



**THE EQUAL SIGN: TEACHERS' SPECIALISED CONTENT KNOWLEDGE AND
LEARNERS' MISCONCEPTIONS.**

By

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ABSTRACT

The equal sign: teachers' specialised content knowledge and learners' misconceptions.

Numerical and algebraic equations require understanding of the equal sign as an equivalence relation. Teachers and learners, however, often have an operational, rather than a relational, understanding of the equal sign. This conception is viewed as a misconception.

This study investigates the extent to which Grade 6 learners at a particular school have this and other misconceptions regarding equality, with the equal sign as focus. It also investigates this school's Grade 1 to 6 teachers' specialised content knowledge (SCK) regarding equality, again focusing on the equal sign. Ultimately the study wishes to establish whether there might be a possible relationship between the level of these teachers' SCK of the equal sign and learners' misconceptions of the equal sign. In particular, it tries to answer the question whether teachers' SCK of the equal sign could possibly promote or prevent the forming of such misconceptions in learners, as well as whether teachers' SCK of the equal sign could possibly help them identify learners' misconceptions and help learners form the correct conceptions. This research project is framed within an interpretive paradigm. It focuses on one school taking the form of a theory-led case study in which a mixed method approach is used. Data collection methods include teacher questionnaires followed by two focus group interviews with teachers, based on data collected from questionnaires. In addition, data is collected through a series of lesson observations on number concepts and assessment. Grade 6 learners answered a set of questions structured in the form of a test to investigate their understanding of equality and the equal sign. Six learners were purposefully selected, based on their answers to the questions, and interviewed.

Although this school is a high-performing academic school, results indicate that few learners have a flexible operational or basic relational view of the equal sign. The same group of learners that struggle with closure seems to struggle with the misconception of using all the numbers in an equation to solve a particular equation. The majority of Grade 6 learners cannot define the equal sign correctly. According to results, the nature of Grade 1- 6 teachers' SCK of the equal sign shows that teachers lack skills to prevent, reduce or correct misconceptions about the equal sign.

DECLARATION

I, Bronwin Colleen Meyer, declare that the content of this thesis represent my own unaided work, and that the thesis has not previously been submitted for academic examination towards any qualification. Furthermore, it represents my own opinions and not necessarily those of the Cape Peninsula University of Technology.

.....

Signed

.....

Date

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- Most of all, I am grateful for the support from my husband, Leigh. I also want to thank to my daughter, Abigail. I love you Abby. Thank you for allowing me to complete this study even though at times that has meant that I haven't always been there when you needed me. I apologise if I missed out on precious time.
- To all my family that had a hand in the success of completing this thesis.

Above all, I would like to thank my Heavenly Father – All Glory; Praise and Honour to The Most High God.

DEDICATION

- In memory of my late father Sydney R Isobell.
- My mother Gloria Williams, my aunty Eleanor Matthysen and my in-laws James and Lorna Meyer.
- To my granny Madeline Prince, 92 this year.
- To all my family members that encouraged me to keep my eye on the goal, for their support and sacrifices they made during the four years of my studies.
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OPERATIONAL DEFINITIONS

Abstraction

Skemp (1989:70; 1987:11) states that abstraction is a process by which we become aware of patterns in our experience, which we can recognize on future occasions: use past experience to determine much of our behaviour in the present. Concepts are mental embodiments of these patterns of behaviour.

Accommodation

According to Ault (1977:19), accommodation involves modifying some elements of an old schema or learning a new schema that is more appropriate for the new object. Ginsburg and Oppen (1979:25) refer to the process of altering existing ways of viewing things or ideas that contradict or do not fit into existing schemas. People modify existing schemas to accommodate new ideas.

Alternative conceptions

Alternative concepts refer to the initial, inaccurate beliefs that remain inaccurate, even after instruction (Chi and Roscoe, 2002:5). Alternative concepts are preconceptions which can be removed with instruction, and not removed if instruction does not address them.

Assimilation

According to Ault (1977:19), assimilation involves the process of applying old schemes to new objects.

Concept

Concepts stand for the common elements between a group of schemas or symbols. Spitzer (1977:23) states that concepts are the mental tools that help us organize our complex experiences and make the world more meaningful to us in our daily lives.

Conceptual change

Conceptual change is the process of correcting learners' misconceptions is called "conceptual change" (Chi, 2008:61; Chi and Roscoe, 2002:4).

Concept images

The term concept image describes the total cognitive structure associated with the concept, which includes all the mental pictures and associated properties and processes (Tall and Vinner, 1981:152; Vinner, 1983:293).

Conceptual understanding

Conceptual understanding denotes knowledge of and a skilful “drive” along particular networks, the elements of which can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

Constructivism

Constructivism is a descriptive theory of learning (how people learn or develop); it is not a prescriptive theory of learning (how people should learn) (Noddings, 1990:7 and Richardson, 1997:3)

Disequilibrium

When newly acquired information does not have the same attributes as existing knowledge, a state of imbalance or disequilibrium occurs (Woolfolk, 2010:33).

Error

Brodie (2013:8) defines errors as systematic, persistent and pervasive mistakes performed by learners across a range of contexts.

Equal Sign

The equal sign (=) is used for communicating equality (Jones, Inglis and Pratt, 2011:2).

Equilibration

Equilibration refers to the act of searching for mental balance between cognitive schemas and new knowledge (Woolfolk, 2010:33 and Lefrançois, 1994:58).

Equality (alt: equivalence)

Equality is a relation expressing the idea that two mathematical expressions hold the same value (Oksuz, nd). Darr (2003:4) agrees that equality is about sameness.

Instrumental understanding

Skemp (1976) refers to instrumental understanding as knowing what to do, but not necessarily why – that is, applying (procedural) “rules without reason”.

Misconception

Misconceptions (1) are strongly held, stable cognitive structures; (2) which differ from expert conceptions; (3) affect in a fundamental sense how students understand natural scientific

explanations; and (4) must be overcome, avoided, or eliminated for students to achieve expert understanding (Hammer, 1996:99).

Naïve Knowledge

Chi and Roscoe (2002:3) refer to “preconception” as naïve knowledge which could be revised and removed easily, while “misconception” is naïve knowledge that is hard to change.

Operational View

Stephens, Knuth, Blanton, Isler, Gardiner and Marum (2013:174) define the operational view of the equal sign as a stimulus to “do something” (compute).

Preconception

A preconception is a concept or idea which a learner has upon entering a course and which has some consequence on the person’s work. Bruner (1960) mentions that a preconception is usually recognized as (the) prior knowledge held by a learner which influences the learner’s learning.

Relational understanding

Skemp (1976) refers to relational understanding as “knowing what to do, and why” – that is, the person possesses rich, integrated schemas of a particular concept.

Relational View

Blanton, Levi, Crities, Dougherty and Zbiek (2011:25) state that a relational view of the equal sign entails understanding that the equal sign represents a relation of equivalence. This relation is often interpreted as meaning “the same as” when expressing a relation between equivalent amounts. Stephens et al. (2013:174) state that learners who understand the equal sign as a relational symbol show a much more flexible ability to work with equations in non-traditional formats such as $15 = 5 + 10$ and $8 + 4 = \Delta + 5$.

Schema

Ault (1977:85), Long (2000:21), Woolfolk (2010:33) and Egodawatte (2011:8) mention that the first unit of thought, called a schema, is a mental representation of events in the world.

Slip

Olivier (1989:3) and Egodawatte (2011:8) state that slips are wrong answers, due to processing, sporadically or carelessly made by both experts and novices; they are easily detected and are spontaneously corrected.

Understanding

'To understand something means to assimilate it into an appropriate schema.' Skemp (1989:41)

ABBREVIATIONS

Cape Peninsula University of Technology	CPUT
Common content knowledge	CCK
Curriculum knowledge	CK
Curriculum and Assessment Policy Statement	CAPS
Department of Basic Education	DBE
Foundation Phase	FP
Intermediate Phase	IP
Learner A	LA
Learner B	LB
Learner C	LC
Learner D	LD
Learner E	LE
Learner F	LF
Mathematical Knowledge for Teaching	MKfT
National Curriculum Statement	NCS
National Council of Teachers of Mathematics	NCTM
Outcome-Based Education	OBE
Pedagogical Content Knowledge	PCK
Postgraduate Certificate in Education	PGCE
Research Question	RQ
Revised National Curriculum Statement	RNCS
Southern and East African Consortium for Monitoring Educational Quality	SACMEQ
Specialised Content Knowledge	SCK
Subject matter content knowledge	SMK
Teacher	T
Trends in International Mathematics and Science Study	TIMSS
Western Cape Education Department	WCED

CHAPTER 1

INTRODUCTION AND OVERVIEW

Chapter one provides the rationale for the problem investigated. The research question and sub-questions are outlined and the methodological and theoretical orientations of the study are presented.

1.1 BACKGROUND AND RATIONALE

In not one of the curriculum documents, National Curriculum Statement (NCS); Revised National Curriculum Statement (RNCS); Outcome-Based Education (OBE) or Curriculum and Assessment Policy Statements (CAPS), are explicit mention is made of the concept of the equal sign, or what teaching time is required to ensure that learners understand this concept. Yet it is required that learners have a well-developed relational understanding of this concept, to master Algebra when they enter High School. However, I have noticed in my teaching at the senior level, that there are differences in learners' abilities when they arrive in class from different teachers or different feeder schools, into the Grade 8 classroom. This prompted the question: is there a link between a teacher's SCK and a learner's understanding and proficiency in Mathematics?

The foundation and intermediate phases of schooling are fundamental for learners to develop an understanding of the symbols used in Mathematics. For the past seven years the researcher has been the Grade 7 Mathematics teacher at a primary school in the Western Cape, South Africa. Even at this high-achieving school several learners have an operational view of the equal sign, which will be revealed in the findings in Chapter 5. This is concerning because virtually all manipulations of equations require a sound understanding of the relational value of the equal sign. Teachers need to use their SCK to prevent misconceptions from developing. Teachers need to identify any misconceptions of the equal sign early so that fundamental building blocks are in place. Such foundational concepts are essential for learners to start making the necessary connections between arithmetic and algebra.

All examples listed below were found in learners' exercise books from the school used for sampling. This is a high performing school. If errors like these below can be accepted as correct, it has to be asked whether the teachers understand the importance of the relational meaning of the equal sign.

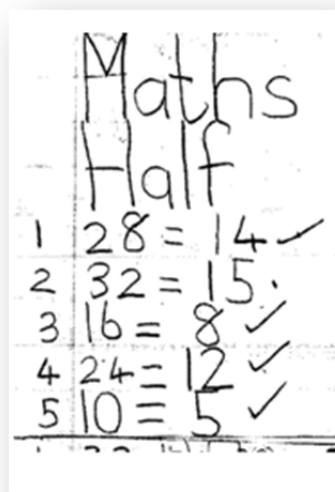


Figure1.1: A Grade 2 Learner's assessment book (marked as correct)

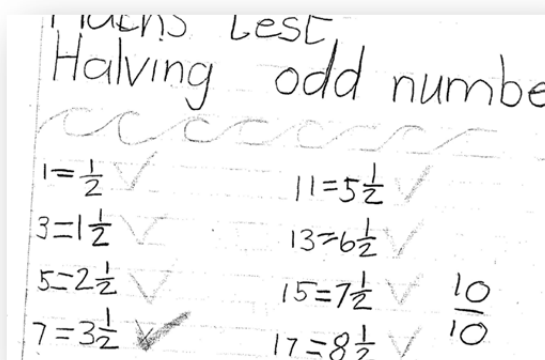


Figure1.2: A Grade 2 Learner's Assessment Book (marked as correct)

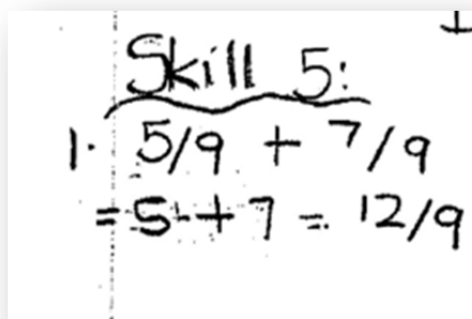


Figure1.3: A Grade 7 learner's workbook- setting out of operation for addition of fractions

1.2 PURPOSE OF THE STUDY

The purpose of this research is to investigate the nature of Grades 1 - 6 teachers' SCK of the equal sign, what misconceptions Grade 6 learners have of the equal sign and whether teachers' SCK of the equal sign could possibly promote, prevent or reduce learners' misconceptions of the equal sign.

1.3 RESEARCH QUESTIONS

The research questions that guided this study were:

- 1.3.1. What misconceptions do Grade 6 learners have of the equal sign?
- 1.3.2. What is the nature of Grade 1 - 6 teachers' SCK of the equal sign?
- 1.3.3. Could the nature of teachers' SCK of the equal sign possibly promote, prevent or reduce learners' misconceptions of the equal sign?

1.4 RESEARCH METHODOLOGY

This theory-led case study was framed within an interpretive paradigm. Both qualitative and quantitative methods were employed to gather the data needed to address the research questions: a convergent parallel mixed method design was used to gather and analyze the data. Data collection methods in this research included: teachers' questionnaires; followed by two focus group interview sessions based on data collected from the questionnaires; one lesson observation session in each of the eleven classes; a test that 57 of the Grade 6 learners completed and individual interview sessions with six learners to allow them an opportunity to explain the thinking behind their errors in the test.

1.5 SIGNIFICANCE OF THE STUDY

The significance of the study is that it could improve South African teachers' SCK pertaining to equality in the foundation and intermediate phase as well as learner proficiency in mathematics. An overview of teacher responses to the questionnaire indicates a weakness in identifying misconceptions and strategies to prevent or reduce learners' misconceptions with regards to the equal sign. Further research is required to gather information about which type of intervention should take place to improve these primary school teachers' knowledge of mathematics. Teachers need support if the goal of mathematical proficiency for all is to be reached (Adler, Ball, Krainer, Fou-lai & Novotna, 2005:361).

According to Skemp (1989:86), teachers cannot construct knowledge for the learners: a good teacher will design and manage learning situations which promote schema construction. Learning situations of this kind use teaching strategies such as structured practical activities or co-operative learning in small groups of learners to foster learners' natural creativity. Learners need to develop a relational understanding of mathematics. Relational understanding supports retention because facts and methods learned with understanding are connected, easier to remember and use, and can be reconstructed when forgotten, according to Kilpatrick, Swafford & Findell (2002:118, 120) and Schneider & Stern (2010:179).

This study investigates Grade 1- 6 teachers' SCK of the equal sign, what misconceptions Grade 6 learners have about the equal sign and whether the nature of teachers' SCK of the equal sign could possibly promote, prevent or reduce learners' misconceptions about the equal sign. This study could assist policy makers, as well as teachers, in formulating strategies for improved learner performance in mathematics and introducing the equal sign correctly at the onset of teaching the concept of equivalence - before misconceptions become entrenched and much harder to correct. This research could help teachers in method courses to identify learners' misconceptions and improve their own SCK sufficiently to assist learners to correct these misconceptions in time. The results of this research project may be used by the Department of Basic Education (DoE) to improve the skills of teachers by offering in-service training. This research may enable the DoE to adjust the CAPS document to allow room for dealing with learners' misconceptions as a platform for conceptual learning to take place.

1.6 LIMITATIONS OF THE STUDY

Rule and John (2011:110) mention that limitations can stem from the following:

- methodology
- data collection methods
- site
- sample
- practical and logistical circumstances
- personal attributes
- the option for a potential participant to choose to take part or not.

The researcher is a teacher at the site of data collection and as such is in an invidious position with regards to gathering data. In the researcher's position as a teacher at the site, sampling and data collection creates apprehension with regards to what will be revealed during observation lessons, questionnaires and focus group interview sessions. Choice of site, sample and position as Grade 6 and 7 mathematics teacher at the site of sampling may have limited the objectivity of this study.

1.7 THESIS OVERVIEW

1.7.1 Chapter Two

Chapter Two provides a theoretical framework and literature review for this study. Three diverse concepts are presented and discussed: teacher knowledge (SCK), learner misconceptions and the equal sign. Literature reviewed was based on the three concepts that form the main structure of this chapter. Part one of this chapter deals with the concept of teacher knowledge in mathematics education. Discussion starts with an overview of the significant work of Shulman (1986; 1987) and Ball, Thames & Phelps (2008). A summary of empirical research on teacher knowledge highlights key aspects of knowledge for teaching related to this study. Part two of the chapter highlights the theory of concept formation: constructivism, teachers and correct formation of the concept of the equal sign. The final part of the chapter focuses on the concept of equality and the symbol that represents it, the equal sign.

1.7.2 Chapter Three

Chapter Three outlines the research design and methodology used in this study. This chapter describes how the study is framed within an interpretive paradigm. The selected methodology, which takes the form of a theory-led case study, is described. Advantages of using a mixed method approach are deliberated upon. A convergent parallel mixed method design is used because qualitative and quantitative methods were employed to gather the data needed to address the research questions. A distinction between qualitative research and quantitative research is made. Reference is made to the positionality of the researcher in this study. The data analysis process is discussed in detail. Matters relating to ethical considerations, reliability, validity, trustworthiness and triangulation conclude this chapter.

1.7.3 Chapter Four

In Chapter Four empirical evidence derived from learner interviews, tests on equivalence, teacher questionnaires, teacher lesson observations and focus group interviews are described, analysed and discussed. This chapter comprises a summary of results of teachers' SCK regarding the equal sign and learner misconceptions with regards to the concept. The SCK of the teachers relating to the equal sign was observed and evaluated using a questionnaire, lesson observations and focus group interviews. One focus group interview session was held with foundation phase teachers and another focus group interview session was held with intermediate and senior phase teachers. Grade 6 learners' misconceptions were evaluated using a test focussing on the use of the equal sign in number sentences. A group of 6 learners were interviewed to allow learners an opportunity to verbalise their thinking process when making an error. The results from the analysis of the data are organised into the following categories:

- Test on equivalence
- Interviews with the six learners
- Questionnaire findings
- Observations of teachers' lessons
- Focus group interviews
- Foundation phase teachers
- Intermediate phase teachers

1.7.4 Chapter Five

This chapter encapsulates results highlighted in chapter four and positions the results within a learning and teaching perspective. Recommendations are made to the learners as well as the teachers. The importance and limitations of the study are highlighted. This chapter concludes with final thoughts with regards to the research study as a whole.

1.8 CONCLUSION

This chapter discusses the research background and rationale of the study, the purpose of the research, research questions, significance of the study, limitations of this study and a summary of the theoretical framework and research methodology. It provides outlines of all

the chapters of the thesis. The next chapter outlines the theoretical framework that informed this study and reviews literature related to this study.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The concept of pedagogical content knowledge (PCK) was introduced by Shulman and later reconstructed into clearer categories by Ball, Thames & Phelps (2008:391). I will discuss the theory of teachers' specialised content knowledge (SCK) in the light of Ball's work and the concept of the equal sign. In this research there are three main areas of discussion: teachers' specialised content knowledge (SCK), learners' misconceptions and the equal sign. These three areas form the body of this chapter.

The first part of this chapter deals with teacher knowledge in mathematics education. Discussion starts with a summary of the work of Shulman (1986; 1987) and Ball et al. (2008). A summary of empirical research on teacher knowledge highlights key aspects of knowledge for teaching related to this study. The second part of the chapter focuses on the theory of concept formation: how it relates to constructivism, the relevance of this relation to teachers and how teachers can prevent learners from forming misconceptions. The third part of the chapter deals with the concept of equality and the symbol that represents it, namely the equal sign; showing how easily learners form misconceptions regarding the equal sign.

2.2 TEACHER KNOWLEDGE IN MATHEMATICS EDUCATION

2.2.1 Background

Shulman (1986) developed a framework for describing teacher knowledge. He set out to answer the question: 'what knowledge is necessary to enable teachers to communicate effectively the most useful forms of representation . . . the most powerful analogies, illustrations, examples, explanations, and demonstrations — in a word, (knowing) the ways of formulating the subject that makes it comprehensible to others?' (1986:9).

Ball, Thames & Phelps (2008:391) mention that teacher knowledge, according to Shulman, includes seven general and content-specific dimensions. The first four dimensions address general areas of teacher knowledge that were the mainstay of teacher education programs at the time. These categories are listed below:

General areas of teacher education:

- General pedagogical knowledge
- Knowledge of learners and their characteristics
- Knowledge of educational contexts
- Knowledge of educational ends, and values

Content-specific dimensions:

- Subject matter content knowledge
- Curriculum Knowledge
- Pedagogical content knowledge

(Shulman, 1986)

The last three categories listed above, defined as content-specific dimensions, comprise what Shulman (1986) referred to as the missing paradigm in research on teaching. What follows is a description of each of these content-specific dimensions.

Subject Matter Content Knowledge (SMK)

Shulman (1986:9) states that SMK “refer[s] to the amount and the organisation of knowledge by itself in the mind of the teacher”. The teacher needs to understand that something is so and understand why it is so. Ball et al. (2008:391) and Prediger (2009:74) agree that teachers need to understand the organizing principles, structures and rules for establishing what is legitimate to do and say in a specific field of learning. Teachers should know why a given topic is central to a subject.

Curriculum Knowledge (CK)

According to Shulman (1986:10), CK refers to the materials and programs that serve as “tools of the trade” for teachers. Ball et al. (2008:391) states that curriculum knowledge is represented by the full range of programs designed for teaching particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the characteristics that serve as indications and contra-indications for the use of particular curriculum or program materials in particular circumstances. Shulman (1986:9) continues that curricular knowledge involves understanding the curriculum being taught. Shulman includes in this type of knowledge an understanding of what has come before and after a particular part of the curriculum being taught: teachers need to know how far learners have progressed and where they are going.

Pedagogical Content Knowledge (PCK)

This is the final content-specific dimension of knowledge which goes beyond knowledge of subject matter to the “dimension of subject matter knowledge for teachers”. Shulman (1987:8; 1986:9) states that PCK is of special interest because it identifies the distinctive bodies of knowledge for teaching. This is the knowledge required for teaching the subject (Shulman 1987:8; 1986:9). Prediger (2009:74) mentions that PCK includes typical difficulties and preconceptions of learners of the most frequently taught topics.

2.2.2 Pedagogical Content knowledge (PCK)

Shulman (1986:9) defines pedagogical content knowledge as subject knowledge for teaching. This entails making the subject accessible to learners. Shulman (1986:8; 1987:9) explains that pedagogical content knowledge (PCK) refers to the knowledge of teaching strategies, learners’ thinking, cognitive demands of task, representations, resources, curriculum and purpose of content knowledge. Knowledge of content and pedagogy is blended into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners, and presented for instruction. PCK bridges subject-matter, content-knowledge and practice of teaching. According to Shulman (1986:8; 1987:9), pedagogical content knowledge is the category most likely to distinguish understanding of content specialist from that of the pedagogue.

When content knowledge and pedagogical knowledge are kept distinct, teachers learn what to teach and best-practice principles that govern how to teach, but not specifically how to teach the “what” (Shulman, 1986:9; 1987:9). Shulman (1986:9; 1987:20) was ultimately concerned about the limited notion of teacher competency which focused too much on “generic teaching behavior”. He proposed a framework for analyzing teacher knowledge, as illustrated below:

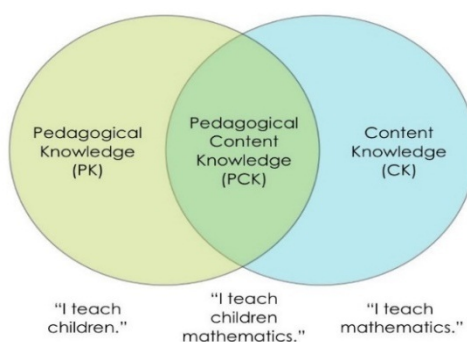


Figure 2.1: Shulman’s illustration of PCK (Shulman, 1986)

Shulman (1987:8; 1986:9) states that PCK includes an understanding of what conceptions and preconceptions learners of different ages and backgrounds bring with them to the learning process. If those preconceptions are misconceptions, teachers need knowledge of the strategies most likely to be of use in re-organising the understanding of learners. Chick & Baker (2005:249) agree; stating that the problem of misconceptions is exacerbated if teachers themselves lack conceptual understanding in key areas.

2.2.3 Mathematical Knowledge for teaching (MKfT)

Building on Shulman's three categories of knowledge, Ball and her colleagues (Ball, Thames, & Phelps, 2008:399) expand and define mathematical knowledge for teaching (MKfT). Ball et al. (2008:399) define MKfT as the mathematical knowledge required to "perform the recurrent tasks of teaching mathematics to learners": the mathematical knowledge "entailed by teaching". According to Ball et al. (2008:399), this framework, illustrated below, expands Shulman's categories of subject matter knowledge and pedagogical content knowledge. Subject matter knowledge consists of common content knowledge (CCK) and specialised content knowledge (SCK), as well as knowledge at the mathematical horizon. Pedagogical content knowledge is sub-divided into knowledge of content and teaching (KCT), knowledge of content and students (KCS), and knowledge of curriculum.

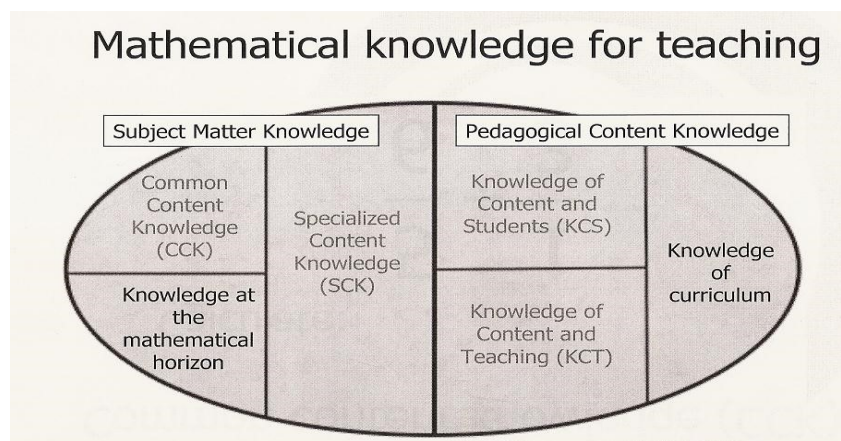


Figure 2.2: Mathematical knowledge for teaching (Ball et al., 2008:403)

According to Thames & Ball (2010:223), common content knowledge (CCK) is important for teaching tasks such as knowing whether a learner's answer is correct, providing learners with the definition of a concept or an object, and demonstrating how to carry out a procedure. Ball et al. (2008:399) mention that examples of common content knowledge include knowledge of algorithms and procedures for adding and subtracting, finding the area and perimeter of a given shape, or ordering a set of decimals. CCK includes knowing when

learners have answers wrong, recognizing when the textbook gives inaccurate definitions, being able to use terms and notations correctly when speaking and writing on the board and the knowledge teachers need in order to be able to do the work that they are assigning their learners. CCK is closely related to content of the curriculum, but not to a particular curriculum (Ball et al. 2008:6; 2008:399).

Thames & Ball (2010:224) mention that some mathematical knowledge entails a specialised knowledge not used by others: for example, being able to define terms in a mathematically correct but inaccessible way. Specialised content knowledge (SCK) is defined as the mathematical knowledge and skill uniquely needed by teachers to conduct their work. Ball et al. (2008:400) state that teachers have to do a kind of mathematical work that others do not. Examples of specialised content knowledge include using mathematical notation and language, critiquing its use, looking for patterns in learners' errors or in sizing up whether a non-standard approach might work in general. Many of the everyday tasks of teaching are peculiar to this special work (Ball et al., 2008:400). Each of these tasks is something teachers routinely do. Taken together, these tasks demand unique mathematical understanding and reasoning. Teaching requires knowledge beyond that being taught to learners. For instance, it requires understanding different interpretations of operations in ways that learners need not explicitly distinguish (Ball et al., 2008:400).

Ball et al. (2008:400) give the following examples of "Mathematical Tasks of Teaching", which could be viewed as aspects of SCK:

- Presenting mathematical ideas
- Responding to students' "why" questions
- Finding an example to make a specific mathematical point
- Recognizing what is involved in using a particular representation
- Linking representations to underlying ideas and to other representations
- Connecting a topic being taught to topics from prior or future years
- Explaining mathematical goals and purposes to parents
- Appraising and adapting the mathematical content of textbooks
- Modifying tasks to be either easier or harder
- Evaluating the plausibility of students' claims (often quickly)
- Giving or evaluating mathematical explanations
- Choosing and developing usable definitions
- Using mathematical notation and language and critiquing its use

- Asking productive mathematical questions
- Selecting representations for particular purposes
- Inspecting equivalencies

Knowledge at the mathematical horizon is a domain of MKfT that affords a kind of mathematical “peripheral vision” needed in teaching: it is an awareness of how mathematical topics are related over the whole span of mathematics included in the curriculum (Ball et al., 2008:403).

Pedagogical content knowledge

Ball et al. (2008:389) suggest that although the term PCK is widely used, the bridge between knowledge and practice is inadequately understood. Ball et al. (2008:391) state that Shulman was concerned with prevailing concepts of teacher competency which focused on generic teaching behaviour. Shulman believes that high-quality instruction requires a sophisticated, professional knowledge that goes beyond a set of simple rules. According to Ball et al. (2008:389) pedagogical content knowledge (PCK) can be sub-divided into:

- Knowledge of Content and Students (KCS)
- Knowledge of Content and Teaching (KCT)
- Knowledge of Curriculum

According to Ball et al. (2008:401), knowledge of content and students (KCS) is knowledge that combines knowing about students and knowing about mathematics. This type of knowledge includes anticipating learners’ difficulties, understanding learners’ reasoning, recognising common errors and misconceptions that learners exhibit with specific material. Ball et al. (2008:401) explain how CCK, SCK and KCS types of knowledge work together in the classroom:

Recognizing a wrong answer is common content knowledge (CCK), while sizing up the nature of the error, especially an unfamiliar error, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialized content knowledge (SCK). In contrast, familiarity with common errors and deciding which of several errors students are most likely to make are examples of knowledge of content and students (KCS).

The final domain that these researchers have expanded on in detail is knowledge of content and teaching (KCT). This type of knowledge combines knowing about teaching with knowing about mathematics. It involves knowing how to sequence a particular set of topics and understanding the power and value of different mathematical representations. Teachers

need to use KCT to help learners during a classroom discussion; they have to decide when to pause for more clarification, when to use a learner's remark to make a mathematical point, and when to ask a new question or pose a new task to further learners' learning (Ball et al., 2008:401). These types of teaching tasks require that the teacher has both a deep understanding of the subject of mathematics as well as an understanding of how a teacher's actions and decisions affect how and what learners learn. Of interest to this study is evidence of teachers' specialised content knowledge (SCK).

2.2.4 Teacher knowledge in South Africa

South Africa has, over the past few years, engaged in various approaches to teacher development, yet little has changed in teacher practices (Jita & Ndlalane, 2009:58). Adler et al. (2005:360) mention that quality teaching is more important at levels where mathematics is a general requirement, such as in primary school. More teachers and better mathematics teaching are needed if mathematical proficiency is to become a widely-held competence.

Venkatakrishnan & Spaul (2007:5) claim that research findings relating to measuring South African teachers' mathematical content knowledge recommend that a focus on content knowledge remain critical. A study conducted by the National School Effectiveness Study (NSES) provides evidence of gaps at the level of doing mathematics at the Grade level of teaching for many teachers across the topics. Venkatakrishnan & Spaul (2007:9) indicate that an overview of South African teacher performance indicates gaps at the level of what Ball et al. (2008) describe as Common Content Knowledge. Many practicing teachers, for different reasons, have not learned some of the content they are now required to teach, or they have not learnt it in ways that enable them to teach what is now required. In particular, curriculum reform processes in mathematics across different countries result in many teachers now having to teach a curriculum that is different from the one for which they were trained, and from one with which they had become experienced, even if successfully so. Teachers need support if the goal of mathematical proficiency for all is to be reached (Adler et al., 2005:361).

Venkatakrishnan & Spaul (2007:9) claim that Chinese teachers display an awareness of connections between concepts and a progression of key ideas whereas there are disconnections evident in teaching at all levels of the South African schooling system (Venkat & Adler, 2012; 2014). Venkatakrishnan & Spaul (2007:10) state that South African teachers generally have gaps of knowledge. Little information exists about what interventions can be used to develop primary school teachers' mathematics knowledge. According to Jita &

Ndlalane (2009:59), the cluster approach to teacher development seems to promote collaboration, construction and sharing of CK and PCK.

2.3 CONSTRUCTIVISM AND CONCEPT FORMATION

2.3.1 Constructivism as a learning theory

Constructivism concerns itself with the construction of making individuals create their own new understanding, based upon the interaction of what they already know and believe, and phenomena or ideas with which they come into contact (Noddings, 1990:7). Constructivism is a descriptive theory of learning (how people learn or develop); it is not a prescriptive theory of learning (how people should learn) (Noddings, 1990:7 and Richardson, 1997:3).

Constructivist learning theory maintains that learning is not the result of teaching but the result of what learners do with new information they are presented with. Learners construct their own knowledge; they are not passive recipients of new information (Sewell, 2002:24). Ball (1988:40), Olivier (1989:10), Prediger (2009:77) and Egodawatte (2011:14) agree that learners are active participants in constructing their knowledge. According to Egodawatte (2011:14), learners construct their own mental representations of situations, events and conceptual structures when they begin schooling. Recognition of this fact is paramount in the constructivist perspective of what should transpire in the classroom. What becomes accepted generally as 'knowledge' is in fact the result of shared consensus over the years. The activities of teaching and learning must be guided by obligations that are created and regenerated through social interaction.

Bodner (1986:2), Nickson (2000:4) agree that knowledge is constructed as the learner strives to organise his or her experience in terms of pre-existing mental structures or schemes. Piaget believes that knowledge is acquired as the result of a life-long constructive process in which we try to organise, structure and re-structure our experiences in light of existing schemas of thought; gradually modifying and expanding these schemas (Olivier, 1989:10; Egodawatte, 2011:14). Skemp (1989:72), Olivier (1989:2) and Kennedy, Tipps & Johnson (2008:50) state that from a constructivist perspective on learning, knowledge does not arise from experience but the interaction between experience and our current knowledge structures. This construction activity involves the interaction of a learner's existing ideas and new ideas. New ideas are interpreted and understood in the light of learners' current knowledge which is built up of his/her previous experiences.

Learners interpret knowledge, organise and structure this knowledge into larger units of interrelated concepts, called schemas. Such schemas are valuable intellectual tools, stored in memory, which can be retrieved and utilised. Learning involves the interaction between a child's schemas and new ideas. Skemp (1989:86) states that the more abstract and hierarchical a schema becomes, the greater the difficulty in constructing it and the more the learner needs direct assistance and explanation.

2.3.1.1 Critics of Piaget's work on constructivism

Lefrançois (1994:70-71) and Woolfolk (2010:41) cite Gelman (2000) and Gelman & Cordes (2001) who mention that it appears that Piaget underestimated the cognitive abilities of learners, particularly younger ones. Saxe (1999), as cited by Woolfolk (2010:41), states that a final criticism of Piaget's constructivism theory is that it overlooks important effects of the learner's cultural and social grouping. Baker, McGaw & Peterson (2007:1-2) argue that constructivism is seldom defined clearly but seems to be used to "distinguish the good guys (constructivists) from the bad guys (traditionalists)." Many critics say that the label 'constructivist teaching' is used by authors as more or less synonymous with any teaching that is somewhat 'child-centered': caring, inclusive, enquiry-based, discovery or any kind of active involvement by the learners. According to Baker et al. (2007:1-2), critics argue that constructivism as a meaningful concept has lost its power.

Despite the above critique, this study utilises constructivism as a theory of learning. It is well known that, from a mathematical perception, constructivism provides a sound explanation for formation of mathematical concepts. This study is interested in how new conceptual knowledge is constructed, given a learner's pre-existing knowledge: whether preconceptions of the equal sign increase misconceptions that become entrenched and resistant to correction later. For this purpose, constructivism as a learning theory provides a suitable theoretical framework.

2.3.2 Concept Formation

2.3.2.1 Formation of new concepts

According to Piaget, adaptation of new knowledge is made possible by a process of assimilation and accommodation (Lefrançois, 1994:58; Woolfolk, 2010:33, Kennedy et al., 2008:50). According to Skemp (1989:86), schemas have to be constructed by the learner in his/her own mind. No teacher can construct knowledge for learners; a good teacher provides

learning situations which are favourable to schema construction. Learning situations of this kind include structured practical activities, co-operative learning in small groups of learners and those which use learners' natural creativity.

Skemp (1989:87) states that because of the importance of learners' schemas for long-term learning, the teacher needs to ensure that at every stage, any new concepts to be learnt can be assimilated into the learner's available schemas. Sometimes ideas are encountered which cannot be assimilated into an available schema: re-construction of the schema is required before learning can take place. This process is known as accommodation and is often unwelcome and difficult. Particular care is needed with the foundational concepts on which a schema is built.

Ault (1977:19), Ginsburg & Oppen (1979:25), Olivier (1989:3), Lefrançois (1994:58) and Woolfolk (2010:33) refer to assimilation as the expansion of existing schemas to incorporate new experiences. According to Lefrançois (1994:58), Long (2000:30) and Woolfolk (2010:33), Piaget believed that much of the time new information is assimilated or "fitted in" with existing schemas. When information does not conform, a state of imbalance occurs. If further information does not fit an existing schema, imbalance becomes too great and forces a process of restructuring or adjustment to the existing schema, known as accommodation. Kennedy et al. (2008:50) agree that when new schemas do not correspond with existing schemas, a state of disequilibrium is created. Disequilibrium ends when learners reconcile new experiences through accommodation or by modifying their existing schemas.

Ault (1977:19), Ginsburg & Oppen (1979:25), Olivier (1989:3), Lefrançois (1994:58) and Woolfolk (2010:33) state that accommodation is the process of altering existing ways of viewing things or ideas that contradict or do not fit into existing schemas. Individual learners may modify their existing schemas to accommodate new ideas. According to Piaget, organising, assimilation and accommodation can be viewed as a complex balancing act. Actual changes in thinking take place through the process of equilibration (Woolfolk, 2010:33). Equilibration refers to the act of searching for mental balance between cognitive schemas and new knowledge (Woolfolk, 2010:33 and Lefrançois, 1994:58).

Skemp (1989:88) concludes that if assimilation and accommodation do not occur, learning with understanding comes to an end, and rote learning alone is possible. Further progress in mathematics is inefficient and the learner often gives up any hope of real understanding.

2.3.2.2 Learners' reaction to new knowledge which cannot be assimilated

Pre-existing understandings are the foundation upon which new knowledge is constructed. New concepts are learnt and retained only if they can be fitted or joined to already existing knowledge. If the foundation is incorrect, new knowledge will not 'stick' according to Sewell (2002:24). Sewell (2002:24) and Risch (2014:2) claim that when learners reach a learning situation where they are presented with new information that differs from their previous understanding or knowledge, they can deal with such new knowledge in one of four different ways:

(a) Delete pre-existing knowledge

This is the most difficult of all four options. "Preconceptions are tenacious; learners hold onto them with great vigor unless forced to change. Even when someone tells us we are wrong, this is usually insufficient reason to change what we know" (Sewell, 2002:25). Major conceptual change occurs here: core concepts are rejected and replaced by new ones (Chinn & Brewer, 1993).

(b) Modify pre-existing knowledge to fit new information

Modifying pre-existing knowledge is easier than deleting pre-existing knowledge. Altering what we know does not require us to forego what we have taken the trouble to learn. Learners are prepared to make minor alterations to existing mental frameworks if this is all that is required for new information to 'fit'. Learners will change their beliefs only if it no longer meets immediate needs; for example, to pass a test (Sewell, 2002:26).

(c) Modify new information to fit old knowledge

According to Sewell (2002:26) and Risch (2014:2), modification of new knowledge often happens in our classrooms. In order to 'fit' existing schemas, incoming information is altered or distorted to such a degree that it is actually wrong. Learners are reluctant thinkers and modification requires work. Given a choice, learners will more often than not choose not to think and instead move to the next, easiest option, namely unequivocal rejection of new information (Sewell, 2002:26). Such resistance to the labour of thinking may be termed mental inertia.

(d) Reject new information

Rejection of new information is the preferred option of many learners and their most common reaction to new information that conflicts with existing beliefs. Such rejection requires no mental effort, no reconstruction of pre-existing knowledge: learners emerge from the lesson content with what they have always known (Sewell, 2002:26; Risch, 2014:2; Chinn & Brewer, 1993).

(e) Form a new schema unrelated to existing schemas

In addition to the four actions mentioned above (2.7.1 to 2.7.4), Chinn & Brewer (1993) mention that “some learners would respond by holding irregular data in abeyance, leaving the original intact. The learners would reinterpret inconsistent data so that incompatible concepts are viewed as complementary and not antagonistic.”

Chinn & Brewer (1993) extend the range of responses to or options for conceptual change available to learners. Based on their analysis of both the history of science and the conceptual change literature, Chinn & Brewer (1993) suggest that, when confronted with a discrepant experience, learners may respond in the following ways:

- 1) Leave the original concept unchanged
- 2) Explain the new data away as being outside the realm of the original concept.
- 3) Change conception peripherally, making minor modifications but leaving basic concepts intact.

2.3.2.3 Understanding

“To understand something means to assimilate it into an appropriate schema” according to Skemp (1989:41,168) and depends on the existence of appropriate ideas and on the creation of new connections, varying with each person. The greater the number of connections to a network of ideas, the better the understanding of the learner will be.

Rittle-Johnson & Alibali (1999:175) and Holmes (2012:59) state that Skemp’s theory classified knowledge into relational and instrumental understanding. Relational understanding is gained from formal language or symbolic representations and is the understanding of rules, algorithms and procedures. Selinger (1994:185) refers to the seminal article written by Skemp (1976) and describes instrumental understanding as ‘rules

without reasons'. Relational understanding involves understanding relations between concepts and principles, including concepts and schemas behind a concept. "Competence in mathematics requires learners to develop and link their knowledge of concepts and procedures" (Rittle-Johnson & Alibali, 1999:175). Selinger (1994:185) continues that a learner who has relational understanding is able to reconstruct forgotten facts and techniques, by means of diagrammatic representations of a concept.

These two types of understanding lie on a continuum and cannot always be separated. The two ends of the continuum represent two different types of understanding. These two types of understanding do not develop independently. It is likely that learners' conceptual understanding influences the procedures they use. Skemp (1976:15) believes that relational and instrumental understandings are spawned from different cognitive activities and produce different cognitive outcomes, as cited by Orton & Frobisher (1996:13). Holmes (2012:58) mentions that relational understanding is defined by the web of conceptual connections which undergird mathematical knowledge. Instrumental understanding isolates concepts as illustrated in the diagram below:

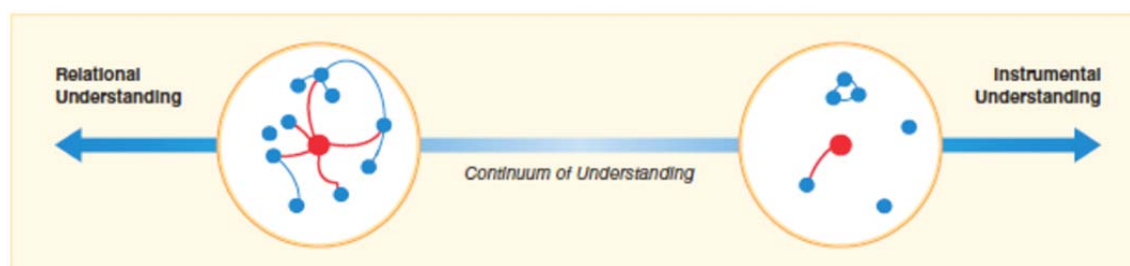


Figure 2.3: Continuum of Understanding (Van De Walle, 2013)

An individual's understanding can be described as existing along a continuum, as illustrated above. At one extreme is a very rich set of connections – relational understanding. The understood idea is linked with many other existing ideas in a meaningful network of concepts and procedures, however on the other extreme, ideas are isolated and, thus, essentially without meaning – instrumental understanding (Skemp 1989:15-16). Figure 2.3 is a metaphor for the construction of concepts: small dots represent existing concepts. Lines joining the ideas represent logical connections that develop between concepts. The large dot is a developing concept; one that is being constructed. When existing concepts are used in the construction of knowledge, they are connected to the new concept and grant meaning to the new concept.

(a) Instrumental Understanding

Instrumental understanding is usually seen as understanding of operators and conditions under which they can be applied to reach certain goals (Schneider & Stern, 2010:179). Automated instrumental understanding can be used with minimal conscious attention or few cognitive resources. This efficiency, however, has the drawback of inflexibility. Automated understanding is partly open to conscious inspection: it can hardly be verbalised or transformed by higher mental processes and is often tied to specific problem types. Skemp (1976:14) mentions that “the kind of learning which leads to instrumental mathematics consists of the learning of an increasing number of fixed plans, by which learners can find their way from particular starting points (the data) to required finishing points (the answers to the questions).” There is no awareness of the overall relation between successive stages and the final goal. In both cases, the learner is dependent on outside guidance for learning each new ‘way to get there’.

Rote memorisation of facts and processes brings about instrumental understanding. This form of knowledge acquisition is a lower level of understanding, termed instrumental understanding (Orton & Frobisher, 1996:13).

(b) Relational Understanding

According to Skemp (1976:14-15), “learning relationally consists of building up a conceptual structure (schema) from which its possessor can (in principle) produce an unlimited number of plans for getting from any starting point within his schema to any finishing point.”

This kind of understanding differs from instrumental understanding in several ways, as outlined below:

- The means becoming independent of particular ends
- Building up a schema within a given area of knowledge becomes an intrinsically satisfying goal in itself
- The more complete a learner’s schema, the greater the feeling of confidence to find new ways of ‘getting there’ without outside help
- A schema is never complete. As our schemas enlarge, so our awareness of greater possibilities is enlarged (Skemp, 1976:15).

Relational understanding refers to an integrated and functional grasp of mathematical ideas, which may be demonstrated in a reflective mode of thinking when, for example, the learner skillfully combines two rules without exactly knowing why they work (Kilpatrick, et al. 2002:118; Schneider & Stern 2010:179 and Holmes, 2012:58). The term “relational” accurately portrays understanding because embedded in meaning are cognitive relations or schemas, where background knowledge and related concepts are connected in the person’s thinking. ‘To understand why is to have learned - relationally’ (Schneider & Stern, 2010:179).

Learners with relational understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which is it useful. They organise their knowledge into a coherent whole which enables them to learn new ideas by connecting such ideas to what they already know. Relational understanding supports retention. Because facts and methods learned with understanding are connected, they are easier to remember and use, and can be reconstructed when forgotten, according to Kilpatrick et al. (2002:118) and Schneider & Stern (2010:179). Kilpatrick et al. (2002:120) continue by stating that relational understanding helps learners avoid many critical errors in solving problems. Relational understanding frequently results in learners having less to learn because they recognise deeper patterns of similarity between superficially unrelated situations. Their understanding has been encapsulated into compact clusters of interrelated facts and principles.

2.3.2.4 COGNITIVE SCHEMAS vs CONCEPTS

(a) Cognitive Schemas

Ault (1977:85), Long (2000:21), Woolfolk (2010:33) and Egodawatte (2011:8) mention that the first unit of thought, called a schema, is a mental representation of events in the world. Long (2000:21) mentions that schemas are useful ways of understanding how we group together and simplify our general knowledge and understanding. Schemas exist because they are ways of achieving cognitive economy. According to Olivier (1989:2), learners organise and structure new knowledge into large units of interrelated concepts called schemas.

According to Skemp (1987:25; 111), our existing schemas are indispensable tools for the acquisition of further knowledge. Everything we learn depends on knowing something else before. Skemp (1987:111) claims that schemas have many uses, which fall into three main groups:

- For integrating knowledge, and making possible understanding.

Skemp (1987:111) mentions that our existing schemas (delta one) are the source from which new schemas (delta-two) are constructed. To make understanding possible, new schemas need to be assimilated into an appropriate schema.

- To help us to co-operate with others in a wide variety of ways.

“The effective working of any society depends on many and several people co-operating in many different ways for a great diversity of tasks. The most successful way of doing this is by existence of widely shared schemas. Co-operation also involves exchange of information, which is to say a shared language for the schema, its concepts and their relationships” (Skemp, 1987:13).

- As agents of their own growth.

“Experiences that fit easily into our existing schemas are more readily learned and better remembered, than those that do not. These experiences that readily fit into our existing schemas sensitise us to experiences that we would otherwise have ignored” (Skemp, 1987:114).

(b) Concepts

Concepts may be described as a mental abstraction of common properties of a group of experiences (Skemp, 1962:22; Ault, 1977:89), of objects (class concepts), actions (operational concepts) and comparisons (relational concepts). Concepts stand for common elements among a group of schemas or symbols. Spitzer (1977:23) and Skemp (1989:52) state that concepts are the mental tools that help us organise our complex experiences and make the world more meaningful in daily life. Skemp (1987:12) mentions that a concept is an idea; the name of a concept is a sound, or a mark on paper, associated with it.

According to Skemp (1989:56), primary concepts are derived directly from sensory experience. Secondary concepts can be formed only if the person already possesses other concepts: primary concepts, or secondary concepts. New concepts cannot be communicated directly in mathematics. Skemp (1989:70) states that if the new concept is of the same order as those in the learner's currently available schema, or of a lower order than these, the method of explanation is suitable. If the new concept is of a higher order than those in the learner's currently available schema, the method of giving carefully chosen examples should be used.

Spitzer (1977:25) suggests that through concept formation, or conceptualisation, we are stream-lining our experiences, but limiting them. The more frequently encountered objects are, the more rapidly they are conceptualised (Skemp, 1987:11). According to Spitzer (1977:23), people who have formed the most functional concepts are able to live their lives most efficiently. According to Skemp (1987:17), conceptual thinking allows learners to adapt behaviour to the environment and shape their environment to suit their own requirements. The power of concepts comes from their ability to combine and relate many different experiences and classes of experiences. The more abstract the concept, the greater its power to be combined and related to many different experiences and classes of experiences. Skemp (1987:17-18) claims that the power of conceptual thinking is related to the brevity of attention span. Our short-term memory can store only a limited number of words or other symbols. The higher the order of the concept which mathematical symbols represent, the greater the stored experience brought to bear. Mathematics is the most abstract, and so the most powerful, of all theoretical systems, according to Skemp (1987:17-18).

2.3.2.5 Formation of mathematical concepts

Skemp (1962:5; 1989:50) mentions that mathematics is much more abstract than any of the other subjects which learners are taught at the same age. This abstract quality can create peculiar difficulties in communication. Mathematics cannot be learnt directly from an everyday environment, but indirectly from other mathematicians, in conjunction with reflective, individual intelligence (Skemp, 1987:18). Such indirect means of acquiring knowledge makes the learner dependent on teachers (including all who write mathematical textbooks) and exposes learners to the danger of a lifelong fear or dislike of mathematics. Mathematics cannot be taught as if it is a collection of facts: it is a structure of concepts and operations.

According to Skemp (1962:6), the development of number concepts and mathematical operations takes place mainly by abstraction from repeated memory and motor experiences with physical objects. Skemp (1989:62) states that this process of abstraction involves becoming aware of something in common among a number of experiences. If a learner lacks the concepts which provide these experiences, he cannot form a new higher order concept from them. A major feature of intelligent learning is the discovery of these regularities and the organization of them into conceptual structures. Concepts are mental embodiments of these regularities. In this way, we are able to make use of our past experiences to guide us in the present (Skemp, 1989:52). Abstraction is a process by which we become aware of regularities in our experience, which we can recognise on future occasions. Past experience

guides present behaviour. Abstraction is a kind of lasting mental change which enables us to recognize similarities between new experiences and a class already formed (Skemp, 1987:11; 1989:70).

2.3.2.6 Preconceptions

Bruner (1960) mentions that a preconception is usually recognised as prior knowledge possessed by a learner which influences the learner's learning. Chi & Roscoe (2002:3) refer to "preconception" as naïve knowledge which could be revised and removed; while "misconception" is naïve knowledge that is resistant to change. Chi & Roscoe (2002:3) state that naïve knowledge has two dominant properties: it is often incorrect (when compared to formal knowledge) and it often (but not always) impedes the learning of formal knowledge with deep understanding. Preconceptions form a learner's current belief about subject material and cannot be removed with ease.

Alternative concepts refer to initial, inaccurate beliefs that remain inaccurate, even after instruction (Chi & Roscoe, 2002:5). Alternative concepts are preconceptions which can be removed with instruction; or not removed if instruction does not address them. "Alternative concepts should be less entrenched and robust, meaning that they should be more readily resolved through learning" Sfard (1991:164). In an attempt to assimilate new information (rather than accommodate it), a so-called synthetic model is created. As a mixture of beliefs and scientific facts, the synthetic model represents a learner's misconception about the subject, which is retained by the learner even after much instructional confrontation (Chi & Roscoe, 2002:5).

2.3.2.7 Misconceptions

According to Hansen (2006:14; 2011:11), the term 'misconception' is commonly used when a learner's understanding of a concept is considered to be in conflict with the accepted meaning and understanding in mathematics. Egodawatte (2011:7) agrees that learners' beliefs, theories, meanings and explanations form the basis of learners' formation of concepts. When those concepts conflict with accepted meaning in mathematics, a misconception occurs (Osborne & Wittrock, 1983). Nesher (1987:33) and Koshy, Ernest & Casey (2000:172) state that a misconception means a series of errors resulting from an incorrect underlying premise rather than sporadic, disconnected and non-systematic errors. Hammer (1996:99) suggests that learner' misconceptions:

- are strongly held, stable cognitive structures;
- differ from expert understanding;
- affect in a fundamental sense how learners understand natural phenomena and scientific explanations; and
- need to be overcome, avoided, or eliminated for learners to achieve expert understanding.

According to Chi (2005:162), misconceptions have been portrayed in one of two ways: either fragmented or coherent. A fragmented view considers misconceptions as a “set of loosely connected and reinforcing ideas.” A coherent view claims that misconceptions are not inaccurate or incomplete isolated pieces of knowledge, but alternative concepts.

Olivier (1989:11) and Brodie (2014:223) state that errors and misconceptions are seen as a result of learners’ efforts to construct their own knowledge: these misconceptions are intelligent constructions but based on incorrect or incomplete previous knowledge. A misconception can be the misapplication of a rule, an over- or under-generalisation, or an alternative conception of the situation. Some errors are symptomatic of a deeper problem: when we listen to learners’ explanations of their errors, we find alternative interpretations of mathematical ideas often called misconceptions, according to Thompson (2003:113). Nesher (1987:35) suggests that misconceptions are not created randomly; although seemingly illogical, they are actually derived from previous instruction. Sewell (2002:24) states that if what learners bring to the classroom is not a scientific view, it can be a misconception or incorrect belief.

According to Hansen (2006:15; 2011:12), misconceptions are more problematic than errors. They are set within deeper levels of knowledge. Misconceptions should be regarded as fundamental understandings in mathematics, essential for development of more sophisticated concepts later on. According to Olivier (1989:3) and Brodie (2014:223), in constructivist theories of learning, errors arise from misconceptions or conceptual structures which learners use to make sense of current knowledge but which are not aligned with conventional mathematical knowledge. Skemp (1989:69) agrees that difficulties in learning a certain concept may be so deeply rooted that other necessary concepts have been blocked out. Learners’ conceptions and misconceptions are what teaching aims to build on or correct. Learners’ errors are often superficial evidence of a basic underlying misconception (Ryan & Williams, 2007:27).

2.3.2.8 Errors

Brodie (2013:8) defines errors as systematic, persistent and pervasive mistakes made by learners across a range of contexts. Errors are systemic, persistent and difficult to correct. Nesher (1987:33) mentions that errors do not occur randomly but originate in a consistent conceptual framework based on earlier knowledge.

Olivier (1989:3) and Egodawatte (2011:8) distinguish between slips, errors and misconceptions. Slips are wrong answers, sporadically or carelessly made by both experts and novices; they are easily detected and spontaneously corrected. Errors are wrong answers made regularly in the same circumstances. Errors can be symptoms of underlying cognitive conceptual structures. Underlying beliefs and principles in the cognitive structure cause systematic conceptual errors known as misconceptions. According to Brodie (2014:224), constructivist theory suggests three important principles in relation to learners' errors:

- errors are reasonable and show reasoning among learners
- they are normal and a necessary part of learning mathematics
- learner errors allow teachers to see how learners think about mathematics and allow teachers to see possibilities for future growth in mathematical thinking and practices.

Brodie (2014:226) claims that errors are seldom taught directly by teachers and yet all learners, even high achieving learners, make errors. Teachers can exacerbate errors through taken-for-granted use of language and concepts or exposing errors and the reasons for them. A deeper understanding of errors suggests that teachers do not deal with errors quickly or easily because transforming errors requires a shift in the criteria of what counts as mathematics. Brodie (2014:226) mentions that errors can arise in interactions between features of mathematics, learning and social practice.

2.3.2.9 Conceptual change model

According to Chi (2008:61) when a learner acquires an idea that is "in conflict with" the to-be-learned concept we refer to this kind of knowledge acquisition as the conceptual change kind of learning. When conceptual change occurs it is not adding new knowledge or a gap filling incomplete knowledge; rather learning is changing prior misconceived knowledge to correct knowledge.

Olivier (1989:10) posits that for the most part learners' errors are rational and meaningful efforts to make sense of mathematics. Such errors derive from what they have been taught, and knowledge they have constructed. These derivations are objectively illogical and wrong, but, psychologically, from the learner's perspective, they make sense. What Olivier (1989:1) wants to show, is that incorrect new learning is often the result of previous correct learning; every misconception (has) an explicable origin in previous correct learning. We need to keep in mind that sometimes learners have existing misconceptions based on incorrect learning carried forward into new learning. Sewell (2002:26) and Winitzky & Kauchalk, as cited by Richardson (1997:72), agree that learners bring with them to any learning experience a firm, coherent, existing knowledge structure that may be in opposition to the concept to be taught. Resistance to change or mental inertia is assumed in most conceptual change models.

Learners enter the classroom with preconceptions due to life experiences or prior instruction. An important task for teachers is to identify learners' misconceptions in order to correct them. This process of correcting learners' misconceptions is called "conceptual change" (Chi, 2008:61; Chi & Roscoe, 2002:4). Misconceptions usually resist change: "any theory of learning must explain not only how people change, but also why people resist change" as cited McNeil & Alibali (2005b:887).

According to Kajander & Lovric (2009:174), the theory of conceptual change is closely aligned with the constructivist view of learning: new knowledge could either be assimilated or accommodated into a learner's existing cognitive framework. The theory of conceptual change makes allowance for situations where new knowledge is incompatible with the learner's existing cognitive framework. According to the constructivist view, the cognitive framework should be adapted to accommodate the new knowledge, rather than trying to "add" it to the existing framework, that is, to assimilate the new knowledge. According to the conceptual change theory, when a learner encounters new knowledge that is incompatible with his existing knowledge, he uses his own ideas and current understanding to create naïve beliefs or alternative conceptions (Kajander & Lovric, 2009:174).

According to Chi (2008:61), and Chi & Roscoe (2002:4), conceptual change does not add new knowledge or complete incomplete knowledge. Learning can mean changing prior misconceived knowledge to correct knowledge. Bodner (1986:8) and Brodie (2014:223) agree that the constructivist model explains why learners form misconceptions about mathematics and why such misconceptions are so remarkably resistant to instruction or correction.

Bodner (1986:9) and Brodie (2014:223) mention that misconceptions are resistant to instruction because once we have constructed knowledge, simply being told that we are wrong is not enough to make us change our (mis)concepts. Constructivists do not argue that there is no role for teachers in restructuring misconceptions, but that the teacher's role is more complex than merely explaining the correct ideas to learners. According to Skemp (1987:22), each (except primary) concept is derived from other concepts and contributes to the formation of yet others, so it is part of a hierarchy of knowledge. Skemp (1987:28) states that a schema is of such value to an individual that the resistance to changing it can be daunting: circumstances or individuals imposing pressure to change may be experienced as threats.

2.3.2.10 The role of misconceptions in teaching and learning

Olivier (1989:3) mentions, from a constructivist perspective, that misconceptions are important to learning and teaching because misconceptions form part of a learner's conceptual structure that interacts with new concepts and affects new learning, mostly in a negative way, because misconceptions generate errors. Koshy et al. (2000:172-173); Thompson (2003:99) and Brodie (2014:221) suggest that learners' errors and misconceptions can be used to improve mathematics teaching and learning. Teachers need to be encouraged to adopt a constructive attitude to their learners' mistakes. Mistakes which are persistent can expose gaps in the learner's basic or foundational knowledge (Koshy et al., 2000:174). Mathematics is a pyramidal discipline; new concepts are built upon the foundation of other basic concepts. Therefore, primary concepts such as the relational nature of the equal sign have to be taught correctly at an early stage.

(a) Possible reasons for learners' misconceptions and mistakes

According to Swan (2001), cited by Thompson (2003:112-113), learners' mistakes include lapses in concentration or memory, hasty reasoning and failure to notice important features of a situation. Brodie (2014:223) mentions that misconceptions often arise from learners' overgeneralisation of a concept from one domain to another.

(b) Careless mistakes

Accidental mistakes are common; they are often the result of inexperience, boredom or carelessness. Learners make mistakes due to pressure or fear (Koshy et al., 2000:175; Hansen, 2006:14; 2011:11).

(c) Reliance on rules

Koshy et al. (2000:175) and Hansen (2006:14; 2011:11) state that a large number of misconceptions and mistakes are caused by reliance on rules which have not been understood in the first place, forgotten or only partly remembered.

(d) Mistakes due to problems with language and mathematical vocabulary

Misunderstanding of vocabulary leads to many learners' mistakes: for example, when learners do not understand that 'subtraction means to take away' (Koshy et al., 2000:177; Hansen 2006:14; 2011:11).

2.3.2.11 Relevance of learners' errors and misconceptions to teachers

Teachers' interventions in the classroom are guided by some theory of how children learn mathematics. A teacher's theory of learning mathematics should enable the teacher to predict what errors learners will typically make; to explain how and why they make these errors and to help them to resolve underlying misconceptions (Olivier, 1989:1). Koshy et al. (2000:172) agree with Olivier that teachers need to use learners' errors and misconceptions to improve mathematics teaching and learning in the classroom by reflecting on mistakes; analysing learners' mistakes collected from class rooms; providing a commentary on what action may be appropriate and considering the role of the teacher in addressing learners' misconceptions.

If learners' misconceptions are anticipated and addressed, it is likely that gaps in their understanding can be remedied. According to Brodie (2013:8), teachers can evaluate errors in different ways. One way is to avoid errors - this approach ignores the need for learners to gain access to appropriate mathematical knowledge. A second way is to correct errors. This makes appropriate knowledge available to learners but depending on how errors are corrected, may not illuminate the criteria by which something is judged to be an error, thus still denying access to important mathematical knowledge. A third possibility is to embrace errors (Swan, 2001) as a point of contact with learners' thinking and as points of conversation, which can generate useful discussion about mathematical ideas. In this way, learners' thinking and mathematical knowledge are brought into contact with each other.

Koshy et al. (2000:174) suggest that for each set of mistakes made by learners, the teacher needs to take note of what the child knows and does; what the possible reasons are for

making the mistakes and what action the teacher will take. Brodie (2014:26) agrees that literature on teachers' work with learners' errors suggests that:

- Teachers' first need to identify, then pay attention to, learners' errors or show an interest in the error.
- Next teachers need to interpret or evaluate the error.
- Finally, they need to decide how to engage with the error.

Research suggests that for this process, teachers need to employ SMK and PCK in order to interpret learners' thinking and errors, and establish the reasoning behind such errors (Brodie, 2014:227). Chick & Baker (2005:249) agree that an understanding of common learner misconceptions, and effective strategies to help learners avoid them, is an important aspect of mathematical PCK.

Holmes (2012:58) suggests that when learners understand the meaning behind a mathematical concept, problem-solving techniques can be employed to retain correct answers and foster further understanding. Skemp (1976), as cited by Holmes (2012:58), states that an elementary mathematics teacher who teaches from an instrumental paradigm cannot produce learners who learn mathematics relationally. Teachers need to make connections about mathematics, learners and pedagogy.

2.4 EQUIVALENCE

Equality is a relation expressing the idea that two mathematical expressions hold the same value (Oksuz, nd). Darr (2003:4) agrees that equality is about sameness. Mathematical equivalence is a fundamental concept in arithmetic and algebra. It incorporates three components:

- The meaning of two quantities being equal
- The meaning of the equal sign as a relational symbol
- The idea that there are two sides to an equation (Rittle-Johnson & Alibali, 1999:177; Matthews, Rittle-Johnson, McEldoon & Taylor, 2012:317).

Baroody & Ginsburg (1983:198) state that learners' understanding of mathematical equivalence is often not challenged until learners learn algebra.

2.4.1 The Equal Sign

The equivalence relation denoted by the "=" symbol is a basic concept in formal arithmetic according to Baroody & Ginsburg (1983:198), Falkner, Levi & Carpenter (1999:233), Chapin & Johnson (2006:195-196), Matthews, Rittle-Johnson & McEldoon (2012:317), Essien & Setati (2006:47), Tilley (2011:19). Jones, Inglis & Pratt (2011:2) agree that the equal symbol (=) is used for communicating equality.

2.4.2 Views of the Equal Sign

Despite the fact that the equal sign denotes equivalence, and relational meaning, many users of mathematics hold an operational view of the equal sign. The operational view is emphasised or promoted in many classrooms (Prediger, 2009:81). When learners see the equal sign as an operational symbol, they exhibit a less sophisticated form of thinking compared to those who see the equal sign as indicating an equivalence relation between two quantities (McNeil & Alibali, 2005a:287).

Prediger (2009:81) mentions that the operational meaning, referred to as the asymmetric use or "operation equal answer", is often employed in teaching elementary arithmetic. Baroody & Ginsburg (1983:198) and Jones & Pratt (2011:2) agree that earlier studies on learners' understanding of the equal sign reflected that the first view contends that learners' inclination to view the equal sign in an operational manner is due to their prior knowledge in arithmetic. In addition, Prediger (2009:81) mentions that the second category, relational meaning, focuses on a symmetrical use of the equal sign, signifying a relation between two mathematical statements. The relational meaning can be sub-divided into four categories; as discussed under 2.19.3.

2.4.2.1 Operational view of the equal sign

The operational view of the equal sign is defined as a stimulus to "do something" (compute) (Stephens, Knuth, Blanton, Isler, Gardiner & Marum, 2013:174). Kilpatrick et al. (2002:270), Molina & Ambrose (2008:61), Haylock (2010:252), Essien (2009:28), Baroudi, (2006:28), Bush & Karp (2013:620) and Ciobanu (2014:14) concur that learners tend to misunderstand the equal sign: they regard it too often as an operator, that is, as a symbol inviting them to "do something", to "find the answer" rather than as a relational symbol signifying equivalence or quantitative sameness. According to Jones (2008:2), many learners view the equal sign as an indicator of computational results rather than an expression of relational equivalence.

Falkner et al. (1999:234) and Jones, Inglis & Gilmore (2011:16) argue that elementary school learners seldom appreciate the equal sign as a symbol that expresses the relation “is the same as.”

Stephens et al. (2013:174) concur: learners with this view are likely to reject equations such as $8 = 8$ as false because there is no obvious action and $15 = 5 + 10$ as “backwards”. Stephens et al. (2013:174), Carpenter, Franke & Levi (2003:10 -11), Kilpatrick et al. (2002:261) and McNeil and Alibali (2005b:887) agree that when given problems such as $8 + 4 = \underline{\quad} + 5$, learners typically respond in one of three ways:

- First, some learners might display an “answer comes next” concept and answer that the missing number, in this case, should be 12.
- Second, some learners might exhibit a “use all the numbers” concept and answer that the missing number should be 17.
- Third, some learners might show an “extend the problem” concept and write 12 on the line, but add a second equal sign after the 5; writing the number 17 to the right of it (i.e., $8 + 4 = \underline{12} + 5 = 17$). These learners, like those who display an “answer comes next” concept, believe the answer to the operation of the left side of the equal sign needs to appear on its right side, but they want to take into account the “+ 5”.

MacGregor & Stacey (1999:3) state that many learners fail to see the algebraic role of the equal sign, as signaling a relation between quantities, such as $9 + 3$ is equivalent to, or the same in value as, 12. McNeil & Alibali (2005b:884) state that early learning constrains later learning; by looking at learners’ prior knowledge, it was found that “the patterns with which people initially gain experience become entrenched, and learning difficulties arise when to-be-learned information overlaps with, but does not map directly onto, entrenched patterns”. They found that there were three operational patterns that hinder learners’ understanding of complex equations: that is, “perform all operations”; “operations = answers”; and “understanding equal sign as total.”

2.4.2.2 Relational view of the equal sign

Blanton, Levi, Crities, Dougherty & Zbiek (2011:25) state that a relational understanding of the equal sign entails understanding that the equal sign represents a relation of equivalence. This relation is often interpreted as meaning “the same as” when expressing a relation between equivalent amounts. Stephens et al. (2013:174) state that learners who understand

the equal sign as a relational symbol “the same as” demonstrate more flexibility working with equations in non-traditional formats such as $15 = 5 + 10$ and $8 + 4 = \Delta + 5$.

MacGregor & Stacey (1999:1) and Blanton et al. (2011:26) suggest that for learners to move towards a more algebraic understanding of equality, they need to learn that the equal sign represents quantitative sameness: e.g. $7 + 2 = 5 + 4$. Learners who do not view the equal sign as relational are less successful in algebra type questions than learners who understand the equal sign as a relation between both sides (Knuth et al., 2006:299).

With reference to the relational meaning, Prediger (2009:81) subdivides it into four categories:

- Symmetric arithmetic identity, for example: $5 + 7 = 7 + 5$
- Formal equivalence describing equivalent terms, for example:
$$x^2 + x - 6 = (x - 2)(x + 3)$$
- Conditional equation characterising unknowns, for example: Solve $x^2 = x + 6$
- Contextual identities in formulae, for example: The Theorem of Pythagoras
$$a^2 + b^2 = c^2$$

2.4.2.3 A construction map for knowledge of the equal sign

In recent studies, Matthews, Rittle-Johnson, McEldeen & Taylor (2012:320-321) have expanded the view of the equal sign to create a construction map for knowledge of the equal sign, detailing four levels of understanding of the equal sign among school-age learners:

- Level 1- Rigid-operational
Learners who display a rigid operational level of knowledge regard the equal sign as an operator, preceding the calculation, often presented to the left. They are successful with equations when the calculation is on the left only.
- Level 2 – Flexible operational
Learners with a flexible operational view believe that the meaning of the equal sign is “get the answer to the question,” but they can successfully evaluate equations where the calculation is either to the left or to the right of the equal sign: $21 = 8 + x$ or $8 + x = 21$.

- Level 3 – Basic Relational

Learners with a basic relational view can evaluate equations involving operations on both sides of the equal sign and recognise the equal sign's relational meaning.

- Level 4 – Comparative relational

Learners with a comparative relational view recognise the relation of equivalence between the two sides of the equation and can employ various strategies to solve tasks such as "Without performing the calculation, can you explain why $56 + 80 = 55 + 81$?"

2.4.3 Misconceptions of the Equal Sign

Knuth, Stephens & McNeil (2006:298) state that the ubiquitous presence of the equal sign at all levels of mathematics highlights its importance. After initial introduction of the equal sign during learners' early elementary school education, however, little if any teaching time is explicitly spent on the concept in later grades. Vela (2011:5) begins by describing the lack of clarity about what the equal sign is and what it means in the classroom: learner misconceptions are often the result of minimal teacher background knowledge, poor textbooks or other materials available (Ball, 1990, Capraro et al., 2007, Davis & Simmt, 2006, Falkner et al. 1999, Jones & Pratt, 2005, Knuth et al., 2006, Prediger, 2009, Sherman & Bisanz, 2009). Bamberger, Oberdorf & Schultz-Ferrell (2010:56) agree: teachers, curricula and many textbooks view arithmetic and algebra as distinct/ different, denying learners the opportunities to connect arithmetic and algebra. This separation impedes learner understanding of critical ideas such as the equal sign.

Powell (2012:3) state that misinterpretation of the equal sign may stem from a misunderstanding of symbols generally. Without formal instruction, learners perform relatively well on verbal story problems yet poorly on corresponding symbolic equations. This difference in performance indicates that symbolic representations and problem structure may hinder learners' ability to solve problems. Learners cannot make sense of questions without meaningful instruction on two crucial yet poorly understood topics namely the equal sign and variables.

Knuth et al. (2006:309) and Stephens (2006:252) state, "Understanding the equal sign is a pivotal aspect of success in solving algebraic equations". If teachers teach an operational meaning of the equal sign only, learners are less successful in future mathematics. It is

imperative that teachers help learners realise the equal sign not as an operation but a relation between both sides of the equal sign. Learners have to see the equal sign as a relation in order to comprehend “complicated equations with operations on both sides of the equal sign (e.g., $5x + 7 = 2x - 11$)” (Knuth et al., 2006:309).

Knuth, Alibali, Hattikudur, McNeil & Stephens (2008:514) mention that the operational view creates a “limited concept of the equal sign, one of the major stumbling blocks in learning algebra, as virtually all manipulations on equations require understanding that the equal sign represents a relation”. Vela (2011:5) suggests that providing teachers with a more in-depth study of the concept of the equal sign gives learners a better understanding of what the equal sign means, which will positively affect learners’ performance in advanced mathematics.

2.4.4 Possible reasons for the Operational View of the Equal Sign

Falkner et al. (1999:234) mention that in elementary school the equal sign usually appears at the end of numerical expressions and only one number comes after it. With number sentences, such as $4 + 6 = 10$ or $67 - 10 - 3 = 54$, learners appropriately infer that the equal sign is a signal to compute. MacGregor & Stacey (1999:3) and Ciobanu (2014:15) agree that for too many learners, the $=$ symbol means to compute the expression to the left and record the answer after it. The repetitive nature of certain arithmetical tasks, in which learners compute an expression then write their answer immediately after the equal sign, can build a misconception in their thinking about what equality really means.

Essien (2009:28) and Welder (2012:8) attribute such narrow understanding of the equal sign to the use of calculators and direct verbal-to-written translation of mathematical sentences. According to Welder (2012:8) researchers believe that these misconceptions stem from the frequency with which learners see the equal sign at the end of numerical expressions with one number appearing afterwards. This superficial usage is sufficient for basic arithmetic computations but will limit learners’ comprehension of the equal sign as a relational symbol.

2.4.5 Inappropriate use of the Equal Sign

Tilley (2011:19) states that the equal sign is too often used in a way that is inaccurate and which creates misconceptions that will need to be resolved when learners apply their understanding of the number system to algebra. Tilley (2011:19) claims that another

common misuse occurs in contexts where learners solve a calculation mentally by breaking it down into manageable steps.

a) Example 1:

$$48 + 36 = 48 + 30 = 78 + 6 = 84$$

The learner clearly understands that they can partition one of the numbers (36 in this case) to simplify the calculation. However, this is incorrect because $48 + 36$ does not equal $48 + 30$. This type of 'relaxed' use of the equal sign has been observed in both learners' work and as part of teachers' demonstrations.

b) Example 2:

When the equal sign is used, where its meaning is not accurate, learners are communicating a result: for example, when learners practise 'doubling' or 'halving' (Tilley, 2011:19).

Doubling and halving

$$6=12$$

$$18=36$$

$$24=48$$

Tilley (2011:19) and MacGregor & Stacey (1999:3) concur that common misuses of the equal sign are illustrated by the following two examples:

c) Example 3:

$$(3 + 5) \times 2 = 3 + 5 = 8 \times 2 = 16.$$

Here equality strings are written down. The teacher needs to identify the misuse of the equal sign and encourage learners to re-write their steps underneath each other as

$$3 + 5 = 8$$

$$8 \times 2 = 16.$$

d) Example 4:

$$4 + 6 = \Delta + 5; \Delta = 10.$$

Here the learner is looking for 'closure'. Blanton (2008:22) agrees: when asking learners to solve the following task: $9 + 3 = \Delta + 4$, learners are most likely to add the 9 and the 3 and write 12 in the triangle. When learners have an operational view of the equal sign, they tend to have difficulty correctly determining the unknown in equations such as $8 + 4 = _ + 5$ (Matthews et al., 2012:318).

2.4.6 The Equal Sign in the Foundation and Intermediate Phase Caps Document

The current curriculum used in South Africa is the Revised National Curriculum Statement (RNCS), which has been sub-divided into the Curriculum and Assessment Policy Statement (CAPS) document for each subject in 2012. According to Essien (2009:29), the RNCS does not foreground the importance of the equal sign in Grade 1 school mathematics. With reference to the CAPS document for foundation phase mathematics (DBE, 2011:22), the first indication of symbolic use of the equal sign is in Topic 1.13 presented in Table 2.1. Relational symbols are not mentioned before this, yet learners are expected to use them appropriately in Grade 1, 2 and 3. CAPS relies too heavily on teachers' assumed SCK for learners to ascertain this concept. Mention is made that a balance scale is a useful apparatus recommended in a foundation phase classroom.

Table 2.1: CAPS document page 22

TOPIC	Grade R	Grade 1	Grade 2	Grade 3
1.13 Addition and subtraction	Solve verbally stated addition and subtraction problems with solutions up to 10	- Add to 20 - Subtract from 20 - Use appropriate symbols (+, −, =, Δ) - Practice number bonds to 10	- Add to 99 - Subtract from 99 - Use appropriate symbols (+, −, =, Δ) - Practice number bonds to 20	- Add to 999 - Subtract from 999 - Use appropriate symbols (+, −, =, Δ) - Practice number bonds to 30
1.14 Repeat addition leading to multiplication		- Add the same number repeatedly to 20 - Use appropriate symbols (+, =, Δ)	- Multiply numbers 1 to 10 by 2, 5, 3, and 4 to a total of 50 - Use appropriate symbols (+, x, =, Δ)	- Multiply any number by 2, 3, 4, 5, 10 to a total of 100 - Use appropriate symbols (x, Δ)
1.15 Division				- Divide numbers up to 100 by 2, 3, 4, 5, 10 - Use appropriate symbols (÷, =, Δ)

In the CAPS document for the intermediate phase - mathematics overview (DBE, 2011:13-31), learners are expected to develop understanding of concepts such as equivalence, conversion in measurement, estimation etc. yet again the concept of equivalence is not explicitly illustrated as a relational symbol. Teachers have to rely on their own understanding of this concept. The CAPS document does not foreground the importance of the equal sign. Neither does it emphasise the importance of teaching/introducing the equal sign to learners correctly. Yet the curriculum expects learners to know how to perform calculations using the equal sign appropriately.

Essien (2009:29) states that the National Council of Teachers of Mathematics' (NCTM) Principles and Standards for School Mathematics, refers to the equal sign symbol as "an important algebraic concept that learners must encounter and begin to understand in the lower grades. "Learners need to understand that the equal sign indicates a relationship - that the quantities on each side are equivalent" [National Council of Teachers of Mathematics (NCTM), 2000:94]. The Principles and Standards note that the common learners' understanding of the equal sign at the foundation phase should be more accurate than the limited understanding of the equal sign as signifying "the answer is coming" (Essien, 2009:29).

2.5 CONCLUSION

Chapter 2 summarises three concepts: teacher knowledge, with a focus on SCK; learner misconceptions and the equal sign. The first unit of this chapter deals with the concept of teacher knowledge in mathematics education. The discussion summarises the work of Shulman (1986; 1987) and Ball et al. (2008). A summary of empirical research on teacher knowledge related to this study is highlighted. The second unit of the chapter focuses on the theory of concept formation as it relates to constructivism and the relevance of this with regards to the formation of misconceptions. The third unit of the chapter focuses on the concept of equality and the symbol that represents it, namely the equal sign; emphasis is placed on misconceptions regarding the equal sign. Chapter 3 provides an outline of the research design and methodology used in this study.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

This chapter outlines the research design and methodology of this study. This research was framed within an interpretive paradigm and took the form of a theory-led case study. A convergent parallel mixed method design was used as both qualitative and quantitative methods were employed to gather the data needed to address the research questions. The site was an ex-model C school in a suburb of Cape Town, South Africa. The research questions were evaluated in this context and not aiming to generalise the findings. The aim of this research was to investigate the nature of Grade 1- 6 teachers' SCK of the equal sign, what misconceptions Grade 6 learners have of the equal sign and whether the nature of teachers' SCK of the equal sign could possible promote, prevent or reduce learners' misconceptions of the equal sign.

3.2 RESEARCH DESIGN

Yin (2009:26) describes research design as a logical sequence that connects empirical data to the initial research questions and conclusion of a study. A research design is a logical plan for progressing from the initial set of questions to the conclusions for these questions. Creswell (2009:5) refers to research design as the plan or proposal to conduct research which involves the intersection between philosophy, strategies of inquiry and specific methods.

The research design used in this study is based on an interpretive paradigm, framed in a theory-led case study, using a mixed method approach. A convergent design is used "to obtain different but complementary data on the same topic" combining the use of qualitative and quantitative data, to best understand the research problem (Creswell & Plano Clark, 2011:70).

3.2.1 Interpretive paradigm

According to Thomas (2011:124) an interpretative paradigm is a form of inquiry that employs a particular approach to answer questions. Such an approach assumes an in-depth understanding and deep immersion in the environment of the subject being studied. Glesne

(2011:5-8) and Maxwell (2013:30) claim the aim of an interpretive paradigm is to understand human ideas, actions and interactions in specific contexts or in terms of the wider culture. The focus of this researcher is the physical events and behaviour that takes place as well as the participants in the study who provide data. How participant understanding affects participant behaviour is a central concern of this study. This focus on meaning is central to what is known as the “interpretive” approach to research in social sciences.

The aim of this research was to investigate the nature of Grade 1- 6 teachers’ SCK of the equal sign, what misconceptions Grade 6 learners have of the equal sign and how teachers’ SCK of the equal sign could possibly promote, prevent or reduce learners’ misconceptions of the equal sign. The intention of the study was to ascertain a possible link between learners’ process of developing concepts (the equal sign) and how interaction with teachers allows for this development.

3.2.2 Case study

Simons (2009:21); Creswell (2009:13); Rule & John (2011:4); Yin (2012:4) and Hamilton & Corbett-Whittier (2013:11) mention that a case study is an in-depth exploration from multiple perspectives of the complexity and uniqueness of a particular project, policy, institution, programme or system in a ‘real-life’ context. It is research-based, inclusive of different methods and evidence-led.

Yin (2003:13; 2014: 16) state that:

- First, a case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not immediately evident.
- Second, because phenomena and context are frequently indistinguishable in real-life situations, a whole set of other technical characteristics, including data collection and data analysis strategies, become the second part of our technical definition.

Simons (2009:24), Morrell & Carroll (2010:81), Thomas (2011:3) and Rule & John (2011:61) indicate that in many situations in which case study research is conducted, formal generalization for policy-making is not the primary aim. The aim is particularisation - to present a rich portrayal of a single setting to inform practice, establish the value of the case and/or add to knowledge of a specific topic. A case study method is a type of research that concentrates on one thing, looking at it in detail, not seeking to generalize from it. Sampling

has to be relevant to the case study. Yin (2012:7) agrees that a “case” is generally a bounded entity (a person, organisation, behavioural condition, event, or other social phenomenon), but the boundary between the case and its contextual conditions - in both spatial and temporal dimensions - may be blurred. Thomas (2011:23) continues that case studies are analyses of persons, events, decisions, periods, projects, policies, institutions or other systems which are studied holistically by one or more methods. The case that is the subject of this present inquiry comprises phenomena that provide an analytical framework within which the study is conducted. Yin (2003:7; 2009:11) continues that the case study is preferred in examining contemporary events, but that relevant behaviours cannot be manipulated.

The aim of this case study was to provide a holistic and meaningful analysis of a possible relation between teachers’ SCK and learners’ misconceptions with regards to the equal sign in the context of the school that I used as my sample. The participants were a group of eleven primary school teachers and fifty-seven Grade 6 learners. The eleven teachers completed a teacher’s questionnaire, allowed the researcher to observe one of the teaching lessons of each teacher and were involved in two separate focus group interviews discussing their opinions around the concept of equivalence. This was done to evaluate their SCK of the concept of equivalence. The learners wrote a test on the equal sign to determine their level of understanding of this concept. Six Grade 6 learners were randomly selected to discuss their errors in the test in an individual interview session, to allow the researcher an opportunity to understand the thinking behind their errors found in their tests.

3.2.2.1 Purpose of a Case Study

According to Yin (2003:2; 2009:4), the case study method allows investigators to retain the holistic and meaningful characteristics of real-life events. A case study need not contain a complete or accurate rendition of actual events; rather, its purpose is to build a framework for discussion and debate among researchers. Thomas (2011:101) suggests that the most common purpose of a case study is to explain the case being researched.

In a case study, the researchers are trading breadth of coverage for depth of understanding and potential explanations based on depth of understanding are what a case study does best relative to other kinds of research. These explanations may be tentative or context-specific, but it is in the multifaceted nature of a case study that you get the opportunity to relate one bit to another and offer explanations based on the interrelationships between these bits (Thomas, 2011:101).

Rule and John (2011:7) agree that a case study approach allows you to study a certain instance in great depth, rather than looking at several points of views superficially. “It focuses on the complex relations within the case and the wider context around the case as it affects the case. It is therefore intensive rather than extensive. The unit of a case study can range from an individual to a country. Case study is therefore very flexible in terms of what it studies can also use a very wide variety of methods, both for data collection and for data analysis, depending on what is appropriate to the case”.

Rule & John (2011:7) continue that case studies can be conducted and used for various purposes. First, they can generate an understanding of, and insight into, a particular instance by providing a rich description of the case and illuminating its relations to its broader context. Second, they can be used to explore a general problem or issue within a limited and focused setting. Third, they can be used to generate theoretical insights, either in the form of grounded theory that arises from the case study itself or in developing and testing existing theory with reference to the case. Fourth, case studies shed light on other, similar cases thus providing a level of generalisation or transferability. Fifth, case studies can be used for teaching purposes to illuminate broader theoretical and/or contextual points.

3.2.2.2 Strength of a Case Study

Yin (2003:8; 2009:11) claims that a case study’s unique strength is its ability to deal with a full variety of evidence: documents, interviews, and observation. Swanborn (2010:33) states that a specific asset of a case study is that it enables us to understand emerging problems and their practical solutions in the social system under study. Gaining insight into these aspects is very profitable in optimising our design or policy advice. In addition, Rule & John (2011:8) indicate that a case study offers versatility in the ways that it can be united with other approaches. The sharp focus of a case study makes it more manageable than a large-scale survey or wide-ranging policy review, especially in a research situation facing constraints of time and resources. According to Yin (2009:145), the ability to trace changes over time is a major strength of case studies.

3.2.2.3 Limitations of a Case Study

Yin (2009:14) states that perhaps the greatest concern has been over the lack of rigour of case study research. Too often, the case study investigator has not followed systematic procedures and, allowed equivocal evidence or biased views to influence the direction of the findings and conclusions. According to Rule & John (2011:105), some writers have claimed

that a case study, as a study of the singular, is limited because its findings cannot be generalized.

Morrell & Carroll (2010:111) state that all studies have limitations. Limitations are conditions that render the outcome of the study less than perfect; factors exist that bias the results and/or generalisability of the study. Rule & John (2011:110) mention that sound and responsible research should state the limitations of the study. The reader then knows the limitations of the study and can determine how dependable and confirmable the study is. Limitations can stem from the following:

- The researcher's methodology
- The researcher's data collection methods
- The researcher's choice of site
- The researcher's sample
- The researcher's practical and logistical circumstances
- Personal attributes of the researcher attributes which could have influenced the data collected
- The option for a potential participant to take part or not (Rule & John, 2011:110).

3.2.3 Theory-Led Case Study

Simons (2009:21-22) mentions that theory-led case studies mean exploring, or even exemplifying, a case through a particular theoretical perspective. Yin (2014:136) states that the theoretical proposition that led to your case study will guide the data collecting plan and your data analysis.

The theory of concept formation as it relates to constructivism and the relevance of this to teachers with regards to the formation of misconceptions was used as the theory to guide this research. The focus of this research is on teachers' SCK, constructivism and concept formation, and the concept of the equal sign. Constructivism is a learning theory which suggests that individuals create their own new understanding, based upon the interaction between what they already know and believe, and the phenomena or ideas with which they come in contact (Noddings, 1990:7 and Richardson, 1997:3). The constructivist learning theory maintains that learning is not the result of teaching but the result of what learners do with fresh information. Learners actively construct their own knowledge; they are not passive recipients of new information. Skemp (1989:72) and Olivier (1989:2) state that, from a

constructivist perspective, knowledge does not simply arise from experience: it arises from the interaction between experience and current knowledge structures. Teachers need SCK to discern patterns in student errors or in sizing up whether a nonstandard approach works in general.

In this study the researcher explored a possible link between teachers' SCK of the equal sign and learners' understanding and misunderstanding of this concept, based on the above mentioned theories.

3.2.4 Mixed Method Approach

Creswell & Plano Clark (2011:4); Yin (2009:62); Creswell (2009:14; 2014:4) and Maxwell (2013:102) suggest that mixed method studies combine qualitative and quantitative approaches into the research methodology of a single study or multi-phased study, for the purpose of breadth and depth of understanding and corroboration. Simons (2009:130) states that methodological triangulation is often advocated as a rationale for mixed methods research. Creswell (2009:66) mentions that theory used in mixed methods research is a theoretical lens or perspective to guide the study.

This mixed method study addresses a possible relation between teachers' SCK and learners' misconceptions with regards to the equal sign. A convergent parallel mixed method design was used. Morell & Carroll (2010:16) and Creswell (2009:213) indicate that a convergent parallel design occurs when the researcher uses concurrent timing to implement the quantitative and qualitative strands during the same phase of the research process, prioritises the methods equally, and keeps the strands independent during analysis and then combines the results during the overall interpretation. The convergent parallel design is known as the triangulated design as cited by Morrell & Carrol (2010:16). Figure 3.1 provides a diagrammatic representation of the convergent parallel design.

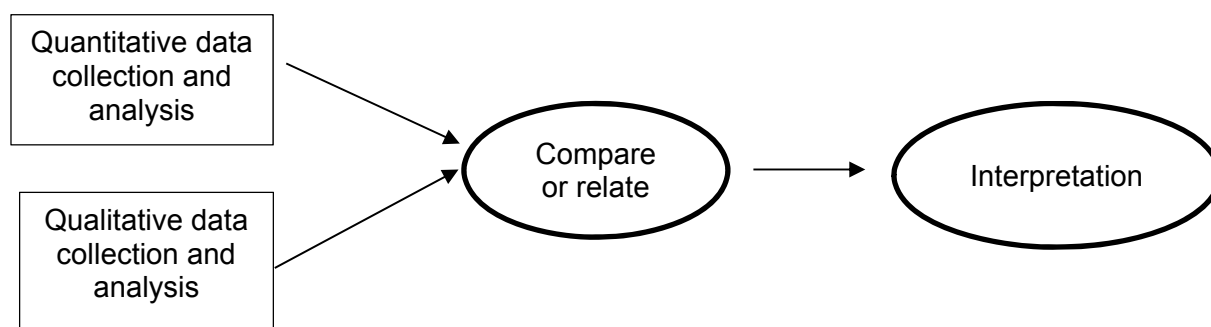


Figure 3.1: The Convergent parallel design (Creswell and Plano Clark, 2011:69)

In this study the researcher combined qualitative and quantitative approaches. The Grade 1 - 6 teachers completed questionnaires individually. Two focus group interviews were conducted with the foundation phase and intermediate phase educators and the researcher observed one lesson in each of the eleven classrooms. This was done to determine the nature of teachers' SCK of the equal sign, whereas a test on the equal sign was used to test the learners' understanding of the concept. Six learners were individually interviewed to discuss the thinking behind their errors, as found in their test. The reason for collecting both quantitative and qualitative data is to converge or compare the different forms of data to bring a greater insight into the research problem than would be obtained by either types of data separately.

Creswell & Plano Clark (2011:77) mention that the purpose of convergent design is "to obtain different but complementary data on the same topic" to best understand the research problem. The design is used when the researcher wants to triangulate the methods by directly comparing and contrasting quantitative statistical results with qualitative findings for corroboration and validation purposes.

3.2.4.1 Advantages of Using a Mixed Methods Approach

Rule & John (2011:61) mention in the mixed methods research tradition that researchers transcend the old debate between qualitative and quantitative research and demonstrate how a combination of traditions (particularly at the level of data-collection methods) can be productive for answering important social questions. If the purpose of research is to gain a holistic understanding of a case, then collecting and analysing both qualitative and quantitative data are often required.

Maxwell (2013:102) states that the purpose for combining methods is to ensure that methods with different strengths and limitations support a single conclusion. This reduces the risk of bias in favour of one specific method. Creswell & Plano Clark (2011:12-13) agree that using a mixed method approach offsets the weaknesses of both quantitative and qualitative research. In mixed method research, the researcher uses all methods possible to address the research question.

3.2.4.2 Qualitative method

Leedy & Ormrod (2005:133) and Salkind (2009:209) mention that qualitative research refers to the study of social or behavioural science. Qualitative research focuses on phenomena

that occur in the “real world” and reviews this investigation in all its complexity. Qualitative researchers rarely try to simplify what they observe. Hamilton & Corbett-Whittier (2013:23) state that to undertake qualitative research usually involves exploring ways of gaining insight into beliefs, attitudes, opinions and practices drawing on data collection tools such as interviews, observations, reflective diaries within such research genres as the case study. According to Morrell & Carroll (2010:10-12), qualitative research design differs from a quantitative research design. Instead of starting with a hypothesis, the researcher ends up with one. Qualitative research tends to be inductive.

3.2.4.3 Quantitative method

Quantitative research, according to Leedy & Ormrod (2005:245) and Salkind (2009:209), makes sense of the world by using numbers that can be summarised and interpreted to find answers to the research question. Quantitative research refers to the use of numbers to represent non-physical phenomena in social science research, such as intelligence and academic achievement. The interpretation of numbers is referred to as statistics. Statistics are an invaluable and often indispensable tool in research. They provide a means through which numerical data can be made more meaningful, so that the researcher may see their nature and better understand their interrelations.

Morrell & Carroll (2010:11) mention that quantitative research ideally requires large, random samples. Quantitative research follows a *deductive model*: the researcher moves from the general to the specific. The researcher has an idea or theory, he/she applies it to the sample and sees if it holds true. The findings tend to be generalisable to a broader population who are similar to those in the study. According to Barbour (2014:39), the deductive approach collects data in order to test a specific theoretical proposition. Rule & John (2011:96) state that deductive reasoning moves from the general to the specific. The theory first approach is useful for testing and developing theories in new contexts. This type of study begins with a theory and applies it to the selected case.

In this research, a deductive approach has been used for collecting data from learners and teachers to test the nature of SCK of teachers with relation to the equal sign and learners’ misconceptions regarding this concept. No statistical analysis was used in this study, however many of the results were quantified as such a quantitative approach has been used to analyse the learners’ test and teachers’ questionnaires

3.3 SITE AND SAMPLING

Rule & John (2011:63-64) and Creswell & Plano Clark (2011:173) state that because it is often impossible for a case study researcher to consult everyone involved in a case, participants are chosen who can shed most light, or the widest range of light, on a case. This is known as purposive sampling: the persons selected as research participants are chosen for their suitability in advancing the purpose of the research. Morrell & Carroll (2010:100-101) and Maxwell (2013:97) agree that in a case study purposeful selection of particular settings, persons, or activities is made to provide information relevant to the nature and ambit of a particular topic. Thomas (2011:62) argues that the aim of a case study is not to find a portion that shows the quality of the whole: in his view, a case study is a choice or selection that is vital to a particular study and consequently not a sample.

This study was a school-based research study. The school chosen is an ex-model C primary school in a middle-class suburb in Cape Town, South Africa. The sample consisted of 11 mathematics teachers (all the Grade 1 to 6 teachers teaching Mathematics to their classes at school) and 57 learners in the Grade 6 classes (there are two Grade 6 classes at the school, in total sixty-one learners but only fifty-seven consented to participate in the study). Because the grade 6 learners are in the final grade before entering the senior phase of school, it is my view that this was the appropriate grade to use in this research. At this stage of schooling, the fundamentals of arithmetic should be established amongst learners and misconceptions should be evident and identifiable.

3.4 DATA COLLECTION METHODS

According to Rule & John (2011:61), the choice of data collection methods is determined more by factors such as the purpose of the study, the key research questions, research ethics and resource constraints than by factors peculiar to case study research as a form of enquiry. Creswell (2014:97-98) states that when collecting the data, it is essential to:

- Respect the site, and disrupt as little as possible
- Make sure that all participants receive benefits
- Avoid deceiving participants
- Respect potential power imbalances
- Avoid exploitation of participants
- Avoid collecting harmful information

Simons (2009:33) mentions that three qualitative methods often used in case study research to facilitate in-depth analysis and understanding are: interviews, observation and document analysis. According to Thomas (2011:124), the interpretive inquirer studies the meaning that people are constructing of the situation in which they find themselves and proceed from these meanings in order to understand the social world. Data needs to be collected in the form of interviews, transcripts and informal observations. Data collection methods in this research included:

- teachers' questionnaires
- two focus group interview sessions based on the data collected from the questionnaires
- one observation session in each of the eleven classes
- a test that 57 of the Grade 6 learners completed
- one-on-one interview sessions with 6 learners to allow them an opportunity to explain their thinking behind the errors in their test

3.4.1 Quantitative instruments

3.4.1.1 Teachers' questionnaires

Rule & John (2011:66) state that questionnaires are printed sets of field questions to which participants respond on their own (self-administered) or in the presence of the researcher. Questionnaires provide an efficient method of collecting data from a large number of people simultaneously (Hamilton & Corbett-Whittier, 2013:108). This method is cheaper and quicker than interviews, however the researcher does not have the opportunity to probe particular responses further or control data exchange. Questionnaires depend on careful construction of clear, unambiguous field questions. The questionnaire used in this study appears as Appendix 7.

Eleven teachers (two in each Grade, I teach the one Grade 6 and the one Grade 7 class) were given a questionnaire to evaluate their understanding of the equal sign. Common themes that arose from the questionnaires were used as a foundation on which to formulate my questions for the focus group discussion.

3.4.1.2 Learners' test

Salkind (2009:130) states that achievement tests are used to measure knowledge of a specific area. Achievement tests are the most commonly used instrument when learning is being tested. They are used to measure the effectiveness of the instruction used to impart knowledge. The learners' test used in this study appears as Appendix 6. Fifty-seven Grade 6 learners consented to their test being used to determine their level of understanding of the equal sign.

3.4.2 Qualitative instruments

3.4.2.1 Learners' interviews

According to Yin (2009:106) and Rule & John (2011:64), interviewing has long been the most popular method in qualitative research and is often used in case studies. Interviews imply one-on-one discussions between the researcher and research participants: a sort of choreographed conversation. Simons (2009:43) concurs that, compared with other methods, interviews enable the researcher to get to core issues in the case more quickly and in greater depth, to probe motivations, to ask follow-up questions and to facilitate individuals telling their stories.

Rule & John (2011:64) indicate that an interview requires preparation, interpersonal skills and communicative competence. Some guidelines for conducting good interviews include:

- Establish a relaxed atmosphere for the interview.
- Explain the nature and purpose of the study.
- Allow interviewees to ask questions about the study and make sure that they are willing to proceed before you begin the interview.
- Inform participants of ethical obligations.
- Adopt a conversational rather than a commanding style in order to build rapport.
- Begin with the least demanding or controversial questions.
- Listen carefully and avoid interrupting the participant.
- Be respectful and sensitive to the emotional climate of the interview.
- Probe and summarise to confirm your understanding.

As far as was humanly possible, the researcher adhered to these guidelines during the interviews.

Simons (2009:43) states that in-depth interviewing has four major purposes:

- To document interviewee perspective on the topic.
- To spark active engagement and learning between interviewer and interviewee
- To adapt to emergent issues, probe a topic or deepen a response, and engage in dialogue with participants.
- To expose hidden feelings or unobserved events.

In interviews, people often reveal more than can be detected or reliably assumed from observing a situation. Simons (2009:49) mentions that unstructured interviewing takes time, especially when unanticipated issues arise. Oliver (2010:106) states that an advantage of interviews is that unanticipated, new and important concerns emerge. A disadvantage, according to Koshy (2006:93) of conducting interviews is that it is more time-consuming than using questionnaires. Typing transcripts requires a large amount of time.

Fifteen-minute interview sessions were conducted with six learners purposefully selected during which they were allowed to explain the thinking behind their errors concerning use of the equal sign. The purpose of the interview sessions was to confirm or substantiate the findings in the test. The interviews were voice recorded and thereafter transcribed, see attached appendix 10. Coding was used to identify themes arising from the transcript.

3.4.2.2 Teachers' lesson observations

Rule & John (2011:67) suggest that observing educational action such as a lesson being taught can provide useful data for a case study. This method is particularly appropriate if the case study is meant to capture and portray the liveliness and situatedness of behaviour. Observation is an important element of case study research and cannot be omitted on some occasions according to Swanborn (2010:73).

Simons (2009:55) and Swanborn (2010:73) mention that through observing, the researcher can gain a comprehensive 'picture' of the site, a 'sense of the setting' which cannot be obtained solely by speaking with people. Observations provide a cross-check on data obtained in interviews. Simons (2009:55) agrees that documenting observed incidents and

events provides a 'rich description' and a basis for further analysis and interpretation. Through observing the researcher can discover the norms and values which are part of an institution or programme's culture or sub-culture. Observation offers another way of capturing the experience of those who are less articulated.

Observation allows the researcher to capture all aspects of the topic of study, offers first-hand data and offers a way of studying, through close scrutiny of behaviour. Disadvantages are that too much information is collected which could pose a challenge at the time of analysis. Background noise and disruptions can compromise important data. There may be a temptation to miss out details if these details do not fit the items on a checklist according to Koshy (2006:103). According to Rule & John (2011:68) as a non-participant observer, the researcher observes the action, while trying to be as unobtrusive as possible.

In this study, the researcher was a non-participant observer, observing one lesson taught by each of the eleven teachers during term 2 (March – May 2015) - a time when number concepts were taught to assess teachers' SCK relating to the equal sign. The term planner of the school prevented me from observing more than one lesson. I am the Grade 7 teacher and could observe only during my administration lesson. Contextual factors such as the June Examination and the general term activities had to be considered when setting up my observation schedule. No observation schedules were used during the observation sessions. Instead I typed out everything I observed during the lessons. The intent of the observation sessions was to see how teachers link the concept taught to the equal sign, and how opportunities are used during lesson time to integrate fundamental concepts like understanding the relational value of the equal sign. Appendix 10 provides a detailed transcription of the teachers' lesson observations.

3.4.2.3 Focus group interviews with teachers

Kvale & Brinkmann (2009:150) mention that in a focus group session, the primary concern is to encourage a variety of viewpoints on the topic in focus for the group. The aim of the focus group is not to achieve consensus about, or solutions to, the issues discussed, but to highlight different view-points on an issue.

Kvale & Brinkmann (2009:150) mention that a focus group consists of six to ten participants led by a facilitator. It is characterised by a non-directive style of interviewing: the prime concern is to encourage a variety of viewpoints on the topic in focus for the group. Group interaction can, however, reduce facilitators' control of an interview. One price of the lively

interaction may be interview transcripts that are somewhat chaotic. Thomas (2011:164) agrees that in a focus group the researcher is a facilitator or moderator. The researcher's aim is to facilitate or moderate discussion between participants, usually using focus materials, such as cards, photographs and newspaper stories, to stimulate discussion. The researcher assumes a marginal rather than a pivotal role in the discussion itself.

Thomas (2011:164) and Rule & John (2011:66) mention that the researcher needs to establish her reason for holding a group rather than an individual interview. If the researcher is interviewing a group, it should be because the group psychology itself has some impact on the situation that is of interest to the researcher. The researcher may, for example, want to find out how the group (as a group) is behaving or compare a group attitude with individual attitudes within the same group, perhaps to judge the power of one or two group members. Focus groups are useful for gaining a sense of the range and diversity of views: which views are dominant and marginal in the group, where resistance and dissent lie and how dialogue shifts the understanding of members in the group. In selecting people to participate in a focus group, attention should be paid to the status of participants.

I conducted the focus group sessions on 12 and 19 June 2015 at a time when all the participating teachers were available. The sessions were held in my classroom after school to avoid interruptions during the sessions. A week prior to each session, all participating teachers signed a confirmation slip that stated when and where the sessions would occur. I needed a data projector, because the discussion was guided by a power point presentation. Two focus group interview sessions were held: one with the Grade 1-4 teachers; six teachers were present. (1B; 2A; 2B; 3A; 3B; 4A). The Grade 1A teacher and the 4B teacher were absent. The Grade 4 teachers requested to be present at both sessions: they consider themselves to be the transition Grade and wanted to hear the outcome of both sessions. In the second focus group interview five teachers were present. (4A; 4B; 5A; 5B; 6A). I teach the one Grade 6 class Mathematics. Each focus group session took 45 min and was voice recorded. Power point slides (see Appendix 8) were used to guide the discussion. Each slide prompted the teachers to discuss a few core issues.

3.5 ROLE OF THE RESEARCHER IN CASE STUDY RESEARCH

Rule & John (2011:35-36) state that researchers should reflect critically on how their research is progressing and on what they are learning as they proceed. If the researcher has no direct role as a participant in the study, the researcher should refrain from sharing his/her feelings as it might slant the perspective of others in the study and distort their voices.

During observation, according to Rule & John (2011: 68), Hamilton & Corbett-Whittier (2013:99) the researcher observes the actions, while trying to be as unobtrusive as possible. When using an open-ended format, the researcher takes notes of all behaviour or action considered significant during the observation. A non-participant observer lends itself to a more structured and measured observation, as the researcher observes from the outside of a situation looking in.

According to Hamilton & Corbett-Whittier (2013:105), during an interview session interview skills need to be developed. It is important that the researcher needs to reflect on his/her role in the interview process. The researcher should not let his or her assumptions and beliefs emerge during the interview. The role of the researcher is to listen, take notes on what is being said and observe verbal and non-verbal cues. Rule & John (2011:113) continue that the position of the researcher's status and authority may influence the data generated. Planning to minimise such influence helps to improve the quality of the study. The researcher needs to be transparent about his/her positionality and its possible contribution to the credibility and confirmability of the study.

I designed both the test on the equal sign completed by the Grade 6 learners and the questionnaires completed by the teachers. Both instruments were discussed in detail with my research supervisor. Empirical research journal articles guided the development of both instruments. I was present in 6B during the test and the class teacher was present in 6A. No help was given to the learners. During the focus group interview, I was a facilitator providing guidelines on what to focus on: a power point presentation was used to direct the discussion and the focus group interviews were audio recorded. The one-on-one interviews held with the learners were audio recorded. During these sessions, I asked learners to explain their thought process behind each incorrect answer in his or her test. During the observation sessions I was a non-participant observer taking written notes as the lesson unfolded.

3.6 DATA ANALYSIS

Creswell & Plano Clark (2007:131-132) claim that data analysis and interpretation constitute a critical stage in the research process which allows you to construct thick descriptions, identify themes, generate explanations of thought and action evident in the case, and theorise the case. Koshy (2006:106); Rule & John (2011:75); Creswell & Plano Clark (2011:207) suggest that the key research question developed at the start of the study should be a guiding force in the analysis process. Swanborn (2010:113) agrees that the basic

challenge of data analysis is to reduce a large amount of data in order to obtain an answer to the research question.

Simons (2009:118) and Creswell (2009:183) mention that during the process of analysing the data, the researcher needs to identify categories or ideas in the data and look for themes and patterns. The researcher needs to decide which data to include as evidence for the story that is developing. The researcher cannot simply present quotations from interviews or observations without any thematic structure, analysis or interpretation. This is unlikely to convey the meaning of the case. Even in a case study where the intent is to portray the authenticity of the setting through using interview excerpts and observations, the researcher needs to select data that will finally tell a story.

Rule & John (2011:76-77) concur that coding requires intelligent, analytic and systematic decisions about “what the data is saying”. Choices made during coding impact on all subsequent analytical processes of generating findings, developing explanations and conclusions, theorising and suggesting recommendations. For this reason, researchers prefer to do their own coding despite the labour-intensive nature of the process. Cohen, Manion & Morrison (2007:144) suggest that when presenting the data all conclusions have to be sustained by the data. Research questions have to be answered. During analysis themes and patterns need to be identified in order to present robust evidence for any claims made.

Creswell & Plano Clark (2007:131-132) mention that qualitative analysis begins with coding the data, dividing the text into small units (phrases, sentences, paragraphs), and assigning a label to each unit. This label can come from the exact words of the participants (in vivo coding), a term composed by the researcher, or a concept in the social or human sciences. If the researcher codes directly on the printed transcript, the transcript pages need to be typed with extra-large margins so that codes can be placed in the margins. In this hand-coding process, researchers assign code words to text segments in the left margin and record broader themes in the right margin. In quantitative research, according to Creswell & Plano Clark (2011:204), the researcher begins by converting raw data into a form useful for data analysis, which means scoring the data by assigning numeric values to each response, cleaning data entry errors from the database and using appropriate statistical tests to address the research question.

Thomas (2011:124) continues that the data collected by an interpretive inquirer needs to be analysed using some kind of constant comparative method. The basic principle governing

the process of constant comparison is that you emerge with themes that capture or summarise the essence of your data.

3.6.1 Data analysis of qualitative instruments

According to Maxwell (2013:105), the initial step in qualitative analysis is reading the interview transcripts, observational notes, or documents that are to be analysed. Listening to interview tapes prior to transcription is an opportunity for analysis, as is the actual process of transcribing interviews or of rewriting and reorganising rough observation notes. Maxwell (2013:105) continues that during this listening and reading, notes and memos on what is seen or heard in your data should be written up, and tentative ideas about categories and relations should be noted. There are three main analytic options that can be followed at this point.

- Memos
- Connecting strategies (such as narrative analysis)
- Categorizing strategies (such as coding and thematic analysis)

In this study, coding was used to analyse qualitative data. In qualitative research, the goal of coding is to “fracture” data and rearrange them into categories that facilitate comparison between data in the same category that aid in the development of theoretical concepts (Maxwell, 2013:107). Categorising analysis begins with the identification of units or segments of data that seem important or meaningful in some way. This identification can be based on prior ideas of what is important, or on an inductive attempt to capture new insights. The later strategy is often called “open coding”.

3.6.1.1 Analysis of the teachers’ focus group interviews

The two focus group interviews were audio recorded and transcribed. Both the audio recording and transcriptions were reviewed several times to analyse the data and to look for common themes arising. I read through the transcripts of the focus group interviews on numerous occasions in order to gain an overall impression of the common themes. The questions utilised during the interviews assisted me to identify themes and categories them. After an extensive process of comparing data, themes and categories emerged from the teachers’ discussions. These themes were compared with the themes that arose from my

observation notes and questionnaire in an attempt to answer the research questions 1 and 3, which informed this research.

3.6.1.2 Analysis of observation of lessons taught by the teachers

Koshy (2006:112) states that significant issues emerging from observations can be included in an appendix so that the reader can refer to it if necessary or if interested. According to Rule & John (2011:68), recording data collected during observations can be obtained in an open-ended format or with the aid of an observation checklist. In an open-ended format, the researcher takes notes of all behaviour or action considered significant during the observation. When a checklist is used, the observations are guided by a set of field questions within the checklist.

According to Rule & John (2011:68) video-recording provides a further opportunity for capturing observations, if practical, and if ethical considerations have been taken care of. Such recordings, however, have to be converted into text-based data for analysis and presentation. This can be a time-consuming process and introduces additional technology and data-management requirements and cost implications. Researchers need to be aware of how their presence as researchers who make observations may be influencing the behaviour and responses of research participants. In research involving non-participant observations, this effect can be reduced to some extent by researchers spending some time in the research environment to allow participants to become familiar with the presence of an observer.

Data was collected during observations using an open-ended format. I recorded everything that occurred during the lesson (written format). Transcribing the document later made it easier to see themes arising from the data. The themes arising from the observation sessions were compared with those of the questionnaire and focus group session to address the research questions 1 and 3, formulated at the beginning of this study. With reference to Creswell & Plano Clark (2011:70); Morrell & Carrol (2010:16) and Creswell (2009:213), a convergent parallel design was used where the quantitative and qualitative strands were collected and analysed during the same phase of the research process. The strands were kept independent during analysis and then the results were combined during the overall interpretation.

3.6.1.3 Analysis of learners' interviews

According to Salkind (2009:155) and Rule & John (2011:76-77), initial steps include typing up interview notes and transcribing the taped interviews. Codes are labels that highlight different themes or foci within data. Coding is a process of choosing labels and assigning them to different parts of data. This is a time-consuming task, often requiring several iterations of reading, coding and recoding. Coding is an integral part of data analysis requiring intelligent, analytic and systematic decisions about “what the data is saying”. Choices made during coding impact on all subsequent analytical processes of generating findings, developing explanations and conclusions, theorising and suggesting recommendations. Coding provides a good opportunity for getting close to the data.

Rule & John (2011:64) state that transcripts of interviews offer a basis for later analysis and a spur to further reflection by participants. According to Rule & John (2011:79) discourse analysis is a relatively well-established method in linguistic studies. The researcher draws inferences from what is said and how it is said. Close attention to the use of language devices, such as metaphor and imagery, helps the researcher to make meaning of the action and the social world conveyed in the text.

The interview session with the learners was transcribed and coded according to similar themes that arose from the test results. To ensure accuracy of the transcripts, I listened to the recordings a few times. If ambiguity was found in the transcribed text, I listened again to that part of the recording. By re-listening to the recordings, I could recreate the interview session, enhancing my interpretation of the evidence. The interview sessions were used as a method to confirm the test results already gained. I used explanation of common misconceptions of the equal sign to confirm the findings of the test. The learners’ test was individually analysed to gain an understanding of the learners’ misconceptions as well as confirmation that the common misconceptions are prevalent in the grade. Finally, the test results and interview sessions were merged to address the second research question that informed this study.

3.6.2 Data analysis of quantitative instruments

Maxwell (2013:107) claims that coding in quantitative research consists of applying a pre-established set of categories to the data according to explicit, unambiguous rules; with the primary goal being to generate frequency counts of the items in each category.

3.6.2.1 Analysis of the teachers' questionnaires

Rule & John (2011:82) mention that with questionnaires, a process of numerical coding usually precedes data analysis. An alternative way of working with this data is to employ open coding and to analyze data in a more qualitative fashion by developing a set of named categories or themes to which the responses appear to belong.

The researcher tabulated the questionnaire results of each individual teacher using the original questionnaire as a guide to see which questions pose the greatest concern among this particular group of teachers. Overall themes arising from the analysis process were compared with the themes arising from the teacher observation sessions and the two focus group sessions. The themes from the questionnaires, teacher observation sessions and two focus group sessions were merged to address the research questions 1 and 3 that guided this study.

3.6.2.2 Analysis of learners' tests on the equal sign

The researcher tabulated the results of each individual learner using the test as a guide to see which questions prompted the greatest concern among this particular group of Grade 6 learners. Overall themes arising from the 57 learners were identified, with regards to the most common misconceptions in this test. The learners' test was coded to see which common errors arose. The researcher compared the individual themes gained from the interview sessions completed with the six learners to see if there were similar themes arising in comparison with the test results. The learner test and learner interviews were merged to address research question 2 that guided this study.

3.7 TRUSTWORTHINESS, VALIDITY AND RELIABILITY

3.7.1 Measure of Quality

Thomas (2011:71) mentions that the quality of a case study depends less on ideas of sample, validity and reliability and more on the conception, construction and conduct of the study. It depends on the initial idea, ways that the case is chosen, the thoroughness with which its context is described, and the care devoted to selecting appropriate methods of analysis and the nature of the arguments deployed in drawing conclusions.

3.7.2 Validity

Validity is about whether the test or instrument used actually measures what it is intended to measure according to Salkind (2009:117) and Simons (2009:127). Validity is concerned with how well work is set up: whether it is sound, defensible, coherent, well-grounded, and appropriate to the case, “worthy of recognition”, according to Simons (2009:127). Morrell & Carroll (2010:77) and Maxwell (2013:122) agree that validity refers to the correctness or credibility of a description, conclusion, explanation, interpretation, or other sort of account. The researcher had to be aware of this throughout as she checked the accuracy and relevance of participants’ perspectives, negotiated meanings, (and) verified accounts. In some contexts, the validity and credibility are evident in a reading of the case. In many kinds of research – in those, for example, where probability samples are used – validity is about the extent to which a piece of research is finding out what the researcher intends it to find out. In a case study there is no probability sample and we may have no idea at all about what we expect to find out from the research. The idea of validity is less meaningful.

Creswell & Plano Clark (2007:146-147) recommend the following methods to address validity in a mixed methods study.

- Report and discuss validity within the context of both quantitative and qualitative research in a mixed methods study.
- Use the term *validity* or *inference quality* to refer to validity procedures in mixed methods research.
- Define validity, within a mixed methods context, as the ability of the researcher to draw meaningful and accurate conclusions from all of the data in the study.
- Validity needs to be discussed from the standpoint of the overall mixed methods design chosen for a study.
- Validity is enhanced when research states openly the potential threats to validity that arise during data collection and analysis.

According to Creswell & Plano Clark (2007:133; 2011:210) another component of all good research is a report on the validity of the data and results. Validity differs in quantitative and qualitative research, but in both approaches, it serves the purpose of checking on the quality of the data and the results. In quantitative research, validity means that the researcher can draw meaningful inferences from the results to a population; reliability means that scores received from participants are consistent and stable over time. The standards are drawn from a source external to the researcher and the participants: statistical procedures or

external experts. In quantitative research, there are two contexts in which to think about validity and reliability. The first pertains to scores from past uses of the instruments and whether the scores are valid and reliable. The second relates to an assessment of the validity and reliability of data collected in the current study.

Cohen, Manion & Morrison (2007:144-145) state that threats to validity can be minimised by:

- selecting an appropriate methodology for answering the research questions; selecting appropriate instrumentation for gathering the type of data required;
- Using an appropriate size (in a qualitative study when sampling; avoiding bias when conducting your research.
- Remembering that respondents behave differently when subjected to scrutiny such as an interview.
- Trying to avoid dropout rates among respondents.

The validity of a test instrument is equally important as its reliability. If a test does not serve its intended function well, it is not valid.

The Grade 6 test on the equal sign was based on empirical research found in journal articles and reference books. The test was discussed and reviewed by my research supervisor, an expert in the field of Mathematics Education at Cape Peninsula University of Technology, on numerous occasions and adjusted according to suggestions to ensure that the research questions were addressed. The teachers' questionnaire was formulated using the same principles stated above. Both the learners' test and teachers' questionnaire were used to formulate the questions used in the focus group interviews. During each process of formulating the instruments, my supervisor reviewed the instruments, before any data was collected. The research questions were constantly kept in mind while designing the data collection instruments. The interviews with the learners were more open-ended. It was a platform to allow learners an opportunity to explain the thinking behind the errors found in their tests.

3.7.3 Reliability

Cohen, Manion & Morrison (2007:146); Yin (2009:45) Salkind (2009:110) mention that reliability is concerned with precision and accuracy, in order to minimize the errors and biases in a study. For research to be reliable, it must demonstrate that if it were to be carried

out on a similar group of respondents in a similar context (however defined), then similar results would be found.

Thomas (2011:62) states that everything he said about reliability applies also to validity. According to Morrell & Carroll (2010:77), qualitative data is reliable when what is observed by the researcher matches what actually occurred in the field of study. The reader needs to be convinced that what the researcher reported, is unbiased and not subjective. To achieve this, qualitative researchers often use video recordings, tape recordings, and multiple observations when collecting data.

In this study, audio recordings were made of both the focus group interviews and the individual interviews held with the learners. During the observations, lesson data was recorded by hand as the lesson unfolded. I was a non-participant observer, merely observing the actions, while trying to be as unobtrusive as possible (Rule & John, 2011:68). The test was deliberated and reviewed by my research supervisor at Cape Peninsula University of Technology, on various occasions and adapted according to recommendations to ensure that the research questions would be answered.

3.7.4 Trustworthiness

Guba (1981), as cited by Rule & John (2011:107), states that the concept of trustworthiness is an alternative to reliability and validity. This concept promotes values such as scholarly rigour, transparency and professional ethics in the interest of qualitative research gaining levels of trust and fidelity within the research community. Guba (1981) suggests that the trustworthiness of qualitative studies is achieved by paying attention to the study's transferability, credibility and dependability.

According to Rule & John (2011:107) transferability in qualitative research serves as an alternative for the generalisability or external validity of a study. The concept of credibility refers to the extent to which a case study has recorded the fullness and essence of the case reality. Credibility in quantitative research reflects the extent to which a study measures what it set out to study. Dependability, on the other hand focuses on methodological rigour and coherence towards generating findings and case accounts which the research community can accept with confidence.

3.7.5 Triangulation

Cohen, Manion & Morrison (2007:141); Plano, Clark & Creswell (2008:108); Yin (2009:115); Morrell & Carroll (2010:77); Rule & John (2011:63) and Maxwell (2013:102) refer to triangulation as the use of two or more methods of data collection, to ensure that the complexity of human behaviour is studied from more than one standpoint thus avoiding bias.

Thomas (2011:68) and Rule & John (2011:108-109) mention that triangulation is an essential prerequisite when using a case study approach. The starting point for the case inquirer is to think small but drill deep, using different methods. According to Maxwell (2013:102), triangulation reduces the risk that your conclusions will reflect the biases of a specific method only, and allow you to gain a more secure understanding of the issues you are investigating. By making use of teacher questionnaires, two focus group interviews, observations, a test for the Grade 6 learners and interviewing six learners, triangulation has been achieved in this case study. I have conducted two focus group interview sessions; one with the foundation phase teachers and one with the intermediate phase teachers. Eleven teaching sessions were observed from Grade 1 to Grade 6. Fifty-seven learners in Grade 6 wrote a test on equivalence and I interviewed six learners to allow them to explain the thinking behind the errors in their tests. Eleven teachers completed a questionnaire, ensuring that this research has been studied from more than one point of view.

3.8 ETHICAL CONSIDERATIONS

Hamilton & Corbett-Whittier (2013:64) state that ethics is defined as 'norms of conduct that distinguish between acceptable and unacceptable behaviour'. Thomas (2011:68) refers to ethics as a principle of conduct about what is right and wrong. Consequently, Creswell (2009:87) and Oliver (2010:122) mention that researchers should at all times respect the human dignity of those who help them. At the heart of research, ethics is the key-principle to treat people with care, consideration, and sensitivity. "Researchers need to protect their research participants; develop a trust with them; promote the integrity of research; guard against misconduct and impropriety that might reflect on their organization or institutions; and cope with new challenging problems." Creswell (2009:87).

Plowright (2011:169) posits that there are issues linked with how the researcher manages the data collected. Confidentiality and anonymity are important factors to take into account once the data have been collected, stored and analysed. Recorded interviews, the recordings and the transcripts, are kept safe and secure and therefore confidential. The

same applies to questionnaires and any other means the researcher used to record and store the data. They should be secure where the researcher can keep discs, tapes, etc. For electronic data, the researcher will need to be confident that no one else has access to your computer. Rule & John (2011:111) suggest that conducting research in an ethically sound manner enhances the quality of research and contributes to its trustworthiness. Research ethics requirements flow from three standard principles, namely autonomy, non-maleficence and beneficence.

- Autonomy refers to the researcher's responsibility to respect and protect any individual's right to be fully informed, to decide whether to participate and choose to withdraw from a study.
- Non-maleficence refers to the researcher's obligation not to cause any harm in the course of the study.
- Beneficence means that the research should aim to contribute to public good.

Confidentiality and anonymity were taken into account during the process of data collection, storage and during the process of analysing the data. Only my supervisor and I had access to any documentation used in this study. The researcher marked the tests of the learners and information obtained from the test and interview sessions were used only for the purpose of this study. The teacher questionnaires, observation schedules and focus group tape recordings are in the safe keeping of the researcher and used for the purpose of this research only. Findings will be made available to the participants if they request the information. Identities are confidential and reference is made only to teachers 1A, B or learners 1, 2. Ethical clearance letters were received from the University (CPUT) and the WCED allowing me to conduct this study. Appendix 1 contains the WCED Ethics Clearance Letter and Appendix 2 the CPUT Ethics Clearance Letter.

3.8.1 CONSENT

Kvale & Brinkmann (2009:70) state that informed consent entails informing the research participants about the overall purpose of the investigation and the main features of the design, as well as of any possible risks and benefits from participation in the research projects. Informed consent involves obtaining the voluntary participation of the people involved, and informing them of their right to withdraw from the study at any time.

Thomas (2011:69-70) agrees that informed consent is needed (in order to conduct a case study). The potential participants should understand what they are agreeing to. Informed consent is centered on the following points:

- The information participants need to know, which will include:
- The nature and purpose of the study, including its methods
- Expected benefits of the study
- Possible harm that may come from the study
- Information about confidentiality, anonymity, how data will be kept and for how long, with details of when data will be destroyed
- Ethics procedures being followed and appeals
- Participants' full name and full contact details
- The presentation of all information in a meaningful and understandable way, explaining any unusual terms simply, in non-technical language
- The option for a potential participant to choose to take part or not.

Because the research was conducted in a WCED school, permission had to be obtained from the WCED (Western Cape Education Department), my school principal, and all eleven teachers who participated in this study as well as the learners in the school. Consent letters were sent home to all the Grade 6 learners' parents and participation in my research was voluntary. Only those learners who returned the consent forms were engaged in the test. Fifty-seven out of the sixty-one Grade 6 learners returned the consent letter. An additional letter was sent to request permission to interview the six learners: all six letters were returned granting me permission to continue with the interview sessions. The purpose of the research was explained to all concerned. All participants were assured of confidentiality. All information gained from this research would be used for the purpose of this research. The participants were informed that they will have access to the results and findings of the study as cited by Koshy (2006:84). Appendix 3, 4 and 5 are the letters requesting permission from my principal, the eleven teachers and all the parents of the learners that took part in my research, as well as the clearance letters received from the WCED and CPUT.

3.9 CONCLUSION

This chapter provides an overview of the research design and methodology of this study. This theory-led case study was framed within an interpretive paradigm. Both qualitative and quantitative methods were employed to gather the data needed to address the research

questions, and a convergent parallel mixed method design was used to gather and analyse the data. The research questions were evaluated in an ex-model C school in a suburb of Cape Town, South Africa. Chapter 4 provides a summary of the results of teachers' SCK regarding the equal sign and learners' misconceptions with regards to the equal sign.

CHAPTER 4

RESEARCH FINDINGS, DISCUSSIONS AND INTERPRETATIONS

4.1 INTRODUCTION

This chapter summarises this study's findings of teachers' SCK regarding the equal sign and learners' misconceptions of the equal sign. The aim of this research was to investigate the nature of Grade 1- 6 teachers' SCK of the equal sign, what misconceptions Grade 6 learners have of the equal sign and whether the nature of teachers' SCK of the equal sign could possibly prevent, promote or reduce learners' misconceptions of the equal sign.

Teachers' SCK of the equal sign was measured using a questionnaire, observations and focus group interviews (one focus group interview session was held with the foundation phase teachers and another with the intermediate and senior phase teachers). Learners' misconceptions were evaluated using a test focussing on the use of the equal sign in number sentences. A group of six learners were interviewed to allow these learners an opportunity to verbalise their thinking processes when making errors. The results from analysis of the data are organised into the following categories:

- Learners' test on equivalence (4.2)
- Interviews with six learners (4.3)
- Teachers' questionnaire findings (4.4)
- Observations of teachers (4.5)
- Focus group interviews with teachers (4.6)
- Foundation phase teachers (4.6.1)
- Intermediate phase teachers (4.6.2)

The first two sections that follow (4.2 and 4.3) deal with learners' misconceptions with regards to the equal sign and the nature of misconceptions. The next three sections (4.4 – 4.6) focus on teachers' SCK relating to the equal sign:

4.2 RESULTS OF LEARNERS' TEST ON EQUIVALENCE

Misconceptions arising from the test data were identified as:

- Closure
- Using all the numbers in the equation
- String operations
- Inability to describe the meaning of the equal sign correctly.

These misconceptions will be discussed in detail below. The test is attached as Appendix 6.

4.2.1 Closure

According to Blanton (2008:22), the learner is looking for 'closure' when his or her response is as follows: $4 + 6 = \Delta + 5$; $\Delta = 10$. This error is due to learners' operational view of the equal sign where the equal sign is interpreted as "the answer comes next" symbol. Stephens et al. (2013:174); Carpenter, Franke & Levi (2003:10-11); Kilpatrick (2002:261) and McNeil & Alibali (2005b:887) concur that when given problems similar to $8 + 4 = __ + 5$, learners typically respond in one of three ways. First, some learners might display an "answer comes next" approach and answer that the missing number in this case should be 12. In Table 4.1 it is evident that in each example some learners seek closure to the equation; they assume that the equal sign signals an operation on the left hand side and that an answer is appears after the equal sign. This signifies an operational view of the equal sign.

Questions 1, 2, 5, 15.2 and 15.3 of the learners' test measured the extent of this misconception. The results are displayed in Table 4.1. The first column of the table contains the questions from the test; the second column contains the correct answer; the third the incorrect answer which indicates 'closure'; the fourth column indicates the number of learners who gave the incorrect response and column five denotes the number of learners who did not respond to the question.

TABLE 4.1. Results for “closure” misconception

Question in the Grade 6 Test	Correct answers	Incorrect answers	Number of learners who made this error	Number of learners who did not answer
Question 1. $7 + 2 = \Delta + 4$	$\Delta = 5$	$\Delta = 9$	2	0
Question 2. $9 + 3 = \Delta - 4$	$\Delta = 16$	$\Delta = 12$	3	0
Question 5. $9 \times 6 = \Delta \div 2$	$\Delta = 108$	$\Delta = 54$	3	4
Which number makes each statement true? Question 15.2. $13 - 7 = \square - 5$	$\square = 11$	$\square = 6$	2	0
Question 15.3. $12 + 9 = \square - 1$	$\square = 22$	$\square = 21$	3	1

Figure 4.1 to 4.3 provide examples of learners’ answers that signify a closure misconception. (The handwritten text is the researcher’s, showing her comments.)

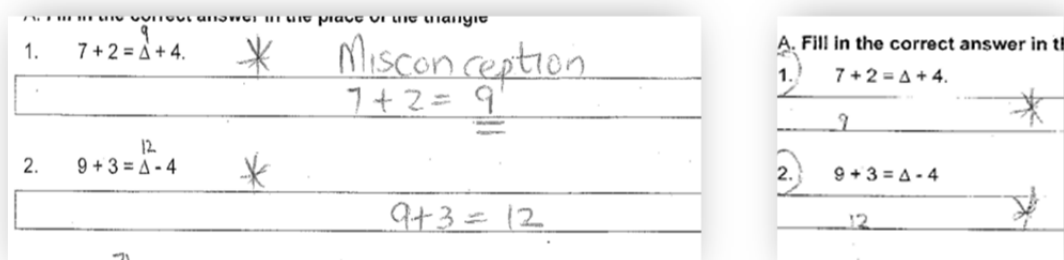


Figure 4.1: Two learners’ responses to Question 1 and 2

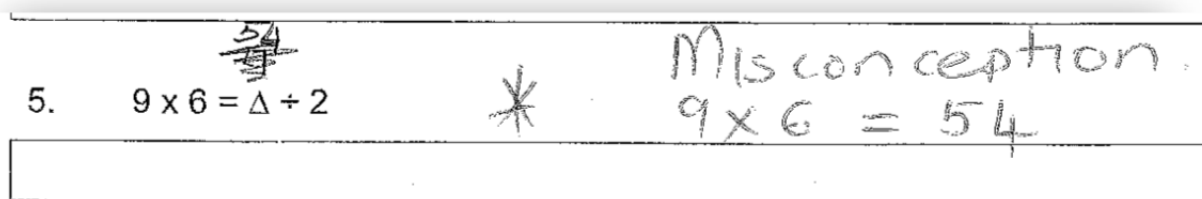


Figure 4.2: A learner’s response to Question 5

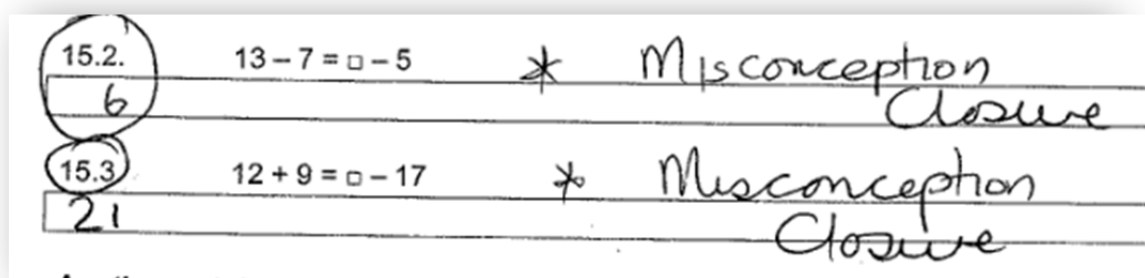
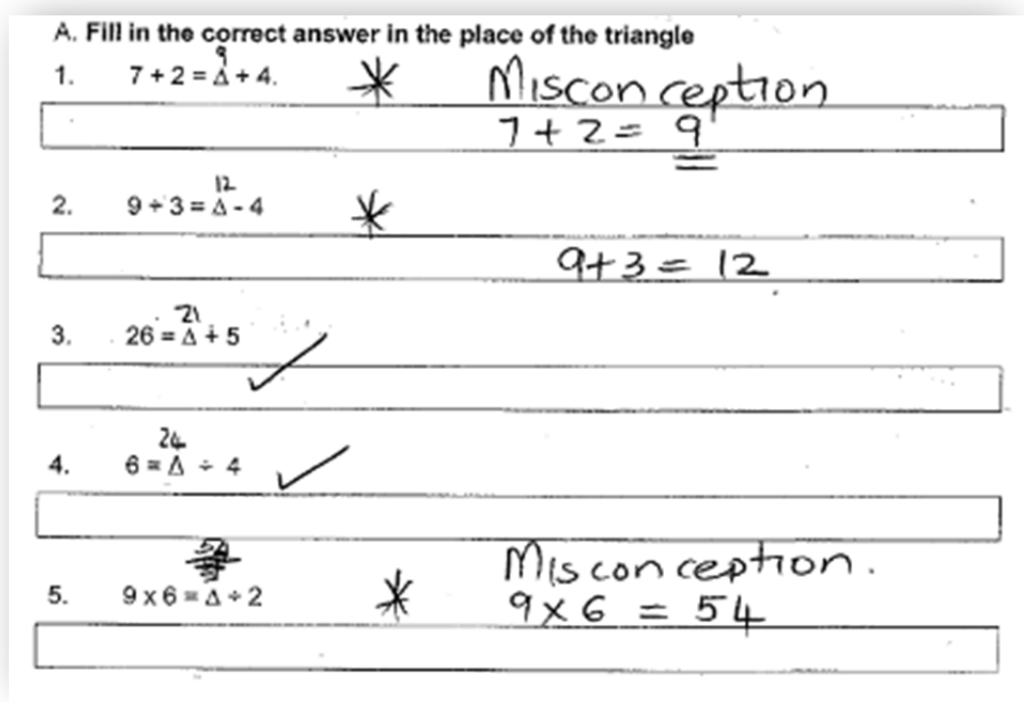


Figure 4.3: A learner's response to Questions 15.2 and 15.3

Four learners in this group of fifty-seven Grade 6 learners were consistent in making errors with regards to closure. Learner A applied closure to question 5, 15.2, and 15.3. Learner B applied closure to questions 1, 2 and 5. Learner C applied closure to 1, 2 and did not answer question 5. Learner D applied it to question 15.2 and 15.3. These responses show that the equal represents a signal that the answer will follow. Learner B's and Learner C's responses appear as Figure 4.4.

Learner B



Learner C

A. Fill in the correct answer in the place of the triangle

1. $7 + 2 = \Delta + 4$. Misconception
 9 ✗ closure.

2. $9 + 3 = \Delta - 4$
 12 ✗ closure.

3. $26 = \Delta + 5$
 21 ✓

4. $6 = \Delta \div 4$

5. $9 \times 6 = \Delta \div 2$

Figure 4.4: Two examples of learners' work who struggle with "closure"

4.2.2 Using all the numbers in the equation

According to Carpenter, Franke & Levi (2003:10-11); Kilpatrick (2002:261) and McNeil & Alibali (2005b:887) another response is that some learners might show an "extend the problem" attitude. In the case of $8 + 4 = _ + 5$ they would write 12 on the line, but add a second equal sign after the 5 and write the number 17 to the right of it (i.e., $8 + 4 = 12 + 5 = 17$). These children, like those who display an "answer comes next" approach, believe the answer to the operation of the left side of the equal sign needs to appear on its right side, but they want to take into account the "+ 5".

In Table 4.2 it is evident that in each example some learners seek to use all the numbers in the equation, in an attempt to find an answer to the equation. This signifies an operational view of the equal sign. Questions 1, 2 and 5 of the learners' test evaluated the extent of this misconception. Question 5 appears to be the most problematic question, with regards to this misconception. The number sentence is more complicated because it requires multiplication, division and knowing that the equal sign represents equivalence. The results are displayed in Table 4.2. The first column of the table contains questions from the test; the second

column contains the correct answer; the third the incorrect answer (which indicates 'using all the numbers'); the fourth column indicates the number of learners who gave the incorrect response and column five shows the number of learners who did not respond to the question.

TABLE 4.2 Results for misconception "using all the numbers in the equation"

Question in the Grade 6 Test	Correct answers	Incorrect answers	Number of learners who made the error	Number of learners who did not answer
Question 1. $7 + 2 = \Delta + 4$.	$\Delta = 5$	$\Delta = 13$	0	0
Question 2. $9 + 3 = \Delta - 4$	$\Delta = 16$	$\Delta = 8$	1	0
Question 5. $9 \times 6 = \Delta \div 2$	$\Delta = 108$	$\Delta = 27$ $\Delta = 90$ $\Delta = 10$ $\Delta = 5$ $\Delta = 104$	6	4

2. $9 + 3 = \Delta - 4$ *

$\Delta = 8$

5. $9 \times 6 = \Delta \div 2$ *

Misconception

$\Delta = 27$

$54 \div 2 = 27$

Figure 4.5: Two learners' responses to Question 2 and 5

One learner in this group of Grade 6 consistently made errors with regards to using all the numbers in the equation. Apart from struggling with "closure", learner A displays a tendency to use all the numbers in the equation. Learner E used all the numbers in the equation for both question 2 and 5. It appears that question 5 is of the greatest concern with regards to using all the numbers in an equation. Of the fifty-seven learners in this grade, learner A; E

and four other learners made this error. Four learners of this particular group did not answer the question. Learner A's and learner E's responses appear as Figure 4.6.

Learner A

5. $9 \times 6 = \Delta + 2$

27

Learner E

5. $9 \times 6 = \Delta + 2$

27

Misconception.

$9 \times 6 = 54 \div 2 = 27$

Figure 4.6: Learner A and learner E's responses, indicating "using all the numbers in an equation"

4.2.3 String operations

Tilley (2011:19) states that another common misuse occurs in contexts where learners solve a calculation mentally by breaking it down into manageable steps: for example, $48 + 36 = 48 + 30 = 78 + 6 = 84$. The learner clearly understands that they can partition one of the numbers (36 in this case) to simplify the calculation. However, the way the solution is written is incorrect because $48 + 36$ does not equal $48 + 30$. This type of 'relaxed' use of the equal sign has been observed by research undertaken by Tilley (2011:19) in both learners' work and as part of teachers' demonstrations. According to Sewell (2002:26) and Risch (2014:2), this relaxed use is the preferred option of many learners; their most common reaction to new information that conflicts with their existing beliefs. This relaxed use requires no mental effort, no reconstruction of pre-existing knowledge: learners emerge from the lesson quite content with what they have always known.

In Table 4.3 this inappropriate use of the equal sign is illustrated as an operational view of the equal sign. Questions 16.3 and 18 of the learners' test evaluated the extent of this misconception. The first column of the table contains the questions from the test; the second column contains the correct answer; the third the incorrect answer; the fourth column

indicates the number of learners who gave the incorrect response and column five shows the number of learners who did not respond to the question.

TABLE 4.3: String operations

Question in the Grade 6 Test	Correct answers	Incorrect answers	Number of learners who made this error	Number of learners who did not answer
Are these statements correct? How do you know? Question 16.3. $4 + 8 = 12 - 6 = 6$	No	Yes	27	3
Question 18. Alan started a problem with a one-digit number. He multiplied the number by 3, added 8, divided by 2 and subtracted 6, and got the same number he started with. What was the number Alan started with? (Show all working out) a. 2 b. 4 c. 6 d. 8	4	8 or 2 or 6	14	3

16.3. $4 + 8 = 12 - 6 = 6$
 True, yes, the final answer is 6 ($4 + 8 = 12$) ($12 - 6 = 6$)

16.3. $4 + 8 = 12 - 6 = 6$
 Correct, $4 + 8 = 12$, $12 - 6 = 6$, so the answer is 6

16.3. $4 + 8 = 12 - 6 = 6$
 Yes, because $4 + 8$ equals 12, 12 minus 6 equals 6

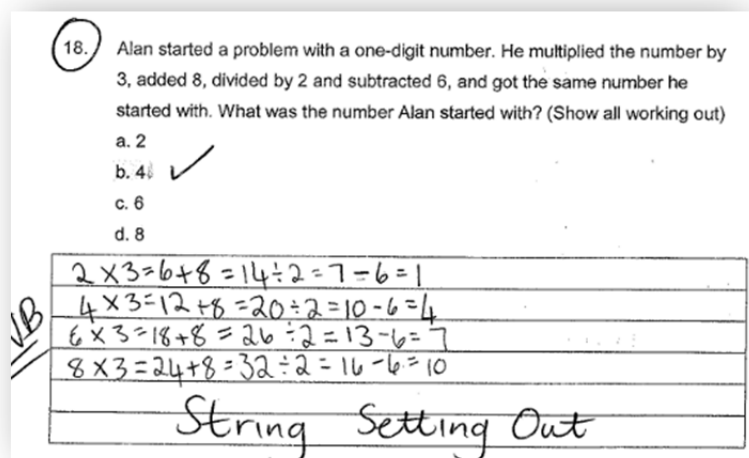


Figure 4.7: Various examples of different learners' accepting string operations as correct (Questions 16.3 and 18)

String operations appear to be the greatest misconception in this particular group of Grade 6 learners. In foundation phase great emphasis is placed on horizontal setting out or decomposing of numbers in a horizontal manner. It appears that this particular school or group of learners do not outgrow this kind of thinking. New methods or strategies to approach operations are not assimilated into their pre-existing knowledge. As the Grade 7 teacher at this particular school, I have seen this problem persisting over the last seven years. New approaches to operations are routinely rejected by learners.

4.2.4 Inability to describe the equal sign correctly

Kilpatrick et al. (2002:270), Molina & Ambrose (2008:61), Haylock (2010:252), Essien (2009:28), Baroudi, (2006:28), Bush & Karp (2013:620) and Ciobanu (2014:14) mention that learners tend to misunderstand the equal sign, thinking of it as an operator, that is, as a symbol inviting them to "do something", to "find the answer" rather than as a relational symbol signifying equivalence or quantitative sameness.

Questions 17.2 and 17.3 of the learners' test evaluated the extent of this inability. Results are displayed in Table 4.4. The first column of the table contains the questions from the test; the second column contains the correct answer; the third learners' incorrect answers; the fourth column indicates the number of learners that gave who incorrect response and column five shows the number of learners that did not respond to the question.

TABLE 4.4: Inability to describe the equal sign correctly

Question in the Grade 6 Test	Correct answer and the number of learners who got it correct	Incorrect answers	Number of learners who made these errors	Number of learners who did not answer
Question 17.2. What does the symbol (=) mean?	LHS = RHS 26 learners answered correctly	The answer What does the two digits add up to Answer is answer The answer is next The sum is equal to the answer Add up to the answer The sum is equal to The number equal the number after it It means the sum can into the answer Adding It means what is that answer	31	0
Question 17.3. Can the symbol mean anything else?	Equivalence 36 learners answered correctly	It joins the numbers together for the answer No the symbol only means the answer	20	1

Interpreting the equal sign

17. The following questions are about this statement

$3 + 4 = 7$
↑

17.1. The arrow above points to a symbol. What is the name of the symbol?

Equal ✓

17.2. What does the symbol mean?

The answer

17.3. Can the symbol mean anything else? If yes, please explain.

No.

Interpreting the equal sign

17. The following questions are about this statement

$$3 + 4 = 7$$

17.1. The arrow above points to a symbol. What is the name of the symbol?

an equals sign ✓

17.2. What does the symbol mean?

the number equals the number after ✓

17.3. Can the symbol mean anything else? If yes, please explain.

no

Interpreting the equal sign

17. The following questions are about this statement

$$3 + 4 = 7$$

17.1. The arrow above points to a symbol. What is the name of the symbol?

equivalent or equal to ✓

17.2. What does the symbol mean?

basically 3+4 equals 7 basically follow + - x ÷

17.3. Can the symbol mean anything else? If yes, please explain.

No

Interpreting the equal sign

17. The following questions are about this statement

$$3 + 4 = 7$$

17.1. The arrow above points to a symbol. What is the name of the symbol?

Equal sign ✓

17.2. What does the symbol mean?

It shows what the answer is. ✓

17.3. Can the symbol mean anything else? If yes, please explain.

No ✓

Interpreting the equal sign

17. The following questions are about this statement

$$3 + 4 = 7$$

17.1. The arrow above points to a symbol. What is the name of the symbol?

The equal sign

17.2. What does the symbol mean?

It means what does the two digits add up to

17.3. Can the symbol mean anything else? If yes, please explain.

No

Figure 4.8: Various learners' response to Question 17.2 and 17.3

From the above it is clear that at this particular school the emphasis is not placed on clearly defining the equal sign. Therefore, the researcher can assume that the concept of the equal sign representing equivalence is not considered as a fundamental concept needed by learners to transcend to a more algebraic manner of thinking. It appears that the majority of learners have a flexible operational view of the equal sign.

4.3 SUMMARY AND GENERAL DISCUSSION OF LEARNERS' TEST RESULTS

Table 4.5 presents an overview of learners' test results. The first column of the table indicates the type of misconception found in the test results; the second column contains the questions that evaluated this misconception; the third points out the number of learners who made the error; the fourth column indicates the total number of learners who answered the question, and column five shows the percentage of learners who made the error.

TABLE 4.5: Summary of Misconceptions

Type of Misconception	Question	No of learners who made the error	Total number of learners who answered the question	% of learners who made the error
Closure	Question 1. $7 + 2 = \Delta + 4$	2	57	3,5
	Question 2. $9 + 3 = \Delta - 4$	3	57	5,2
	Question 5. $9 \times 6 = \Delta \div 2$	3	53	5,6

Type of Misconception	Question	No of learners who made the error	Total number of learners who answered the question	% of learners who made the error
	Which number makes each statement true? Question 15.2. $13 - 7 = \square - 5$	2	57	3,5
	Which number makes each statement true? Question 15.3. $12 + 9 = \square - 17$	3	56	5,3
Use all the numbers in the equation	Question 1. $7 + 2 = \Delta + 4$.	0	57	0
	Question 2. $9 + 3 = \Delta - 4$	1	57	1,8
	Question 5. $9 \times 6 = \Delta \div 2$	6	53	11,3
	Are these statements correct? How do you know? Question 16.3. $4 + 8 = 12 - 6 = 6$	27	54	50
	Question 18. Alan started a problem with a one-digit number. He multiplied the number by 3, added 8, divided by 2 and subtracted 6, and got the same number he started with. What was the number Alan started with? (Show all working out) a. 2 b. 4 c. 6 d. 8	14	55	25,5
Could not define the equal sign correctly	Question 17.2. What does the symbol (=) mean?	31	57	54,3
	17.3. Can the symbol mean anything else?	20	57	35

Results from this study, illustrated in the summary above, confirm previous research into learners' understandings of the equal sign. Although the school at which research for this study was undertaken is considered a high-performing school many Grade 6 learners

misunderstood the equal sign. String operations, using all the numbers in an equation in an attempt to solve a problem, when combined with a failure to define the equal sign correctly, are of great concern at this particular school. A group of learners still view the equal symbol as an operational sign, to carry out the calculation from left to right. Data from this study support the finding that learners consider the following as a string operation: for example:

$4 + 8 = 12 - 6 = 6$ and accept it as true, seeking closure and accept this relaxed manner of using the equal sign in string setting out (Tilley, 2011:19). Data collected from this study confirm findings that show how learners think the answer comes right after the “=” sign. Such learners place the answer right after the equal symbol. Such learners’ understanding of the concept of equality and the equal sign representing it, appears to be limited.

Learners’ responses to question 4 ($6 = \Delta \div 4$) and question 10 (is this statement True or False: $6 \times 5 = 120 \div 4$) is a matter of concern. Nine learners approached Q4 in various ways and for Q10, two learners did not answer the question and six stated false. Both these questions indicated that the learners are not proficient with regard to their times tables and, as a result made calculation errors. It appears that when equations are written in an unfamiliar way, learners struggle to approach the equations in the correct manner. Learners are used to seeing the answer on the right hand side of the equal sign. It is evident that a group of learners struggles with their times and division tables, which leads to errors. With regards to this case study, string operations seem to be the major difficulty among this group of learners. The fact that 54, 3 % of these learners cannot clearly describe the meaning of the equal sign allows room for error in their mathematical proficiency. The test is attached as Appendices 6.

4.4 RESULTS FROM LEARNER INTERVIEWS

I scheduled 15-minute interview sessions with each of the six learners. In the interviews the researcher gained a deeper understanding of the thinking behind the errors, revealing learners’ misconceptions with regards to the equal sign. Interviews have been transcribed, see appendix 9.

4.4.1 Learner 1

At the start of the interview session, the researcher greeted the learner and explained the purpose of the interview session. “We are going to look at your test ... and I want you to explain to me how you made the error.”

Question 16.3: $4 + 8 = 12 - 6 = 6$

Researcher: Why is there no answer here?

Learner: “I knew $4 + 8 = 12$ but I was not sure how to explain it properly but I think it’s true.”

Researcher’s comment: This learner accepts horizontal setting out. Question 16.3 indicates that this learner has a misconception regarding use of the equal sign. String setting out is accepted by this learner.

4.4.2 Learner 2

Question 2: $9 + 3 = \Delta - 4$

Researcher: “How did you get to 13?”

Learner: “ $9 + 3$ is 12, and $13 - 4 = 9$ ”

Researcher’s comment: Learner 2 used all the numbers in her/his attempt to solve the equation: indicative of an operational view to find an answer after the equal sign. In the process a calculation error occurred.

Question 5: $9 \times 6 = \Delta \div 2$

Researcher: “How did you get 90 for no. 5?”

Learner: “ 9×6 is 54 and 90 divided by 2

Researcher’s comment: This learner used all the numbers and made a calculation error ($90 \div 2$ is not equal to 54). It appears that this learner does have a relational view because in his/her calculation ($9 \times 6 = 54$ and $90 \div 2 = 45$), the learners swopped the 4 and 5 around when making the error.

Question 16.3: $4 + 8 = 12 - 6 = 6$

Researcher: “Why did you say yes this is correct?”

Learner: “Because 4 plus 8 equals 12 and 12 minus 6 equals to 6.

Researcher’s comment: Learner 2 accepts horizontal setting out: i.e. he displays string setting out.

Question 17.2: What does the equal sign mean?

Researcher: "What does the equal sign mean to you?"

Learner: "the equal sign means that it is equal to something else or the sum before the equal to sign, meaning that the equal to sign is telling us the answer."

Researcher's comment: Learner 2 has an operational view of the equal sign. Learner 2 confirmed the following during the interview session. In question 5 the learner's thinking was correct; he/she just made a calculation error, possibly as a result of not knowing his/her time tables adequately. In question 12 the learner approached the operation in a horizontal manner, from left to right. Therefore, the string setting out created an opportunity for an error to occur during calculations. In 16.3 the horizontal setting out of the sum was accepted as correct therefore the learner answered yes, displaying the misconception – accepting string operations. For question 17.2 his/her response is the answer to the meaning of the equal sign. This learner presents concerns that need to be addressed in his/her schemas with regards to viewing the equal sign in an operational manner. This learner lacks mathematical proficiency according to Kilpatrick et al. (2002:15). He/She displays no conceptual understanding (comprehension of mathematical concepts, operations, and relations) or strategic competence (the ability to formulate, represent and solve mathematical problems) when approaching mathematical problems.

4.4.3 Learner 3**Question 15.3: $12 + 9 = \square - 17$**

Researcher: "How did you get 31?"

Learner: "first I plussed the 12 with the 9 and then I got 21 + 17"

Researcher's comment: Which is 38 not 31. The learner used all the numbers in the equation, instead of subtracting, he/she added 21 and 17 incorrectly: indicating an operational view of the equal sign.

Question 17.2: What does the equal sign mean?

Researcher: "What does the symbol mean?"

Learner: "two numbers are equal to a specific number"

Researcher's comment: The learner's response indicates an operational view of the equal sign. This learner accepts an answer follows the equal sign. The misconception presented is closure.

Question 17.3: Can the symbol mean anything else? If yes, please explain.

Researcher: "Can the symbol mean anything else? If yes, please explain."

Learners: "No"

Researcher's comment: This learner does not have a relational view of the equal sign: according to the learner's response an answer follows the equal sign, seeking closure after the equal sign.

4.4.4 Learner 4

Question 5: $9 \times 6 = \Delta \div 2$

Researcher: "Tell me what you were thinking to get to 27?"

Learner: "Well, I said 9×6 is 54 and then I thought I could halve it, but then you say 27 is wrong."

Researcher's comment: The learner is using all the numbers in the equation: this is an operational view of the equal sign.

Question 16.3: $4 + 8 = 12 - 6 = 6$

Researcher: "why did you say yes?"

Learner: "Well, ma'm $4 + 8$ is 12 and 12 minus 6 is 6. And I thought it was correct because they said the answer."

Researcher's comment: According to Learner 4 she was taught to set out her sums horizontally so if she does it they are correct. If the teacher taught and accepted this form of setting out this particular teacher has a lack of SCK. In question 5 this particular learner used all the numbers in the number sentence in an attempt to solve the equation. In question 16.3 string operation is accepted.

4.4.5 Learner 5

This is learner C referred to earlier in section 4.2.

Question 1: **$7 + 2 = \Delta + 4$.**

Researcher: “how did you get to 9?”

Learner: “Am see, I just plussed the 7 plus the 2”. $\Delta = 9$

Researcher’s comment: This is a clear indication that this learner has an operational view of the equal sign, giving the answer 9 immediately after the equal sign. Misconception: Closure.

Question 2: **$9 + 3 = \Delta - 4$**

Researcher: “how did you get the 12 for number 2?”

Learners: “I plussed the 9 and the 3. $\Delta = 12$.”

Researcher’s comment: Once again the learner gives the answer immediately after the equal sign. Misconception: Closure.

Question 15.2 **$13 - 7 = \square - 5$.**

Researcher: “how did you get 20?”

Learner: “I think instead of minus I plussed.”

Researcher’s comment: There is a calculation error as well as the misconception - closure. Operations that learner 5 found difficult, were left out so that he could return to it later: it was never completed. Question 2 was approached in the same way as Question 1 seeking closure. Question 15.3 once again indicates a need for computing seeking closure. This learner clearly indicates a misconception regarding the equal sign, viewing it in an operational manner.

4.4.6 Learner 6

Question 4. **$6 = \Delta \div 4$**

Researcher: “how did you get to 16?”

Learner: “Because I have been thinking that you must count in 6’s, I should have counted in 4’s.”

Researcher's comment: This learner displayed an elementary way of solving this equation, his/her attempt was to count in 4's or 6's and as a result an error occurred. The learner appears to have a relational view of the equal sign as he/she attempted to solve the equation by counting in 6's. He/she knew that something divided by 6 would give him/her the answer. There is no misconception regarding the equal sign present in this error.

Question 16.3. **$4 + 8 = 12 - 6 = 6$**

Researcher: "why do you think the statement is right?"

Learner: "I said it's right because I calculated, I plussed 4 and 8, which makes 12 then I minus the 6 from the 12 which is equal to 6."

Researcher's comment: In question 16.3 this learner accepted the horizontal manner in which this equation was set out, indicating an operational view of the equal sign. The misconception present here is solving an equation in a string operation.

Question 18. **Alan started a problem with a one-digit number. He multiplied the number by 3, added 8, divided by 2 and subtracted 6, and got the same number he started with. What was the number Alan started with? (Show all working out)**

Researcher: "explain how you got to you answer?"

Learners: "I did BODMAS, I times 3 by 8 then divided it by 2 minus 6. I first started with the times 3×8 was 24 divided by 6 then I would go to the minus then it made 4. 24 divided by 4 equals 6."

Researcher's comment: This indicates that he/she lacks proficiency in procedural fluency – a skill in carrying out procedures flexibly, accurately, efficiently and appropriately according to Kilpatrick et al. (2002:15). Both in question 16.3 and 18 string setting out is accepted.

4.4.7 Summary of learners' interviews

A comparative study of the results from the test and learner interview sessions confirmed the learners' thinking behind their errors. There appears to be an elementary understanding of the use of the equal sign. It appears as if the learners at this particular school have a flexible operational view of the equal sign. Therefore, in some instances it is not used appropriately, for example in string operations. With reference to Olivier (1989:11) and Brodie (2014:223),

errors and misconceptions are seen as a natural result of learners' efforts to construct their own knowledge: such misconceptions are intelligent constructions based on incorrect or incomplete previous knowledge. Themes that arose from these sessions were: certain learners still approach an equation from left to right seeking closure at the end of the equation, as reported by Kilpatrick et al. (2002:270), Molina & Ambrose (2008:61), Haylock (2010:252), Essien (2009:28) and Bush & Karp (2013:620). They use all the numbers to complete an operation as reported by Carpenter, Franke & Levi (2003:10-11) and Kilpatrick (2002:261).

String operations, working from left to right, seem to be the greatest concern at this specific school. There is evidence of careless mistakes not displaying a proficiency with regards to certain concepts in mathematics, as reported by Brodie (2014:223). Learners do not know their multiplication tables which affects their answers (Kilpatrick et al., 2002:15). In the foundation phase these learners are exposed to horizontal, expanded notation. However, the results show that they never progressed to a more sophisticated form of setting out in Grade 6 and 7. I teach the Grades 6 and 7 learners' mathematics at this school and have seen this inappropriate use of the equal sign even among high-achieving learners indicating that there is not a clear understanding that the equal sign is a relational symbol.

4.5 RESULTS FROM THE TEACHERS' QUESTIONNAIRES

All relevant results were summarised in table format listed in Table 4.6. Only ten of the eleven questionnaires were handed in, so the data reflect ten sets of information collected. The questionnaires which appear as Appendix 7, were given to the eleven teachers in the first week of March 2015 with an expected due date of 23 March. The one intermediate teacher, the Grade 4 B teacher, did not return her questionnaire. The following themes arose from the teachers' questionnaires, and will be discussed in detail later:

- Cannot identify whether an operation contains an error/misconception (4.4.1)
- Left out questions not relevant to their Grade (4.4.2.)
- Do not know how to correct, prevent or reduce the misconceptions regarding the equal sign (4.4.3.)

I observed that questionnaires were left until the last week of the second term which created apprehension among teachers, especially foundation phase teachers. On two occasions I witnessed questions being discussed in the staff room by some of the Grade 1 – 3 teachers.

On both occasions they were told that it was an individual questionnaire. They handed in the questionnaires in an anonymous manner and answers on the questionnaire list appear to be the same among certain Grade partners. One of the Grade 1 teachers stated that he/she did not answer certain questions on the questionnaire because it was not content taught in her/his particular Grade, therefore not relevant to the teacher. This indicated a lack of knowledge at the mathematical horizon, or “peripheral vision” needed in teaching; an awareness of how mathematical topics cover the span of mathematics included in the curriculum (Ball et al., 2008:403).

For each question in the questionnaire, there was a section where the teacher needed to indicate whether they realised there was an error or misconception. Most of these questions were left unanswered.

4.5.1 Cannot identify whether an operation contains an error/misconception

Question 2.1 – 2.17 required that teachers identify whether the learners’ response indicated an error or possible misconception in learners’ understanding of the equal sign. Most of the teachers had the ability to explain the learners’ thinking during the process of trying to solve the equation, but few of the teachers could identify whether the learners’ response contained an error or misconception; revealing a lack in SCK with regards to identifying errors, or misconceptions in learners’ work by class-based teachers. It was surprising to see that even questions relevant to Grade 5 were left out by one of the Grade 5 teachers.

2.5. Instruction given to learner: What does the following symbol mean (=)?
Learners' response: Answer of the sum is.

Error/	Sum means add.??
Misconception	
Possible cause	?
How to prevent	
How to reduce	

Figure 4.9: Question 2.5. Did not indicate whether there is an error or misconception

Researcher’s comment: this teacher answered that the “sum means add”, but did not indicate that the learners’ response to the given question is an incorrect view. From her/his answer(s), however, it is difficult to assume that she has a well-developed relational view.

2.3. Instruction given to learner: Fill in the correct answer in the place of the triangle: $7 + 2 = \Delta + 4$

Learner's response: $\Delta = 9$

Error/ Misconception	Δ is answer to "7+2"
Possible cause	
How to prevent	Solve sums separately, spend time on number sentences.
How to reduce	

Figure 4.10: Teacher 5A - Question 2.3 did not state whether there is an error or misconception.

Researcher's comment: this teacher knows that the learner added 7 and 2 to get to the answer 9. From her/his answer(s), however, it is difficult to determine whether s/he recognises this as an error/misconception, and therefore to determine whether s/he has a well-developed relational view.

2.8. Instruction given to learner: Calculate: $3 \times 9 - 5 + 11$

Learner's response: $= 3 \times 9 - 5 + 11 = 27 - 5 + 11$
 $= 27 - 5 = 22 + 11 = 2$

Error/ Misconception	/
Possible cause	
How to prevent	
How to reduce	

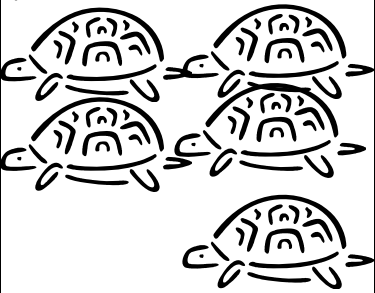
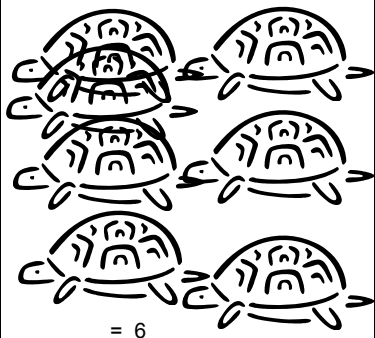
Figure 4.11: Teacher 5A – No error present according to the teachers' response

Researchers' comment: I can assume that s/he accepts string operations as correct; indicating an operational view. According to this teacher's response, there was no error in this particular learner's response in question 2.8.

TABLE 4.6: Summary of teachers who could identify the error or misconception in the question and the number of teachers who did not give a response to the question.

QUESTION	No. of teachers who identified the error or misconception		No. of teachers who did not give an answer	
	Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire	Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire
2. In each of the following scenarios determine whether the learner's response contains an error/misconception. If so, explain the possible cause and how to prevent or how to reduce the misconception. 2.1. Instruction given to learner: Halve the numbers 28 and 32 Learner's response: $28 = 14$ $32 = 16$	Error: 1 Misconception: 2	Misconception 2	3	2
2.2. Instruction given to learner: Decompose the number 11. Learner's response: $11 - 10 + 1$	Error: 1 Misconception: 3	Misconception: 1	2	3
2.3. Instruction given to learner: Fill in the correct answer in the place of the triangle: $7 + 2 = \Delta + 4$ Learner's response: $\Delta = 9$	Error: 4	Error: 1	2	3
2.4. Instruction given to learner: Calculate $2 \times 3 \times 8 \div 4$ Learner's response: $(2 \times 3) \times (8 \div 4)$ $= 2 \times 3 = 6 \times (8 \div 4)$ $= 6 \times (8 \div 4) = 2$ $= 6 \times 2 = 12$	Error: 3 Misconception: 1	Misconception: 1	2	3
2.5. Instruction given to learner: What does the following symbol mean (=)? Learners' response: Answer of the sum is.	Misconception: 2 Error: 1	Misconception: 1	2	3
2.6. Instruction given to learner: Choose the correct symbol (<; >; =) $3 + 5 \times 2 \dots\dots\dots 16$ Learner's response: $3 + 5 \times 2 = 16$	Error: 3	Error: 1	3	3
2.7. Instruction given to learner: Fill in the correct answer in the place of the Δ: $9 + 3 = \Delta - 4$ Learner's response: $9 + 3 = 12 - 4 = 8$	Error: 3	Error: 1	3	3

QUESTION	No. of teachers who identified the error or misconception		No. of teachers who did not give an answer	
	Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire	Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire
2.8. Instruction given to learner: Calculate: $3 \times 9 - 5 \div 11$ Learner's response: $= 3 \times 9 - 5 \div 11 = 27 - 5 \div 11$ $= 27 - 5 = 22 \div 11 = 2$	Error: 2	Error: 1	4	3
2.9 Instruction given to learner: Calculate: 10% of 240 Learner's response: $10 \div 240 = 24$		Misconception: 1	6	3
2.10. Instruction given to learner: Calculate: $\frac{5}{9} + \frac{7}{9}$ Learner's response: $\frac{5}{9} + \frac{7}{9}$ $= 5 + 7 = \frac{12}{9}$		Error: 1	6	3
2.11. Instruction given to learner: Give the Multiples of 3 Learner's response: $3 = 3; 6; 9; 12; 15$	Error: 1 Misconception: 2	Misconception: 1	3	3
2.12. Instruction given to learner: State whether the following is true or false $2 \times 8 = 16 + 1$ Learner's response: True	Error: 4	Error: 1	2	3
2.13. Instruction given to learner: State whether the following is true or false $25 - 5 = 17 + 3$ Learner's response: True	Learner's answer is correct: 3	Learner's answer is correct: 2	3	2
2.14. Instruction given to learner: Complete the number sentence. $2 + \square = 5$ Learner's response: He writes 7 in the empty box	Error: 3	Error: 1	3	3
2.15. Instruction given to learner: How old is John? Learners' response: John = 8.	Error: 2	All 4 teachers did not indicate Error or Misconception	3	4

QUESTION	No. of teachers who identified the error or misconception		No. of teachers who did not give an answer	
	Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire	Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire
2.16. Instruction given to learner: How many tortoises are in the picture?  Learner's response:  = 6	Error: 2 Answer Correct: 2	Misconception: 1	3	3
2.17. Instruction given to learner: calculate $20 + 30 + 7 + 8$ Learners' response: $20 + 30 = 50 + 7 = 57 + 8 = 65$	Misconception: 1 Error: 1	Misconception: 1	4	3

4.5.2 Do not know what could have caused the misconception, or how to prevent or reduce misconceptions regarding the equal sign

Question 2.1 – 2.17 required that teachers show their SCK with regards to the equal sign, by explaining how to correct, prevent or reduce the error/misconception identified in the questionnaire. The sections were generally left out: “practise” or “re explain” was given to assist learners in overcoming their misconceptions. The teachers did not explain how they intended to “re explain” the concept. This indicates a lack in SCK with regards to correcting error or a lack of strategies about how to deal with misconceptions (Ball et al., 2008:401). There is a need to develop teachers’ SCK, to assist them to develop the skill of correcting, preventing or reducing errors/misconceptions in learners. “Re-explain” or practise as a solution to errors is not an option, because, according to Bodner (1986:9) and Brodie

(2014:223), misconceptions are resistant to instruction. Once we have constructed knowledge, simply being told that we are wrong is not enough to make us change our (mis)concepts.

2.12. Instruction given to learner: State whether the following is true or false

$$2 \times 8 = 16 + 1$$

Learner's response: True

Error/ Misconception	
Possible cause	Doesn't know times table.
How to prevent	Know times table.
How to reduce	Practice

Figure 4.12: Question 2.12 from the questionnaire

Researcher's comment: Teacher 1A: according to this teacher's comment, this learner only has a concern with regards to the times table. No mention is made about the equal sign or any other relation symbol that could be used to make the equation true. It is not clear what this teacher's view is about relational symbols, especially the equal sign in this particular study.

2.17. Instruction given to learner: calculate $20 + 30 + 7 + 8$

Learners' response: $20 + 30 = 50 + 7 = 57 + 8 = 65$

Error/ Misconception	Errors
Possible cause	$20 + 30 \rightarrow 50$ $50 + 7 \rightarrow 57$ $57 + 8 \rightarrow 65$
How to prevent	
How to reduce	

Figure 4.13: Question 2.17 from the questionnaire

Researchers' comment: Teacher 3A knew how to set out the operation in a more appropriate way (correctly) but could not identify the correct cause for this learner's setting out. S/he did not answer the sections on preventing or reducing the learners' manner of setting out the equation.

2.14. Instruction given to learner: Complete the number sentence. $2 + \square = 5$

Learner's response: He writes 7 in the empty box

Error	Number sentence not balanced. Sees it as $7-5=2$.
Misconception	
Possible cause	
How to prevent	
How to reduce	

Figure 4.14: Teacher 5A, explaining the learner's thinking behind the error.

Researchers' comment: This teacher knows that the equation needed to be balanced and could identify the learner's thinking behind the error. However, s/he did not answer the section on preventing or reducing this misconception in the learner's thinking. It appears that this teacher lacks knowledge at the mathematical horizon since she/he could not provide a possible cause for this error.

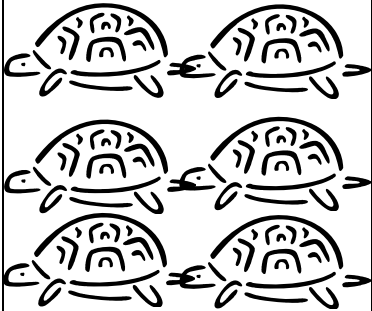
4.5.3 Left out questions not relevant to their Grade (SCK limited to the Grade they teach)

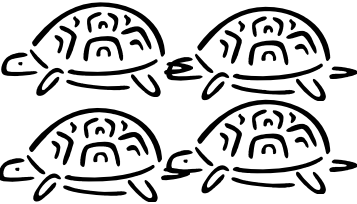
Table 4.7 presents a summary of the number of teachers who left out questions not relevant to their Grade (SCK limited to the Grade they teach). The first column of the table indicates the question listed in the questionnaire; the second column states whether it is a foundation phase or intermediate phase question; the third points out how many teachers left out the question not relevant to their Grade; column four is subdivided into foundation phase and intermediate phase teachers who left out the question.

TABLE 4.7: Number of Teachers who completely left out questions not relevant to their Grade

QUESTION	FP Question Or IP Question	Number of teachers who left out the question not relevant to their grade	
		Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire
2. In each of the following scenarios determines whether the learner's response contains an error/misconception. If so, explain the possible cause and how to prevent or how to reduce the misconception. 2.1. Instruction given to learner: Half the numbers 28 and 32 Learner's response: $28 = 14$ $32 = 16$	FP	0	0
2.2. Instruction given to learner: Decompose the number 11. Learner's response: $11 \text{ — } 10 + 1$	FP	0	1
2.3. Instruction given to learner: Fill in the correct answer in the place of the triangle: $7 + 2 = \Delta + 4$ Learner's response: $\Delta = 9$	FP or IP	0	0
2.4. Instruction given to learner: Calculate $2 \times 3 \times 8 \div 4$ Learner's response: $(2 \times 3) \times (8 \div 4)$ $= 2 \times 3 = 6 \times (8 \div 4)$ $= 6 \times (8 \div 4) = 2$ $= 6 \times 2 = 12$	IP	1	1
2.5. Instruction given to learner: What does the following symbol mean (=)? Learners' response: Answer of the sum is.	FP or IP	0	0

QUESTION	FP Question Or IP Question	Number of teachers who left out the question not relevant to their grade	
		Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire
2.6. Instruction given to learner: Choose the correct symbol (<; >; =) $3 + 5 \times 2 \dots\dots\dots 16$ Learner's response: $3 + 5 \times 2 = 16$	IP	1	0
2.7. Instruction given to learner: Fill in the correct answer in the place of the Δ: $9 + 3 = \Delta - 4$ Learner's response: $9 + 3 = 12 - 4 = 8$	FP or IP	1	0
2.8. Instruction given to learner: Calculate: $3 \times 9 - 5 \div 11$ Learner's response: $= 3 \times 9 - 5 \div 11 = 27 - 5 \div 11$ $= 27 - 5 = 22 \div 11 = 2$	IP	2	1
2.9 Instruction given to learner: Calculate: 10% of 240 Learner's response: $10 \div 240 = 24$	IP	4	0
2.10. Instruction given to learner: Calculate: $\frac{5}{9} + \frac{7}{9}$ Learner's response: $\frac{5}{9} + \frac{7}{9}$ $= 5 + 7 = \frac{12}{9}$	IP	4	1

QUESTION	FP Question Or IP Question	Number of teachers who left out the question not relevant to their grade	
		Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire
2.11. Instruction given to learner: Give the Multiples of 3 Learner's response: 3 = 3; 6; 9;12;15	IP or FP	1	0
2.12. Instruction given to learner: State whether the following is true or false $2 \times 8 = 16 + 1$ Learner's response: True	IP or FP	0	0
2.13. Instruction given to learner: State whether the following is true or false $25 - 5 = 17 + 3$ Learner's response: True	IP or FP	2	3
2.14. Instruction given to learner: Complete the number sentence. $2 + \square = 5$ Learner's response: He writes 7 in the empty box	IP or FP	1	0
2.15. Instruction given to learner: How old is John? Learners' response: John = 8.	FP	1	0
2.16. Instruction given to learner: How many tortoises are in the picture? 	FP	1	2

QUESTION	FP Question Or IP Question	Number of teachers who left out the question not relevant to their grade	
		Foundation Phase 6 FP teachers completed the questionnaire	Intermediate Phase 4 IP teachers completed the questionnaire
Learner's response:   = 6			
2.17. Instruction given to learner: calculate $20 + 30 + 7 + 8$ Learners' response: $20 + 30 = 50 + 7 = 57 + 8 = 65$	IP	2	1

The results in Table 4.7 above indicate that most of the teachers have only SCK limited to the curriculum that they interact with, and that they do not feel the need to expand their SCK beyond the grade that they are teaching. One intermediate phase teacher could not respond to questions relevant to the grade that she/he is teaching at present. This particular teacher completed a Postgraduate Certificate in Education (PGCE) and has been teaching for the past 2 years which creates the question of whether the PGCE prepared her/him adequately enough to teach mathematics in the intermediate phase. Most teachers at primary school level are generalist teachers, not specialised mathematics teachers. Some of the teachers at this particular school have been teaching for over 30 years and have developed knowledge based on their experience in teaching. At the other end of the spectrum there are new teachers with a Postgraduate Certificate in Education (PGCE). None of the teachers that took part in this study are specialists in the field of mathematics education.

Figure 4.15 to 4.19 are examples of questions not completed by teachers because it was not relevant to the grade that they were teaching. This may indicate a lack of knowledge at the mathematical horizon.

2.10. Instruction given to learner: Calculate: $\frac{5}{9} + \frac{7}{9}$

Learner's response:

$$\frac{5}{9} + \frac{7}{9}$$

$$= 5 + 7 = \frac{12}{9}$$

Error?	
Misconception	
Possible cause	
How to prevent	
How to reduce	

Figure 4.15: Teacher 3B, addition of fractions is done in Grade 4

Researcher's comment: Addition of fractions in this abstract manner is not part of the foundation phase curriculum, therefore it was just left out by this teacher. This omission displayed a lack of knowledge at the mathematical horizon. It appears that this teacher does not have a good number concept, because s/he accepts that whole numbers can be equal to fractions.

2.4. Instruction given to learner: calculate $2 \times 3 \times 8 \div 4$

Learner's response:

$$(2 \times 3) \times (8 \div 4)$$

$$= 2 \times 3 = 6 \times (8 \div 4)$$

$$= 6 \times (8 \div 4) = 2$$

$$= 6 \times 2 = 12$$

Error?	
Misconception	
Possible cause	
How to prevent	
How to reduce	

Figure 4.16: Teacher 1A, question 2.4

Researcher's comment: Order of operations in this format is not part of the foundation phase curriculum requirements. The teacher wrote N/A, so according to this teacher it was not applicable to her; displaying a lack of knowledge at the mathematical horizon.

2.8. Instruction given to learner: Calculate: $3 \times 9 - 5 \div 11$

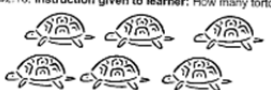
Learner's response: $= 3 \times 9 - 5 \div 11 = 27 - 5 \div 11$
 $= 27 - 5 = 22 \div 11 = 2$

Error/	
Misconception	
Possible cause	
How to prevent	
How to reduce	

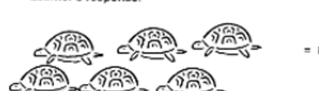
Figure 4.17: Teacher 1B, question 2.8

Researcher's comment: Order of operations in this format is not part of the foundation phase curriculum requirements and was left out by this teacher. This teacher cannot identify that the setting out of this operation is incorrect; displaying a lack of knowledge at the mathematical horizon.

Q2.16. Instruction given to learner: How many tortoises are in the picture?



Learner's response:



Error/	
Misconception	
Possible cause	
How to prevent	
How to reduce	

Figure 4.18: Teacher 4A, question 2.16

Researcher's comment: This example indicates that teachers use the equal sign in this manner to give the answer. It does not display the relational quality, promoting an operational view. Two foundation phase teachers accepted this example as correct.

2.11. Instruction given to learner: Give the Multiples of 3

Learner's response: 3 = 3; 6; 9; 12; 15

Error/ Misconception	No cut off. Needs to extend to infinity.
Possible cause	
How to prevent	properties of numbers, focus on specific wording of question
How to reduce	

Figure 4.19: Teacher 5A, question 2.14

Researcher's comment: This teacher does not display a relational view of the equal sign in question 2.14. She/he could not identify that multiples of 3 should not be displayed in this manner and accepts that the answer follows the question (operational view). According to this teacher, counting of the multiples was the error.

2.12. Instruction given to learner: State whether the following is true or false

$2 \times 8 = 16 + 1$

Learner's response: True

Error/ Misconception	error
Possible cause	
How to prevent	
How to reduce	

Figure 4.20: Teacher 6A, question 2.12

Researcher's comment: This teacher could identify that there was an error, but did not elaborate on the type of error, cause or methods to prevent or reduce the error. The researcher could not ascertain his/her SCK with regards to the equal sign.

4.5.4 Summary of teachers' questionnaires

Of the ten questionnaires, five teachers answered the questionnaire in detail, the rest of the questionnaires were poorly answered which limited use of questionnaires as an instrument in

this study. Grade 1 A and B and 3 A and B teachers left out most of the answers not relevant to the phase that they are teaching. The Grade 5 A teacher also left out most of the answers relevant to grade 5 or answered it incorrectly; demonstrating a lack of SCK and knowledge at the mathematical horizon. Sections on how to prevent, correct or reduce errors were left out by most of the ten teachers which suggested a need for skills development in primary school teachers with regards to dealing with learners' misconceptions. According to Ball et al. (2008:400) framework of "Mathematical Tasks of Teaching", there appears to exist a lack of SCK and mathematical horizontal knowledge among the class teachers of this particular school, which needs to be developed. The following needs to be addressed: recognising what is involved in using a particular representation, linking representations to underlying ideas and to other representations, connecting a topic being taught to topics from prior or future years, inspecting equivalencies.

4.6 RESULTS FROM THE LESSON OBSERVATIONS

4.6.1 Brief Summary of the lesson observations

Observations of the eleven teachers took place over a period of five weeks during my administration lessons and corresponded with their teaching time. Each observation lesson took 30 minutes. Observation lessons took place during the second term of 2015, from 30 April until 28 May. The aim of the observations was to gauge how the equal sign concept is addressed or used in a general arithmetic lesson. I recorded everything I observed during the lesson. A transcription of the recordings was made: view appendix 10. No observation sheet was used and I observed a general lesson, not specifically a lesson on the concept of the equal sign. The intent was to see how any lesson was used to incorporate this fundamental concept. Below is a summary of the observations relevant to this study.

4.6.1.1 The Grade 1 and Grade 2 observations

Four teachers selected displayed the same method of teaching their learners. Carpet work was done with one ability group, while the rest worked at their tables, completing an activity in silence. During this time the teacher was explaining a new or revising a taught concept to the group of learners on the carpet. The groups did not rotate in the 30 minutes that I was present in the classroom. The concept was estimation of the number of breads, then counting in groups of 3s or 2s to see if their estimation was correct. The 2A and 2B teachers used the same method; except that a different concept was covered during my observation

lessons. None of the four lessons observed in this four classes above were linked to the equal sign.

4.6.1.2 The Grade 3 observations

The teachers did not teach Mathematics lessons during the observations even though they knew what the study was dealing with. None of the lessons taught was relevant to the equal sign.

4.6.1.3 The Grade 4 observations

Both Teachers 4A and 4B introduced fractions during observation lessons. Teacher 4B briefly linked concepts of fractions to time but did not mention the equal sign or equivalence.

4.6.1.4 Grade 5A observation

Capacity and Volume were explained during this observation lesson. This lesson type (concrete apparatus could have been used to illustrate equivalence) was a good opportunity to bring in equivalence using the equal sign but the opportunity was not taken by the teacher.

4.6.1.5 Grade 5B observation

This observation lesson was an introductory lesson to fractions, which is introduced in Grade 4. This lesson was well presented, using circular diagrams representing fractions. To some extent, a limited amount of time was spent discussing equivalent fractions.

4.6.1.6 Grade 6A observation

The researcher did not observe anything of significance regarding the equal sign during this observation lesson.

4.6.2 Themes that arose from the observation lessons are:

4.6.2.1 Apprehension to reveal their SCK relating to Mathematics

Teachers in general are not keen to have observers present in their classrooms. The question relating to content knowledge of teachers is a sensitive topic and not well received

by most teachers. All eleven teachers allowed me to observe their teaching methods, however most of the lessons were introductory lessons to a new concept, where the teacher did most of the explaining. This part of my research was not easy. I was working with my colleagues and I am the Grade 7 teacher at this particular school. Both Grade 3 teachers avoided teaching a mathematics lesson during my observation time even though they were aware what the research entailed. In the Grade 6 class learners were correcting a previous activity, so the teacher was a facilitator allowing learners to complete the activities on the whiteboard.

4.6.2.2 Methods used in the foundation phase are completely different to those used in the intermediate phase.

According to one Grade 3 teacher, skills are trained in this phase, rules and structure are emphasised. In the foundation phase, teaching takes place in ability groups, whereas in the intermediate phase one general lesson is taught to all the learners. Learning barriers in the foundation phase are dealt with by a remedial teacher at this particular school. The intermediate phase has a learner support teacher present in the mathematics lessons to assist learners with learning barriers. This particular learner support teacher is a trained foundation phase teacher yet lacks the necessary skills to deal with intermediate SCK. During the observation lessons, misconceptions were not used as a point of learning.

4.6.2.3 Curriculum driven – no time for diagnostic marking/teaching

Both phases spent time on the concept that needed to be taught and completed it in a 30-minute lesson. I did not see diagnostic teaching take place during these observation lessons. However, I am aware that this school is in a fortunate position: there are teachers' assistants and remedial staff to deal with learning barriers of individual learners in both phases of the school. The CAPS document used in South Africa as the curriculum guide for teaching is structured with regards to time and concepts that need to be completed within a particular timeframe, not leaving room to deal with learners' misconceptions.

4.6.2.4 Teachers did not link the concept being taught to the equal sign

Most of the teachers displayed a compartmentalised manner of teaching during the time that I was observing. Only the topic being taught was being discussed. The 4B teacher linked her concept of fractions to time, demonstrating some form of knowledge on equivalence. Emphasis was not placed on the concept of equivalence, it was verbally mentioned, but not

displayed visually on the board so that learners could link the two concepts. The 5B teacher mentioned equivalent fractions in her/his introduction lesson of fractions.

4.6.3 Summary of teachers' observation lessons

The fact that I observed one lesson per teacher, in total eleven observation lessons, could limit the data collected. In my view, a series of observation lessons over a course of time would better reflect results related to teachers' SCK. However, during the limited observation sessions, it was evident that there is a need to assist primary school teachers develop knowledge at the mathematical horizon, which is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum (Ball et al., 2008:403). There is a need to slow down the pace in teaching, to allow room to deal with learners' misconception as a platform for sound learning to take place.

4.7 RESULTS FROM THE FOCUS GROUP INTERVIEWS

On 12 June 2015, the first focus group interview session was held with the foundation phase teachers. The following teachers were present: 1B, 2A, 2B, 3A, 3B, 4A (1A was absent). The second session was held on 19 June 2015 and the following teachers were present: 4A, 4B, 5A, 5B and 6A. In both sessions a Power Point presentation was used to facilitate discussion. The teachers were asked, as each Power Point slide appeared, to use the guidelines to keep them focussed on the things they needed to discuss. Themes that arose from the discussions were:

4.7.1 Grade 1 teachers introduce learners only to examples where the answer is on the right hand side for the first 9 months of the year.

Researcher: How do you set out your number sentences?
For example, is it always set out as $3 + 4 = 7$?

Teacher 1B: In Grade 1, we only do it this way, toward the end of the third term we introduce number sentences $10 = 3 + 7$. At the beginning of the year it would be too confusing. The sums in Grade 1 are too basic for us to see misconceptions with regards to the equal sign. Because the number sentences are so basic, misconceptions are not picked up in Grade 1. I do not think it's a good idea to introduce a variety of ways

of setting out with regards to number sentences in Grade 1. I am prepared to do that in June with the top group.

Researcher's comment: The fact that the Grade 1 learners are exposed to elementary operations only, requiring that they give the answer on the right hand side for the first six months of schooling, creates an opportunity that an operational view could develop among certain learners. With reference to chapter 2, Falkner, Levi, Thomas and Carpenter (1999:234) remark that in the elementary school the equal sign usually appears at the end of numerical expressions and only one number comes after it.

Researcher: With reference to slide 9 and 10 of the Power Point.

Fill in the correct answer in the place of the triangle:

$$7 + 2 = \Delta + 4$$

Teacher 2A: "The learners are so used to seeing the box on the right-hand side that they think it's the answers."

Teacher 2A: "Often in Grade 1 and 2 the box is used to place in the answer. It's always on the right-hand side."

Researcher's comment: In Grade 2 we can see that the box or triangle is mostly placed on the right-hand side, encouraging learners to develop an operational view of the equal sign. The fact that these teachers do not use their own SCK to rectify this concern, indicates that these particular teachers do not realise the importance of teaching their learners the relational view of the equal sign. An awareness of the "Mathematical Tasks of Teaching" (Ball et al., 2008:400), which includes finding an example to make a specific mathematical point, recognising what is involved in using a particular representation, and modifying tasks to be either easier or harder, are not displayed by these teachers. Instructional material is just taken and used without evaluating them.

4.7.2 The balance scale is used to introduce the concept of equivalence but is not linked to the symbol (equal sign)

Researcher: How is the concept of the equal sign introduced in foundation phase?

Teacher 3A: In Grade 1 we start teaching the equal sign with a balance scale, without having the equal sign in the middle.

Researcher's comment: The teacher lacks the SCK to link the equal sign in her/his explanation of equivalence. The balance scale should be used as a representation of the equal sign, to show the learners the relational meaning of the symbol.

4.7.3 Teachers acknowledge that learners refer to (=) as indicating the answer

Researcher: With reference to slide 3 of the Power Point, the teachers responded as follows.

Teacher 3A: Some Grade 3 learners still refer to the (=) as the answer.

Teacher 4A: Learners treat the equal sign the same way they treat punctuation in language where it is a waste of their time: they see it so often that it has lost its meaning to the children, even though we have taught them what it means. In Grade 4, we see a lot of learners using the equal sign to indicate the answer.

Teacher 3A: They like to say the equal sign means the answer.

Researcher's comment: Teachers are aware that certain learners have a misconception with regards to the equal sign but have not addressed this misconception; they were not aware that it was such a stumbling-block in later algebraic learning. The importance of the equal sign has not been emphasised sufficiently. This study brings this fundamental element of mathematical education to light.

4.7.4 Textbook errors (CAPS material) and teachers' setting out on the blackboard encourage an operational view of the equal sign

Handwritten work by a Grade 2 learner titled "Halving odd num". The work shows a list of odd numbers from 1 to 19, each followed by an equals sign and a fraction representing half of the number. Each equation is marked with a checkmark.

$1 = \frac{1}{2}$ ✓	$11 = 5\frac{1}{2}$ ✓
$3 = 1\frac{1}{2}$ ✓	$13 = 6\frac{1}{2}$ ✓
$5 = 2\frac{1}{2}$ ✓	$15 = 7\frac{1}{2}$ ✓
$7 = 3\frac{1}{2}$ ✓	$17 = 8\frac{1}{2}$ ✓
$9 = 4\frac{1}{2}$ ✓	$19 = 9\frac{1}{2}$ ✓

Figure 4:21: Grade 2 learner's work from her/his workbook accepted as correct (slide 6 of the Power Point)

- Researcher: Look at slide 5 and 6. What do you think created this error?
- Teacher 4A: The teacher might have set it out incorrectly on the board.
- Teacher 2A: Some of the assessments received from the Department set it out like that. Even in the Blue Book it is set out like that.

Researchers' comment: The teachers could see this learner set out the activity the way her/his teacher set it out on the board. Therefore, we can conclude that sometimes teachers in their presentation on operations on the board display (and strengthen) an operational view of the equal sign.

4.7.5 Summary of Teacher's focus group discussions

The focus group interviews proved informative: both myself and teachers realised that the equal sign is not emphasised appropriately and that instruction of this concept is taken for granted. In the first session (foundation phase session) one teacher hardly contributed to the discussion. Teachers know that the equal sign is a relational symbol but in their instruction of the concept fail to link the equal sign and equivalence when introducing the concept. Grade 1 and 2 teachers admit that most of the number sentences have the answer on the right-hand side and, according to them, this is correct. Grade 1 learners are introduced to too many new concepts in Grade 1. After June teachers present the number sentences in a variety of ways: where the answer could appear on the left. This tends to promote an operational view among learners.

During the second session, it was clear that these teachers have seen their learners displaying an operational view of the equal sign but they did not realise the significance of the equal sign. Teachers lacked strategies in place to reduce misconceptions regarding the equal sign.

4.8 CONCLUSION

This chapter presented, analysed and discussed the results of the research. The data obtained from learners' tests, interviews with six learners, teachers' questionnaires, lesson observations and focus group interviews were analysed to answer the research questions. Results have revealed that this research study was consistent with previous research findings: that there are learners who view the equal sign as an operational symbol which indicates that the calculation should be carried out from left to right. This confirms the need to place the answer right after the equal symbol. It confirms that teachers lack SCK with

regards to introducing the equal sign concept as well as strategies to prevent or reduce misconceptions in learners. Chapter 5 provides conclusions and recommendations based on the findings of the research.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter provides an overview of the study. Results presented in Chapter 4 are interpreted and discussed. Recommendations are made in respect of learners' mathematical proficiency and teachers' SCK with regards to the equal sign. The aim of this study is to investigate the nature of Grade 1- 6 teachers' SCK of the equal sign, what misconceptions Grade 6 learners have of the equal sign and whether the nature of teachers' SCK of the equal sign could possibly prevent, promote or reduce learners' misconceptions of the equal sign. Data was collected using a learners' test as well as individual interviews with six learners, teachers' questionnaires, lesson observations and two focus group interviews. The research was guided by the following research title and questions:

TITLE

The equal sign: teachers' specialised content knowledge and learners' misconceptions.

RESEARCH QUESTIONS

1. What misconceptions do Grade 6 learners have of the equal sign?
2. What is the nature of Grade 1- 6 teachers' SCK of the equal sign?
3. Could the nature of teachers' SCK of the equal sign possibly promote, prevent or reduce learners' misconceptions of the equal sign?

5.2 DISCUSSION

5.2.1 Results from learners' tests and interviews

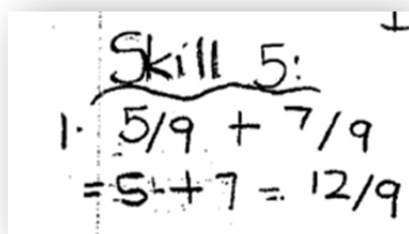
RQ 1: What misconceptions do Grade 6 learners have of the equal sign?

In an attempt to answer research question 1 of this study, learners' test results and the outcome of learners' interview sessions were combined. Themes identified in the test and interview sessions were merged to confirm that a group of this particular Grade 6 learners struggle with the following misconceptions regarding the equal sign.

- Closure
- Using all the numbers in the equation
- String operations
- Inability to define the equal sign correctly

It appears that some of the Grade 6 learners in this particular school struggle mostly with string operations and an inability to define the equal sign correctly. Despite the fact that it is a high performing academic school, several learners lack a clear relational view of the equal sign. I have identified that the same group of learners who struggle to understand closure, struggle with the misconception of using all the numbers in an equation to solve that particular equation. String operations and the inability to define the equal sign seem to be a general concern with this particular group of Grade 6 learners. According to studies conducted by Matthews, Rittle-Johnson, McEldoon & Taylor (2012:320-321), it appears that this group of Grade 6 learners often have a flexible operational view (refer to chapter 2), believing that the meaning of the equal sign is “get the answer to the question,” yet they can successfully evaluate equations where the calculation is either to the left or to the right of the equal sign. Alternatively, this same group of Grade 6 learners may have a basic relational view: learners can evaluate equations involving operations on both sides of the equal sign and be able to recognise the equal sign’s relational meaning.

Research by Nickson (2000:101-102) reveals that learners’ interpretation of equality changes and develops gradually. Initially the equal sign is taken to be a command which indicates that the numbers given have to be acted upon; only later did learners make the connection between ‘quantitative sameness on both sides’ of the symbol. However, as the Grade 7 teacher at this particular school, I still find that a number of learners’ interpretation of equality has not developed beyond an operational view. They use the equal sign to signal that the answer follows or in string operations as displayed below.



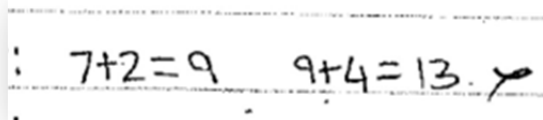
The image shows a handwritten note on a piece of paper. At the top, it says "Skill 5:" with a horizontal line underneath. Below this, the student has written the equation $1. \frac{5}{9} + \frac{7}{9}$. Underneath that, they have written $= 5 + 7 = 12/9$. This demonstrates a string operation where the student is adding the numerators (5 and 7) to get 12, and then placing that over the common denominator 9, without properly applying the fraction addition rule.

Figure 5.1: Grade 7 learner’s workbook - setting out of operation for addition of fractions (mentioned in Chapter 1)

or

The question was: Give the correct value of Δ :

$$7 + 2 = \Delta + 4$$



$7 + 2 = 9$ $9 + 4 = 13$ ✓

Figure 5.2: Learner's Calculation - Grade 6. (mentioned in Chapter 1)

5.2.2 Results from teachers' questionnaires, lesson observations and focus group interviews

RQ 2: What is the nature of Grade 1- 6 teachers' SCK of the equal sign?
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In an attempt to answer research question 2, the teachers' questionnaire, lesson observations and focus group interviews themes were merged to ensure triangulation of the results of this research. The results indicate that the nature of the Grade 1-6 teachers' SCK of the equal sign pose the following concerns:

- The majority of teachers could not identify an error or misconception in learners' work regarding the equal sign.
- The majority of teachers do not know how to correct, prevent or reduce misconceptions regarding the equal sign.
- Teachers left out questions not relevant to their Grade (SCK limited to the Grade they teach).
- Teachers did not use opportunities during my observation lesson to link the concept taught to the equal sign.

According to Thames & Ball (2010:223) and Ball et al. (2008:6; 2008:399) common content knowledge (CCK) refers to knowing when learners have answers wrong, recognising when textbooks give inaccurate definitions and being able to use terms and notations correctly when speaking and writing on the board. Common content knowledge is the knowledge teachers need in order to be able to do the work that they are assigning their learners and

which is closely related to the content of the curriculum. SCK, according to Ball et al. (2008:400), is defined as the mathematical knowledge and skill uniquely needed by teachers to conduct their work. Specialised content knowledge include using mathematical notation and language and critiquing its use, looking for patterns in learners' errors or in sizing up whether a nonstandard approach would work in general. Many of the everyday tasks of teaching are distinctive to this special work (Ball et al., 2008:400). Each of these tasks is something teachers routinely do. Taken together, these tasks demand unique mathematical understanding and reasoning. Teaching requires knowledge beyond that being taught to learners. For instance, it requires understanding different interpretations of the operations in ways that learners need not explicitly distinguish (Ball et al., 2008:400). The results in Chapter 4 indicate that certain teachers at this particular school lack both CCK and SCK with regards to equality and the equal sign. Certain teachers do not possess the ability to recognise what is involved in using a particular representation, to link representations to underlying ideas and to other representations, or to connect a topic being taught to topics from prior or future years

One Grade 5 teacher could not identify the wrong answer. One of the Grade 2 teachers admitted that the textbook or Blue CAPS books display the use of the equal sign incorrectly. That could be a reason why the teacher displayed it in this manner to her/his learners during her/his explanation and accepted the learners' answers as correct in the Power Point (slide 5 and 6) which displayed the operation in the same manner.

Maths
Half

1	$28 = 14$ ✓
2	$32 = 15$ ✓
3	$16 = 8$ ✓
4	$24 = 12$ ✓
5	$10 = 5$ ✓

Figure 5.3: Slide no 5

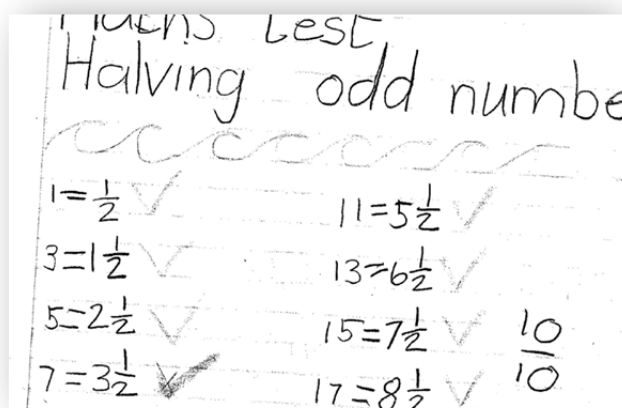


Figure 5.4: Slide no 6

From figures 4.12 - 4.14, it is clear that teachers at this particular school could not provide strategies to rectify, prevent or reduce misconceptions about the equal sign. Table 4.7 indicates that, of the six foundation phase teachers, four teachers did not answer questions 2.9 and 2.10 which cover intermediate phase content. One Grade 5 teacher could not answer questions 2.4; 2.8 and 2.10, which is content relevant to the grade she/he teaches. This confirms a lack of CCK, SCK and mathematical knowledge at the horizon among certain teachers at this particular school.

RQ 3: Could the nature of teachers' SCK of the equal sign possibly promote, prevent or reduce learners' misconceptions of the equal sign?

Kilpatrick et al. (2002:373) mention that learners' achievements can be related to teachers' knowledge of their subject. According to Bansilal, Brijlall & Mkhwanazi (2014:30) in South Africa many studies suggest that mathematics teachers struggle to comprehend the content they teach. In fact, studies point to the teachers' poor content knowledge as one of the reasons for South African learners' poor results in both national and international assessments.

Galant (2013:46-47) mentions that, based on a study (SPADE project) conducted in the poorer communities in the Western Cape during 2012, several teachers display a weak 'specialised content knowledge' and 'mathematical knowledge at the horizon'. Venkat and Spaul (2015:5) agree, based on research findings relating to measure South African teachers' mathematical content knowledge, that a focus on content knowledge remains critical. Studies conducted by Taylor & Taylor (2013), as reported by Venkat & Spaul

(2015:5), have analysed the SACMEQ 3 (2007) mathematics teacher data and concluded that: “Most of the South African grade six teachers were unable to respond to a question that their learners should be able to answer. If most of these teachers were unable to answer the question, what do they teach their learners in the mathematics class?”

Research Question 3 has been subdivided into two sections to provide clarity about the results. Sub-section 5.2.2.1 that follows deals with whether the nature of teachers’ SCK of the equal sign could possibly promote learner’s misconceptions of the equal sign. Subsection 5.2.2.2 deals with whether the nature of teachers’ SCK of the equal sign could possibly prevent or reduce learners’ misconceptions of the equal sign.

5.2.2.1 Could the nature of teachers’ SCK of the equal sign possibly promote learners’ misconceptions of the equal sign?

It is clear from the evidence gathered in the focus group discussion in Chapter 4 that the one Grade 1 teacher have not introduced the equivalence concept and the symbol that represents it correctly to the learners. For the first nine months, s/he only uses number sentences with a box or triangle on the right hand side, promoting an “answer follows” perception (operational view) of the equal sign. According to the teacher, this is done because so many new concepts are introduced to learners in Grade 1. During the focus group discussion, one of the Grade 2 teachers agreed that because the box or triangle is on the right-hand side, this could imply to the learners that the answer follows the equal sign. One of the Grade 3 teachers stated that balance scales are used to represent equivalence, but balance scales are not linked to the equal sign or the concept of equivalence.

Even though it appears that all the teachers at this school are implicitly aware that the equal sign is a relational symbol, teachers were not aware of the importance of the equal sign as a concept that could later affect learners’ performance in algebra. The Grade 1 teacher mentioned during the focus group interview that for the first nine months of Grade 1 the answer appears on the right-hand side, promoting an operational view of the equal sign. This is an indication that this concept is taught incorrectly to learners in grade 1. Teachers’ SCK of the equal sign may have caused learners to form misconceptions of the equal sign. Textbooks used can increase use of string setting out of equations.

5.2.2.2 Could the nature of teachers' SCK of the equal sign possibly prevent or reduce learners' misconceptions of the equal sign?

From results displayed in Chapter 4, it is concerning that this section of the questionnaire was generally not completed by teachers. It was left out and the only options that were mentioned were to "re-explain" or "practise". We can assume that this particular group of teachers needs assistance in developing strategies in how to prevent and reduce learners' misconceptions of the equal sign in training workshops.

5.3 IMPORTANCE OF OVERCOMING THESE MISCONCEPTIONS

Kilpatrick et al. (2002:271), Chapin & Johnson (2006:195-196) and Leavy, Hourigan & McMahon (2013:247) mention that equality is a fundamental concept in algebra which is indicated by the equal sign and can be modeled by thinking of a level balance scale. Equality is important for learners to understand for the reasons listed below. The idea that two mathematical expressions can have the same value is at the heart of developing number sense.

Example: $9 \times 4 = 2 \times 3 \times 6$
 $7 + 6 = 6 + 6 + 1$
 $7 + 6 = 7 + 7 - 1$

Understanding these number sentences and the relations expressed by them is linked to correct interpretation of the equal symbol. Research by Chapin & Johnson (2006:195-196) reveals that failing to understand the equal sign is one of the major stumbling blocks for learners when solving algebraic equations. To solve: $3x + 2 = 14$, the following is done to the equation, always maintaining equivalence on both sides of the equal sign.

$$\begin{aligned}3x + 2 &= 14 \\3x + 2 - 2 &= 14 - 2 \\3x &= 12 \\3x \div 3 &= 12 \div 3 \\x &= 4\end{aligned}$$

In addition, Chapin & Johnson (2006:195-196) state that if learners do not understand the idea of equality of expressions, they may perform computations on one rather than on both

sides of an equal sign. Jones (2008:6) states that “the notion of equivalence goes hand in hand with the important mathematical idea of replacement. Meaning, if two expressions are equivalent then one may be used to replace the other at any time. For example: $3 \times (2 \times 4) = (3 \times 2) \times 4$ therefore $3 \times (2 \times 4)$ can replace $(3 \times 2) \times 4$ or (24, the counting number) can be used to replace the expression”. Substitution in algebra requires this kind of thinking in learners.

Manipulation and simplification of algebraic expressions are underpinned by the principle of equivalence. E.g. to write $2(x + 3) = 2x + 6$ syntactically means that these two expressions are equivalent, i.e. although they appear different, they are equal in value should x be replaced by any (real) value. Vermeulen (2007:16; 2008:200) agrees that in algebra the aim of manipulation and simplification is not so much the finding of an “answer” in a simplified form, as the replacement of one algebraic expression by another more useful, yet still equivalent, expression. According to Vermeulen (2007:16; 2008:200) learners experience problems in simplifying expressions such as $2a + 5b$, because they interpret the equal sign as an operational symbol, leading to closure giving an answer of $7ab$. The $=$ sign in $(x - 1)(x - 2) = x^2 - 3x + 2$, simply indicates that these two expressions are equivalent.

5.4 RECOMMENDATIONS FROM THIS STUDY

Knowledge of the content to be taught is the cornerstone of teaching for proficiency, therefore improving teachers’ mathematical knowledge and their capacity to use it to do the work of teaching is crucial in developing learners’ mathematical proficiency (Kilpatrick et al. 2002:372). Brodie (2014:222) agrees that in order to be able to teach learners effectively and increase learner achievement, teachers need a solid understanding of mathematics, recognise the importance of higher level thinking tasks and be conscious of research concerning learner misconceptions. Many teachers, however, lack sufficient background knowledge of algebra because most have not been in an algebra class since high school (Kilpatrick et al., 2002:372 and Vela, 2011:8).

Stephens (2006:274) and Prediger (2009:77) suggest that when teachers are evaluating learners’ work in methods courses, they need to focus on learner misconceptions, in order to gain insights into how learners think and learn. Prediger (2009:77) continues that this focus will increase teachers’ background knowledge so they can teach their learners in a variety of ways instead of just learning mathematics as a procedure. Hansen (2011:13) agrees that rather than simply correcting an error, it is more productive for teachers to investigate why a learner gives an incorrect answer. Teachers need to adopt a constructive attitude to their

learners' mistakes, recognising that analysis and discussion of mistakes or basic misconceptions can develop learners' mathematical skills.

As mentioned in Chapter 2, Skemp (1987:25; 111) asserts that existing schemas are essential tools for gaining further knowledge. Almost everything we learn depends on knowing something else already. From a constructivist point of learning, errors are assumed to arise from misconceptions or concepts constructed by learners to relate their existing knowledge to new knowledge. But such individual constructions are not necessarily aligned with conventional mathematical knowledge (Olivier, 1989:3 and Brodie, 2014:223). Skemp (1989:69) agrees that obstacles to learning a certain concept may lie in a learner's past experiences. Learners' conceptions and misconceptions should be what teachers engage with or expose in the classroom. Koshy et al. (2000:180) state that, if learners' misconceptions are anticipated and addressed in time, it is likely that gaps in learner understanding can be remedied, which in turn helps them to develop a more robust framework of conceptual understanding. Enhanced understanding of concepts enables learners to recall rules correctly; or at least reconstruct forgotten or partially remember rules.

5.4.1 Recommendations for teachers

5.4.1.1 Adaptive teaching

Kilpatrick et al. (2002:375-377) state that adaptive teaching requires teachers to adapt their teaching to learners' individual needs. Teachers need to be able to identify learners' misconceptions (i.e. learners' incorrect thinking and learning processes) and decide on a plan of action to help learners overcome these misconceptions. Adaptive teaching motivates teachers to plan how to teach material in meaningful ways tailored specifically to their own learners, thus increasing learners' understanding of the equal sign. First, teachers need to be aware of common misconceptions, and then identify misconceptions which are difficult for learners to correct. If misconceptions are not addressed, it will lead to confusion in more advanced mathematics, such as algebra (Stephens, 2006:253).

Orton & Frobisher (1996:59) and Nickson (2000:102) suggest that discussion is seen as important in a constructivist approach to teaching and learning in the classroom as a means of reaching a shared understanding; such discussion provides teachers and researchers with an insight into how learners reach the understandings they do.

5.4.2 Suggested methods to prevent and reduce misconceptions and promote correct understanding of the equal sign

Teaching algebra begins at primary school level when learners are taught basic concepts such as equivalence. The structural rule that relates to the use of the equal sign needs to be recognized and used so that misunderstanding and misuse can be avoided in proceeding from the arithmetical to the algebraic level (Nickson, 2000:123 and Knuth et al., 2006:309). Mann (2004:65) agrees that exposing learners to this important algebraic concept in the lower grades is essential to an understanding of equality [National Council of Teachers of Mathematics (NCTM) 2000]. Falkner et al. (1999:232-234) propose that the idea of equivalence could be introduced in foundation phase using a balance scale.

Chapin & Johnson (2006:196) believe that explaining the meaning of the equal sign alone is not as effective as providing activities that foster this understanding. This belief in follow-up activities is heavily supported by past research in which models, including the use of actual balance scales, have been successfully used to demonstrate equivalence. Research by Knuth et al. (2008:10) suggests that using physical and pictorial models can be used to consolidate the idea of equality as a balance. Their work highlights how equalities can be read from right to left (which begins to occur in algebra) in addition to left to right (the predominant use in arithmetic). A balance scale is generally not introduced until learners are doing formal algebra. Such a late illustration causes confusion: learners think new content is being introduced which is disconnected from their experiences with arithmetic. Kieran (1981) as cited by Knuth et al. (2008:10) correctly points out that if the concept of an equal sign was built from learners' arithmetic knowledge, learners could develop an intuitive understanding of an equation and gradually transform their understanding into that which is required for algebra. Blanton (2008:23-24) agrees that a scale can be used to help learners visualise equivalent quantities. The scales allow learners to work simultaneously with both sides of the equation (as opposed to operating only on one side, usually the left).

What follows is a more detailed description of teaching strategies that could be used to prevent and reduce misconceptions of the equal sign and to promote a correct or proper understanding of the equal sign.

5.4.2.1 The same but different

Based on studies by Darr (2003:6) and Knuth et al. (2008:11), it is important to vary the ways in which equations are represented. Instead of writing equations with the unknown as the

answer, for instance in the form $3 + 4 = \Delta$, vary it so that learners can see the empty box presented in other positions, for example as $\Delta = 3 + 4$. The teacher might simply need to offer a set of tasks: for example, $9 + 3 = \Delta + 4$ or $9 + 3 = 4 + \Delta$ and draw learners' attention to misconceptions in their thinking. Alternatively, the teacher can ask learners to express their answers to computations as unexecuted operations, rather than as single-numerical-value answers. For example, instead of giving 37 as an answer to $23 + 14$, they could express their answer as the sum of two numbers. They might write $23 + 14 = 10 + 27$, or $23 + 14 = 19 + 18$.

Blanton (in Van der Walle et al. 2013:262) agrees that a subtle shift in the way teachers approach teaching computation can alleviate such a major misconception. Instead of asking learners to solve the problem $45 + 61 =$, ask them to find an equivalent expression and use that expression to write an equation e.g. $45 + 61 = 40 + 66$. It is important for learners to understand and symbolise relations in our number system. The equal sign is a principal method of representing these relations: for example: $6 \times 7 = 5 \times 7 + 7$. In this way it is easy to help learners develop an algebraic view of equality. Leavy et al. (2013:249) agree that teachers need to emphasise that expressions may have the same value but look different.

5.4.2.2 Combinations may still have equal value

Leavy, et al. (2013:249) continues that using examples as illustrated below can assist learners to develop a relational view of the equal sign. The use of money, for instance, can provide an almost tangible aid for learners to grapple with the notion that quantities that appear different may have the same value.

$$R1 = 50c + 50c$$

$$25c \times 4 = R1$$

$$50c + 50c = 25c \times 4$$

5.4.2.3 Use of correct mathematical language

Kieran (1981) and Fielker (2008:38) as cited by Knuth et al. (2008:10) suggest that teachers should refrain from reading the equal sign in arithmetic as “makes,” as in “2 plus 3 makes 5,” which reinforces an operational view. Instead, teachers should read the equation $2 + 3 = 5$ as “2 plus 3 is equivalent to 5” emphasising the equal sign as a symbol of a relation (Booth, 1986). This idea is echoed today by researchers and teacher educators who encourage teachers to use the phrase “is the same as” when reading the equal sign (Knuth et al. 2008; Van de Walle, Karp, & Bay-Williams, 2010).

5.4.2.4 True or false statements

Another way of developing learners' conceptions of equality is by asking them to determine if number sentences are **true or false**. Teachers can start with number sentences following the familiar form of an operation of two numbers on the left of the equal sign, followed by the answer, such as $3 + 5 = 8$. Number sentences are then gradually introduced in less familiar forms, such as $8 = 3 + 5$, $3 + 5 = 5 + 3$, and $3 + 5 = 4 + 4$. In determining whether these number sentences are true or false, learners are forced to examine their understanding of how the equal sign can be used (Carpenter et al., 2003:16; Knuth et al., 2008:11; Chapin & Johnson, 2006:196).

5.4.2.5 The use of concrete apparatus

To help learners move from "the answer is" to the "is the same as" mode of thinking, Mann (2004:66-67) initiated a discussion about seesaws. Darr's (2003:6) research confirms that picturing equations as a kind of balance beam is another way to challenge learners' ideas about the equal sign. Learners can draw, imagine, or even use a balance beam to model how an equation works. To be even or equal, the balance beam must of course have the same value (or weight) on both sides.

5.4.2.6 Conjectures and refutations

Based on the research conducted by Darr (2003:6), teachers can present learners with equations such as $\Delta + 2 = 5$ and ask them for any statements they can make about the value of the missing number, compared with 5. For instance, should it be bigger or smaller? Older learners could be given equations such as $\Diamond = \Delta + 2$ and asked to discuss what can be said about the number represented by the diamond, compared with the number represented by the triangle, if the equation is going to be true.

5.4.2.7 Creating new names for numbers

Another activity that emphasises the meaning of the equal sign involves learners in creating different ways of naming or expressing a number, and using the equal sign to link them: for instance, $3 + 4 = 7 = 5 + 2 = 1 + 6$, etc. as cited by Darr (2003:6). MacGregor & Stacey (1999) describe a more practical version of this activity. They show how learners could use a geoboard to investigate how many rectangles can be made with a piece of string 24 units long. An equal sign could then be used to connect all the expressions that describe the

perimeter of each rectangle. For instance: $24 = 4 \times 6 = 6 + 6 + 6 + 6 = 12 + 12 = 2 + 10 + 10 + 2$. Making the units on the geoboard some size other than one, for instance 0.2 or $\frac{1}{2}$, could make this activity more suitable for older children.

5.4.2.8 Using the equal sign in notation (string operations)

Darr (2003:6) suggests challenging learners about the way the equal sign is used in learners' working: that is, as a way of linking a series of computations, for instance: $3 + 7 = 10 \div 2 = 5$. Kieran (1981:323) comments that this technique is often observed even at university level. In terms of conventional mathematics, it is inappropriate. When teachers notice learners doing this, it could be a good opportunity to introduce and talk about other symbols that can be used to link steps, such as an arrow. Using an arrow, the example above could then be rewritten as:

$$3 + 7 = 10 \longrightarrow 10 \div 2 = 5$$

5.4.2.9 More methods

Based on research by Leavy et al. (2013:249) further methods that teachers can use to help learners develop a relational view of the equal sign include:

- Explore the relational concept of equality further by developing human equations (tug-of-war activities).
- Using children's literature to explore equality (Leavy et al., 2013:250).
- Kinetic activities reinforcing the idea that the value of quantities on either side of the equal sign must balance each other (Leavy et al., 2013:250).
- A drawstring bag can be used to represent something unknown. Place marbles in the drawstring bag, to represent the unknown quantities in the equation. In the problem situation, $8 = c + 3$, place five marbles in the drawstring bag to ensure that the quantities balance (Leavy et al., 2013:251).
- Encourage learners to represent problems as equations and represent unknown quantities with a triangle: Δ (Leavy et al., 2013:251).

Knuth et al. (2008: 519) posits that intentional opportunities need to be created by teachers to provide learners with arithmetical (or algebraic) equations to solve in which numbers and operations (or symbolic expressions and operations) appear on both sides of the equal sign.

Such equation formats may promote more appropriate interpretations and uses of the equal sign.

5.4.3 Recommendations for learners

According to a study conducted by Vela (2011:7), learning mathematics conceptually rather than learning a set of rules or actions, encourages learners to become successful problem solvers. In this study learners were expected to discuss their mistakes and the reasons for these mistakes. Encouraging learners to think aloud in the classroom, deterred them from making similar mistakes. To be successful in mathematics, learners need to have mathematical proficiency. Mathematical proficiency, according to Kilpatrick et al. (2002:15), consists of five strands, namely:

- Conceptual understanding - comprehension of mathematical concepts, operations, and relations.
- Procedural fluency - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence - the ability to formulate, represent and solve mathematical problems. According to Kilpatrick et al. (2002:22), strategic competence involves using conceptual and procedural knowledge to solve problems.
- Adaptive reasoning - capacity for logical thought, reflection, explanation, and justification. According Kilpatrick et al. (2002:23), by thinking about the logical relations between concepts and situations, learners can navigate through the elements of a problem and see how they fit together.
- Productive disposition - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These five strands are interwoven and interdependent, according to Kilpatrick et al. (2002:23)

Learners need to be proficient in all five of the strands mentioned above, to make the transition from arithmetical to algebraic thinking smoother. With reference to the current research study, many of these Grade 6 learners lack the ability to comprehend mathematical concepts and display an elementary style of setting out operations. Many learners do not possess sufficient conceptual and procedural knowledge to approach their number sentences. They rely on elementary skills to solve Grade 6 equations. Consequently, they

cannot formulate, represent their thoughts in a logical manner or provide a clear justification for their thought processes.

5.5 STRENGTHS AND POTENTIAL LIMITATIONS OF THIS RESEARCH

The first limitation experienced in this study was that I chose to use the school that I teach at as the site for my research. As the Grade 7 mathematics teacher, the Grade 1- 6 teachers were apprehensive about allowing me to investigate their SCK.

“Research represents a shared space, shaped by both researcher and participants. Therefore, the identities of both researcher and participants have the potential to impact the research process. Identities come into play via our perceptions, not only of others, but of the ways in which we expect others will perceive us. Our own biases shape the research process, serving as checkpoints along the way.”

Bourke (2014:1).

Through recognition of our biases, we gain insight into how we approach a research setting, members of particular groups, and how we seek to engage with participants. “Within positionality theory, it is acknowledged that people have multiple overlapping identities. Thus, people make meaning from various aspects of their identity....” (Kezar, 2002:96).

Foundation phase teachers shared answers in the questionnaire. Certain dominant teachers spoke during focus group interviews. New teachers or those that appeared uncertain about a question limited their contributions. One Grade 4 teacher did not hand in her questionnaire and one Grade 1 teacher was absent from focus group interviews.

Inexperience as a researcher limited how I interacted during my focus group discussions. I should have probed certain important issues more. For example, in hindsight, I realise that I could have questioned the teachers on why so many of them left out the section on preventing, reducing and promoting learners’ misconceptions of the equal sign.

A strength of the research was that the dropout rate of teachers and learners during this study was low: only two incidents, mentioned in Chapter 3, occurred with regards to the teachers. The co-operation and willingness of all participants to return questionnaires and consent forms was a strength in this study.

I am, however, concerned about the validity of the questionnaires among the foundation phase teachers: information was shared on two occasions that I am aware of. Questions that were not related to the grade that they teach in were not answered. It appears that

Grade partners shared information: some answers seemed the same. This was more evident among the foundation phase grade partners. Figure 5.3 is an illustration of information shared among certain grade partners.

2.3. Instruction given to learner: Fill in the correct answer in the place of the triangle: $7 + 2 = \Delta + 4$

Learner's response: $\Delta = 9$

Error/ Misconception	Error
Possible cause	Has not read the whole question, did first step first
How to prevent	Teach the the value of both sides =
How to reduce	

Teacher 3A

2.3. Instruction given to learner: Fill in the correct answer in the place of the triangle: $7 + 2 = \Delta + 4$

Learner's response: $\Delta = 9$

Error/ Misconception	Error
Possible cause	Learner doing 1st operation and writing down answer before reading whole sum
How to prevent	Teach learner to check the value of both sides of =
How to reduce	

Teacher 3B

Figure 5.5: Grade partners shared information

The intention of the learners' test was to determine what misconceptions the learners have of the equal sign and why. From the analysis of the test results certain questions did not serve the intended purpose; however they revealed other concerns among the Grade 6 group of learners.

Question 9 required learners to respond by stating true or false to the following statement: $49 = 7^2$: most Grade 6 learners responded 'false' to this question. From the analysis, it was clear that learners did not know the meaning of "square". For 7^2 in most cases they interpreted it as 7×2 , indicating that 49 is not equal to 7^2 . As a result, this question was not included in the results. Question 12 ($5 \times 7 + 49 \div 7$) was not included in the results of this study. Question 12 required learners to calculate an operation using order of operations. The researcher needed to know how the equal sign is used during setting out and whether learners would set it out in a string setting. It appears that the Grade 6 learners struggled because they did not apply the order of operations correctly and therefore calculated the

answer incorrectly. They approached the number sentence from left to right. Question 14 was an addition of mixed numbers operation; requiring learners to show their calculations, to see how the equal sign is used by learners. This question revealed that learners were not proficient when working with fractions. It did not indicate an inappropriate use of the equal sign nor was it included in the results of this study.

5.6 CONCLUSION AND FINAL THOUGHTS

In spite of the fact that this school is a high-performing academic school, the results discussed in Chapter 4 indicate that few learners have a flexible operational or basic relational view of the equal sign. The same group of learners that struggle with closure seem to struggle with the misconception of using all the numbers in an equation to solve a particular equation. The majority of Grade 6 learners cannot define the equal sign correctly. According to the results, the nature of Grade 1-6 teachers' SCK of the equal sign shows that teachers lack skills to correct, prevent or reduce misconceptions of the equal sign. The aim of this research was to ascertain whether there is a link between teacher's SCK and learner's misconception of the equal sign but as discussed in Chapter 4, other concerns emerged and teachers at this particular school need to develop CCK, SCK and mathematical knowledge at the horizon.

Nowhere in the CAPS document for Mathematics Grade 1-7, the curriculum document used in South Africa, is time set aside to teach understanding of mathematical symbols, in particular the equal sign. It is taken for granted that understanding will develop along the way. Learners' misconceptions are not used as a platform to promote learning. Time and resources are not made available for teachers to do diagnostic marking and deal with learners' misconceptions. Most of the time, these learners are lost in the system of South African education. Recommendations to curriculum developers are that diagnostic competence in the domain of mathematical knowledge needs to be developed in the primary classroom. The equal sign is only one example among many others which are crucial for classrooms that aim to facilitate understanding instead of rote learning (Prediger, 2009:90). This study can be used for future research, resulting in effectively improving teachers' SCK, as well as in the amendment of the Curriculum and Assessment Policy Statement (CAPS) document. This study will contribute to the body of knowledge by publishing it in peer-reviewed journals and made available to policy makers and the Department of Basic Education (DBE).

Finally, teachers are reminded by Leavy et al. (2013:252) that developing relational understanding of equality in primary school is not easy. Suggested methods mentioned in 5.4.1 should be sequential in nature, building on learners' prior knowledge of the concept of equality. To do this successfully, teachers need to possess mathematical knowledge at the horizon, SCK and CCK. Teachers should be able to identify gaps in learners' prior knowledge. They should be able to adapt their teaching methods when identifying a misconception regarding the equal sign in their learners' understanding, and put strategies in place to help learners overcome this misconception.

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APPENDIX 1: WCED ETHICS CLEARANCE LETTER



**Western Cape
Government**
Education

Directorate: Research

Audrey.wyngaard@westerncape.gov.za

tel: +27 021 467 9272

Fax: 0865902282

Private Bag x9114, Cape Town, 8000

wc.ed.wcape.gov.za

REFERENCE: 20140730-33841

ENQUIRIES: Dr A T Wyngaard

Mrs Bronwin Meyer
4 Utrecht Street
Monte Vista
7460

Dear Mrs Bronwin Meyer

RESEARCH PROPOSAL: THE EQUAL SIGN: TEACHERS' SPECIALISED CONTENT KNOWLEDGE AND LEARNER'S MISCONCEPTIONS

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **17 January 2015 till 30 June 2015**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

**The Director: Research Services
Western Cape Education Department
Private Bag X9114
CAPE TOWN
8000**

We wish you success in your research.

Kind regards.

Signed: Dr Audrey T Wyngaard

Directorate: Research

DATE: 31 July 2014

Lower Parliament Street, Cape Town, 8001
tel: +27 21 467 9272 fax: 0865902282
Safe Schools: 0800 45 46 47

Private Bag X9114, Cape Town, 8000
Employment and salary enquiries: 0861 92 33 22
www.westerncape.gov.za

APPENDIX 2: CPUT ETHICS CLEARANCE LETTER



APPENDIX II: FACULTY OF EDUCATION ETHICS FOR ORIGINAL RESEARCH

This form is to be completed by the student, member of staff and other researchers intending to undertake research in the Faculty. It is to be completed for any piece of research the aim of which is to make an original contribution to the public body of knowledge.

For students this type of work will also have educational goals and will be linked to gaining credit - it is the type of work that will be the basis for a Masters/Doctoral thesis or any research project for which ethical clearance is deemed necessary:

Name(s) of applicant	Bronwin Meyer
Project Title	The equal sign: teachers' specialised content knowledge and learners' misconceptions.
Is this a staff research project?	No
Degree	M Ed in Mathematical Education
Supervisor(s)	Prof C. Vermeulen
Funding sources	N/A

Attached: Information sheet ☐ Consent form ☒ Questionnaire ☐ Other (Specify)

Summary

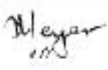
(i) Participants (teachers and learners) will be from the school that I am teaching at. 50% of Gr 1 – 6 teachers and 50% of Grade 6 learners will be randomly selected. They will be under no obligation to participate.

(ii) I will discuss the research beforehand with teachers and learners. Learners will only do a small test (for data collection) which will be administered to them by their class teachers. I will be present to explain to learners once more what the purpose of the test (and research) is, and that they are under no obligation to take part. Teachers will complete a questionnaire, take part in focus group interviews, and a maximum of three lessons per teacher will be observed. Before each of these episodes I will explain to the teachers the purpose of the research, and the specific purpose of the current data collection activity, and that they are free to withdraw at any stage.

- (iii) When introducing the research project to the participants, as well as before each data collection activity, I will inform participants that they are under no obligation to participate, that participation is voluntary, and that they may withdraw at any stage.
- (iv) Learners' test is attached.
- (v) There will be no risk of harm to participants or their community, to the researcher or research community, or to the university.
- (vi) Learner tests will be answered anonymously. Teacher questionnaires will have teachers' names on, in order to guide and focus my observations of teachers as well as the focus group interviews. However, no names will be mentioned in the research report. The name of the school where the research will be taking place will not be disclosed either.
- (vii) Although I am a teacher at the school where the research will take place, there will be no conflict of interest. My class (Grade 7) will not be involved, and in addition, the findings of the study will be to the advantage of the entire school, and not only for me.
- (viii) The findings will be shared with the staff of the school during staff development sessions, and if measures need to be taken to address problems that emanate from the research, proper planning will be done to deal with these.
- (ix) To the best of my knowledge, no other ethical issues will be raised by this research.
- (x) I have been a professional teacher for 18 years, and should be able to identify and deal with any ethical issues that may arise during this research. If in doubt, I will discuss the matter(s) at hand with my principal and/or supervisor.

Research Checklist:		Yes	No
1:	Does the study involve participants who are unable to give informed consent? Examples include children, people with learning disabilities, or your own students. Animals?		X
2:	Will the study require the co-operation of a gatekeeper for initial access to the groups or individuals to be recruited? Examples include students at school, members of self-help groups, residents of nursing homes — anyone who is under the legal care of another.	X	
3:	Will it be necessary for participants to participate in the study without their knowledge and consent at the time — for example, covert observation of people in non-public places?		X
4:	Will the study with the research subject involve discussion of sensitive topics? Examples would include questions on sexual activity or drug use.		X
5:	Will the study involve invasive, intrusive, or potentially harmful procedures of any kind (e.g. drugs, placebos or other substances to be administered to the study participants)?		X
6:	Will the study involve prolonged or repetitive testing on sentient subjects?		X
7:	Will financial inducements (other than reasonable expenses and compensation for time) be offered to participants?		X
8:	Does your research involve environmental studies which could be contentious or use materials or processes that could damage the environment? Particularly the outcome of your research?		X

Signatures:

Researcher/Applicant:	 Bronwin Colleen Meyer	Supervisor/Senior investigator (if applicable):  Prof C. Vermeulen
Date: 24 June 2014		Date: 24 June 2014

Please note that in signing this form, supervisors are indicating that they are satisfied that the ethical issues raised by this work have been adequately identified and that the proposal includes appropriate plans for their effective management.

Education Faculty Ethics Committee comments:

EFEC unconditionally grants ethical clearance for the study titled, "The equal sign: teachers' specialised content knowledge and learners' misconceptions". The certificate is				
Valid for 3 years from the date of issue.				
Approved			 Chairperson: Cina P Mosito, PhD	Date: 5/8/2014
Approval Certificate/Reference: EFEC 19-8/2014				

APPENDIX 3: Consent Letter: Principal



4 Utrecht Street
Monte Vista
7460
23 February 2015
Email: bromey@phps.org.za
Cell no.0728700846

The Headmaster and SGB (XXX Primary School)
Rhone Street
Pinelands
7760

Dear Sir

Request for permission to conduct my research at Pinehurst Primary

I am currently conducting research for a Masters degree in Education at CPUT (Cape Peninsula University of Technology) in the Faculty of Education. The title of my research study is: **"The equal sign: teachers' specialised content knowledge and learners' misconceptions."**

Permission has been granted by the Western Cape Education Department (WCED) and the ethics committee at CPUT. The research study will be conducted from March to May 2015. The study will include a teachers' questionnaire, a learners' test and two focus group interviews: one with the foundation phase teachers and one with the intermediate phase teachers as well as observations of teachers' lessons in

Mathematics. I hope that you will allow me to audio record, both focus group interviews and lesson observations. I will need to transcribe the information gathered for further analysis. Pseudonyms will be used to ensure the confidentiality and anonymity of the participants and the school. The audio recording is solely for the research purpose of this study. In this research-based study confidentiality of all information obtained will be a priority and at no time will any information be disclosed to a third party.

I request your permission to conduct the research at your school. If you grant permission, please complete the letter of consent, confirming your school's participation in this research study. Once completed, the research study will be available for you to view.

Should you wish to speak to me directly I am available at the following no. 0728700846.

Yours sincerely

Bronwin Meyer

APPENDIX 4: Consent Letter: Teacher



4 Utrecht Street
Monte Vista
7460
23 February 2015
Email: bromey@phps.org.za
Cell no.0728700846

Dear.....

I am currently conducting research for a Masters degree in Education at CPUT (Cape Peninsula University of Technology) in the Faculty of Education. The title of my research study is: **"The equal sign: teachers' specialised content knowledge and learners' misconceptions."**

Permission has been granted by the headmaster, Western Cape Education Department (WCED) and the ethics committee at CPUT. The research study will be conducted from March to May 2015. The study will include a teachers' questionnaire, a learners' test and two focus group interviews: one with the foundation phase teachers and one with the intermediate phase teachers as well as observations of teachers' lessons in Mathematics. Since the above will be used in my study I am hoping what you will allow me to audio record both focus group interviews and lesson observations. I will need to transcribe the information gathered for further analysis. Pseudonyms will be used to ensure the confidentiality and anonymity of the participants and the school. The audio recordings are solely for the research purpose of this study. In this research-based study confidentiality of all information obtained will be a priority and at no time will any information be disclosed

Consent Letter: Bronwin Meyer

Page 3 | 8

to a third party. Please be assured that you can withdraw consent at any time by contacting me via email or at school (tel: 021 531 2783). If consent is withdrawn none of your contributions will be included in my study.

I hereby request your permission to participate in the research study. If you grant permission, please complete the letter of consent, confirming your participation in this research study. Once completed, the research study will be available for you to view.

Should you wish to speak to me directly I am available at the following no. 0728700846.

Yours sincerely

Bronwin Meyer

APPENDIX 5: Consent Letter: Learners



4 Utrecht Street
Monte Vista
7460
23 February 2015
Email: bromey@phps.org.za
Cell no.0728700846

Dear Parents

Learner:

I am currently conducting research for a Masters degree in Education at CPUT (Cape Peninsula University of Technology) in the Faculty of Education. The title of my research study is: **"The equal sign: teachers' specialised content knowledge and learners' misconceptions."**

Permission has been granted by the headmaster, Western Cape Education Department (WCED) and the ethics committee at CPUT. The research study will be conducted from March to May 2015. The study will include a teachers' questionnaire, a learners' test and interviews with six learners from each Grade 6 class, two focus group interviews: one with the foundation phase teachers and one with the intermediate teachers as well as observations of teachers' lessons in Mathematics. Participation in this study is voluntary. Your child's comments, work or test will only be used with your consent.

Since the above will be used in my study, I am hoping what you will allow me to audio record lesson observations as I will need to transcribe the information gathered for further analysis. Pseudonyms will be used to ensure your confidentiality and anonymity of the participants and the school. The audio recordings are solely for the research purpose of this study. In this research-based study, confidentiality of all information obtained will be a priority and at no time will any information be disclosed to a third party. Please be assured that you can withdraw consent at any time by contacting me via e-mail or at school (tel: 021 531 2783). If consent is withdrawn none of your child's contribution will be included in my study.

I hereby request your permission to allow your child to participate in the research study. If you grant permission, please complete the letter of consent, confirming his/her participation in this research study. Once completed, the research study will be available for you to view.

Should you wish to speak to me directly I am available at the following no. 0728700846.

Yours sincerely

Bronwin Meyer

Consent slip

Ihereby give my son/daughter

in Gradepermission to participate in the research study conducted by Mrs Meyer.

Date:

Signature: Name:

APPENDIX 6: Learners' Test

Grade 6 Learners' Test



A. Fill in the correct answer in the place of the triangle

1. $7 + 2 = \Delta + 4.$

2. $9 + 3 = \Delta - 4$

3. $26 = \Delta + 5$

4. $6 = \Delta \div 4$

5. $9 \times 6 = \Delta \div 2$

B. State whether the following are TRUE or FALSE. If false state why:

Operation	True or False	Explanation
6. $4 + 5 = 8 + 1$		
7. $6 - 3 = 7 - 4$		
8. $8 = 10 - 2$		
9. $49 = 7^2$		
10. $6 \times 5 = 120 \div 4$		

C. Calculate each of the following, showing all working out:

11. $(8 \times 2) + 5$

12. $5 \times 7 + 49 \div 7$

13. $2^2 + 5$

14. $5\frac{3}{4} - 2\frac{1}{2}$

Which number makes each statement true?

15.1. $5 + 6 = \square + 3$

--

15.2. $13 - 7 = \square - 5$

--

15.3. $12 + 9 = \square - 17$

--

Are these statements correct? How do you know?

16.1. $15 = 15$

--

16.2. $10 = 5 \times 2$

--

16.3. $4 + 8 = 12 - 6 = 6$

--

16.4. $10 + 4 = 9 + 5$

--

Interpreting the equal sign

17. The following questions are about this statement

$$\begin{array}{c} 3 + 4 = 7 \\ \uparrow \end{array}$$

17.1. The arrow above points to a symbol. What is the name of the symbol?

--

17.2. What does the symbol mean?

--

17.3. Can the symbol mean anything else? If yes, please explain.

--

18. Alan started a problem with a one-digit number. He multiplied the number by 3, added 8, divided by 2 and subtracted 6, and got the same number he started with. What was the number Alan started with? (Show all working out)

- a. 2
- b. 4
- c. 6
- d. 8

APPENDIX 7: Teachers' Questionnaires

Teachers' Questionnaire



1. You are the teacher of a learner who worked on a problem and got an answer.
- 1.1 Please choose the best option from A, B or C. If you chose B or C give a reason for your answer.

Splendiferous bought 4 apples and 3 oranges. She ate 1 apple and squeezed 2 oranges for juice. How many pieces of fruit does she have left?

$$4 + 3 = 7 - 1 = 6 - 2 = 4$$

- A) The learner's answer is correct
- B) The learner's answer is incorrect
- C) The answer is correct but the working is incorrect.
- 1.2 Your reason for choosing B or C

- 1.3 Now:
- Explain what you think was the learner's problem solving process.

2. In each of the following scenarios determine whether the learner's response contains an error/misconception. If so, explain the possible cause and how to prevent or how to reduce the misconception.

2.1. **Instruction given to learner:** Half the numbers 28 and 32

Learner's response: $28 = 14$

$$32 = 16$$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.2. **Instruction given to learner:** Decompose the number 11.

Learner's response: $11 \text{ — } 10 + 1$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.3. **Instruction given to learner:** Fill in the correct answer in the place of the triangle:
 $7 + 2 = \Delta + 4$

Learner's response: $\Delta = 9$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.4. **Instruction given to learner:** calculate $2 \times 3 \times 8 \div 4$

Learner's response:

$$(2 \times 3) \times (8 \div 4)$$

$$= 2 \times 3 = 6 \times (8 \div 4)$$

$$= 6 \times (8 \div 4) = 2$$

$$= 6 \times 2 = 12$$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.5. **Instruction given to learner:** What does the following symbol mean (=)?

Learners' response: Answer of the sum is.

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.6. **Instruction given to learner:** Choose the correct symbol (< ; > ; =)

$$3 + 5 \times 2 \dots\dots\dots 16$$

Learner's response: $3 + 5 \times 2 = 16$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.7. **Instruction given to learner:** Fill in the correct answer in the place of the Δ :

$$9 + 3 = \Delta - 4$$

Learner's response: $9 + 3 = 12 - 4 = 8$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.8. **Instruction given to learner:** Calculate: $3 \times 9 - 5 \div 11$

Learner's response: $= 3 \times 9 - 5 \div 11 = 27 - 5 \div 11$

$$= 27 - 5 = 22 \div 11 = 2$$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.9 **Instruction given to learner:** Calculate: 10% of 240

Learner's response: $10 \div 240 = 24$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.10. **Instruction given to learner:** Calculate: $\frac{5}{9} + \frac{7}{9}$

Learner's response:

$$\frac{5}{9} + \frac{7}{9}$$

$$= 5 + 7 = \frac{12}{9}$$

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.11. **Instruction given to learner:** Give the Multiples of 3

Learner's response: 3 = 3; 6; 9;12;15

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.12. **Instruction given to learner:** State whether the following is true or false

$$2 \times 8 = 16 + 1$$

Learner's response: True

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.13. **Instruction given to learner:** State whether the following is true or false

$$25 - 5 = 17 + 3$$

Learner's response: True

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.14. **Instruction given to learner:** Complete the number sentence. $2 + \square = 5$

Learner's response: He writes 7 in the empty box

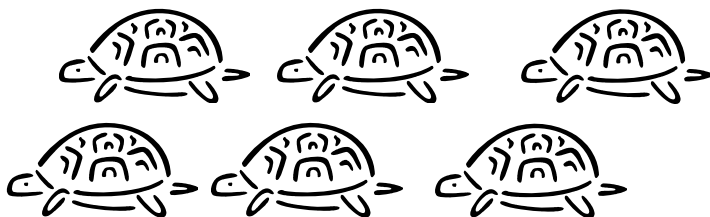
Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.15. **Instruction given to learner:** How old is John?

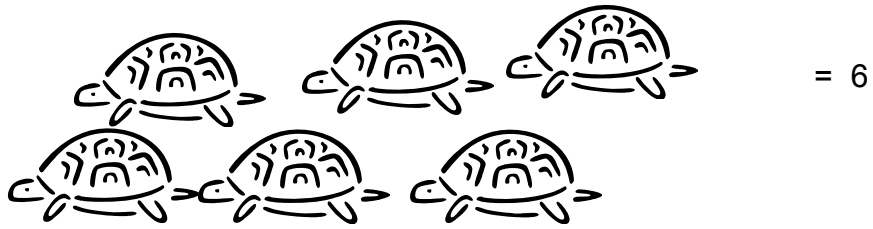
Learners' response: John = 8.

Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.16. **Instruction given to learner:** How many tortoises are in the picture?



Learner's response:

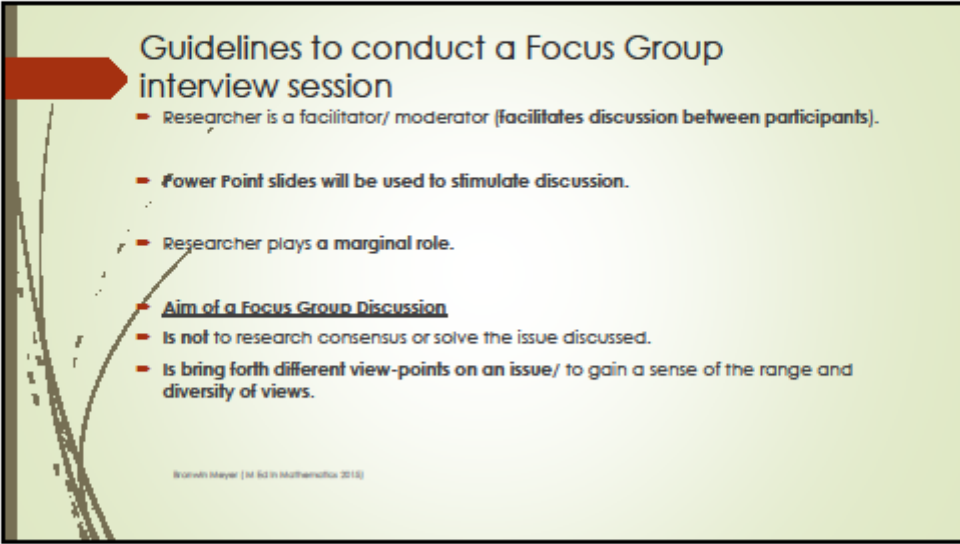


Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	

2.17. Instruction given to learner: calculate $20 + 30 + 7 + 8$

Learners' response: $20 + 30 = 50 + 7 = 57 + 8 = 65$

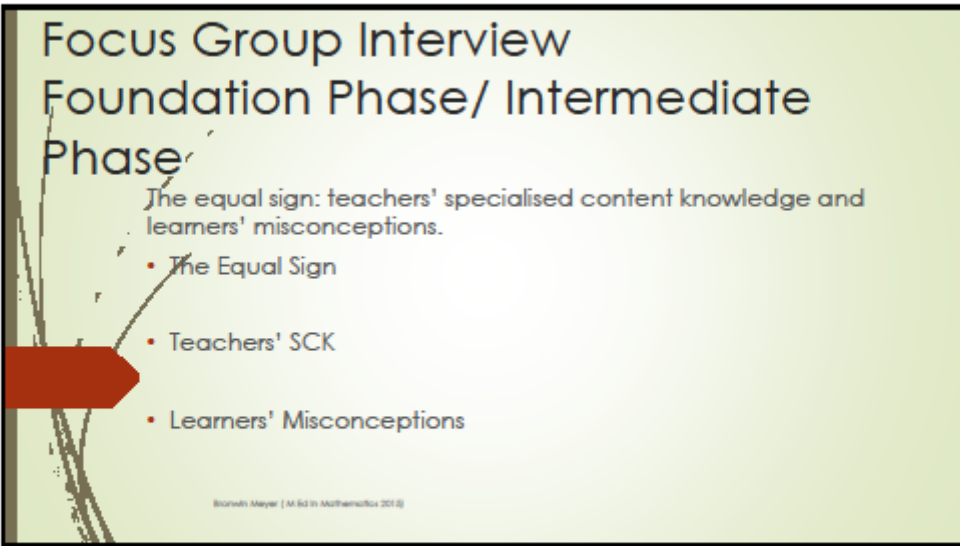
Error/ Misconception	
Possible cause	
How to prevent	
How to reduce	



Guidelines to conduct a Focus Group interview session

- Researcher is a facilitator/ moderator (facilitates discussion between participants).
- Power Point slides will be used to stimulate discussion.
- Researcher plays a marginal role.
- Aim of a Focus Group Discussion
 - Is not to research consensus or solve the issue discussed.
 - Is bring forth different view-points on an issue/ to gain a sense of the range and diversity of views.

Brunetti Meyer (M Ed in Mathematics 2010)



**Focus Group Interview
Foundation Phase/ Intermediate
Phase**

The equal sign: teachers' specialised content knowledge and learners' misconceptions.

- The Equal Sign
- Teachers' SCK
- Learners' Misconceptions

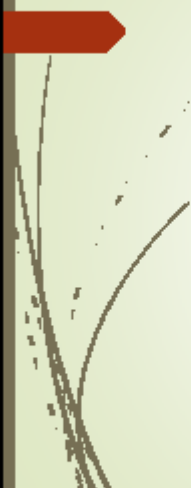
Brunetti Meyer (M Ed in Mathematics 2010)



Equal Sign

- What are the different meanings of the equal sign?
- In how many different ways can the equal sign be used in different exercises? Give some examples.
- Have you observed your learners using the equal sign?
- In what way(s)?
- How have they understood?
- Have you noticed anything worth mentioning?

Brown Meyer | M Ed in Mathematics 2016



Equal Sign

- What do you think your students will do with: $8 + 4 = ___ + 5$?
- What do your learners do if the equal sign is in the middle of a number sentence?
- If you noticed a common learner error, and have talked to the class about it, and corrected it with them, can you give an indication (approx. % of the class) of how many do it correctly from then onwards?
- Do you ask your learners to write their own number sentences? If so, what sorts of sentences do they use?

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Teachers Questionnaire

Maths Test

Half

$28 = 4 \checkmark$ $6 + 7 = 13 \checkmark$ $16 + 9 = 24 \checkmark$
 $32 = 15 \checkmark$
 $16 = 8 \checkmark$ $13 - 5 = 8 \checkmark$
 $24 = 12 \checkmark$
 $10 = 5 \checkmark$ $8 + 4 = 12 \checkmark$

Halving odd num

$1 = \frac{1}{2} \checkmark$ $11 = 5\frac{1}{2} \checkmark$
 $3 = 1\frac{1}{2} \checkmark$ $13 = 6\frac{1}{2} \checkmark$
 $5 = 2\frac{1}{2} \checkmark$ $15 = 7\frac{1}{2} \checkmark$ $\frac{10}{10}$
 $7 = 3\frac{1}{2} \checkmark$ $17 = 8\frac{1}{2} \checkmark$
 $9 = 4\frac{1}{2} \checkmark$ $19 = 9\frac{1}{2} \checkmark$

Teachers Questionnaire

Instruction given to learner: How many tortoises are in the picture?



Learner's response:



= 6

Brownell Meyer / M Ed in Mathematics 2012 ©

From Teachers Questionnaire

Instruction given to learner:

How old is John?

Learners' response: John = 8.

Brownell Meyer / M Ed in Mathematics 2012 ©



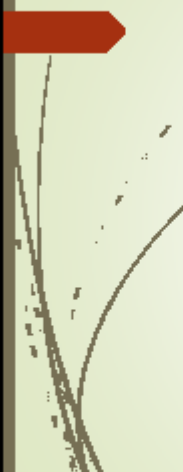
In Grade 6 Learners' Test

- Instruction given to learner:
/ Fill in the correct answer in the place of the triangle: $7 + 2 = \Delta + 4$

Learner's response: $\Delta = 9$

TEACHERS RESPONSE – this is an error

Brown Moyer (M Ed in Mathematics 2012)



From Teachers Questionnaire

- Instruction given to learner: Complete the number sentence. $2 + \square = 5$
- Learner's response: He writes 7 in the empty box

Brown Moyer (M Ed in Mathematics 2012)

Teachers Questionnaire

Skill 5: Incorrect Use of Equal Sign

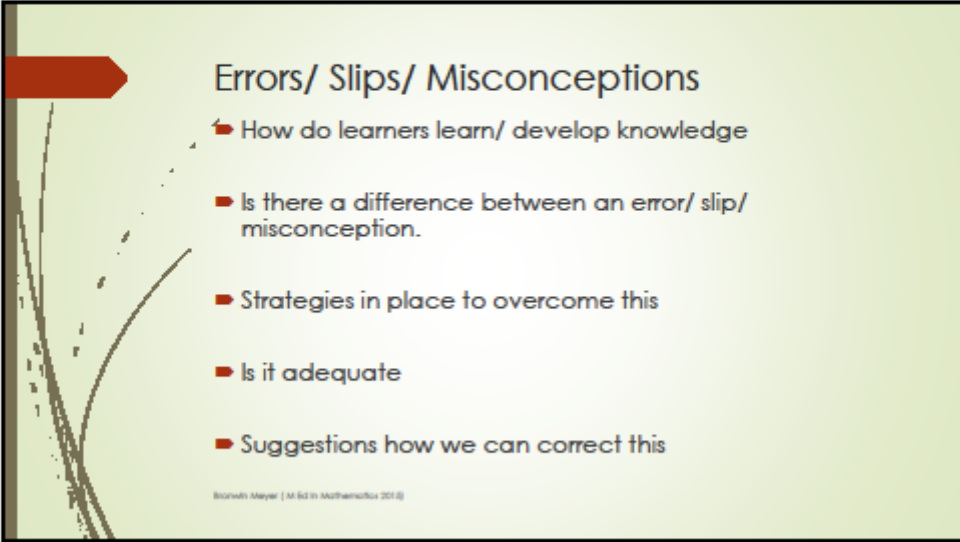
1. $\frac{5}{9} + \frac{7}{9}$ $= 5 + 7 = \frac{12}{9}$	2. $\frac{5}{7} + \frac{6}{7}$ $= 5 + 6 = \frac{11}{7} (1\frac{4}{7})$
3. $\frac{11}{13} + \frac{5}{13}$ $= 11 + 5 = \frac{16}{13} (1\frac{3}{13})$	4. $\frac{1}{7} + \frac{6}{7}$ $= 1 + 6 = \frac{7}{7} (1 \text{ whole})$

Brown Meyer | M Ed in Mathematics 2016

From Teachers Questionnaire

- Instruction given to learner: Fill in the correct answer in the place of the Δ :
- $9 + 3 = \Delta - 4$
- Learner's response: $9 + 3 = 12 - 4 = 8$

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Errors/ Slips/ Misconceptions

- How do learners learn/ develop knowledge
- Is there a difference between an error/ slip/ misconception.
- Strategies in place to overcome this
- Is it adequate
- Suggestions how we can correct this

Brown Moyer (M Ed in Mathematics 2016)

APPENDIX 9: LEARNERS INTERVIEWS

Learner 1: - 27 March 2015 (13:18)

- R: Ok Good afternoon we are going to look at your test that you wrote for me, we are going to have a look at all the one that you got incorrect and I want you to explain to me how you made the error ok.
- R: So the first error that I see on your page is number 9: and Number 9 says: $49=7^2$. Why did you say false?
- K: Well, I though 7 to the power two means 7×2
- R: Which gives you?
- K: 14
- R: And that is why you said it's not true. Thank you for that.
- R: Let's go to section C. And Section C says calculate each of the following showing all working out. Now number 11: says $(8 \times 2) + 5$ and your answer is correct but you're setting out is not correct so just explain you setting out to me. Why were you setting your sum out like that?
- K: well I did the BODMAS first then plus the 5.
- R: Who showed you that setting out?
- K: I forgot how to set it out properly
- R: I just want to know where in school has this setting out been ok. Grade 4 Grade 5. Where has it been fine?
- K: Nowhere.
- R: **Let's look at the next one number 12. $5 \times 7 + 49 \div 7$. How did you get to 84?**
- K: **Well I thought of BODMAS and 5×7 is 35 then I added 49 and divided by 7**
- R: **Did you divided it by 7**
- K: **I must have forgot.**
- R: BODMAS says: Brackets of division multiplication addition and subtraction Should you not have said 5×7 is 35; $49 \div 7$ is 7; and should you not have added the two together to get an answer. Do you understand?
- R: Let's go to the next one. Why did you not do number 13

-
- K: Because aham, well I knew on the first page it's probably not right I wasn't sure.
- R: What answer was not right?
- K: That true or False, I knew I wasn't sure how to do that
- R: So you did not know the part 2 to the power. Ok
- R: now over here is your fraction question which is number 14. And it says $5\frac{3}{4} - 2\frac{1}{2}$. Explain to me that is happening over here.
- K: Ok, well I took $5 - 2 = 3$ and the 3 over 4 and I converted the half to quarters. And I know a half is the same as two quarters. I wanted the denominator to be the same. 3 over 4 minus 2 over 4 is 1 over 4 then I added the 3 to the quarter.
- R: Again your answer is right but you're setting out is horizontally. Where did you set out your sums like this? Why is it easier for you to set out your sums like this?
- K: ah, I think I am still thinking like in Grade 5. The method we taught in Grade 5
- R: so you were allowed to write your sums out horizontally like this in Grade 5.
- K: Sometimes
- R: **Let's move on. Let's look at 16.3. You did not give me an answer there. Why is there no answer?**
- K: **I knew $4 + 8 = 12$ but I was not sure explain it properly. $4 + 8 = 12 - 6 = 6$. But I was not sure how to explain it**
- R: **if you look at it now that horizontal statement now do you think it's true or false.**
- K: **I think it's true**
- R: so you think 16.3 is true. Ok
- R: now look at 17.2. It says what does the symbol means? And you said the name of the symbol is equal to sign. What does it mean to you?
- K: **$3 + 4 = 7$ so $3 + 4$ is the same at 7.**
- R: **Does the equal to sign always just give you an answer.**
- K: **not always**
- K: **it can be $3 + 4 = 4 + 3$**
- R: **Thank you so much for helping me with the analysing of your paper.**
-

Learner 2: - 31 March 2015 (09:06)

R: 1. $7 + 2 = \Delta + 4$.

Ok, Good morning Paige, we are going to have a look at your test on the Equal to sign that we did two weeks ago and or a week ago and I want you to help me understand how you got to certain of your answers. Ok so ok, if we look at no. 1 your answer was 3. How did you get to 3? Explain to me how did you get to 3.

P: So $7 + 2$, oh, I made a silly mistake.

R: Ok but, tell me what your mistake was.

P: My mistake, so $7+2$ is 9, and I made a mistake because I said $3+4=7$ and not 9.

R: So you you said $3 + 4$ is equal to 7 ok.

R: 2. $9 + 3 = \Delta - 4$

And over here for number 2, you got 13. How did you get to 13?

P: $9 + 3$ is 12, and $13 - 4$ is 9

R: Ok so again you said, $13 - 4$ equal 9 and $3+4$ is 7. Ok I understand what you did there. 13. Sorry

R: I am going to go back to number 1. $4+3$ is equal to 7 that's where you got the 3 from and $13 - 4 = 9$. That's what you did. Ok let's go to no. 5. How did you get to 90 for no. 5?

5. $9 \times 6 = \Delta \div 2$

P: 9×6 is 54 and 90 divided 2 is ...

R: O, you were under the impression that 90 divided by 2 would give you 54. Ok. Let's turn the page around. Ok, are you from our school or did you come from another school.

P: I came from another school.

R: Which school did you come from?

P: Gene Louw Primary.

R: Gene Louw Primary ok. Just have a look at no. 11

11. $(8 \times 2) + 5$.

You are setting out your sum. You are setting the right answer but do you think that your setting out is right because you are doing 8×2 as one part of your sum then you are adding your 5 on. So in my thinking you haven't progressed to a linear form of setting out. So are you happy with your setting out?

P: Yes

R: Ok, and is this how you did it at your previous school.

P: Yes, we were taught for a little sum like this in the exam or like a test we always had to write it out. That's how we got our full marks.

R: Ok, this is also a BODMAS sum no. 12.

12. $5 \times 7 + 49 \div 7$

Is said 5 times 7 plus 49 minus 7. And you got an incorrect answer using your method, ok. Just help me go through that steps to get to that incorrect answer.

P: ok, so must I just get to

R: Explain to me how you got to that answer.

P: Ok, so 5 times 7 is 35

R: That part of it is correct.

P: And then I got 49 divided by 7 which is 7 and then I plus them and I got to hundred and twelve.

R: 35 plus 7 is a hundred and twelve so it's an addition error.

R: Ok my understanding is it's an addition error and a setting out error because it's not set out in a linear form.

P: Ok

R: Vertical form. Have a look at this one here. That you go right that you got right we are not worried about the ones you got right.

R: 16.3 you said yes for that equation. Why did you say yes?

16.3. $4 + 8 = 12 - 6 = 6$

P: Because 4 plus 8 equals to 12 and 12 minus six equals to six

R: so you are fine with that horizontal setting out. According to you that's right

P: Yes

R: Ok, ah. Let's look at 17.2 wants to know what the equal to sign means to you. And you said the equal to sign means that it is equal to something else or the

sum before the equal to sign, meaning that the equal to sign is telling us the answer. This that the only, only definition you have for the equal to sign.

17.2. What does the symbol mean?

P: Yes

R: ok so it means nothing else to you

P: It means it's equal to something.

R: Ok, so the one part is equal to another something, it will give you an answer. Ok and you this one correct. Ah, thank you so much for your time and for just explaining to me your thinking. I really appreciate it.

Learner 3: - 27 March 2015 (13:18)

R: Ok Good afternoon we are going to go through the test that we wrote on the equal to sign and I just want you to explain to me every question as I go along we are going to start at number 3. So just tell me how did you get to 11

H: Amah. I thought what equal 26 so I minused the 5 from the 26.

R: And that is how you got what?

H: now that I look at it I get 19.

R: Ok so you made a subtraction error is that right.

R: Now if you look at number 4. It says $6 = \Delta \div 4$. How did you get 2?

H: amah I think I times it 6 times 4.

R: So how much does 6×4 give you?

H: 24.

R: Ok so it's not 2.

R: What does it mean when you see $26 = () + 5$. What do you understand in your mind when you see an equation like that?

H: I see 26 as a number and two or three other numbers that equal 26

R: Ok, the same here by number 4. Explain that to me quickly.

H: 6 is a number and two or three or more numbers that equals is

R: Ok, so can we say at number 3 you made a calculation error,

H: Yes

R: and can we say for number 4 you also made a calculation error

H: yes

R: **Now if you look at number 12. It says. $5 \times 7 + 49 \div 7$. How did you get 12?**

H: **5×7 give me 35 and then plus the 49 divided by 7**

R: **can I understand that your order of your operations was wrong**

H: Yes.

R: your answer is right its 9 for number 13 but you did not show your calculations

H: I thought 2 to the power of 2 means you double it and I just plussed the 5 on

R: number 14 is a fraction sum it says $5\frac{3}{4} - 2\frac{1}{2}$. Explain your operation for me here.

H: so I made it into an improper fraction first

R: so how did you get $\frac{8}{4}$ from $5\frac{3}{4}$

H: I plussed the 5 with the 3

R: the 5 and the 3 gives you 8, ok, and how did you do this one here $2\frac{1}{2}$ is $\frac{3}{2}$

H: the same I plussed the 2 with the 1

R: ok, so this is also not understanding how to convert a mixed number to an improper fraction. What you did was wrong. In $5\frac{3}{4}$ it is $5 \times 4 + 3$ which is $20 + 3$; and this gives you $\frac{23}{4}$ and for $2\frac{1}{2}$ We say 2×2 which is $4 + 1$ will give you $\frac{5}{2}$. A calculation error and not knowing how to convert a mixed number to an improper fraction.

R: Let's move on. Here for number 15.3 again we have $12 + 9 = \square - 17$ and you got 31. How did you get 31?

H: First I plussed the 12 with the 9 and then I got $21 + 17$

R: Which is 38 not 31. Ok is this a calculation error - actually closure

H: Let's look at 17.2. The question states (What does the symbol mean?)

R: two number are equal to a specific number

R: does the equal to sign mean anything else to you

H: No

R: Have you hear of the word equivalent?

H: yes

R: what does equivalent mean to you?

H: two numbers are equal to one number

R: thank you so much for helping me understand your paper

Learner 4: - 27 March 2015 (12:55)

R: Ok Good afternoon, how are you doing?

S: Fine, thank you

R: We are going to look at your test and look at all the sums that you got incorrect and then I want you to explain to me how you got to your error or how you got to your answer. Let's start at number 5 that was your first error. The statement says. Fill in the correct answer in the place of the triangle and in the place of your triangle you had 27. So just tell me you're thinking to get to 27.

5. $9 \times 6 = \Delta \div 2$

S: well I said 9×6 is 54 and then I thought I could halve it, but then you say 27 is wrong

R: can I just repeat that, you say 9×5 is 54 and then you halve the 54 to get the 27. *Used all the numbers in the statement.*

R: Can you actual see where your error was?

S: Ahm. I don't know what the answer. I should have said 54×2

R: now you understand that you should have timed it instead of halving it.

R: Let's look at number 9. $49 = 7^2$ that was your next error. You said false. Why do you think 49 is not equal to 7^2 ?

S: well Aham, I did not exactly know 7^2 , I just though 7×2 is 14 so then I thought it was false.

R: Ok so you did not understand the concept of 7^2 . Ok. I just want to back to no. 5. That equal to sign between the 9×6 and the $\Delta \div 2$. What does that tell you about the LHS and the RHS? What do you understand now?

S: That both of the two sums has to equal to the same answer

R: But you did not understand it during the test

S: Ja, aham I think I was not aware and did not do it properly

R: Not a problem, let's go to this one over here. This is Section C. And it says Calculate each of the following showing all you're working out. All you're working out is correct. No. 11; 12; 13 and 14 is correct but your setting out is not vertical, your setting out is horizontal. Why do you set sums out horizontally?

S: Because aham I go to this place called Master Maths and they teach you how to do it horizontally and I just thought it was right.

R: So no one has ever should you the vertical setting out of a sum like that

S: No

R: Ok, Let's go to the next section and here you only have 16.3 wrong
Just have a look at 16.3. You said yes. The sum says 4 plus 8 equals 12
minus 6 equals 6. And you said yes. Why did you say yes?

16.3. $4 + 8 = 12 - 6 = 6$ *getting a final answer is important*

S: Well, Aham 4 + 8 is 12 and 12 minus 6 is 6. And I thought it was correct
because they said the answer.

R: Ok that setting out again is a horizontal setting out. Is this correct?

S: Aham, Yes

R: Now over here the question says interpret the following sign, and they have
Interpreting the equal sign

17. The following questions are about this statement

$3 + 4 = 7$



The arrow point to a symbol and you said equal. Is that the correct name?

S: Aham, you can also say equivalent or maybe the answer *incorrect definition*

R: For this one you said it means the answer. So whenever you see a sum like
that you assume that the answer follows the equal to sign

S: Aham, ja.

R: Is the any other way we can use the equal to sign?

S: Aham, well I have seen the sum where they say like a 100 then they do this
wobble equal to sign. I think that means rounding off. So I thought there was
no other meaning for it.

R: Ok, now just to get back over here you said another word for the equal to sign
is equivalent. I want to tell you now another answer for the equal to sign is
equivalence. Which actually means the LHS is equal to the RHS.

R: Let's quickly move on now over here you already said that you believe
horizontal setting out is correct. So according to you the setting out that you
have done here is correct. Your answer is right. But I want to know is your
setting out correct. This is no. 18.

S: Aham, yes, again I was taught like that, Ja. So I thought I could do it like that test *SCK- taught to use horizontal method*

R: Ok, since when has people taught you to set out your sums horizontally?

S: Aham, they, I don't know but they, taught me to do that.

R: You don't know which Grade?

S: No I think it was in Grade 5 last year

R: Ok, who was your teacher?

S: Ms Burr.

R: Your answer is correct but again over here your setting out is not so I am going to try and fix that when we get to class.

Thank you so much for coming to chat to me about your misconceptions.

Thank you.

Learner 5: - 31 March 2015 (08:56)

R: Ok, Good morning.

K: Morning

R: Ok fine, we are just going through the test that you wrote for me on the Equal to sign and I just want you explain to me there were there was an error, how you got to that error ok. So explain to me how you got 9 as an answer for number 1

1. $7 + 2 = \Delta + 4$.

K: Am See, I just plused the 7 plus the 2.

R: And that gave you 9. Ok. And that is called closure.

R: How did you get the 12 for number 2?

2. $9 + 3 = \Delta - 4$

12 as an answer for number 2.

K: I plused the 9 and the 3.

R: Ok, why did you leave out number 4?

4. $6 = \Delta \div 4$

K: Am aah, I think maybe because there was no plus maybe I was going to come back to that

R: Ok, so you did not understand at that point and you wanted to come back

K: Yes

R: Ok, and number 5

5. $9 \times 6 = \Delta \div 2$

K: No. 5. Am, I just had to times it but I think I wanted to do the same thing just to be sure that I am right

R: Ok and you forgot to come back

K: Yes

R: Look at number 9. Number 9 says 49 equals to 7 to the power 2. You said False. They did you say False?

9. $49 = 7^2$

K: False. Am

R: When you looked at that equation, what made you think it was false?

K: Am, Am for the 49, for the 2 times 7 which is 14, I thought it was wrong.

R: Ok so you thought that was 14.

K: Yes

R: **7^2** so 7 times 2 is 14

R: Ok look at number 13. What was your error there? What do you think? Ok there isn't actually and error it's just a setting out. How do you feel about the setting out over here? 2 to the power 2 is equal to 2 times 2 equal to 4. How do you feel about that horizontal setting out? Do you feel it is right? Or don't you think it is right?

13. $2^2 + 5$

K: Am I think it is right but maybe I had to just add the sum here and not break it up.

R: Ok, so you are fine with horizontal setting out.

K: Ja.

R: Ok, let's go to the next one, 15.2 you got the answer 20. For 13 minus 7 equals what minus 5. So why did you get 20?

15.2. $13 - 7 = \square - 5$

K: Am I don't.

R: Just think. Just have a little think there quickly. How did you get to the 20?

K: Ma, Am I think I just instead of a minus I plused maybe.

R: Ok Like a calculation error

K: Yes

R: Ok, look at 17.2. It refers to the equal to sign. What does this symbol mean? And you said it means that does the two digits add up to. Are you telling me it gives you the answer or what are you trying to tell me with that statement?

17.2. What does the symbol mean?

K: Am you have your two digits or your three-digit number sentence and then when you want to put the equal to sign it means you are going to add up that. Or you are going to times that

R: And what is it going to give you? If you add it or times it?

K: Am it will give you the answer of the two digits or three digits together.

R: Ok, so whenever you see an equal to sign are you expecting an answer?

K: Am, Ja

R: Ok, Thank you. Do you think that the equal to sign means anything else? Because you said. Can the symbol mean anything else? And you said no. So do you think the equal to sign can mean anything else but the answer?

K: I think it can mean that like 2 to the power of 2 like I did here. To break it up.

R: 2 time 2 then you get 4.

K: Yes

R: Ok if you look at number 18. Why did you not do it?

K: Am

R: Was the word sum too difficult for you. Did you not understand that? What was the reason you did not attempt number 18.

K: Am, I think I ran out of time maybe, or Ja I think I did not have enough time.

R: And you needed more time to think

K: Hap, I like to double check my work.

R: Ok Sure,

R: Thank you so much for giving me some of your time, I really appreciate it ok.

Learner 6: - 27 March 2015 (13:25)

R: Ok, Good afternoon.

T: Good afternoon Mrs M.

-
- R: We are going to look at the test that you did for me for my studies and we are just going to look at all the ones that were not answered correctly. Then you must just explain how you got to your answer. Ok
- R: So the first one that was not answered correctly was number 4. And number 4 says 6 equals triangle divided by 4. How did you get to 16? Explain to me how did you get to 16.
- 4. $6 = \Delta \div 4$**
- T: Because I have been thinking that you must count in 6's
- R: Count in 6's ok.
- T: Count in 6's to get 4 but I wasn't actual quite sure about the answer.
- R: Ok
- T: So I made a mistake by because I should have made it count in 4's
- R: But if you count in 4's you do get 16. If you count in 6's you won't get 16. So how did you get to 16?
- T: I know that I did not get to 16. So both of them actually equals 24.
- R: So why did you get 16.
- T: I got 16 because I have been thinking I was counting in 6's instead of 4's
- R: Ok, now let's look at number 9. No 9 says 49 equals 7 to the power 2. You said false. So they did you say false?
- 9. $49 = 7^2$**
- T: I said false, because, I counted in 7's and since I saw 2 I thought it would be 7 times 7 equals 14
- R: Do you think that 2 means 7 times 2
- T: Yes
- R: 7 times is 14 so they are not the same.
- T: Yes
- R: Ok, Section c you got correct and you set it out very nicely. Who showed you this method of setting out?
- T: O it was my dad
- R: Ok, did your dad show you to set it out like this
- T: When he visited in Cape Town
- R: okay look at number 14. No. 14 is a fraction sum and let just look at what you did here, according to me, you made an error because you added instead of

subtracting your conversion from a mixed number to a to an improper number is correct and your conversion but you just and error over here 5 and 3 to the power 4 should actually be **23 over 4 and not 44 over 4**. That is a calculation error.

R: **Look at no 16.3. You say that statement is right $4 + 8 = 12 - 6 = 6$. Why do you say its right?**

T: I said its right because I calculated, I plused 4 and 8 which makes 12 then I minuses the 6 from the 12 which is equal to 6.

R: So you think it's right

T: Yes I though it's right

R: So why do you now think it's not right anymore

T: Because you told me that its wrong

R: Ok. Now if you look at that sum, who sets out sum like that in school for you. Who set out sums horizontally for you? Who showed you to set out your sum like this?

T: Actually no one

R: Ok but you think its fine to set out sums like that

T: A Ja

R: Ok

R: Let's look at this one over here, here you say looking at the equation $3 + 4 = 7$ They wanted to know that is that and you said it's the equal to sign which is correct. **And what does this symbol mean to you .and you said it means the sum can into a number**

T: I was supposed to say can go into a number

R: What sum can go into a number?

T: Any sum can actually go into the number it is equal to.

R: Explain it using that numbers.

T: Basically $3 + 4$ which will make 7 but you need the equal sign to convert it the 3 and 4 into a 7

R: Ok so must you always have two number is equal to one number?

T: Ahom. Actually I think no because you don't really need two numbers you to become one number you could ahom you could add it into

-
- R: Ok you got for number 18. 6 Explain to me. The sum says.
Allen started a problem with one-digit number he multiplied the number by 3
added 8 divided by 2 and subtracted 6 and got the same answer he started
with. What was the number Allen started with? Show all you're working out.
How did you get to six?
- T: Well I time 3. I did BODMAS I times 3 by 8 hen divided then 2 minus 6. I first
started with the times. 3×8 was 24 divided by 6 then I would go to the minus
then it made 4. $24 \div 6 = 4$
- R: $2 - 6$ is 4
- T: Ja
- R: Thank you so much for explaining you work to me we will have a look at it
again in class. Then I will explain why I think it's wrong. Ok Thank you so
much.

APPENDIX 10: Observations of Teachers' Lessons

APPENDIX 10 – OBSERVATIONS

Teacher 1 – 1A (20/5 Wednesday)

Lesson on Number Patterns

Act 8 pg. 73

The entire class was sitting on the mat while the teacher explained the instruction
Teacher – each caterpillar has 10 pieces to his body.

Example 1: 10; ; 8; ; 6; ; 4; ; 2;

First pattern – counting in 1's backwards.

Example 2: Second pattern explained.

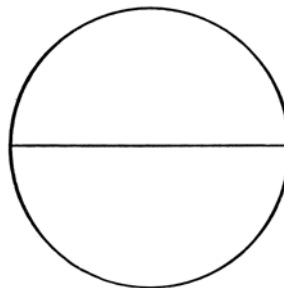
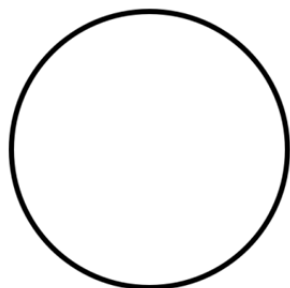
10; 20;

Counting in 10's

The diamond group remains on the mat and the rest of the class is asked to return to their seats and complete activity 8 in the textbook.

Group work is done with the DIAMOND GROUP

- They are reminded that Estimate: - means a good guess. With a box with a few sticks in it the teacher asks the learners individually to have a look and estimate the amount of sticks they think are in the box.
Answer = 23
Closest number guessed was 20 (3 less than 23 - explained)
- Each learner is asked to count out 15 sticks and arrange them in 2's. 1 stick does not have a partner he is odd, therefore 15 is an odd number.
- Now learners are asked to count out 20 sticks on the mat. Again they are asked to count in 2's.
- Put the 20 sticks in groups of 5. How many groups of 5 is there in 20.
- Now make the 20 sticks in groups of 10.
- Each learner is given a paper plate. The one side is clear the other side has



be halved. Concrete apparatus + verbal explanation.

- The 1st side without the line is used to explain and illustrate doubling of numbers.
- Double 1 (explained add another 1) is 2
- Double 2 (explained add another 2) is 4
- Double 4 (explained add another 4) is 8
- Double 8 (explained add another 8) is 16

NB an opportunity to explain the Equal sign not used.

- The 2nd side of the plate is used to explain and illustrate halving of numbers.
- 4 sticks placed on the middle line now halve it
- What is half of 4 (learners response is 2) **NB Equal to sign left out**
- 10 Sticks placed on middle line now, halve it.
- What is half of 10 (learners response is 5) **NB Equal to sign left out**
- 20 sticks placed on middle line now halve it
- What is half of 20 (learners response is 10) **NB Equal to sign left out**

In the 30 min lesson only 1 group worked like this on the mat.

Teacher 2 – 1B

- Group work is done in Grade 1- Ability group (group of 6 learners on the mat with the Teacher – she explains a concept)
- Rest of learners was working at their tables from a worksheet. (in silent – well controlled)

The teacher randomly took out a number of blocks

Activity 1

Then ask the learners to estimate the number of blocks she had.

The learners each get a choice to say how many she has on the mat.

Then she counted out blocks out, and then she asks:

1. Who estimated too many?
2. Who estimated too little?

One learner estimated correctly 22

Activity 2

She took a few blocks away and asked now estimate again how many there are on the mat.

Learners guess again

Then she counts out and asks:

1. Who estimated too many?
2. Who estimated too little?
3. Who came close to the correct answer?

Learners indicate by showing their hands.

Activity 3

The teacher gives each learner on the a pile of block and ask them to

1. Estimate how many they have
2. Then to count out in 2's how many they have.

The teacher only focused on estimation in this lesson, the opportunity was not used to speak about the equal to sign – This is a Grade 1 class.

Teacher asks the learners to take out 16 blocks (she checks on the rest of the glass). She checks on the rest of the class.

Activity 4

From the 16 block, she asks the following question

1. How many groups of 2 can you make? 8 groups of 2
2. How many groups of 5 can you make? 3 groups of 3 and 1 reminder.

Every leaner thinks differently about addressing the activity.

Teacher 3 - Grade 2 A (Tuesday 12/5)

Books handed out. Ability groups are called to the front to get their workbooks.

Learners are giving work cards from which a list of story sums are done. Story sum

What are the steps we follow when doing story sums?

- Write what you have
- Draw what you have
- The sum with a block at the end (**good place to emphasize the equal sign not used**)
- The answer (**could have mentioned answer in a sentence**)
- Then a pattern

Very busy group, tables has too many books on learners are taking very long to settle and start reading the sums. Teacher changed her lesson from group activities in front to story sums. Teacher is moving around to help settle her learners. Children

are starting to focus. Lots of noise yet this is an individual lesson. Each learner has their own work card.

Teacher calls individual groups to the mat to assist with counting in 3s. **Einstein (group name)**. Same group counting in 4s. The other groups are completing their individual story sums. The learners need to pack the Cards in correct order. Also counting backwards in 4s.

Teacher 4 - Grade 2 B (21/5 Thursday)

The rest of class is doing Blue book activities its book check today

Rest of Class – Addition Sum from Blue Book (WCED)

Pg. 50; 51, 48, 49.

Again: call calculations are on the LHS = Answer

Promotion a perception that the equal sign means Answer

- **The jelly tots were called to the front**
- Group work- Playing a game on adding by grouping numbers together

$$7 + 8 + 23 =$$

$$6 + 24 + 7 =$$

$$5 + 8 + 25 =$$

$$21 + 8 + 9 =$$

$$7 + 29 + 1 =$$

$$28 + 4 + 2 =$$

NB calculation on the left hand side = Answer can create misconception on Equal to Sign

- The teacher flashers the cards and learner need to speed answer
- Mental activity- teacher reads out the operations learners needs to mentally calculate and answer. (reading from flash cards)

Teacher reads out

- Start with 40 half it add 4 divide by 2
- Start with 100 half it divide by 5 add 6
- Start with 30 multiply by 2 subtract 20 half it
- Start with 63 add 2 subtract 10 divide by 5
- Start with 60 half it add 4 subtract 10

Next activity (bingo card given)

Numbers on the card, the teacher calls out a number that is halve of one of the numbers on the board. The first learner to complete board calls out bingo

Second Group (Astros)

Same activity as before

Teacher 5 – Grade 3A (Thursday 30/4)

The learners arrived from the CAMI lesson. Busy books was taken out, pencils was sharpen to start the lesson. Lots of rules was reinforced as this is a Foundation phase class and according to the teacher here they train skills. The class is in arranged in groups of four. The lesson was on Cursive Writing.

However the previous Mathematics lesson was still on the board.

The 2 times table was set out as expanded notation. When as the 2 times table.

$$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 22$$

$$11 \times 2 = 22$$

Learners had to set out the following in their busy book

$9 \times 2 =$

$5 \times 2 =$

$11 \times 2 =$

$12 \times 2 =$

$8 \times 2 =$

$4 \times 2 =$

$1 \times 2 =$

$3 \times 2 =$

$7 \times 2 =$

$10 \times 2 =$

$6 \times 2 =$

$2 \times 2 =$

The learners also practiced their hand writing the small g and Capital G in cursive

Teacher 6 – Grade 3B (Thursday 14/5)

- Learners were doing a Formal Mathematics assessment task during this observation lesson.
- Each individual question was explained in detail before learners could start, not sure if this is too guided. NB
- Class was very responsive to teacher, when the question was completed, they had to place their head on table until everyone else was done.
- Foundation Phase. NB Q2- again explained in detail then learners had to do it and once completed had to wait until everyone was done. Very guided approach.

-
- Maybe this method was used to facilitate the weaker group that has reading difficulties.
 - 2nd Lang. Xhosa learners in class.
 - Every sum was done in this manner. Assessment takes very long.
 - Learners was seated according to behavior issues and strengths and weaknesses.
 - Mat work is normally used to teach concepts in ability groups
 - This style of assessment could be spoon feeding and not a good preparation for the intermediate phase.
 - Clear that this assessment is not a true reflection of learners' ability. Too much guidance given, leading to the answer.

Teacher 7 – Grade 4 A (Thursday (23/4)

As I arrived in the class room, the class was verbally marking the activity done the day before. Stars as a reward was given to everyone that had 10/10. Mental Maths. This teacher is a Class based grade 4 teacher not a specialist in Mathematics. The class is divided into groups. There was 29 Learners present during this lesson. A you tube clip (Mr. R's Fraction Song) was used as an introduction to Fractions. This was in the learner ZPD. It was child friendly. This was an American you tube video so the English used for fractions differed example: three quarters (they said three fourths). The teacher used the Board well. Heading was placed on the board. **Fractions** while the you tube video played, the learners were listening attentively. The learners repeated after the video three fourth instead of three quarters and this was not corrected by the teacher or highlighted. The teacher presented her though pattern beautifully on the board.

- Fractions
- Fractions are parts of a whole- definition (When we speak of fractions a whole is divided into equal parts.
- **NB:** One paper plate was use for this lesson.
 - ✓ First divided into halves/ then the same plate into quarters/ then into thirds/ then into eights
 - ✓ **NB: I would have use different plate maybe colour coded to link to equivalent fractions.**
- The parts of the whole are equal parts
- Referred to numerator and denominator
- $\frac{1 \text{ numerator}}{2 \text{ denominator}}$ written on the board
- The denominator tells us how many parts the whole is divided into
- The numerator tells us we ate 1 part of the whole.

When learners was ask what does the denominator tell us?

They struggled to verbalize their thought process.

It was a learner's birthday in the class she brought along 33 cupcakes this was represented as a whole. The teacher asked if we eat 20 of the 33 cupcakes how would we present it in fraction form. A girl responded $\frac{20}{33}$.

This explanation took 30 minutes. At the end of the lesson the key concepts was summarized and learners was asked to take of their textbooks. This was their first lesson on the concept: fractions.

Text book Used

Shutters Premier

Mathematics Grade 4

Caps Compliment

C Jackson/ J Raubenheimer

Note and Fraction wall handed out to learners. Activity 2 pg. 65 will be done following this explanation

My opinion: Opportunities during lesson time is not used to emphasize the importance of the equal to size

Teacher 8 – Grade 4B (Thursday 23/4)

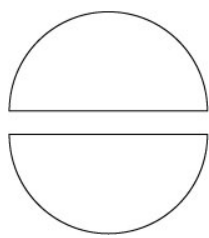
As I entered the class room Ms. Corné introduced me to her class and explained why I was observing the lesson. She asked her learners to bring along 7 paper plates each. She started the lesson asking why we brought paper plates today. Learners responded because we are moving on to Fractions. In her explanation she said it's sometimes easy to explain a new concept using physical objects.

Teacher Explained- My mom always shared things equally between me and my sister. If she left 1 whole pizza at home, who would we share it equally between my sister and I

- **First paper plate-** fold in half
1 whole pizza between 2 people

$$\frac{1}{1} = 1 \text{ whole pizza written on the board}$$

- The learners cut the first plate in half the two halves is placed on the board



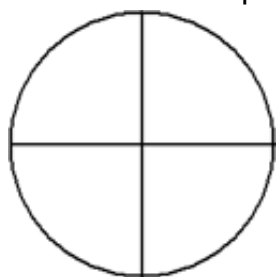
- $\frac{1}{2} + \frac{1}{2}$ You get 1 piece of the two halves.
($\frac{1}{2}$ for your sister and $\frac{1}{2}$ for you)

- Next Day your brother brings home a friend and mom leaves a cake on the table to be shared equally between the 3 of you.

- **Second Paper plate** – Teacher shows learners how to fold the second plate and they cut it into 3 equal parts.



- The whole is $\frac{3}{3}$
- How many are you getting of the whole
- $\frac{1}{3}$ for you $\frac{1}{3}$ for your brother and $\frac{1}{3}$ for his friend
- The bottom number tells us how many parts you need to cut your cake into.
- Would you prefer to share between 3 or 2? Learners Answer between two.
- So a half is bigger than a third.
- **Third paper plate** - So on Friday night we have quiche for supper. We are four people in the family. Mom, Dad, You and your bother. We have to share the quiche into four equal parts. So the class folds the third place into half then half again.
- **This was a very practical lesson on share into equal parts.**
- I observe for 30 minute of the hour lesson
- They had 7 plates. Cut each plate as the teacher explained. So plate no.3 was cut into 4 equal parts.



- How many pieces out of the four will you get? Learners response $\frac{1}{4}$
- Teacher links this to time and refers to this fraction as a quarter

-
- Refers to time quarter part one placing the quarter onto the clock
 - Now placing 2 quarters onto the one side of the clock and a half on the other side showing them that $\frac{2}{4}$ is the same as a $\frac{1}{2}$.

NB. But we will not explain that now. Grade 4 level

- Very good linking of concepts (time linked to Fractions)
- A bit of equivalent fractions
- **NB: Teach did not use used like equal to or equivalent**

I left the class at this point 30 min was up.

I felt that her explanation was sound but there was no clear note on the board to capture her thought pattern.

A grade 4 class based teacher not a Mathematics teacher. This teacher did not hand in her questionnaire.

Teacher 9 – Grade 5A (Tuesday 21/4)

I arrived in the teachers' class at the start of her lesson. It took the class a few minute to settle down. The teacher started her lesson by handing out a mental test on multiplication. A timer was displayed on the board after the instructions was explained the learners was given 5 minutes to complete a 100-mark test on times table. At the end of the 5 minute papers was swopped and the teacher read out the answers while the learners marked. In this Grade a rough book and a new book is used. According to my observation this is a follow up lesson on Capacity and Volume. The textbook used in this Grade is Study and Master by Cambridge. The teacher introduced her lesson by drawing a water jug half full on the board. She told the learners that the capacity of the jug is 1 litre and asked them to estimate how full it could be. Their response was $\frac{1}{2}$ full/ more or less 500ml. She referred them to pg. 82 in the textbook which is a table in which ml were converted to litre. And told them that this was needed to complete their Activity on pg. 84. She verbally did the Mental Math on pg. 84 that had to do with $\times 1000$ and $\div 1000$. I felt that at this point the learners became distracted and started fidgeting. **I felt that her explanation was sound but there was no clear note on the board to capture her thought pattern.** She would explain by writing on the board then erasing it to write a new example. As her lesson was on conversion of ml to litre, this was a good opportunity to link it to equivalence that was not used. In my opinion it is not easy for teachers to have an observer in their class and I am sure this young teacher (her 2nd year in

teaching) was intimidated by my presence in her class. Resulting in her lack of structure on the board. Explaining the concept very verbally, to a Grade 5 group of learners. Not on the ZPD level to abstract.

(Definitely not on their level- they require to see and handle a topic like volume and capacity). It was a 1-hour lesson, I only stayed for the first 30 minute. **This teacher is not a Mathematics qualified teacher. She is a Grade 5 Class base teacher teaching a variety of subjects.** The lesson took on a traditional form. Teacher explains, learners respond to question. Teacher's level of proficiency. She taught the learners tricks when multiplying with 1000 (add on 3 zero) and when dividing by 1000 (you take away 3 places) not sure if comprehension of the concept was experienced by the learners. In their neat book there are lots of photocopied worksheets. Not a lot of time is therefore given to learners to get use to correct setting out of work as the worksheets only require an answer. I am of the opinion what this was a good concept in which the importance of the equal to sign could have been highlighted. However it was never brought up in the lesson. Showing that the understanding of the symbol is taken for granted. I also feel that because there was no structured settling out of work on the board, it leaves to much room for learners to set out their work in an incorrect manner.

Suggestion:

- Make the equal to sign on cardboard A4 size.
- Have different measuring units
- Convert from ml – l in the concrete apparatus and place the equal size in between them, therefore drawing attention to the symbol as well as physically demonstration conversion.

Teacher 10 – Grade 5B (Tuesday 28/4)

Introduction to fractions this concept has been covered in Grade 4.

Circular Fraction parts apparatus was used in different colours

Fractions part of a whole Use 1 whole circle then use a whole circle divided into two parts. Another whole circle divided into 4 parts. Describes denominators and numerator. Played a game in class Andila has 10 choices to throw a paper ball in the paper bin. 1 out of the ten shoots into the bin. The second attend 5 out of 10. $\frac{1}{2}$ is

and equivalent. (Did not link this). 18 out of 36 more equivalent to a half is mentioned.

Fold paper into two equal parts. Fold it into 4 equal parts. $\frac{8}{8}$ will make a whole. How many will make a half. $\frac{4}{8}$ will make a $\frac{1}{2}$. A pupil was given half a pizza that was divided into 8ths. How many 8^{th} do I need to be equal to $\frac{3}{4}$? What do we call this?

Learners say equivalent fractions. On the board $\frac{3}{4} = \frac{6}{8}$.

Colour coded pizza fractions models are used to demonstrate this lesson. Tone of teachers' voice low. Good.

$\frac{1}{2} > \frac{2}{8}$

Relational signs related to the lesson. Chocolate lab. How many parts are there in a slab? 14 parts. In a slab. If I give away the whole chocolate how many am I giving away. Learners struggled. Some answered 14 parts are given away. If I give a way half of my chocolate. $\frac{7}{14}$ are given away. Easter eggs. 4 was eaten. The whole container has 6 eggs. Write it on the board. $\frac{2}{6}$. How many did I eat already? $\frac{2}{6}$ left. $\frac{4}{4}$ eaten. 30 min introduction over.

Exercise. – A Note was handed out at this point. I stayed for 30 minutes. The first 30 minutes was just explaining.

Teacher 11 – Grade 6A (21/5 Thursday)

When I arrived in the class a general discussion took place on compound interest.

This is a 1H30 Math lesson – Not conducive for learners.

Lesson for the day

This is an introduction Lesson to this concept

- Subtracting of Decimal Number

Th H U, th h

Th	H	U	t	h	th
	8	7	6		
-	3	3	2	0	5

- What will confuse you? Add zeros above

-
- Dayne does the subtraction verbally. Borrowed from the 6.
 - A decimal worksheet was given to the learners.
 - Very playful lesson.
 - Why do we use decimal (we use money, therefore we need to know how decimals work)

Learners called individually to the board to do the activity. Not very constructive, not sure how much learning is taking place. Doing it in a vertical setting out on the board.

Kate

(88, 99 – 62, 50)

Kauter

(70, 332 – 43, 726)

Yethu

(56,928 – 16, 743)

Micheala

(74, 66 – 66, 915)

Done as a class activity.

Very noisy rest of class not focused at all doing nothing ###

The Teacher silenced the class, re-explained the reason they decimal comma needs to be below each other.

Learners very distracted not focused at all. I think the first few minutes - created a space where learners were lost.

APPENDIX 11: Letter of confirmation – Language Editing

Letter of Confirmation

22/01/2016

Department of Research

Faculty of Education

Mowbray Campus

Cape Peninsula University of Technology

This is to confirm that the thesis written by Bronwyn Mayer has been edited to the required standard by myself, Dr M.A. Curt

