSFORMATIONS

Most students acknowledge that the velocity remains constant, but a significant proportion drew a *v-t* graph that represented constant *negative* velocity values to match the observed negative displacement values. 7 % conserved the sign as well as the gradient, indicating a complete lack of understanding of the relationship between displacement and velocity.

The *v-t* $\rightarrow$ *s-t* transformation illustrated in table 5.8 <u>also</u> indicates a preference for maintaining an observed sign:

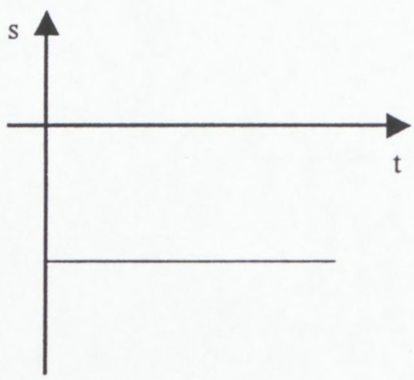
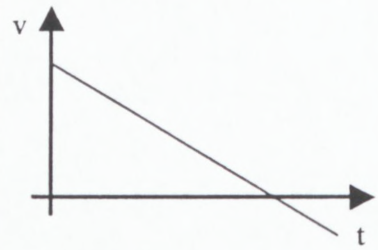
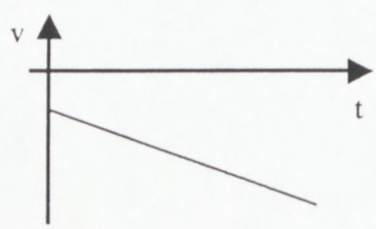
<table>
<tr><th>GIVEN GRAPH</th><th>STUDENT'S RESPONSE</th><th>CODE</th><th>RESPONSE RATE</th></tr>
<tr><td rowspan="2"></td><td></td><td>a</td><td>31 %</td></tr>
<tr><td></td><td>b</td><td>25 %</td></tr>
<tr><td></td><td></td><td>c</td><td>5 %</td></tr>
</table>

TABLE 5.8
CONSERVATION OF SIGN INVOLVING NEGATIVE KINEMATIC VALUES

While only 31 % of all respondents agreed (correctly) that code (a) does in fact represent a valid graph transformation, a further 30 % (codes b & c) believed that the required graph **had** to be drawn for negative values of the displacement. When only those students who *succeeded* with the graph transformation of figure 5.35 below are considered, it transpires that 24 % of these students *still* maintained that response codes (b) or (c) of table 5.8 constitute the *only* valid transformation.

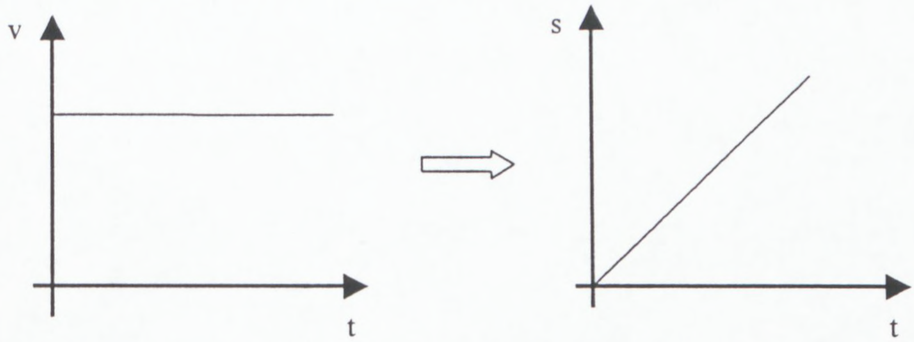


FIGURE 5.35

This means that these students made a β-type error by reasoning that if a positive but constant *v-t* graph yields an *s-t* graph with positive slope, starting at the origin, then a negative but constant *v-t* graph yields an *s-t* graph with negative slope, starting at the origin.

Code (c) of table 5.8 represents students who conserved both the *vertical axis intercept* as well as the *sign* of the given graph. Not surprisingly most of *these* students drew *s-t* graphs with reference to figure 5.35 that started at *positive* s-values.

Even teachers of physics are sometimes tempted by a conservation of intercept. One physics lecturer who assisted in validating the main questionnaire was asked for his interpretation of the graph transformation involving a negative but increasing velocity. His corresponding *s-t* graph is shown in figure 5.36.

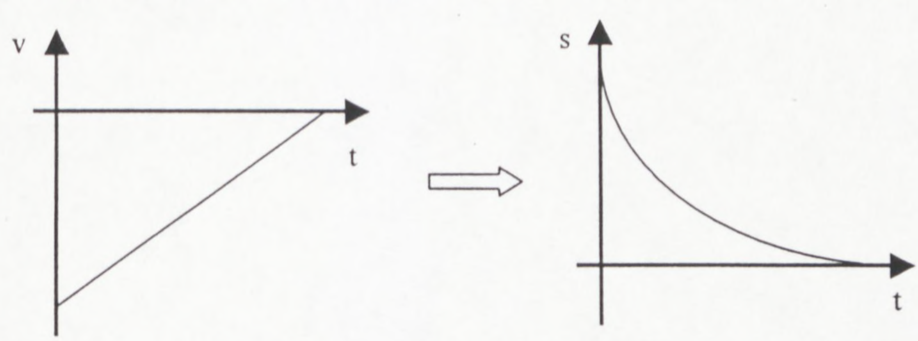


FIGURE 5.36

This lecturer, when asked about a motivation for drawing the illustrated *s-t* graph, stated simply: *I felt a tremendous urge to have the graph end up on the time axis.*

Although he subsequently acknowledged that his *s-t* graph could be moved any number of units in a vertical direction, this *does* illustrate the clearly developed tendency to maintain the given intercepts of a graph.

(c) *conservation of shape*

When required to make a graph transformation, students' graphs often have a similar shape to the given graph. This appears to be at least partially due to a tendency towards a conservation of slope or sign or both. The following distinctions can be made as far as the conservation of *shape* is concerned:

- the shapes are either **similar** or **identical**;

- the shapes of the graphs are similar (or identical), either **globally** or for **parts** of the graph.

The most striking evidence of shape conservation is found in the responses to question 14 of the main questionnaire. The given graph together with the correct, as well as two incorrect responses, is repeated in table 5.9.

Code (b) of table 5.9 indicates that 26 % of all respondents conserved the basic shape of the given graph by drawing an *s-t* graph, which *resembled* the original shape. A further 12 % (code c) drew graphs which can be described as *congruent* (i.e. identical) to the given graph. Clement (1989:83) refers to this type of response as *a global correspondence error.*

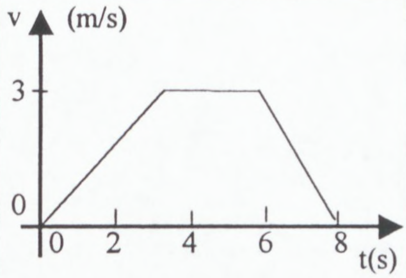
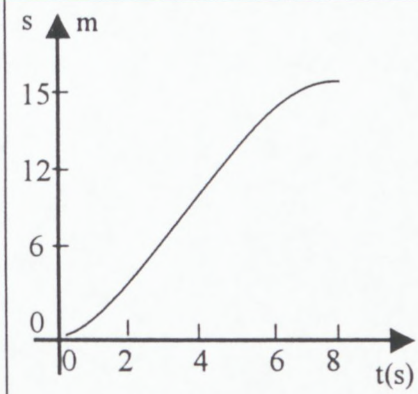
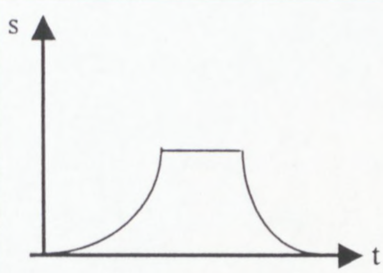
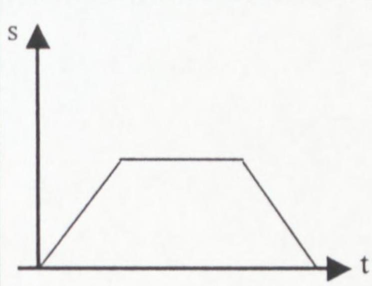
<table>
<tr><th>GIVEN GRAPH</th><th>STUDENT'S RESPONSE</th><th>CODE</th><th>RESPONSE</th></tr>
<tr><td rowspan="3"></td><td>
Comment: shape correct with or without values</td><td>a</td><td>20 %</td></tr>
<tr><td>
Comment: similar shape to given graph</td><td>b</td><td>26 %</td></tr>
<tr><td>
Comment: identical shape to given graph</td><td>c</td><td>12 %</td></tr>
</table>

TABLE 5.9
CONSERVATION OF SHAPE IN GRAPH TRANSFORMATIONS

The tendency to conserve a given slope often appears to be the underlying reason for the fact that certain basic graphs or *part* of more complex graphs are transformed so that the basic shape remains intact. Table 5.9 reveals that 38 % of the respondents drew *s-t* graphs which were decreasing during the last two seconds so that the shape during this time interval resembled the *shape* of the given *v-t* graph. Only 29 % drew some form of increasing function during this time interval.

One student who was interviewed made the transformation shown in figure 5.37.

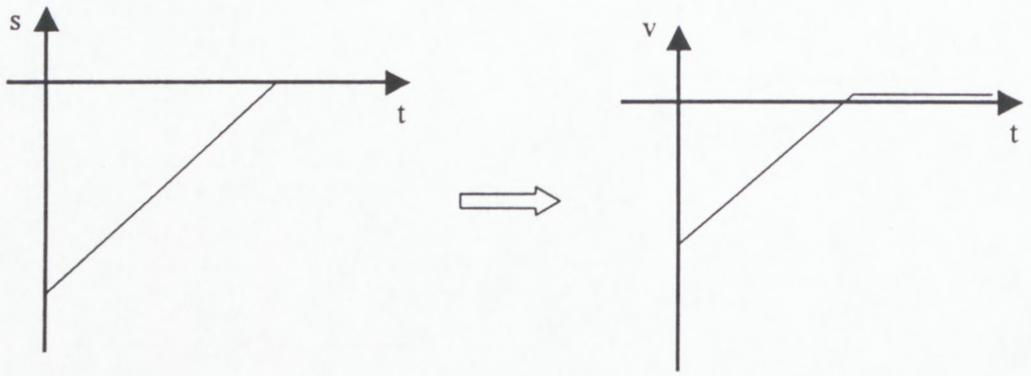


FIGURE 5.37

When asked for his reasoning, he replied: *It* (the *v-t* graph) *would be the same because if s is negative then v is also negative and it is the same motion. The graph stays on the t-axis after it strikes it.*

This student conserved both *sign* (negative), *slope* (positive) as well as *intercepts* with the axes, resulting in a conservation of *shape*. The inference that the graphical representation of different kinematic quantities will appear identical because they described the *same* motion is a disturbing one, because it points to a serious lack of conceptual understanding of translation E as well as an inability to interpret graphs correctly.

A number of students were asked to explain the *v-t* → *s-t* graph transformation of figure 5.38. Graph (a) is the correct *s-t* graph while graph (b) represents a typical student response.

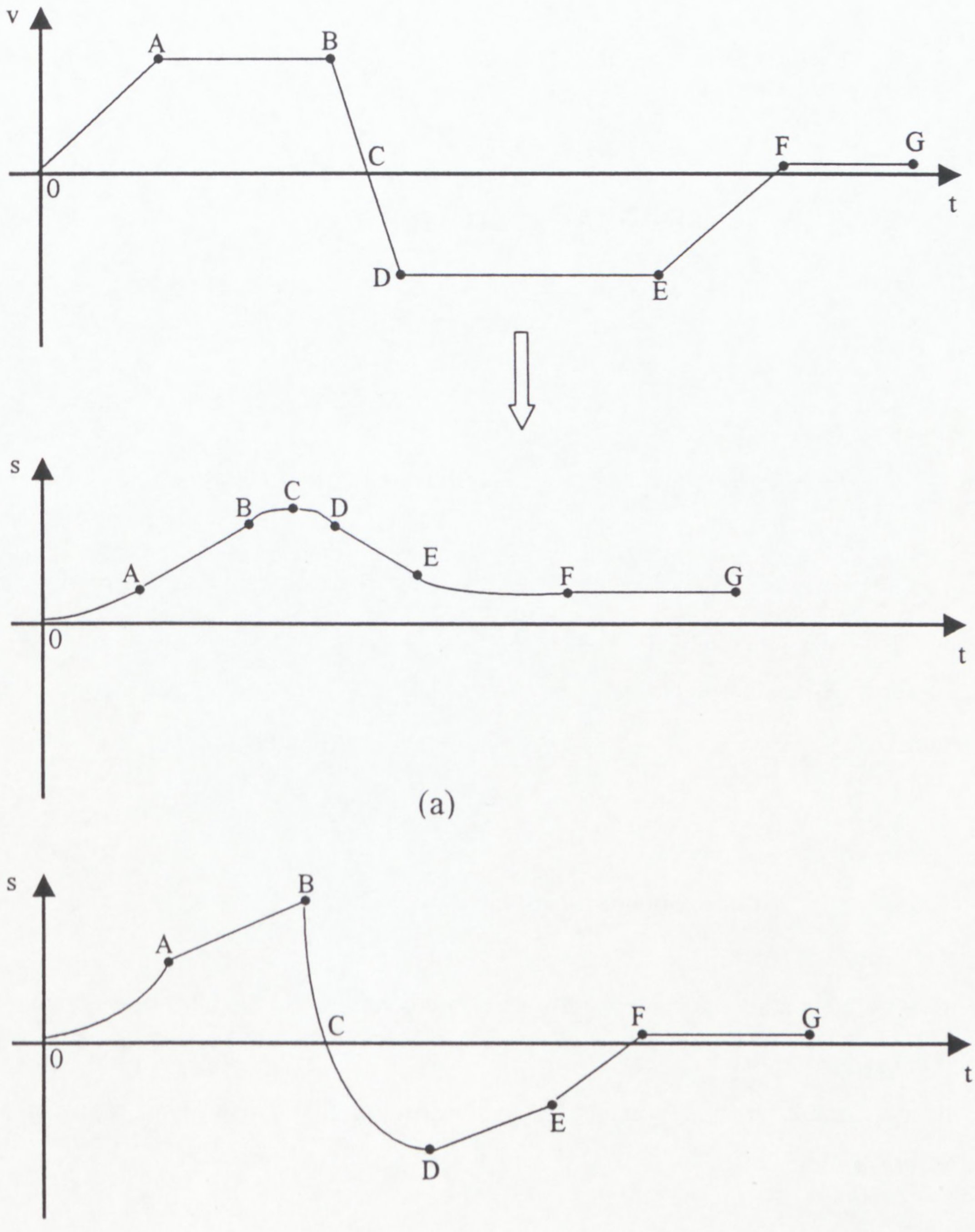


FIGURE 5.38

It is immediately clear that the students' *s-t* graphs often resembled the general shape of the given *v-t* graph. The required *s-t* graph was usually drawn fairly promptly but with a tendency to stay clear from any *verbal* explanation. When asked about the reasons for drawing various sections, they often had to think about the graphs carefully before verbalising their thoughts, the following being some typical comments:

- *as time passes the displacement becomes less* (referring to section BC of graph (b)).
- *s-t cuts t-axis at C because v-t cuts t-axis at C.*
- *at DE the displacement increases just like at AB because in both cases the velocity is constant.*
- *no velocity - no displacement* (referring to section FG).

From these comments it is again evident that for these students a conservation of slope and intercept, together with a belief that a *constant* velocity (irrespective of sign) causes an *increasing* displacement and that a *zero* velocity implies a *zero* displacement, is the underlying **reason** why the shapes of the graphs appear similar.

5.3.4 Starting graphs at the origin

s vs t graphs may start anywhere on the s-axis, unless the initial displacement or reference point is prescribed. Students, however, show a strongly developed preference for starting their *s-t* graphs at the origin, usually without stating that a displacement of zero at t = 0 s is *assumed*.

Even *v-t* graphs are often incorrectly drawn from the origin, for example in an *s-t* → *v-t* transformation where the gradient of a given curved *s-t* graph at t = 0 is non-zero.

The following table (table 5.10) summarises the incidence of students' *s-t* graphs starting at the origin as observed in the responses to the main questionnaire.

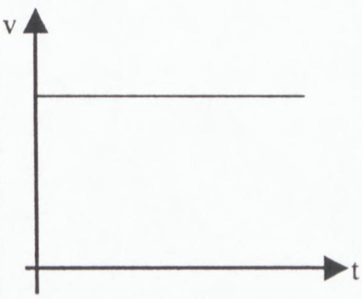
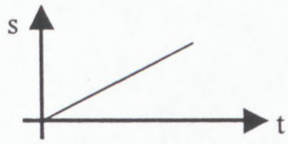
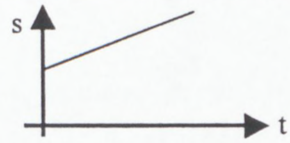
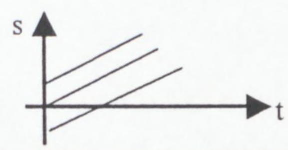
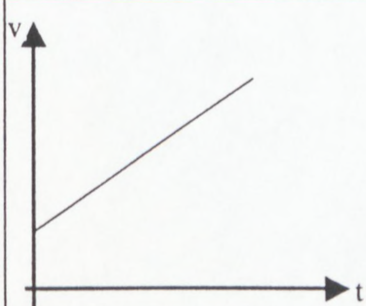
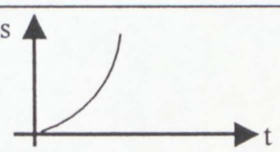
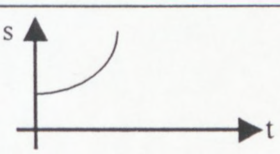
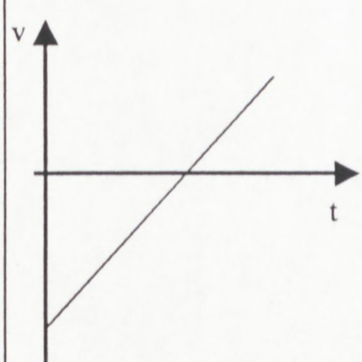
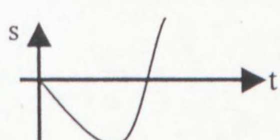
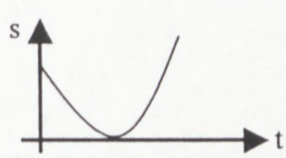
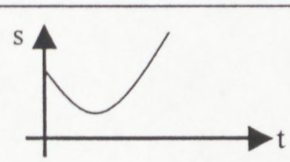
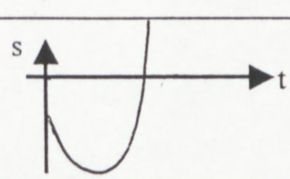
NO	GIVEN GRAPH	STUDENT RESPONSES	RESPONSE RATE
1			74 %
			3 %
			2 %
2			36 %
			17 %
3			70 %*
			20 %*
			5 %*
			5 %*

TABLE 5.10
PROPORTION OF RESPONDENTS' GRAPHS STARTING AT THE ORIGIN
(*In no 3 the percentages refer to the relative proportion of those who made the correct translation)

It is immediately evident that the notion to start *s-t* graphs at (0;0) is very strongly developed. The argument in favour of this seems to be centred in the belief that *the displacement at the start of a physical motion* ***must*** *be zero.*

In some cases a conflict between *intercept conservation* and tendency to start graphs at the *origin* has been encountered. This is typified by the second year technikon student who was requested to draw the *s-t* graph which would accompany the *v-t* graph of figure 2 in table 5.10. He initially drew the correctly shaped *s-t* graph starting at the origin and when asked why he did this, replied:

S: *No! I am changing my mind, the s-t graph must also start higher up.* (Draws the second graph of figure 2 in table 5.10) *but if you have a graph like this* (draws transformation of figure 5.39 below) *you* ***must*** *start your displacement at (0;0).*

I: *Why?*

S: *I don't know - it's just logical that the displacement starts at 0 for* ***constant*** *velocity.*

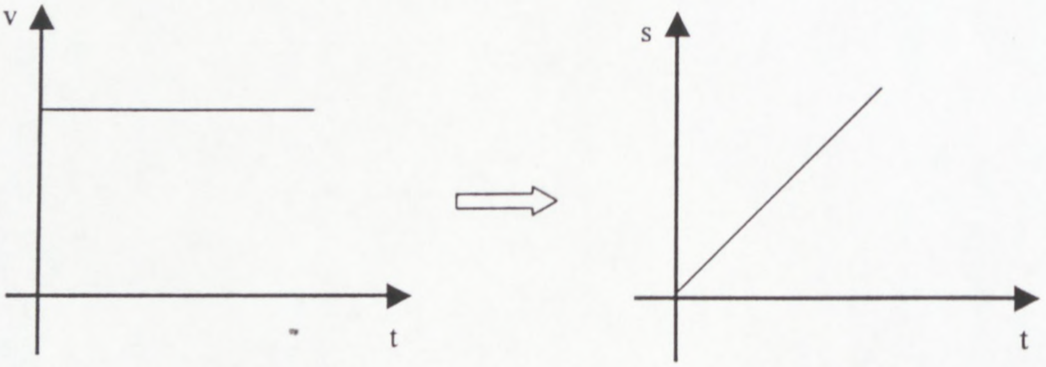


FIGURE 5.39

A large majority of the respondents who drew correctly shaped *s-t* graphs for graph no 3 started their graphs at the origin, indicating that the intercept conservation was not as compelling in this case.

Starting *s-t* graphs at (0;0) should not always be viewed as erroneous. The real problem seems to be that students believe that this **has** to happen instead of that this **may** happen (depending on which point has been taken as the reference point).

In the case of *v-t* graphs, however, starting at the origin is often *incorrect* as the transformation of figure 5.40 demonstrates:

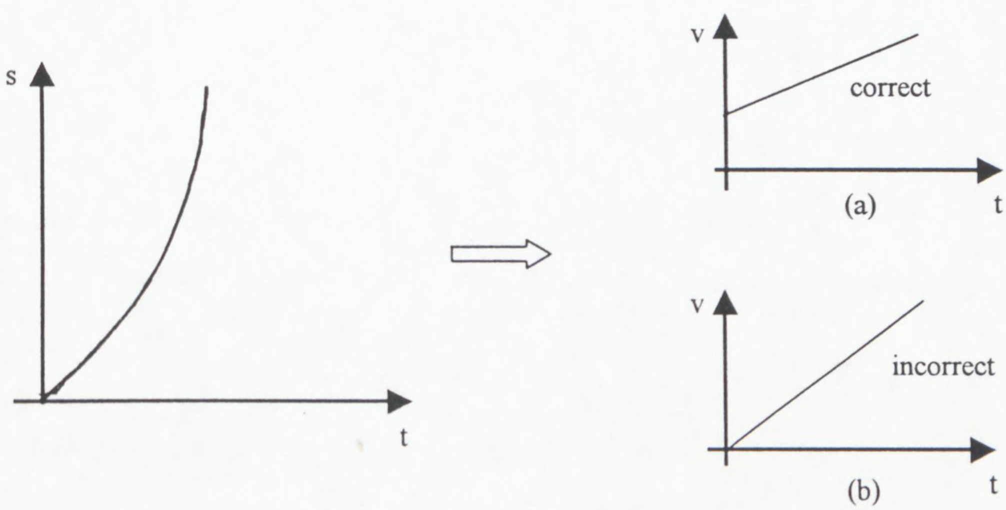


FIGURE 5.40

Students at both ends of the academic spectrum were quite convinced that graph (b) of figure 5.40 represents the same motion as the given *s-t* graph, because to them it was *quite natural and obvious that the velocity at t = 0 had to be zero because s = 0 at t = 0.*

Once again, a reluctance to observe changes in the given gradient and more specifically the positive gradient of the *s-t* graph at t = 0, leads students to incorrect conclusions about features of the required graph.

5.3.5 cusps in s-t graphs

Students' *s vs t* graphs often contain cusps which represent sudden discontinuous changes in the values of the displacement. This has been observed to occur especially when a straight line changes into a curve or vice versa.

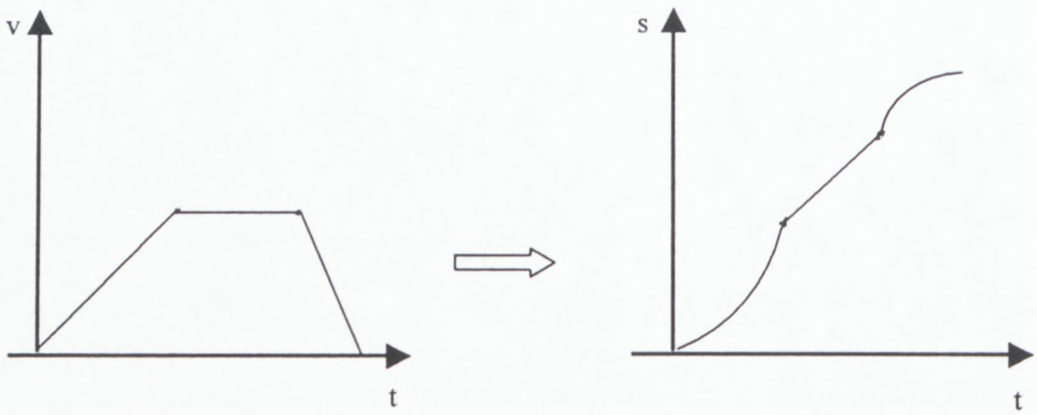


FIGURE 5.41

The results of the main questionnaire, as well as the pilot study, indicated that approximately 10 % of all respondents tended to include these cusps which are illustrated in figure 5.41 above.

A possible reason for this mistake could be that students want to emphasise what they believe to be a *sudden* change in the curvature at point A and B.

5.4 EXTERNAL CAUSES OF STUDENTS' POOR UNDERSTANDING OF MOTION GRAPHS

Many student misconceptions about kinematic graphs find their origin in factors other than those that can be directly related to the conventions of kinematics. Aspects that will be discussed in this section relate to problems with *terminology*, a poor understanding of what a motion graph *means* and the failure to *integrate* knowledge gained in the mathematics class with the content of the physics curriculum. A number of **syllabus-related** causes, which have been identified in the literature, have been confirmed by the present investigation. These will be discussed under the following headings:

- a possible mismatch between the syllabus and the students' intellectual development;
- inadequacies in teaching, textbooks and examination papers;

- more specific *instructional* shortcomings.

5.4.1 Language and terminology

Marais (1997:216) singled out the *meaning of words* to be one of the most important aspects of science learning and one that is responsible for many of the problems encountered in the process. He goes on to classify the vocabulary used in science into *non-technical terms, technical terms* and *everyday words*.

The meaning that students attach to technical terms such as *positive velocity* or *negative acceleration* has a profound effect on their understanding of kinematics and has already been discussed in the previous section. There are, however, a number of non-technical terms and every day words, which are normally used in the teaching, and assessing of kinematics. These words, such as "constant", "uniform", "rate" and "representation" when used in a physics context, can cause great confusion amongst students, especially for those whose home language is not English (see Marais 1997:179).

The word "uniform" as in *uniform velocity* is an example of a word that was given a number of different meanings by different students. The present study revealed that students thought of "uniform" as meaning either "staying constant", "steadily increasing" or "zero". The Grade 12 student who commented that: *velocity is constant, therefore the acceleration is uniform*, probably interpreted "uniform" in terms of "maintaining a constant value of zero".

However, the boy who stated: *the velocity is constant, therefore the* ***displacement*** *is uniform* was likely to attach the meaning of "steadily increasing" to the same word in this particular context.

The results of the questionnaire revealed that some students did not attach an appropriate meaning to the words "decreasing" and "increasing" when talking about graphs. A number of students called a graph such as the one in figure 5.42 (a) a "decreasing" graph, while the one in figure 5.42 (b) was termed an "increasing" graph.

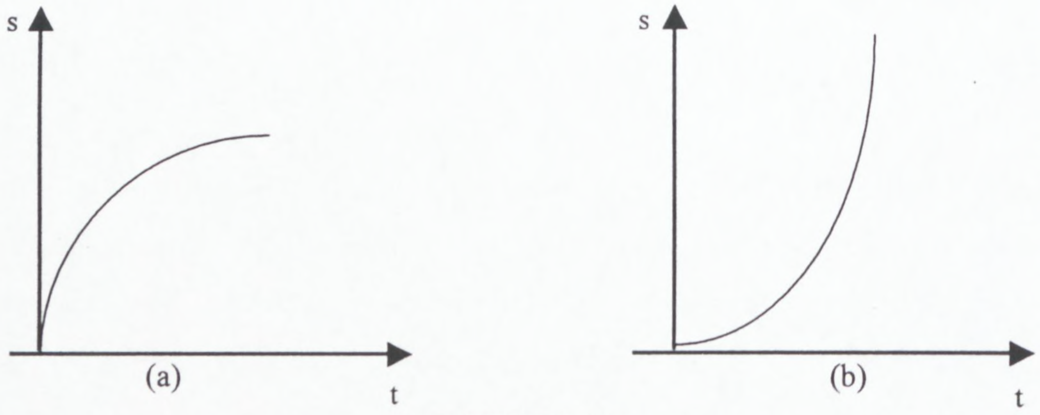


FIGURE 5.42

Students have to give the same meaning to words as those given by the scientific community if they want to communicate effectively about kinematics and kinematic graphs. This is important, both for the learning and mastering of the basic principles involved as well as for effective oral or written communication on the subject material with someone else.

The present study revealed that a number of other words, which are frequently used in kinematics, were incorrectly interpreted by a number of students. These words were:

- *deceleration* - sometimes used to indicate an acceleration.
- *time interval* - often confused with specific *values* on the time axis.
- *slope* - some students did not realise this means the same as "gradient".
- *proportional* - many students did not accept that for two quantities to be proportional, the dependent variable has to be zero when the independent variable is zero.
- *rectilinear motion* - a technical term which caused widespread confusion.
- *rest* - this was not always associated with a zero value for the velocity or speed.

- *rest* - this was not always associated with a zero value for the velocity or speed.

5.4.2 The meaning of kinematic graphs

In order to find out whether students understood a kinematic graph as a symbolic representation of the continuous covariation of a kinematic quantity and time, a number of graphs such as those illustrated in figure 5.43 were given to the students.

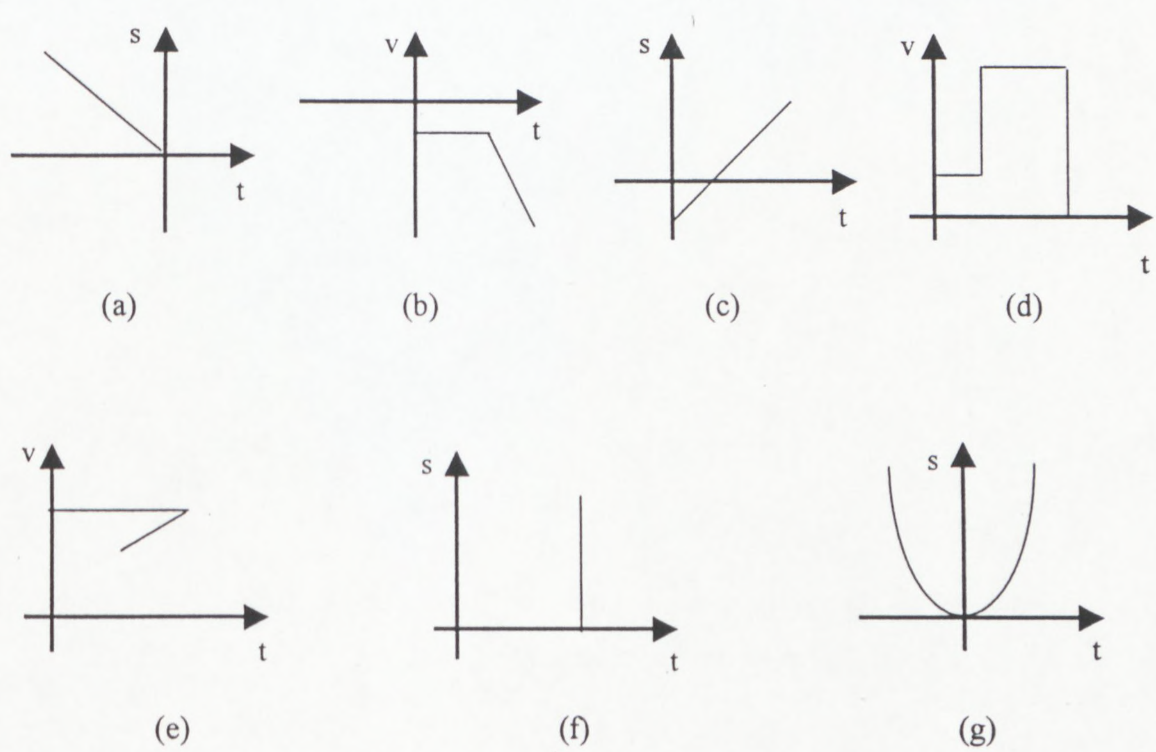


FIGURE 5.43

They were requested to explain which ones represented real motions and asked to motivate their reasoning. Nearly all those interviewed were in agreement that graphs (b) & (c) represented real motion and expressed no concern about the negative values, while most questioned the existence of "negative time" depicted by graph (a). However, many students verbalised the fact that they had no problem with graph (d) and simply interpreted the vertical sections of the graph as representing an *incredibly high acceleration*.

Graph (e) was accepted by about one third of those interviewed, with a number of students stating that... *the object travels in a straight line and then suddenly goes back to its starting point.* These students had no idea about the *meaning* of a graph and merely viewed it as a picture of the actual motion of the object. Some of these students interpreted the second part of the *v-t* graph in (b) as *something that is being thrown downward.*

Most of the students interviewed rejected graph (f), with the exception of one girl who claimed that the body was moving with constant displacement. She seemed to confuse the axes with each other and simply recalled that a parallel line to an axis represents a constant value for the kinematic quantity represented. The same error was made by a black student who spontaneously made the following transformation:

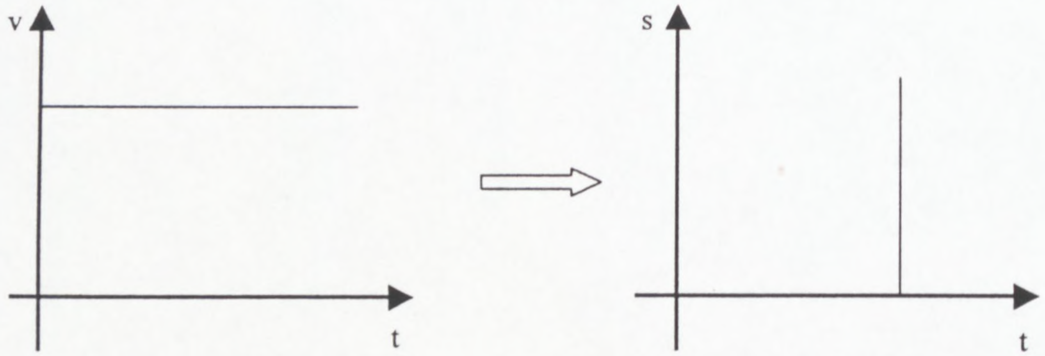


FIGURE 5.44

He gave as his reason the argument that the given graph is drawn "horizontally", therefore the related graph has to be drawn "vertically".

Graph (g) was accepted by a surprisingly large number of students with comments ranging from: *Yes, this one is all right, the negative displacement becomes positive again...* to: *it* (the body) *starts from a high displacement, comes down and goes up again.* The first student confused the two axes with each other, whilst the second student ignored the negative time values, and then proceeded to interpret the curve iconically.

A number of black students assigned compass directions to certain graphical representations, thus confusing the graph with the actual vector of the kinematic quantity.

Typical responses included a reference to the fact that the body in the graphs of figure 5.45 (a) and (b) moved in a north-easterly and an easterly direction respectively!

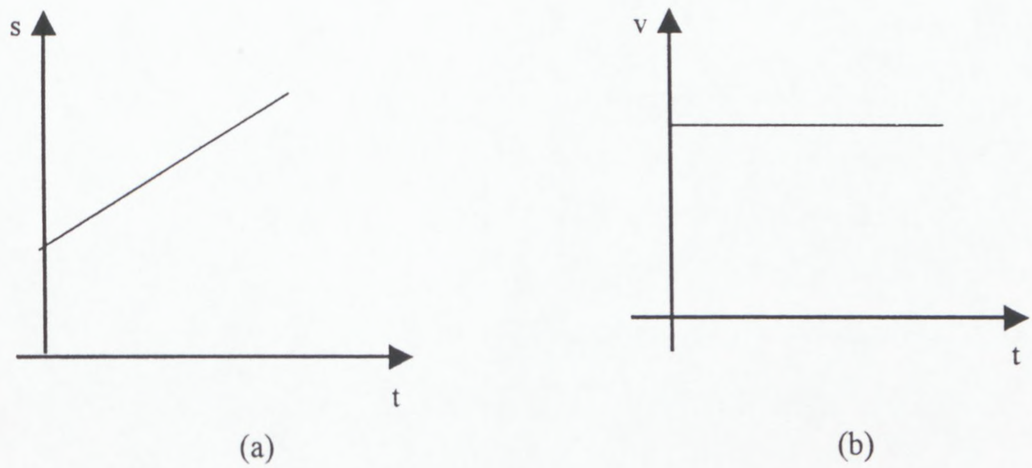


FIGURE 5.45

It transpires therefore that students do not always know what a graph *means* - a factor which can seriously impair their ability to represent real motions graphically or to transform one kinematic graph into a related one.

5.4.3 Lack of integration with the mathematics syllabus

Some of the problems that students experience with kinematic graphs could be related to a lack of associating certain topics, concepts, and procedures with those taught in the mathematics syllabus. The extent to which their lack of mastering mathematics topics influences their progress in science and particularly their mastery of kinematics graphs has not been fully researched in this study.

A number of studies, however, (for example McDermott *et al.* 1983 and Leinhardt *et al.* 1990) have reported that students who can solve graphing and other problems in mathematics often seem to be unable to use it from a different perspective in a different situation (such as motion graphs). The following topics being taught in the secondary mathematics curriculum, have a direct bearing on the facility that learners have with kinematic graphs in the senior science curriculum. (The topics are listed *without* further comment about the nature and extent of their influence on the understanding of kinematics).

(1) Gradients

Gradient of linear graphs

Negative gradients

Gradients of tangents to curved functions.

(2) Graphical representations of functions

Linear functions

Quadratic functions

Dependent & independent variables

Decreasing & increasing functions.

(3) Formulas

Changing the subject of a formula

Substitution into formulas.

(4) Operators

Note the analogy here that if sin x = y then it does not follow that sin y = x and, similarly, if the gradient of a *v-t* graph gives acceleration, then the gradient of an *a-t* graph does not give velocity

(5) Areas

Areas of triangles & rectangles

(6) Absolute values & magnitudes

$-6 < -5$, but:

$|-6| > |-5|$

(7) Calculus

Rate of change

Derivative as the gradient of the tangent to a graph

Application of derivatives to kinematics.

Calculus is normally taught in the mathematics syllabus *after* the section on kinematics has already been completed in the science class. This means that learners have to study kinematic graphs and aspects such as instantaneous rate of change without a knowledge of limits or differentiation techniques. Furthermore, integration is not included in the South African high school mathematics syllabus, a fact which seriously impedes an understanding of the reason why areas under *v-t* and *a-t* graphs give actual numerical values for the change in displacement and the change in velocity respectively.

5.4.4 Students' intellectual development and the syllabus content

Students are often not at the appropriate stage of intellectual development to cope with the idea of a graph as an abstract symbolic representation of the covariance between two variables. Helm (1978, 1980) refers to this problem when he reports on his investigation into typical students' errors in science:

> *.the misconceptions revealed by students' responses may be calling attention to a general mismatch between students' intellectual development and the demands made upon them by the content of their course.*
>
> (1980:97)

This view is also supported by authors such as Padilla *et al.* (1986:25); Wavering (1989:377); Leinhardt *et al.* (1990:54) and Brasell and Rowe (1993:69) who all agree that both the syllabus content and method of instruction in science should take into account the

Piagetian stage of development of the particular learners to whom this instruction is directed.

Leinhardt *et al.* (1990:16) describe various translation tasks which require either interpretation of a given representation or construction of a new representation, given a particular representation. The intellectual demands made on the learner are viewed by these authors as of a higher level in the case of construction tasks than in the case of pure interpretation tasks:

> *Whereas interpretation relies on and requires reaction to a given piece of data (e.g. a graph, an equation or a data set), construction requires generating **new** parts that are not given.* (1990:12)

Leinhardt *et al.* (1990:16) next describe three types of *translation* tasks, which all require students to be in the "formal operational" stage in order to perform them successfully. These are described as:

a) the act of recognising the same function in different forms of representation,

b) identifying for a specific transformation of a function in one representation, its corresponding transformation in another representation,

c) constructing one representation of a function, given another one.

In the light of these three categories a fourth category is proposed, which describes the action involved in graph → graph transformations:

Given one (graphical) representation, constructing a similar (graphical) representation of a different function, which describes the same physical situation.

This action clearly involves both interpretation and construction tasks and when this is linked to two algebraic functions, students often fail to come to grips with these questions.

Of all the students interviewed, none could fully master the algebraic rigour demanded by the *v-t → s-t graph* transformation described below. The kinematic situation is initially described by the *v-t* graph of figure 5.46. The idea is that students must deduce an algebraic expression for s in terms of t and then draw the appropriate s-t graph. The solution is provided here as an illustration of the level of abstraction required.

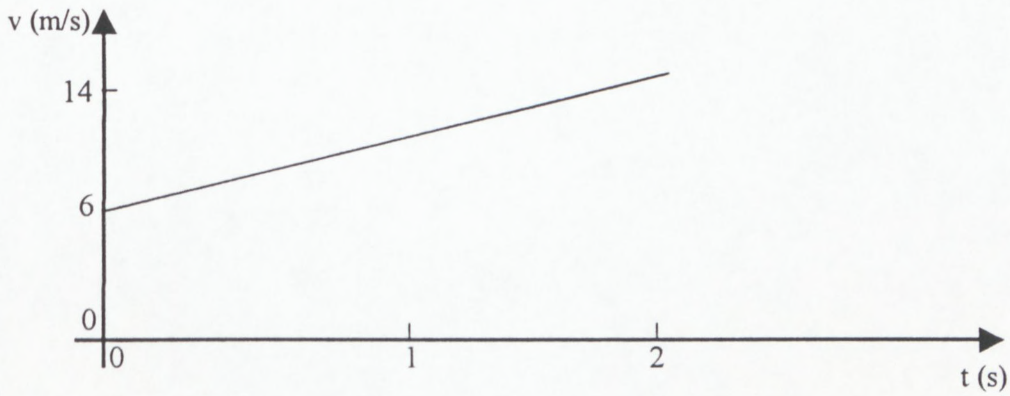


FIGURE 5.46

Now: $v = u + at$

and $u = 6$ (the vertical intercept)

$a = \frac{8}{2} = 4$ (the gradient of the graph)

hence $v = 6 + 4t$

But $s = ut + ½at^2$ which is a *different* function describing the *same* event.

substituting the u and a values: $s = 6t + 2t^2$

and factorising: $= 2t\,(3 + t)$

Now if s = 0 then t = 0 or -3, leading to the graphical representation of s vs t: (see figure 5.47)

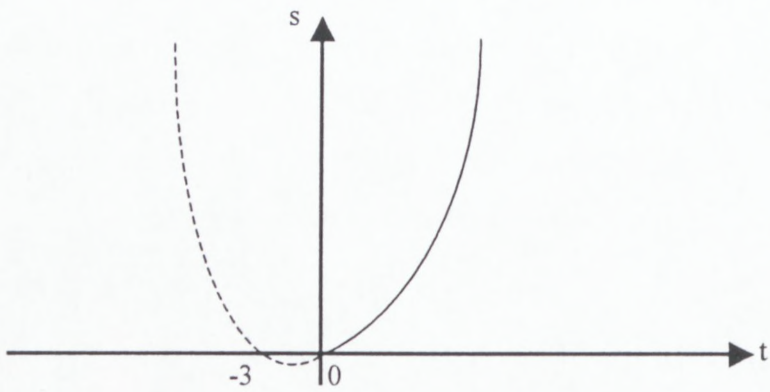


FIGURE 5.47

[Note that negative t-values, although having no real meaning, do assist with the construction of the graph.]

Personal experience confirms the findings of the present study: The high level of abstraction in terms of both interpretation and construction tasks which is required by the example above, is often beyond the level of mental development of the students who are required to understand it.

5.4.5 Inadequacies in teaching, text books and examination papers

A possible reason for learners' poor performance in tasks relating to the construction and interpretation of kinematic graphs could be deficiencies in the mathematics and science syllabus with regard to topics related to graphing skills in general.

The nature and extent of the problems that students experience with graphs have been extensively described in paragraph 3.5, so that only a few additional points will be made here to underline the importance of sufficient emphasis on graphing in the curriculum.

Padilla *et al.* (1986:25) call for... *the need for more training beyond the plotting of points, especially if we intend to use graphs for interpreting information in high school science* *courses*, while Wavering (1989:379) stresses the importance of training young learners to understand graphing procedures by using data generated by the students themselves. He goes on to call for a less abstract approach to graphing, even in the mathematics class. Brasell and Rowe (1993:67) undertook a study which identifies among learners a serious lack of confidence and competence with graphing techniques and conventions **as a direct result of inadequate instruction.**

Students might therefore be struggling with graph $\rightarrow$ graph transformations because they have not been properly guided (both in the mathematics and the science class) how to *interpret* graphs. In the words of Leinhardt *et al.*:

> *What has been well documented is that the presence of one element, such as construction of graphs from tables, does not produce capability in another area, such as interpretation of qualitative graphs. This suggests that, regardless of entry, the sequences* (of teaching graphs) *must contain all significant elements.* (1990:53)

Inadequacies in instruction as a contributing factor to commonly occurring misconceptions can be related more specifically to shortcomings with regard to some or all of the following aspects of the curriculum:

- the curriculum content;
- textbooks;
- the teaching approach and subject knowledge of the instructor;
- examination papers.

Helm (1978:288) investigated students' and teachers' understanding of certain key aspects of mechanics and concluded that many teachers share the same misconceptions as their students. Textbook writers and examiners of science also have been seen to contribute to students' misconceptions:

> *It is tempting to suggest that in these cases at least, acquisition of misconceptions by pupils may derive from or be reinforced by three main influences: teaching, textbooks and examinations* (Helm 1978:288).

Often it is not blatant errors, but rather an incomplete exposition of the material or the omission of certain important information in examination papers, which gives rise to the acceptance of incorrect ideas in science in general and kinematics in particular. Helm (1978:296) describes how this could have influenced the performance of the respondents in his investigation:

> *In such a milieu it is to be expected that if certain misconceptions or errors of commission or omission appear in textbooks and examination papers, they might be reflected in pupil and teacher performance on a suitably designed test.*

Thijs (in Thijs *et al.* 1987:44) points out that traditional textbook exercises (and examination questions!) often refer to situations where students only have to reproduce the *application* of mathematical formulas and algorithms given to them in the theoretical introduction. When these students have to interpret *real-life* situations they often resort to their intuitive beliefs, thus failing to apply the principles and definitions learned in the theory class.

The present study revealed a number of shortcomings in the prescribed syllabus for kinematic graphs, inadequacies in textbooks and incidents of confusing external question papers. The impact of these shortcomings on students' understanding of kinematic graphs is especially noticeable in the case of textbooks and examination papers, as most students interviewed stated that they rely almost exclusively on these in order to study and understand the subject material. Examples of specific shortcomings in local textbooks and external examination papers, which could play a vital role in the establishment of specific misconceptions, include the following:

- Initial displacements are <u>usually</u> taken as **zero** and the initial direction of motion is <u>always</u> taken as **positive**.

- External exam-papers often do not prescribe reference points and sign conventions and textbook writers seldom encourage students to establish these for themselves.

- The terms *deceleration* and *negative acceleration* are frequently confused.

- A negative velocity of *increasing magnitude* is sometimes referred to as an increasing velocity. This causes learners not to realise that in the case of vectors *depicting negative quantities, an increasing magnitude of this quantity should be associated with a decrease in the corresponding vector.*

- Experiments controlled and executed by students are not sufficiently encouraged, resulting in lost opportunities to implement the principles advocated by the constructivist theory.

Leinhardt *et al.* (1990:46) refer to the problem of deficiencies in the subject knowledge of the teacher and stress the negative effect that this can have on the teaching and learning of subject specific material such as graphs:

> *Limitations on subject matter knowledge on the other hand, often reduce the flexibility and creativity of a teacher as well as create a kind of authoritarianism toward the subject and student that permits little or no exploration of ideas.*

Weaker students tend to study examples of certain principles and laws and view these as prototypes for solving other similar problems. Even when these examples are incorrect they will be remembered...*as hooks for further misconceptions* (Leinhardt *et al.* 1990:52).

Thijs (in Thijs *et al.* 1987) studied the influence of teaching on learners' understanding of the basic properties of *forces* and, since a proper grasp of the nature of forces can be directly related to an understanding of velocity and acceleration, he is quoted here:

> *In talking on force in physics lessons we should be very precise, since many misconceptions are caused by incoherent teaching.* (1987:54)

The fact that ineffective teaching consolidates learners' widely held misconceptions with regard to *kinematic graphs* in particular is reported by, among others, Brasell (1987), McDermott *et al.* (1987), Mokros and Tinker (1987), Goldberg and Anderson (1989) and Leinhardt *et al.* (1990). The main hypothesis put forward by these researchers can be briefly stated as follows: **Students' poor grasp of graphical representation can, to some extent, be blamed on ineffective instruction.**

Interviews conducted with individual students in the present study, suggest a number of specific shortcomings in the *teaching* of kinematic graphs. One student stated:

> *My teacher teaches kinematics from the textbook and does not give us enough different graphs to practise drawing. He makes the stuff seem a bit abstract so that I can't picture what it looks like.*

When asked about the integration of "algebraic" graphs with motion graphs, the student replied that he has never been encouraged to "link" the two topics and added that...*the teacher does not talk much about reference point and choice of positive direction.*

Inadequacies in the curriculum, together with shortcomings in the instruction of kinematics encourage the continued interpretation of graphs as pictures or "icons" of the physical problem scene. Not understanding, accepting or knowing the conventions used to describe vector quantities can thus cause learners to interpret all kinematic graphs as position-time graphs.

5.4.6 Specific shortcomings in instruction

The findings of the empirical investigation together with a study of the transcripts of individual interviews conducted with students, lead to the following identified shortcomings in the teaching of kinematic graphs in local schools:

- learners are not encouraged to investigate a large number of different graphical representations representing different physical situations;

- a teaching approach that is too textbook-bound;

- not emphasising the role of gradient and area - particularly with regard to graph-graph transformations and not allowing sufficient practise in selecting these and other features, such as height and co-ordinates, to learn more about the real situation which is being investigated;

- assuming that students understand the meaning of certain key words such as *slope*, *interval* and *uniform*;

- not allowing students to discover for themselves the important role of sign convention and reference point in kinematics;

- a reluctance to give clear instruction in those areas of kinematics where students have been shown to exhibit serious deficiencies in their understanding. The present study suggests that particular instruction is required in the following areas:

  - representation of *negative* kinematic quantities.
  - the tendency to *conserve* the properties of a given graph.
  - confusion between *acceleration* and *velocity*.
  - graphs involving a *reversal in direction* of the motion.

- a failure to integrate the knowledge that students acquired about graphs in *other disciplines*, particularly mathematics;

- a teaching approach that concentrates on an abstract mathematical treatment of the study material and does not always allow for basic definitions and conventions to be stated in clear uncluttered language.

Other shortcomings with respect to the *teaching* of motion graphs that were identified by researchers such as McDermott *et al.* (1983, 1987), Brasell (1987), Mokros and Tinker (1987), Goldberg and Anderson (1989) and Leinhardt *et al.* (1990) include the following:

- a tendency to teach **procedural**, rather than **conceptual** issues;

- not emphasising **qualitative graphs** (i.e. sketch graphs) sufficiently which means learners do not get adequate training in aspects such as *pattern detection* and *the meaning of gradient*;-

- students are not given sufficient experience in applying basic concepts of graphing to the **motion of real objects**;

- not allowing students to design and perform experiments for themselves and using the results from these **student-controlled investigations** for graphical displays and analysis;

- not devoting sufficient attention to the **meaning** of the **basic kinematic quantities**;

- explanations do not always build on the present state of **students' knowledge and intuition;**

- graphing exercises are often confined to the topic under discussion (in this case kinematics) instead of relating these exercises also to **topics outside the physics curriculum;**

- **limited subject matter knowledge on the part of the instructor**, which often leads to certain incorrect student preconceptions being reinforced, instead of *altered.*

These shortcomings with respect to the way in which kinematic graphs are traditionally taught are likely to be further enhanced by a vague and often confusing curriculum as well as deficiencies in the available textbooks and external examination questions It is possible that inexperienced and underqualified science teachers are more likely to be relying on what is written in textbooks and what is asked in external examination papers than their experienced and well-qualified colleagues and could thus be more inclined to display the above-mentioned questionable teaching practices (see for example Helm 1980:96).

5.5 INSTRUCTIONAL STRATEGIES FOR THE TEACHING OF GRAPHS, WITH SPECIAL REFERENCE TO KINEMATIC GRAPHS

This section describes certain techniques and approaches to the teaching of kinematic graphs. It is suggested that these (untested) teaching techniques might assist in eliminating or reducing some of the more commonly occurring misconceptions and/or mistakes identified by the present study. In order to discuss this meaningfully, it is necessary to first report on some of the research findings with respect to the teaching of science in general and of graphs in particular.

5.5.1 The need for instruction in science in general

Most researchers agree on the importance of *active and deliberate instruction* to expose and clarify the subject material that has to be mastered (see for example Reif *et al.* 1976;

Trowbridge and McDermott 1981; Treagust 1988; Jordaan 1990 and Leinhardt *et al.* 1990).

Reif *et al.* (1976:212) emphasise however that the teaching of science should involve much more than the mere transmission of factual information. To them, a more important goal is to:

> *...teach students to use basic facts and concepts flexibly so that they can deal with new situations, predict various consequences, and solve problems.*

They go on to point out that learners usually lack systematic strategies to apply appropriate facts and principles to fresh situations. According to Reif *et al.* (1976:216) the specific learning and problem-solving skills involved in activities such as *description*, *planning*, *implementation* and *checking* ***can and must be taught.***

Treagust (1988:167) stresses that the identification of specific misconceptions should inspire and motivate a science teacher to attempt to remedy the problem by developing and/or utilising alternative teaching approaches which endeavour to address these misconceptions. Leinhardt *et al.* (1990:51) investigated those features that good instructional explanations have in common and came to the following conclusion:

> *Expert teachers tend to include specific elements in their explanations. Characteristically, these explanations include carefully selected representations and well-developed verbal explanations. The problem and the principles used in the solution are clearly flagged and marked. Specific errors are deliberately investigated rather than easily overlooked.*

Jordaan (1986:19-24) suggests a number of ways in which instructors can go about improving learners' poor understanding of a law (in this case Newton's 3rd law). The instructional strategies that he proposes are based on the following points. (The implication for the teaching of kinematic graphs is included by way of illustration).

1 *The ability to define (kinematic) concepts and quantities does not guarantee that a student understands the concept. They should be given ample examples in which they have the opportunity to identify the quantities.* This means that learners should be encouraged to do more than merely learn the definitions of the important kinematic quantities. They must demonstrate their ability to recognise their role in describing many different kinematic situations, together with the ability to differentiate between the various graphical representations.

2 *Clarify the mutual relationships between the processes or concepts.* Instructors need to ensure that learners understand the relationships between related quantities such as velocity and acceleration or velocity and speed.

3 *Students should be given an opportunity to state a law or procedure in their own words.* This encourages students to move away from set-piece type memorisation of the definitions of the various kinematic quantities. It also suggests that learners must be given the opportunity to explain important principles, like the role of gradient in graph transformations, in their own words

4 *Provide students with as many applications to everyday situations as possible.* Students should be encouraged to analyse the motion of everyday objects, such as motor cars, athletes or trains, as well as the motion of objects in the laboratory, in order to describe the time-rate-of-change of the various kinematic quantities.

5.5.2 Constructivism and instruction

The importance of students constructing their own meaning about a scientific topic has been explored in section 3.2. The serious implication that this has for the teaching of science has been acknowledged by a number of researchers. Leinhardt *et al.* (1990:54) observe in this connection that ... *we need to understand what students know and the power and utility of their intuitions at different age levels and after different kinds of instruction. The power of anchoring instruction around core concepts needs to be explored.*

Strong support for the necessity of finding out what the existing knowledge and understanding about a topic is and then making sure that explanations link up with this knowledge comes from Trowbridge and McDermott (1980), Halloun and Hestenes (1985), Freyberg and Osborne (1985), Thijs (1987), Hewson (1987) and others. Hewson (1988:44), for example, stresses that instructors, through questioning strategies and analysis of homework and tests, must find out what conceptions students bring to a learning situation and organise instruction so that it starts with that knowledge. Thijs (1987:44) warns in this connection that:

> *conventional physics instruction superposes a superficial layer of knowledge of physical laws and concepts on top of the intuitive beliefs, without really modifying them... students are therefore not always challenged to restructure their preconceptions.*

Freyberg and Osborne (1985:82) quote the views of five prominent researchers with respect to what the response of instructors to the constructivist theory should be:

- *find the subskills that a learner has, then develop the learner sequence starting from these subskills (Gagne and White, 1978);*

- *find the logical structures of thought the child is capable of and match the logical demands of the curriculum to them (Shayer and Adey, 1981);*

- *find the prior concepts of the learner and determine the necessary links between what is to be taught to what the learner already knows (Ausubel, 1968);*

- *find the alternative viewpoints possessed by the child and provide material in such a way so as to encourage the child to reconsider or modify these viewpoints (Driver, 1980);*

- *find the meanings and concepts that the learner has generated already from his or her background, attitudes, abilities and experiences to determine ways so that the* *learner will generate new meanings and concepts that will be useful to him or her (Wittrock, 1974).*

The implication that these ideas have for an instructor of physical science who has to teach kinematic graphs is simply this: **Take note of the prevailing alternate viewpoints of learners with regard to motion and graphs and organise the teaching so as to link up with these pre-conceptions.** Examples of questions that should be asked include the following:

- *What is the present state of their knowledge about graphs in general?*
- *What is their understanding of the concepts of speed, distance and other "everyday" quantities?*
- *What meaning (if any) do they attach to the idea of rate of change?*
- *Is their understanding of gradient merely instrumental or do they understand it in terms of rate of change?*
- *What is their current understanding of the meaning of tangents to curved graphs.*

5.5.3 The teaching of graphs in general

In order to obtain some idea of how instructors should go about teaching *kinematic graphs*, it is necessary to report on a few research findings about the teaching of *graphs in general*. Mansfield (1985) strongly endorses the fact that students carry with them intuitions of graphing from both their real-world experience and their past formal mathematics education and, in keeping with the constructivist theory, he contends that this should be recognised when starting the topic of graphing. Leinhardt *et al.* (1990:51) quote Krabbendam (1982) who agrees that all explanations should be built from the child's own self-explanation.

According to Leinhardt (1990:51) Krabbendam uses naturally occurring situations (familiar to the learner) to establish a common base and then has the students justify their qualitative graphs. The idea is that they are encouraged to acknowledge the need to modify their original preconceptions, in order to establish a more acceptable conception about graphs and graphing.

Clement (1989:85) contends that the idea that a vertical line represents the magnitude of a physical quantity, is easily accepted by even 10 year olds. This, to Clement, constitutes a natural subskill or prior concept upon which the learner can *construct* his understanding of what a graph is, namely (in the words of Clement):

> *The curve of a graph represents the infinite set of uppermost points of all vertical lines representing for example the speed of a car at a certain instant.* (Clement 1989:85).

Although some children have a natural "feel" for graphs as representations of physical phenomena and to a lesser extent mathematical relations, *graphing literacy* does not develop spontaneously. Brasell and Rowe (1993:69) for example point out that students need to have repeated experience with a wide variety of graphs *used as an integral part of communicating information in many courses and contents*. They call for graphs not only to be merely present in the curriculum, but to become <u>cognitively available</u> to students and stress that...*room must be made in our already overcrowded curriculum to teach adequate graphing skills*. McDermott *et al.* (1987:513) support the notion that graphing skills should be explicitly taught:

> *It has been our experience that literacy in graphical representations often does not develop spontaneously and that intervention in the form of direct instruction is needed.*

The instruction should attempt to *integrate* the use of graphs in mathematics and other subjects as well as different topics within science itself. McDermott *et al.* (1987:512) stress this point when they claim that instructors should... *take advantage of the increased depth* *of understanding that comes from using the same procedure and reasoning in different contexts,* while Brasell and Rowe (1993:69) state: *Students must practice interpreting graphs in a number of different contexts before they are able to transfer their graphing skills to new topics.*

Instructors should point out to their students that their ability to work with graphs is often more important than their knowledge about a specific topic which is represented graphically (see for example Schuster 1983, McKenzie and Padilla 1986 and McDermott *et al.* 1987). Schuster (1983:324) refers to representations in general when he states ... *for students not majoring in physics, their grasp of representations may ultimately be more useful to them than their knowledge of kinematics.*

The question can now be asked: **How should the teaching of graphs be started and in what sequence should the material be presented?** Leinhardt *et al.* (1990) addressed this question for graphs in mathematics and, because of the direct implication for the teaching of graphs in science, their views will be briefly summarised here. Leinhardt *et al.* examined the available literature on the question of entry and sequencing with regard to the teaching of graphs and distinguished three different ways of starting the topic, namely (a) discover the rule; (b) generate the data and plot; and (c) interpret qualitative graphs of situations. The first of these (discover the rule) is based on students attempting to discover an appropriate function which matches certain given "input" and "output" values. The idea is that students are confronted with a table of data values for which they are supposed to discover an algebraic rule in the form of an equation. The resulting function (often linear or quadratic) is then graphed according to familiar conventions learned in the mathematics class.

The second possible entry consists of an activity which takes data gathered in a *real context*, organises it in some way and then builds a graph from this data. The data should preferably be generated by a problem or an experiment that students understand and can control.

The use of qualitative graphs, originally proposed as an entry by Krabbendam (1982), requires some knowledge of the conventions of graphing; it does, however, stimulate the natural curiosity of young learners about a relation such as, for example, growth as function of time. In the science curriculum this approach would imply that students are encouraged to interpret given qualitative graphs of physical variables in terms of general (or global) tendencies or patterns *before* proceeding to plot data points on a system of axes.

On the question of sequencing, Leinhardt *et al.* (1990:49) distinguish two possible routes. The first, proposed by Janvier (1982) and Swan (1982), moves from tabulation of data, through plotting and reading of values, to sketch graphs and curve fitting. The emphasis of this approach is on capitalising on student interest in that it moves from displaying specific real data to the recognition of functions and the sketching thereof.

The second approach builds on students' intuitions about qualitative phenomena, having students construct *non-quantitative, low-detail but measurable covarying variables* (i.e. sketch graphs). Students are then encouraged to describe *global tendencies* of their graphical representation before a quantitative, point-wise, more mathematical focus is encouraged. (see Larkin and Reif 1979 and Krabbendam 1982). Pechenik and Tashiro (1992:432-434) put it strongly by suggesting that instructors should actively *discourage* students (who start working with graphs) to focus on specific data points or comparison of data points, but rather to encourage them to focus on *general tendencies* in order to see if it makes sense in terms of the specific investigation which was undertaken.

> *Students can become thoughtful consumers of scientific information only if they learn to evaluate data critically, along with the experimental designs that give rise to the data.* (Pechenik and Tashiro, 1992:432)

Wavering (1989:379) believes that specific pregraphing skills need to be identified so that they can be positioned in the curriculum at the appropriate grade level. He goes on to suggest (p. 379) that ... *before quantitative understanding can occur the teaching of graphing should begin with activities that develop one-to-one correspondence between data, serration activities and recognition of pattern in a series of changes.*

Leinhardt *et al.* (1990:49) stress that all the research points to an approach to graphing that moves **from the less formal, less abstract, more global and intuitive to the formal,**

notationally rigorous system. Furthermore, they believe that there should be an active attempt by instructors to move back and forth between qualitative and quantitative representations of graphs, as ... *research consistently supports a lack of relationships between the specific, notational, analytical capabilities and the global qualitative ones.* In practice this means that students must be made aware of the difference between qualitative sketch graphs and accurate quantitative graphs. They should be encouraged to understand the value of the former in describing global tendencies of the variable and the latter in giving specific point-wise information.

On the question of the origin or source of the data to be represented graphically, there is considerable agreement that it should be generated from experiments or activities that students themselves can control or at least can identify with (see for example Helm 1980; Schuster 1983; Mokros and Tinker 1987; Wavering 1989 and de Jager 1990). Mokros and Tinker (1987:381) put it best when they observe ... *learning about graphs may be particularly meaningful when done in the context of experiments that students understand and control.* De Jager (1990:161) recognises the value of drawing and interpreting graphs in the context of **everyday real life situations** and claims that this helps to overcome many graphing problems that students have. This, according to de Jager (1990), is especially true in cases where the link between the behaviour of the graph and the nature of the physical system that it attempts to describe, is not immediately obvious.
McDermott *et al.* (1983:18) in their investigation of students' understanding of kinematic graphs have this to say about teaching graphs in general:

> *instruction is best conducted in the laboratory where students can observe and manipulate physical systems that are being represented graphically.*

From the preceding discussion a number of key points can now be highlighted with regard to the teaching of graphs to students studying science:

- *Specific graphing skills and conventions should be taught* before topics are taught where these skills are *applied*. Examples of these specific skills include:

- an understanding of what a graph is;
- the use of scaling and labelling axes;
- working with sketch graphs and accurate graphs;
- the significance of gradient.

▪ Instructors must be aware of learners' *previously acquired knowledge bases* about graphs and build on (and modify where necessary) their preconceptions about graphs.

▪ Learners must be made aware that graphing strategies used in a specific context are applicable to *other subject areas* outside of science.

▪ Learners could be introduced to graphs by means of two possible routes, each having particular advantages. These can be described as a *"generate the data and plot"* approach or an *"interpret qualitative graphs of real situations"* approach.

▪ The *variables* that are represented should be such that *learners can identify with* them.

▪ When data points need to be generated, these should, where possible, come from *experiments which learners themselves perform.*

▪ Once learners have mastered the basic concepts of graphing, instructors should encourage them to move *back and forth between a quantitative and qualitative description* of the particular problem scene.

5.5.4 The teaching of kinematic graphs

In section 5.3.3 it was indicated that a number of studies suggested that certain specific graphing skills should be taught explicitly, as students do not always possess the necessary insights into graphing when a topic such as kinematics is taught.

Researchers such as Schuster (1983), De Jager (1990) and Leinhardt *et al.* (1990) are in agreement that graphing problems in general can be addressed by encouraging students to interpret and construct graphs in the context of kinematics, which often relates to a real-life situation that most learners can understand and identify with. The reverse, however, has also been acknowledged as true, namely that difficulties with kinematic concepts and conventions can be addressed through an increased emphasis on graphing (see for example Orton 1984 and McDermott *et al.* (1983, 1987). McDermott *et al.* (1987:13) state in this regard: *facility with graphing can play a critical role in helping students deepen their understanding of certain kinematic concepts*, while Orton (1984:25) reports an improved understanding of the "rate of change" concept if it is taught in a graphical context.

Peters (1982:508), Schuster (1983:334) as well as Orton (1984:25), report that most instructors are not aware of many of the conceptual difficulties with respect to kinematics that impede the progress of their students and that they are often surprised by their poor performance. Orton (1984:25) refers to the idea of tangents and rate of change when he states ... *the situation appears to be one where students need more help than might have been expected.*

Trowbridge and McDermott (1981:253) and Rosenquist and McDermott (1987:415) believe that it is important for instructors to devote special attention to those concepts and definitions in kinematics with which students have been found to have particular difficulty. Only after these concepts are understood and clearly differentiated from different but related concepts, should the graphical representations of kinematic quantities be investigated. The difference between velocity and acceleration, instantaneous velocity as a limit and acceleration as the ratio of change of instantaneous velocity to elapsed time, are examples of topics that have to be addressed by observing the motion of *real objects*. Students are then confronted, through laboratory exercises, class discussions and individual discussions, with situations to help them to differentiate between various kinematic concepts and kinematic quantities in order to represent them graphically.

Poduska and Phillips (1986:847) presents evidence which indicates that topics in kinematics should be presented in the order: *distance, speed, time and proportional reasoning* in order to be in keeping with the natural order of the mental development of young learners. Boulanger (1976:8) stresses that care should be exercised to ensure that a concept such as speed (or velocity) is not simply a formula which students memorise ... *but is well-founded on competence in recognising all the relationships summarised, quantified and symbolised in the formula*.

Kinematics is an ideal vehicle for illustrating translations between different representations in a specific context. The physics education group at the university of Washington (see McDermott *et al.* 1983, 1987) have suggested that the specific translations between real-life situations and their graphical representation is best conducted in the laboratory, where students can actually observe and manipulate physical systems that are being represented graphically (see also Schuster 1983 and Goldberg and Anderson 1989).

The "physical system" suggested by these authors usually consists of a number of aluminium tracks held at various angles on which steel balls are allowed to roll down..

Students are encouraged to go in both directions, i.e. draw kinematic graphs for certain observed motions or arrange the tracks in order to produce an appropriate motion to match a given graphical representation of that motion. (see Trowbridge and McDermott 1981:253). Instructors have the important task of ensuring that students make the proper correspondence between the features of each graph and the various characteristics of the observed motion. In view of the fact that they frequently underestimate their difficulties with kinematic graphs (see for example Brasell and Rowe 1993:65) and are thus unaware of their own graphing deficiencies, some form of active intervention in the form of instruction is necessary to expose the relationship between physical phenomena and the concepts and conventions of graphs and graphing. Examples of kinematic graph *features* that require instruction in order to relate them to the relevant *characteristics* of a motion, include the following:

- maximum velocity is represented by the *steepest slope* of the displacement-time graph and the *greatest height* on the velocity-time graph.

- speeding up or slowing down is represented by the *increasing or decreasing steepness of the slope* on the displacement-time graph. On the velocity-time graph this is represented by a *moving away from or toward the time axis*.

- a uniform change in velocity is represented by *a horizontal line* on the acceleration-time graph.

The idea is that learners are given practice to set up motions in the laboratory to accompany a given graph. Students should, for instance, be able to set up their tracks so that the steel ball moves with (roughly) constantly increasing speed to accompany the *s-t* graph of figure 5.48.

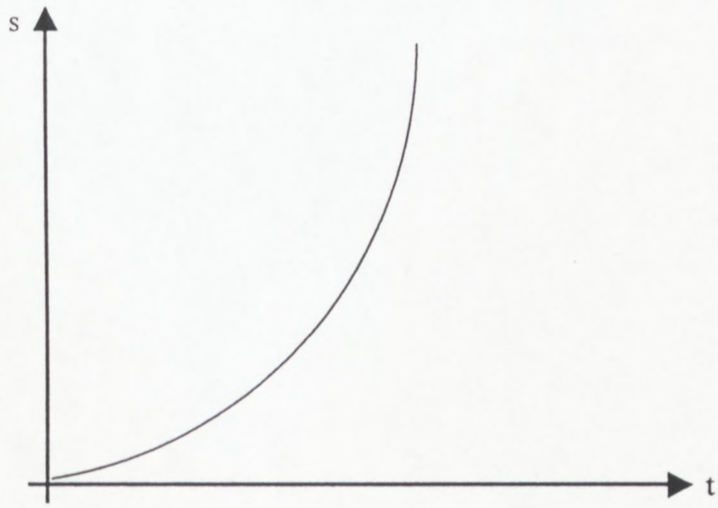


FIGURE 5.48

A specific instructional technique suggested by McDermott *et al.* (1987:571) where tertiary students are given practice in selecting the appropriate feature of a graph to obtain the information required, is summarised below. (The original example used by these authors has been slightly adapted to make it more suitable for secondary school students.)

Students are presented with three identically shaped motion graphs representing different kinematic quantities, similar to those shown in figure 5.49. They are told that *the initial* *velocity in each case is 10 m.s^{-1}* and are asked to find the velocity at t = 9 s for each of the three graphs.

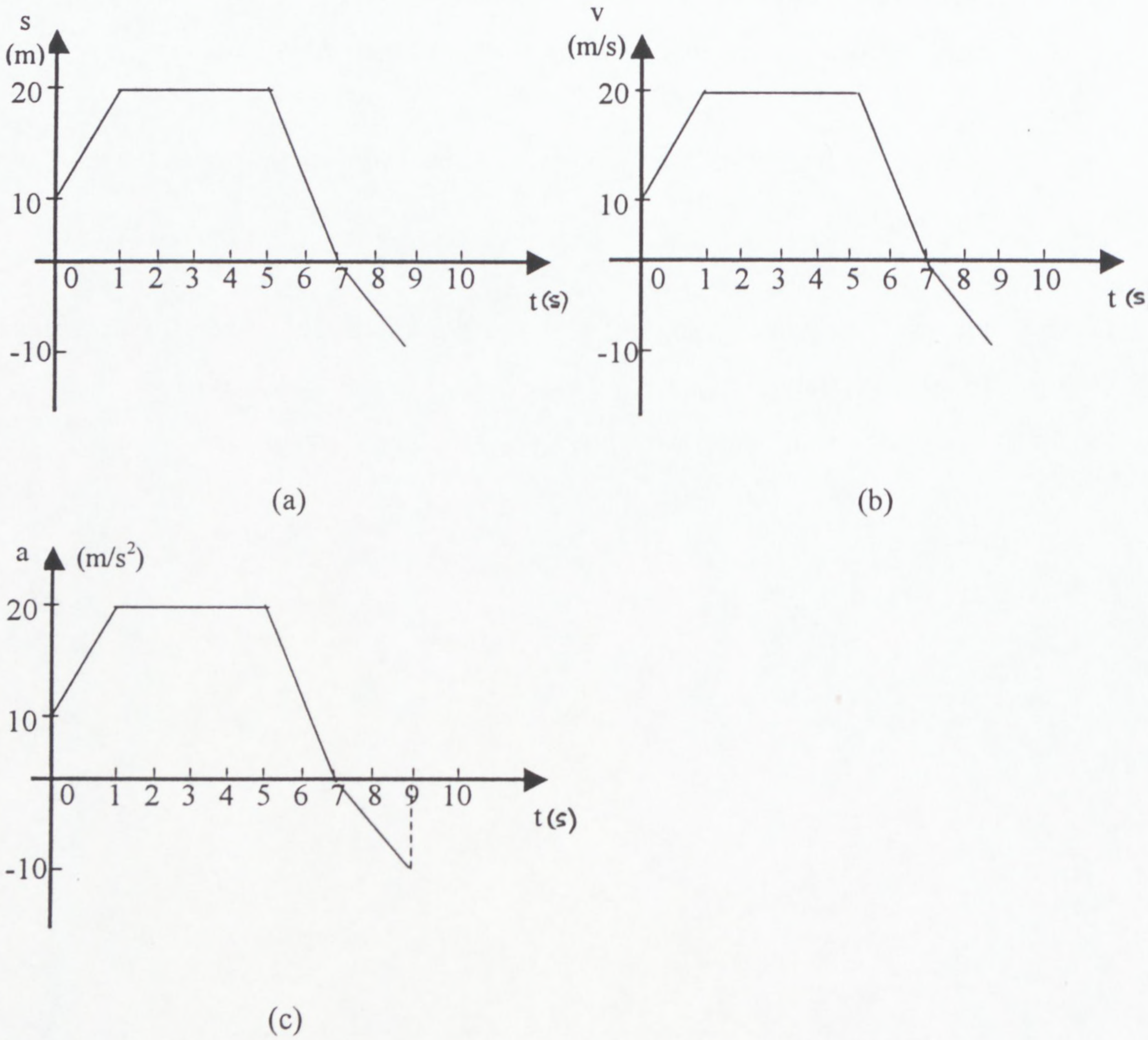


FIGURE 5.49

Being confronted with all three types of motion graphs at the same time helps to impress upon students that three very different motions can be represented by identically shaped *s-t*, *v-t* and *a-t* motion graphs. **Information about the velocity must be extracted from a different feature of each graph**. The students find the velocity from the *s-t* graph by calculating the **slope** at t = 9 s (-5 m.s^{-1}) and from the *v-t* graph by calculating the **height** or vertical co-ordinate (-10 m.s^{-1}). For the *a-t* graph they must examine the **area** under the curve between t = 0 and t = 9 s. The sum of the positive and negative areas gives

105 - 10 = 95 units, which represents the total change in velocity. The velocity at t = 9 s is thus 10 + 95 = 105 $m.s^{-1}$.

Instead of allowing learners to investigate the *different motions* of **identically shaped** *s-t*; *v-t* and *a-t* graphs instructors can also encourage students to consider the reverse scenario, namely investigating how the *same motion* is represented by **very differently shaped** *s-t*, *v-t* and *a-t* graphs. One strategy would be to encourage students into describing verbally the motion represented by a *v-t* graph such as that of figure 5.49(b), and use this description to draw the appropriate *a-t* and *s-t* graphs. (Students might even be encouraged to make use of the basic kinematic equations to obtain key values for the semi-accurate graphs). Alternatively, students might be explicitly asked to make use of gradient and area in order to transform the given *v-t* graph. The instructor might prompt them by suggesting that they ask themselves: *How can I draw an s-t curve so that the gradient of my curve is always equal to the height (positive* **or** *negative) of the given v-t graph?*

In this way a number of different strategies are made available to learners in order to arrive at the appropriate *s-t* and *a-t* graphs, which students should note are very different in shape to the original *v-t* graph. An example of a correct explanation for the motion of figure 5.49(b), as given by a first year technikon student together with the appropriate *s-t* and *a-t* graphs, is given next (see figure 5.50 (a) and (b)).

A motorbike going at 10 $m.s^{-1}$ accelerates within 1 second to 20 $m.s^{-1}$. This speed is maintained for 3 seconds, after which he slows down to stop after a total time of 7 seconds. He then turns around to accelerate in the opposite direction, reaching a speed of 10 $m.s^{-1}$ after another 2 seconds.

For s-t graph:

change in displacement	=	*successive areas under v-t graph*	
	=	*+15 m*	*(0 - 1 seconds)*
and		*+60 m*	*(1 - 4 seconds)*
and		*+30 m*	*(4 - 7 seconds)*
and		*-10 m*	*(7 - 9 seconds)*

For v-t graph

acceleration		=	*gradient of v-t graph*	
		=	*10 m.s^{-2}*	*(0 - 1 seconds)*
	and		*0 m.s^{-2}*	*(1 - 4 seconds)*
	and		*-20/3 m.s^{-2}*	*(4 - 7 seconds)*
	and		*- 5 m.s^{-2}*	*(7 - 9 seconds)*

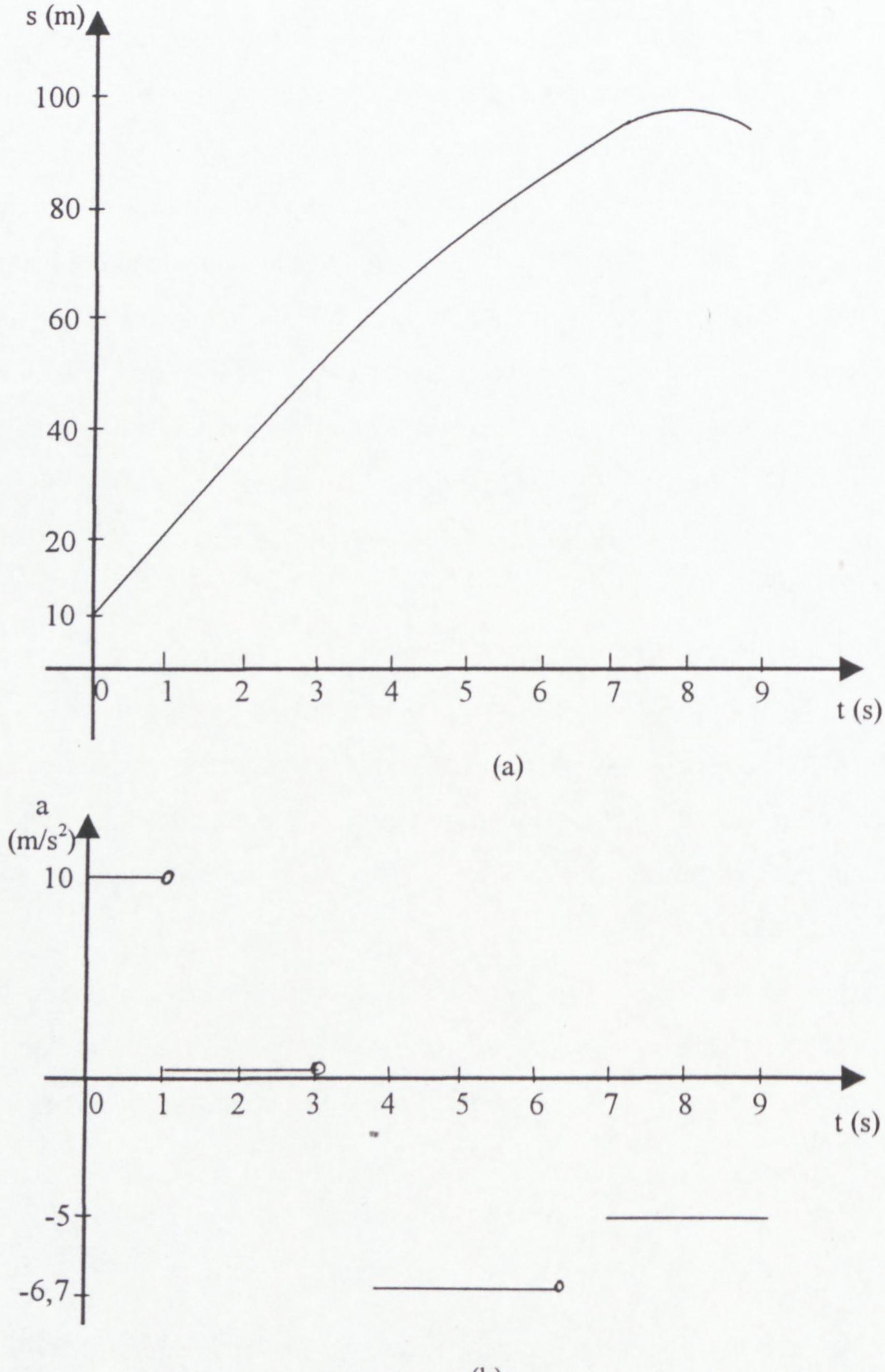


FIGURE 5.50

In view of the identified student misconceptions with regard to kinematic graphs, instructors have to ensure that the following points are clearly understood by learners. [This should be done by exposing learners to a large number of similar translations followed by testing of the appropriate skills and techniques. In many cases sketch graphs are sufficient to emphasise basic principles.]

a) The *s-t* graph may start anywhere on the s-axis. In the case of figure 5.50(a) the initial position of the motorbike was + 10 m from an *arbitrarily*[2] chosen reference point.

b) The gradient of the *s-t* graph at t = 0s is **not** zero but + 10 because v = + 10 $m.s^{-1}$ at t = 0s.

c) No cusps appear in the *s-t* graph.

d) The displacement increases for positive v-values and decreases only when the velocity becomes negative (i.e. after t = 7 s).

e) The following *v-t* ⇒ *s-t* graph shape transformation is noted in terms of change of *height* ⇒ change of *gradient* (figure 5.51)

[2] The word "arbitrary" should be explained to students to ensure that they all attach the same correct meaning to it.

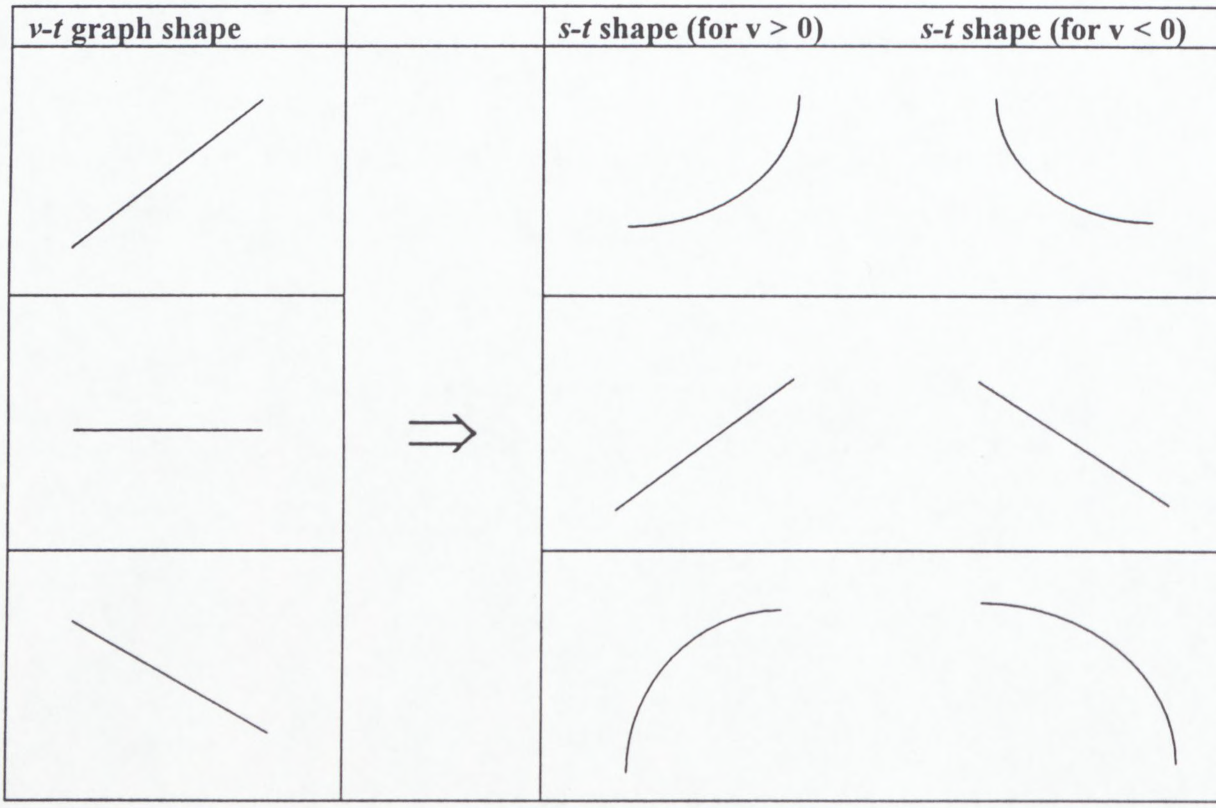


FIGURE 5.51

f) The *a-t* graph is "easier" because it merely involves the determination of the gradients of the various sections of the *v-t* graph. It should be pointed out here that for uniform acceleration, the *a-t* graph will always be a *horizontal line or lines*, because the various sections of the *v-t* graph consist of straight lines with *constant* gradient.

As with the teaching of graphs in general, McDermott *et al.* (1987:512) propose that learners are exposed to graphs in contexts other than kinematics in order to take advantage of the increased depth of understanding that comes from using the same procedures in different problem areas:

> *From careful monitoring of student performance on examination questions, we have found that the ability to choose correctly whether to extract information from the height, slope, or area of a graph develops*

slowly but steadily as students gain experience with graphs in different contexts.

(1987:512)

Schuster (1983), Orton (1984), Padilla *et al.* (1986) and McDermott *et al.* (1987) stress the need for instructors to actively address the following specific identified problem areas (A number of different graphs from various subject areas should be used as examples).

- what is a graph?
- the need for scaling the axes;
- slope as rate of change;
- height-slope relationships;
- the difference between positive and negative slopes;
- the meaning of a slope to a curved graph;
- zero slopes - maximum and minimum values;
- areas under a graph.

Goldberg and Anderson (1989) maintain that the serious identified student misconceptions in connection with *negative kinematic* quantities (see section 4.7.4.2) can be corrected through a teaching approach that gives particular attention to the "negative cases" of kinematic graphs. Goldberg and Anderson (1989:259-260) refer specifically to **negative velocity** representations when they suggest the following:

To help students develop a better facility with graphical representations of negative kinematic quantities there should be a greater focus, through *classroom demonstrations and discussion, laboratory work, and assigned homework, on having the students make connections between real-life motion of an object reversing direction and the graphical representation of the motion.*

Goldberg and Anderson (p 260) suggest that students should be encouraged to "act out" graphically represented motions (involving a reversal of direction) by either moving their fingers along a flat surface or using balls and tracks. Alternatively they can write a description of the motion. The students' replies are criticised by both instructor and fellow students, after which they should become aware of how the crossing of the time axis by a *v-t* graph is associated with a reversal in direction of the actual motion.

Schuster (in Helm and Novak 1983:331-337) argues that **representations** should be made an explicit part of the physics curriculum and that the translation between two representations in *both directions* should be emphasised during instruction. Schuster points out in this regard that: *it is interesting to note that students are particularly inexperienced in translating between real observations and representations thereof. Perhaps schooling does not sufficiently connect the real and representational worlds.*

In order to familiarise students with different forms of representations, Schuster (in Helm and Novak 1983) offers a special course in kinematics. The motion of a body is demonstrated, after which students are encouraged to act out or simulate the observed motion. Students are also asked to describe the motion in words using the terms *position, velocity* and *acceleration*. After this they are provided with stroboscopic photographs which are used to illustrate the motion graphically in terms of displacement-time, velocity-time and acceleration-time graphs. The motion is also represented symbolically by using formulas and equations and students are given numerical values in tabular form. The idea is that they are not only exposed to the different forms of representation, but they also see the *connection* between these representations.

Clement (1989:80) contends that instructors should initially concentrate on assisting students to generate or interpret the basic features of a kinematic graph *qualitatively*, i.e.

without any numerical values on the axes. In this way students are made aware of how specific aspects of a motion are represented by different features on the various kinematic graphs. A possible teaching strategy involving *graph transformations* which focuses on different *features* of graphs is given as an example:

The *v-t* graph of figure 5.52 is handed out to each student with the request to draw the corresponding *s-t* and *a-t* graphs representing the same motion. Students are also asked to act out the motion by running their fingers along the surface of their desks.

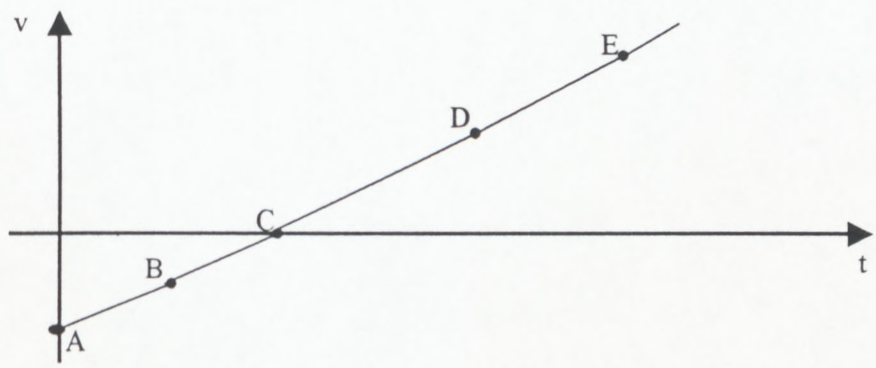


FIGURE 5.52

Before students attempt to draw their graphs, it is agreed through a class discussion that the ordinate (or vertical co-ordinate) of the given *v-t* graph must give the *gradient* of the required *s-t* graph and that the gradient of this *v-t* graph must yield the *height* of the *a-t* graph. After allowing the students an opportunity to draw their sketch graphs, the instructor draws the correct graphs as in figure 5.53, where certain key positions are indicated with letters to highlight the different features on the different graphs that represent the same aspect of the motion.

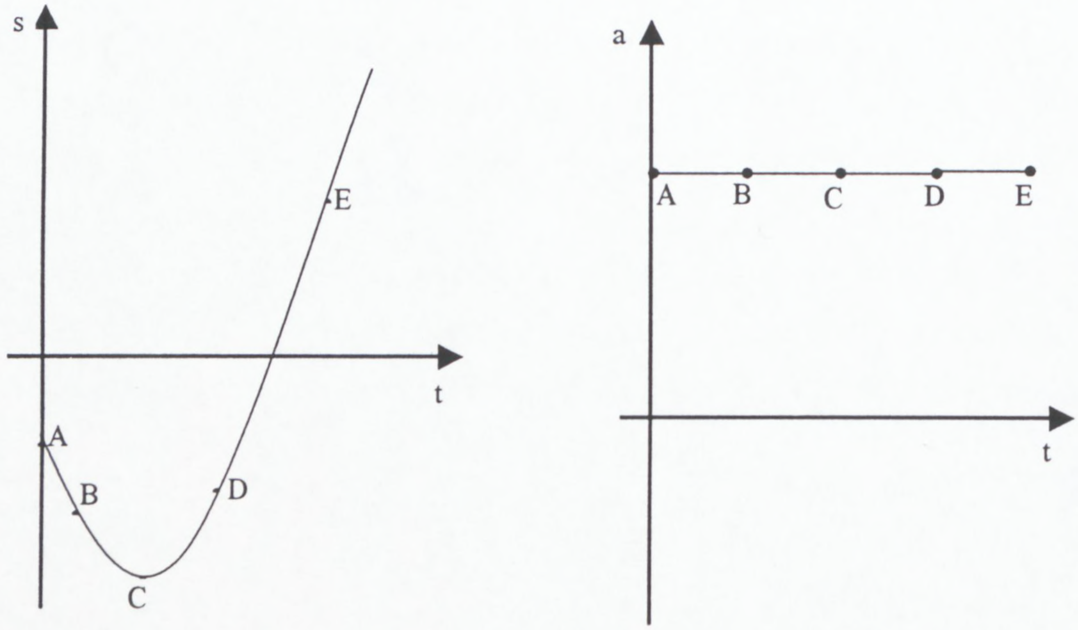


FIGURE 5.53

Tangent lines to the *s-t* graph at the indicated letters can also be drawn to illustrate how the changing gradient values correspond to the ordinates of the *v-t* graph. The acted out finger motions are compared until agreement is reached about the nature of the motion. Finally, the instructor could highlight a number of other important aspects such as:

- the reversal in direction is indicated by the crossing of the t-axis in the *v-t* graph and the minimum value on the *s-t* graph;

- the reversal in direction cannot be observed on the *a-t* graph because "slowing down in one direction has the same acceleration *sign* as speeding up in the opposite direction";

- the *s-t* graph could start **anywhere** on the s-axes. [Learners should be encouraged to come to accept this important fact by a discussion on the role of reference point];

- the crossing of the t-axis by the *s-t* graph has little significance, except that the object has returned to the chosen reference point at s = 0;

- a reference to the role of area between a graph and the t-axis and more specifically how the units assist in making correct inferences about which kinematic quantities are represented by this area (for example $m.s^{-2} \times s = m.s^{-1}$ and $m.s^{-1} \times s = m$);

- the fact that areas are negative *below* the t-axis and positive *above* the t-axis.

5.5.5 Integrating mathematics graphs and kinematic graphs

Students seldom make the connection between graphs studied in the mathematics class and the graphical representation of scientific measurable quantities such as displacement, velocity and acceleration (see for example McDermott *et al.* (1983, 1987). Little is written about *how instructors can go about helping learners to apply their knowledge of "mathematics graphs" to the study of kinematics and other topics in science.* Equally little is said about how a study of kinematic graphs can clear up misconceptions students might have about algebraic graphs representing linear and quadratic relationships. In an attempt to illustrate how the connection between "maths graphs" and "science graphs" can be made, a teaching approach is next proposed with respect to a specific example. Other aims of this teaching approach include the following:

- To emphasise the importance of linear and quadratic functions, in other words to show how *parabolas* and *straight lines* are used in kinematics.

- To encourage students to come to *identify* themselves with the real system which is being represented. This is done by ensuring that they themselves control the experiment that is to be represented graphically.

- To show learners that the use of "x" and "y" for the independent and dependent variable respectively, is purely conventional and that nothing is lost when these are replaced by time and a kinematic quantity such as "v", "s" or "a".

STEP 1

Students are requested to sketch the following semi-accurate graphs as they would in the mathematics class, indicating intercepts with the axes:

$y = 2x^2 - 2$ ------------------------------- ①

$y = 4x$ ------------------------------------ ②

y -= 4------------------------------------ ③

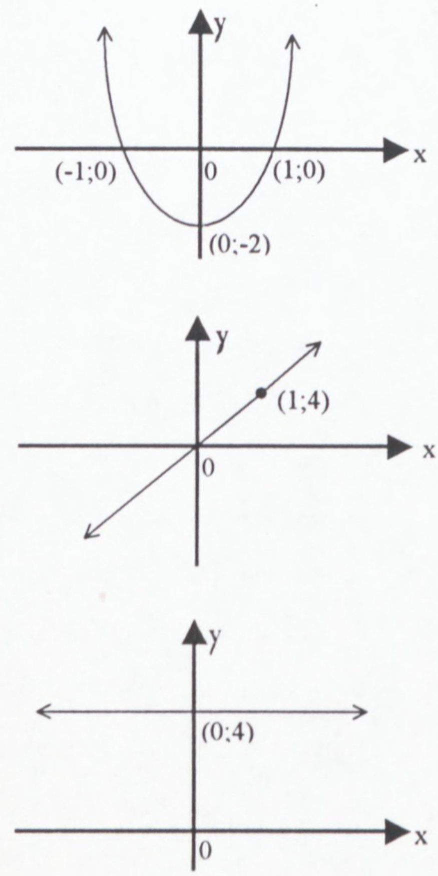


FIGURE 5.54
A few commonly occurring "mathematics graphs"

STEP 2

A steel ball is allowed to roll down an inclined aluminium track from rest with constant acceleration of 4 $m.s^{-2}$, with an observer at reference point X, 2 m from the point of release A. The positive direction is as indicated in the sketch (figure 5.55).

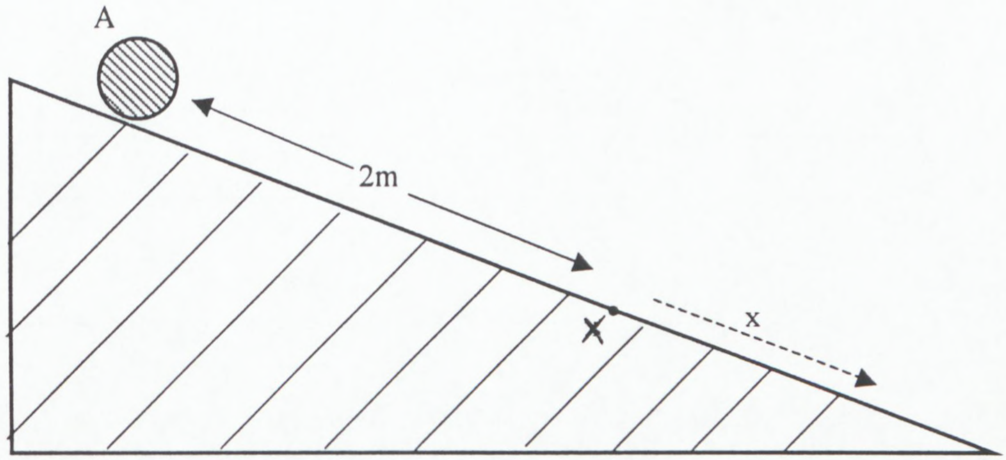


FIGURE 5.55

Students are requested to draw semi-accurate *s-t*, *v-t* and *a-t* graphs of the motion by making use of the algebraic equations of motion.

STEP 3

Students are guided to accept that the familiar equation $s = ut + \frac{1}{2}at^2$ should be adapted to read $s = ut + \frac{1}{2}at^2 + k$ for the case where the displacement at t = 0 is non-zero. They are then encouraged to apply the equations to the observed physical problem, namely taking $u = 0\ m.s^{-1}$, $a = 4\ m.s^{-2}$ and $k = -2$ m:

$$s = ut + \tfrac{1}{2}at^2 + k = 0 + \tfrac{1}{2}(4)(t)^2 + (-2)$$
$$\therefore s = 2t^2 - 2$$
$$v = u + at = 0 + 4t$$
$$\therefore v = 4t$$
$$\text{and} \quad a = 4$$

This is accompanied by a brief discussion on the *signs* of s, v and a on either side of the reference point X as well as at X itself. Students are required to draw up a table such as the one illustrated in table 5.11.

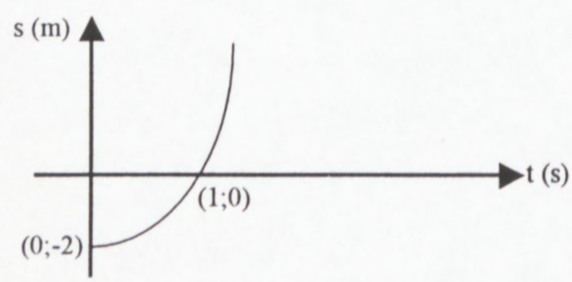
	left of X	X	right of X
s	-	0	+
v	+	+	+
a	+	+	+

TABLE 5.11
SIGNS OF KINEMATIC QUANTITIES AT VARIOUS POSITIONS WITH RESPECT TO A REFERENCE POINT

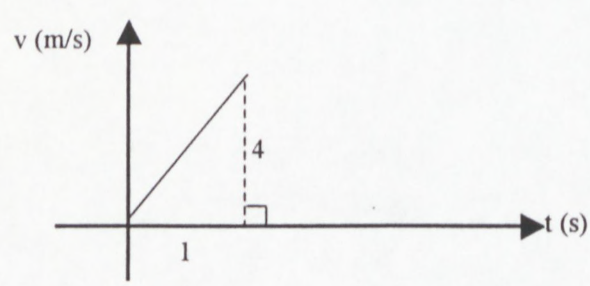
STEP 4

Students are asked to compare the functions represented by the "maths equations" 1, 2 and 3 with the kinematic equations 4, 5 and 6 and discuss their findings in groups. Once consensus is reached, the appropriate kinematic graphs are drawn. Negative time values are ignored and students are encouraged to notice the resemblances and differences between the kinematic graphs and their mathematical counterparts (see figure 5.56).

$s = 2t^2 - 2$ ------------------------------- ④



$v = 4t$ ------------------------------------ ⑤



a = 4-------------------------------------- ⑥

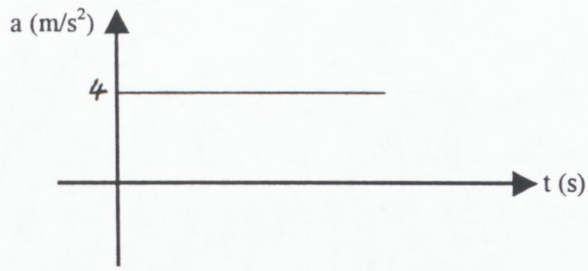


FIGURE 5.56

STEP 5

It should now be pointed out to students that their *s-t* graph could have been transformed into the corresponding *v-t* graph by considering the steadily increasing gradient of the tangents drawn to the *s-t* graph. It is suggested that this is illustrated by drawing a number of successive tangents as shown in figure 5.57.

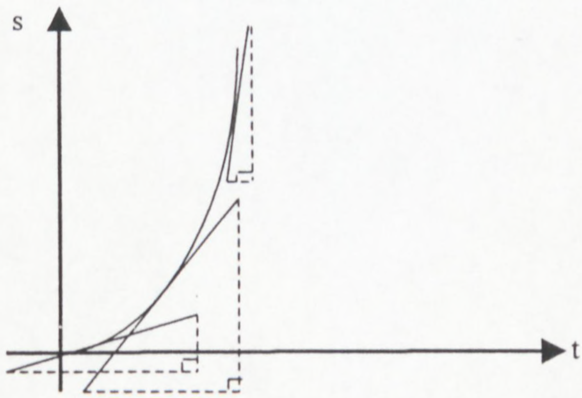


FIGURE 5.57

Finally, students are asked whether they can make similar (but more elementary!) inferences about the role of gradients in transforming their *v-t* graph (graph no 5 of figure 5.56) into the corresponding *a-t* graph.

5.5.6 Microcomputer-based Laboratories (MBL's)

Brasell (1987), Linn *et al.* (1987), Mokros and Tinker (1987) and others have investigated the effect of so-called MBL's on learners' understanding of the graphical representation of real-life motions. It involves the use of a special sonar-motion device that attaches to the game port of a microcomputer and allows students to make real-time *s-t*, *v-t* and *a-t* graphs of their own motion or that of a moving object, such as a toy car, **while the motions are being performed.**

Mokros and Tinker (1987:381) report that such MBL activities brought about a significant improvement in viewing graphs as symbolic representations of physical phenomena rather than pictures of the physical problem scene. Mokros and Tinker (1987:381) give the following illustration of a pre-test, post-test gain achieved after exposing a group of students to MBL experiences:

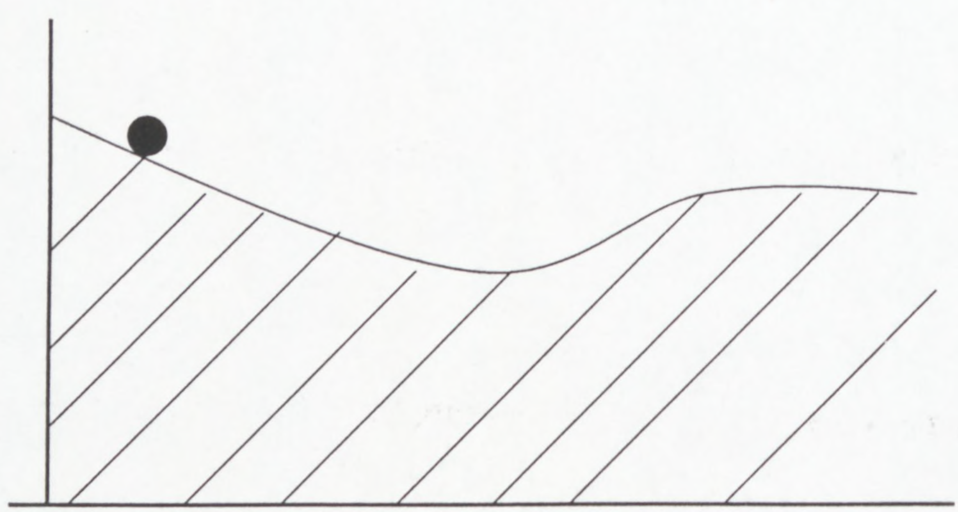


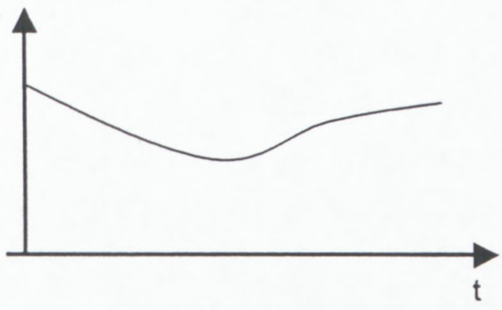
FIGURE 5.58

A ball rolls down a ramp, then up a hill and along a flat horizontal frictionless surface as depicted in figure 5.58. Choose the appropriate velocity vs time graph. The responses, which were recorded in a "pre-test", "post-test" analysis, were as follows:

Incorrect graph-as-picture

pre-test 61 %

post-test 30 %



Correct graph

pre-test 23 %
post-test 62 %

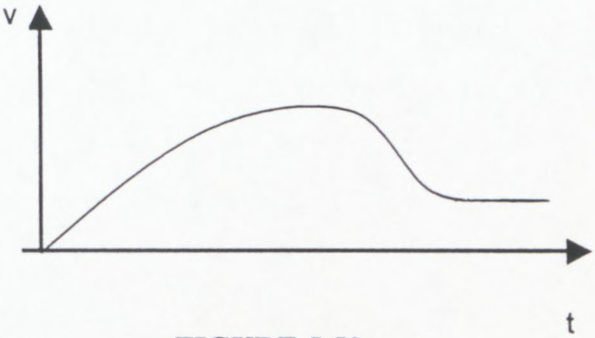


FIGURE 5.59

Figures 5.55(a) and (b) clearly show that, although the incorrect graph-as-picture idea still persisted after exposure to MBL for about one third of the students, the overall improvement was quite significant.

Mokros and Tinker (1987:381) discuss the possibility that, in view of this improvement, *the real-time graphing may operate (within Piagetian theory) as a link between a concrete experience and the more formal, symbolic representation of that experience.*

Linn *et al.* (1987) undertook similar investigations and report that improved graphing skills due to exposure to MBL activities are a result of the fact that the graphs are formed as the experiment is carried out and are **immediately** related to a physical activity that the student may have designed or set up for themselves.

> *Thus they (the graphs) are less likely to be seen as static pictures and more likely to be seen as dynamic relationships.* (Linn *et al.*, 1987:245)

Brasell (1987) found that significant gains in the ability to translate between verbal and graphic descriptions of motion events was achieved in a single class period where learners used MBL's, and suggests that this was mainly due to improvements in the understanding of the *conventions* of graphing. She states further (1987:387):

> *MBL relieves the students of some of the more tedious aspects of collecting, manipulating and displaying data that otherwise divert their attention from interpreting and understanding the data.*

Not all science educators are equally enthusiastic about the use of computers to teach graphing skills. Wavering (1989:379) for instance warns that the relationship between a real understanding of graphs and the ability to have graphs made instantly on computers needs to be explored before students are exposed to MBL's at earlier and earlier ages.

> *With the increasing use of computers to generate graphs for students, if students are not given opportunities to work their way through their own graphs, it could be that logical development and understanding of graphing may be short-circuited.*

It seems therefore that more research needs to be undertaken to study and evaluate the role of microcomputer-based laboratories on the teaching and learning of graphs, especially in the field of kinematics.

5.6 PROPOSED ADJUSTMENTS TO THE PRESENT SYLLABUS FOR KINEMATIC GRAPHS IN SOUTH AFRICAN SCHOOLS

5.6.1 Reasons for the adjustment.

The various teaching strategies which are proposed in the previous section necessitate a number of adjustments to the present South African syllabus for kinematic graphs in Grade 11. The changes proposed here are described in terms of a number of *additions* or *extensions* to the present syllabus, in order to make it possible for instructors to work towards the following ultimate goals with regards to kinematic graphs:

- Students should be able to use graphs to help clear up difficulties experienced with certain kinematic concepts.

- Students should gain an improved understanding about graphing in general by applying the conventions of graphing to real - life every day kinematic situations.

- The elegance and beauty of motion graphs and transformations between graphs should be seen to have academic merit in itself.

5.6.2 Adaptations to the present syllabus

The following important aspects are at present not explicitly mentioned in the Senior Secondary Science syllabus for South African schools:

- Representations of data in physics with special reference to the advantages gained from *graphical* representations.

- The use of sketch graphs, semi - accurate graphs and accurate quantitative graphs.

- A brief reference to the use of graphs in other contexts than kinematics, especially with reference to the role of gradient and area.

- The following important translations (with appropriate examples):
  - verbal description of physical situation → graph (translation A)
  - graph → verbal description of physical situation (translation B)
  - algebraic formula → graph (translation C)
  - graph algebraic → formula (translation D)
  - graph → graph (translation E)

- *a-t* graphs for uniform and accelerated motion.

- The role of gradient in transforming *s-t* graphs into the corresponding *v-t* graphs and *v-t* graphs into the corresponding *a-t* graphs.

- The role of area in transforming *v-t* graphs into the corresponding *s-t* graphs and *a-t* graphs into the corresponding *v-t* graphs.

- The importance of reference level and sign convention in order to...

    - assist in the understanding of the meaning of negative kinematic quantities;

    - facilitate the construction and interpretation of motion graphs involving a reversal in direction of motion.

5.7 SUMMARY AND CONCLUSION

A summary of the main points of this chapter is given below.

Student misconceptions with regard to kinematic graphs can be classified in terms of factors related to **kinematics**, factors that have to do with the **curriculum** and how it is taught and factors related to areas **outside** of kinematics.

Kinematics-related categories include the following:

- A poor perception of **reference point** and **sign convention**, leading to particular problems with graphing negative kinematic quantities and representing a reversal in direction of a motion;

- Not differentiating adequately between certain kinematic quantities such as **acceleration and velocity**;

- **Conserving** the properties of a given graph, when a related graph has to be drawn. Properties that tend to be conserved include the slope, shape, sign and intercepts of the given graph;

- A tendency to commence all *s-t* graphs at the **origin.**

- A reluctance to make use of **gradient and area**, (a) when a given graph has to be transformed into one representing a related kinematic quantity, and (b) to obtain certain key information about a motion from its graph.

- Graphs are often viewed as **iconic representations** of the physical problem scene.

Curriculum-related factors include the following:

- The stubborn nature of students' **preconceptions** and the fact that instructors do not always acknowledge the implications of the **constructivist theory**.

- The fact that many students are not "**intellectually ready**" for the abstract approach to kinematics that is required by the study of kinematic graphs.

- Inadequacies in the **teaching** of graphs.

- A **science curriculum** which does provide adequate detail about the role of graphs and other representations of physical phenomena.

The following **external factors** have been found to play an important role in students' understanding of kinematic graphs:

- An inability or unwillingness on the part of students to **connect the real and representational worlds**.

- Problems with giving the proper **meaning to certain terms**, both within kinematics, as well as to some words in general.

- Not understanding the **meaning of a kinematic graph** as a representation of the co-variance of two variables.

- An inability to relate linear, curvy-linear and parabolic curves in kinematics to the same representations covered in the **mathematics curriculum**.

The following instructional strategies are suggested to address identified misconceptions:

- Instructors should be **aware of generally occurring misconceptions**, which are specifically mentioned by researchers.

- A real effort should be made to **link existing preconceptions about graphing to the accepted more conventional language and notation of graphing.**

- Learners should be **actively instructed** about what a graph is and how it relates to other representations.

- **Student experiments,** where a physical system is manipulated by learners themselves, should as far as possible be used for graphical representation.

- In cases where a poor understanding of the meaning of certain **kinematic quantities and relations** leads to inadequacies in graphing, the kinematic problems have to be addressed directly by instructors.

- A more formal **algebraic interpretation** of and approach to graphs should only be employed once learners are properly schooled in the **conventions of graphing**.

- Learners must be encouraged, whenever it is possible, to **translate in both directions**: from a physical system to a graph (translation A) and from a graph to the physical situation that it represents (translation B).

- The role of **gradient and area** in translation E (graph → graph) is crucial and should be emphasised with many examples for many different shapes of graphs.

- Learners should note how very **different shapes** of the various types of kinematic graphs describe the **same motion**. Conversely they must also be shown how **identical graphs** representing different kinematic quantities describe **very different motions**.

- Graphs in kinematics **should not be taught in isolation** from graphs in other science topics or even other disciplines. Students should be made aware of how an aspect such as rate of change is handled and interpreted graphically in different contexts and other subjects.

- **Graph instruction in mathematics and physics should be more integrated,** so that learners can be aware of the connection between specific graphical representations (such as linear and quadratic functions) in mathematics and kinematics.

- The use of **microcomputers-based laboratories**, where a graph is seen to be generated at the same time as a motion, has assisted many students to improve their comprehension of graphing.

- Instructors have to bear in mind that for many students, their **knowledge gained about graphing conventions, construction and interpretation is more important than their understanding of kinematics.**

A number of researchers have endeavoured to research typical student errors and preconceptions in kinematics. A few of these have included some reference to graphical representations (see Chapter 3). This chapter attempts to classify those misconceptions and mistakes which relate exclusively to kinematic graphs and more particularly to graph-graph transformations — an area which has received no attention in the literature to date.

A thorough analysis of graph-graph transformations is deemed necessary in view of the important role that transformations play in the understanding of not only kinematics, but graphs in general.

It is hoped that a knowledge of the misconceptions (and their causes), together with the suggestions for minimising them in the classroom, will be of some use to teachers of physical science.

CHAPTER 6

OVERVIEW OF THE STUDY AND SUGGESTIONS FOR FURTHER RESEARCH

This chapter provides a brief synopsis of the study by highlighting the important points of the various chapters. This will be followed by a critical appraisal of the study as a whole, a look at possible shortcomings and finally some suggestions for further research.

6.1 GENERAL OVERVIEW OF THE STUDY

6.1.1 Summary of Chapter 1

Chapter 1 deals with the motivation for the study, the formulation of the problem and the ultimate aim of the study. It also provides a broad outline of the research methodology that was used.

6.1.2 Summary of Chapter 2

In this chapter a careful analysis is made of the syllabus content for kinematic graphs which secondary students can reasonably be expected to master. In order to make a meaningful content analysis, the following functions of motion graphs were mentioned:

(a) The generation, communication and consolidation of information about the physical world.

(b) Improved understanding of a physical situation.

(c) Improved understanding of graphs.

(d) An appreciation of the inherent elegance of graphs.

A number of representations of physical phenomena are identified, the most important ones being verbal descriptions, algebraic equations, ordered pairs and finally graphs. Graphical representations are categorised as accurate, semi-accurate and sketch graphs and the following translations between representations are identified:

Translation A: Verbal description → Graph

Translation B: Graph →Verbal description

Translation C: Algebraic formula → Graph

Translation D: Graph → Algebraic formula

Translation E: Graph → Graph

A number of important points are made with respect to these translations:

- The importance of gradients as a measure of the rate of change.
- The link between certain features of a graph and key aspects of a motion.
- The role of "mathematics" graphs to facilitate the understanding of kinematic graphs.
- The significance of a reference point for displacement and sign convention for displacement, velocity and acceleration.

- Translation E is called a *transformation* and the use of gradient and area in making appropriate transformations is stressed.

- Chapter 2 also analyses the syllabus for kinematic graphs for the senior secondary science course in South African schools, the interpretation that local textbook writers give to this syllabus, as well as the interpretation that external examiners give to it. This is described in the context of the five translations mentioned earlier. A number of shortcomings are identified with respect to the prescribed syllabus and the content of the textbooks that were studied. These include:

- The absence of detail in the official syllabus.

- A poor exposition of the meaning of negative kinematic quantities.

- A lack of a proper explanation of reference level and sign convention.

- No explicit reference to *a-t* graphs in the syllabus.

- An insufficient emphasis on the role of sketch graphs.

- A lack of integration with graphs in mathematics and other subject areas.

- Insufficient handling of graph $\rightarrow$ graph transformations.

- Insufficient handling of graphical representations of real data from real experiments.

The study of a number of overseas textbooks revealed a degree of similarity with the local approach, except for a heavy emphasis on the use of ticker tape charts of motions which, when placed side-by-side, act as a first primitive motion graph.

6.1.3 Summary of Chapter 3

Chapter 3 focuses on some of the research articles written on students' understanding of graphs in general and motion graphs in particular. Initially a few theories relating to the learning of science are described. The constuctivist theory and the generative model of learning which emphasise the role of learners in generating views of the world and so constructing their own knowledge bases in science are particularly highlighted.

The views of learners, though often at variance with those of subject experts, are strongly rooted in their (the learners') knowledge bases and it is therefore important to devise techniques to find out more about the nature of these student preconceptions. Among the most commonly used methods are unstructured clinical interviews and questionnaires which rely on paper-and-pencil activities. The latter can either be of the open-ended free response type or the multiple-choice type of question.

Several researchers have attempted to describe the nature of understanding. The views of Skemp (1976), Reif *et al.* (1976), Ormell (1979), Trowbridge and McDermott (1980) and Nickerson (1985) are reported on, with the main points being the following:

- Understanding is very difficult to describe or define.
- What is meant by understanding depends on the nature of the content.
- The goal is for students' understanding to correspond with the understanding of experts in the field.
- Learners should be able to distinguish between different but similar concepts.
- Real understanding involves solving original non-textbook style problems.

Understanding of kinematic graphs is defined in terms of the ability of students to translate between the various representations of kinematics in a manner acceptable to a subject expert in the field of kinematics.

The role of ratios, proportion and rate of change is described in terms of the following key points:

- An understanding of ratio and proportion is necessary for the understanding of graphs.

- Graphs can assist in clearing up misconceptions relating to ratio and proportion.

- Students experience great difficulty in understanding the concept of rate of change, especially its graphical representation as the gradient of the tangent at a point in the case of a curved graph.

A survey of the literature into students' difficulties with graphs in general reveals the following common misconceptions:

- an inability to understand the meaning of a graph as a representation of the dynamic functional relationship between two (usually) continuous variables;

- a failure to differentiate between the independent and the dependent variable;

- serious difficulties with the comprehension of slope as a measure of rate, leading to slope-height confusion, problems with the slope of curved graphs and confusion between positive and negative gradients;

- an inability to draw sketch graphs of an observed or verbally described physical situation;

- an inability to describe a given graph in terms of an algebraic formula;

- problems with scaling and interpolation;
- the tendency to concentrate on specific coordinates of a graphical representation;
- an inability to differentiate between discreet and continuous graphs;
- the idea that a graph is a picture of the physical situation that it represents;
- an inability to integrate graphs in different contexts.

Graphing problems in kinematics are sometimes the result of a poor understanding of certain kinematic quantities and relationships. The following student misconceptions with respect to kinematics in general are described in Chapter 3:

- the tendency to believe that when two objects reach the same position they must have the same speed and the same acceleration;
- confusing velocity and acceleration;
- a failure to use a sign convention and reference point appropriately;
- the failure to associate a reversal in direction with a change in sign of the velocity vector;
- the tendency to view velocity as a property of an object, instead of relating it to a frame of reference;
- the tendency to experience greater difficulty with acceleration than with velocity and greater difficulty with velocity than with displacement

The science education research group at the University of Washington in Seattle has undertaken a detailed investigation into students' graphing skills in kinematics. Rosenquist, McDermott, Trowbridge, Popp and van Zee (1981, 1983 and 1987), as well as other researchers such as Peters (1982), Schuster (1983), Clement (1989) and Brasell and Rowe (1993) report a number of commonly occurring misconceptions or difficulties, which include the following:

- the inability to translate an observed motion into its graphical representation, resulting in errors such as the slope of a graph resembling the path of the motion;
- a more serious problem relating to the inability to demonstrate a graphically represented motion;
- a general tendency to draw graphs in such a way that they resemble the physical orientation of the experimental set-up which produces the motion;
- the tendency to plot discreet points on a *v-t* graph, each representing a different value for the velocity;
- the belief that *s-t* and *v-t* graphs are not allowed to cross the time-axis;
- the tendency to draw horizontal *a-t* graphs, both above and below the time-axis for a motion that involves a reversal in direction;
- a confusion between the slope and the height of an *s-t* graph, resulting in the tendency to use position criteria to draw conclusions about relative velocities in the case of the *s-t* graphs of two different motions;
- the inability to identify the appropriate feature of a graph (such as slope, change in slope, or height) which relates to a required kinematic quantity;

- a reluctance to use areas under kinematic graphs to gain information about changes in displacement or changes in velocity;

- the inability to associate graphs such as parabolas in the mathematics curriculum with the corresponding kinematic graph such as an *s-t* graph for accelerated motion;

- a failure to recognise that in graph-graph transformations, it is often the changing slope of the given graph that translates into a changing height of the required graph, or vice versa;

- the tendency to draw a required graph of a related kinematic quantity in such a way that it matches the shape of a given kinematic graph.

6.1.4 Summary of Chapter 4

Chapter 4 reports on the empirical investigation undertaken as part of the study. The main aim of the investigation is to find out more about the nature and extent of students' preconceptions and misconceptions about kinematic graphs. The specific problems which surround graph-to-graph transformations are focused on because of the absence of sufficient literature on this important aspect of graph transformations.

6.1.4.1 A description of the questionnaires, the test sample and the test procedure

A series of audiotaped personal interviews provided the material for a pilot questionnaire which was answered by 32 senior high school students. The results of the pilot study were used to draw up a final free-response-type questionnaire which was distributed to 278 students from different cultural back-grounds in Grades 11 and 12, and the first two academic years at the Cape Technikon. A further series of clinical interviews provided additional information about the way students think about kinematic graphs.

6.1.4.2 The test results

The main part of this chapter is devoted to a question by question analysis and discussion of the results of the empirical investigation. A brief report on the quality of the responses as a function of race, academic achievement and present educational level is also included.

6.1.5 Summary of Chapter 5

The present study confirms a large number of the misconceptions revealed by the literature study, but also brings to light some important, previously unreported and/or unrecognised student perceptions of motion graphs. Chapter 5 categorises **all** these misconceptions, which makes a measure of overlap with the categories reported on in Chapter 3 unavoidable. The causes or origins of certain widespread misconceptions are also researched. Finally, a number of untested teaching suggestions are included in an attempt to minimise the described problem areas.

6.1.5.1 Categories of student misconceptions

The following five categories or classes of misconceptions are identified:

(a) <u>The meaning of kinematic quantities</u>

- a particular confusion between the meaning of acceleration and velocity;
- a resulting confusion between *v-t* and *a-t* graphs;
- not understanding the real meaning of negative displacement, velocity and acceleration;

- a belief that negative acceleration is synonymous with "deceleration";

- a belief that negative velocity indicates the slowing down of an object.

(b) Reference point and sign convention

- an inability to deal with the graphical representation of negative kinematic quantities;

- not dealing adequately with motion involving a reversal in direction;

- a tendency to start all graphs at the origin;

(c) Gradient and area

- a general reluctance to use gradient and area to interpret graphs.

(d) Iconic representations

- graphs are often drawn so that they appear like “pictures” of the actual problem scene;

- a physical change in direction is often represented by a sudden directional switch on the graph;

- graphs have slopes which copy the observed slope of a physical problem scene.

(e) Graph → graph transformations.

- a general tendency to conserve certain observed properties when attempting graph transformations ;

- more specifically a tendency to conserve the following observed properties of the given graph:

  * the algebraic sign;
  * the gradient;
  * the shape (both locally and globally);
  * the intercepts with the axes.

- a reluctance to use gradient and area considerations for transforming sketch graphs;

- difficulty with transformations where either or both graphs represent negative values;

- the tendency to include cusps in *s-t* graphs when this graph is derived from a *v-t* graph.

6.1.5.2 Origins of poor graphing skills

It is suggested that the following external factors contribute to the causes of the identified misconceptions and mistakes:

- the fact that learners have a poor understanding of some of the terminology used in the instruction of kinematic graphs;

- students often do not know what a kinematic graph means or represents;

- a lack of interaction and integration between that which is taught in the mathematics class and the content of the science curriculum;

- students are often not intellectually ready for the abstract approach which is necessary for the mastering of the subject content;

- problems with teaching, text books and examination papers, as well as a curriculum which is lacking in clarity and detail.

6.1.5.3 Some ideas on the teaching of motion graphs

A number of untested suggestions are made regarding ways in which instructors can improve (a) the general graphing ability of their students and (b) their students' understanding of motion graphs. The following points are mentioned with respect to general graphing instruction in science:

- Science teachers should be aware of the typical student preconceptions with regard to graphs and should tailor their teaching to link these preconceptions with the way that subject experts view graphs.

- Students must receive instruction about graphs and graphing conventions and how these relate to other forms of representations.

- A real effort should be made to integrate graphs in mathematics with the equivalent graphs in physics.

As far as the teaching of kinematic graphs in particular is concerned, the following strategies are suggested:

- Allow students to design and control their own experiments to generate data for the construction of kinematic graphs.

- Learners should be made explicitly aware of those kinematic quantities and conventions which give rise to graphing problems.

- Translations in both directions should be encouraged: from the verbal description of a physical situation to a graph, as well as from a graph to a verbal description of the motion.

- The important role of gradient should be stressed in *s-t* and *v-t* graphs in general, in graph-graph transformations as well as in graphs relating to other disciplines. Similarly, the role of area under a graph should be explained to students.

- Students must be shown how the same motion is represented by very different graphs and how identically shaped *s-t*, *v-t* and *a-t* graphs describe very different motions.

- When teaching kinematic graphs, instructors have to bare in mind that graphing skills acquired by students are often more important than their specialised knowledge about kinematics and kinematic graphs.

- A number of researchers acknowledge the value of microcomputer-based laboratories in assisting students to make proper connections between a motion and a graph as a representation of that motion.

Chapter 5 concludes with a number of suggestions for elaborating on and adjusting the present syllabus for kinematic graphs in the senior secondary course in South African schools.

6.2 CRITICAL OVERVIEW OF THE STUDY

6.2.1 Shortcomings of the study

The aim of this study was to research the nature and extent of learners' graphing problems with regard to kinematics and propose ways in which these problems could be addressed. Although the investigation did reveal a number of areas where misconceptions are widespread, the following shortcomings can nevertheless be identified:

- The fact that the graph-graph transformations of the main questionnaire focused mainly on *v-t* $\rightarrow$ *s-t* transformations, thus largely ignoring other important transformations.

- A number of students ignored certain questions in the main questionnaire which were perceived to be "difficult", thus lowering the statistical significance of the results.

- The study did not provide for a means of empirically testing any of the recommendations made with regard to the teaching of kinematic graphs.

6.2.2 Value of the study

It is believed that this study is of value to teachers of graphs, not only in kinematics, but also in other areas of science and even other subjects where the graphical representation of a problem scene helps learners to understand the subject material more fully.

The study emphasises the need for teaching graphing skills in a specific context such as kinematics. It is therefore hoped that instructors will become more aware of their students' shortcomings in order to instruct and guide them so as to eliminate these misconceptions according to the teaching techniques suggested in Chapter 5. It is believed that the results of the empirical study could serve as an important database of students' ideas about motion and its graphical representation.

The specific action of transforming one graph into another related one is experienced as very difficult by many learners, yet once they have acquired this skill, the general graphing ability of these learners should improve considerably. The reason for this lies in the fact that students, through graph transformations, are explicitly forced to focus on and understand the significance of aspects such as gradient, area, extreme points and intercepts with the axes in order to transform a given graph correctly.

The literature study has not revealed another investigation that has, as its main focus, student misconceptions with regard to graph transformations, and it is for this reason that it is believed that all science educators should take note of the present study.

6.3 RECOMMENDATIONS FOR FURTHER STUDY

The study of the available literature undertaken in Chapter 3 indicates that a substantial amount of research has been undertaken into graphically related aspects of science teaching. However very little research is reported on the question of learners' perception of the relationship between graphs of functions studied in the mathematics class, and those studied within a context such as kinematics.

The present study presents some tentative evidence that many students view graphs taught in different disciplines in isolation from each other. However more research needs to be undertaken to investigate to what extent this impairs their understanding of graphs as a tool to represent measurable quantities in science.

Another important issue which should be a topic for future research, is a follow-up study to the present investigation, in order to find out whether the teaching strategies proposed here do in fact result in improved understanding of kinematic graphs and graph transformations.

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APPENDIX A

PHYSICAL SCIENCE

QUESTIONNAIRE ON KINEMATICS AND KINEMATIC GRAPHS

SURNAME:

INITIALS:

STD:

HOME LANGUAGE :

MALE	
FEMALE	

Approx. % for last Physical Science exam:

Approx. % for last Maths exam
(fill in the appropriate block)

HG	
SG	

INSTRUCTIONS:

1 This questionnaire forms part of a research project into pupils' and students' understanding of motion and its graphical representation.

2 Please attempt to answer the questions as honestly and thoroughly as possible.

3 You may write on the blank pages if you run out of space.

4 Your personal information will be treated as strictly confidential.

QUESTION 1

Explain briefly how you would explain the concepts of displacement, velocity and acceleration to a friend who has very little knowledge about Science.

Explanation:

QUESTION 2

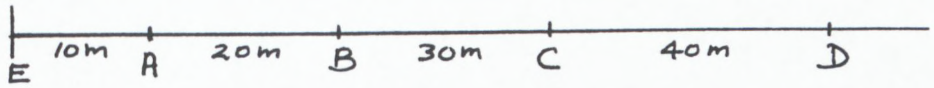
A pupil makes the following statement: *A car* **can** *travel at a velocity of -20 $m.s^{-1}$*.
Cross the appropriate block to indicate your view on this statement:

Agree ☐ Disagree ☐ Unsure ☐

Reason:

QUESTION 3

Consider the following figure representing the positions of four cars A; B; C and D.

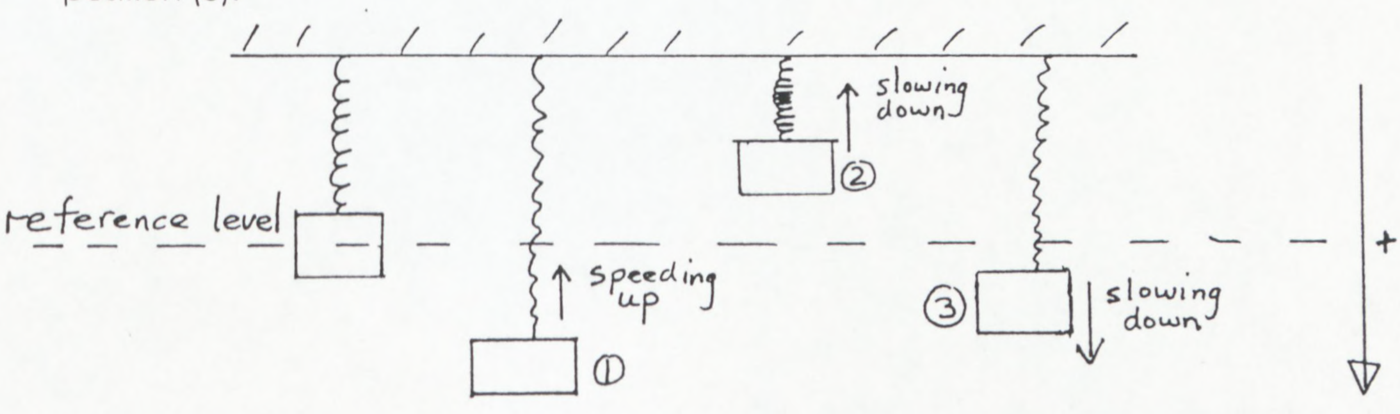


A student states that the displacement of car D is 100 m.

Comment:

QUESTION 4

The sketch shows a wooden block suspended from the ceiling by means of a coiled spring. It is stretched from its position of rest to position (1) after which it is released, moving upwards through position (2) and eventually downwards through position (3).



Taking down as a positive direction and noting the reference-level as indicated, fill in the signs (i.e. + or -) of the displacement, velocity and acceleration at positions (1), (2) and (3) in the following table:

	1	2	3
displacement			
velocity			
acceleration			

QUESTION 5

State the importance of and mention all the information that can be obtained from **gradients** in *displacement-time*, *velocity-time* and *acceleration-time* graphs.

QUESTION 6

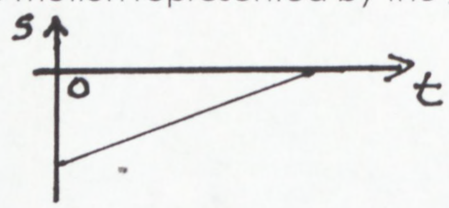
An object moving with constant negative acceleration can be "speeding up"

Agree ☐ Disagree ☐ Unsure ☐

Reason:

QUESTION 7

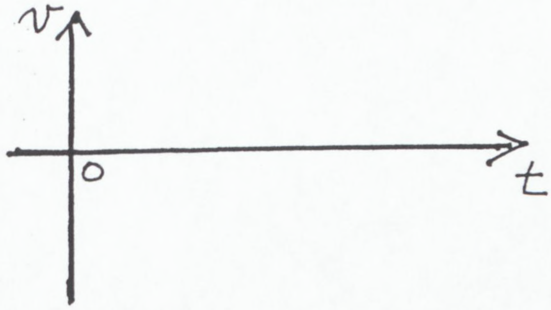
Briefly describe the motion represented by the given *displacement-time* graph.



Description:

QUESTION 8

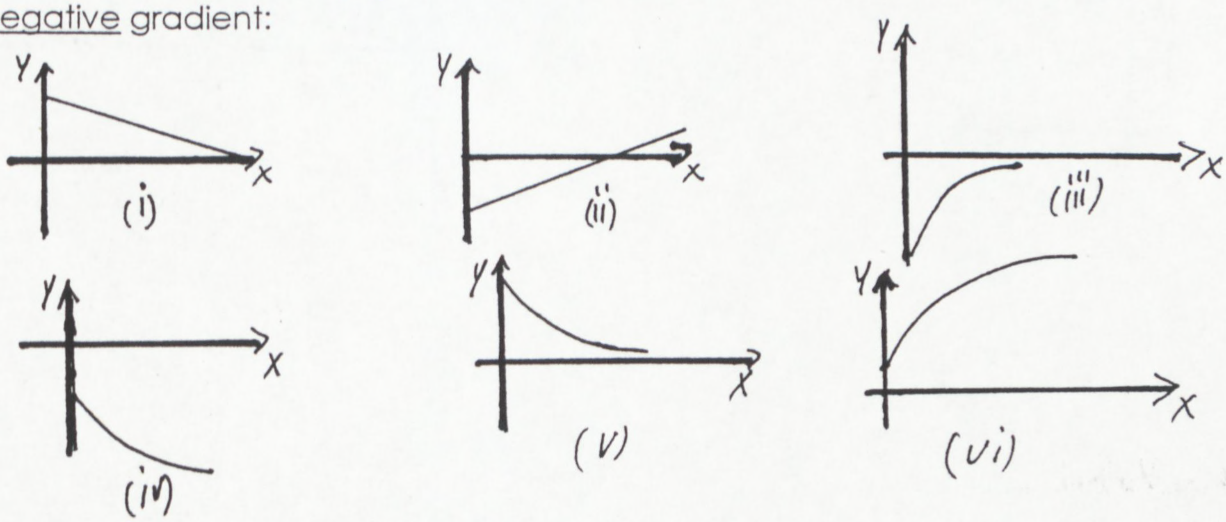
Draw the *velocity-time* graph which describes the same motion depicted by the graph in question 7.



Reason:

QUESTION 9

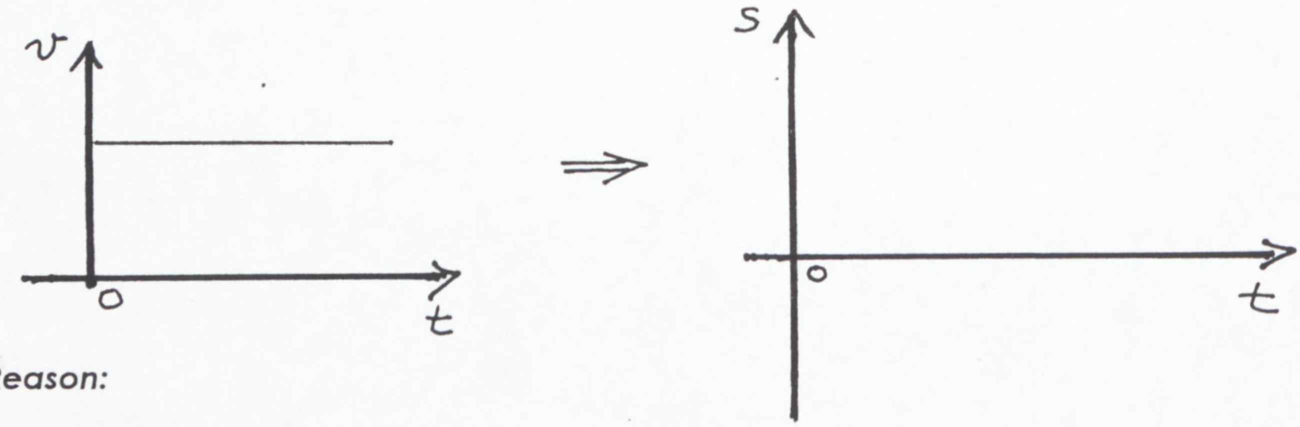
Consider the following graphs, and write down the numbers of those with a negative gradient:



those with a negative gradient are

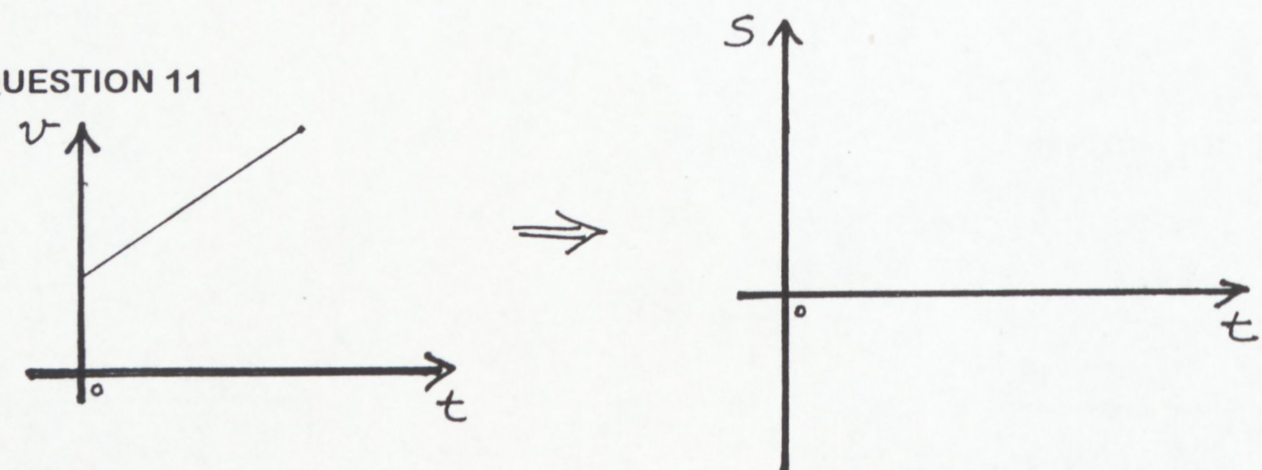
In each of the questions 10, 11, 12 and 13, draw a *displacement-time* graph which matches the given *velocity-time* graph (in other words it describes the same motion). Give a brief reason for your answer in each case.

QUESTION 10



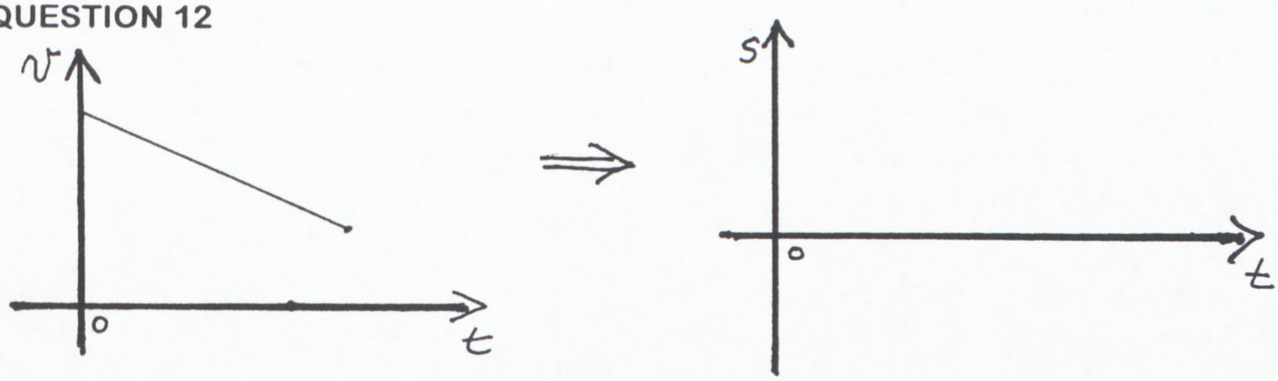
Reason:

QUESTION 11



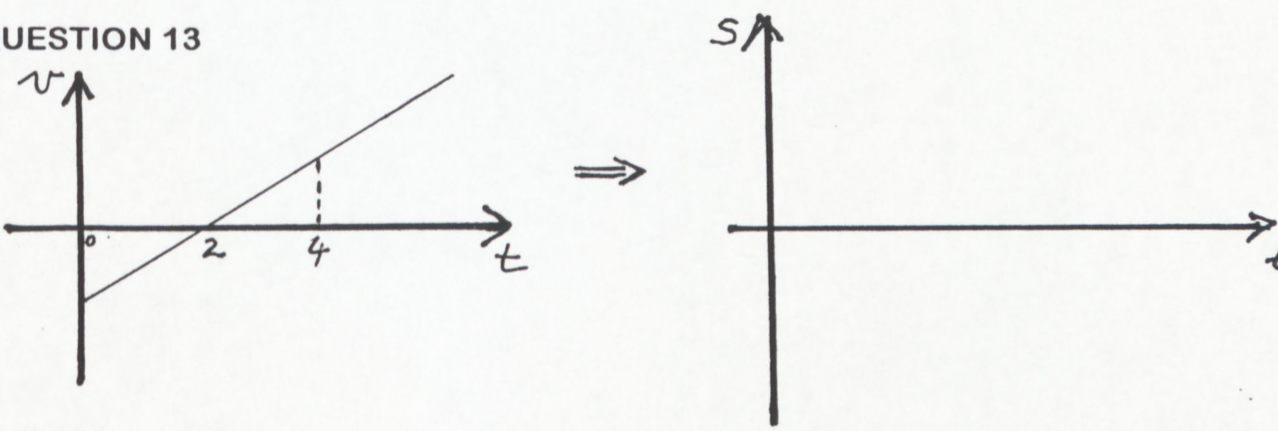
Reason:

QUESTION 12



Reason:

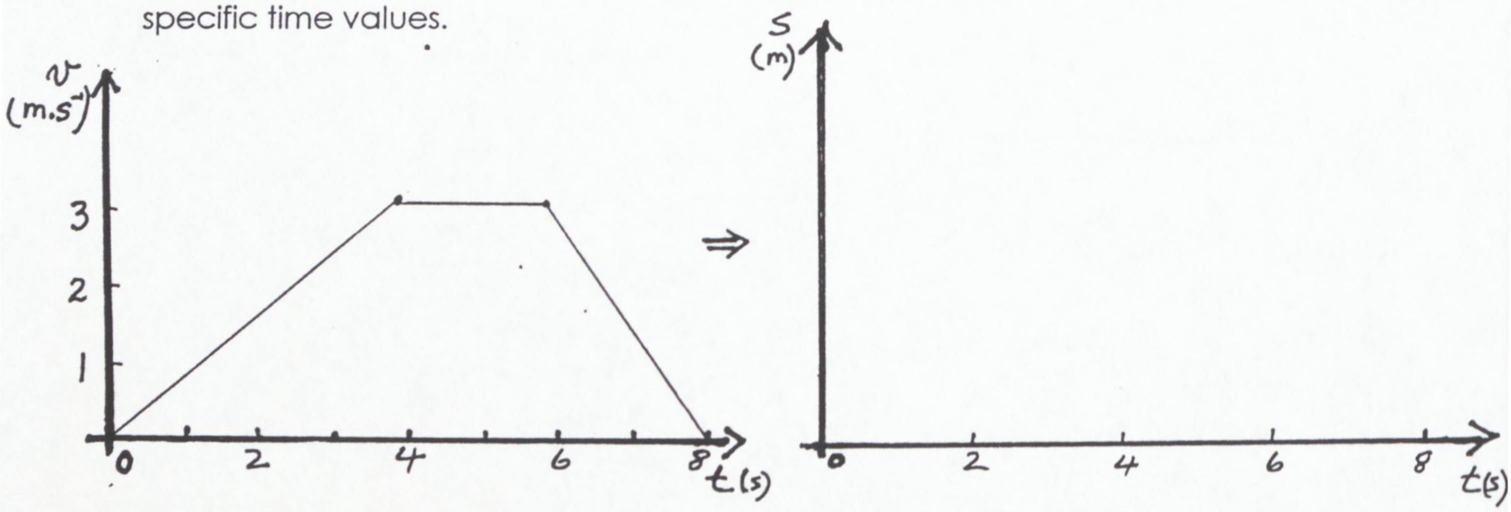
QUESTION 13



Reason:

QUESTION 14

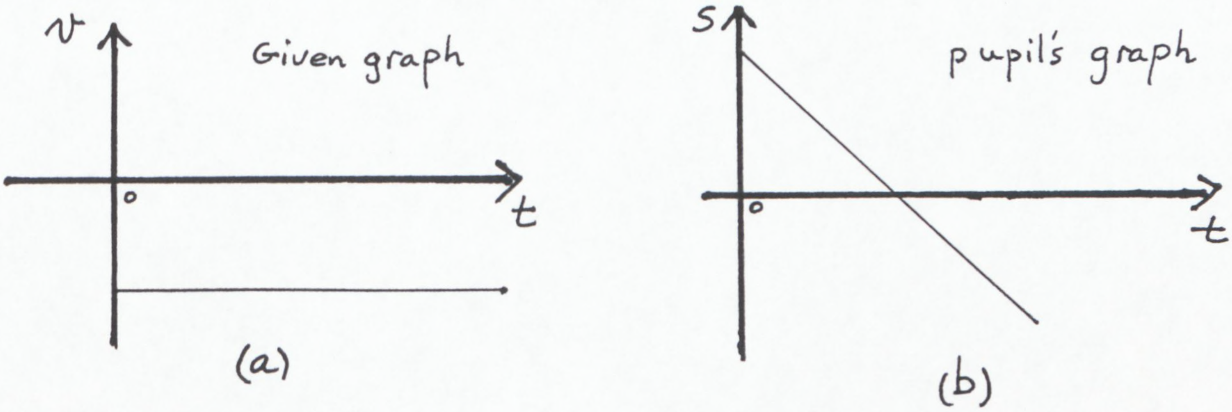
You are given the following *velocity-time* graph of a moving car. Draw the corresponding *displacement-time* graph on the given system of axes, taking s = 0 at t = 0. Indicate actual numerical values on the vertical axes corresponding to specific time values.



Describe briefly how you arrived at your answer:

QUESTION 15

A pupil was asked to draw a *displacement-time* graph corresponding to a given *velocity-time* graph as in fig(a) below. His answer is illustrated in fig(b).



The student's answer is

Completely correct ☐

Partly correct ☐

Completely wrong ☐

Reason: